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HEAT SINKS
A Study of Variable Types of Heat Sinks

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HANOVER, NEW HAMPSHIRE

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Twelve various types of heat sinks are included in this investigation, which is stressing a comparison between the profiles rather than a comparison between manufactures. It has been examined as to how much power can be added to the heat sink at intermittent service without influencing the junction temperature of the power component to exceed a tolerated maximum temperature.

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HEAT SINKS: A STUDY OF VARIABLE TYPES OF HEAT SINKS. EFFECTS OF VARIOUS FACTORS ON TEMPERATURE PROFILES

U. Fabricius

SUMMARY

Twelve different types of heat sink are included in this investigation, which stresses comparison between the profiles rather than comparison between products of different manufacturers.

The test plan includes, for instance, measurements of temperature rise depending on load. In connection with this measurement the importance of the heat sink's having a black anodized finish rather than a bright surface was examined. In addition, the temperature distribution of the heat sink was studied.

Besides the practical measurements, some theoretical studies were carried out, and some hypotheses put forward whose correctness was verified by tests and measurements. Among other things, it was found that cooling of the heat sink by convection is strongly dependent on the space between the fins.

Computer calculations of the thermal resistance and the temperature gradient of the heat sink were carried out. Those calculations were used to find out which proportion between height of profile and height of fin gives the lowest thermal resistance compared to the space taken up by the profile.

A simple method of calculation is indicated to find out the thermal resistance of any cooling profile.

Finally, a study was done of how much power can be added to the heat sink in intermittent service without influencing the junction temperature of the power component to exceed a maximum permissible temperature. It appears that in certain cases the influence tolerated is far above the permissible influence according to the data sheet.

This study was carried out with the support of the Technology Council and the Society for Testing and Environmental Engineering (Sammenslutningen for Pålideligheds- og Miljøteknik).

1. INTRODUCTION

Thermal problems with electronic equipment rarely attract attention until too late in the development phase. Frequently no attempt is made to solve the problems until the prototype stage, with the result that to solve the thermal problems at all it is necessary to adopt a considerably more costly solution.

We may have to be content, willingly or unwillingly, if the equipment works. But many internal and external conditions may cause it not to work satisfactorily or adequately; in stowing the equipment away it is necessary to pay close attention to the maximum ambient temperature and exposure to solar radiation, and blocking of radiation by boxing the apparatus in.

A poor design can reduce the lifetime, lower the long-term stability, and decrease the efficiency of the equipment. At the worst it may result in gross defects, actually dangerous products, and costly or even fatal damage.

It may be stated as a rule of thumb that the lifetime of a transistor, for example, is halved by a temperature rise of about 15°C.

The problem of dimensioning and choice of the right cooling disk is a part of the complex system that is called thermal design.

What will the cooling plate look like that exactly satisfies the requirements of the job in hand? How big will it be? Shall it be bulky or slender? High or low? Broad or narrow? Black or shiny?

If the model is in the prototype stage, the possibility of choice is often greatly restricted, whereas there is considerably more freedom if the choice is made at an earlier stage of development.

This report is intended to cover certain theoretical considerations and practical experiments which will give the designer a better chance to choose the right cooling plate for a given purpose.

2. THE TYPES INVESTIGATED

The investigation includes ten different types made by two manufacturers and two types of unknown make. Nine of the types are anodized black, one is shiny, and two types are represented by both a black anodized and a shiny type.

In the selection of types for study, the primary emphasis was placed on studying the various existing designs within a certain power range, rather than on making a comparative study of brands.

The types studied, generally speaking, cover the range within the field of cooling plates which is used for cooling low-power components which give off up to about 100 watts as heat, e.g. diodes, transistors, thyristors, and triacs.

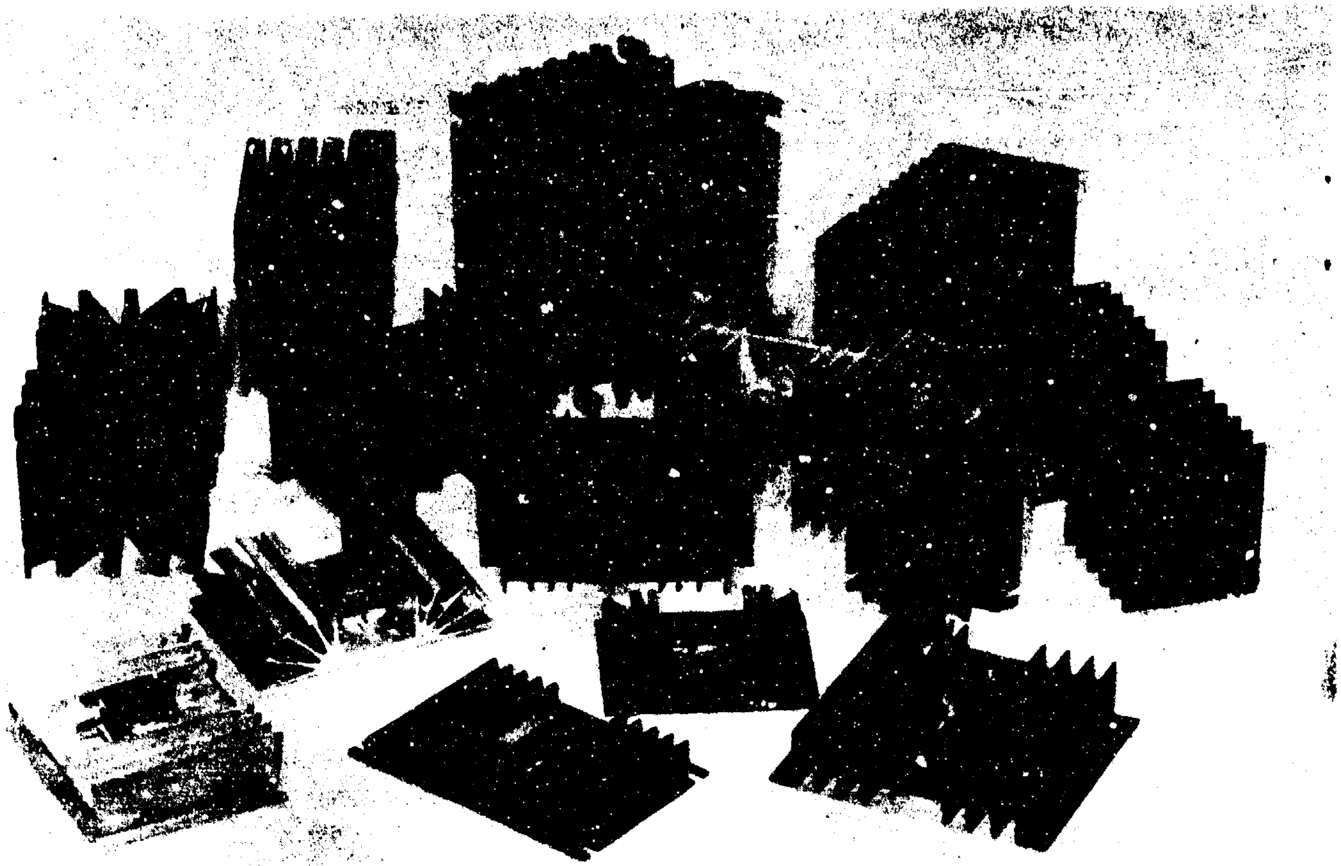
These cooling plates cover the thermal resistance range from about 0.5°C/W to about 3.5°C/W.

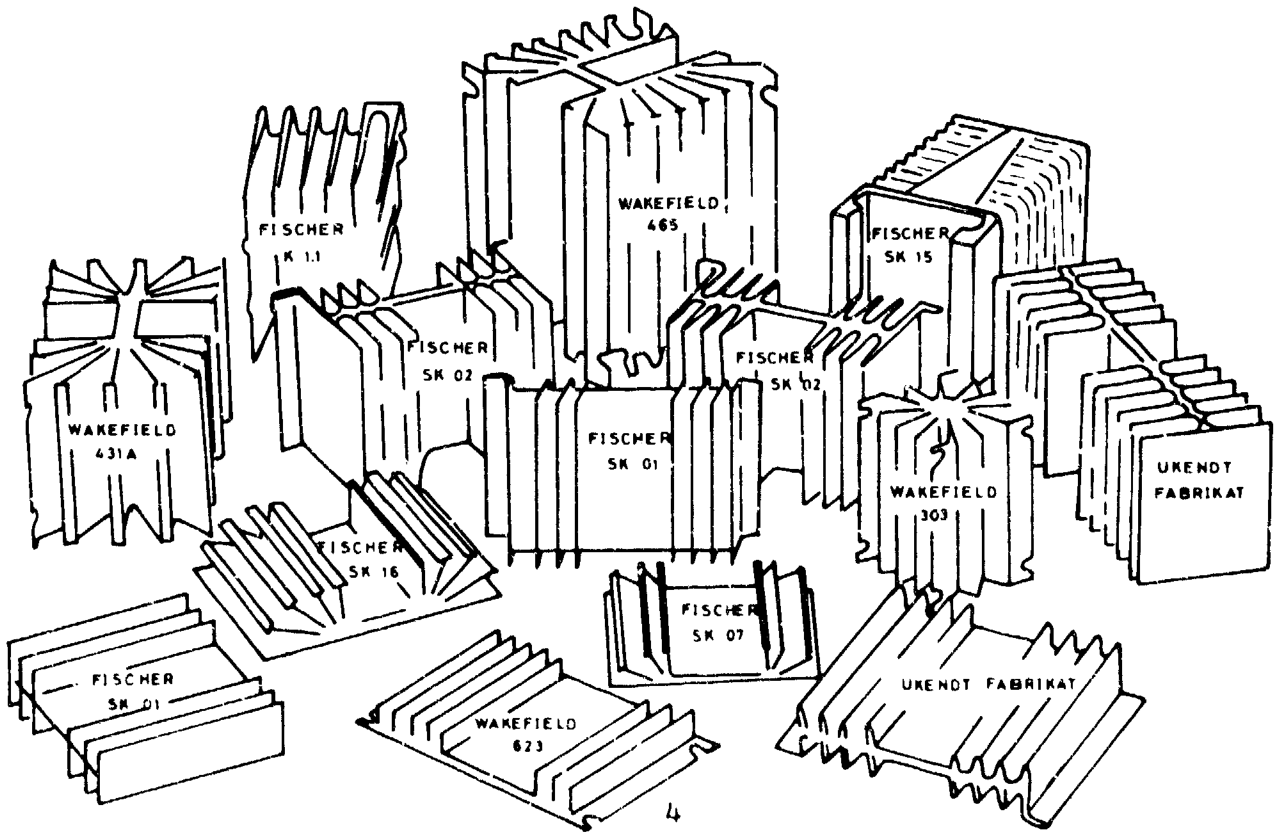
A list of variants of the types studied is given in the table below. The thermal resistances shown are measured values.

2.1. Table of Types Investigated

No.	Manufacturer	Type	Danish Supplier	Anodized?	Thermal Resistance at $\Delta T = 100°C$	Price per Hundred 1 Sept. 1974
1	Fischer	SK07	Inotec	Yes	3.1	4.93 kr.
2	Wakefield	623K	Scansupply	Yes	2.3	8.90 kr.
3	Fischer	SK01	Inotec	Yes	2.3	5.36 kr.
3a	Fischer	SK01	Inotec	No	2.9	4.21 kr.
4a	Fischer	SK16	Inotec	No	2.7	4.79 kr.
5	Wakefield	NC-303K	Scansupply	Yes	2.3	9.25 kr.
6	Unknown			Yes	1.7	
7	Fischer	SK02	Inotec	Yes	1.3	11.89 kr.
7a	Fischer	SK02	Inotec	No	1.6	9.72 kr.
8	Fischer	SK15	Inotec	Yes	1.2	30.31 kr.
9	Fischer	K1.1	Inotec	Yes	1.1	19.72 kr.
10	Unknown			Yes	1.1	
11	Wakefield	431K	Scansupply	Yes	0.7	17.05 kr.
12	Wakefield	465K	Scansupply	Yes	0.6	37.80 kr.

2.2. Photograph of Types Investigated





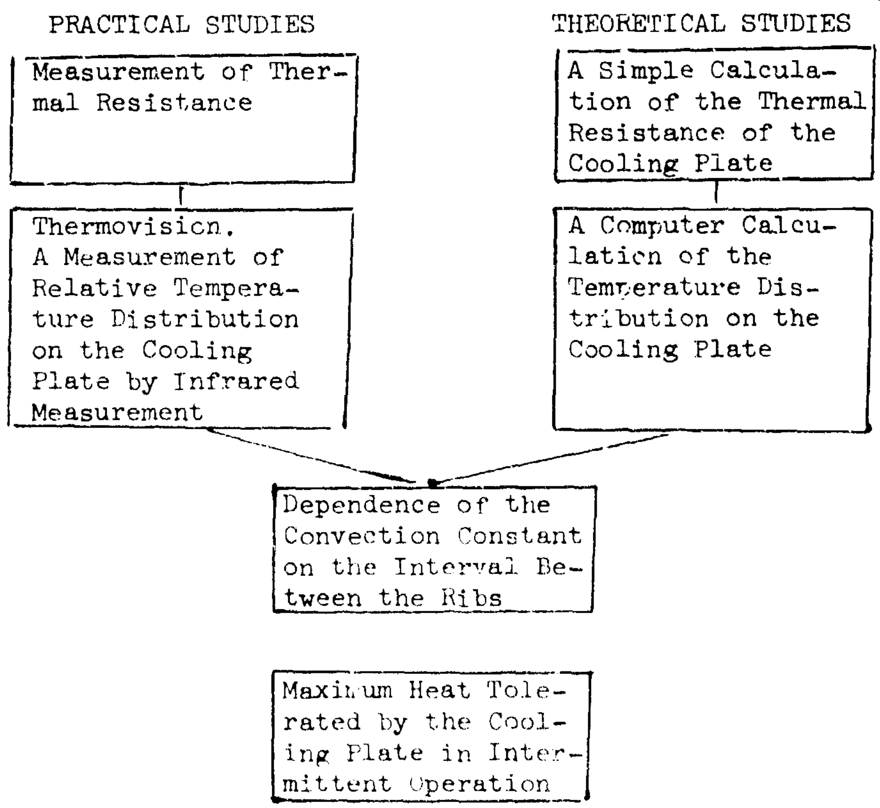
3. TEST PLAN

3.1. Survey

The block diagram below shows the test plan used, which consists in part of practical studies and in part of theoretical studies.

For the two studies in question -- the dependence of the convection constant on the interval between the ribs and the intermittent operation of the cooling plate -- this means that certain calculations were done and a hypothesis was set up, which was verified by tests.

A more detailed description of these studies and tests can be found under the respective headings.



4. POSSIBILITIES OF HEAT TRANSFER

For any electronic equipment to which power is applied, this power can be divided into effective power and waste power; i.e., there is a question of the efficiency of the equipment. The waste power, which often amounts to more than 50% of the power supplied, must be removed by heat transfer. There are three basic principles of heat transfer, namely conduction, radiation, and convection.

A brief description of their nature is given below. If further information is desired, the reader is referred to ECR-15.

4.1. Heat Conduction

All materials allow more or less heat transfer by conduction, depending on the specific thermal conductivity of the material.

For a given material the amount of heat transferred by conduction is dependent on the temperature difference as well as the dimensions of the material and its specific thermal conductivity.

$$P_L = \frac{\Delta T}{R_L} \qquad [W]$$

where R_L is the thermal resistance.

Starting from the specific thermal conductivity of the material, the thermal resistance to conduction can be defined as:

$$R_L = \frac{1}{K \cdot A},$$

where L is the length of the material in m,
A is the area of cross section of the material in m^2, and
K is the specific thermal conductivity in W/m/°C.

4.2. Radiation

Heat transfer by radiation occurs with the emission of electromagnetic radiation. For the temperatures which actually occur in connection with electronic equipment, the wavelength falls in the lower end of the infrared range.

The heat given off by radiation can be expressed by the formula

$$P_S = C \cdot A\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right],$$

where C is the radiation number in $W/m^2/k^4$,
A is the area of the heat-emitting surface in m^2,
T is the temperature of the object in °C, and
T is the radiation temperature of the environment in °C.

The formula is valid on the assumption that the object is completely surrounded by the surface which is to receive the heat radiation and that that surface has a high emission factor and an area that is much greater than the area of the object in question.

These conditions will almost always be satisfied.

4.3. Convection

If a warm body is placed in motionless air, the air is heated, and the resultant change in specific gravity sets the air in motion. New cold air is thus brought in, and a transfer of heat is under way. This mechanism of heat transfer is called free convection or natural convection.

Figure 1. Isotherms Around a Horizontal Pipe With Free Convection.

Heat transfer by free convection may be expressed by the following formula:

$$P_K = K \cdot C \cdot A \cdot \sqrt{\frac{\Delta T}{L}} \cdot \Delta T,$$

where K is a constant,

C is a constant determined by the orientation and geometric shape of the object,

A is the area of the heat-emitting surface in m^2,

ΔT is the difference in temperature between the heat-emitting surface and the object in °C, and

L is the characteristic dimension in meters.

5. TEMPERATURE RISE AS A FUNCTION OF LOAD

On the pages that follow, the rise in temperature is shown as a function of the power applied. All measurements are done at an ambient temperature of 20°C under free-field conditions; i.e., in the measurement chamber the only circulation of air is that which is created by free convection from the cooling plate.

During the experiment the cooling plate was freely suspended in the room, with such a distance between the various subjects of the experiment that they could not, from the thermal point of view, "see" each other.

The individual measurement points are shown on the sketches which accompany the graphs. The temperature was measured with iron-constantan thermocouples.

For the various types curves are presented for the relatively warmest point and coldest point measured on the cooling plates. Where the type is under study in both a bright and a black anodized version, the two sets of curves are drawn on the same graph to facilitate comparison. In addition, the curve given in the data sheet is drawn in if available.

5.1. Comparison With the Specifications of the Data Sheet

For most types it is found that the measured values fall within those given in the data sheet; e.g., No. 1 and No. 2 are considerably better than their respective data sheets indicate. It should be noted that these two are small cooling plates. No. 8 is poorer than its specifications. No. 9, which in the data sheet is merely specified with ΔT = 100°C, meets the specification at that point, but below ΔT = 100°C, where the cooling plate would normally be used, the thermal resistance is higher than stated (R_T = 1.1°C/W). No. 8 and No. 9 are relatively large cooling plates. There is thus a tendency for small cooling plates to be better than is claimed for them, while larger cooling plates are poorer than claimed.

5.2. Comparison Between Bright and Black Anodized Finish

Comparison of the bright and the anodized versions shows a rise in thermal resistance of about 20 to 25% for the bright versions as against the anodized ones.

Since the surface treatment can have significance only for heat transfer by radiation -- the emission factor is reduced from about 5.5 (anodized) to 0.5 (bright) -- we can get an impression from this of the part that radiation plays in the heat transfer for the types available in both an anodized and a bright version.

5.3. Temperature Gradients in the Heat Sink

The table below shows the maximum measured temperature gradient in the heat sink itself at a maximum absolute temperature of 100°C. The less the temperature gradient, the more fully the cooling plate is utilized and the more effective it is.

Type No.	1	2	3	3A	4A	5	6	7	7A	8	9	10	11	12
Temperature Gradient in °C	9	14	14	11	8	21	13	/	7	37	26	31*	18	20*

*At 80°C.

The next section, on thermovision, gives further information about the temperature distribution on the various cooling plates.

6. THERMOVISION

The pages following the temperature gradient graphs show thermovision pictures of the various types. It should be noted that No. 4 and No. 6 are not included.

Figure 2. AGA Thermovision Apparatus, Type 680.

The thermovision pictures were taken with the AGA thermovision system 680. That apparatus operates at a wavelength of 2 to 5.6 μm, and without filters it can measure temperatures from -30° to +850°C. The apparatus scans with a frequency of 16 pictures per second.

In the present case we are not concerned with measuring the absolute temperature, but merely with getting an impression of the temperature distribution on the cooling plate.

It should be mentioned that the scale is not the same in the various pictures; the object was simply focused on in such a way as to fill out the frame as well as possible.

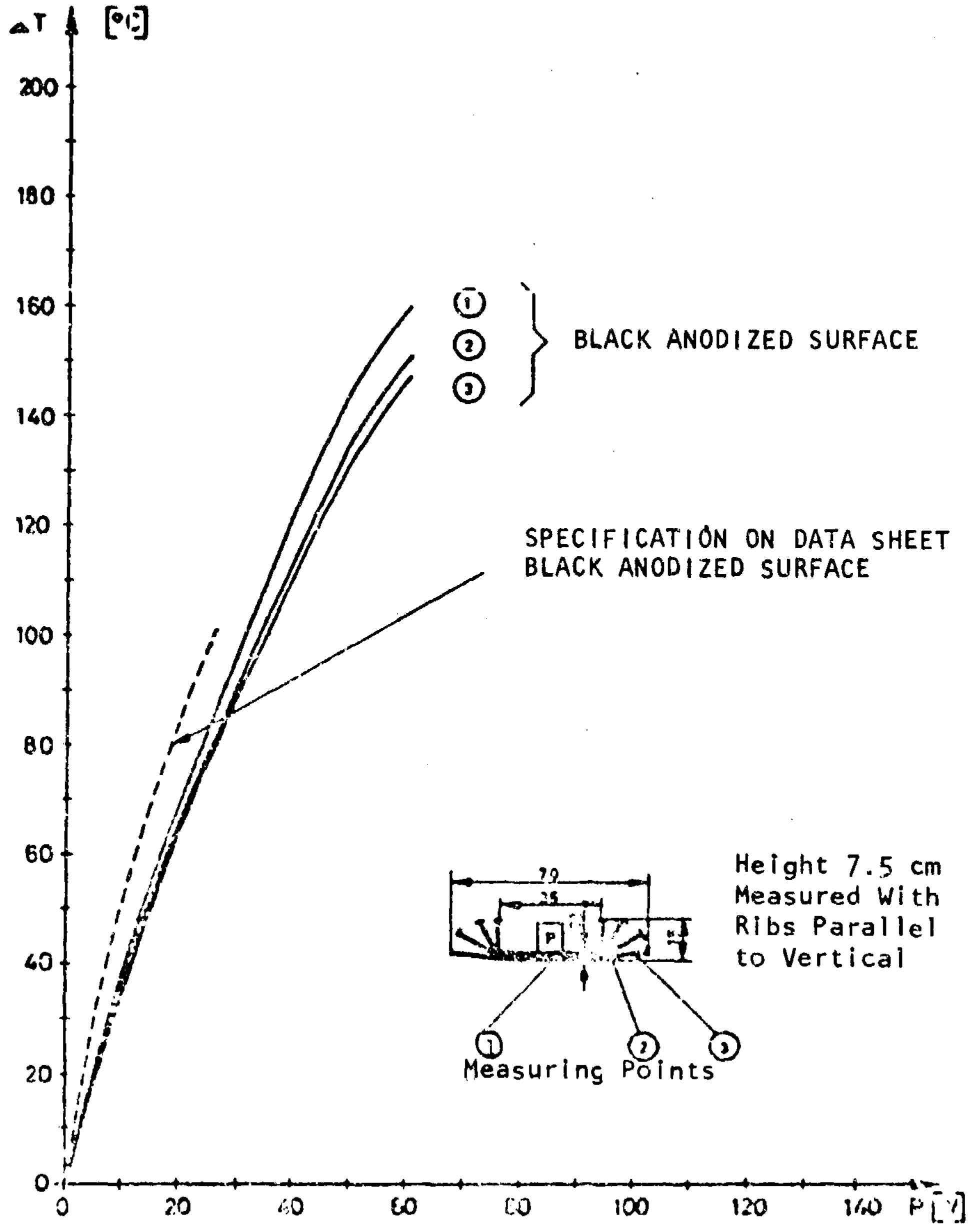
Since the apparatus measures the radiation of heat, it is very difficult to make measurements on the bright cooling plates, which have a very low emission factor (0.2 to 0.5) and a very high reflection factor (about 0.95). The measured value is the sum of the characteristic radiation and the reflection; see Figure 3 [page 22].

TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

Number 1
Manufacturer: Fischer
Type: SK 07

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 76.1 cm^3

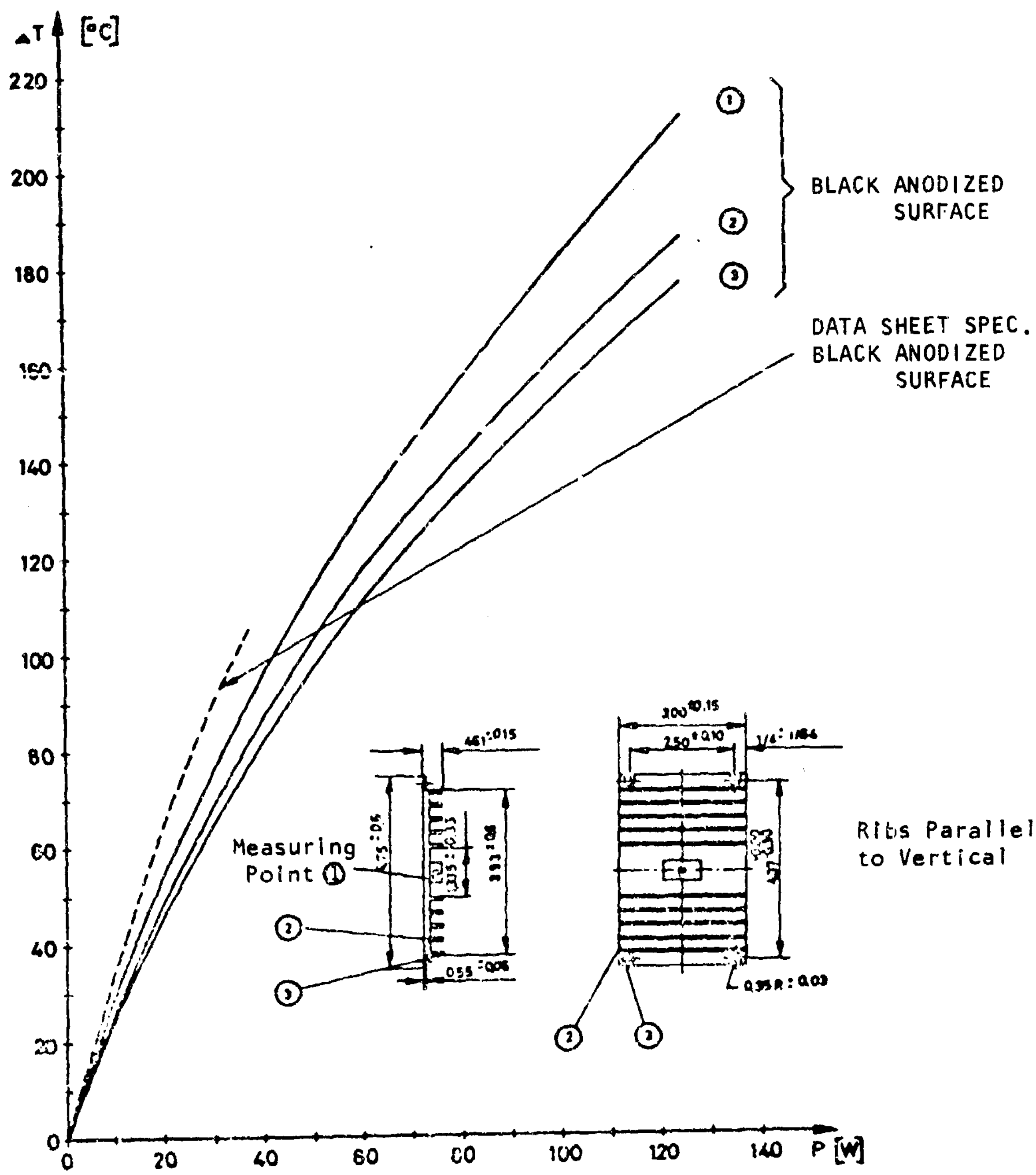


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 2
Manufacturer: Wakefield
Type: 623

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 98.96 cm^3

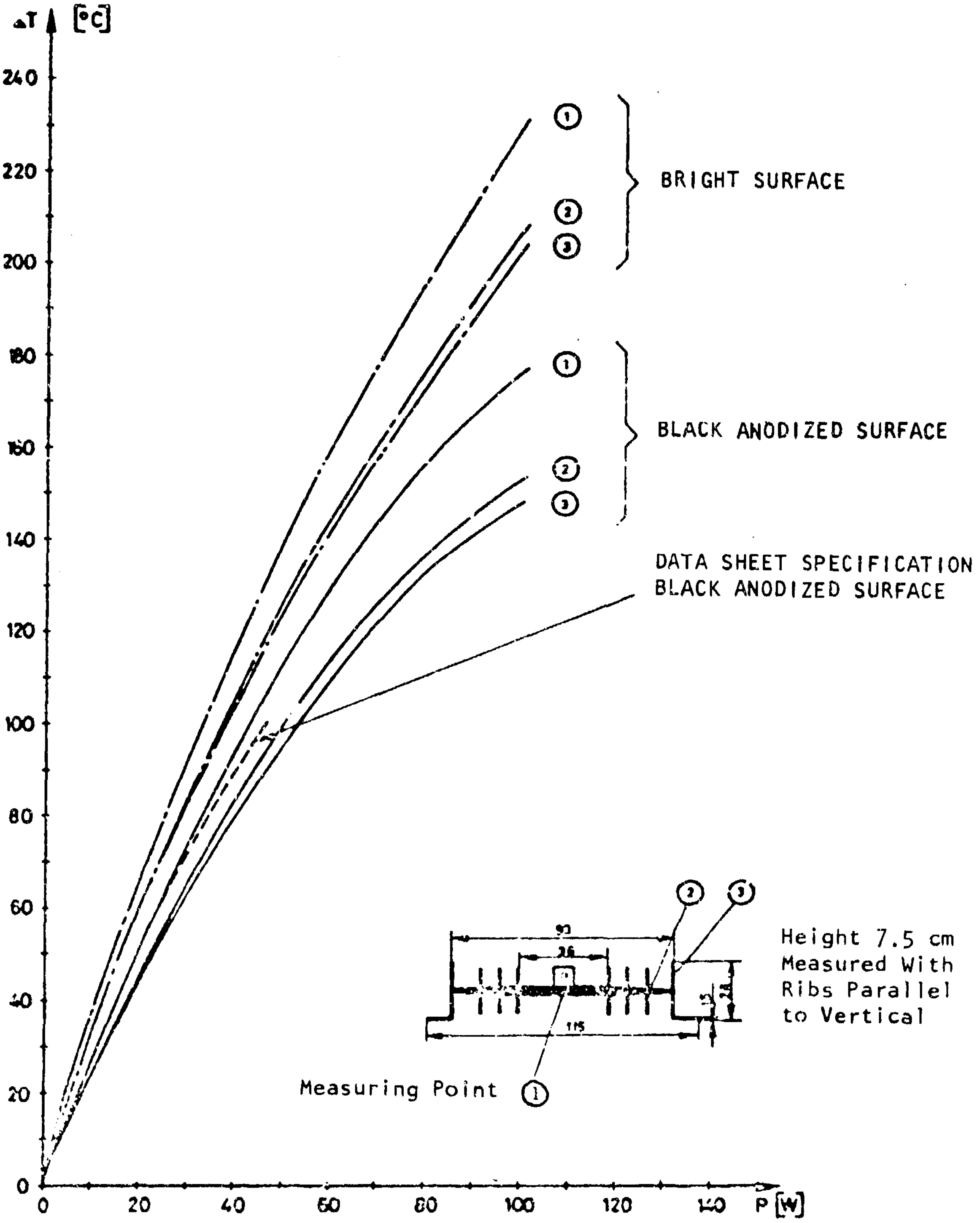


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 3
Manufacturer: Fischer
Type: SK 01

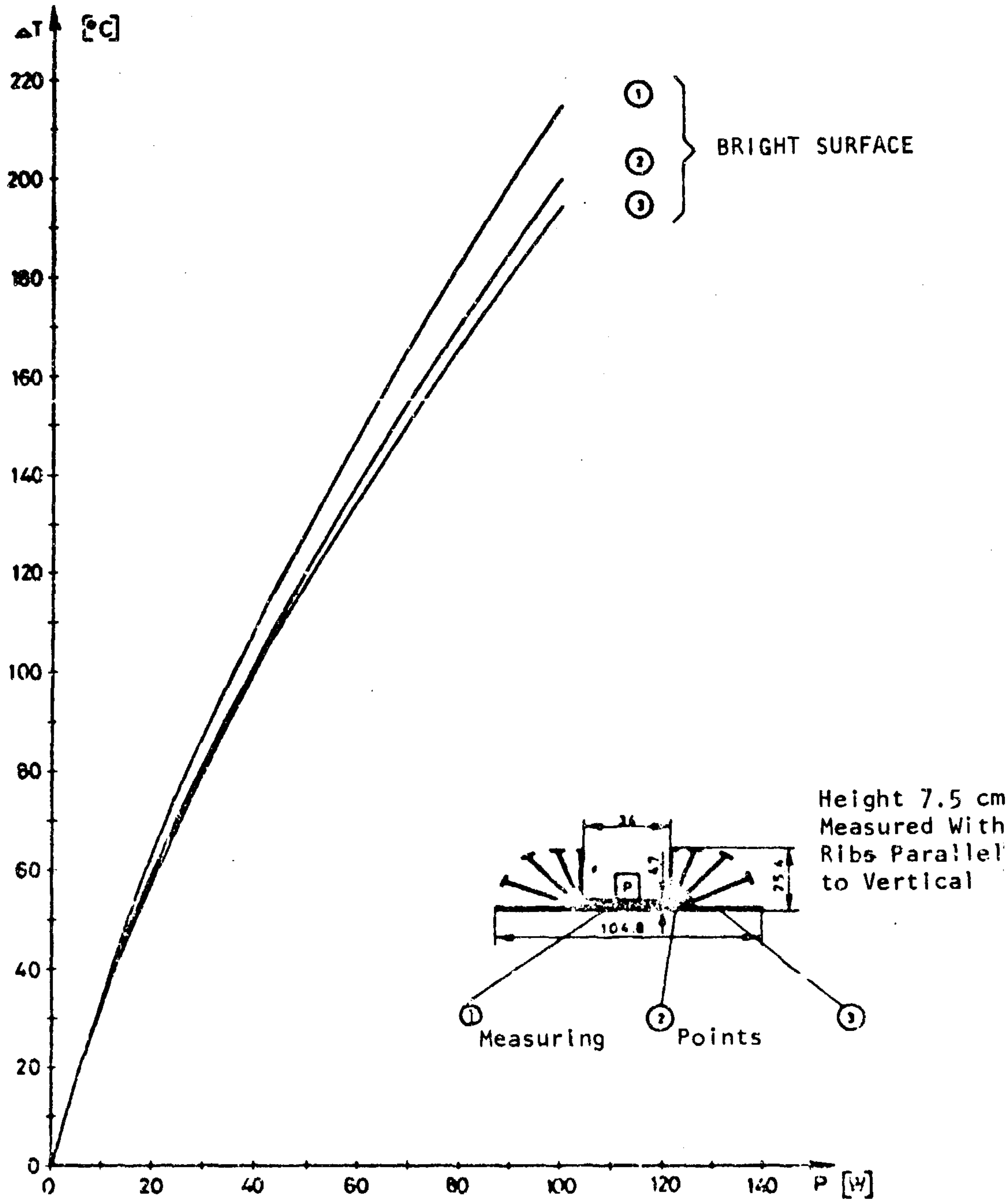
Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 181.35 cm^3



TEMPERATURE RISE AS A FUNCTION OF LOAD
Free-Field Measurements

No. 4
Manufacture: Fischer
Type: SK 16 (Bright)

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 200 cm^3

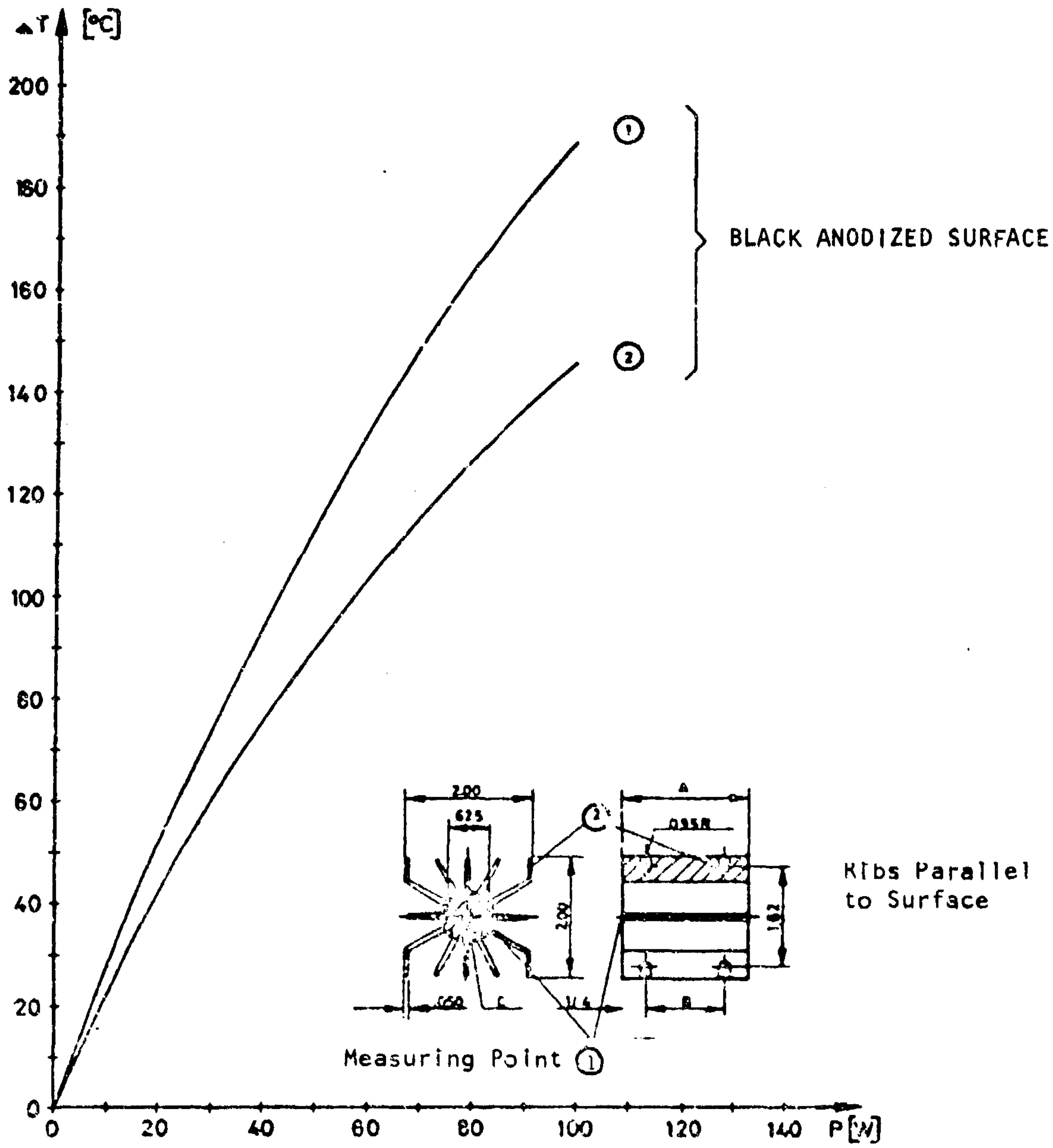


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 5
Manufacturer: Wakefield
Type: 303

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 198 cm^3

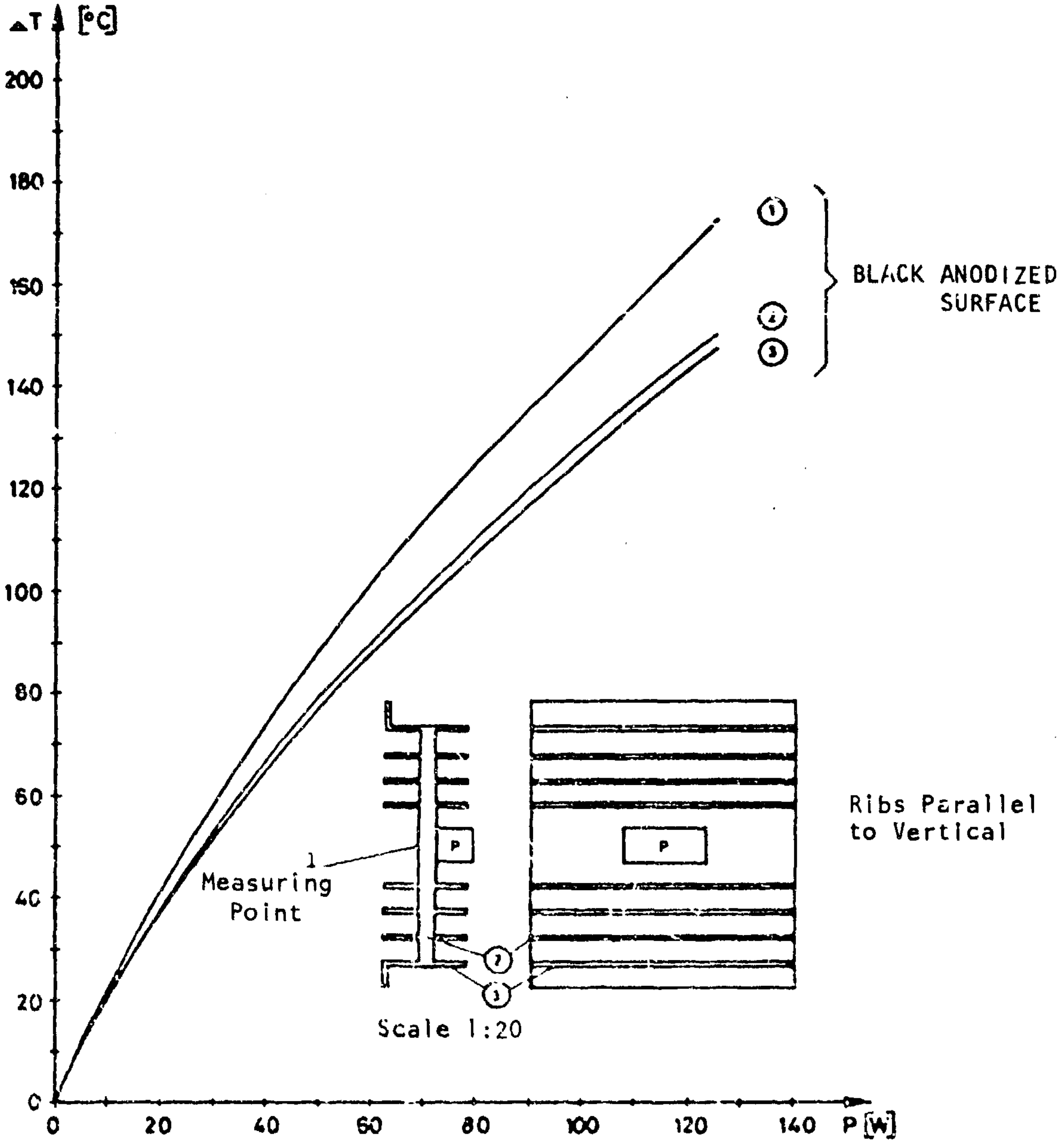


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 6
Manufacturer: unknown
Type: unknown

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 297.6 cm^3

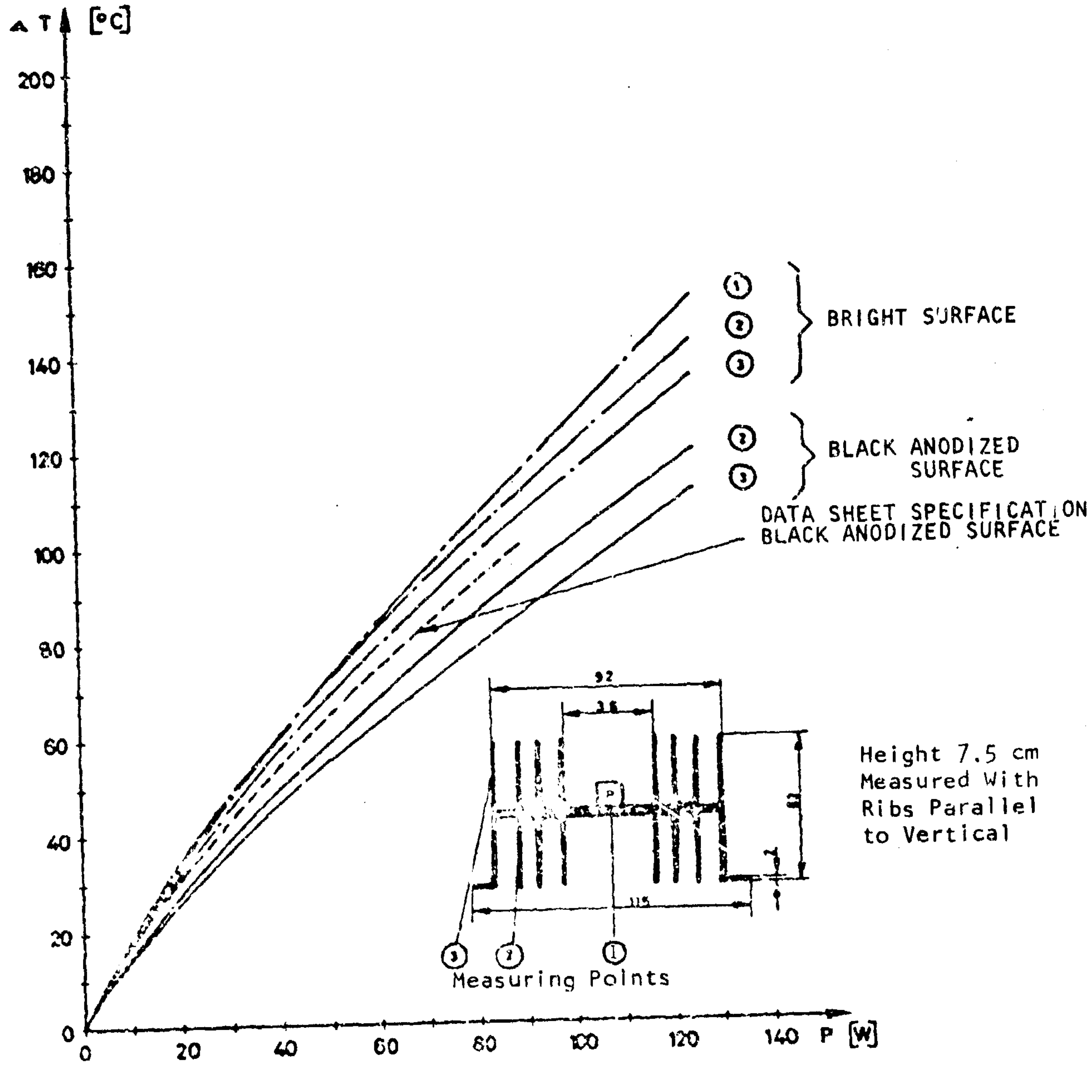


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 7
Manufacturer: Fischer
Type: SK 02

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 434.7 cm³

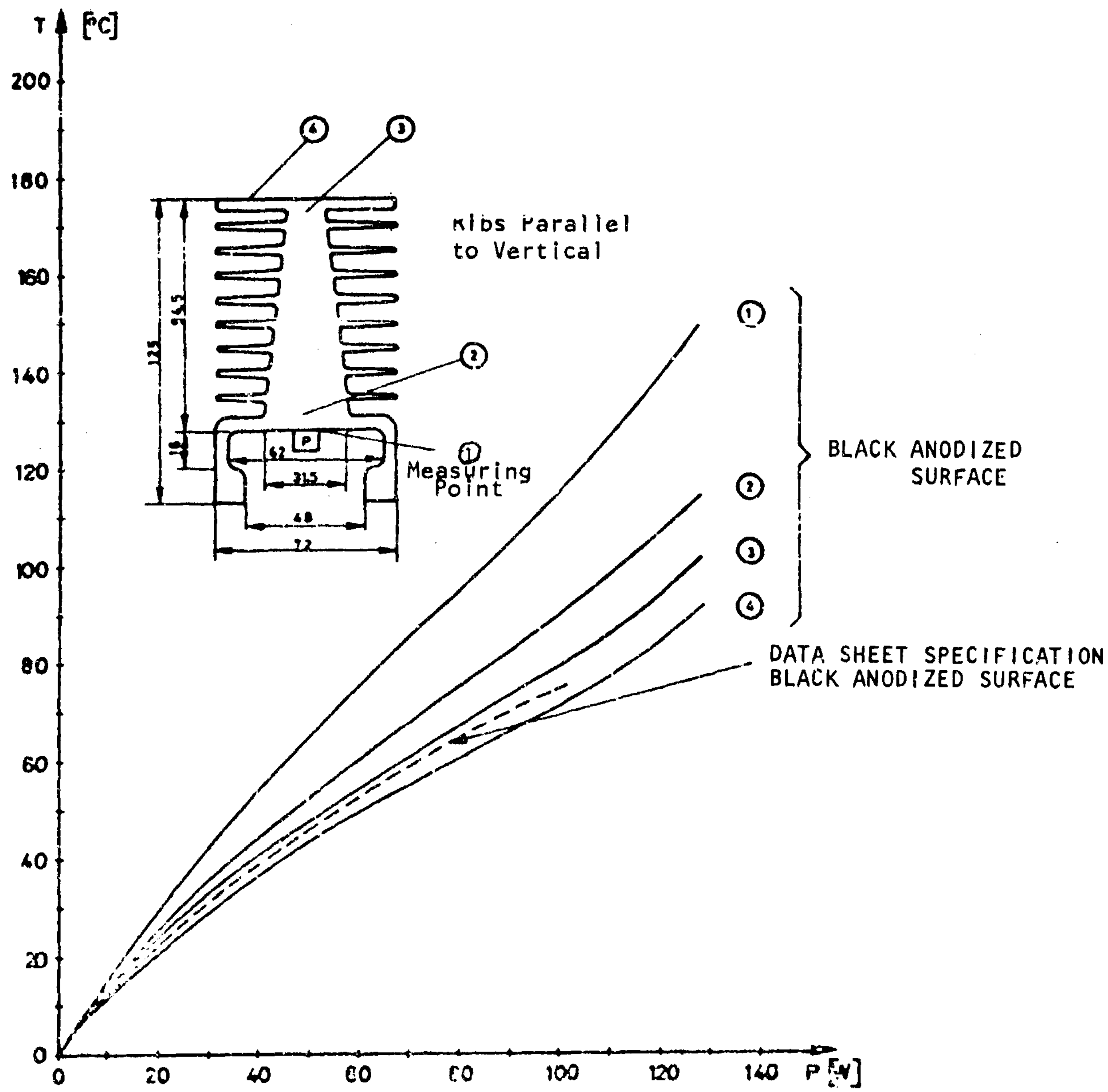


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 8
Manufacturer: Fischer
Type: SK 15

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 675.0 cm^3

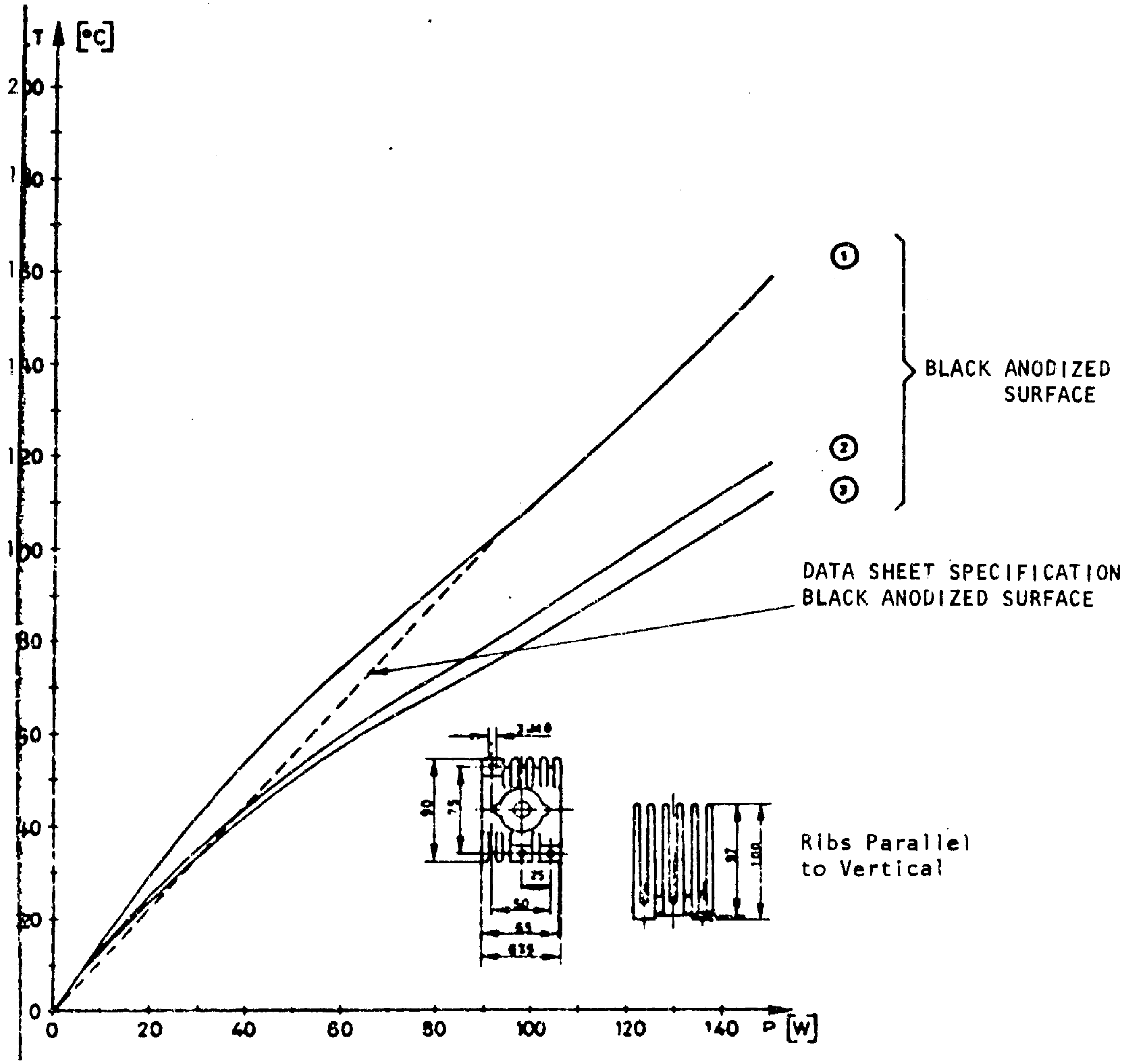


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 9
Manufacturer: Fischer
Type: K 1.1

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Specification: 1.1°C/W
Volume = 607.5 cm³

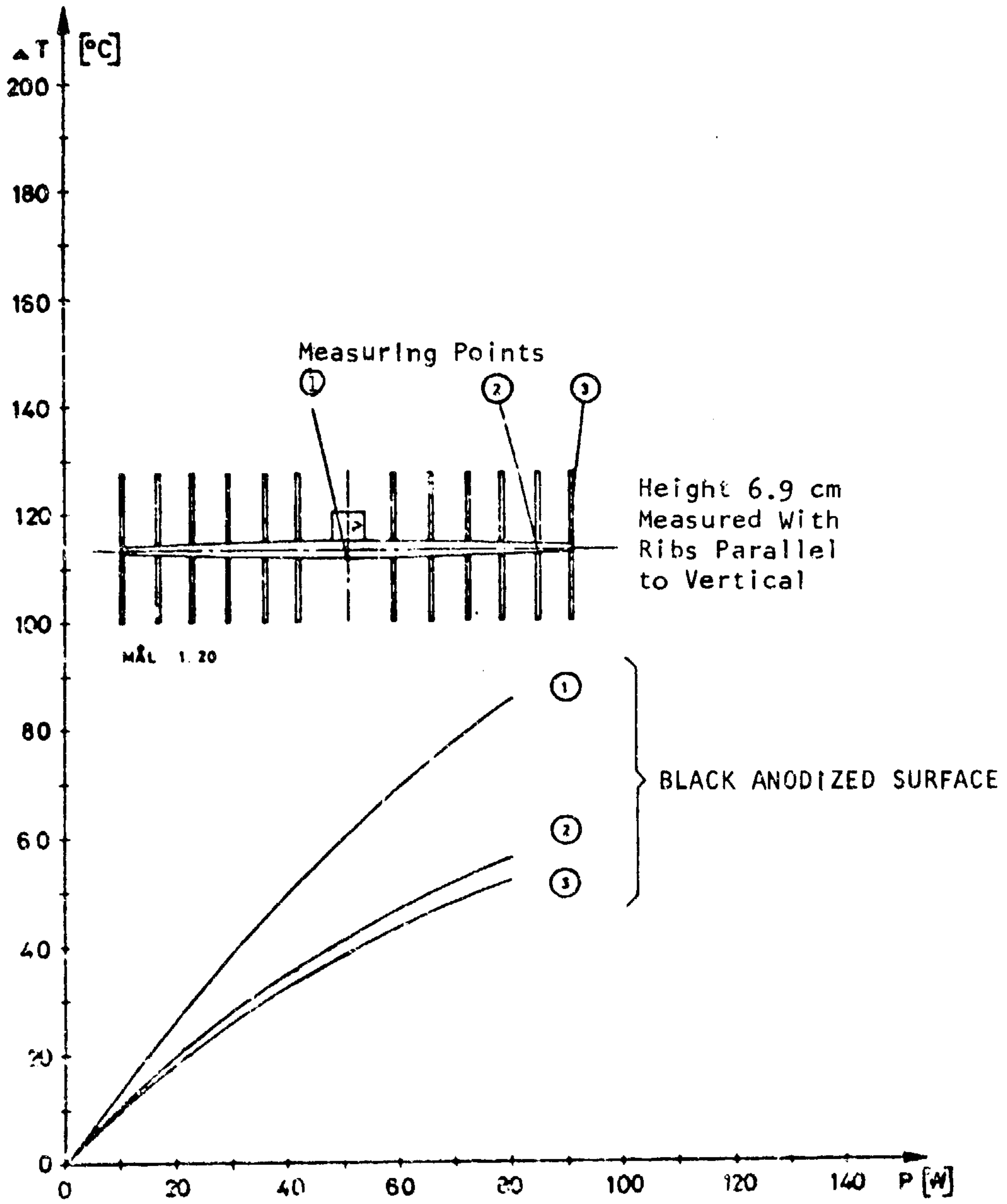


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 10
Manufacturer: unknown
Type: unknown

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 633.7 cm^3

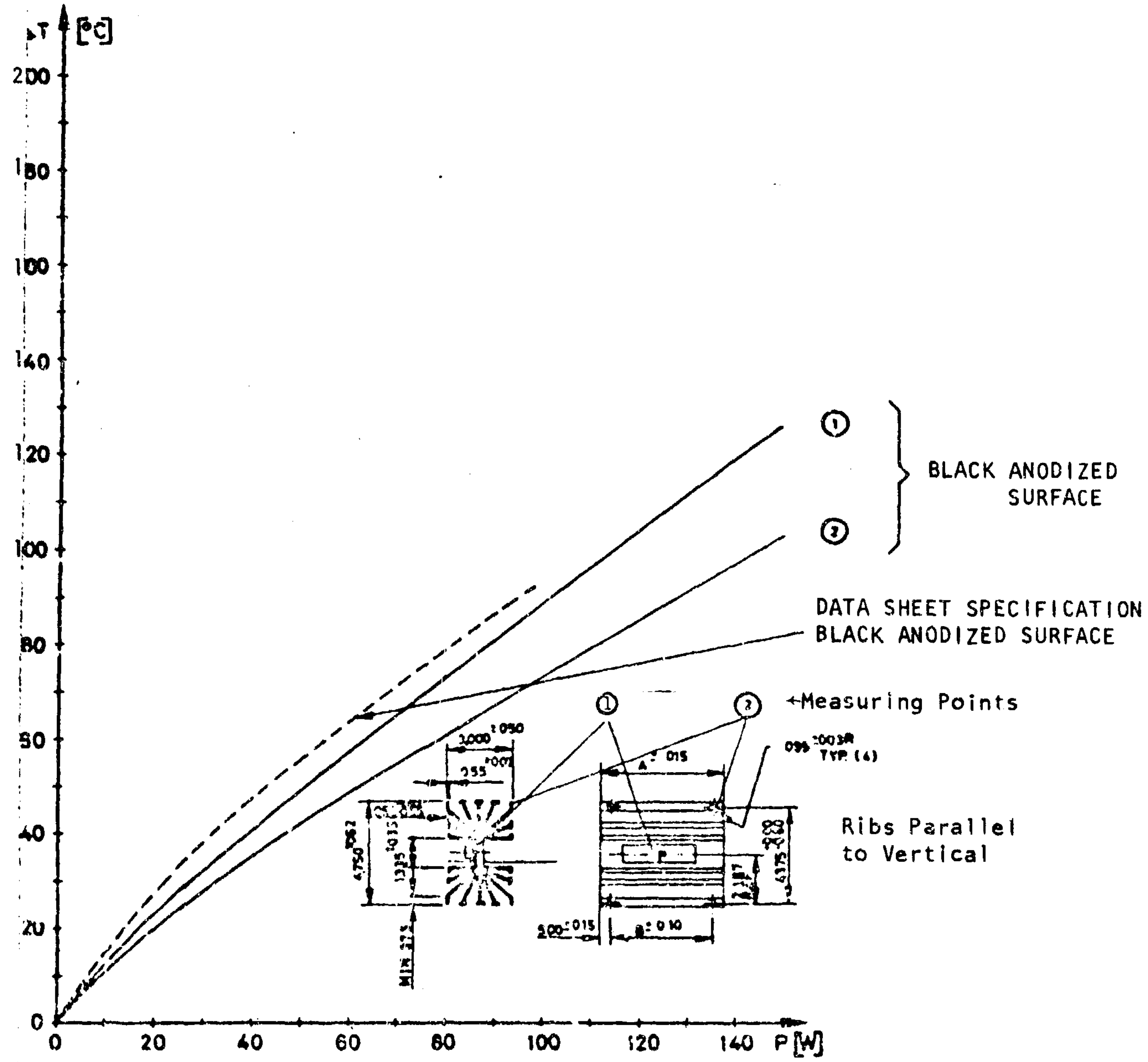


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 11
Manufacturer: Wakefield
Type: 431A

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 695 cm^3

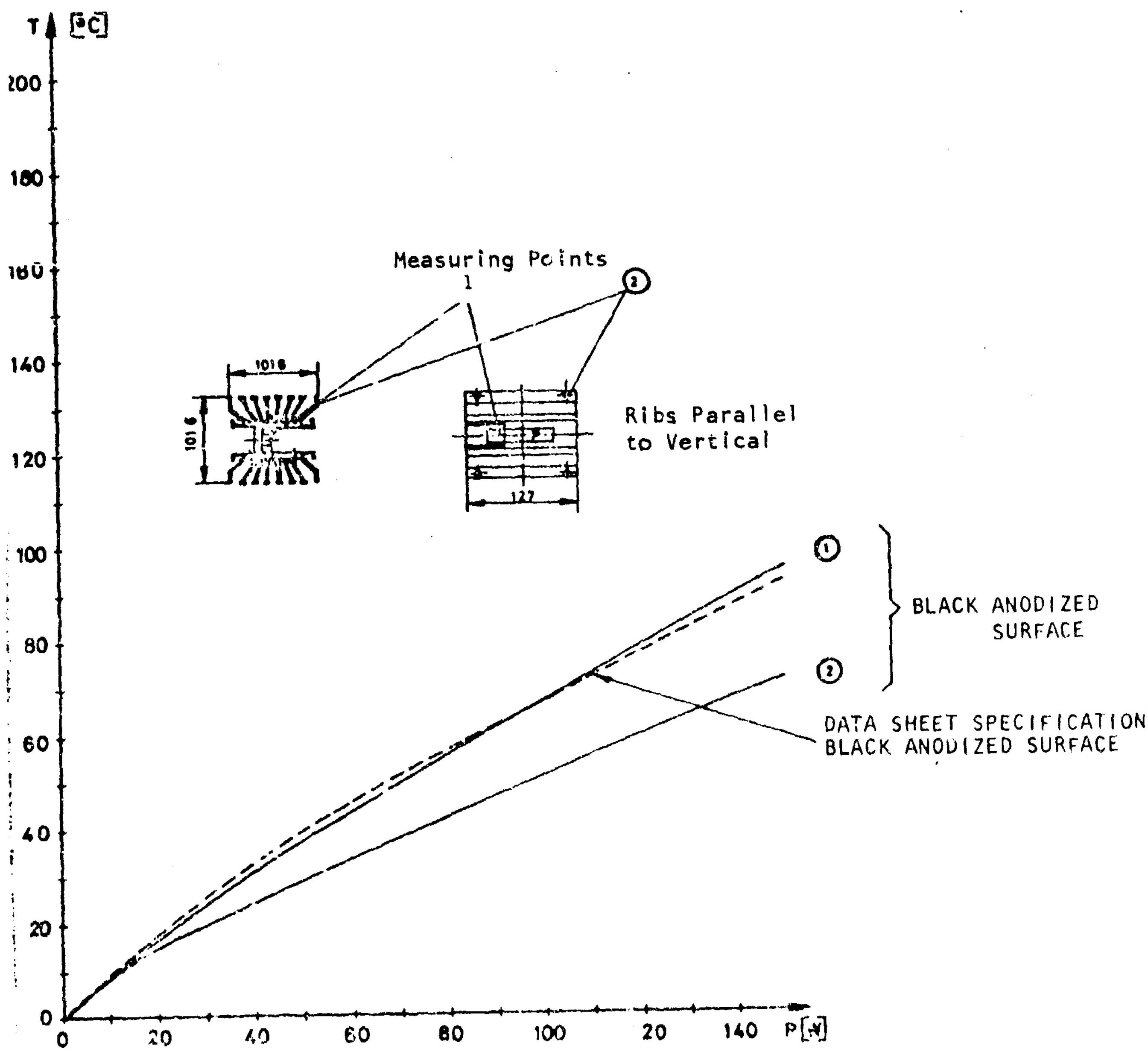


TEMPERATURE RISE AS A FUNCTION OF LOAD

Free-Field Measurements

No. 12
Manufacturer: Wakefield
Type: 465

Ambient Temperature = 20°C
Air Velocity = 0 m/second
Volume = 1,311 cm^3



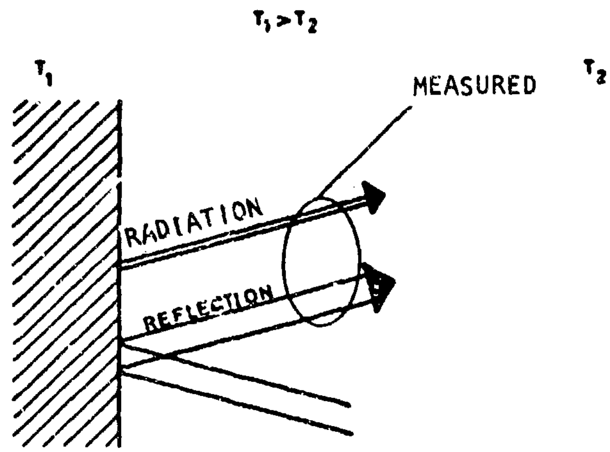
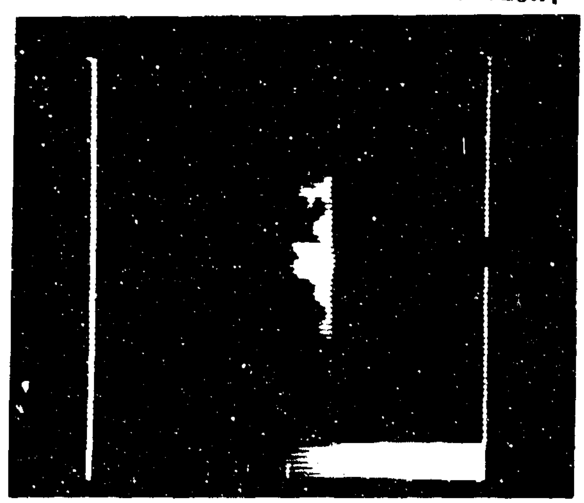
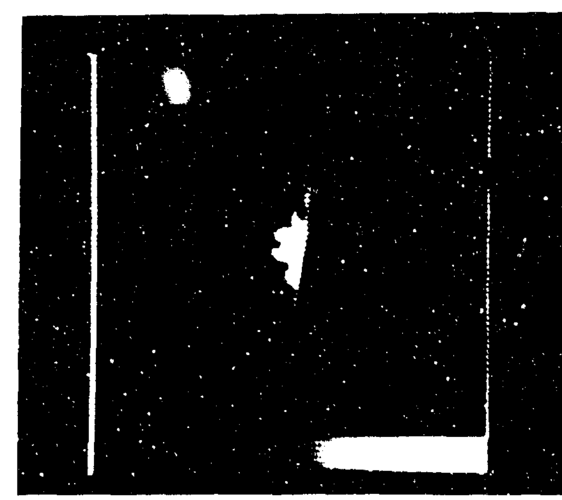


Figure 3. In Measurements of Thermal Radiation, What is Measured is Characteristic Radiation + Reflection.

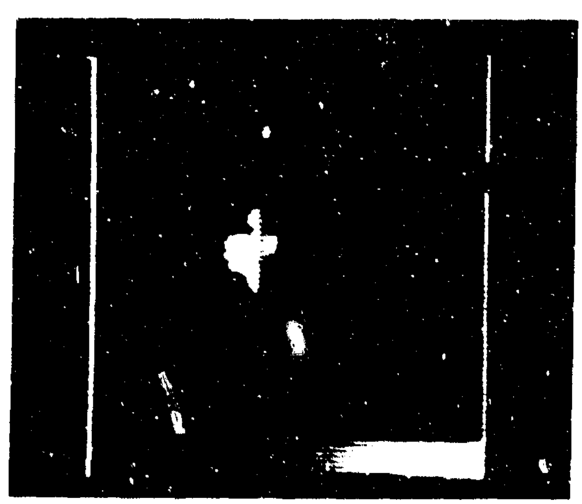
It is evident that in the main the small types assume the same temperature throughout their surface, while the temperature gradient on the larger ones is more pronounced, especially in the case of No. 8 and No. 10. This is in harmony with the difference measured under 5.4 above.



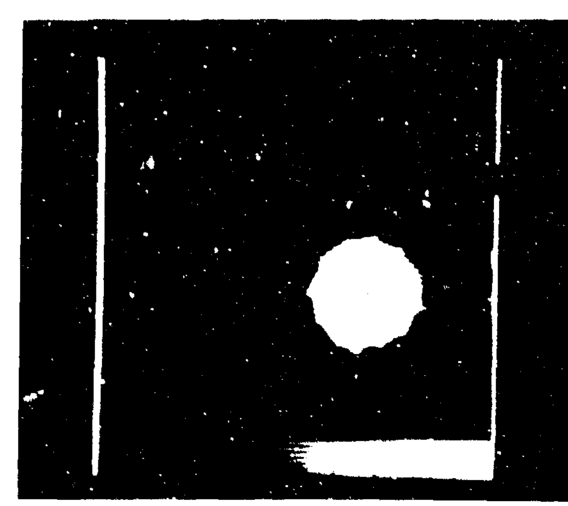
Fischer type SK07



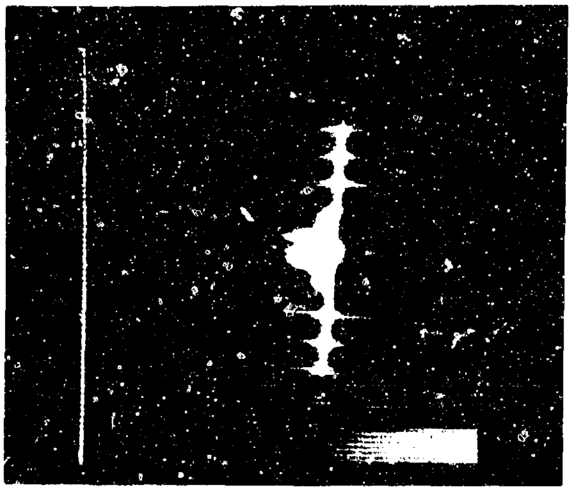
Wakefield type 623

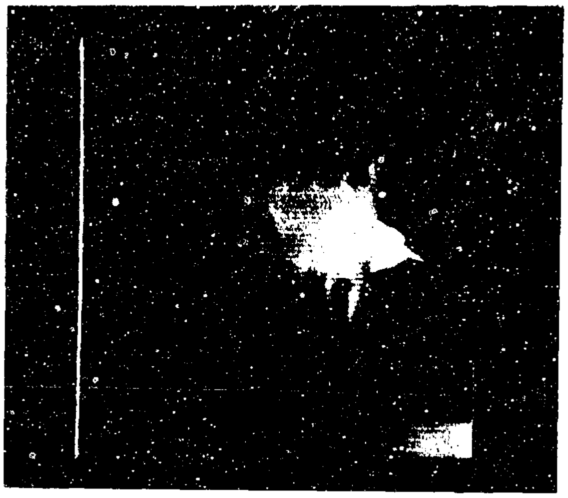


Fischer type SK01



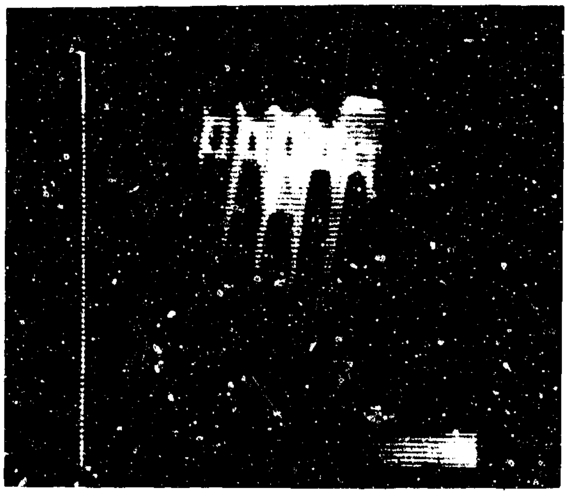
Wakefield type 303

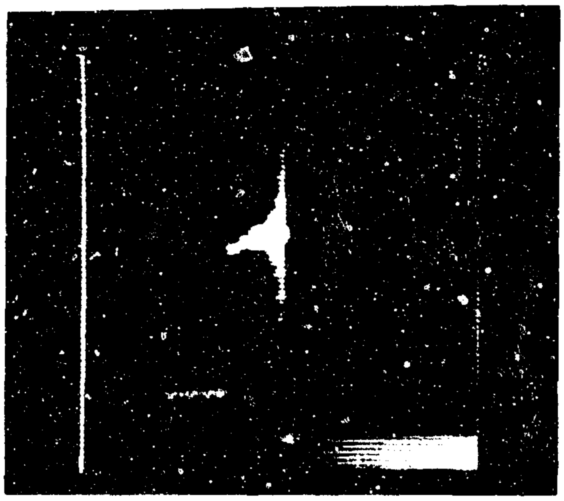




Fischer type SK02

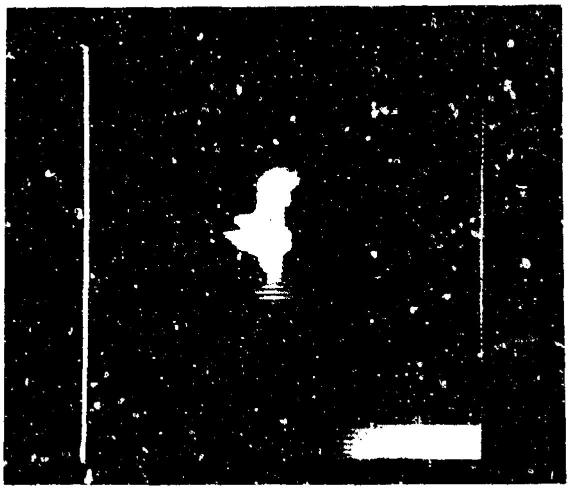
Fischer type SK15

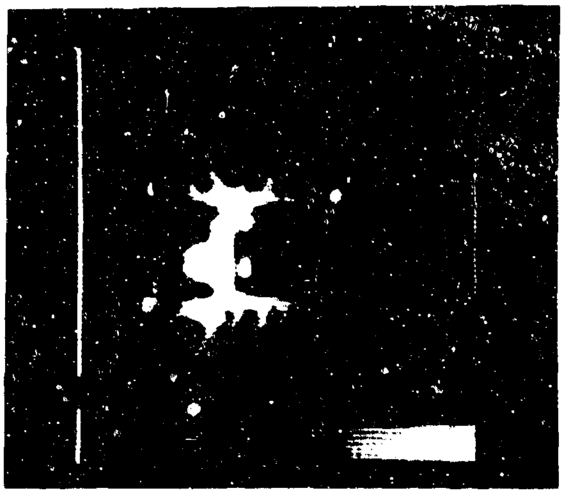




Fischer type K 1.1

Ukendt





Wakefield type 431 A

Wakefield type 465

7. COMPARISON OF QUALITY

A comparison of quality may be based on the principle that the least possible thermal resistance is desired within a given space.

If we plot the thermal resistance of the various types as a function of their volume on a simple logarithmic coordinate system, it turns out that we get a straight line. What is meant here by the volume of the heat sink is the volume or the space that the heat sink takes up.

The table below gives the characteristic values for the various types.

No.	Manu-facturer	Type	Thermal Re-sistance at ΔT = 100°C	Thermal Capac-ity	Thermal Time Const.	Volume	Weight	Ano-dized?	"Specific Gravity"
			°C/W	Ws/°C	min.	cm^3	grams		g/cm^3
1	Fischer	SK07	3.1	80	4.2	76	89	yes	1.17
2	Wakefield	623	2.3	98.5	3.8	99	110	yes	1.11
3	Fischer	SK01	2.3	111	4.2	181	124	yes	0.68
3a	Fischer	SK01	2.9	111	5.4	181	124	no	0.68
4a	Fischer	SK16	2.7	140	6.4	200	156	no	0.78
5	Wakefield	303	2.3	146	5.6	198	163	yes	0.82
6	Unknown		1.7	183	5.1	298	205	yes	0.69
7	Fischer	SK02	1.3	244	5.4	435	272	yes	0.63
7a	Fischer	SK02	1.6	244	6.5	435	272	no	0.63
8	Fischer	SK15	1.2	753	14.6	675	841	yes	1.25
9	Fischer	K1.1	1.1	565	10.4	608	631	yes	1.04
10	Unknown		1.1	269	5.0	634	300	yes	0.47
11	Wakefield	431A	0.6	334	4.8	695	373	yes	0.54
12	Wakefield	465	0.6	722	7.6	1,311	806	yes	0.61

The straight line (see Figure 4) can be used as a basis for a comparative evaluation of quality, since we can say that for heat sinks whose measurements fall above the straight line the thermal resistance is great, while for those which fall below the line the thermal resistance is low, everything being taken in relation to the space taken up by the heat sink.

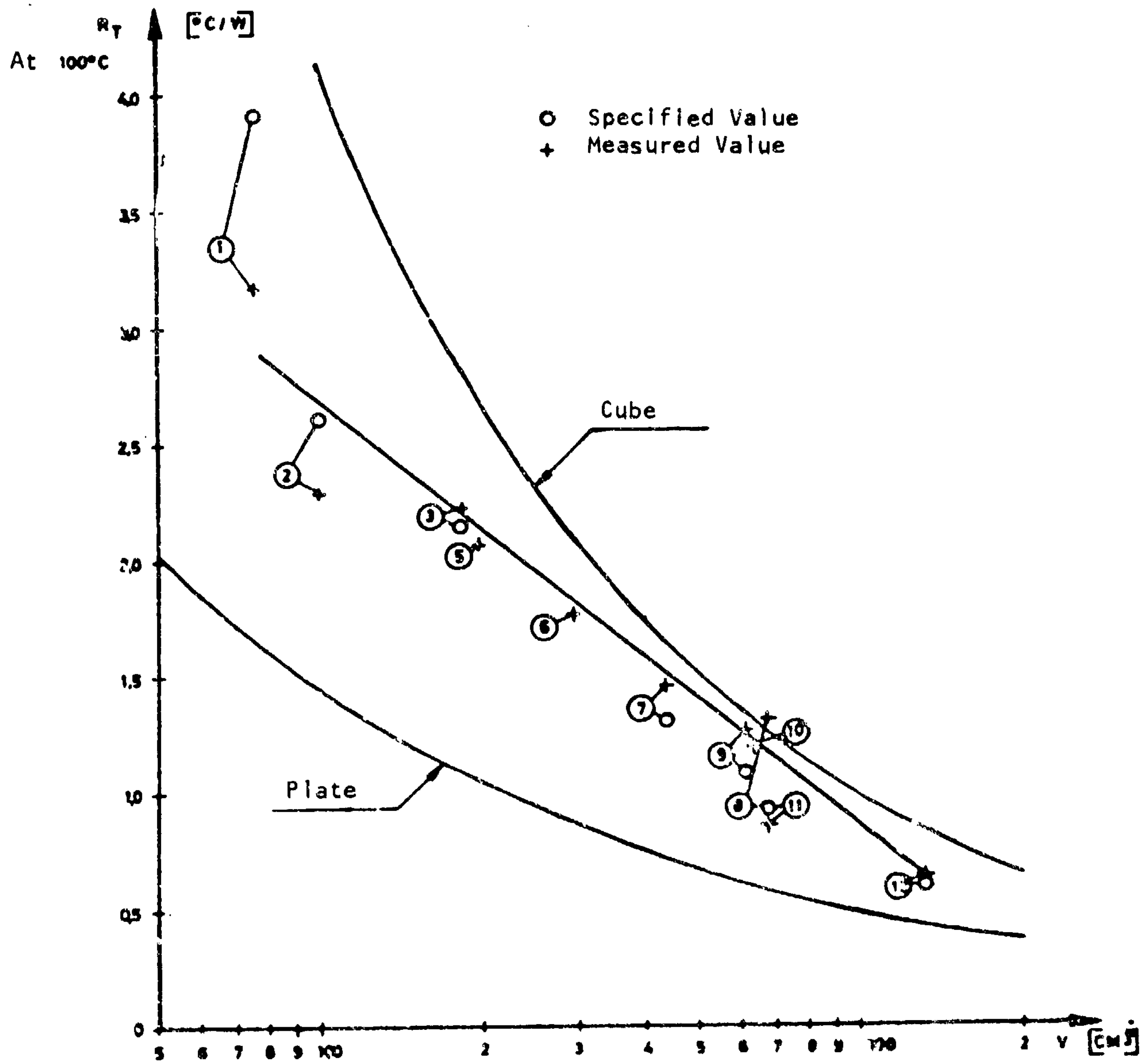


Figure 4. Thermal Resistance as a Function of the Volume.

For comparison, the thermal resistance is plotted in for a black anodized aluminum cube and also for a similarly black anodized Al plate, where the thermal resistance from the center of the plate to the outer edge is only one third of the thermal resistance from the plate to the surrounding air. These two curves may be considered as extreme limits, since the cube is one of the worst possible designs, from the thermal point of view, and the plate is the design that may be considered in practice as the best. There was no attempt to optimize the thickness chosen, however.

For small values of thermal resistance the area of the plate is impractically large, so that it does not represent a realistic solution. It is included here only in order to provide an extreme limit.

A general appraisal shows that small heat sinks lie in the range between a plate and a cube, approaching the cube for a size of about 1°C/W in thermal resistance and then shifting back toward a plate design. If we look at the geometric form of the heat sinks, too, we see that small heat sinks approach a plate in design, while the medium size with a thermal resistance of about 1°C/W is more massive and approaches a cube in shape. Lastly, the large sizes (No. 11 and No. 12) have a design in which the individual cooling fins are set at such a distance from each other and in such a geometric arrangement that they may be regarded as composite plates.

In conclusion it should be noted that the two limiting curves are purely theoretically calculated curves, based on the assumption that the whole mass has the same temperature.

8. THEORETICAL INVESTIGATIONS OF THE HEAT SINK

The preceding sections are based on practical measurements. In the following sections theoretical reflections and investigations will be undertaken, both in order to facilitate the calculation of a heat sink and in order to approach a possible optimal design of a heat sink.

8.1. The Convection Resistance as a Function of the Distance

The heat released from a warm body by convection may be expressed by the formula

$$P = K \cdot C \cdot A \cdot \sqrt[4]{\frac{\Delta T}{L}} \cdot \Delta T.$$

This expression contains a number of parameters, some of which are measurable and others very difficult to measure.

It is generally true for electronic apparatus, however, that the "constant" K, which covers many of the quantities that are hard to measure, can be set equal to 2.5, so that we get the simplified expression

$$P = 2.5 \cdot C \cdot A \sqrt[4]{\frac{\Delta T}{L}} \cdot \Delta T.$$

However, for heat sinks, among other things, we cannot always allow ourselves to simplify the expression as shown above. It is found that when we have two heat-emitting surfaces parallel to each other, the factor K is greatly dependent on the distance between the two surfaces.

If we look at the air velocity curve for a vertical heated surface with free convection (Figure 5), we see that the curve reaches a maximum value at a distance of about $\frac{1}{3}\partial$ from the heat-emitting surface, and the velocity drops to zero at the distance $y = \partial$.

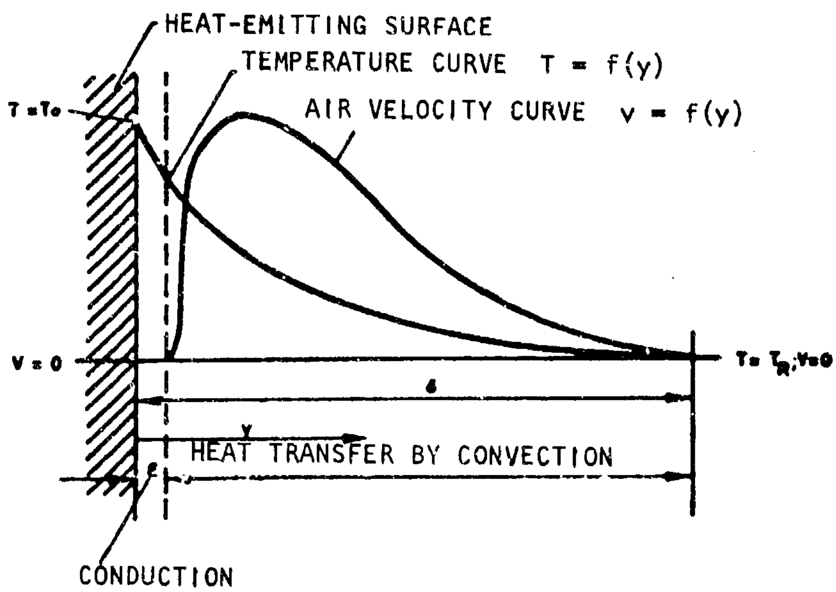


Figure 5. Temperature and Air Velocity Plotted as Functions of Distance From a Heat-Emitting Surface With Free Convection.

This limiting distance ∂ for a laminar flow is defined by the following expression:

$$\partial_{lam} = 5.83 \sqrt{\frac{\beta \cdot l}{v}},$$

where	∂_{lam}	is the limiting distance in laminar flow	$1 \ \mathrm{m}$
	β	is the dynamic viscosity of the air	$1 \ \frac{\mathrm{m}^2}{\mathrm{s}}$
	l	is the distance to the edge of inflow	$1 \ \mathrm{m}$
	v	is the velocity of the airflow	$1 \ \frac{\mathrm{m}}{\mathrm{s}}$

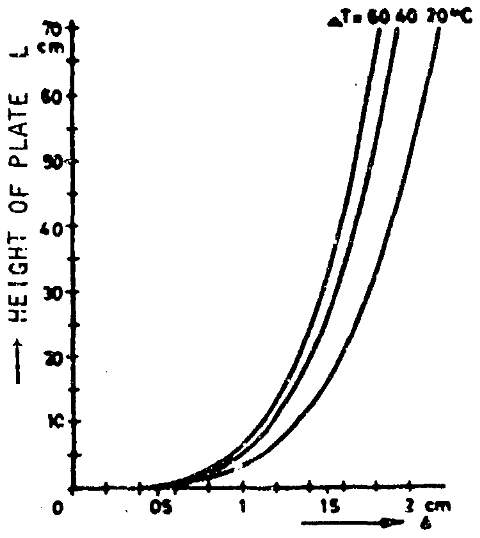


Figure 6. Graphic Representation of the Limiting Distance as a Function of the Height and Temperature of the Surface.

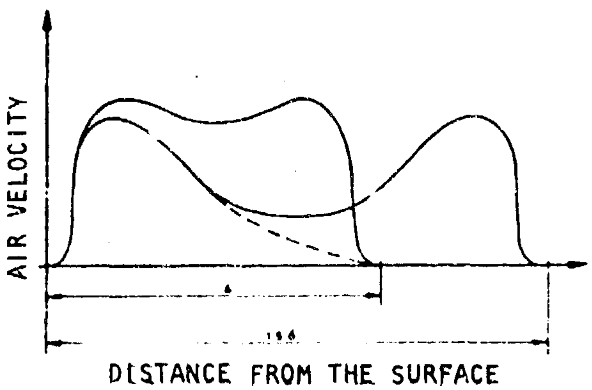
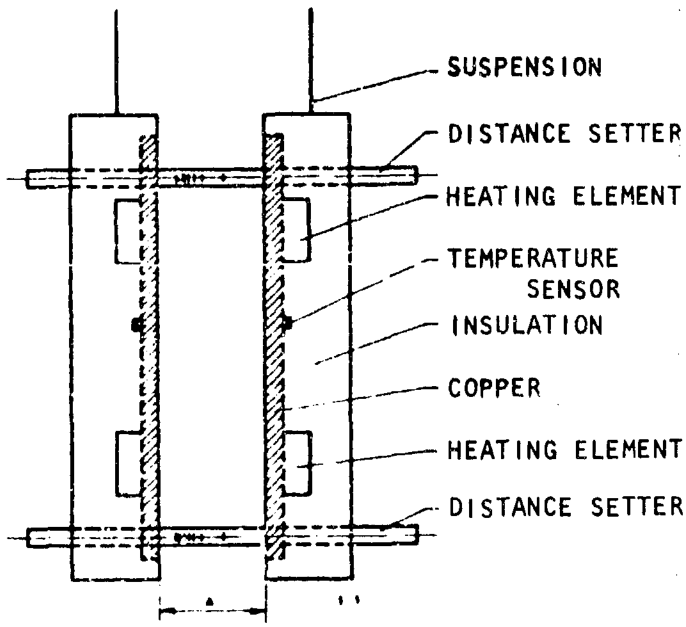


Figure 7. Curve of Velocity of Movement of Air Between Two Vertical Heated Surfaces.

Now if we consider the air velocity between two heat-emitting surfaces, we will theoretically get curves like those shown in Figure 7. We shall now show that in practice we cannot allow an overlapping of the two velocity curves, if free convection is to be maintained.

To establish the exact dependent relationship between the heat that can be given off in free convection, i.e. the convection resistance, and the distance between two heat-emitting surfaces, experiments were carried out in which a constant and equal amount of heat was applied to two parallel surfaces and the temperature rise was measured.

Figure 8 is a sketch of the experimental arrangement. By applying equal amounts of heat to the two plates, so that they took on the same temperature, we eliminated the possibility of a heat transport between the two surfaces by radiation. Loss of heat from the back of the plates was reduced as much as possible by effective insulation; emission of heat from the back cannot be entirely eliminated, but since the thermal resistance here is constant, it can be eliminated in the calculations.



POWER SUPPLIED TO EACH PLATE $P = 4$ W
A = (2.5, 1.5, 1.0, 0.8, 0.7, 0.6, 0.5, 0.4)
Area of the Plate: $A = 30$ cm^2
CONVECTION RESISTANCE ($K = 2.5$)
$R_C = 43.1$°C/W

Figure 8. Sketch of the Experimental Set-Up for Study of the Convection Constant's Dependence on the Distance Between Heat-Emitting Surfaces.

The result of the experiment can be seen from the two curves of Figure 9, showing the temperature rise on the plates and the size of the constant K as a function of the distance between the surfaces.

As the figure shows, the temperature begins to rise when the distance between the plates is less than $2 \times \partial$; this exactly corresponds to the point where an overlapping of the air velocity curves of the two surfaces begins.

When the rise in temperature is inserted in the formula for free convection, we get the indicated dependence of the size of K on the distance between the surfaces.

8.2. Thermal/Electric Analogy

Since part of the tools for computing thermal designs are based on electrical magnitudes, and since these magnitudes are generally easier to compute with, it will be a great advantage if we can convert the thermal magnitudes into analogous electrical magnitudes.

It turns out that a very simple analogy can be found between the thermal and electrical magnitudes.

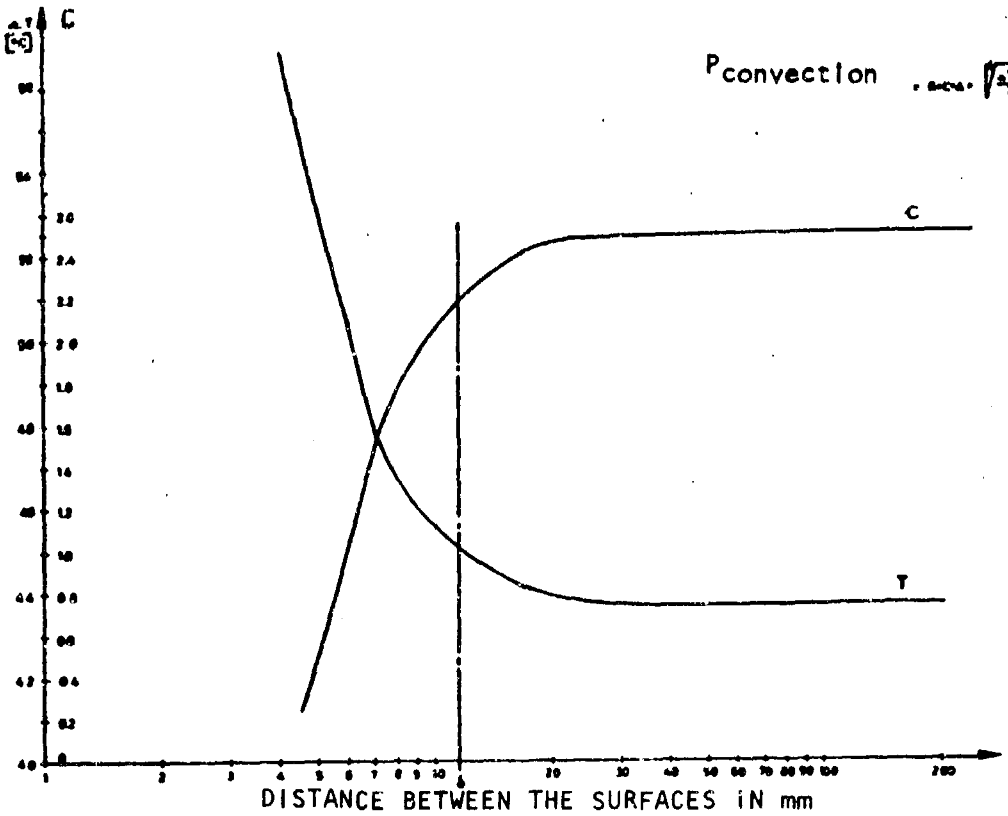


Figure 9. The Curves Show the Dependence of the Temperature [T] and Convection [C] on the Distance Between Two Heat-Emitting Surfaces.

The correspondence is as follows:

Thermal Magnitude				Electrical Analogy		
P	power	in watts	∿	I	current	in amperes
T	temperature	in °C	∿	V	voltage	in volts
R	thermal resistance	in °C/W	∿	R	electrical resistance	in ohms
C	thermal capacity	in Ws/°C	∿	C	electrical capacity	in farads

Transfer of heat by conduction:

$$P_L = \frac{\Delta T}{\frac{1}{K \cdot A}} \qquad R_L = \frac{1}{K \cdot A}$$

Transfer of heat by convection:

$$P_K = K \cdot C \cdot A \sqrt[4]{\frac{\Delta T}{L}} \cdot \Delta T \qquad R_K = \frac{1}{K \cdot C \cdot A \sqrt[4]{\frac{\Delta T}{L}}}$$

Transfer of heat by radiation:

$$P_s = C \cdot A \cdot \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] \qquad R_s = \frac{(T_1 - T_2)}{C \cdot A \cdot \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right]}$$

8.3. A Simple Calculation of the Thermal Resistance of the Heat Sink

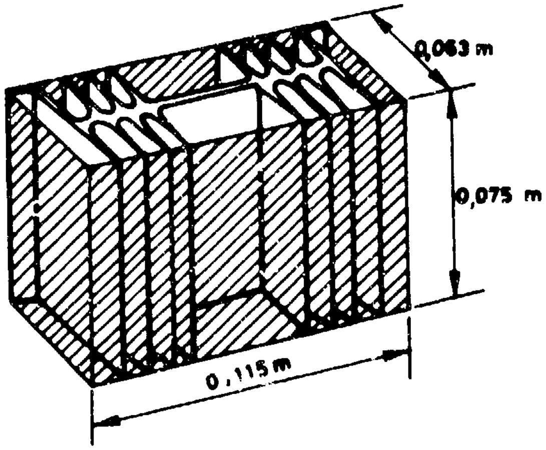


Figure 10. Radiation Area of No. 9.

A very simple method is given below for calculating the thermal resistance of a heat sink.

It is assumed that the heat sink has the same temperature throughout. No. 7 is used as an example for the calculation.

For this type, the area effective for radiation is limited to the area shaded in Figure 10. That is, the area is

$$A = 2 \times 0.115 \times 0.075 + 2 \times 0.063 \times 0.075 = 0.0267 \text{ m}^2.$$

On the basis of Section 8.1, we find the radiation resistance to be

$$R_s = \frac{1}{C \cdot A \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] / (T_1 - T_2)} .$$

Since we are calculating with a black anodized surface, $C \simeq 5.7$, and if we wish to find the thermal resistance at $\Delta T = 100$ and we assume an ambient temperature $T_2 = 20°C$, →293K we get $T_1 = 120°C$. →393K With those values substituted in the formula, we get

$$R_s = \frac{100}{5.7 \times 0.0267 \times 164.9} = 3.98°C/W.$$

In convection it is the entire area of the heat sink that contributes.

A relatively simple way of calculating the area is as follows (see Figure 11):

$A = 2 \times A_2 + 16 + A_1 - 30 \times A_3$
$= 2 \times 0.075 \times 0.092 + 16 \times 0.075 \times 0.063 - 30 \times 0.075 \times 0.003 = 0.0827\ m^2.$

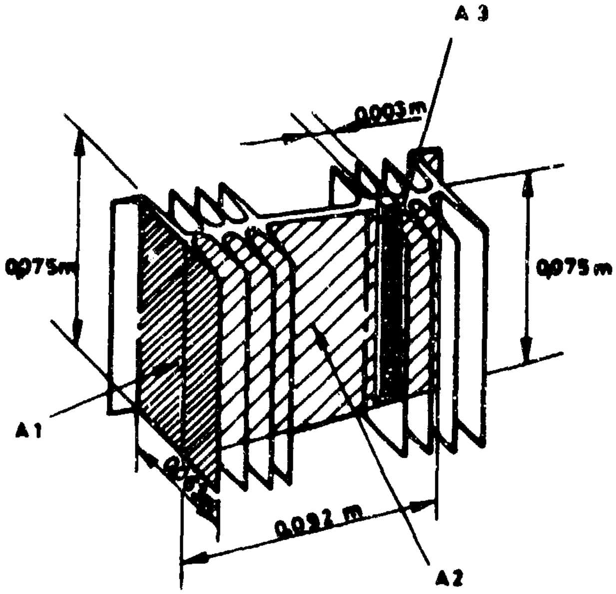


Figure 11. Areal Contribution to the Area Effective for Convection for No. 7.

According to Section 8.1, the thermal convection resistance is

$$R_K = \frac{1}{K \cdot C \cdot A \cdot \sqrt[4]{\frac{T}{L}}},$$

where the characteristic length in this case is L = 0.075 m and the constant C = 0.55, since the surfaces are vertical. From Section 8.2, K = 1.95; therefore

$$R_K = \frac{1}{1.95 \cdot 0.55 \cdot 0.0827 \cdot \sqrt[4]{\frac{100}{0.075}}}$$

$$= 1.87°C/W.$$

Radiation resistance and convection resistance may be regarded as a parallel relationship, and we get

$$R = \frac{R_K \cdot R_s}{R_K + R_s} = \frac{1.87 \cdot 3.98}{1.87 + 3.98} \simeq 1.3°C/W.$$

The measured thermal resistance at $\Delta T = 100°C$ is R = 1.33°C/W. Thus in this case there is close agreement between the calculated and the measured results.

This method of measurement does not show the temperature gradient in the heat sink.

The next section will show the application of a computer program to calculation of the thermal resistance of the heat sink. This calculation gives not only the thermal resistance (indirectly) but also the temperature distribution in the heat sink.

8.4. Computer Calculation

The principle of the use of a computer in thermal design of electronic apparatus will be described in greater detail in this section and will be illustrated both by a little example and by calculation of the temperature distribution in a heat sink.

The technique is based on the use of circuit programs; i.e., the converted magnitudes described in Section 8.1 will be used.

Since the calculation is rather complex and may not seem very clear at first glance, the principle will first be explained by means of a little hypothetical example.

8.4.1. The Principle of the Use of Circuitry Programs

Figure 12a shows a simple example of a thermal model and Figure 12b the analogous electrical model.

The power is generated, for example, by a power transistor in a TO-3 housing. The transistor is mounted in one end of the structure and the heat is carried by conduction down through the structure, part of it being given off to the surroundings by radiation and convection on the way.

The analogous electrical model in Figure 12b is divided into nodes in such a way that between two nodes there is either a resistance or a current generator. Node 0 is the "frame."

Between nodes 1 and 2 the resistance corresponds to the thermal resistance from the junction to the housing of the transistor; the resistance between nodes 2 and 3 corresponds to the thermal contact resistance from the housing; and the resistance between 3 and 4 corresponds to the thermal conduction resistance in the first half of Section 1. The two resistances between 4 and 0 correspond to the convection resistance and radiation resistance to the surroundings respectively, and so on.

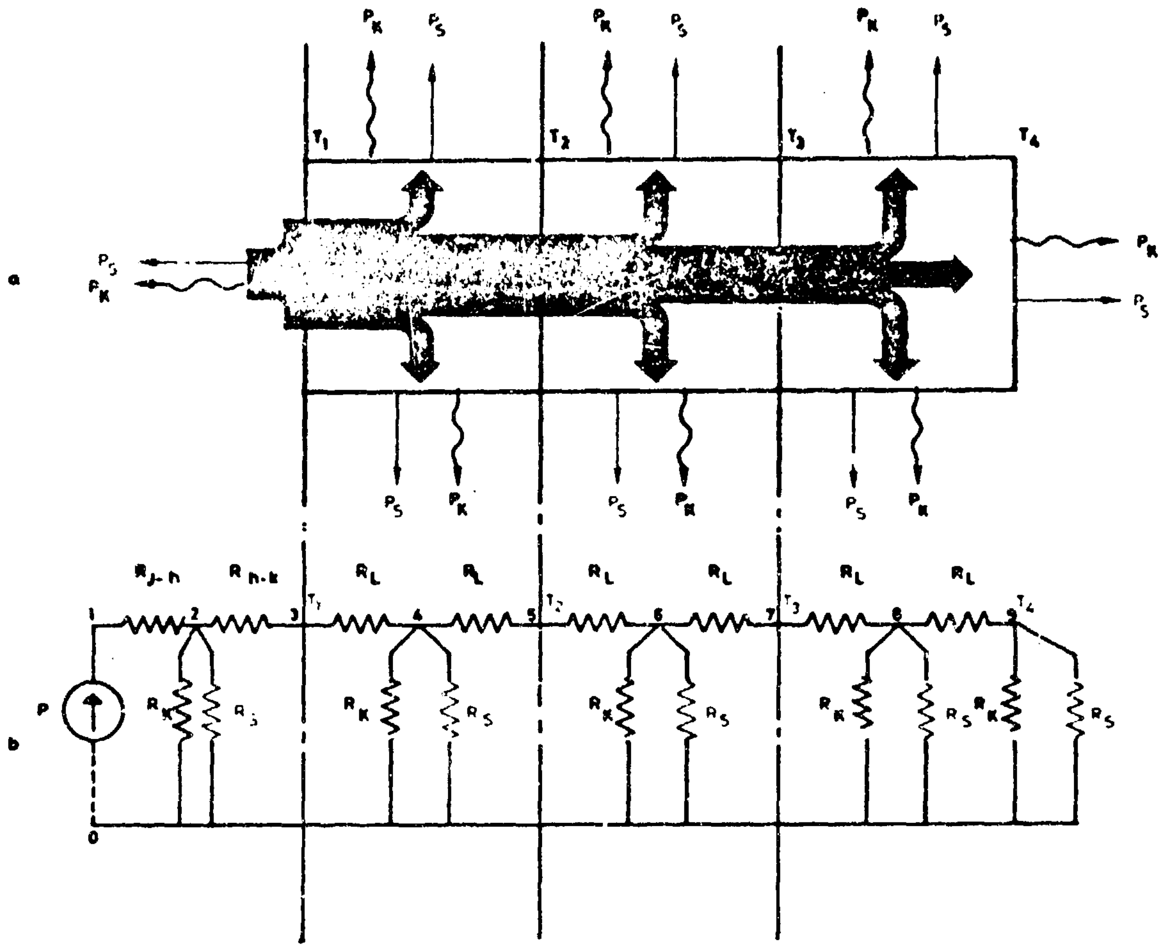


Figure 12. Example of a thermal model (a) and the analogous electrical model corresponding to it (b).

The voltage in relation to the frame, i.e. the temperature in relation to the ambient temperature for the various nodes can be calculated by means of a circuitry program.

It will be found that the more sections the structure is divided into, the more accurate the calculations will be.

8.4.2. Computer Calculation of a Heat Sink.

A corresponding description of the thermal calculation of a heat sink will be given below.

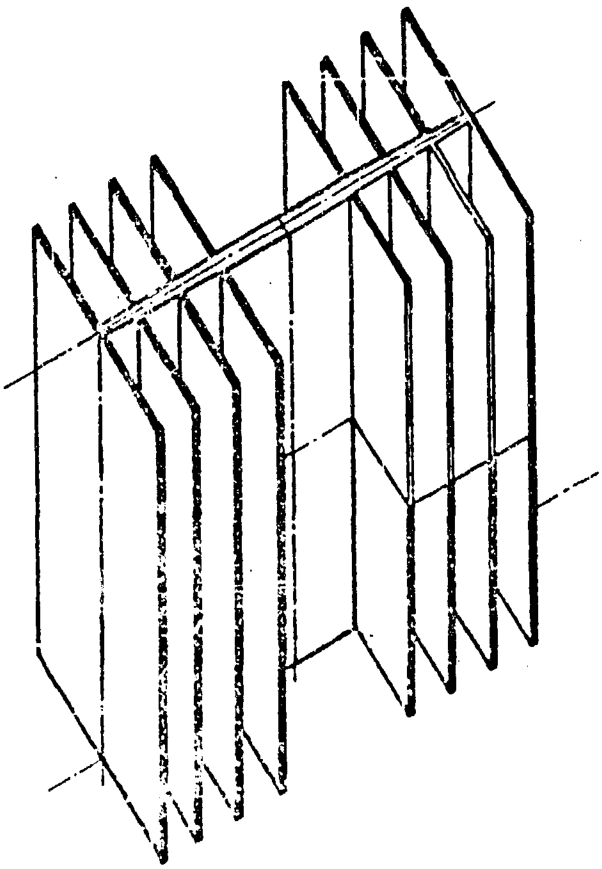


Figure 13. The Type of Heat Sink Used for the Computer Calculations.

Because of the symmetrical design of the heat sink (see Figure 13), we can content ourselves with considering $\frac{1}{8}$ of the whole, although this means leaving the difference in temperature between top and bottom out of consideration.

The heat sink is divided up into sections, such that the first section includes the part from the center line to the first rib inclusive, the second section includes the part between the first and second ribs plus the second rib, etc. The individual sections are then further divided into a number of nodes, and the necessary resistance is plotted.

In calculating the individual resistances, two "types" will be considered, namely the radiation resistance and the convection resistance between the ribs.

Radiation Resistance. -- There is a very low temperature gradient between the ribs, and since the radiation resistance is temperature-dependent, this will be taken into account.

Convection Resistance. -- It follows from what was said in Section 8.I that there is a very strong dependent relationship between the interval between the ribs and the convection constant. To get as flexible a method of calculation as possible, a program was devised that makes these calculations.

The program can generate the necessary input data for the circuitry program, but the input program can only cope with a heat sink design such as is shown in Figure 13.

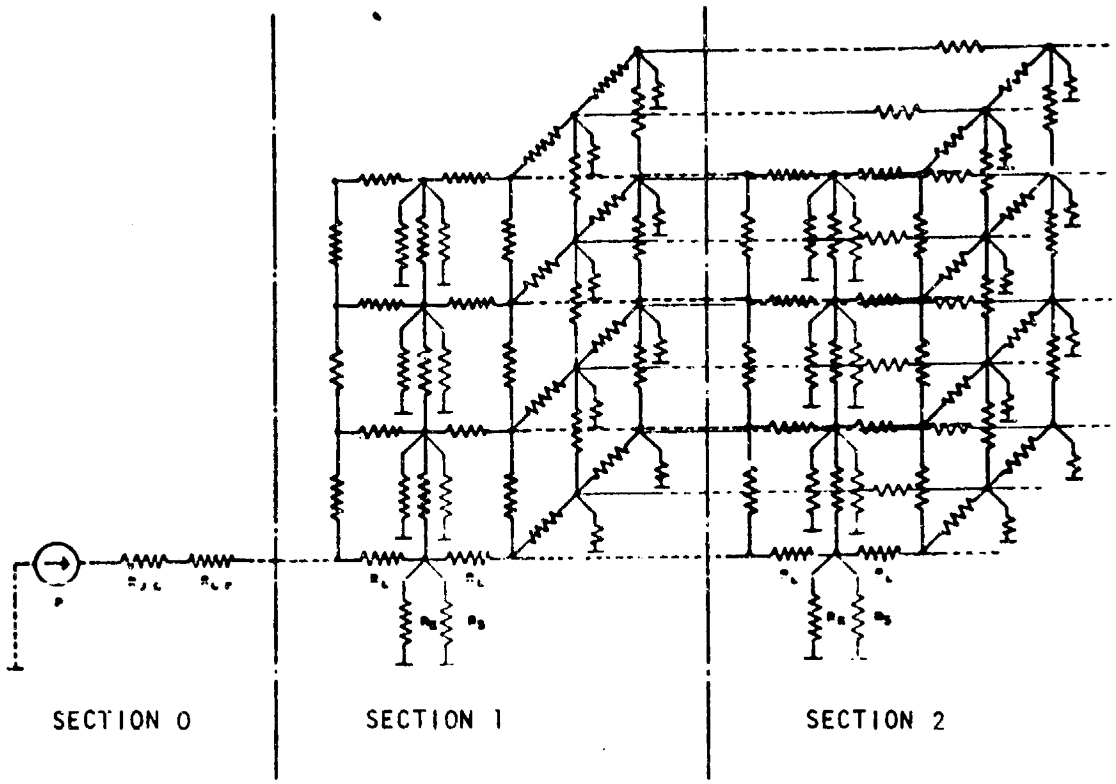


Figure 14. Analogous Electrical Model of a Heat Sink. R_L = Conduction Resistance, R_k = Convection Resistance, and R_S = Radiation Resistance.

The necessary input data will be given as answers to the questions asked by the program, such for example as the dimensions, radiation number, and load.

As a test of the computer calculation just described, a calculation was done on heat sink No. 7, the result of which can be seen in Figure 16. As the graph shows, there is close agreement between the measured and the calculated values. At the coldest measuring point (Point 5) there is a slight discrepancy, which is due to the fact that the outermost rib of the heat sink has two projections provided for mounting the heat sink. These two projections, which were not taken into account in the calculation, will cool the outermost rib still further.

A more detailed description of the input program is to be found in Appendix 1.

```
WHAT IS THE PASSWORD FOR FILE DCFIL
PASSWORD

     DETTE ER ET PROGRAM TIL BEREGNING AF
     TEMP=FORDELINGEN PAA ET KOLEPROFIL

HVAD ER PROFILENS LAENGDE - L.RIBBEHOJDEN - H.
RIBBEAFSTANDEN - A.AFSTANDEN FRA CENTERLINIEN
TIL FORSTE RIBBE - C,TYKKELSEN AF RIBBEN 1. VED
FODEN - S1, OG 2. YDERST - S2, MAAL I METER.
? 0.0375.0.03.0.007.0.018.0.003.0.0015
HVOR MANGE RIBBER ER DER PAA HVER HALVDEL AF PROFILEN
?.4
HVAD ER KROPPENS HALVE TYKKELSE
MELLEM CENTER OG 1. RIBBE, MELLEM 1. OG 2. O.S.V.
VERDIERNE INDTASTES EN AF GANGEN
? 0.002
? 0.002
? 0.002
? 0.0012
LANGDE ER DELT OP I 1 KNUDEPUNKTER
RIBBEN ER DELT OP I 3 KNUDEPUNKTER
ER DETTE OK JA ELLER NEJ? NEJ
HVAD SKAL ANTALLET AF KNUDEPUNKTER VARE I
HENHOLDSVIS LAENGDE OG RIBBE? 4,3
HVAD ER OMGIVELSESTEMPERATUREN.
HVAD ER KOLEPROFILENS OVERTEMPERATUR,
SAMT HVAD ER TEMPERATURFORSKELLEN MELLEM RIBBERNE.
? 29, 35, 1
ER KOLEPROFILEN MONTERET LODRET ELLER VANDRET? LODRET
HVAD ER STRAALINGSTALLET? 5.7
HVAD ER VARMELEDNINGSEVNEN FOR MATERIALET? 200
HVOR STOR ER EFFEKTEN I WATT (1/8)? 3 24
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Figure 15. The Questions Asked by the input program.

8.5. Optimization of the Heat Sink

In the choice of a heat sink there are a number of factors connected with the apparatus in question which restrict the possibilities of choice. The optimal solution is an infinitely thin and infinitely conductive plate with a high emission factor. This theoretical solution is obviously impossible in practice. One of the solutions that come closest, as mentioned above, is a black anodized aluminum plate with a reasonably low conduction resistance. This solution is applicable for the great majority of apparatus, however, only if a very small cooling surface is to be used, since otherwise the dimensions of the plate will be of an order of magnitude that

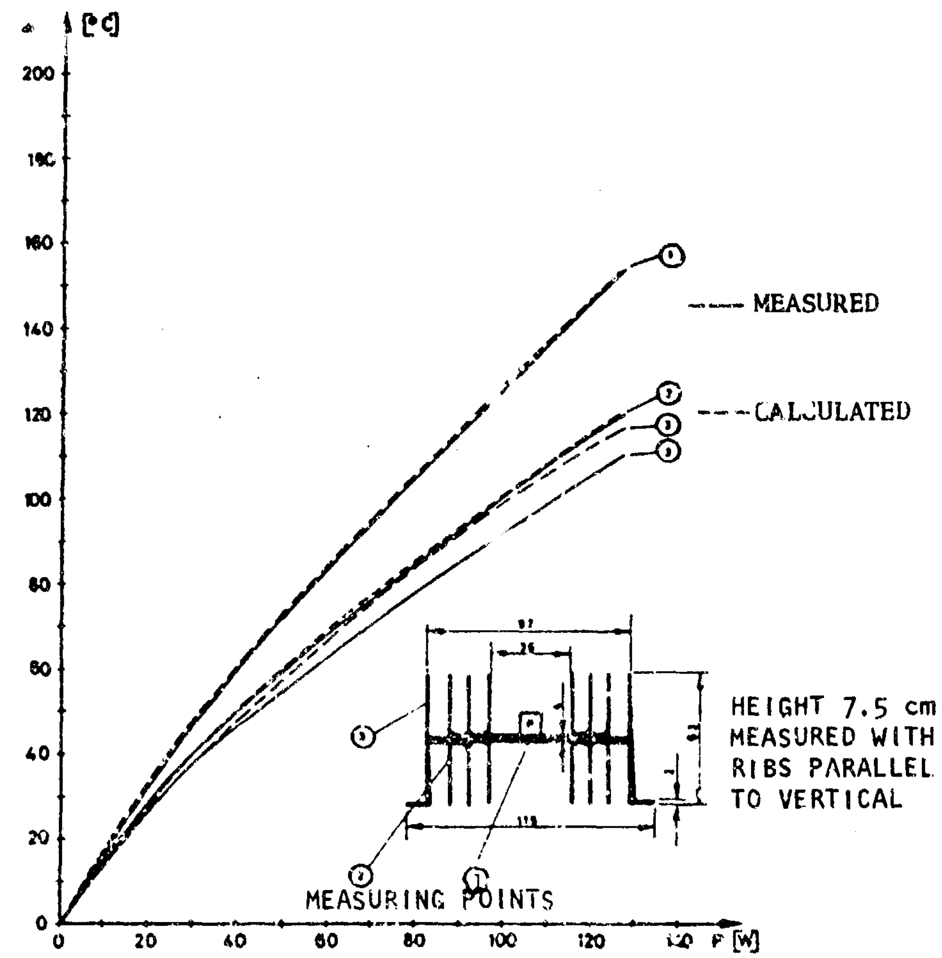

Figure 16. The Result of the Computer Calculations.

is not realistic.

If in the various heat sink designs the plate is cut up into larger and smaller pieces and put together in a different way, an attempt is made to preserve the thermal properties of the plate by providing possibilities of heat transfer by radiation and convection while getting a handier geometric pattern.

In this change from plate to profile the aim must be the best solution; i.e., the greatest possible part of the total area of the cooling profile must be used as a radiation area (this obviously can be significant only if the profile is black anodized) and the distance between the ribs must be

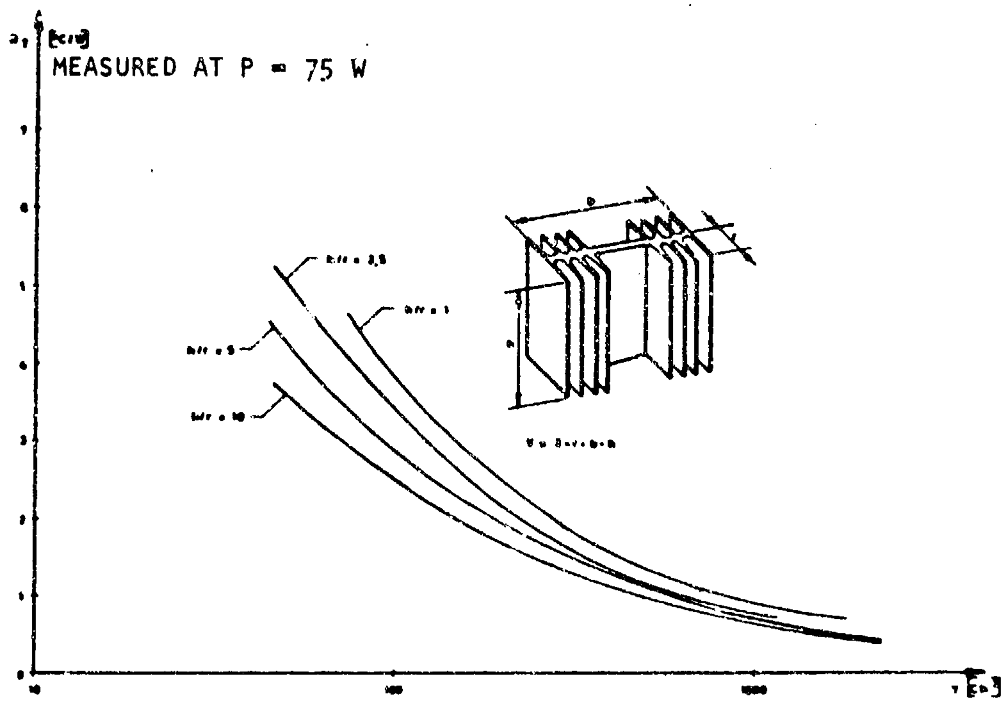

Figure 17. Significance of the Height of the Profile and the Height of the Ribs.

great enough so that the convection resistance is not significantly increased, but not so great as to result in poor utilization of the volume of the heat sink. In addition, the "plates" must be of such thickness that the temperature gradient through the profile does not become too great.

What design and what mutual ratio between the dimensions of the heat sink will give the lowest thermal resistance in relation to the volume of the heat sink, or in other words, the best utilization of the given space?

Figure 17 shows some calculated curves of a heat sink, which is shown above the curves. The calculations were done as indicated in Section 8.4. The ratio of the height of the profile h to the rib height r is taken as a parameter. The greater the ratio the lower the thermal resistance. This is most marked for small heat sinks (small volume), while for the larger heat sinks the ratio of the height of the profile to the height of the rib is of less significance.

If a star-shaped design is used instead, as shown for example in Figure 18, it is found that a ratio of 2.5 between the height of the profile and the radius gives a curve that coincides with the curve in Figure 17 with the ratio 5.

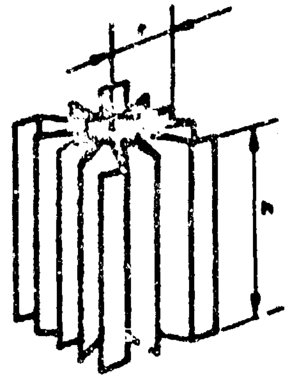

Figure 18. Example of a Star-Shaped Design.

It may thus be concluded that, especially in the case of small profiles, as high a profile as possible may be used and by way of compensation a smaller rib, while for larger profiles the ratio between the height of the profile and the height of the rib is of less significance.

It may also be concluded that the star-shaped profile is a better design than the profile with perpendicular ribs.

9. INTERMITTENT SERVICE

Up to this point the heat sink has been studied only where there was a constant input of heat from the semiconductor, but often the latter is used in intermittent service.

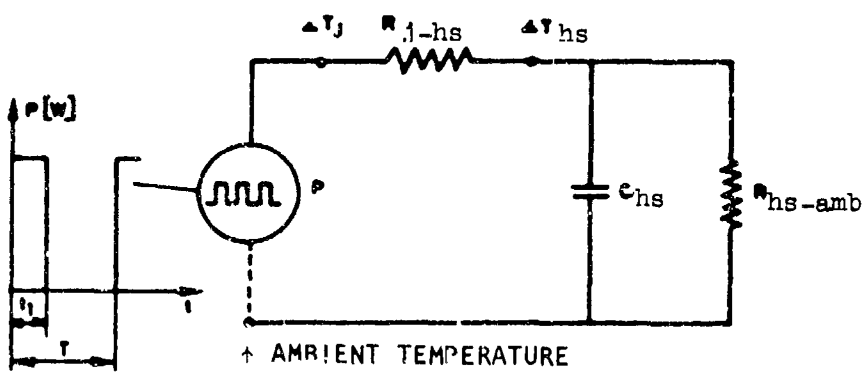

Figure 19. Equivalent Diagram for the Heat Sink and the Load to Which the Heat Sink is Subjected.

This means that the heat sink has a chance to cool off more or less between the individual pulses, so that the same maximum temperature is not attained in the heat sink and the semiconductor as under a constant load.

It is thus possible to apply a greater amount of power in using the semiconductor in pulse operation than in constant operation, without exceeding a specified maximum temperature at the junction.

We shall discuss below how much greater load it is permissible to put on the heat sink in pulse service as compared to steady service.

From the thermal/electrical equivalence, the analogous electronic equivalent diagram for the heat sink can be drawn [Figure 19], where

P	is the power developed in the semiconductor	W
ΔT_j	the temperature rise at the junction in comparison to the ambient temperature	°C
ΔT_{hs}	the maximum temperature rise in the heat sink in comparison to the ambient temperature	°C
T_{amb}	the ambient temperature	°C
R_{j-hs}	the thermal resistance from junction to heat sink	°C/W
R_{hs-amb}	the thermal resistance from heat sink to ambience	°C/W
C_{hs}	the thermal capacity of the heat sink	J/°C
t	the time	min.
t_1	the pulse width or "on" time	min.
T	the period time	min.

In the equivalent diagram the thermal capacity of the semiconductor is disregarded, since it is much lower than the thermal capacity of the heat sink and is therefore significant only in the case of very small pulse times.

The thermal resistance from junction to heat sink includes both the thermal resistance from junction to case and the thermal contact resistance to the heat sink. Provided a heat-conducting paste is used between semiconductor and heat sink, experience shows that R_{j-hs} may be set equal to R_{hs-amb}, since in the overwhelming majority of cases there will be the same thermal resistance for the heat sink selected as for the semiconductor in question.

When a load is applied, there will be a very rapid heating up of the semiconductor, which will give rise to heating up of the heat sink with a time constant corresponding to the product of the thermal resistance of the heat sink and its thermal capacity.

The temperature will approach its maximum value more or less closely depending on the relation between time constant and pulse width.

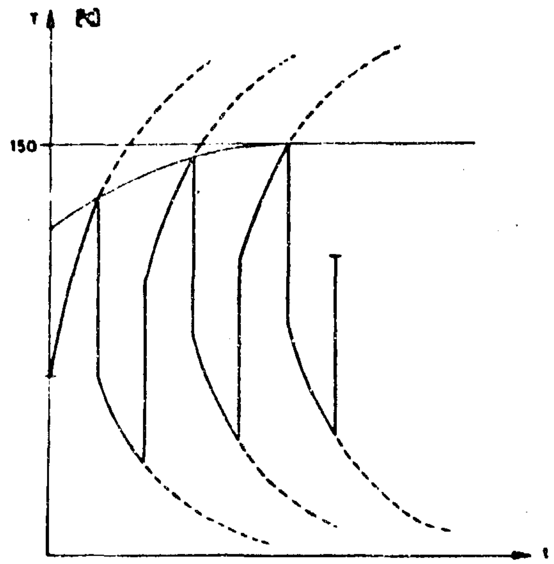

Figure 20. Temperature Variation at the Junction.

When the power is cut off, the junction will very rapidly approach the temperature of the heat sink, and after that its temperature will decline with the aforementioned time constant for the heat sink. How far the temperature will drop depends on the ratio between the "off" time and the thermal time constant.

Figure 20 shows a theoretical temperature variation for the junction.

From what has been said here, the load applied can be calculated as a function of the pulse time T, and with the junction temperature, the two thermal resistances, and the heat capacity of the heat sink as parameters. If the junction temperature is set at the maximum permitted (in the calculations that follow, 150°C), and with a defined maximum ambient temperature (40°C), the maximum permissible load can be calculated, since R_{hs-amb} and C_{hs} are determined by the heat sink chosen as a function of the pulse time T and with the "on-off" duty cycle as primary parameter, according to the formula:

$$P = \frac{\Delta T_{jmax}}{R_{j-hs} + R_{hs-amb} \dfrac{1-e^{-t_1/R_{hs-amb} \cdot C_{hs}}}{1-e^{-T/R_{hs-amb} \cdot C_{hs}}}} .$$

A derivation of the expression is given in Appendix 2.

Figure 21 shows the relation between the maximum permissible load in constant service (P_{DC}) and the maximum permissible load with a specified pulse time (P) as a function of the pulse time (T) in relation to the given time constant for the heat sink (τ) and with the pulse duty cycle t_1/T as a parameter -- $P/P_{DC} = f(T/\tau, t_1/T)$. The time constant τ of the heat sink is equal to the product $R_{hs-amb} \cdot C_{hs}$. The curves are calculated and are valid only with the stipulation that the thermal resistance from heat sink to ambience is equal to that from junction to heat sink.

All curves give a temperature rise from junction to ambience of 110°C.

Since the junction temperature is difficult to measure, in order to verify the theory the same curves are plotted in Figure 22 with the thermal resistance from the junction to the heat sink left out. Naturally these curves give considerably higher permissible loads, since the momentary rise in junction temperature in relation to the temperature of the heat sink when the load is applied is left out. The curves show a temperature rise from the heat sink (the hottest point) to the ambience of 55°C.

Measurements were done on this basis for a number of heat sinks, and various points on the curves were rechecked. It was found that all measurements gave temperatures within ±2°C of the permissible temperatures. Figures 21 and 22 are shown on the next two pages.

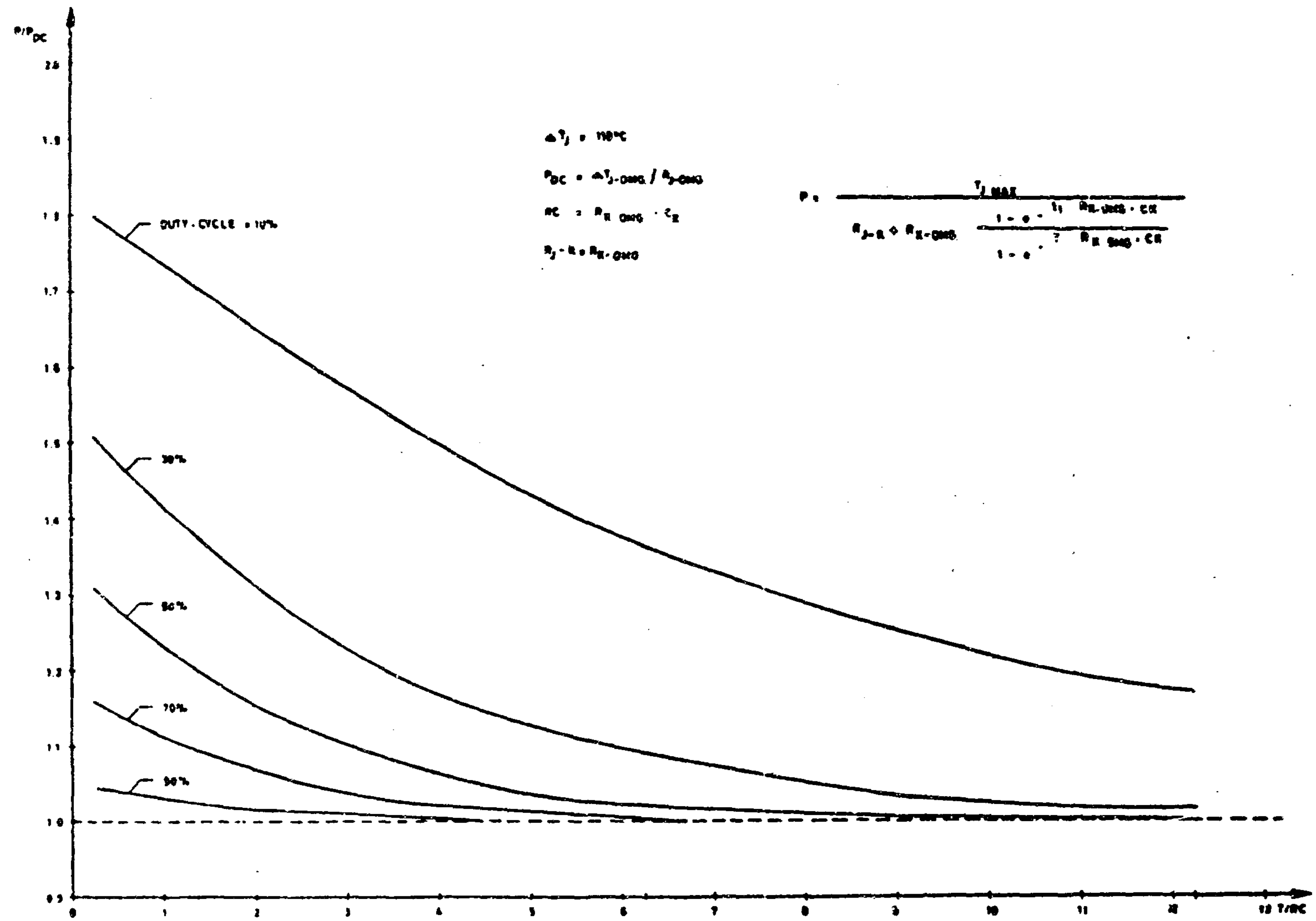

Figure 21. The curves show the maximum permissible load in intermittent service in relation to the maximum permissible load in constant service as a function of the ratio of pulse time to the time constant of the heat sink. All curves give a temperature rise of 110°C at the junction. [The subscripts in the legend signify: J - junction, OMG - ambience, K - heat sink, DC - continuous service.]

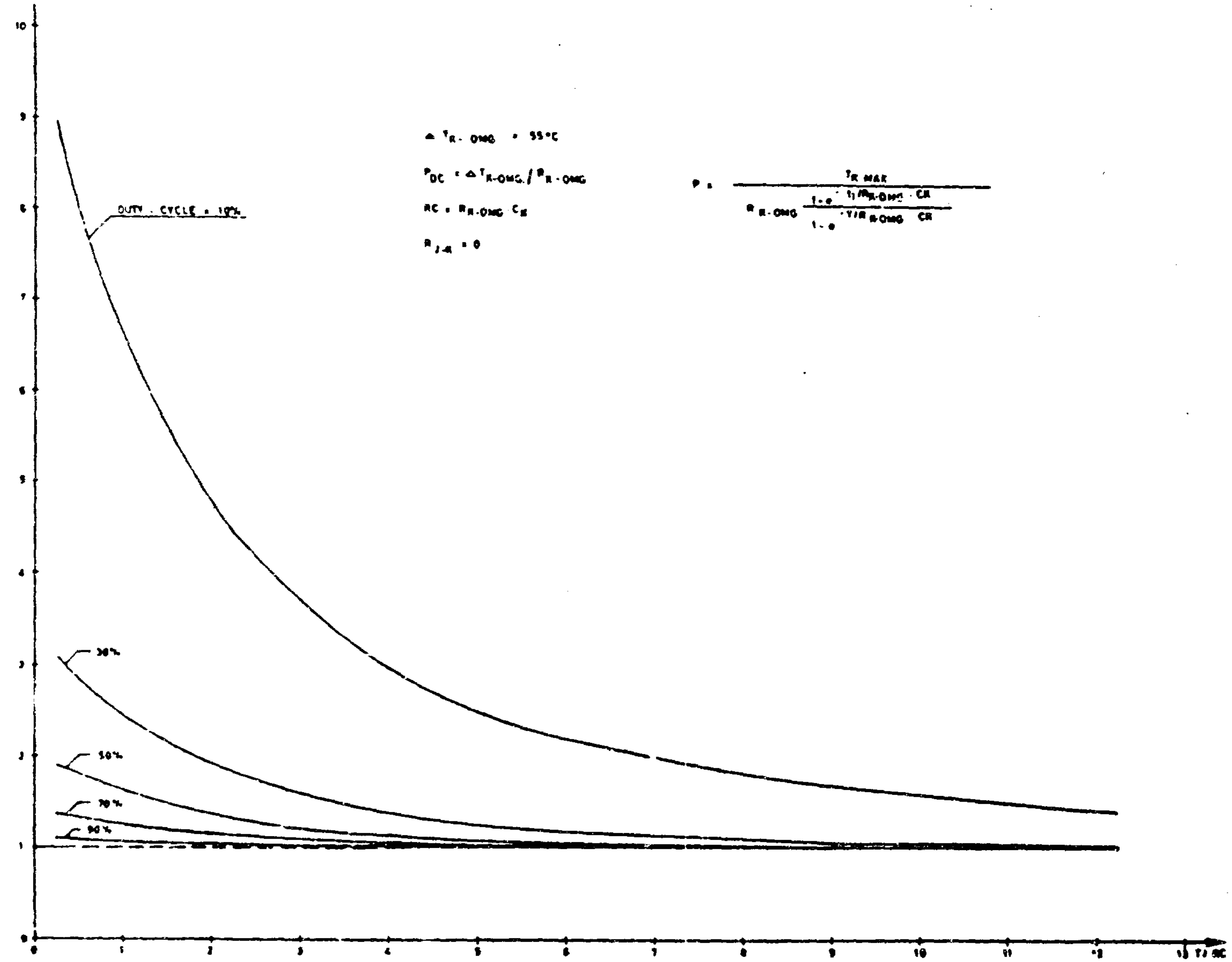

Figure 22. In principle the curves show the same thing as the curves of Figure 21, but the thermal resistance from junction to heat sink is not included. The curves give a temperature rise at the heat sink of 55°C. [The subscripts in the legend signify: J - junction, OMG - ambience, K - heat sink, DC - continuous service.]

10. TEN EFFECTIVE WAYS OF DECREASING THE LIFETIME OF A POWER COMPONENT

1. Forget that an increase in load produces the greatest temperature rise across the greatest thermal resistance (possibly not the heat sink).

2. Forget the thermal contact resistance between semiconductor and heat sink.

3. Dispense with the use of a thermal compound.

4. Cover the heat sink on the top side.

5. Forget the possibility that the apparatus will be "packed down" while in service.

6. Mount the heat sink horizontally or in such a way that the ribs are parallel to the horizontal.

7. Use a bright-surfaced heat sink and calculate with the data sheet's curves for a black anodized one.

8. Forget the possibility of solar radiation.

9. Forget the radiation from other hot components or surfaces.

10. Calculate with a maximum ambient temperature of 25°C, especially when the heat sink is mounted in a cabinet.

APPENDIX 1. COMPUTER CALCULATION

The input program discussed in Section 8.4 is shown on this and the next two pages. The program is in "Basic." Lines 90 to 620 and 1300 and 1310 are input lines. Lines 620 to 960 are calculation of the necessary constants and the rest are output lines. An example of such an output is shown on page 50. The first symbol after the line number indicates whether an impedance, admittance, capacity, or current is involved; the next two figures indicate the nodes between which the admittance, e.g., is situated; the last number shows the size of the component. This output can be used directly as input to a circuitry program.

```
COL 1         08:40        09/30/74

90 FILES DCFIL
100 PRINT "     THIS IS A PROGRAM FOR CALCULATION OF"
110 PRINT"      TEMP. DISTRIBUTION ON A HEAT SINK   "
120 PRINT
130 PRINT "WHAT IS THE PROFILE LENGTH - L, RIB HEIGHT - H,"
140 PRINT "RIB INTERVAL - A, DISTANCE FROM CENTER LINE"
150 PRINT "TO FIRST RIB - C, THICKNESS OF RIB 1ST AT"
160 PRINT "THE BASE - S1, AND 2ND AT THE OUTER END - S2, MEAS. IN METERS."
170 INPUT H3,H4,H1,H2,S1,S2
180 PRINT "HOW MANY RIBS ARE THERE ON EACH HALF OF THE HEAT SINK";
190 PRINT
200 INPUT N
210 PRINT "WHAT IS HALF THE THICKNESS OF THE BODY"
230 PRINT "BETWEEN CENTER AND 1ST RIB, BETWEEN 1ST AND 2ND ETC"
235 PRINT "VALUES ARE PUNCHED IN ONE AT A TIME"
240 FOR K=1 TO N
250 INPUT M(K)
270 NEXT K
280 E=INT(H3/0.03+0.5)
290 F=INT(H4/0.01+0.5)
300 PRINT "LENGTH IS DIVIDED INTO I";E;"NODES"
310 PRINT "RIBS ARE DIVIDED INTO I";F;"NODES"
320 PRINT "IS THIS OK YES OR NO";
330 INPUT XS
340 IF XS="YES" THEN 380
350 PRINT "WHAT SHALL THE NUMBER OF NODES BE IN"
360 PRINT "RELATION TO LENGTH AND RIBS";
370 INPUT E,F
380 PRINT "WHAT IS THE AMBIENT TEMPERATURE,"
390 PRINT "WHAT IS THE TOP TEMPERATURE OF THE HEAT SINK,"
400 PRINT "AND WHAT IS THE TEMPERATURE GRADIENT BETWEEN THE RIBS."
410 INPUT T3,T1,T2
415 T3=T3+273
420 01=(T3+T1)^3+(T3+T1)*T3^2+T3*(T3+T1)^2+T3 3
430 02=(T1+T2+T3)^3+(T1+T2+T3)*(T3+T1)^2+(T3+T1)*(T1+T2+T3 2+(T3+T1)^3
```

```
440 PRINT "IS THE HEAT SINK MOUNTED VERTICALLY OR HORIZONTALLY";
450 INPUT X$
460 IF X$= "VERTICALLY" THEN 510
470 X5=2
480 V1=0.71
490 L2=H4/(F-1)
500 GO TO 540
510 L1=H3/E
520 V1=0.55
530 L2=L1
540 V2=0.55
550 PRINT "WHAT IS THE RADIATION NUMBER";
560 INPUT Z1
570 Z2=11.54*Z1/(5.77-2*Z1)
580 Z1=Z1*1E-8
590 Z2=Z2*1E-8
600 PRINT "WHAT IS THE THERMAL CONDUCTIVITY OF THE MATERIAL";
610 INPUT G
620 X1=2*G*H3/E
630 X2=G*(E-1)/H3
640 X3=G*H4*(E-1)/(F-1)/H3
650 X4=G*H3*(F-1)/H4/E
660 X6=2.5*H3*V1/E
670 X7=2*V2*H4*H3/E/(F-1)
680 X8=Z1*H3*O1/E
710 Y(4)=Z1*H4*H3*O1/(E*(F-1))
720 Y(8)=Z2*H4*H3*O2/(E*(F-1))
730 Y(9)=2*G*M(1)*H3/H2/E
740 Y(10)=G*H2*M(1)*(E-1)/(2*H3)
760 A=F*E
770 P=A+E
780 SCRATCH#1
790 FOR K=1 TO N
800 Q=E*(F+1)*(K-1)+E
820 IF K>1 THEN 850
830 H5=H2
840 GO TO 860
850 H5=H1
860 FOR I=1 TO E
870 FOR J=1 TO (F+1)
880 IF J>2 THEN 930
890 X(1,2)=X1*M(K)/H5
895 IF K>1 THEN 905
900 X(1,1)=X2*M(K)*H5/2
902 GO TO 910
905 X(1,1)=X2*M(K)*H5
910 X(2,1)=X2*M(K)*S1
920 GO TO 940
930 X(J,1)=X3*(S2+(S1-S2)*(1-(J-2)/F))
940 IF J<2 THEN 960
950 X(J,2)=X4*(S2+(S1-S2)*(1-(J-2)/F))
960 IF I=1 THEN 980
970 WRITE#1,"Y";Q+(I-1)+(J-1)*E;Q+(I-1)+(J-1)*E+1;X(J,1);
980 IF J=(F+1) THEN 1140
990 WRITE#1,"Y";Q+I+(J-1)*E;Q+I+J*E;X(J,2);
1000 IF J=1 THEN 1070
1010 D=Q+I+E*(J-1)+E
1020 Y(3)=X7*SQR(SQR(T1/L2))
1030 WRITE#1,"Y";D;0;Y(3);
1040 IF K<N THEN 1060
1050 WRITE#1,"Y";D;0;Y(4);
1060 GO TO 1130
1070 IF X5<>2 THEN 1090
1080 L1=(H5*H3/E)/(H3/E+H5)
1090 Y(5)=X6*H5*SQR(SQR(T1/L1))
1100 Y(6)=X8*H5
1110 WRITE#1,"Y";Q+I;0;Y(5);
```

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```
1120 WRITE#1,"Y";Q+I;0;Y(6);
1130 NEXT J
1140 NEXT I
1150 FOR J=1 TO E
1160 IF K=1 THEN 1195
1170 B=2*E+J+P*(K-2)
1180 Y(7)=X1*M(K)/H1
1190 WRITE#1,"Y";B;B+A;Y(7);
1195 IF K=N THEN 1280
1200 FOR I=1 TO (F-1)
1210 C=3*E+(K-1)*P+J+E*(I-1)
1220 WRITE#1,"Y";C;C+P;Y(8);
1230 NEXT I
1240 IF K>1 THEN 1280
1250 WRITE#1,"Y";J;J+E;Y(9);
1260 IF J=E THEN 1290
1270 WRITE#1,"Y";J;J+1;Y(10);
1280 NEXT J
1290 NEXT K
1300 PRINT "HOW GREAT IS THE POWER IN WATTS (1/8)";
1310 INPUT I
1320 WRITE#1,"I";0;1;I/2;
1325 WRITE#1,"I";0;2;I/2;
1330 RESTORE#1
1340 END
```

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```
KOL2        14:26    ON 74-02-27 D

1000   Y   5   9   1.83333
1010   Y   5   0   4.55161E-03
1020   Y   5   0   3.41459E-04
1030   Y   9   13      .635556
1040   Y   13      0   7.58602E-03
1050   Y   13      17      .537778
1060   Y   17      0   7.58602E-03
1070   Y   5   6   .147273
1080   Y   6   10      1.83333
1090   Y       0   4.55161E-03
1100   Y       0   3.41459E-04
1110   Y   9   10      3.27273E-02
1120   Y   10      14      .635556
1130   Y   14      0   7.58602E-03
1140   Y   13      14      .141818
1150   Y   14      18      .537778
1160   Y   18      0   7.58602E-03
1170   Y   17      18      .12
1180   Y   6   7   .147273
1190   Y   7   11      1.83333
1200   Y   7   0   4.55161E-03
1210   Y   7   0   3.41459E-04
1220   Y   10      11      3.27273E-02
1230   Y   11      15      .635556
1240   Y   15      0   7.58602E-03
1250   Y   14      15      .141818
1260   Y   15      19      .537778
1270   Y   19      0   7.58602E-03
1280   Y   18      19      .12
1290   Y   7   8   .147273
1300   Y   8   12      1.83333
1310   Y   8   0   4.55161E-03
1320   Y   8   0   3.41459E-04
1330   Y   11      12      3.27273E-02
1340   Y   12      16      .635556
1350   Y   16      0   7.58602E-03
1360   Y   15      16      .141818
1370   Y   16      20      .537778
1380   Y   20      0   7.58602E-03
1390   Y   19      20      .12
1400   Y   13      29      8.89840E-04
1410   Y   17      33      8.89840E-04
1420   Y   1   5   1.83333
1430   Y   1   2   .147273
1440   Y   14      30      8.89840E-04
1450   Y   18      34      8.89840E-04
```

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APPENDIX 2. DERIVATION OF THE EXPRESSION $P = \dfrac{\Delta T_{jmax}}{R_{j-hs} + R_{hs-amb}\dfrac{1-e^{-t_1/R_{hs-amb}\cdot C_{hs}}}{1-e^{-T/R_{hs-amb}\cdot C_{hs}}}}$

In the following derivation of the above expression the thermal/electrical equivalents shown in the table below are used.

Thermal Magnitude				Electrical Analogy		
P	power	W	∿	I	current	A
T	temperature	°C	∿	V	voltage	V
R	thermal resistance	°C/W	∿	R	electrical resistance	Ω
C	thermal capacity	J/°C	∿	C	electrical capacity	F

This means that in the present case (cf. Figure 19) we have the following analogy:

$$P \sim i$$
$$v' \sim \Delta T_j$$
$$v \sim \Delta T_{hs}$$
$$R_1 \sim R_{j-hs}$$
$$R_2 \sim R_{hs-amb}$$
$$C \sim C_{hs}$$

The figure below gives the analogous electrical equivalent diagram.

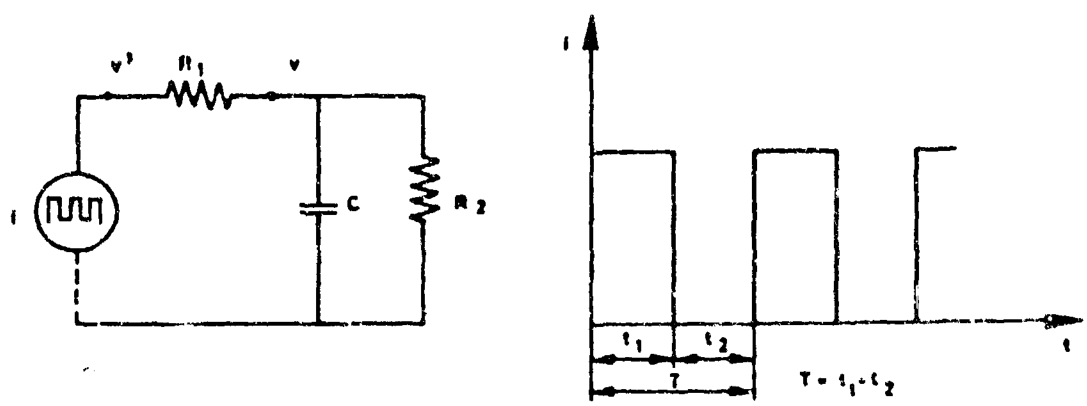

Figure 23. Analogous Electrical Equivalent Diagram for Intermittent (Pulse) Service of the Heat Sink.

Since we are only interested in the maximum temperature rise in the junction, in the derivation of the expression we shall concentrate only on V'_{max} and V_{max} respectively and not v' and v.

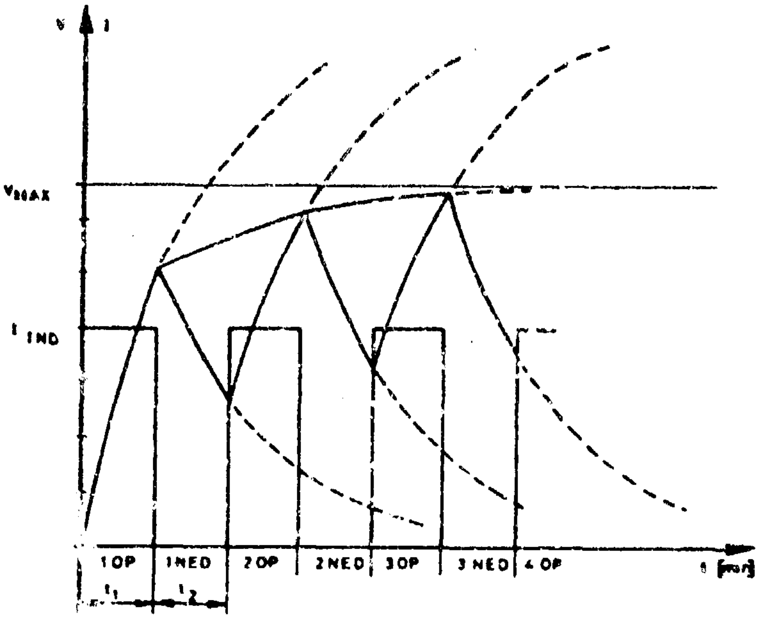

Figure 24. Analogous Maximum Load for Heat Sink as a Function of the Time. [For OP read TOP; for NED read BOTTOM.]

Since what we have here is a current generator, the voltage at R_1 will contribute with a constant voltage drop in the period t_1 and no voltage drop in the period t_2.

To facilitate the calculations, V_{max} is first found instead of V'_{max}; i.e., we get V_{max} expressed in terms of i, t_1, and t_2. Then the voltage over R_1 can be added to the result to get V'_{max}.

Lastly, from this expression i can be found as a function of V'_{max}, R_1 and R_2, C and t_1 and t_2, and these can be converted back to the thermal magnitudes.

Figure 24 shows the voltage v as a function of the time.

The calculation is shown below.

The first voltage maximum can be expressed as

$$V_{1max} = i \cdot R_2 (1-e^{-t_1/R_2C}) ,$$

The first minimum

$$V_{1min} = V_{1max} \cdot e^{-t_2/R_2C} ,$$

The second maximum

$$V_{2max} = (i \cdot R_2 - v_{1min}) \; (1-e^{-t_1/R_2C}) + v_{1min}$$

$$= i \cdot R_2 (1-e^{-t_1/R_2C}) + v_{1min} e^{-t_1/R_2C} ,$$

The second minimum

$$V_{2_{min}} = V_{2_{max}} \cdot e^{-t_2/R_2C} ,$$

The third maximum

$$V_{3_{max}} = i \cdot R_2 (1-e^{-t_1/R_2C}) + V_{2_{min}} e^{-t_1/R_2C} ,$$

The third minimum

$$V_{3_{min}} = V_{3_{max}} \cdot e^{-t_2/R_2C} ,$$

The fourth maximum

$$V_{4_{max}} = i \cdot R_2 (1-e^{-t_1/R_2C}) + V_{3_{min}} e^{-t_1/R_2C} ,$$

etc.

Substituting V_3 for V_4, V_2 for V_3, etc., we get

$$V_{4_{max}} = i \cdot R_2 (1-e^{-t_1/R_2C}) + (i \cdot R_2 (1-e^{-t_1/R_2C}) + (i \cdot R_2 (1-e^{-t_1/R_2C}) +$$

$$i \cdot R_2 (1-e^{-t_1/R_2C}) \cdot e^{-t_1/R_2C} \cdot e^{-t_2/R_2C}) e^{-t_1/R_2C} \cdot e^{-t_2/R_2C}) \cdot$$

$$e^{-t_1/R_2C} \cdot e^{-t_2/R_2C} .$$

Reducing this expression, we get

$$V_{4_{max}} = i \cdot R_2 (1-e^{-t_1/R_2C}) \; [\; 1+e^{-(t_1+t_2)R_2C} + e^{-2(t_1+t_2)/R_2C}$$

$$+ e^{-3(t_1+t_2)/R_2C}].$$

Since the expression in square brackets is a quotient series, V_{max} can be found by allowing the series to approach the infinite.

$$V_{max} = i \cdot R \; \frac{1-e^{-t_1/R_2C}}{1-e^{-(t_1+t_2)/R_2C}} \cdot$$

The voltage field across R_1 is added.

$$V_{j\ max} = i \; , R_1 + R_2 \; \frac{1-e^{-t_1/R_2C}}{1-e^{-(t_1+t_2)R_2C}}) \cdot$$

Since we wish to find i as a function of t_1 and t_2, we divide the parenthesis through and we have

$$i = \frac{V_{jmax}}{R_1 + R_2 \frac{1-e^{-t_1/R_2C}}{1-e^{-(t_1+t_2)/R_2C}}} .$$

The electrical magnitudes are replaced with the equivalent thermal magnitudes, and we have the expression

$$P = \frac{\Delta T_{jmax}}{R_{j-hs} + R_{hs-amb} \frac{1-e^{-t_1/R_{hs-amb} \cdot C_{hs}}}{1-e^{-T/R_{hs-amb} C_{hs}}}} .$$

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