

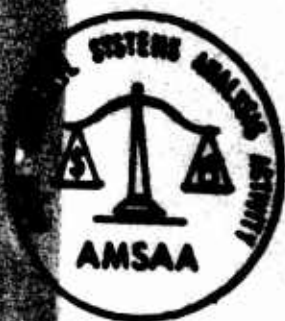
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A KILL PROBABILITY MODEL FOR A MULTIPLE-BURST
ATTACK OF A VEHICLE, WHERE THE PROBABILITY OF
IGNITING SPILLED FUEL IS TIME DEPENDENT

ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY

JUNE 1975



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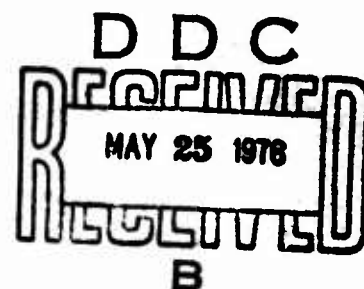
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A KILL PROBABILITY MODEL FOR A MULTIPLE-BURST ATTACK OF
A VEHICLE, WHERE THE PROBABILITY OF IGNITING SPILLED
FUEL IS TIME DEPENDENT

ARTHUR D. GROVES

JUNE 1975



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A Kill Probability Model for a Multiple-Burst Attack
of a Vehicle, Where The Probability of Igniting
Spilled Fuel is Time Dependent

1. INTRODUCTION

Military vehicles form a very important class of targets against which kill probabilities must be computed for various weapons. There are generally two types of damage which can result in a kill of a vehicle. These are (1) a specified type of mechanical damage, and (2) fire. A fire can generally be started either as the instantaneous result of puncturing some component of the fuel system with a high energy projectile, or by igniting fuel which was spilled but not ignited by earlier punctures. Since the amount, and perhaps the location, of the spilled fuel would generally depend on the number and times of occurrence of previous punctures, the probability of igniting such spilled fuel might depend on these factors. The method presented in this report allows for this dependence to be taken into account.

The model to be presented is not a vehicle vulnerability model, but merely describes a method to put together certain basic vulnerability-related probabilities to obtain the overall probability that the vehicle is killed. The basic probabilities, which are related to various aspects of vehicle vulnerability, are assumed to be available, and are not generated in the model.

There are three basic damage events that may occur when a burst of rounds are fired at a vehicle. These are (1) the vehicle may receive disabling mechanical damage, (2) some part of the fuel system may be punctured, with an associated spillage of fuel, and (3) fuel which was spilled but not ignited by earlier bursts may be ignited by the burst under consideration.

Two conditions of dependence among these three basic events are treated in this report. The first is that of complete independence among the events, whereby the probability of occurrence of any one of them on a given burst does not depend on whether or not any of the

others occur on the same burst. This case might be appropriate for the situation where the number of rounds in the burst is large. The second condition is that of complete dependence among the events, whereby the occurrence of any one of them precludes the occurrence of any of the others on the same burst. This case might be appropriate when consecutive single rounds are fired at the vehicle, that is, when each burst consists of only a single round.

2. DERIVATION OF METHOD

Let t_i be the time at which the rounds from the i^{th} burst arrive at the target vehicle. It is assumed that all the rounds of a given burst arrive at the vehicle at the same time, but the time varies from burst to burst. This is not a restrictive assumption, because a burst that extends over a significant interval in time can be subdivided into shorter bursts, even to single-round bursts, if desired, with each shorter burst assumed to have all its rounds impact at the same time.

The condition of the vehicle at any time can be expressed using the following vector-like notation;

$$v = (F; g),$$

where F describes the condition of the vehicle relative to fire, and g describes its condition relative to disabling mechanical damage. If the vehicle is not on fire, F is the set of times at which punctures of the fuel system have occurred. If no punctures have occurred, $F = \phi$, the empty set. If the vehicle is on fire, the indicator $F = *$ will be used. If the vehicle has suffered disabling mechanical damage, $g = 1$; if not, $g = 0$. Therefore, using this notation, a completely undamaged vehicle would be denoted by the "vector" $(\phi; 0)$.

Before any computations can be made, several basic probability tables or functions must be provided. These relate to the vulnerability of the vehicle, the characteristics of the ammunition being fired at the vehicle, the delivery accuracy of the weapon, and the number of rounds in a burst.

- $P_1(i)$ = Probability that a puncture of the fuel system occurs on the i^{th} burst.
- $P_2(i)$ = conditional probability that the i^{th} burst causes a fire, given that it has punctured the fuel system. This type of fire will be referred to as a Type I fire, and is distinguished from one caused by a burst igniting fuel which was spilled but not ignited by rounds of a previous burst. This second kind of fire will be referred to as a Type II fire.
- $P_3(i,F)$ = probability that the i^{th} burst ignites fuel that was spilled by the previous punctures identified in the set F . This is the Type II fire. Since there are many possible compositions of the set F for each i , there will be as many values of $P_3(i,F)$. Since F is a subset of the times $t_1, t_2, \dots, t_{i-1}$, there are 2^{i-1} possible subsets. For example, if $i = 4$, there are eight possible F 's. These are:

$\emptyset$
 t_1
 t_2
 t_3
 $t_1 t_2$
 $t_1 t_3$
 $t_2 t_3$
 and $t_1 t_2 t_3$

- $P_4(i)$ = probability that the i^{th} burst causes disabling mechanical damage.
- $P_5(i) = P_1(i)P_2(i)$ = probability that a Type I fire is started on the i^{th} burst.

For any of these probabilities, Q , with the same subscripts and arguments, will be used to represent $1 - P$. For example:

$$Q_3(i, F) = 1 - P_3(i, F).$$

In addition, note that if $F = \phi$, that is, if there have been no punctures prior to time t_j , then $P_3(i, \phi) = 0$.

Now let S_j denote the set of possible "vectors" (vehicle states) after the i^{th} burst. For $i = 0$, that is, before the first burst, S_0 consists of the single state $(\phi; 0)$. After the first burst, which occurs at time t_1 , six states are possible, so S_1 consists of six "vectors." These are:

$(\phi; 0)$	which occurs if the first burst does no damage
$(t_1; 0)$	which occurs if the first burst punctures the fuel system without causing a Type I fire, and does not cause disabling mechanical damage
$(*; 0)$	which occurs if the burst punctures the fuel system and causes a Type I fire, but does not cause disabling mechanical damage
$(\phi; 1)$	which occurs if the burst does not puncture the fuel system, but does cause disabling mechanical damage
$(t_1; 1)$	which occurs if the burst punctures the fuel system without causing a Type I fire, but causes disabling mechanical damage, and
$(*; 1)$	which occurs if the burst punctures the fuel system, causes a Type I fire, and also causes disabling mechanical damage.

Note that if the basic damage events are completely dependent, so that the burst can cause at most one of the events, vectors $(t_1; 1)$ and $(*; 1)$ cannot occur in S_1 . The probabilities for the transition from S_0 to S_1 are shown in Table 1.

TABLE 1

TRANSITION PROBABILITIES FROM S_0 TO S_1

VECTOR IN S_0	VECTOR IN S_1	INDEPENDENT DAMAGE EVENTS	DEPENDENT DAMAGE EVENTS
$(\phi; 0)$	$(\phi; 0)$	$Q_1(1) Q_4(1)$	$1 - P_1(1) - P_4(1)$
	$(t_1; 0)$	$P_1(1) Q_2(1) Q_4(1)$	$P_1(1) Q_2(1)$
	$(*; 0)$	$P_5(1) Q_4(1)$	$P_5(1)$
	$(\phi; 1)$	$Q_1(1) P_4(1)$	$P_4(1)$
	$(t_1; 1)$	$P_1(1) Q_2(1) P_4(1)$	0
	$(*; 1)$	$P_5(1) P_4(1)$	0

On the second burst, the possible transitions from the vectors in S_1 to vectors in S_2 , with the associated probabilities, are shown in Table 2. Note that S_2 consists of only ten different "vectors." These are:

$(\phi; 0)$	$(\phi; 1)$
$(t_1; 0)$	$(t_1; 1)$
$(t_2; 0)$	$(t_2; 1)$
$(t_1 t_2; 0)$	$(t_1 t_2; 1)$
$(*; 0)$	$(*; 1)$

On the third burst, the possible transitions from the "vectors" in S_2 to "vectors" in S_3 , with the associated probabilities are shown in Table 3. Note that S_3 consists of only 18 different "vectors." These are:

$(\phi; 0)$	$(\phi; 1)$
$(t_1; 0)$	$(t_1; 1)$
$(t_2; 0)$	$(t_2; 1)$
$(t_3; 0)$	$(t_3; 1)$

TABLE 2

TRANSITION PROBABILITIES FROM S_1 TO S_2

VECTOR IN S_1	VECTOR IN S_2	INDEPENDENT DAMAGE EVENTS	DEPENDENT DAMAGE EVENTS
$(\phi; 0)$	$(\phi; 0)$	$Q_1(2) Q_4(2)$	$1-P_1(2)-P_4(2)$
	$(t_2; 0)$	$P_1(2) Q_2(2) Q_4(2)$	$P_1(2) Q_2(2)$
	$(*; 0)$	$P_5(2) Q_4(2)$	$P_5(2)$
	$(\phi; 1)$	$Q_1(2) P_4(2)$	$P_4(2)$
	$(t_2; 1)$	$P_1(2) Q_2(2) P_4(2)$	0
	$(*; 1)$	$P_5(2) P_4(2)$	0
$(t_1; 0)$	$(t_1; 0)$	$Q_1(2) Q_3(2, t_1) Q_4(2)$	$1-P_1(2)-P_3(2, t_1)-P_4(2)$
	$(t_1 t_2; 0)$	$P_1(2) Q_2(2) Q_3(2, t_1) Q_4(2)$	$P_1(2) Q_2(2)$
	$(*; 0)$	$[1-Q_5(2) Q_3(2, t_1)] Q_4(2)$	$P_5(2) + P_3(2, t_1)$
	$(t_1; 1)$	$Q_1(2) Q_3(2, t_1) P_4(2)$	$P_4(2)$
	$(t_1 t_2; 1)$	$P_1(2) Q_2(2) Q_3(2, t_1) P_4(2)$	0
	$(*; 1)$	$[1-Q_5(2) Q_3(2, t_1)] P_4(2)$	0
$(*; 0)$	$(*; 0)$	$Q_4(2)$	$Q_4(2)$
	$(*; 1)$	$P_4(2)$	$P_4(2)$
$(\phi; 1)$	$(\phi; 1)$	$Q_1(2)$	$Q_1(2)$
	$(t_2; 1)$	$P_1(2) Q_2(2)$	$P_1(2) Q_2(2)$
	$(*; 1)$	$P_5(2)$	$P_5(2)$
$(t_1; 1)$	$(t_1; 1)$	$Q_1(2) Q_3(2, t_1)$	$1-P_1(2)-P_3(2, t_1)$
	$(t_1 t_2; 1)$	$P_1(2) Q_2(2) Q_3(2, t_1)$	$P_1(2) Q_2(2)$
	$(*; 1)$	$1-Q_5(2) Q_3(2, t_1)$	$P_5(2) + P_3(2, t_1)$
$(*; 1)$	$(*; 1)$	1	1

TABLE 3
TRANSITION PROBABILITIES FROM S_2 TO S_3

VECTOR IN S_2	VECTOR IN S_3	INDEPENDENT DAMAGE EVENTS	DEPENDENT DAMAGE EVENTS
$(\phi; 0)$	$(\phi; 0)$	$Q_1(3) Q_4(3)$	$1-P_1(3)-P_4(3)$
	$(t_3; 0)$	$P_1(3) Q_2(3) Q_4(3)$	$P_1(3) Q_2(3)$
	$(*; 0)$	$P_5(3) Q_4(3)$	$P_5(3)$
	$(\phi; 1)$	$Q_1(3) P_4(3)$	$P_4(3)$
	$(t_3; 1)$	$P_1(3) Q_2(3) P_4(3)$	0
	$(*; 1)$	$P_5(3) P_4(3)$	0
$(t_1; 0)$	$(t_1; 0)$	$Q_1(3) Q_3(3, t_1) Q_4(3)$	$1-P_1(3)-P_3(3, t_1)-P_4(3)$
	$(t_1 t_3; 0)$	$P_1(3) Q_2(3) Q_3(3, t_1) Q_4(3)$	$P_1(3) Q_2(3)$
	$(*; 0)$	$[1-Q_5(3) Q_3(3, t_1)] Q_4(3)$	$P_5(3) + P_3(3, t_1)$
	$(t_1; 1)$	$Q_1(3) Q_3(3, t_1) P_4(3)$	$P_4(3)$
	$(t_1 t_3; 1)$	$P_1(3) Q_2(3) Q_3(3, t_1) P_4(3)$	0
	$(*; 1)$	$[1-Q_5(3) Q_3(3, t_1)] P_4(3)$	0
$(t_2; 0)$	$(t_2; 0)$	$Q_1(3) Q_3(3, t_2) Q_4(3)$	$1-P_1(3)-P_3(3, t_2)-P_4(3)$
	$(t_2 t_3; 0)$	$P_1(3) Q_2(3) Q_3(3, t_2) Q_4(3)$	$P_1(3) Q_2(3)$
	$(*; 0)$	$[1-Q_5(3) Q_3(3, t_2)] Q_4(3)$	$P_5(3) + P_3(3, t_2)$
	$(t_2; 1)$	$Q_1(3) Q_3(3, t_2) P_4(3)$	$P_4(3)$
	$(t_2 t_3; 1)$	$P_1(3) Q_2(3) Q_3(3, t_2) P_4(3)$	0
	$(*; 1)$	$[1-Q_5(3) Q_3(3, t_2)] P_4(3)$	0
$(t_1 t_2; 0)$	$(t_1 t_2; 0)$	$Q_1(3) Q_3(3, t_1 t_2) Q_4(3)$	$1-P_1(3)-P_3(3, t_1 t_2)-P_4(3)$
	$(t_1 t_2 t_3; 0)$	$P_1(3) Q_2(3) Q_3(3, t_1 t_2) Q_4(3)$	$P_1(3) Q_2(3)$
	$(*; 0)$	$[1-Q_5(3) Q_3(3, t_1 t_2)] Q_4(3)$	$P_5(3) + P_3(3, t_1 t_2)$
	$(t_1 t_2; 1)$	$Q_1(3) Q_3(3, t_1 t_2) P_4(3)$	$P_4(3)$
	$(t_1 t_2 t_3; 1)$	$P_1(3) Q_2(3) Q_3(3, t_1 t_2) P_4(3)$	0
	$(*; 1)$	$[1-Q_5(3) Q_3(3, t_1 t_2)] P_4(3)$	0

TABLE 3 (CONT)

TRANSITION PROBABILITIES FROM S_2 TO S_3

VECTOR IN S_2	VECTOR IN S_3	INDEPENDENT DAMAGE EVENTS	DEPENDENT DAMAGE EVENTS
(*; 0)	(*; 0)	$Q_4(3)$	$Q_4(3)$
	(*; 1)	$P_4(3)$	$P_4(3)$
(ϕ ; 1)	(ϕ ; 1)	$Q_1(3)$	$Q_1(3)$
	(t_3 ; 1)	$P_1(3) Q_2(3)$	$P_1(3) Q_2(3)$
	(*; 1)	$P_5(3)$	$P_5(3)$
(t_1 ; 1)	(t_1 ; 1)	$Q_1(3) Q_3(3, t_1)$	$1 - P_1(3) - P_3(3, t_1)$
	($t_1 t_3$; 1)	$P_1(3) Q_2(3) Q_3(3, t_1)$	$P_1(3) Q_2(3)$
	(*; 1)	$1 - Q_5(3) Q_3(3, t_1)$	$P_5(3) + P_3(3, t_1)$
(t_2 ; 1)	(t_2 ; 1)	$Q_1(3) Q_3(3, t_2)$	$1 - P_1(3) - P_3(3, t_2)$
	($t_2 t_3$; 1)	$P_1(3) Q_2(3) Q_3(3, t_2)$	$P_1(3) Q_2(3)$
	(*; 1)	$1 - Q_5(3) Q_3(3, t_2)$	$P_5(3) + P_3(3, t_2)$
($t_1 t_2$; 1)	($t_1 t_2$; 1)	$Q_1(3) Q_3(3, t_1 t_2)$	$1 - P_1(3) - P_3(3, t_1 t_2)$
	($t_1 t_2 t_3$; 1)	$P_1(3) Q_2(3) Q_3(3, t_1 t_2)$	$P_1(3) Q_2(3)$
	(*; 1)	$1 - Q_5(3) Q_3(3, t_1 t_2)$	$P_5(3) + P_3(3, t_1 t_2)$
(*; 1)	(*; 1)	1	1

$(t_1 t_2; 0)$	$(t_1 t_2; 1)$
$(t_2 t_3; 0)$	$(t_2 t_3; 1)$
$(t_1 t_3; 0)$	$(t_1 t_3; 1)$
$(t_1 t_2 t_3; 0)$	$(t_1 t_2 t_3; 1)$

The pattern should now be apparent. In general, the set S_i consists of $2(2^i + 1)$ "vectors." These are shown in Table 4.

Before generalizing the probabilities for the transition from set S_{i-1} to set S_i , the notation to be used will be described. L_i will be used to denote the set of times $\{t_1, t_2, t_3, \dots, t_i\}$. Then L_{i-1} is the set $\{t_1, t_2, \dots, t_{i-1}\}$. F will be used to denote any subset of L_{i-1} , that is, any set of times prior to t_i . $F + t_i$ will be used to denote the set consisting of the elements of F and the time t_i . Thus, for example, if $F = \{t_1, t_2, t_4\}$, then $F + t_6 = \{t_1, t_2, t_4, t_6\}$. Similarly, if $F = \phi$, then $F + t_6 = \{t_6\}$. The notation F in L_i will signify "for all subsets F of L_i ." F' will be used to denote any non-empty subsets of L_i . Thus the notation F' in L_i means "for all subsets of L_i except the empty set ϕ ." With this notation the transition probabilities, and later the recursive formulas, can be very concisely presented. Table 5 shows the generalized probabilities of transition from S_{i-1} to S_i . (Recall that $P_3(i, \phi) = 0$ for all i .)

The probabilities in Table 5 will be used to develop a set of recursive formulas for determining the probabilities of occurrence of the various vectors of S_i , given the probabilities for the vectors of S_{i-1} . First, we introduce the notation $P_i(F;g)$ to denote the probability of occurrence of the vector $(F;g)$ in the set S_i . To illustrate how the recursive formulas are developed, consider the determination of $P_i(\phi;1)$. Table 5 shows that the vector $(\phi;1)$ in S_i can result from either one of two possible transitions from vectors in S_{i-1} . These are:

- (1) $(\phi;0)$ in S_{i-1} , with only disabling mechanical damage occurring on the i^{th} burst,

TABLE 4
LIST OF VECTORS IN S_i ; FOR $i \geq 1$

<u>NUMBER OF VECTORS</u>	<u>FORM OF VECTORS</u>
$\binom{i}{0}$	$(\phi; 0)$
$\binom{i}{0}$	$(\phi; 1)$
$\binom{i}{1}$	$(t_j; 0)$
$\binom{i}{1}$	$(t_j; 1)$
$\binom{i}{2}$	$(t_j t_k; 0)$
$\binom{i}{2}$	$(t_j t_k; 1)$
$\binom{i}{3}$	$(t_j t_k t_m; 0)$
$\binom{i}{3}$	$(t_j t_k t_m; 1)$
	.
	.
	.
	.
$\binom{i}{i}$	$(t_1 t_2 t_3 \dots t_i; 0)$
$\binom{i}{i}$	$(t_1 t_2 t_3 \dots t_i; 1)$
1	$(*; 0)$
1	$(*; 1)$

$$\binom{i}{j} = \frac{i!}{j!(i-j)!} = \text{Number of combinations of } i \text{ things taken } j \text{ at a time}$$

$$\text{Total number of vectors} = 2 \sum_{j=0}^i \binom{i}{j} + 2 = 2(2^i + 1)$$

TABLE 5
TRANSITION PROBABILITIES FROM S_{i-1} TO S_i

VECTOR IN S_{i-1}	VECTOR IN S_i	INDEPENDENT DAMAGE EVENTS	DEPENDENT DAMAGE EVENTS
(F; 0)	(F; 0)	$Q_1(i)Q_3(i,F)Q_4(i)$	$1-P_1(i)-P_3(i,F)-P_4(i)$
	(F+t _i ; 0)	$P_1(i)Q_2(i)Q_3(i,F)Q_4(i)$	$P_1(i)Q_2(i)$
	(*; 0)	$[1-Q_5(i)Q_3(i,F)]Q_4(i)$	$P_5(i) + P_3(i,F)$
	(F; 1)	$Q_1(i)Q_3(i,F)P_4(i)$	$P_4(i)$
	(F+t _i ; 1)	$P_1(i)Q_2(i)Q_3(i,F)P_4(i)$	0
	(*; 1)	$[1-Q_5(i)Q_3(i,F)]P_4(i)$	0
(*; 0)	(*; 0)	$Q_4(i)$	$Q_4(i)$
	(*; 1)	$P_4(i)$	$P_4(i)$
(F; 1)	(F; 1)	$Q_1(i)Q_3(i,F)$	$1-P_1(i)-P_3(i,F)$
	(F+t _i ; 1)	$P_1(i)Q_2(i)Q_3(i,F)$	$P_1(i)Q_2(i)$
	(*; 1)	$1-Q_5(i)Q_3(i,F)$	$P_5(i) + P_3(i,F)$
(*; 1)	(*; 1)	1	1

F in L_{i-1}

or (2) $(\phi;1)$ in S_{i-1} , with no puncture of the fuel system on the i^{th} burst.

Therefore, the probability of occurrence of $(\phi;1)$ in S_i is the sum of two probabilities. These are:

(1) the product of the probability of occurrence of $(\phi;0)$ in S_{i-1} and the probability of transition from $(\phi;0)$ in S_{i-1} to $(\phi;1)$ in S_i ,

and (2) the product of the probability of occurrence of $(\phi;1)$ in S_{i-1} and the probability that $(\phi;1)$ remains unchanged by the i^{th} burst.

Thus, for the case of independent damage events,

$$P_i(\phi;1) = Q_1(i) [P_{i-1}(\phi;0) P_4(i) + P_{i-1}(\phi;1)]$$

This same reasoning has been applied to all vectors to obtain the complete set of recursive formulas shown in Tables 6 and 7. To use these tables to obtain the probabilities of occurrence of the various vectors in S_i , for any i , it only remains to specify starting values $P_0(F;g)$ for the possible vectors in S_0 . But S_0 consists of only the single vector $(\phi;0)$. Thus $P_0(\phi;0) = 1$ and $P_0(F;g) = 0$ for all other $(F;g)$.

The $2(2^i + 1)$ states whose probabilities of occurrence are listed in Tables 6 and 7 are generally not of individual tactical interest. For example, if a vehicle is afire after a certain number of bursts have been fired at it, it is probably of no tactical interest whether or not the vehicle has also suffered disabling mechanical damage. Table 8 shows a list of nine damage categories that might be of tactical interest, the list of vectors that correspond to each of these categories, and the probabilities of occurrence of each of these categories in terms of the recursively defined probabilities in Tables 6 and 7. Note that the probability of occurrence of any damage category is simply the sum of the probabilities of occurrence of the vectors (states) that comprise that

TABLE 6 RECURSIVE FORMULAS INDEPENDENT DAMAGE EVENTS

1. $P_i(F;0) = P_{i-1}(F;0) Q_1(i) Q_3(i,F) Q_4(i) \quad (F \text{ in } L_{i-1})$
2. $P_i(F+t_i;0) = P_{i-1}(F;0) P_1(i) Q_2(i) Q_3(i,F) Q_4(i) \quad (F \text{ in } L_{i-1})$
3. $P_i(*;0) = Q_4(i) \left\{ P_{i-1}(*;0) + \sum_{F \text{ in } L_{i-1}} P_{i-1}(F;0) [1-Q_5(i) Q_3(i,F)] \right\}$
4. $P_i(F;1) = Q_1(i) Q_3(i,F) \left\{ P_{i-1}(F;0) P_4(i) + P_{i-1}(F;1) \right\} \quad (F \text{ in } L_{i-1})$
5. $P_i(F+t_i;1) = P_1(i) Q_2(i) Q_3(i,F) \left\{ P_{i-1}(F;0) P_4(i) + P_{i-1}(F;1) \right\} \quad (F \text{ in } L_{i-1})$
6. $P_i(*;1) = P_{i-1}(*;0) P_4(i) + P_{i-1}(*;1) + \sum_{F \text{ in } L_{i-1}} [P_{i-1}(F;0) P_4(i) + P_{i-1}(F;1)] [1-Q_5(i) Q_3(i,F)]$

TABLE 7 RECURSIVE FORMULAS DEPENDENT DAMAGE EVENTS

1. $P_i(F;0) = P_{i-1}(F;0) \left\{ 1 - P_1(i) - P_3(i,F) - P_4(i) \right\} \quad (F \text{ in } L_{i-1})$
2. $P_i(F+t_i;0) = P_{i-1}(F;0) P_1(i) Q_2(i) \quad (F \text{ in } L_{i-1})$
3. $P_i(*;0) = P_{i-1}(*;0) Q_4(i) + \sum_{F \text{ in } L_{i-1}} P_{i-1}(F;0) [P_5(i) + P_3(i,F)]$
4. $P_i(F;1) = P_{i-1}(F;0) P_4(i) + P_{i-1}(F;1) \left\{ 1 - P_1(i) - P_3(i,F) \right\} \quad (F \text{ in } L_{i-1})$
5. $P_i(F+t_i;1) = P_{i-1}(F;1) P_1(i) Q_2(i) \quad (F \text{ in } L_{i-1})$
6. $P_i(*;1) = P_{i-1}(*;0) P_4(i) + P_{i-1}(*;1) + \sum_{F \text{ in } L_{i-1}} P_{i-1}(F;1) [P_5(i) + P_3(i,F)]$

category.

This model calls for many values of $P_3(i,F)$. For example, when $i = 4$, there are eight possible F's (seven not counting $F = \phi$) for which values of $P_3(i,F)$ are required. As i increases, the number of associated F's increases greatly. (When $i = 8$, there are 128 possible F's). It is unlikely that vehicle vulnerability will ever be known in enough detail to provide all of these probabilities. However, the model is still good. Since it provides for the greatest possible detail with respect to the puncture history, any less detailed history is embedded in the model. For example, if the only vulnerability data that are available give the probability of igniting spilled fuel in terms of the elapsed time since the first puncture, this can be handled by making $P_3(i,F)$ a function only of the t_i and the earliest time in the set F.

3. NUMERICAL EXAMPLE

Find the probabilities of occurrence of the damage categories listed in Table 8 for one, two, and three bursts, where the basic damage events are assumed to be independent, and the basic input probabilities are as follows:

$$P_1(i) = .1 \quad \text{for } i = 1, 2, 3 \quad ,$$

$$P_2(i) = .4 \quad \text{for } i = 1, 2, 3 \quad ,$$

$P_3(i,F)$ is given by the following table:

$P_3(i,F)$	F			
	ϕ	t_1	t_2	t_1, t_2
$i = 1$	0	---	---	---
$i = 2$	0	.03	---	---
$i = 3$	0	.04	.02	.05

and $P_4(i) = .2 \quad \text{for } i = 1, 2, 3.$

TABLE 8

<u>DAMAGE CATEGORY</u>	<u>CORRESPONDING STATES</u>	<u>PROBABILITY OF OCCURRENCE OF DAMAGE CATEGORY</u>
Undamaged	($\phi;0$)	$P_i(\phi;0)$
Fuel System Puncture Only	($F';0$) $F' \text{ in } L_i$	$\sum_{F' \text{ in } L_i} P_i(F';0)$
Disabling Mechanical Damage Only	($\phi;1$)	$P_i(\phi;1)$
Fire Only	($*;0$)	$P_i(*;0)$
Fuel System Puncture Disabling Mechanical Damage; No Fire	($F';1$) $F' \text{ in } L_i$	$\sum_{F' \text{ in } L_i} P_i(F';1)$
Disabling Mechanical Damage and Fire	($*;1$)	$P_i(*,1)$
Disabling Mechanical Damage	($\phi;1$) $F' \text{ in } L_i$ ($F';1$) ($*;1$)	$P_i(\phi;1) + \sum_{F' \text{ in } L_i} P_i(F';1) + P_i(*;1)$
Fire	($*;0$) ($*;1$)	$P_i(*;0) + P_i(*;1)$
Either Disabling Mechanical Damage or Fire	($\phi;1$) $F' \text{ in } L_i$ ($F';1$) ($*;0$) ($*;1$)	$P_i(\phi;1) + \sum_{F' \text{ in } L_i} P_i(F';1) + P_i(*;0) + P_i(*;1)$

$$\begin{aligned}
\text{Then, } P_5(i) &= P_1(i) P_2(i) = .04 & \text{for } i = 1, 2, 3 & , \\
Q_1(i) &= 1 - P_1(i) = .9 & \text{for } i = 1, 2, 3 & , \\
Q_2(i) &= 1 - P_2(i) = .6 & \text{for } i = 1, 2, 3 & ,
\end{aligned}$$

$Q_3(i, F)$ is given by the following table:

$Q_3(i, F)$	F			
	ϕ	t_1	t_2	$t_1 t_2$
$i = 1$	1.0	---	---	---
$i = 2$	1.0	.97	---	---
$i = 3$	1.0	.96	.98	.95

$$Q_3(i) = 1 - P_4(i) = .8 \quad \text{for } i = 1, 2, 3 ,$$

$$\text{and } Q_5(i) = 1 - P_5(i) = .96 \quad \text{for } i = 1, 2, 3.$$

Initially, $P_0(\phi; 0) = 1$ and L_0 consists only of $F = \phi$. Then, as a result of the first burst, using the formulas in Table 6:

$$\begin{aligned}
P_1(\phi; 0) &= P_0(\phi; 0) Q_1(1) Q_3(1, \phi) Q_4(1) & [\text{Formula 1}] \\
&= (1) \quad (.9) \quad (1) \quad (.8) = .720
\end{aligned}$$

$$\begin{aligned}
P_1(t_1; 0) &= P_0(\phi; 0) P_1(1) Q_2(1) Q_3(1, \phi) Q_4(1) & [\text{Formula 2}] \\
&= (1) \quad (.1) \quad (.6) \quad (1) \quad (.8) = .048
\end{aligned}$$

$$\begin{aligned}
P_1(*; 0) &= Q_4(1) \left\{ P_0(*; 0) + P_0(\phi; 0) [1 - Q_5(1) Q_3(1, \phi)] \right\} & [\text{Formula 3}] \\
&= (.8) \left\{ 0 + (1) [1 - (.96)(1)] \right\} = .032
\end{aligned}$$

$$\begin{aligned}
P_1(\phi; 1) &= Q_1(1) Q_3(1, \phi) \left\{ P_0(\phi; 0) P_4(1) + P_0(\phi; 1) \right\} & [\text{Formula 4}] \\
&= (.9) \quad (1) \left\{ (1) \quad (.2) + 0 \right\} = .180
\end{aligned}$$

$$\begin{aligned}
P_1(t_1; 1) &= P_1(1) Q_2(1) Q_3(1, \phi) \left\{ P_0(\phi; 0) P_4(1) + P_0(\phi; 1) \right\} & [\text{Formula 5}] \\
&= (.1) \quad (.6) \quad (1) \left\{ (1)(.2) + 0 \right\} = .012
\end{aligned}$$

$$\begin{aligned}
 P_1(*;1) &= P_0(*;0) P_4(1) + P_0(*;1) && \text{[Formula 6]} \\
 &\quad + [P_0(\phi;0) P_4(1) + P_0(\phi;1)] [1-Q_5(1) Q_3(1,\phi)] \\
 &= (0) (.2) + 0 + [(1) (.2) + 0] [1-(.96) (1)] = .008
 \end{aligned}$$

The effect of the second burst is determined by again using the recursive formulas of Table 6 to update the state probabilities. Since the purpose of this section is to provide an example of how the formulas are used, eight decimal places will be carried. This may assist the reader who is working through the example himself. Of course, it is generally meaningless to report hit probabilities to this many places, so in the table of final results, only three places are given.

Since $L_1 = \{t_1\}$, there are only two F's ($F = \phi$ and $F = t_1$). Thus,

$$\begin{aligned}
 P_2(\phi;0) &= P_1(\phi;0) Q_1(2) Q_3(2,\phi) Q_4(2) && \text{[Formula 1]} \\
 &= (.72) (.9) (1) (.8) = .5184
 \end{aligned}$$

$$\begin{aligned}
 P_2(t_1;0) &= P_1(t_1;0) Q_1(2) Q_3(2,t_1) Q_4(2) && \text{[Formula 1]} \\
 &= (.048) (.9) (.97) (.8) = .0335232
 \end{aligned}$$

$$\begin{aligned}
 P_2(t_2;0) &= P_1(\phi;0) P_1(2) Q_2(2) Q_3(2,\phi) Q_4(2) && \text{[Formula 2]} \\
 &= (.72) (.1) (.6) (1) (.8) = .03456
 \end{aligned}$$

$$\begin{aligned}
 P_2(t_1 t_2;0) &= P_1(t_1;0) P_1(2) Q_2(2) Q_3(2,t_1) Q_4(2) && \text{[Formula 2]} \\
 &= (.048) (.1) (.6) (.97) (.8) = .00223488
 \end{aligned}$$

$$\begin{aligned}
 P_2(*;0) &= Q_4(2) \left\{ P_1(*;0) + P_1(\phi;0) [1-Q_5(2) Q_3(2,\phi)] \right. \\
 &\quad \left. + P_1(t_1;0) [1-Q_5(2) Q_3(2,t_1)] \right\} && \text{[Formula 3]} \\
 &= (.8) \left\{ (.032) + (.72) [1-(.96) (1)] \right. \\
 &\quad \left. + (.048) [1-(.96) (.97)] \right\} = .05128192
 \end{aligned}$$

$$\begin{aligned}
 P_2(\phi;1) &= Q_1(2) Q_3(2,\phi) \left\{ P_1(\phi;0) P_4(2) + P_1(\phi;1) \right\} && \text{[Formula 4]} \\
 &= (.9) (1) \left\{ (.72) (.2) + (.180) \right\} = .2916
 \end{aligned}$$

$$P_2(t_1;1) = Q_1(2) Q_3(2,t_1) \{P_1(t_1;0) P_4(2) + P_1(t_1;1)\} \quad [\text{Formula 4}]$$

$$= (.9) (.97) \{(.048) (.2) + (.012)\} = .0188568$$

$$P_2(t_2;1) = P_1(2) Q_2(2) Q_3(2,\phi) \{P_1(\phi;0) P_4(2) + P_1(\phi;1)\} \quad [\text{Formula 5}]$$

$$= (.1) (.6) (1) \{(.72) (.2) + (.18)\} = .01944$$

$$P_2(t_1 t_2;1) = P_1(2) Q_2(2) Q_3(2,t_1) \{P_1(t_1;0) P_4(2) + P_1(t_1;1)\} \quad [\text{Formula 5}]$$

$$= (.1) (.6) (.97) \{(.048) (.2) + (.012)\} = .00125712$$

$$P_2(*;1) = P_1(*;0) P_4(2) + P_1(*;1) \quad [\text{Formula 6}]$$

$$+ [P_1(\phi;0) P_4(2) + P_1(\phi;1)] [1-Q_5(2) Q_3(2,\phi)]$$

$$+ [P_1(t_1;0) P_4(2) + P_1(t_1;1)] [1-Q_5(2) Q_3(2,t_1)]$$

$$= (.032) (.2) + .008 + [(.72) (.2) + (.18)] [1-(.96) (1)]$$

$$+ [(.048) (.2) + (.012)] [1-(.96) (.97)] = .02884608$$

Finally, to continue the recursive computation to account for the effect of the third burst, where $L_2 = \{t_1, t_2\}$ and, therefore, the four possible sets F and ϕ , t_1 , t_2 , and $t_1 t_2$, we have:

$$P_3(\phi;0) = P_2(\phi;0) Q_1(3) Q_3(3,\phi) Q_4(3) \quad [\text{Formula 1}]$$

$$= (.5184) (.9) (1) (.8) = .373248$$

$$P_3(t_1;0) = P_2(t_1;0) Q_1(3) Q_3(3,t_1) Q_4(3) \quad [\text{Formula 1}]$$

$$= (.0335232) (.9) (.96) (.8) = .02317123$$

$$P_3(t_2;0) = P_2(t_2;0) Q_1(3) Q_3(3,t_2) Q_4(3) \quad [\text{Formula 1}]$$

$$= (.03456) (.9) (.98) (.8) = .02438553$$

$$P_3(t_1 t_2;0) = P_2(t_1 t_2;0) Q_1(3) Q_3(3,t_1 t_2) Q_4(3) \quad [\text{Formula 1}]$$

$$= (.00223488) (.9) (.95) (.8) = .00152865$$

$$P_3(t_3;0) = P_2(\phi;0) P_1(3) Q_2(3) Q_3(3,\phi) Q_4(3) \quad [\text{Formula 2}]$$

$$= (.5184) (.1) (.6) (1) (.8) = .0248832$$

$$P_3(t_1 t_3; 0) = P_2(t_1; 0) P_1(3) Q_2(3) Q_3(3, t_1) Q_4(3) \quad [\text{Formula 2}]$$

$$= (.0335232) (.1) (.6) (.96) (.8) = .00154474$$

$$P_3(t_2 t_3; 0) = P_2(t_2; 0) P_1(3) Q_2(3) Q_3(3, t_2) Q_4(3) \quad [\text{Formula 2}]$$

$$= (.03456) (.1) (.6) (.98) (.8) = .0016257$$

$$P_3(t_1 t_2 t_3; 0) = P_2(t_1 t_2; 0) P_1(3) Q_2(3) Q_3(3, t_1 t_2) Q_4(3) \quad [\text{Formula 2}]$$

$$= (.00223488) (.1) (.6) (.95) (.8) = .00010191$$

$$P_3(*; 0) = Q_4(3) \left\{ P_2(*; 0) + P_2(\phi; 0) [1 - Q_5(3) Q_3(3, \phi)] \right. \quad [\text{Formula 3}]$$

$$+ P_2(t_1; 0) [1 - Q_5(3) Q_3(3, t_1)] + P_2(t_2; 0) [1 - Q_5(3) Q_3(3, t_2)]$$

$$+ P_2(t_1 t_2; 0) [1 - Q_5(3) Q_3(3, t_1 t_2)] \left. \right\}$$

$$= (.8) \left\{ (.05128192) + (.5184) [1 - (.96) (1)] \right.$$

$$+ (.0335232) [1 - (.96) (.96)] + (.03456) [1 - (.96) (.98)]$$

$$+ (.00223488) [1 - (.96) (.95)] \left. \right\} = .061511$$

$$P_3(\phi; 1) = Q_1(3) Q_3(3, \phi) \left\{ P_2(\phi; 0) P_4(3) + P_2(\phi; 1) \right\} \quad [\text{Formula 4}]$$

$$= (.9) (1) \left\{ (.5184) (.2) + .2916 \right\} = .355752$$

$$P_3(t_1; 1) = Q_1(3) Q_3(3, t_1) \left\{ P_2(t_1; 0) P_4(3) + P_2(t_1; 1) \right\} \quad [\text{Formula 4}]$$

$$= (.9) (.96) \left\{ (.0335232) (.2) + .0188568 \right\} = .02208508$$

$$P_3(t_2; 1) = Q_1(3) Q_3(3, t_2) \left\{ P_2(t_2; 0) P_4(3) + P_2(t_2; 1) \right\} \quad [\text{Formula 4}]$$

$$= (.9) (.98) \left\{ (.03456) (.2) + .01944 \right\} = .02324246$$

$$P_3(t_1 t_2; 1) = Q_1(3) Q_3(3, t_1 t_2) \left\{ P_2(t_1 t_2; 0) P_4(3) + P_2(t_1 t_2; 1) \right\}$$

$$\quad [\text{Formula 4}]$$

$$= (.9) (.95) \left\{ (.00223488) (.2) + .00125712 \right\} = .001457$$

$$P_3(t_3; 1) = P_1(3) Q_2(3) Q_3(3, \phi) \left\{ P_2(\phi; 0) P_4(3) + P_2(\phi; 1) \right\} \quad [\text{Formula 5}]$$

$$= (.1) (.6) (1) \left\{ (.5184) (.2) + .2916 \right\} = .0237168$$

$$\begin{aligned}
 P_3(t_1 t_3; 1) &= P_1(3) Q_2(3) Q_3(3, t_1) \{P_2(t_1; 0) P_4(3) + P_2(t_1; 1)\} \\
 &\quad \text{[Formula 5]} \\
 &= (.1) (.6) (.96) \{(.0335232) (.2) + .0188568\} = .00147233
 \end{aligned}$$

$$\begin{aligned}
 P_3(t_2 t_3; 1) &= P_1(3) Q_2(3) Q_3(3, t_2) \{P_2(t_2; 0) P_4(3) + P_2(t_2; 1)\} \\
 &\quad \text{[Formula 5]} \\
 &= (.1) (.6) (.98) \{(.03456) (.2) + .01944\} = .00154949
 \end{aligned}$$

$$\begin{aligned}
 P_3(t_1 t_2 t_3; 1) &= P_1(3) Q_2(3) Q_3(3, t_1 t_2) \{P_2(t_1 t_2; 0) P_4(3) + P_2(t_1 t_2; 1)\} \\
 &\quad \text{[Formula 5]} \\
 &= (.1) (.6) (.95) \{(.00223488) (.2) + .001257512\} \\
 &= .00009713
 \end{aligned}$$

$$\begin{aligned}
 P_3(*; 1) &= P_2(*; 0) P_4(1) + P_2(*; 1) \quad \text{[Formula 6]} \\
 &\quad + [P_2(\phi; 0) P_4(3) + P_2(\phi; 1)] [1 - Q_5(3) Q_3(3, \phi)] \\
 &\quad + [P_2(t_1; 0) P_4(3) + P_2(t_1; 1)] [1 - Q_5(3) Q_3(3, t_1)] \\
 &\quad + [P_2(t_2; 0) P_4(3) + P_2(t_2; 1)] [1 - Q_5(3) Q_3(3, t_2)] \\
 &\quad + [P_2(t_1 t_2; 0) P_4(3) + P_2(t_1 t_2; 1)] [1 - Q_5(3) Q_3(3, t_1 t_2)] \\
 &= (.05128192) (.2) + .02884608 \\
 &\quad + [(.5184) (.2) + .2916] [1 - (.96) (1)] \\
 &\quad + [(.0335232) (.2) + .0188565] [1 - (.96) (.96)] \\
 &\quad + [(.03456) (.2) + .01944] [1 - (.96) (.98)] \\
 &\quad + [(.00223488) (.2) + .00125712] [1 - (.96) (.95)] = .05862767
 \end{aligned}$$

The formulas in Table 8 are then used to give the results shown in Table 9.

The damage categories whose probabilities are given in Table 9 fall into three classes. These are:

- Class 1 - Categories that can never be entered from other categories, only left in moving to other categories.
- Class 2 - Categories that can be both entered from other categories and left in moving to other categories.

Class 3 - Categories that can only be entered from other categories, but never left.

Of the categories listed, the only Class 1 category is "Undamaged." This is the initial state of the target before any firing. Class 2 categories are those representing only partial damage. These are "Fuel System Puncture Only," "Disabling Mechanical Damage Only," "Fire Only," and "Fuel System Puncture, Disabling Mechanical Damage; No Fire." Class 3 categories are those representing either partial or complete damage. These are "Disabling Mechanical Damage," "Fire," and "Either Disabling Mechanical Damage or Fire." As the number of bursts increases, the probabilities associated with categories in these classes will differ. Probabilities for Class 1 categories will always decrease, for subsequent bursts will always carry the chance of moving the target out of a Class 1 category, never into it. Probabilities for Class 2 categories will generally increase at first, but finally decrease, and, in the limit, become zero, as will those for Class 1 categories. This is because Class 2 categories can only be entered from Class 1 categories, and, as the probabilities associated with the Class 1 categories steadily decrease, there is less chance of the target entering a Class 2 category; it can only leave it. Probabilities for Class 3 categories always increase, approaching unity in the limit. If an "infinite" number of bursts are fired at the target, it will with complete certainty suffer both "Fire" and "Disabling Mechanical Damage."

These trends are noted in Table 9, although the probabilities for some of the Class 2 categories have not yet started to decrease after three bursts.

TABLE 9

PROBABILITIES OF TARGET BEING IN VARIOUS DAMAGE CATEGORIES
FOR INDEPENDENT DAMAGE EVENTS

<u>DAMAGE CATEGORY</u>	PROBABILITY OF DAMAGE CATEGORY AFTER		
	<u>ONE BURST</u>	<u>TWO BURSTS</u>	<u>THREE BURSTS</u>
Undamaged	.720	.518	.373
Fuel System Puncture Only	.048	.090	.077
Disabling Mechanical Damage Only	.180	.292	.356
Fire Only	.032	.051	.062
Fuel System Puncture Disabling Mechanical Damage; No Fire	.012	.040	.074
Disabling Mechanical Damage; Fire	.008	.029	.059
Disabling Mechanical Damage	.200	.360	.488
Fire	.040	.080	.120
Either Disabling Mechanical Damage or Fire	.232	.411	.550