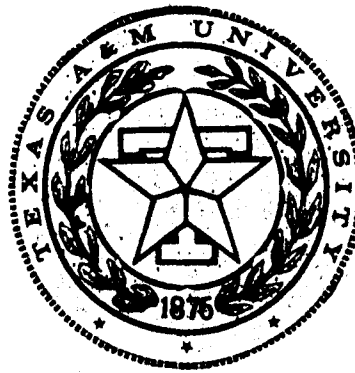


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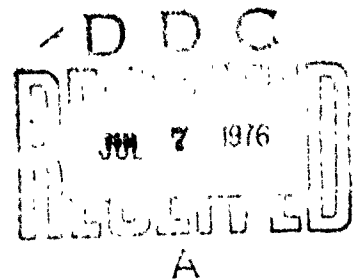
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REGRESSION WITH DIFFERENTIAL EQUATION MODELS

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DARCOM Intern Training Center
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April 1976

Final Report



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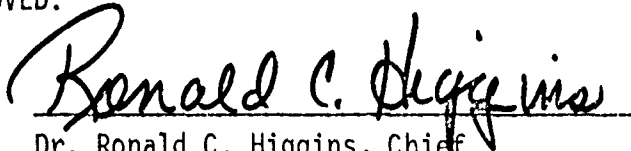
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Texarkana, Texas 75501

FOREWORD

The research discussed in this report was accomplished as part of the Maintenance Effectiveness Engineering Graduate Program conducted jointly by the DARCOM Intern Training Center and Texas A&M University. ~~As such,~~ the ideas, concepts and results herein presented are those of the author and do not necessarily reflect approval or acceptance by the Department of the Army.

This report has been reviewed and is approved for release. For further information on this project contact Dr. Ronald C. Higgins, Intern Training Center, Red River Army Depot, Texarkana, Texas 75501.

APPROVED:


Dr. Ronald C. Higgins, Chief
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For the Commander


James L. Arnett, Director
Intern Training Center

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During the course of this study, my employer was the U.S. Army as a career intern in the DARCOM Maintenance Effectiveness Engineering Graduate Program. I am grateful to the U.S. Army for the opportunity to participate in this program.

The ideas, concepts, and results herein presented are those of the author(s) and do not necessarily reflect the approval or acceptance by the Department of the Army.

ABSTRACT

Research Performed by Craig D. Hunter

Under the Supervision of Dr. S. Bart Childs

Regression analysis normally implies the use of algebraic equations to describe a system; however, some cases would better be modeled by differential equations. This is accomplished by assuming a differential equation model for a given set of data and estimating the values of the unknown parameters within the model. These values are then systematically perturbed to generate particular solutions which are superimposed to yield a better estimate of the unknowns. This process is repeated until a specified accuracy is met.

Through an analysis of variance, the statistical characteristics of linear regression can be generated for most n^{th} order differential equations. This provides a basis for evaluating the 'acceptance or rejection' of the regression.

The characteristics generated consist of an ANOVA table (uncorrected), general F test on the regression, the R^2 value, covariance matrix of the superposition constants, an estimate of the variance about the regression, an estimate of the variance of the parameters, and the confidence intervals on these estimates.

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CHAPTER I

INTRODUCTION

Regression analysis usually implies determining a set of independent variables and corresponding coefficients for algebraic equations such that these equations best portray the data or system given. Examples are presented in which algebraic models are inadequate and can be appropriately modeled by differential equations.

This report is an investigation into the validity and accuracy of a differential equation regression package, both linear and non-linear (Childs, 1970). It is not the intent to deal with extensions of the established theory behind such an approach (Bellman and Kalaba, 1965), (Roberts and Shipman, 1972). Nomenclature and background material are given in Chapter II.

A control program was written to provide a means for evaluating QUASII (Childs, 1970). The control program yields solutions of a specified accuracy (within the computational restrictions of the computer) providing a means for comparison. QUASII would be used instead of a program yielding "exact" solutions because of ease of use.

The control program is based on power series evaluation and integration of differential equations; whereas, QUASII utilizes a 4th order Runge-Kutta integrator. The Runge-Kutta integrator allows simpler programming of non-linear differential equations.

Two solution areas are of interest:

Case I: The number of boundary conditions (m) equals the order of the differential equation (n), therefore all boundary conditions are to be

met exactly (exact system).

Case II: The number of boundary conditions (m) exceeds the order of the differential equation (n), therefore some boundary conditions are to be met exactly and others to be met in a least squares sense (over-determined system).

Both QUASII and the control program utilize a least squares fit of the non-exact boundary conditions in Case II. Further study in this area is being done by Walker (Ref. 21). These results will be available in April 1976 and entails meeting these boundary conditions in a least squares and other criteria.

Case I is of theoretical interest. This report contains results for Case II studies.

CHAPTER II

DIFFERENTIAL REGRESSION

The following example of a linear case from Childs et. al. 1971, (Ref. 6), is for clarification of terms and background. For information on non-linear cases, refer to Reference 5 and 17.

Given: A simple mass-spring-damper system governed by the equation;

$$\ddot{x} + \mu \dot{x} + \xi x = \sin(\omega t) \quad (2-1)$$

where $(\dot{})$ denotes the total derivative of that term with respect to t , the independent variable.

Denoting the following state variables;

$$y_1 = x \quad y_2 = \dot{x} \quad (2-2)$$

equation 2-1 can be rewritten as two non-trivial first order differential equations:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -\xi y_1 - \mu y_2 + \sin(\omega t) \end{aligned} \quad (2-3)$$

The state variables y_1 and y_2 are functions of t , $y_i(t)$ (t has been omitted for simplicity).

If μ and ξ are known, equations 2-1 and 2-3 are linear differential equations. Assuming that μ and ξ are unknown, which is not an unreasonable assumption, two more state variables must be added;

$$y_3 = \mu \quad y_4 = \xi$$

or which the following four equations result;

$$\dot{y}_3 = 0 \quad (2-4)$$

$$\dot{y}_4 = 0 \quad (2-5)$$

$$\dot{y}_1 = y_2 \quad (2-6)$$

$$\dot{y}_2 = -y_1 y_4 - y_2 y_3 + \sin(wt) \quad (2-7)$$

the frequency w could have been assumed unknown, introducing a fifth state variable, $y_5 = w$. Except for equation 2-7, the four differential equations are linear with 2-4 and 2-5 being trivial. Since equation 2-7 is non-linear, the system is non-linear.

At least two Newton type or Taylor series methods used in solving such systems are perturbation and quasilinearization methods.

Perturbation methods are used in the control program and QUASII for ease of programming. For information regarding this theory, see Reference 5, Doiron (1970), or any of Childs' writings on non-linear differential equation solutions.

The right hand side (RHS) of equation 2-3 is unknown (except for μ and ξ), therefore, avenues are opened for the implementation of boundary conditions or constraints which must be met. In a mass-spring-damper system governed by equation 2-1, the use of an accelerometer to measure the acceleration of the mass at certain points in time (t_i) is within experimental procedure. An accumulation of constraints (boundary values) could be obtained in the following form;

$$\ddot{x}(t_i) = b_i \quad i = 1 \dots m \quad (2-8)$$

where b_i is the measured acceleration at time t_i and m is the number of boundary values. The LHS of equation 2-3, y_2 ($y_2 = \ddot{x}$), can now be replaced by b_i ;

$$b_i = -\xi y_1(t_i) - \mu y_2(t_i) + \sin(wt_i) \quad (2-9)$$

in more general terms;

$$q_i(y(t_i)) = b_i \quad i = 1 \dots m \quad (2-10)$$

where q_i is a boundary condition operator. Equation 2-10 defines m boundary conditions resulting in m linear (or as in equation 2-7, non-linear) equations. From Chapter I, m must be equal to or greater than n , the order of the differential equation (number of state variables). These m differential equations will either be met exactly or at least in a least square (best fit) manner.

A brief summary of a "shooting" technique of solving linear differential equations follows. Given;

$$\dot{y} = Ly + f \quad (2-11)$$

where L is a linear operator and f is a forcing function.

A set of homogeneous solutions is usually superimposed upon a particular solution. The number of homogeneous solutions is equal to n , the order of the differential equation (see equation 2-1, $n=2$). Assuming that no initial conditions are known, $n = r$ (let r equal the number of unknown initial conditions, where $r \leq n$). When only r initial conditions are unknown, r homogeneous equations are required. The result of the superposition of a particular solution (2-12) and a set of homogeneous equations (2-13) is:

$$\dot{p} = Lp + f \quad (2-12)$$

$$\dot{H} = LH \quad (2-13)$$

$$y = p + H\beta = p + \sum_{k=1}^r h^{(k)} \beta_k \quad (2-14)$$

It is an established theory that the sum of a particular solution and a set of homogeneous equations yields another particular solution. An alternative is to superimpose only particular solutions. This scheme

is used in QUASII and the control program;

$$y = Pa = \sum_{k=0}^r p^{(k)} a_k \quad (2-15)$$

where

$$\dot{p}^{(k)} = Lp^{(k)} + f \quad (2-16)$$

There will be $r+1$ particular solutions, where r is the number of unknown initial values.

A restriction on the superposition constants is:

$$\sum_{k=0}^r a_k \equiv 1 \quad (2-17)$$

This is a result of equation 2-11, where the differential equation is given by:

$$\dot{y} = Ly + f$$

If particular solutions of the form of equation 2-16 are superimposed without the constraint 2-17, the resulting differential equation could be

$$\dot{y} = Ly + Cf \quad (2-18)$$

where

$$c = \sum_{k=0}^r a_k \neq 1 \quad (2-19)$$

The original differential equation has not been satisfied and necessitates the restriction on the superposition constants to yield

$$\dot{y} = Ly + f \sum_{k=0}^r a_k \quad (2-20)$$

The computational strategy requires choosing initial arbitrary values for the r unknown state variables. These values are used in defining the unperturbed particular solution (denoted by $p^{(0)}$). Each of the unknown variables (r of them) will in turn be individually perturbed by a

predetermined (arbitrary) constant to yield an appropriately perturbed particular solution. These particular solutions, obtained by integration, yield values corresponding to the boundary values at the appropriate value of the independent variable.

The result is a matrix equation of the form

$$Sa = d \quad (2-21)$$

where S is a function of the integrated solutions, a is the vector of superposition constants, and d is a vector of boundary values. The resulting $m+1$ equations (m due to the boundary conditions and one due to the restriction on the superposition constants) contain $r+1$ unknown a_k 's ($r+1 \leq m+1$), which can be determined as discussed in Chapter III.

The resulting y solutions (state variables) are the initial arbitrary estimates plus the addition of the corresponding superposition constant times the perturbation constant.

Evaluation of the a 's was stated to be a rather simple process. This entails a Gauss-Jordan reduction utilizing a least squares fit of the over-determined system. A more in-depth discussion of the theory is presented in Chapter III and Reference 4.

CHAPTER III

THE OVER-DETERMINED SYSTEM

When the number of boundary conditions (m) equals the order (n) of the differential equation, the boundary conditions can theoretically all be met exactly, however, in the case of the over-determined system, this is impossible. Thus, several boundary values will have to be met by some other means.

The problem of Chapter II was narrowed to the final equation:

$$Sa = d \quad (3-1)$$

This equation and the role of a least squares fit are the topics of this chapter.

Examining the composition of the S matrix, two subdivisions can be shown;

$$\left[\begin{array}{c} S_A \\ \hline S_B \\ r+1 \end{array} \right] \begin{array}{c} k+1 \\ \hline m-k \end{array}$$

where k is the number of boundary conditions to be met exactly. The S_A matrix contains the exact conditions and S_B the least squares boundary conditions.

A further reduction or partitioning can be;

$$\left[\begin{array}{c|c} S_1 & S_2 \\ \hline S_3 & S_4 \end{array} \right] \begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} d \end{bmatrix} \quad (3-2)$$

where

S_1 is order $(k+1) \times (k+1)$

S_2 is order $(k+1) \times (r-k)$

S_3 is order $(m-k) \times (k+1)$

S_4 is order $(m-k) \times (r-k)$

By a Gauss-Jordan reduction, the S_1 matrix can be transformed into an identity matrix resulting in S_3 being a null matrix. The results are;

$$\left[\begin{array}{c|c} I & S_2' \\ \hline 0 & S_4' \end{array} \right] \begin{bmatrix} a_e \\ a_1 \end{bmatrix} = \begin{bmatrix} d_e' \\ d_1' \end{bmatrix} \quad (3-3)$$

where the subscripts 1 and e refer to least square and exact boundary conditions respectfully.

The null matrix (S_3) allows for an equation (3-4) containing only least squares conditons.

$$S_4' a_1 = d_1' \quad (3-4)$$

Equation 3-4 yields $m-k$ equations each with an unknown.

Premultiplying by the transpose of S_4' gives the normal equation (refer to Appendix B);

$$(S_4')^T S_4' a_1 = (S_4')^T d_1' \quad (3-5)$$

resulting in the solution for a_1 :

$$a_1 = \left[(S_4')^T S_4' \right]^{-1} (S_4')^T d_1' \quad (3-6)$$

Having solved for a_1 in equation 3-6, a_e (from exact conditions) can be solved by referring to equation 3-3;

$$I a_e + S_2' a_1 = d_e' \quad (3-7)$$

or

$$a_e = d_e' - S_2' a_1 \quad (3-8)$$

Equation 3-4 and 3-5 are important and will be used extensively in Chapter IV.

CHAPTER IV

ANALYSIS OF VARIANCE

The boundary conditions are:

$$q(y(t_i)) = b_i \quad i = 1 \dots m \quad (4-1)$$

This defines an operator q operating on the $y(t_i)$'s to yield the desired boundary values. The statistical basis for 'accepting or rejecting' the regression is the nearness of the equality 4-1.

In Appendix B, the equation;

$$z = Wd + \epsilon \quad (4-2)$$

is analyzed through an analysis of variance.

From equations 4-1 and 4-2, the following results can be obtained;

$$b = \hat{q} + \epsilon \quad (4-3)$$

where

$$\hat{q} = q(\hat{y})$$

$$b \approx z$$

$$\epsilon = \epsilon$$

$$\hat{q} \approx Wd$$

Note that $q(y(t_i))$ may not be linear as required for an analysis of variance as described in Appendix B, but the solution process is iterative using linearized equations. Because the steps are small, the assumption of $\hat{q}$ being linear is a good approximation.

An analysis of variance can be accomplished as shown in Table 4.1.

TABLE 4.1

ANOVA TABLE

Source	Sum of Squares	Degrees of Freedom	Mean Square
Due to regression	$\hat{q}^T b$	r	SS/r
About the regression (residual)	$b^T b - \hat{q}^T b$	$m-k-r$	s^2
Total (uncorrected)	$b^T b$	$m-k$	

The following are calculated directly from Table 4.1:

$$R^2 = \frac{SS_{\text{regression}} - (m-k)(\text{mean of } b_i's)^2}{SS_{\text{total}} - (m-k)(\text{mean of } b_i's)^2} \quad (4-4)$$

$$F_{\text{cal}} = \frac{MS_{\text{regression}}}{MS_{\text{residual}}} \quad (4-5)$$

$$s^2 = MS_{\text{residual}} = \text{estimated variance of system} \quad (4-6)$$

The F_{cal} value will be compared to a Fisher's F with:

Probability of $1-\alpha$ (α is the producer's risk)

Numerator degrees of freedom r

Denominator degrees of freedom $(m-k-r)$

If F_{cal} exceeds Table F, the regression is 'accepted'. (Note: The producer's risk is defined as the probability of rejecting the null hypothesis (H_0) (see Appendix B) when in fact the null hypothesis is true.

Confidence limits for the estimated least square boundary values can be determined once the individual variances are known. Referring to Appendix B, the variance of the estimated observation is given by equation B-10:

$$\text{est. var}(\hat{z}_i) = W_i (W^T W)^{-1} W_i^T S^2 \quad (4-7)$$

Equation 4-3 shows that $\hat{q}$ is synonymous with Wd of Appendix B, but $\hat{q}$ cannot be subdivided into a matrix of independent variables and a vector of parameters, therefore another approach must be used.

Using the system of equation 3-5, a relation between d_1 (least squares boundary values of b) and d_1' (transformation of d_1) can be determined.

The variances of the individual least squares boundary values;

$$\begin{bmatrix} S_3 \\ S_4 \end{bmatrix} \begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} d_1 \end{bmatrix} \quad (4-8)$$

is equal to the variances of the transformed system:

$$\begin{bmatrix} 0 \\ S_4' \end{bmatrix} \begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} d_1' \end{bmatrix} \quad (4-9)$$

i.e.

$$\text{var}(d_1) = \text{var}(d_1') \quad (4-10)$$

The variances of the estimated least squares boundary values are given by;

$$\text{est. var}(\hat{b}_i) = S_{4_i}' \left[(S_4')^T S_4' \right]^{-1} (S_{4_i}')^T S^2, \quad i=1 \dots m \quad (4-11)$$

where S_{4_i}' is the row vector of S_4' corresponding to the i^{th} least square boundary condition.

The resulting confidence limits are;

$$\hat{b}_i \pm t(v, 1 - \frac{\alpha}{2}) \sqrt{\text{est. var}(\hat{b}_i)} \quad (4-12)$$

where

$$v = m - k - r \quad (4-13)$$

$$i = k+1 \dots m$$

A main objective of the regression package is to obtain the solutions

for the y_i 's given in Chapter II. Table 4.1 does not always provide a means for investigating this because it involves the boundary condition functions. By doing a partial analysis of variance on equation 4-10, the variance of the a_1 's can be determined;

$$\text{est. var}(a_1) = ((S_4^1)^T S_4^1)^{-1} s_1^2 \quad (4-14)$$

where

$$s_1^2 = \frac{(d_1^1)^T (d_1^1) - a_1^T (S_4^1)^T d_1^1}{m-k-r} \quad (4-15)$$

Referring to Chapter II, the y_i solutions are equal to the previous value (initial guess) plus the perturbation times a superposition constant;

$$\hat{y} = \hat{y}_{\text{previous}} + (a_1 * \text{perturbation}) \quad (4-16)$$

therefore

$$\text{var}(\hat{y}) = \text{var}(a_1) * (\text{perturbation})^2 \quad (4-17)$$

This results in a confidence interval for these solutions as follows:

$$\hat{y}_i \pm t(v, 1 - \frac{\alpha}{2}) \sqrt{[\text{est. var}(\hat{y}_i)]} \quad i = 1 \dots r \quad (4-18)$$

CHAPTER V

RESULTS-COASTING DATA

The statistical study outlined in Chapter IV was accomplished by a subroutine package PSTAT (Appendix D contains PSTAT and supportive subroutines).

Since experimental data was not available on the system of equation 2-1, a problem of determining the aerodynamic drag coefficient and rolling friction coefficient of an automobile was used (Ref. 21). The nonlinear equation governing this system is;

$$\ddot{x} + \frac{A_f \rho}{2M} C_d \dot{x}^2 + \mu_f g = 0 \quad (5-1)$$

where

ρ = air density (slugs/ft³)

A_f = frontal area of vehicle (ft²)

M = mass of vehicle (slugs)

g = acceleration of gravity (ft/sec²)

C_d = coefficient of drag

μ_f = rolling friction coefficient

$\dot{x}$ = velocity (ft/sec)

$\ddot{x}$ = acceleration (ft/sec²)

This problem demonstrates the advantages of modeling by differential equations with a regression package like QUASII. Attempts to determine C_d and μ_f by other than numerical methods (such as wind tunnel testing, treadmills, etc.) would be more laborious, less accurate, and more expensive.

Coasting data (value of $\dot{x}$) for a Sunbeam Alpine was obtained from

Road and Track Road Test Annual for 1966. Figure 5.1 is a listing of input for such a system. Figure 5.2 is an output of the statistical analysis from PSTAT. Figure 5.1 shows only one non-trivial differential equation. Although equation 5-1 is a 2nd order differential equation, it does not contain an x term, allowing a single first order representation.

Except for three parameters, the PSTAT output (Figure 5.2) has been previously defined. The three additional parameters are:

1) Mean of the observations is the sum of the least squares observations (boundary values) divided by the total number of these observations:

$$\bar{b} = \frac{\sum_{i=1}^{m-k} b_i}{m-k} \quad (5.2)$$

2) Coefficient of variation (cv) is a measure of the dispersion of the data (Hald, 1952) and is expressed as:

$$c = \frac{\text{MS residual}}{\text{mean of observations}} = s/\bar{b} \quad (5-3)$$

The importance of this simple parameter is questionable.

3) $P(F(\text{ALPHA}).GT.FCAL)$ is the probability that a random variable that is F distributed, with the same degrees of freedom as $F(\text{ALPHA})$, will be greater than $FCAL$, thus the minimum producer's risk that can be assumed.

The PSTAT output (Figure 5.2) is an ANOVA table of the uncorrected sum of squares plus the corresponding degrees of freedom and mean square terms. Next is R^2 and the coefficient of variation value. A R^2 value of 1 indicates a 'perfect' regression. The TEST OF OVERALL REGRESSION statement is a test of the hypothesis stated in Appendix B for a risk of $\text{ALPHA} (\alpha)$. If the $FCAL$ exceeds $F(\text{ALPHA})$, H_0 is rejected and the alternative, H_1 , is accepted. Under these conditions,

```
*****  
FOLLOWING IS THE OUTPUT OF INPUT  
*****  
LINEAR= 0 STAT= 1 ALPHA= 0.50000000D-01  
LINEAR= ZERO IMPLIES NON-LINEAR STAT= ONE IMPLIES POST STATISTICAL STUDY TO BE EXECUTED  
LINEAR= NON-ZERO IMPLIES LINEAR STAT= ZERO IMPLIES NO POST STATISTICAL STUDY  
*****  
ACCURACY ACC= 0.10000D-05  
FOR GJWRLS CHECK= 0.10000D-19 DET= 1.0000  
FUDDGE FACTOR FOR DTIND RF= 0.50000  
*****  
NUMBER OF DIFFERENTIAL EQ NDE= 3  
NUMBER OF BOUNDARY CONDITIONS NBV= 9  
NUMBER OF TERMS IN PWR SERIES NTERMS= 20  
NO. EXACT BOUNDARY CONDITIONS NEM'X= 1  
NO. OF NCN-TRIVIAL DIFF EQ NEO= 1  
*****  
BOUNDARY VALUES APPLIED ON(1BV) IQBV TIME OF BV  
117.30000 1 1 0.0  
104.90000 1 1 5.0000000  
95.30000 1 1 10.0000000  
87.30000 1 1 15.0000000  
79.20000 1 1 20.0000000  
71.90000 1 1 25.0000000  
65.70000 1 1 30.0000000  
59.80000 1 1 35.0000000  
54.00000 1 1 40.0000000  
*****  
INITIAL TIME OF INTEGRATION TO= 0.0  
*****  
INITIAL GUESSES OF Y ICEX LOWER LIMIT UPPER LIMIT  
Y 1 Y( 1)= 110.00000 1 0.0 0.0  
Y 2 Y( 2)= 0.40000000 1 0.0 0.0  
Y 3 Y( 3)= 0.30000000D-01 1 0.0 0.0  
*****  
PERTURBATION SCALER PTBS= 0.10000000D 00  
ABSOLUTE VALUE LIMITS PTMIN= 0.0  
PTMAX= 2.0000  
*****  
IOUT= 0  
QUIT= 15  
*****  
CONCLUDES FORMAL OUTPUT OF THE INPUT  
*****
```

FIGURE 1

 FOLLOWING IS THE OUTPUT OF THE POST STATISTICAL STUDY

SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE
REGRESSION	63716.493	3	21238.831
RESIDUAL	0.96692256	6	0.16115376
TOTAL(UNC)	63717.460	9	= S**2

PER-CENT VARIATION R**2= 99.973342 COEFFICIENT OF VARIATION CV= 0.49129134D-02
 TEST OF OVERALL REGRESSION FCAL= 131792.34 F(ALPHA)= 4.7571592 P(F(ALPHA).GT.FCAL)= 0.83446503D-06
 FOR RISK OF ALPHA= 0.50000000D-01
 *****ACCEPT REGRESSION*****

SPECIFICS OF THE BOUNDARY CONDITIONS		T(ALPHA)= 2.4469080	ALPHA= 0.50000000D-01	UPPER LIMIT
OBSERVED VALUES	ESTIMATED VALUE	RESIDUAL	ESTIMATED S.E.	LOWER LIMIT
117.30000	116.89493	0.40507263	0.35231109	116.03285
104.90000	105.52164	-0.62164188	0.20683878	105.01353
95.30000	95.605284	-0.30928430	0.19612970	95.129373
87.30000	86.856482	0.44351834	0.20538341	86.353927
79.20000	79.037830	0.16217005	0.19521417	78.560159
71.90000	71.985065	-0.85064950D-01	0.17431129	71.558541
65.70000	65.568615	0.13138498	0.17133949	65.149363
59.80000	59.682176	0.11782363	0.21489984	59.156336
54.00000	54.240515	-0.24051461	0.30186196	53.501886
MEAN OF OBSERVATIONS=	81.71111			
SUM OF THE RESIDUALS=	0.34640960D-02			

SPECIFICS OF THE YI SOLUTIONS		T(ALPHA)= 2.4469080	ALPHA= 0.50000000D-01
ESTIMATED VALUE	EST VARIANCE	CONFIDENCE INTERVAL	
116.89493	0.12412310	116.89493 (+-) 0.86207283	
0.50251918	0.79454589D-03	0.50251918 (+-) 0.68972685D-01	
0.1692418D-01	0.15727466D-05	0.16924118D-01 (+-) 0.30686476D-02	

COVARIANCE MATRIX OF THE SUPERPOSITION CONSTANTS
 0.31031D-01 0.31974D-01 -0.65375D-01
 0.61974D-01 0.31464 -0.40651
 -0.65375D-01 -0.40651 0.54909

CONCLUDES STAT PACKAGE

FIGURE 2

*****ACCEPT REGRESSION***** is printed. This is not to imply this is the 'best' possible regression, but for the stated hypothesis and the given data, it is an 'acceptable' regression.

Following the hypothesis test are two sections showing confidence intervals on the observations and state variables respectfully. These limits are all based on the same producer's risk (ALPHA) and t value (student's t distribution). In addition, the observed values (given boundary values) are shown with the estimated values from the regression. The residual terms;

$$(\text{observed value}) - (\text{estimated value}) \quad (5-4)$$

and the sum of the differences are included. Both sections contain the estimated variance, or standard error, of the individually estimated solutions.

The last section is the covariance matrix of the superposition constants (least squares). The diagonal elements are the variances of those constants.

A regression cannot be 'accepted' or 'rejected' on the basis of just one statistic. The programmer must qualify an acceptable solution and evaluate the overall regression, confidence limits, variances, hypothesis, etc. for the given system to determine whether a resulting model is acceptable. PSTAT does furnish enough information to evaluate the regression validity in most cases.

CHAPTER VI

CONCLUSIONS ON STATISTICAL RESULTS

PSTAT, a subroutine from the control program, provides a means of evaluating the acceptability of differential equation regressions. The control program and QUASII are based on the same theory and procedures (except for the integration methods). PSTAT is presently not directly compatible with QUASII, but does provide a basis on which QUASII solutions can be evaluated.

This statistical package can be used for linear and non-linear systems (see Reference 4 for types of non-linearities).

PSTAT cannot handle multiple observations for distinct value(s) of the independent variable(s). The ANOVA table and procedures should be modified to include pure error and further hypothesis testing.

The assumption of the linearity of equation 4-2 for the purpose of an analysis of variance, does not appear to be valid for examples worked with non-zero forcing functions (see equation 2-1). This area should be investigated from the possible approach of a non-linear analysis of variance.

In its present form, PSTAT coupled with QUASII provides a powerful tool for statistically evaluating many systems not previously modeled in a sufficient manner by standard algebraic regression packages.

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A P P E N D I C E S

APPENDIX A

NOMENCLATURE

A_f	Frontal area of a vehicle.
a	Superposition constant.
b	Boundary condition.
c	Coefficient of variation.
d	Transformation of boundary condition.
E	Expected value of.
f	Forcing function.
F	Theoretical value from Fisher's F Distribution Tables.
F_{cal}	Variable that is F distributed.
g	Acceleration of gravity.
H	Set of homogeneous solutions.
I	Identity matrix.
k	Number of boundary conditions to be met exactly.
L	Linear operator.
M	Mass.
m	Number of boundary conditions.
MS	Mean-square.
n	Order of the differential equation (number of state var.).
p	A particular solution.
P	Set of particular solutions.
q	Operator, either linear or non-linear.
r	Number of unknown initial conditions.
R^2	Percent variation due to regression.
s^2	Estimate of variance.

s^2	Estimate of variance.
S	Matrix of the integrated values of the particular solutions.
SS	Sum of squares.
s.e.	Standard error of.
t	Independent variable time or a student's t statistic.
var	Variance of.
y	State variable.
μ	Coefficient of damping.
ξ	Spring constant.
w	Frequency.
α	Producer's risk.
β	Superposition constant.
μ_f	Rolling friction coefficient.
ρ	Air density.
C_d	Coefficient of drag.
v	Degrees of freedom.
est.	Estimated.
e	Exact.
l	Least squares.
T	Transpose.
$'$	Transformation of by matrix operations.
$\wedge$	Estimate.
$-$	Mean.

APPENDIX B

GENERAL LINEAR REGRESSION THEORY (Draper and Smith, 1966)

To handle multiple linear regression, a matrix approach is desirable. Unless otherwise stated, all unsubscripted letters are matrices or vectors.

Consider a linear problem (only in the coefficients) with n observations and p independent variables, the following linear equation could arise;

$$z = W\beta + \epsilon \quad (B-1)$$

where

z is a $(n \times 1)$ vector of observations

W is a $(n \times p)$ matrix of independent variable

β is a $(n \times 1)$ vector of parameters to be estimated

ϵ is a $(n \times 1)$ vector of errors

Certain basic assumptions must be made on the error terms, ϵ ;

ϵ_i 's are random variables

$$E(\epsilon_i) = 0$$

$$\text{var}(\epsilon_i) = \sigma^2 \text{ (unknown)}$$

$$\epsilon_i \text{ and } \epsilon_j \text{ are uncorrelated (i \neq j)} \quad (B-2)$$
$$\text{cov}(\epsilon_i, \epsilon_j) = 0$$

ϵ_i 's are normally distributed; $N(0, \sigma^2)$

ϵ_i 's are therefore independent

As a result of $\text{cov}(\epsilon_i, \epsilon_j) = 0$, z_i and z_j are also uncorrelated and

$$E(z_i) = W\beta$$

$$\text{var}(z_i) = \sigma^2$$

By evaluating the error sum of squares and substituting the least squares estimates of β (d an n vector), the following normal equation results:

$$(W^T W)^{-1} d = W^T z \quad (B-3)$$

Using the decomposition of the total sum of squares (uncorrected), the ANOVA (analysis of variance) Table B.1 results.

TABLE B.1			
Source	Sum of Squares	Degrees of Freedom	Mean Square
Due to regression	$d^T W^T z$	p	MS_{REG}
About regression (residual)	$z^T z - d^T W^T z$	$n-p$	$MS_E = s^2$
Total (uncorrected)	$z^T z$	n	

MS_E is an estimate of the variance about the regression.

Once the ANOVA table is constructed, several conclusions can be deduced:

- I. A percent measurement of the total variation about the mean attributed to the regression (square of the multiple correlation coefficient) is given by;

$$R^2 = \frac{d^T W^T z - n \bar{z}^2}{z^T z - n \bar{z}^2} \quad (B-4)$$

where $\bar{z}$ is the mean of the observations.

A value of $R^2 = 1$ implies that the prediction is 'perfect'.

- II. To test the overall regression equation, the hypothesis:

$$H_0: \beta_0 = \beta_1 = \dots = \beta_p = 0 \quad (B-5)$$

must be tested against the alternate hypothesis:

$$H_1: \beta_i \neq 0 \quad i = 0 \dots p \quad (B-6)$$

This employs the Fisher's F distribution. The F_{cal} statistic is:

$$F_{cal} = \frac{MS_{REG}}{MS_E} \quad (B-7)$$

This value is to be compared to a table value of;

$$F_{(p, n-p)}^{1-\alpha}$$

where:

$1-\alpha$ is a probability

p is the degree of freedom for the numerator

$n-p$ is the degree of freedom for the denominator

If F_{cal} exceeds the table value, the null hypothesis is rejected implying that the variation in the data is more than would be expected by chance in $(1-\alpha)100\%$ of similar data sets.

III. The covariance matrix of the least squares estimates (d) is;

$$\text{est. var}(d) = (W^T W)^{-1} s^2 \quad (B-8)$$

where the i^{th} diagonal elements is the estimated variance of d_i . Equation B-8 coupled with the student's t distribution yields a confidence interval for d_i with a risk of alpha (α) of:

$$d_i \pm t(n-p, 1-\alpha/2) [\text{estimate s.e. } (d_i)] \quad i=1 \dots n \quad (B-9)$$

IV. Similarly, a confidence interval for the estimates of z ($\hat{z}$) can be determined. Based on linear combinations of random variables, the estimated variance of $\hat{z}_i$ is;

$$\text{est. var}(\hat{z}_i) = W_i (W^T W)^{-1} W_i^T S^2 \quad i=1 \dots n \quad (\text{B-10})$$

where W_i is the vector of W corresponding to the i^{th} row or boundary condition (least square). The resulting confidence interval is:

$$\hat{z}_i \pm t(n-p, 1-\alpha/2) \sqrt{[\text{est. var}(\hat{z}_i)]} \quad i=1 \dots n \quad (\text{B-11})$$

APPENDIX C

APPLICATIONS TO ALGEBRAIC REGRESSION

Illustration of the versatility of QUASII and verification of the control program can be shown utilizing the following algebraic regression:

$$x=b_0+b_1t+b_2t^2+b_3t^3 \quad (C-1)$$

Equation C-1 is a linear third order regression equation.

Utilizing the following state variables:

$$\begin{aligned} y_1 &= x = b_0 + b_1t + b_2t^2 + b_3t^3 \\ y_2 &= \dot{x} = b_1 + 2b_2t + 3b_3t^2 \\ y_3 &= \ddot{x} = 2b_2 + 6b_3t \\ y_4 &= \dddot{x} = 6b_3 \end{aligned} \quad (C-2)$$

Four differential equations result:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ \dot{y}_3 &= y_4 \\ \dot{y}_4 &= 0 \end{aligned} \quad (C-3)$$

Equations C-3 are linear with $\dot{y}_4$ being trivial.

By solving the set of differential equations C-3, the values of the y_i 's at $t=0$ can be determined. Equation set C-2 yields the following results:

$$\begin{aligned} b_0 &= y_1(0) & b_2 &= y_3(0)/2 \\ b_1 &= y_2(0) & b_3 &= y_4(0)/6 \end{aligned} \quad (C-4)$$

or in the more general case:

$$b_n = y_{n+1}(0)/n!$$

(C-5)

APPENDIX D

THE CONTROL PROGRAM WITH OUTPUT OF THE NON-LINEAR MASS-SPRING-DAMPER SYSTEM

The control program is based on the same theory as QUASII and both are quite similar in structure. Several subroutines are common (with slight differences) to both programs. However, QUASII is capable of handling a larger variety of problems. As stated earlier, the main difference is that QUASII utilizes a 4th order Runge-Kutta integrator opposed to power series integration employed by the control program. Because of this point a subroutine RECURL must be rewritten for each system to be modeled.

The following pages contain a copy of the control program main-line, supporting subroutines, and post-statistical package for the non-linear system of equation 2-1. The output does not include the results from the PSTAT package.

The program is well documented for both Case I and II problems.

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H A S P J O B L O G

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//HUNTER JOB (S978, RRA, 001, 006, CH), 'HUNTER',
 PASSWORD **
 ***JOBPARM REGION=192

// EXEC FORTG, TIME=(, 20), REGION=192K

XXGO EXEC ?GM=FORTRANG, REGION=110K
 *** COMPILER DD CARDS
 XXSYSPT DD SYSOUT=A
 XXSYSPLN DD SYSOUT=B
 XXSYSPLN DD DSN=SYSLOADSET, DISP=(MOD, DELETE), UNIT=SYSDA,
 XX SPACE=(2240, (800, 100), RLSE), DCB=(BLKSIZE=2240, BUFNO=3, OPTCD=C)
 *** LOADER DD CARDS
 XX DD DDNAME=OBJECT DEFINES HEX DECK FOR LOADER
 //SYSLIB DD
 X/SYSLIB DD DSN=SYS1.FORTLIB, DISP=SHR
 // DD
 X/ DD DSN=SYS1.DPCLIB, DISP=SHR
 // DD
 X/ DD DSN=SYS1.SSPLIB, DISP=SHR
 // DD
 X/ DD DSN=USER.PLOTLIB, DISP=SHR
 // DD
 X/ DD DSN=SYSDPC.IMSLO.LOAD, DISP=SHR
 // DD
 X/ DD DSN=SYSDPC.IMSLS.LOAD, DISP=SHR
 XXSYSLOUT DD SYSOUT=A
 *** TAPU SUPPLIED GO DD CARDS
 XXFT05F001 DD DDNAME=SYSIN
 XXFT06F001 DD SYSOUT=A
 XXFT07F001 DD SYSOUT=B
 //SOURCE DD *
 //SYSIN DD *

// IEF236I ALLOC. FOR HUNTER GO
 IEF237I A06 ALLOCATED TO SYSPT
 IEF237I AA3 ALLOCATED TO SYSPLNCH
 IEF237I 251 ALLOCATED TO SYSPLN
 IEF237I 150 ALLOCATED TO SYSPLB
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 IEF237I A0C ALLOCATED TO SYSLOUT
 IEF237I A52 ALLOCATED TO FT05F001
 IEF237I A10 ALLOCATED TO FT06F001
 IEF237I AA4 ALLOCATED TO FT07F001
 IEF237I A53 ALLOCATED TO SOURCE
 IEF142I - STEP WAS EXECUTED - COND CODE 0000
 IEF285I SYS76097.T154830.RV000.HUNTER.LOADSET DELETED
 IEF285I VOL SER NOS= WORK32.
 IEF285I SYS1.FORTLIB
 IEF285I VOL SER NOS= VS2RES.
 IEF285I SYS1.DPCLIB
 IEF285I VOL SER NOS= VS2RES.
 IEF285I SYS1.SSPLIB
 IEF285I VOL SER NOS= USER02.
 IEF285I USER.PLOTLIB
 IEF285I VOL SER NOS= USER02.
 IEF285I SYSDPC.IMSLO.LOAD
 IEF285I

N O T E

IEF285I VOL SER NOS= USER02.
IEF285I SYSOPC.IMSLS.LOAD
IEF285I VOL SER NOS= USER02.
TAM001I STEP /GO/ REGION SIZE = 192K - CORE USED = 192K

TAM002I STEP /GO/ EXEC TIME = 5.14 SEC - I/O TIME = 1.98 SEC
TAM003I STEP /GO/ STEP SETUP = 1.67 SEC - I/O OVERHEAD = .00 SEC
TAM004I STEP /GO/ PAGES IN = 452 - PAGES OUT = 148


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0021      INPT=5
C-----
C  YI ARE THE INITIAL GUESSES OF THE Y VALUES
C  THE FIRST NEQ VARIABLES ARE TIME DEPENDENT
C  THE REMAINING :DE-NEQ VARIABLES ARE NON-TIME DEPENDENT CONSTANTS
C  ICXZ GREATER THAN 0 IMPLIES NOT EXACT KNOWN INITIAL CONDITION
C  ICXZ EQUAL TO ZERO IMPLIES THAT THE INITIAL CONDITION
C  HAS AN UPPER AND LOWER BOUND(LLIM AND ULLIM)
C  ICXZ LESS THAN 0 IMPLIES EXACT INITIAL CONDITION
C  IF A VARIABLE HAS AN ICXZ LESS THAN ZERO, THUS EXACT KNOWN
C  INITIAL CONDITION THE YI GUESS SHOULD BE THAT EXACT KNOWN
C  INITIAL CONDITION ALSO, IF KNOWN EXACTLY THERE CANNOT BE ANY
C  BOUNDARY CONDITIONS ON THIS VARIABLE MET EXACTLY
C  IF IT IS NON-TIME DEPENDENT, A CONSTANT
C  LLIM IS A LOWER BOUND ON THE INITIAL CONDITION
C  ULLIM IS AN UPPER BOUND ON THE INITIAL CONDITION
C  LLIM AND ULLIM ONLY APPLY IF ICXZ=0
C  PTBS IS A PERTURBATION SCALAR TO DETERMINE THE SIZE OF PTB(1)
C  PTMIN IS THE MIN VALUE OF THE ABSOLUTE VALUE OF THE PERTURBATION
C  PTMAX IS THE MAX VALUE OF THE ABSOLUTE VALUE OF THE PERTURBATION
C-----
      IPRT=6
C-----
C  ICUT IS AN OUTPUT-DEBUG VARIABLE
C  .EQ. -1 IMPLIES OUTPUT OF PSTAT PACKAGE
C  .EQ. 0 IMPLIES NO DEBUG OPTIONS IN AFFECT
C  .GT. 1 IMPLIES PIVOT VALUES FROM GJRWLS
C  .GT. 3 IMPLIES P MATRIX BEFORE CALCR
C  .EQ. 4 IMPLIES PARTICULAR SOLUTIONS FROM CALCSR**VOLUMINOUS**
C  .GT. 5 IMPLIES OUTPUT FROM EVAL OR DEVAL
C  QUIT IS A VARIABLE WHICH WILL TERMINATE THE PROGRAM
C  BEFORE THE OPTIMAL SOLUTION IS DETERMINED. THE REASON
C  FOR THIS IS TO BE ABLE TO RUN LESS THAN THE MINIMUM
C  TIME IF THE PROGRAM CONVERGES SLOWLY.
C  QUIT THEREFORE SPECIFIES THE NUMBER OF ITERATIONS
C  IF SPECIFIED ZERO, IT HAS NO AFFECT ON THE PROGRAM
C-----
C  INPUT
C-----
C 1  CONTINUE
C  ITER=1
C  NOSTT=0
C  ACC=1.0D-06
C  CHECK=1.0D-20
C  DET=1.0D0
C  RF=0.500
C  READ(INPT,600,END=520)LINEAR,STAT,ALPHA
C 600  FORMAT(215,G10.0)
C  READ(INPT,610)NDE,NBV,NTERMS,NEMAX,NEQ
C 610  FORMAT(515)
C  READ(INPT,620)(TBV(I),BV(I),IBV(I),IOBV(I),I=1,NBV)
C 620  FORMAT(2G10.0,10X,215)
C  READ(INPT,630)TO
C 630  FORMAT(G10.0)
C  READ(INPT,640)(YI(I),ICEX(I),LLIM(I),ULLIM(I),I=1,NDE)
C 640  FORMAT(G10.0,5X,15,5X,G10.0,5X,G10.0)
C  READ(INPT,650)PTBS,PTMIN,PTMAX
C 650  FORMAT(3G10.0)

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MAIN

FURTRAN IV G LEVEL 21

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0042 READ(INPT,660)ICUT,QUIT
0043 FORMAT(15,15)
C
C CONCLUDES INPUT
C-----
C OUTPUT OF THE INITIAL INPUT
C
0044 CONTINUE
0045 WRITE(IPRT,670)
0046 FORMAT(11,2(110(1H*))1,6X,32H)FOLLOWING IS THE OUTPUT OF INPUT)
0047 WRITE(IPRT,680)LINEAR,STAT,ALPHA
0048 FORMAT(1H0,5X,7H)LINEAR=,15,30X,5H)STAT=15,10X,6H)ALPHA=,G15.8,/,
C
0049 A10X,30H)LINEAR= ZERO IMPLIES NON-LINEAR,12X,
0050 B5H)STAT= ONE IMPLIES POST STATISTICAL STUDY TO BE EXECUTED,/,
0051 C10X,31H)LINEAR= NON-ZERO IMPLIES LINEAR,11X,
0052 D4H)STAT= ZERO IMPLIES NO POST STATISTICAL STUDY)
0053 WRITE(IPRT,690)ACC,CHECK,DET,RF
0054 FORMAT(1H0,5X,8H)ACCURACY,23X,4H)ACC=,G12.5,/,
0055 A6X,10H)FOR GJWALS,21X,6H)CHECK=,G12.5,5X,4H)DET=,G12.5,/,
0056 B6X,23H)FUDGE FACTOR FOR DIFF,8X,3H)RF=,G12.5)
C
0057 WRITE(IPRT,700)NUE,NBV,NTERMS,NEMAX,NEQ
0058 FORMAT(1H0,5X,25H)NUMBER OF DIFFERENTIAL EQ, 6X,4H)NDE=,3X,15,/,
0059 14X,29H)NUMBER OF BOUNDARY CONDITIONS,2X,4H)NBV=,3X,15,/,
0060 26X,29H)NUMBER OF TERMS IN PWR SERIES,2X,7H)NTERMS=,15,/,
0061 36X,29H)NO. OF NON-TRIVIAL DIFF EQ,5X,4H)NEC=,3X,15)
0062 WRITE(IPRT,710)IBV(J),IBV(J),IOBV(J),IBV(J),J=1,NBV)
0063 FORMAT(1H0,5X,15H)BOUNDARY VALUES,5X,15H)APPLIED ON,18V),6X,
0064 16H)18V,5X,10H)TIME OF BV,
0065 225(1,5X,G15.8,10X,15,10X,15,07X,G15.8)
0066 WRITE(IPRT,720)TO
0067 FORMAT(1H,5X,27H)INITIAL TIME OF INTEGRATION,6X,3H)TO=,G15.8,/,
0068 17,6X,20H)INITIAL GUESSES OF Y,20X,4H)ICEX,
0069 208X,1H)LOWER LIMIT,10X,11H)UPPER LIMIT)
0070 WRITE(IPRT,730)J,J,YI(J),ICEX(J),LLIM(J),ULIM(J),J=1,NDE)
0071 FORMAT(1H,10X,1HY,12,2X,3HY)1,12,2H)=,G15.8,5X,15,
0072 210X,G15.8,6X,(15.8)
0073 WRITE(IPRT,740)PTBS,PTMIN,PTMAX
0074 FORMAT(1H0,5X,19H)PERTURBATION SCALER,10X,5H)PTBS=,G15.8,/,
0075 A10X,21H)ABSOLUTE VALUE LIMITS,4X,6H)PTMIN=,G12.5,/,
0076 B35X,6H)PTMAX=,G12.5)
0077 WRITE(IPRT,750)ICUT,QUIT
0078 FORMAT(1H0,5X,5H)ICUT=,15,/,6X,5H)QUIT=,15)
0079 WRITE(IPRT,760)
0080 FORMAT(1H0,5X,36H)CONCLUDES FORMAL OUTPUT OF THE INPUT,
0081 A2(1,110(1H*))
C
C CONCLUDES THE OUTPUT OF THE INPUT
C-----
C THIS LOOP PROVIDES A SAFEGUARD ON LINEAR INDEPENDENCE OF CONSTRAINTS
C IF A NON-TIME DEPENDENT VARIABLE IS TO BE MET EXACTLY IN A
C BOUNDARY CONDITION, THIS CAUSES THE BOUNDARY CONDITION
C TO BECOME AN EXACT KNOWN INITIAL CONDITION AND THAT THE
C BOUNDARY CONDITION WOULD ALSO BE MET IN A LEAST SQUARE SENSE
C
0082 ID=0

```

0065

ID=0

MAIN1160
MAIN1170
MAIN1180
MAIN1190
MAIN1200
MAIN1210
MAIN1220
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MAIN1690
MAIN1700
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MAIN1730

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MAIN

FORTRAN IV G LEVEL 21

```

0067 IF(I8V(I).LE.NEQ.OR.I8BV(I).NE.O) GO TO 5
0068 NEMAX=NEMAX-1
0069 ID=I8V(I)
0070 YI(ID)=8V(I)
0071 I8BV(I)=1
0072 ICX(ID)=-1
0073 WRITE(IPT,770)ID,YI(ID),I,ICX(ID),I8BV(I),NEMAX
0074 770 FORMAT(1H0,5X,38HCORRECTION DUE TO EXACT BC ON CONSTANT,/
110X,11H AFFECTS YI,12,1H),5X,10HNOV EQUALS,2X,G15.8,/
210X,27HFROM BOUNDARY CONDITION NO.,15,/
326X,5HICEX=,15,/,26X,5HI8BV=,15,/,26X,6HNEMAX=,15,/,
46X,37H THEREFORE REPRINT THE CORRECTED INPUT,/,2(110(1H*))//)

0075 5 CONTINUE
0076 IF(ID.NE.O) GO TO 665
-----
0077 9876 CONTINUE
-----
C NCNT IS JUST A COUNTER
C NCNTEX IS NCNT FOR BOUNDARY CONDITIONS TO BE MET EXACTLY
C EXACT CONDITIONS GO IN FIRST ROWS OF S MATRIX
C NCNTLS IS NCNT FOR BOUNDARY CONDITIONS TO BE MET IN LEAST SQUARES
C SINCE THESE OCCUPY ROWS AFTER EXACT CONDITIONS IN S MATRIX
C NBVAT IS THE DESIGNATION OF THE BOUNDARY VALUE YOU ARE AT
C NROWA,NROU,NRS ARE ALL NUMBER OF ROWS IN APPROPRIATE MATRIX
C
0078 NCNT=1
0079 NCNTEX=1
0080 NCNTLS=NEMAX
0081 NBVAT=1
0082 NRS=NBV+1
0083 NROU=NBV+1
0084 NROWA=NBV+1
0085 TC=10
-----
C PLACES INITIAL GUESSES AND PERTURBED ONES IN THE P MATRIX
C LOOP TO SETS FIRST COLUMN OF P MATRIX EQUAL TO
C UNPERTURBED INITIAL GUESSES OF YI
C NP IS THE NUMBER OF PARTICULAR SOLUTIONS
C NP IS EQUAL TO THE NUMBER OF DIFFERENTIAL EQUATIONS (VARIABLES)
C PLUS ONE MINUS THE NUMBER OF EXACT KNOWN INITIAL CONDITIONS
C
0086 NP=1
0087 DO 10 I=1,NDE
0088 P(I,1)=YI(I)
C THE FOLLOWING IF STATEMENT ALLOWS FOR THE PERTURBATION
C SIZE TO DEFAULT TO ZERO IF THE VARIABLE HAS AN EXACT
C KNOWN INITIAL CONDITION
0089 IF(ICEX(I).LT.O) GO TO 8
0090 NP=NP+1
C THE PERTURBATION WILL BE (PTBS*100) OF THE INITIAL YI(I) GUESS
C THE ABSOLUTE VALUE OF PTB(I) WILL NOT BE LESS THAN PTMIN
C OR GREATER THAN PTMAX
0091 PTB(I)=PTBS*YI(I)
0092 IF(DABS(PTB(I)).LT.PTMIN) PTB(I)=DSIGN(PTMIN,PTB(I))

```


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MAIN

FORTRAN IV G LEVEL 21

```

0207      ZBAR=ZBAR/DFLOAT(NRZ)
0208      GO TO 230
0209      145 CONTINUE
C
C      ESTABLISHES VECTORS OF THE YI SOLUTION AND THEIR ASSOCIATED
C      PERTURBATIONS. ONLY THOSE NOT KNOWN EXACTLY ARE CONSIDERED
C      IF(STAT.EQ.0) GO TO 152
      II=0
      DO 150 I=1,NDE
        IF(ICEX(I).LT.0) GO TO 150
        II=II+1
        PB(II)=PTB(I)
        YI(II)=YI(I)
      CONTINUE
150 CONTINUE
152 CONTINUE
C-----
C      OUTPUT THE S MATRIX
      WRITE(IPRT,840)ITER
      840  FORMAT(1H1,5X,9HITERATION,15)
      WRITE(IPRT,850)
      850  FORMAT(1H0,5X,48HS MATRIX(CONTAINS UNPERTERBED AND PERTURBED SOL))
      JEND=NP+1
      DO 155 JB08=1,NRS
        155  WRITE(IPRT,860)(S(JB08,JBIG),JBIG=1,JEND)
        860  FORMAT(10X,9(1X,G12.5))
      ITER=ITER+1
C-----
C      GURSS-JORDAN REDUCTION WITH LEAST SQUARES
C
      CALL GJRWLS
      1(S,UU,NRS,NRDL,NEMAX,DET,CHECK,IOUT,NP,NROWA,IPUNT,IQUM,
      2STAT,M,OLS)
C
C      FOLLOWING GIVES DESIRED OUTPUT
C      C      OUTPUTS SUPERPOSITIONING CONSTANTS
C
      WRITE(IPRT,870)
      870  FORMAT(1H0,5X,23HSUPERPOSITION CONSTANTS)
      DO 160 NT=1,NP
        160  WRITE(IPRT,880) NT,S(NT,NX)
        880  FORMAT(1H ,10X,2HNT(,12,4H) = ,G15.8)
C
C      OUTPUT THE VALUES OF Y
C      C      ALSO DETERMINES THE NEW PERTURBED VALUES OF YI
      WRITE(IPRT,885)
      885  FORMAT(1H0,5X,33 SOLUTION FOR STATE VARIABLES)
      IQQ=1
      DO 165 IQ=1,NDE
        165  IQQ=IQQ+1
        YI(IQ)=YI(IQ)+S(IQ,NX)*PTB(IQ)
        WRITE(IPRT,890) IQ,YI(IQ)
        IF(ICEX(IQ).NE.0) GO TO 165
        IF(YI(IQ).GT.ULIM(IQ)) YI(IQ)=ULIM(IQ)
        IF(YI(IQ).LT.ULIM(IQ)) YI(IQ)=ULIM(IQ)
      CONTINUE

```

[illegible]

PAGE 0010

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MAIN

ORTRAN IV G LEVEL 21

0279 510 CONTINUE
0280 GO TO 1
0281 520 CONTINUE
0282 CALL EXIT
0283 END

MAIN5220
MAIN5230
MAIN5240
MAIN5250
MAIN5260

FORTRAN IV G LEVEL 21

MAIN

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OPTIONS IN EFFECT ID,ERCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = MAIN , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 283,PROGRAM SIZE = 16108
STATISTICS NO DIAGNOSTICS GENERATED

FORTRAN IV G LEVEL 21

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PAGE 0001

```

0001      BLOCK DATA
0002      C  BLOCK DATA ENABLES ALL DATA IN COMMON TO BE INITIALIZED
0003      IMPLICIT REAL*8(A-H,O-Z)
0004      REAL*8      YI(10),P(10,11),PC(20,10,11),TBV(25)
0005      REAL*8 W(25,10),ZHAT(25,1),ZEE(25,1),PB(10),YIR(10)
0006      REAL*8 ALS(10,1),DLS(25,1)
0007      COMMON/PYPS/YI,P,PC,TBV
0008      COMMON/REG/ZEE,ZHAT,W,PB,YIR,ALS,DLS,ZBAR
0009      DATA YI/10*0.000/
0010      DATA P/110*0.000/
0011      DATA PC/2200*0.000/
0012      DATA TBV/25*0.000/
0013      DATA ZEE/25*0.0000/
0014      DATA ZHAT/25*0.0000/
0015      DATA W/250*0.0000/
0016      DATA YIR/10*0.000/
0017      DATA PB/10*0.000/
0018      DATA ALS/10*0.000/
0019      DATA DLS/25*0.000/
0020      END
0021      MAIN5270
0022      MAIN5280
0023      MAIN5290
0024      MAIN5300
0025      MAIN5310
0026      MAIN5320
0027      MAIN5330
0028      MAIN5340
0029      MAIN5350
0030      MAIN5360
0031      MAIN5370
0032      MAIN5380
0033      MAIN5390
0034      MAIN5400
0035      MAIN5410
0036      MAIN5420
0037      MAIN5430
0038      MAIN5440
0039      MAIN5450
0040      MAIN5460

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FORTRAN IV G LEVEL 21

BLK DATA

DATE = 76097

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OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = BLK DATA, LINECNT = 60
STATISTICS NO DIAGNOSTICS GENERATED

FORTRAN IV G LEVEL 21 DTFOUND DATE = 76097 15/49/10 PAGE 0001

```

0001 SUBROUTINE DTIND(A,NEQ,NTERMS,RF,ACC,DI,DT)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(20,20)
-----
C DTIND DETERMINES THE LARGEST STEP THAT CAN BE TAKEN
C IN INTEGRATING A POWER SERIES
C A IS THE POWER SERIES IN QUESTION
C NEQ IS THE NUMBER OF NON-TRIVIAL DIFFERENTIAL EQ.
C NTERMS IS THE NUMBER OF TERMS IN THE POWER SERIES
C RF IS A FUDGE FACTOR, REDUCING THE STEP BY FACTOR OF RF
C ACC IS AN ACCURACY CHECK
C DI IS THE DESIRED STEP TO TAKE
C DT IS THE LARGEST STEP ALLOWED FOR THE GIVEN ACCURACY
-----
      PMAXM1=NTERMS-2
      N=NTERMS-1
      SPALL=1.0D-30
      VS=1.0D06
      DT=DABS(DI)
      DO 50 J=1,NEQ
C
C      THIS IF STATEMENT IS NEEDED TO AVOID DIVISION BY ZERO
C      THIS CAN ARISE WHEN DEALING WITH UNKNOWN CONSTANTS
C      THE REASON BEING THE FIRST TERM IS A CONSTANT, AND MAY BE ZERO,
C      THE REMAINDER OF THE TERMS ARE ZERO IN THE POWER SERIES
C
C      IF(A(N,J).EQ.0.0D0) GO TO 50
C
C      V=DABS(A(1,J)/A(N,J))
C      IF(V.LT.SMALL) V=SMALL
C      IF(VS.GT.V) VS=V
C      CONTINUE
50      OTT=DEXP(DLOG(ACC*VS)/PMAXM1)*RF
C      IF(DT.GT.OTT) DT=OTT
C      DT=DT*DI/DABS(DI)
C      RETURN
C      END

```

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CTFIND

FORTRAN IV G LEVEL 21

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = DTFIND * LINECNT = 60
STATISTICS SOURCE STATEMENTS = 19,PROGRAM SIZE = 890
STATISTICS NO DIAGNOSTICS GENERATED

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DATE = 76097

DEVAL

FORTAN IV G LEVEL 21

```

0001 SUBROUTINE DEVAL(A,SUM,NEQ,NTERMS,ACC,DT)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(20,20),SUM(20)
0004 COMMON/LOG/INPT,IPRT
-----
C DEVAL EVALUATES THE DERIVATIVE OF A POWER SERIES
C A IS THE VECTOR OF TERMS OF WHICH THE DERIVATIVE IS DESIRED.
C SUM IS THE VALUE OF THE DERIVATIVE
C NEQ IS THE NUMBER OF NON-TRIVIAL DIFFERENTIAL EQ.
C NTERMS IS THE NUMBER OF TERMS IN THE POWER SERIES
C ACC IS AN ACCURACY CHECK
C DT IS THE TIME INCREMENT FOR INTEGRATION FROM CENTER OF EXPANSION
-----
      DO 10 J=1,NEQ
        TPOW=1.D00
        PSA=A(2,J)
        ICHK=0
        DO 5 I=3,NTERMS
          FI=DFLOAT(I-1)
          TPOW=TPOW*FI
          TT=A(I,J)*TPOW*FI
          PSA=PSA+TT
        10 CONTINUE
      C THIS IF STATEMENT IS NEEDED TO AVOID DIVISION BY ZERO
      C WHERE THERE MAY BE ONLY ONE NON-ZERO TERM IN THE POWER SERIES
      C THIS CAN ARISE WHEN DEALING WITH UNKNOWN CONSTANTS
      IF(PSA.EQ.0.000) PSA=1.0D-70
      IF(DABS(TT/PSA).LT.ACC) ICHK=ICHK+1
      IF(ICHK.EQ.2) GO TO 10
      5 CONTINUE
      WRITE(IPRT,900) NEQ,J,ICHK,TT
      900 FORMAT(1X,10HNEQ,J,1CHK,3I4,3X,4HTT =,G15.8,/)
      10 SUM(J)=PSA
      RETURN
      END

```

DEVAL000
DEVAL010
DEVAL020
DEVAL030
DEVAL040
DEVAL050
DEVAL060
DEVAL070
DEVAL080
DEVAL090
DEVAL100
DEVAL110
DEVAL120
DEVAL130
DEVAL140
DEVAL150
DEVAL160
DEVAL170
DEVAL180
DEVAL190
DEVAL200
DEVAL210
DEVAL220
DEVAL230
DEVAL240
DEVAL250
DEVAL260
DEVAL270
DEVAL280
DEVAL290
DEVAL300
DEVAL310
DEVAL320
DEVAL330
DEVAL340
DEVAL350
DEVAL360

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FORTRAN IV G LEVEL 21

DEVAL

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OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = DEVAL , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 22,PROGRAM SIZE = 864
STATISTICS NO DIAGNOSTICS GENERATED

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DATE = 76097

EVAL

FORTRAN IV G LEVEL 21

```

0001 SUBROUTINE EVAL(A,SUM,NEQ,NTERMS,ACC,DT) EVAL0000
0002 IMPLICIT REAL*8(A-H,O-Z) EVAL0010
0003 REAL*8 A(20,20),SUM(20) EVAL0020
0004 COMMON/LOG/INPT,IPRT EVAL0030
0005 ----- EVAL0040
0006 C EVAL EVALUATES A POWER SERIES EVAL0050
0007 C A IS THE POWER SERIES TO BE INTEGRATED EVAL0060
0008 C SUM IS THE INTEGRATED VALUE OF THE POWER SERIES EVAL0070
0009 C NEQ IS THE NUMBER OF NON-TRIVIAL DIFFERENTIAL EQ. EVAL0080
0010 C NTERMS IS THE NUMBER OF TERMS IN THE POWER SERIES EVAL0090
0011 C ACC IS AN ACCURACY CHECK EVAL0100
0012 C DT IS THE TIME INCREMENT FOR INTEGRATION FROM CENTER OF EXPANSION EVAL0110
0013 ----- EVAL0120
0014 C GO TO J=1,NEQ EVAL0130
0015 C TPOW=1.D00 EVAL0140
0016 C PSA=A(1,J) EVAL0150
0017 C ICHK=0 EVAL0160
0018 C GO 5 I=2,NTERMS EVAL0170
0019 C TPOW=TPOW*DT EVAL0180
0020 C TT=A(I,J)*TPOW EVAL0190
0021 C PSA=PSA+TT EVAL0200
0022 C EVAL0210
0023 C EVAL0220
0024 C EVAL0230
0025 C EVAL0240
0026 C EVAL0250
0027 C EVALC260
0028 C EVAL0270
0029 C EVAL0280
0030 C EVAL0290
0031 C EVAL0300
0032 C EVAL0310
0033 C EVAL0320
0034 C EVAL0330
0035 C EVAL0340
0036 C EVAL0350

0005 C GO 10 J=1,NEQ
0006 C TPOW=1.D00
0007 C PSA=A(1,J)
0008 C ICHK=0
0009 C GO 5 I=2,NTERMS
0010 C TPOW=TPOW*DT
0011 C TT=A(I,J)*TPOW
0012 C PSA=PSA+TT

0013 C
0014 C THIS IF STATEMENT IS NEEDED TO AVOID DIVISION BY ZERO
0015 C THIS CAN ARISE WHEN DEALING WITH UNKNOWN CONSTANTS
0016 C WHERE THERE MAY BE ONLY ONE NON-ZERO TERM IN THE POWER SERIES
0017 C
0018 C IF(PSA.EQ.0.0L0) PSA=1.0D-78
0019 C
0020 C IF(DABS(TT/PSA).LT.ACC) ICHK=ICHK+1
0021 C IF(ICHK.EQ.2) GO TO 10
0022 C 5 CONTINUE
0023 C WRITE(IPRT,900) NEQ,J,ICHK,TT
0024 C 900 FORMAT(1X,10HNEQ,J,ICHK,314,3X,4HTT =,G15.8,/)
0025 C 10 SUM(J)=PSA
0026 C RETURN
0027 C END

```

FORTRAN IV G LEVEL 2:

EVAL

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OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 OPTIONS IN EFFECT NAME = EVAL , LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 21, PROGRAM SIZE = 804
 STATISTICS NO DIAGNOSTICS GENERATED

FORTRAN IV G LEVEL 21

GJRWLS

DATE = 76097

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OPTIONS IN EFFECT ID,E9CDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = GJRWLS * LINECNT = 60
STATISTICS SOURCE STATEMENTS = 92,PROGRAM SIZE = 3600
STATISTICS NO DIAGNOSTICS GENERATED

FORTRAN IV G LEVEL 21 LSTSQS DATE = 76097 15/49/10 PAGE 0001

```

0001 SUBROUTINE LSTSQS(A,B,NRA,NRB,K,J,LSTOP,NROW,N,NROWA,IGRAD)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(20,20),B(20,20)
0004 REAL A(NRA,1), B(NRB,1), HBIG( 270)
0005 INTEGER JZC(20)
0006 INTEGER J(1)
0007
0008 C-----
0009 C THIS ROUTINE WHEN THE SAVE MATRIX IS OVERDETERMINED
0010 C FORMS THE SQUARE PRODUCT OF THE SAVE MATRIX AND ITS
0011 C TRANSPOSE
0012 C-----
0013 KPI=K+1
0014 NPI=N+1
0015 DO 100 L=1,N
0016 JZC(L) = 0
0017
0018 BTB=0.00
0019 DO 110 M=KPI,NROWA
0020 BTB=BTB+A(M,NPI)**2
0021 IRA = 0
0022 IRA=IRA + 1
0023 IRB = 0
0024
0025 IRB=IRB + 1
0026
0027 IF(IRA.GT.JCA) GO TO 130
0028 TEMP=0.000
0029 DO 120 KR=KPI,NROWA
0030 TEMP=TEMP+A(KR,IRAT)*A(KR,JCA)
0031
0032 IF(IRA.LT.JCA.AND.JCA.LE.N) B(IRB,IRA) = TEMP
0033 B(IRA,IRB) = TEMP
0034
0035 IRB = 0
0036 IRB = IRB + 1
0037 JCB = 0
0038
0039 JCB = JCB + 1
0040 A(L,JCA) = B(IRB,JCB)
0041
0042 DO 150 M=KPI,NPI
0043 JCA=J(M)
0044
0045 DO 160 CONTINUE
0046 LSTOP=N
0047 NROW=N
0048 RETURN
0049 END
0050
0051 LSTSQ000
0052 LSTSQ010
0053 LSTSQ020
0054 LSTSQ030
0055 LSTSQ040
0056 LSTSQ050
0057 LSTSQ060
0058 LSTSQ070
0059 LSTSQ080
0060 LSTSQ090
0061 LSTSQ100
0062 LSTSQ110
0063 LSTSQ120
0064 LSTSQ130
0065 LSTSQ140
0066 LSTSQ150
0067 LSTSQ160
0068 LSTSQ170
0069 LSTSQ180
0070 LSTSQ190
0071 LSTSQ200
0072 LSTSQ210
0073 LSTSQ220
0074 LSTSQ230
0075 LSTSQ240
0076 LSTSQ250
0077 LSTSQ260
0078 LSTSQ270
0079 LSTSQ280
0080 LSTSQ290
0081 LSTSQ300
0082 LSTSQ310
0083 LSTSQ320
0084 LSTSQ330
0085 LSTSQ340
0086 LSTSQ350
0087 LSTSQ360
0088 LSTSQ370
0089 LSTSQ380
0090 LSTSQ390
0091 LSTSQ400
0092 LSTSQ410
0093 LSTSQ420
0094 LSTSQ430
0095 LSTSQ440
0096 LSTSQ450
0097 LSTSQ460
0098 LSTSQ470

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LSTSQS

FORTRAN IV G LEVEL 21

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NONAP
OPTIONS IN EFFECT NAME = LSTSQS , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 42,PROGRAM SIZE = 1506
STATISTICS NO DIAGNOSTICS GENERATED

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DATE = 76097

CALCSR

FORTRAN IV G LEVEL 21

```

0001 SUBROUTINE CALCSR(IQUT)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 YI(10),P(10,11),PC(20,10,11),TBV(25)
0004 COMMON/PYPS/YI,P,PC,TBV
0005 COMMON/PAARM/TO,TC,NDE,NP,NBV,NTERMS,LINER
0006 COMMON/LOG/INPI,IPRT
0007
0008
0009 IF(IQUT.NE.4) GO TO 100
0010 WRITE(IPRT,900)I
0011 FORMAT(1H1,11HFROM CALCSR,3X,15,3X,19HPARTICULAR SOL COEF)
0012 DO 25 J=1,NTERMS
0013 WRITE(IPRT,910)(PC(J,K,I),K=1,NDE)
0014 CONTINUE
0015 FORMAT(1H,5X,10G12.5)
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CALCSR

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OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP

OPTIONS IN EFFECT NAME = CALCSR * LINECNT = 60

STATISTICS SOURCE STATEMENTS = 18,PROGRAM SIZE = 672

STATISTICS NO DIAGNOSTICS GENERATED

CRTRAN IV G LEVEL 21 RECURL DATE = 76097 15/49/10 PAGE 0001

```

0001 SUBROUTINE RECURL(Y, YC, IND)
0002 IMPLICIT REAL*8(A-H, O-Z)
0003 REAL*8 Y(10), P(10, 11), PC(20, 10, 11), TBV(25)
0004 REAL*8 SN(20), CS(20), TT(20)
0005 REAL*8 Y(10), YC(20, 10)
0006 REAL*8 TEMP2(20)
0007 COMMON/PAARM/TO, TC, NDE, NP, NBV, NTERMS, LINEAR
0008 COMMON/PPPS/YI, P, PC, TBV
-----
C RECURL GOES THROUGH THE RECURSIVE RELATIONSHIPS FOR THE
C POWER SERIES AND SENDS IT BACK TO CALCSR
C Y IS AN INPUT MATRIX
C YC IS THE RESULTING MATRIX
C INC INDICATES FOR WHICH PARTICULAR SOLUTION
C FOR MASS-SPRING-DAMPER SYSTEM
-----
C
      NTERMS=NTERMS-1
      IF(IND.GT.1) GO TO 50
      DO 10 I=1, 20
        TT(I)=0.0DC
        CONTINUE
      10 TT(I)=YC
        TT(2)=1.0DC
        CALL TRIGF(TT, SN, CS, 20, 0, 1)
      50 DO 60 I=1, NDE
        YC(I, 1)=Y(I)
        CONTINUE
      60
-----
C DO 100 LOOP CHANGES FOR GIVEN PROGRAM
C THEREFORE MUST BE REPROGRAMED
C
      DO 100 I=1, NTERMS
        FI=CFLOAT(I)
        YC(I+1, 1)=YC(I, 2)/FI
        IF(IND.GT.1-A.D.LINEAR.EQ.0) GO TO 70
        YC(I+1, 2)=(SN(I)-Y(I, 4)*YC(I, 1)-Y(3)*YC(I, 2))/FI
        TEMP2(I)=SN(I)+YC(I, 1)*Y(4)+YC(I, 2)*Y(3)
        GO TO 100
      70 CONTINUE
        TEMP1=0.0DC-P(4, 1)*YC(I, 1)-P(3, 1)*YC(I, 2)-PC(I, 2, 1)*Y(3)
        A-PC(I, 1, 1)*Y(4)
        YC(I+1, 2)=(TEMP1+TEMP2(I))/FI
      100 CONTINUE
-----
C RETURN
C END

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RECRL090
RECRL010
RECRL020
RECRL030
RECRL040
RECRL050
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RECRL430
RECRL440
RECRL450

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RECURL

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OPTIONS IN EFFECT IO,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 OPTIONS IN EFFECT NAME = RECURL , LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 32,PROGRAM SIZE = 1512
 STATISTICS NO DIAGNOSTICS GENERATED

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TRIGF

FORTRAN IV G LEVEL 21

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0001 SUBROUTINE TRIGF(TH,SN,CS,INDX,IFLAG,ISTR) TRIGF000
0002 IMPLICIT REAL*8(A-H,O-Z) TRIGF010
0003 REAL*8 TH(20),SN(20),CS(20) TRIGF020
----- TRIGF030
C TH ARE VALUES OF THE COEFFICIENTS OF THE POWER SERIES TRIGF040
C FOR EITHER THETA OR THETA-DOT TRIGF050
C SN IS THE VALUE OF THE FIRST COEF IN SINE POWER SERIES SIN(THETA(0)) TRIGF060
C CS IS THE VALUE OF THE FIRST COEF IN COSINE POWER SERIES C/S(THETA(0)) TRIGF070
C INDX IS THE INDEX NUMBER OF THE POWER SERIES AND IS I+1 TRIGF080
C IFLAG INDICATES WHETHER INPUT IS THETA OR THETA DOT TRIGF090
C IFLAG=C IMPLIES THETA TRIGF100
C IFLAG=1 IMPLIES THETA DOT TRIGF110
C ISTR IS THE POINT IN THE SERIES AT WHICH YOU DESIRE TO START TRIGF120
C MINIMUM VALUE FOR ISTR IS 1 TRIGF130
----- TRIGF140
IF (IFLAG.EQ.1) GO TO 20 TRIGF150
IF (ISTR.GT.1) GO TO 5 TRIGF160
CS(1)=DCOS(TH(1)) TRIGF170
SN(1)=DSIN(TH(1)) TRIGF180
IF (INDX.LE.1) RETURN TRIGF190
JSTR=2 TRIGF200
IF (ISTR.GT.JSTR) JSTR=ISTR TRIGF210
C THETA INPUT CALCULATIONS TRIGF220
DO 15 J=JSTR,INDX TRIGF230
  SN(J)=0.000 TRIGF240
  CS(J)=0.000 TRIGF250
  I=J-1 TRIGF260
  DO 10 KJ=1,I TRIGF270
    JSS=J-KJ TRIGF280
    CS(J)=CS(J)+DFLOAT(KJ)*SN(JSS)*TH(KJ+1) TRIGF290
    SN(J)=SN(J)+DFLOAT(KJ)*CS(JSS)*TH(KJ+1) TRIGF300
  CS(J)=CS(J)/DFLOAT(J-1) TRIGF310
  SN(J)=SN(J)/DFLOAT(J-1) TRIGF320
GO TO 35 TRIGF330
C THETA DOT INPUT CALCULATIONS TRIGF340
CONTINUE TRIGF350
DO 30 J=ISTR,INDX TRIGF360
  SN(J)=0.000 TRIGF370
  CS(J)=0.000 TRIGF380
  I=J-1 TRIGF390
  DO 25 KJ=1,I TRIGF400
    JSS=J-KJ TRIGF410
    CS(J)=CS(J)+SN(JSS)*TH(KJ) TRIGF420
    SN(J)=SN(J)+CS(JSS)*TH(KJ) TRIGF430
  CS(J)=CS(J)/DFLOAT(J-1) TRIGF440
  SN(J)=SN(J)/DFLOAT(J-1) TRIGF450
GO TO 35 TRIGF460
C CONTINUE TRIGF470
RETURN TRIGF480
END TRIGF490
      TRIGF500
      TRIGF510

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TRIGF

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OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = TRIGF , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 35,PROGRAM SIZE = 1286
STATISTICS NO DIAGNOSTICS GENERATED

[illegible]

FORTRAN IV G LEVEL 21

PSTAT

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OPTIONS IN EFFECT ID,EBDCIC,SOURCE,NOLIST,NGDECK,LOAD,NOMAP
 OPTIONS IN EFFECT NAME = PSTAT * LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 173, PROGRAM SIZE = 12044
 STATISTICS NO DIAGNOSTICS GENERATED

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DMATA

FORTRAN IV G LEVEL 21

```

0001 SUBROUTINE DMATA(A,R,N,M,MS)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(1),R(1)
-----
C DMATA PREMULTIPLIES A MATRIX BY ITS TRANSPOSE
C TO FORM A RESULTANT MATRIX
C
C A IS THE INPUT MATRIX
C R IS THE OUTPUT MATRIX
C N IS THE NUMBER OF ROWS IN A
C M IS THE NUMBER OF COLUMNS IN A. ALSO THE NUMBER
C OF ROWS AND COLUMNS OF R
C MS IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF MATRIX A
C 0-GENERAL 1-SYMETRICAL 2-DIAGONAL
C-----
      DO 60 K=1,M
      KX=(K*K-K)/2
      DO 60 J=1,M
      IF(J-K) 10,10,60
      10 IR=J+KX
      R(IR)=0.000
      DO 60 I=1,N
      IF(MS) 20,40,20
      20 CALL LOC(I,J,IA,N,M,MS)
      CALL LOC(I,K,IB,N,M,MS)
      IF(IA) 30,60,30
      30 IF(IB) 50,60,50
      40 IA=N*(J-1)+I
      IB=N*(K-1)+I
      50 R(IR)=R(IR)+A(IA)*A(IB)
      60 CONTINUE
      RETURN
      END

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DMATA

GRTRAN IV G LEVEL 21

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = DMATA , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 21,PROGRAM SIZE = 840
STATISTICS NO DIAGNOSTICS GENERATED

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DMCPY

FCRTRAN IV G LEVEL 21

```

0001 SUBROUTINE DMCPY(A,R,N,M,MS)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(1),R(1)
-----
C DMCPY COPIES AN ENTIRE MATRIX (USED BY DMTRA)
C
C A IS THE INPUT MATRIX
C R IS THE OUTPUT MATRIX
C N IS THE NUMBER OF ROWS IN A AND R
C M IS THE NUMBER OF COLUMNS IN A AND R
C MS IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF A(AND R)
C 0-GENERAL 1-SYMETRICAL 2-DIAGONAL
-----
C COMPUTE VECTOR LENGTH,IT
CALL LOC(N,M,IT,N,M,MS)
C COPY MATRIX
DO 1 I=1,IT
R(I)=A(I)
RETURN
END
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DMCPY000
DMCPY010
DMCPY020
DMCPY030
DMCPY040
DMCPY050
DMCPY060
DMCPY070
DMCPY080
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DMCPY

FORTRAN IV G LEVEL 21

OPTIONS .N EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = DMCPY , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 8,PROGRAM SIZE = 478
STATISTICS NO DIAGNOSTICS GENERATED

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DMPRO

GRTRAN IV G LEVEL 21

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0001 SUBROUTINE DMPRO(A,B,R,N,M,MSA,MSB,L)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(1),B(1),R(1)
C-----
C DMPRO MULTIPLIES TO MATRICES TO FORM A RESULTANT MATRIX
C
C IS THE FIRST INPUT MATRIX
C B IS THE SECOND INPUT MATRIX
C R IS THE OUTPUT MATRIX
C N IS THE NUMBER OF ROWS IN A AND R
C M IS THE NUMBER OF COLUMNS IN A AND ROWS IN B
C MSA IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF A
C MSB IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF B
C 0-GENERAL 1-SYMETRICAL 2-DIAGONAL
C 1 IS THE NUMBER OF COLUMNS IN B AND R
C-----
C SPECIAL CASE FOR DIAGONAL BY DIAGONAL
C MS=MSA*10+MSB
C IF(MS-22) 30,10,30
C 10 DO 20 I=1,N
C 20 R(I)=A(I)*B(I)
C RETURN
C ALL OTHER CASES
C 30 IR=1
C 40 DO 90 K=1,L
C 50 DO 90 J=1,N
C 60 R(J,R)=0.0D0
C 70 DO 80 I=1,M
C 80 IF(MS) 40,60,40
C 90 CALL LOC(J,I,IA,N,M,MSA)
C 100 CALL LOC(I,K,IB,M,L,MSB)
C 110 IF(IA) 50,80,50
C 120 IF(IB) 70,80,70
C 130 IA=N*(I-1)+J
C 140 IB=M*(K-1)+I
C 150 R(J,R)=R(J,R)+A(IA)*B(IB)
C 160 GO TO 110
C 170 RETURN
C 180 END

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DMPRO

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OPTIONS IN EFFECT* ID,EBDCIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT* NAME = DMPRO , LINECNT = 60
STATISTICS* SOURCE STATEMENTS = 25,PROGRAM SIZE = 1010
STATISTICS* NO DIAGNOSTICS GENERATED

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DMIRA

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SUBROUTINE DMTRA(A,R,N,M,MS)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8  A(1),R(1)
-----
C  DMTRA DETERMINES THE TRANSPOSE OF A MATRIX
C
C  A IS THE MATRIX TO BE TRANSPOSED
C  R IS THE TRANSPOSED MATRIX
C  N IS THE NUMBER OF ROWS OF A AND COLUMNS OF R
C  M IS THE NUMBER OF COLUMNS OF A AND ROWS OF R
C  MS IS ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A(AND R)
C  0-GENERAL 1-SYMMETRICAL 2-DIAGONAL
C-----
C      IF MS IS 1 OR 2, COPY A
      IF(MS) 10,20,10
      10 CALL DMCPY(A,R,N,N,MS)
      RETURN
C      TRANSPOSE GENERAL MATRIX
      20 IR=0
      DO 30 I=1,N
      IJ=I-N
      DO 30 J=1,M
      IJ=IJ+N
      IR=IR+1
      30 A(IR)=A(IJ)
      RETURN
      END

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DMTRA

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OPTIONS IN EFFECT IO,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP

OPTIONS IN EFFECT NAME = DMTRA , LINECNT = 60

STATISTICS SOURCE STATEMENTS = 15,PRJGRAM SIZE = 618

STATISTICS NO DIAGNOSTICS GENERATED

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USMPY

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0001 SUBROUTINE DSMPY(A,C,R,N,M,MS,IT)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 REAL*8 A(1),R(1)
C-----
C DSMPY MULTIPLIES EACH ELEMENT OF A MATRIX BY A SCALAR TO
C FORM A RESULTANT MATRIX
C
C A IS THE INPUT MATRIX
C C IS THE SCALAR
C R IS THE OUTPUT MATRIX
C N IS THE NUMBER OF ROWS IN MATRIX A AND R
C M IS THE NUMBER OF COLUMNS IN A AND R
C MS IS A ONE DIGIT NUMBER FOR STORAGE MODE OF A(AND R)
C 0-GENERAL 1-SYMETRICAL 2-DIAGONAL
C IT IS THE TOTAL LENGTH OF VECTOR A AND R
C-----
C MULTIPLY BY SCALAR
C DO 1 I=1,IT
C 1 R(I)=A(I)*C
C RETURN
C END
DSMPY000
DSMPY010
DSMPY020
DSMPY030
DSMPY040
DSMPY050
DSMPY060
DSMPY070
DSMPY080
DSMPY090
DSMPY100
DSMPY110
DSMPY120
DSMPY130
DSMPY140
DSMPY150
DSMPY160
DSMPY170
DSMPY180
DSMPY190
DSMPY200

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DSMPY

FORTRAN IV G LEVEL 21

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOWAP
OPTIONS IN EFFECT NAME = DSMPY , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 7, PROGRAM SIZE = 492
STATISTICS NO DIAGNOSTICS GENERATED

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DATE = 76097

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SUBROUTINE DTPRD(A,B,R,N,M,MSA,MSB,L)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 A(1),B(1),R(1)
-----
C
C DTPRD TRANSPOSES A MATRIX AND POSTMULTIPLIES BY
C ANOTHER TO FORM A RESULTANT MATRIX
C
C A IS THE FIRST INPUT MATRIX(TO BE TRANSPOSED)
C B IS THE SECOND INPUT MATRIX
C R IS THE OUTPUT MATRIX
C N IS THE NUMBER OF ROWS IN A AND B
C M IS THE NUMBER OF COLUMNS IN A AND ROWS IN R
C MSA IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF MATRIX A
C MSB IS A ONE DIGIT NUMBER FOR THE STORAGE MODE OF MATRIX B
C 0-GENERAL 1-SYMETRICAL 2-DIAGONAL
C L IS THE NUMBER OF COLUMNS IN B AND R
-----
C
C SPECIAL CASE FOR DIAGONAL BY DIAGONAL
MS=MSA*10+MSB
IF(MS-22) 30,10,30
10  DO 20 I=1,N
20  R(I)=A(I)*B(I)
RETURN
C
C MULTIPLY TRANSPOSE OF A BY B
IR=1
30  DO 90 K=1,L
DO 90 J=1,M
R(IR)=0.000
DO 80 I=1,N
IF(MS) 40,60,40
CALL LOC(I,J,IA,N,M,MSA)
CALL LOC(I,K,IB,N,L,MSB)
IF(IA) 50,80,50
IF(IB) 70,80,70
IA=N*(J-1)+1
IB=N*(K-1)+1
R(IR)=R(IR)+A(IA)*B(IB)
80  CONTINUE
IR=IR+1
90  RETURN
END
DTPRD000
DTPRD010
DTPRD020
DTPRD030
DTPRD040
DTPRD050
DTPRD060
DTPRD070
DTPRD080
DTPRD090
DTPRD100
DTPRD110
DTPRD120
DTPRD130
DTPRD140
DTPRD150
DTPRD160
DTPRD170
DTPRD180
DTPRD190
DTPRD200
DTPRD210
DTPRD220
DTPRD230
DTPRD240
DTPRD250
DTPRD260
DTPRD270
DTPRD280
DTPRD290
DTPRD300
DTPRD310
DTPRD320
DTPRD330
DTPRD340
DTPRD350
DTPRD360
DTPRD370
DTPRD380
DTPRD390
DTPRD400

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DTPRO

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
OPTIONS IN EFFECT NAME = DTPRO , LINECNT = 60
STATISTICS SOURCE STATEMENTS = 25,PROGRAM SIZE = 1010
STATISTICS NO DIAGNOSTICS GENERATED
STATISTICS NO DIAGNOSTICS THIS STEP

VS LOADER

PTIONS USED - PRINT,MAP,NOLET,CALL,RES,NOTERM,SIZE=159744,NAME=**GO

NAME	TYPE	ADDR	NAME	TYPE	ADDR	NAME	TYPE	ADDR	NAME	TYPE	ADDR	NAME	TYPE	ADDR		
MAIN	SD	4E1010	PYPS	SD	4E4F00	REC	SD	4E9848	OTFIND	SD	4EA368	DEVAL	SD	4EA6E8		
EVAL	SD	4EAA48	GJRWLS	SD	4EAD70	LSTSQS	SD	4EBB80	CALCSR	SD	4EC168	RECURL	SD	4EC408		
TRIGF	SD	4ECA28	PSTAT	SD	4ECF30	DMATA	SD	4EFE40	DMCPY	SD	4F0188	DMPRD	SD	4F0368		
DMTRA	SD	4F0760	DSMPY	SD	4F09D0	DTPRD	SD	4F0BC0	LOC	**	SD	EC5934	IHCCECMH*	SD	4F0F88	
I8COM	*	LR	FOIOCS	*	LR	4F1074	INTSWICH*	LR	4F1EFE	IHCCEMH2*	SD	4F1F20	SEQDASD *	LR	4F2298	
IHCCEXIT*	SD	4F2540	EXIT	*	LR	4F2580	IHCLEXP *	SD	4F25A0	OEXP	*	LR	4F25A0	SD	4F2828	
DLOGLO *	LR	4F2828	DLOG	*	LR	4F2840	IHCLSCN *	SD	4F2A28	DCOS	*	LR	4F2A28	SD	4F2A42	
MOFI *	SD	4F2C98	MDSTI	*	SD	4F2E90	MOFI	SD	4F33F8	LINVIF *	SD	4F3D58	IHCLSQRT*	SD	4F3FF0	
USQRT *	LR	4F3F40	IHCFCVTH*	SD	4F4150	ADCON *	LR	4F4150	FCVACUTP*	LR	4F41FA	FCVLOUTP*	LR	4F428A		
FCVZOUTP*	LR	4F43E2	FCVIOUPT*	LR	4F4796	FCVEOUTP*	LR	4F4C98	FCVCCUTP*	LR	4F4EB2	INT6SMCH*	LR	4F519B		
IHCLEFIOS*	SD	4F4308	FIUCS	*	LR	4F5308	FIUCS8EP*	LR	4F530E	IHCLEFIOS2*	SD	4F6230	IHCLEFNTH*	SD	4F6760	
ARITH *	LR	4F6760	ADJSWICH*	*	LR	4F6AFC	IHCUIPT *	SD	4F6CA8	IHCERRM *	SD	4F6FA8	ERRMON *	LR	4F6FA8	
IHCERRE *	LR	4F6FC0	MOBETI *	SD	4F7588	UERTST *	SD	4F7818	IHCSEXP *	SD	4F7A90	EXP	*	LR	4F7A90	
IHCERXPR*	SD	4F7C28	FRXPR *	LR	4F7C28	IHCSTNGT*	SD	4F7D30	COTAN	*	LR	4F70B0	TAN	*	LR	4F70C6
UTAN *	LR	4F7F38	IHCSSQRT*	SD	4F8018	SQRT *	LR	4F8018	PERFI *	SD	4F8160	MONRIS	*	LR	4F8174	
PERFCI *	LR	4F818A	IHCSTATN2*	SD	4F8470	ATAN2 *	LR	4F8470	ATAN	*	LR	4F8484	IHCSERF *	SD	4F8640	
ERFC *	LR	4F8640	ERF *	LR	4F8656	IHCLOG *	SD	4F8830	ALOGLO *	LR	4F8830	ALOG	*	LR	4F8848	
LEQTF *	SD	4F89E8	IHCUIATBL*	SD	4F8C90	IHCETRCH*	SD	4F92C8	IHCIRCH *	LR	4F92C8	ERKTRA *	*	LR	4F92D0	
MODETA *	SD	4F9558	LUDATF *	SD	4F9C80	LUELME *	SD	4FA6A0	IHCLGAMA*	SD	4FAA58	DLGAMA	*	LR	4FAA58	
LGAMMA *	LR	4FAA74	IHCFOXPI*	SD	4FAE48	FOXPI *	LR	4FAE48	LOG	CM	4FAF98	PARM	CM	4FAFA0		

TOTAL LENGTH 19FB4
ENTRY ADDRESS 4E1010

ITERATION 1

S MATRIX CONTAINS UNPERTURBED AND PERTURBED SOL

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
-0.64228	-0.66568	-0.74448	-0.64035	-0.72161	-0.22000
-0.49241	-0.45223	-0.56472	-0.48344	-0.42572	0.350000-01
-0.51667	-0.46350	-0.48175	-0.51154	-0.33497	-0.47400
-0.687260-01	-0.627610-01	0.244950-01	-0.741490-01	0.677760-01	-0.58900
1.0073	0.96399	1.0500	0.99151	0.95145	0.39300
1.6095	1.5696	1.5591	1.5937	1.3335	1.5970
0.58079	0.58825	0.50209	0.58413	0.25038	1.4520
-1.5115	-1.4695	-1.5285	-1.4801	-1.5760	-0.38800
-2.4989	-2.4722	-2.4413	-2.4623	-2.0932	-2.3240
-0.93638	-0.95315	-0.87507	-0.93819	-0.30496	-2.2740
1.9457	1.9082	1.9424	1.8904	2.1686	0.880000-01
3.1756	3.1609	3.1175	3.1139	2.5807	2.7110
1.1797	1.2019	1.1364	1.1841	0.18644	2.9970
-2.2683	-2.2373	-2.2501	-2.1834	-2.6331	0.40100
-3.6462	-3.6417	-3.5926	-3.5601	-2.8052	-2.8160

SUPERPOSITION CONSTANTS

A(1) = -23.496468
 A(2) = 4.5826473
 A(3) = -3.2130453
 A(4) = 24.602487
 A(5) = -1.4756211

SOLUTION FOR STATE VARIABLES

TRUE IC OF Y(1) IS 0.72913236
 TRUE IC OF Y(2) IS 0.67869547
 TRUE IC OF Y(3) IS 0.34602487
 TRUE IC OF Y(4) IS 1.1081693

ITERATION 2

S MATRIX/CONTAINS UNPERTURBED AND PERTURBED SOL)

PRICE CONTAINS UNPERTURBED AND PERTURBED SOL.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
-0.28400	-0.30875	-0.34368	-0.27680	-0.32611	-0.22000	-0.22000
-0.10934	-0.733010-01	-0.14369	-0.10041	-0.60240D-01	-0.35000D-01	-0.35000D-01
-0.52390	-0.47562	-0.51101	-0.51597	-0.43741	-0.47400	-0.47400
-0.44776	-0.43203	-0.41245	-0.44100	-0.36722	-0.58900	-0.58900
0.53576	0.51504	0.55679	0.52034	0.57264	0.39300	0.39300
1.4436	1.4148	1.4366	1.3975	1.3700	1.5970	1.5970
1.0295	1.0196	1.0387	0.99392	0.82706	1.4520	1.4520
-0.62154	-0.60964	-0.63440	-0.58872	-0.81733	-0.38800	-0.38800
-1.9529	-1.9357	-1.9491	-1.8563	-1.9219	-2.3240	-2.3240
-1.4983	-1.4921	-1.4860	-1.4271	-1.1795	-2.2740	-2.2740
0.50384	0.49703	0.51169	0.45960	0.86618	0.88000D-01	0.88000D-01
2.1947	2.1844	2.1927	2.0530	2.7110	2.9970	2.9970
1.8772	1.8732	1.8699	1.7699	1.4797	2.9970	2.9970
-0.26560	-0.26170	-0.27038	-0.22084	-0.77395	0.40100	0.40100
-2.2557	-2.2456	-2.2547	-2.0810	-2.3812	-2.8160	-2.8160

SUPERPOSITION CONSTANTS

POSITION
A(1) = 3.8263155
A(2) = 5.2942813
A(3) = -2.1706495
A(4) = -4.3846907
A(5) = -1.5652568

SOLUTION FOR STATE VARIABLES

ION FOR STATE VARIABLES	
TRUE IC QF Y (1) IS	1.1151556
TRUE IC QF Y (2) IS	0.53137447
TRUE IC QF Y (3) IS	0.19430367
TRUE IC QF Y (4) IS	0.93471231

ITERATION 3

S MATRIX(CONTAINS UNPERTURBED AND PERTURBED SOL)

1.0000	1.0000	1.0000	1.0000
-0.26299	-0.30925	-0.30601	-0.25780
0.158210-01	0.536290-01	-0.203290-01	0.190820-01
-0.46096	-0.38362	-0.46306	-0.45911
-0.58687	-0.53773	-0.55929	-0.58120
0.36125	0.34858	0.39161	0.35862
1.5921	1.5385	1.6009	1.5649
1.5573	1.5121	1.5414	1.5201
-0.20990	-0.21269	-0.23361	-0.21022
-2.2593	-2.2249	-2.2708	-2.1949
-2.4772	-2.4392	-2.4696	-2.3899
-0.28135	-0.27024	-0.26402	-0.26147
2.5169	2.4971	2.5286	2.4212
3.2457	3.2161	3.2437	3.0989
0.96133	0.94683	0.94951	0.90198
-2.4385	-2.4292	-2.4491	-2.3286
		1.0000	1.0000
		-0.31348	-0.22000
		0.736830-01	0.350000-01
		-0.35464	-0.47400
		-0.48996	-0.50700
		0.41187	0.39300
		1.5326	1.5970
		1.3369	1.4520
		-0.49802	-0.38800
		-2.3554	-2.3240
		-2.1736	-2.2740
		0.28847	0.880000-01
		2.8723	2.7110
		2.9600	2.9970
		0.14009	0.40100
		-3.1189	-2.8160

SUPERPOSITION CONSTANTS

A(1) = 1.3517611
 A(2) = -1.1611008
 A(3) = -0.48508048
 A(4) = 0.63423510
 A(5) = 0.66018502

SOLUTION FOR STATE VARIABLES

TRUE IC OF Y(1) IS 0.98567475
 TRUE IC OF Y(2) IS 0.50559853
 TRUE IC OF Y(3) IS 0.20662709
 TRUE IC OF Y(4) IS 0.99642062

ITERATION 4

S MATRIX (CONTAINS UNPERTURBED AND PERTURBED SOL)

1.0000	1.0000	1.0000	1.0000	1.0000
-0.21557	-0.25624	-0.25822	-0.21059	-0.22000
0.23854D-01	0.63661D-01	-0.96796D-02	0.26537D-01	0.81132D-01
-0.48682	-0.41452	-0.48517	-0.48462	-0.38917
-0.59250	-0.55364	-0.56361	-0.58564	-0.50506
0.39681	0.37631	0.42395	0.39377	0.44332
1.5982	1.5463	1.6014	1.5686	1.5363
1.4510	1.4166	1.4321	1.4140	1.2293
-0.38023	-0.37196	-0.40146	-0.37416	-0.65396
-2.3051	-2.2690	-2.3107	-2.2337	-2.3590
-2.2628	-2.2339	-2.2510	-2.1797	-1.9157
0.70663D-01	0.69730D-01	0.86804D-01	0.71959D-01	0.62427
2.6704	2.6460	2.6767	2.5579	2.9215
2.9700	2.9468	2.9631	2.8314	2.5612
0.42242	0.41932	0.41048	0.39875	-0.40697
-2.7546	-2.7387	-2.7608	-2.6115	-3.2646

SUPERPOSITION CONSTANTS

A(1) = 1.2542553
 A(2) = 0.14870376
 A(3) = -0.11856874
 A(4) = -0.32043791
 A(5) = 0.36047558D-01

SOLUTION FOR STATE VARIABLES

TRUE IC OF Y(1) IS 1.0003321
 TRUE IC OF Y(2) IS 0.49960372
 TRUE IC OF Y(3) IS 0.20000597
 TRUE IC OF Y(4) IS 1.00000125

ITERATION 5

S MATRIX (CONTAINS UNPERTURBED AND PERTURBED SOL)

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
-0.21984	-0.26149	-0.26211	-0.21482	-0.26524	-0.22000
0.350670-01	0.759210-01	0.159700-02	0.375470-01	0.947620-01	0.350000-01
-0.47327	-0.39891	-0.47164	-0.47161	-0.37377	-0.47400
-0.58918	-0.54936	-0.56018	-0.58280	-0.50300	-0.58900
0.39301	0.37137	0.42026	0.39032	0.43622	0.39300
1.5971	1.5432	1.6002	1.5689	1.5326	1.5970
1.4520	1.4166	1.4327	1.4163	1.2294	1.4520
-0.38817	-0.37892	-0.40968	-0.38227	-0.66096	-0.38800
-2.3238	-2.2857	-2.3293	-2.2543	-2.3740	-2.3240
-2.2739	-2.2440	-2.2617	-2.1931	-1.9200	-2.2740
0.682370-01	0.865810-01	0.10476	0.888820-01	0.64776	0.880000-01
2.7113	2.6851	2.7175	2.6004	2.9593	2.7110
2.9971	2.9727	2.9897	2.8612	2.5730	2.9970
0.40068	0.39803	0.38830	0.37894	-0.44600	0.40100
-2.8164	-2.7990	-2.8225	-2.6736	-3.3272	-2.8160

SUPERPOSITION CONSTANTS

A(1) = 0.99774769
 A(2) = -0.279602350-02
 A(3) = 0.359004120-02
 A(4) = 0.130394710-02
 A(5) = -0.451653600-03

SOLUTION FOR STATE VARIABLES

TRUE IC OF Y(1) IS 1.0000524
 TRUE IC OF Y(2) IS 0.49978308
 TRUE IC OF Y(3) IS 0.20004417
 TRUE IC OF Y(4) IS 0.99996731

ITERATION 6

S MATRIX (CONTAINS UNPERTURBED AND PERTURBED SOL)

1.0000	1.0000	1.0000	1.0000	1.0000
-0.21984	-0.26213	-0.21483	-0.26525	-0.22000
0.348100-01	0.756480-01	0.133020-02	0.372950-01	0.944670-01
-0.47351	-0.39518	-0.47189	-0.37402	-0.47400
-0.58922	-0.54940	-0.56021	-0.58283	-0.58900
0.39314	0.37152	0.42040	0.39044	0.39300
1.5972	1.5433	1.6004	1.5328	1.5970
1.4521	1.4166	1.4328	1.2294	1.4520
-0.38813	-0.37890	-0.40965	-0.38224	-0.38800
-2.3238	-2.2057	-2.3292	-2.2543	-2.3240
-2.2740	-2.2440	-2.2617	-2.1931	-2.2740
0.880500-01	0.864020-01	0.10458	0.886990-01	0.880000-01
2.7110	2.6849	2.7173	2.6001	2.7110
2.9971	2.9726	2.9997	2.8611	2.9970
0.40098	0.39833	0.38860	0.37923	0.40100
-2.8159	-2.7985	-2.8221	-2.6732	-2.8160

SUPERPOSITION CONSTANTS

A(1) = 0.99999570
 A(2) = -0.431698820-06
 A(3) = 0.722168560-06
 A(4) = 0.720612370-08
 A(5) = -0.1648.8120-08

SOLUTION FOR STATE VARIABLES

TRUE IC OF Y(1) IS 1.0000524
 TRUE IC OF Y(2) IS 0.49978311
 TRUE IC OF Y(3) IS 0.20004417
 TRUE IC OF Y(4) IS 0.599996731

ITERATION 6 CONVERGED TO THE ACCURACY SPECIFIED FOR THE SOLUTION

BOX	JOB NAME	COMMENTS	DATE	JOB NUMBER
RRA	HUNTER	HUNTER99999999999999		3388
***** I T E M I Z E D J O B C O S T *****				
I T E M	QUANTITY	COST	PREVIOUS BALANCE	
CPU	16.62 SECONDS	5.98		105.30
CARD INPUT	1424 CARDS	0.00		
PRINTED OUTPUT	31 PAGES	0.00		

			TOTAL CHARGES	5.98
			CURRENT BALANCE	99.32

[illegible][illegible]