DEVELOPMENT OF A TIME-VARIANT FIGURE-OF-MERIT FOR USE IN ANALYSIS OF AIR COMBAT MANEUVERING ENGAGEMENTS

NAVAL AIR TEST CENTER

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Technical Memorandum

DEVELOPMENT OF A TIME-VARIANT FIGURE-OF-MERIT FOR USE IN ANALYSIS OF AIR COMBAT MANEUVERING ENGAGEMENTS

by

Mr. W. R. Simpson

Strike Aircraft Test Directorate

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Recent emphasis on Air Combat Maneuvering Testing has led to the development of improved analytic tools for the analysis of air combat data. These improved tools are characterized by a time-variant figure-of-merit or performance index for the determination of engagement treads in one-on-one and multiaircraft air combat. A derivation of the performance index, together with a comparison to present analytic tools, is given. Application to a variety of analysis problems, including design, training, test and evaluation, and operations, are discussed.

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PREFACE

Recent emphasis on Air Combat Maneuvering (ACM) including the congressionally directed AIMVAL/ACEVAL trials has created interest in developing methods of analysis to assess aircraft capabilities, pilot proficiency, force requirements, etc. Several methods are in use by industry, but many emphasize specific aircraft characteristics and are therefore limited in their application. This report describes a continuous ACM performance index developed by Mr. W. S. Stewart (Naval Weapons Center), Dr. R. A. Oberle (Center for Naval Analysis), and Mr. W. R. Simpson (Naval Air Test Center). Data from the VF-51/VF-111 (Carrier Air Group 15) deployment to the Air Combat Maneuvering Range at Yuma, Arizona, were used to validate the concept. Several aspects of the performance index and other analysis methods are being explored jointly by the Naval Fighter Weapons School, the Naval Weapons Center, the Center for Naval Analyses. and the Naval Air Test Center. The Naval Air Test Center participation was funded by the Joint Test Coordinating Group, Joint Munitions Effectiveness Meeting, chaired by the Naval Weapons Center.

APPROVED FOR RELEASE

J. G. WISSLER, CAPT, USN
ACTING COMMANDER
NAVAL AIR TEST CENTER

A

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INTRODUCTION

BACKGROUND

1. The development of adequate analysis techniques in the area of air combat maneuvering (ACM) is vital to the definition of the mission effectiveness of fighter aircraft or of performance devices installed on fighter aircraft. For the most part, however, the development of ACM testing and analysis procedures has lagged other evaluation areas such as air-to-ground or aircraft performance and flying qualities testing. NAVAIRTESTCEN evaluation methods for ACM has historically been primarily qualitative. While the qualitative aspects of ACM are, and will continue to be, an important area of any ACM analysis, a quantitative basis is required for extrapolation and prediction by any method other than conjecture.

2. Detailed procedures for quantitative evaluations have been available in the industry for some time, and NAVAIRTESTCEN began quantitative ACM evaluation with a program to determine the tactical improvement available with inflight thrust reversing on an F-11A airplane (reference 1).

PURPOSE

3. The purpose of this memorandum is to present the experiences gained in ACM testing by NAVAIRTESTCEN personnel and to introduce the derivation and application of a new time-based analysis parameter which is presently under development.

METHOD OF TESTS

4. The test methods for ACM should include both qualitative and quantitative testing. These tests should be done separately so that the pilot is not afraid to try innovative maneuvers during the trials. The qualitative trials allow for an exploration of tactics and techniques while familiarizing the pilot with his aircraft performance and response under ACM conditions.

5. The quantitative trials should be completed under engagement rules which are realistic (within safety limits), and the criteria are victory and survival. Quantification may come from a variety of sources (radar, onboard tape, etc.) or a combination of sources, but the easiest form of quantification is by the use of military ranges configured for ACM evaluations.

1) NAVAIRTESTCEN Report SA-C3R-76, Navy Evaluation of F-11A Inflight Thrust Control System, Confidential Supplement to NAVAIRTESTCEN Technical Report SA-75R-75, of 26 Jan 1976.

TEST INSTRUMENTATION (RANGES)

6. The Air Combat Maneuvering Range (ACMR) was developed for the U. S. Navy by the Cubic Corporation for use in pilot training and research, development, and operational test and evaluation of ACM problems. The total system consists of an area of controlled airspace about 60 miles (100 km) east of Yuma, Arizona, with tracking stations and radio link to a computer complex for display and communications. The system is capable of handling up to four aircraft for real-time ACM analysis. A complete description of the ACMR is contained in reference 2. An east-coast ACMR is under construction at Cherry Point, North Carolina.

7. The Air Combat Maneuvering Instrumentation (ACMI) Range is a similar facility under development for the U. S. Air Force by the Cubic Corporation for use in pilot training and ACEVAL (as well as other) tests. The system is basically an extension of the ACMR and is located at the Nell's Air Force Base about 10 miles (15 km) north of Las Vegas, Nevada. The ACMI Range is capable of handling up to eight aircraft for real-time ACM analysis. A complete description of the ACMI Range is contained in reference 3.

8. The ACMR and ACMI systems are the same in basic concept and may be characterized as inertially aided multilateration systems designed to accurately track, monitor, and record high performance aircraft data. Using simultaneous range measurements from multiple ground station, a real-time multilateration computation uniquely determines the position of the aircraft with respect to the ground reference network. Inertial data communicated from an aircraft pod via an integral data link permits determination of aircraft attitude. These data are resolved into a situation display and alphanumerics of the engagement particulars and are made available for display at the control center. Such resolved data are recorded on magnetic tape for later playback and debriefing. These functions are provided by the following subsystems:

a. Display and Debriefing Subsystem (DDS) which includes three-dimensional situation displays, alphanumeric, and status displays (reference 2).

b. Computation and Control Subsystem (CCS), a large multiprocessor for real-time computation (reference 4).

2) Cubic Corporation Report No. P-74000, The Air Combat Maneuvering Range (ACMR) AN/USQ-T2(V), of 1974.

3) ACEVAL-AIMVAL Preliminary Test Plan, Volume III (Engineering), of 22 Feb 1976 (for official use only)

4) Cubic Corporation Report No. SP/525-5A, Performance Specification for the Control and Computation Subsystem (CCS), of Oct 1974.

c. Tracking Instrumentation Subsystem (TIS), a high-speed phase-comparison ranging system (reference 5).

d. Airborne Instrumentation Subsystem (AIS), a self-contained pod with sensors for measuring and transmitting airplane parameters (reference 6).

ANALYSIS PARAMETERS

9. The primary parameters to be used in the analysis are given in table I. These data are obtained directly from the ranges.

Table I

Primary ACM Parameters

Parameter	Definition
Airplane Parameters	
Angle of Attack (AOA)	Angle between the free stream flow and the airplane reference line
Normal Acceleration (N_z)	The load factor taken perpendicular to the flight path
Altitude (ALT)	Geometric altitude above ground level
Indicated Airspeed (IAS)	Airspeed measured by AIS uncorrected for position error
Specific Energy (E_S)	Sum of the weight specific kinetic and potential energies
Target Mach Number (MT)	Mach number of the target airplane
Interairplane Parameters	
Range (R)	Line of sight distance between the c. g. of two airplanes
Closing Velocity (VC)	Time rate of change of range
Antenna Train Angle (ATA)	The angle between the aircraft reference line forward of the c. g. and any sight line
Angle Off Tail (AOT)	The angle between the aircraft reference line aft of the c. g. and any sight line

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5) Cubic Corporation Report No. SP/006-201, Specification for Tracking Instrumentation Subsystem (TIS), of 1974.

6) Cubic Corporation Report No. SP/525-38C, Specification for Airborne Instrumentation Subsystem (AIS), of Oct 1974.

10. The basic interairplane parameters defined in table I are shown geometrically in figure 1, together with a cone of lethality.

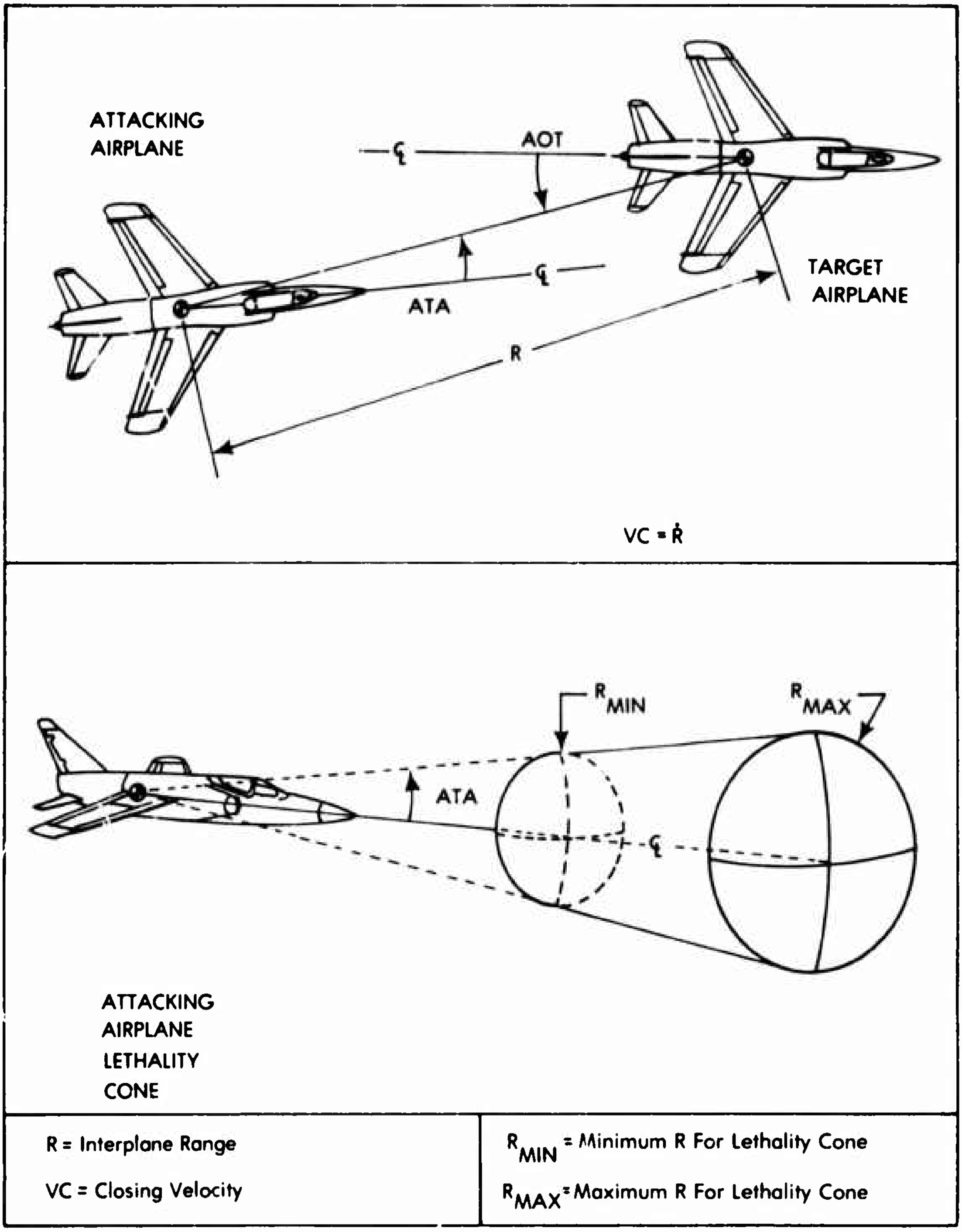

Figure 1
Interairplane ACM Geometry

ACM ANALYSIS CRITERIA

11. Three distinct analysis techniques have been developed for use in the analysis of ACM engagements. The technique used is dependent upon the intended use of the ACM data. The first type of analysis stems from a desire for simplicity in computation and usability of the data by people not involved in the actual computations. Such an analysis technique is represented by a discrete computation based upon a given set of conditions. The advantages of this type of computation are its inherent simplicity, basically self-explanatory conclusions, and ease of use. The disadvantages are that it does not account for the total range of possibilities, and it may be difficult to distinguish the effect of isolated occurrences in the data. Such an analysis technique is used in most tactical manuals for pilots and is typified by the use of kill probabilities and average energy states or box scoring of kills (reference 7).

12. The second technique stems from the desire to continuously monitor airplane performance during an ACM encounter and follow the progress of events before, during, and after a specific occurrence (such as slat deployment). This type of analysis is characterized by the development of a performance index which is purported to be indicative of the airplane's relative ACM performance. The advantage of such an analysis technique is the ease in which numerical data can be used to form conclusions and recommendations. The disadvantages of such a system of analysis are the inherent complexity, the difficulty in assigning a proper form of the performance index, and the required detailed knowledge of the computations in order to draw conclusions. An example of such as analysis technique is the airplane directional angle computation developed by the British, McDonnell-Douglas, and NASA, Langley, for use with the HARRIER (AV-8A) Vectoring in Forward Flight (VIFF) program (reference 8).

13. The final analysis technique combines the simplicity of the discrete analysis (and some of its disadvantages) with the increased analysis capability of the continuous analysis. The technique is characterized by taking the continuum of interairplane relationships and breaking it into discrete segments or ACM states. The airplane or hardware is then evaluated against its ability to maintain or change the ACM state of the airplane. The ACM state analysis technique lies between the continuum approach and the discrete approach and can be driven to either extreme. By defining a very large number of states, the analysis becomes nearly continuum and carries with it the advantages and disadvantages previously noted. By taking only one state of interest (such as kills), the analysis becomes discrete and carries with it the associated advantages and disadvantages of the discrete analysis. The ACM state analysis technique is common throughout the industry and has been used

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7) Northrop Corporation Report NB74-72, F-5E Combat Tactics Manual, Part 3 Air-to-Air Combat Effectiveness, of Sep 1974.

8) McDonnell-Douglas Corporation Report, Analysis of HARRIER ACM Flight Trials, Contract No. N00019-73-C-00118, of 1974 (Secret Report).

on flight data, simulator data, and computer generated data (an example is reference 1). The technique defines ACM states (e.g., offensive weapons, offensive, neutral, defensive, and defensive fatal). Such an array of states provides simplicity of computation and easily used self-explanatory results. Because the analysis is not continuous, it does not allow for optimization of events or time dependent analysis.

14. The analysis technique developed for use in this research falls in the category of the continuous analysis techniques. A figure-of-merit, or performance index, is computed at each point in the engagement. The time variance of the figure-of-merit is then given as the engagement trend. Figure 2 gives two performance indices presently in use for reference. The two measures of figure 2 are tailored to turning performance and thus include only angular terms. Northrop also uses an additional continuous index given as the differential energy integral:

$$I_{\Delta E_S} = \int_0^t (Es_2 - Es_1)\, dt \qquad (1)$$

This is designed to measure the time advantage of a thrust minus drag or thrust-to-weight differential. Other indices exist but are also tailored to a specific aircraft performance trait and are not indicative of interaircraft interaction.

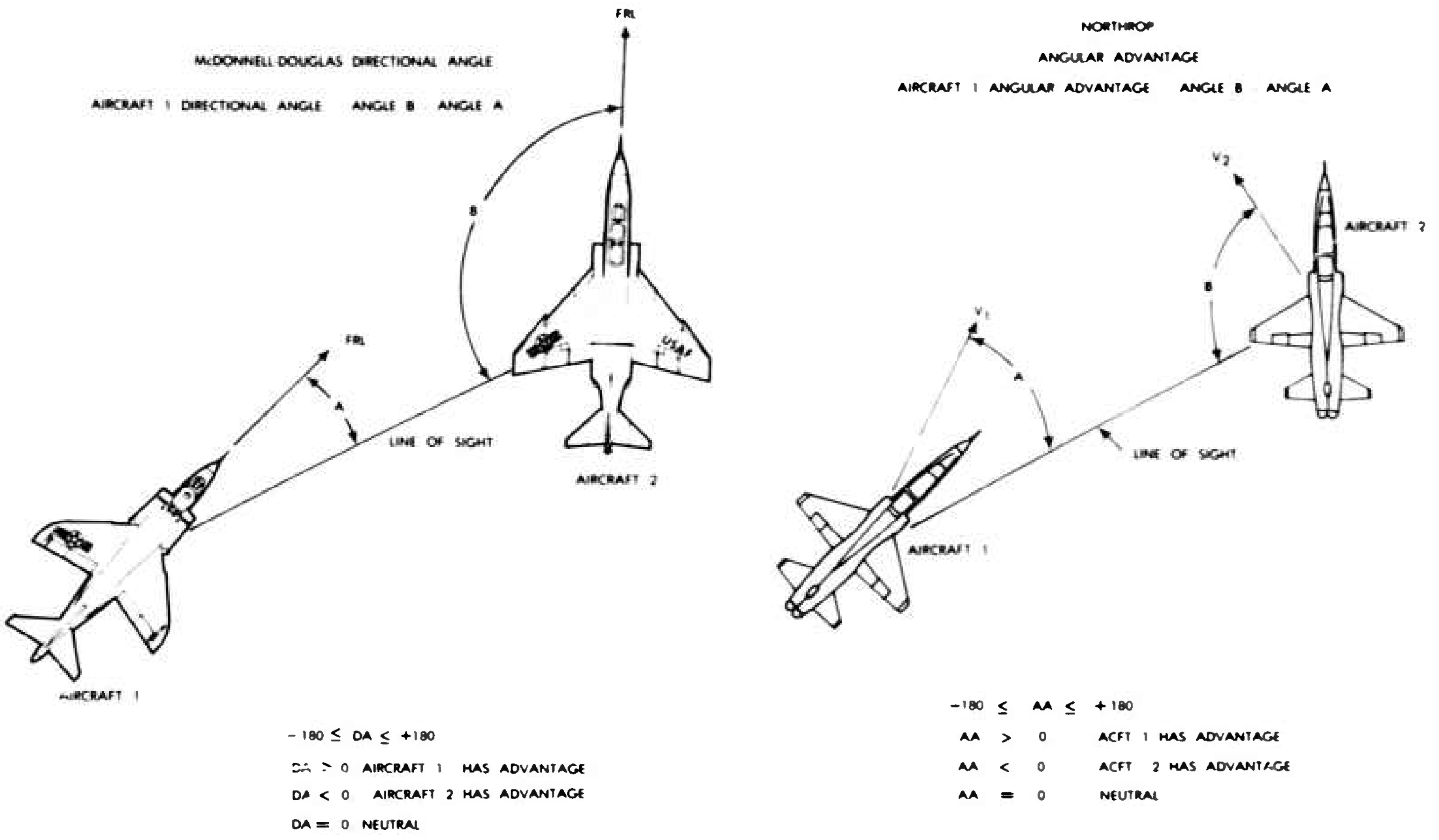


Figure 2
Examples of Continuous Analysis Performance Indices

RESULTS OF STUDY

THE ACM PERFORMANCE INDEX

15. The figure-of-merit to be used for the analysis should be indicative of the relative advantage/disadvantage of a particular set of circumstances existing in time and should not include the aircraft capability or performance which should be reflected in a time-rate-of-change of the performance index. The derivation of a performance index will be approached from an examination of the angular geometry, interairplane distances, and interairplane dynamics. Figure 3 shows the development of the angular geometry. Figure 3 shows a definite relation between the sum of the antenna train angle and the angle off tail. This sum is also directly related to the Directional Angle (DA) term of figure 2. This DA makes a reasonable starting point for a performance index if it is normalized in order that the numerical value will have intuitive meaning.

$$DA_N = 100 \left[\frac{180 - (AOT + ATA)}{180} \right] \qquad (2)$$

Equation 2 yields +100 for the best angular geometry, -100 for the worst angular geometry, and a value of 0 for the neutral condition.

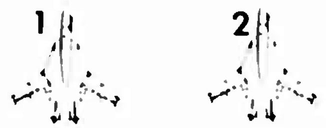
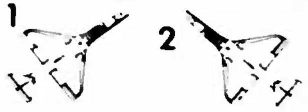
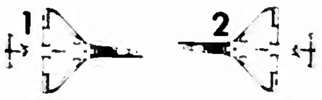
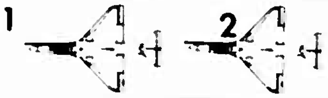
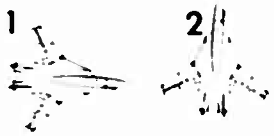
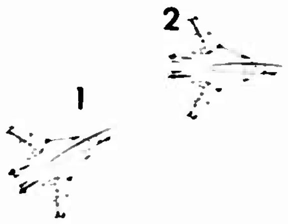
AIRCRAFT RELATIVE POSITION	ANGULAR RELATION (DEG)		QUALITATIVE ACM POSITION	
	AIRCRAFT 1-2	AIRCRAFT 2-1	AIRCRAFT 1	AIRCRAFT 2
	AOT = 90 ATA = 90 AOT + ATA = 180	AOT = 90 ATA = 90 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 135 ATA = 45 AOT + ATA = 180	AOT = 135 ATA = 45 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 180 ATA = 0 AOT + ATA = 180	AOT = 180 ATA = 0 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 180 ATA = 180 AOT + ATA = 360	AOT = 0 ATA = 0 AOT + ATA = 0	FATAL	EXCELLENT
	AOT = 90 ATA = 0 AOT + ATA = 90	AOT = 180 ATA = 90 AOT + ATA = 270	ADVANTAGE	DISADVANTAGE
	AOT = 45 ATA = 0 AOT + ATA = 45	AOT = 180 ATA = 135 AOT + ATA = 315	ADVANTAGE	DISADVANTAGE

Figure 3
Aircraft Relative Position Chart

16. The total geometry and interairplane dynamics must then modify the normalized directional angle. Figure 4 shows that the addition of the range term alters the basic conclusions. The two situations are identical in terms of angular geometry alone, but the ACM situations are radically different from a standpoint of total geometry.

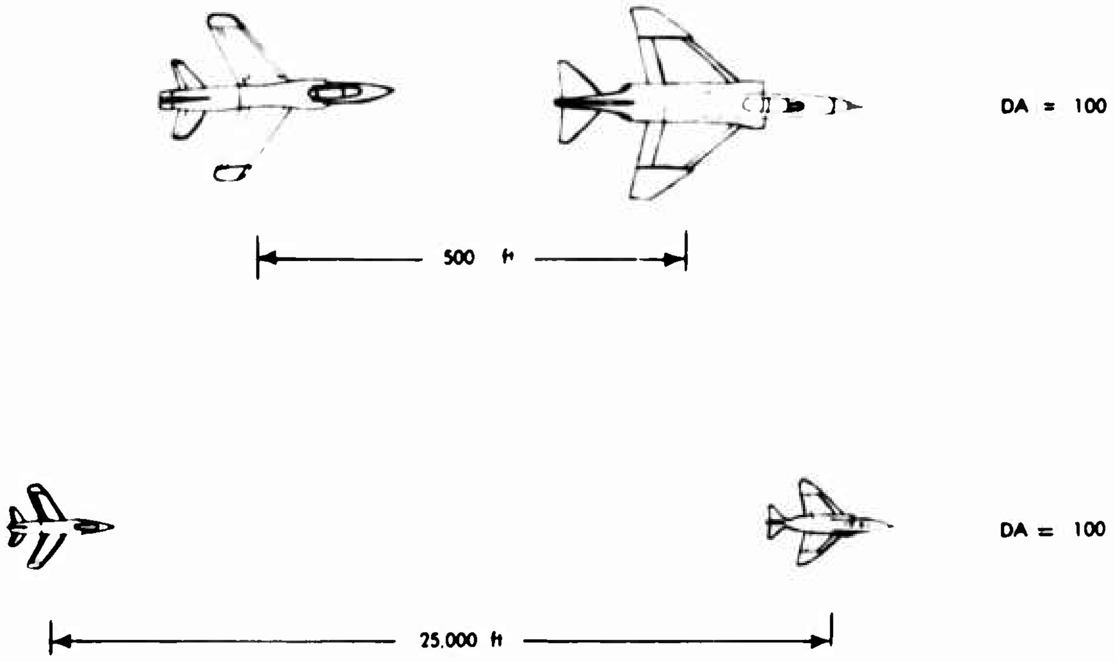
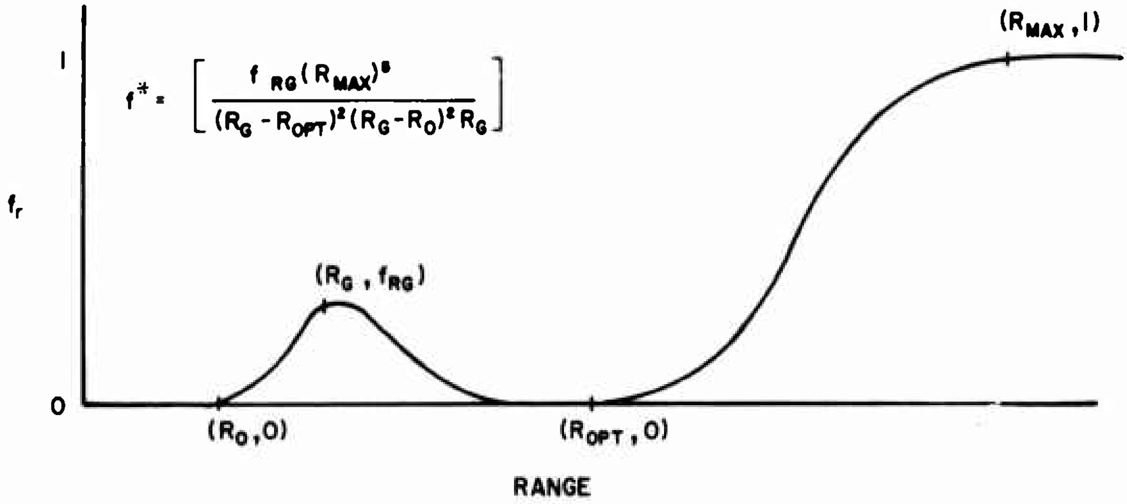


Figure 4
Influence of Range

17. To account for this reality, a range performance penalty function (f_r) is introduced. The f_r is shaped so that there is no penalty ($f_r=0$) when the ranges are suitable for weapons delivery but which penalizes the performance index when the range is beyond the weapon's capability and neutralizes the fight as range becomes too large for offensive maneuvering. An optional penalty may be imposed when the offensive aircraft is at a range between a guns and a missile envelope. Such a penalty function, together with the analytic equation it represents, is given in figure 5.

$$f_r = f^* \frac{R}{R_{MAX}} \left[\left(\frac{R - R_{OPT}}{R_{MAX}} \right) \left(\frac{R - R_O}{R_{MAX}} \right) e^{-8\left(\frac{R - R_G}{R_{OPT}} \right)^2} \right]^2 + \left[\frac{1}{1 + 500\, e^{-\left(\frac{R - R_{OPT}}{R_{MAX}} \right) 12}} \right]$$



$$f^* = \left[\frac{f_{RG} (R_{MAX})^5}{(R_G - R_{OPT})^2 (R_G - R_O)^2 R_G} \right]$$

Figure 5
Range Performance Penalty Function

18. The range penalty function has the general form of a sigmoid curve and would modify the DA to yield a figure-of-merit (FM) as follows:

$$FM = DA_N (1 - f_r) \qquad (3)$$

19. The figure-of-merit is further affected by interairplane dynamics. The effect of closing velocity is illustrated in figure 6. Just what an advantage or disadvantage is in this instance is a complicated function of range and offensive/defensive position. Certain statements can be made for a positive or negative energy increment as tabulated in table II.

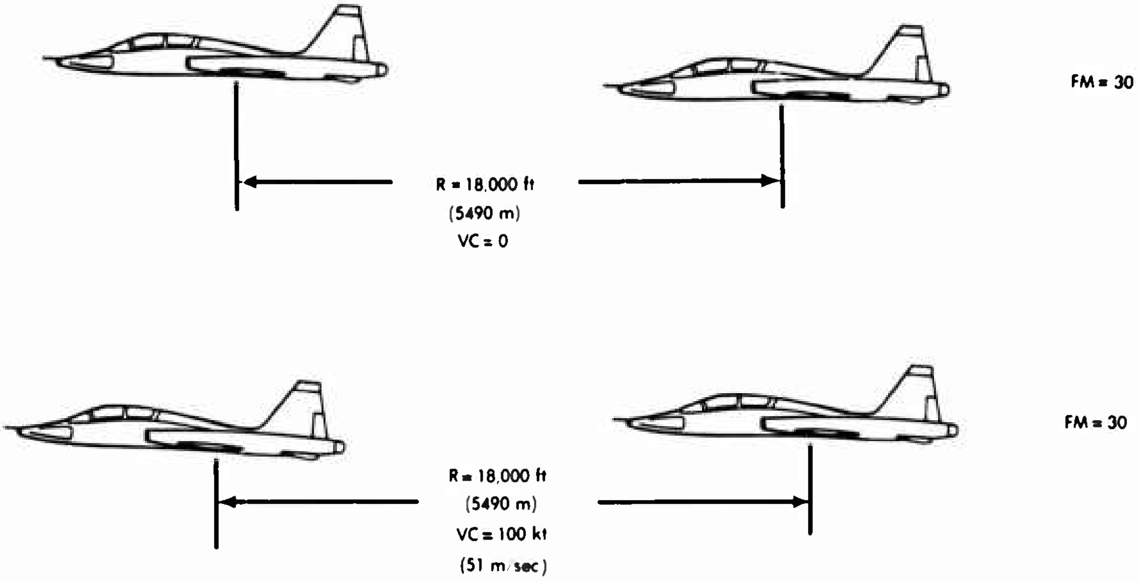


Figure 6
Influence of Closing Velocity

Table II

Energy Influence on Performance Index

Range	Energy Increment	ACM State	Conclusion
Very Close	Positive	Offensive	Disadvantage due to move toward overshoot
		Neutral	No effect
		Defensive	Advantage due to move toward overshoot
	Negative	Offensive	Advantage due to increase of range
		Neutral	No effect
		Defensive	Disadvantage due to increase of range
Weapon Opportunity	Positive or Negative	Offensive	No effect unless weapon parameters are affected
		Neutral	No effect
		Defensive	No effect unless weapon parameters are affected
Larger than weapon opportunity but less than very far	Positive	Offensive	Advantage due to move toward weapon opportunity
		Neutral	No effect
		Defensive	Disadvantage due to move toward weapon opportunity
	Negative	Offensive	Disadvantage due to move away from weapon opportunity
		Neutral	No effect
		Defensive	Advantage due to move away from weapon opportunity
Very far	Positive or Negative	All	No effect

20. These complicated relations can be somewhat simplified by noting that for everything else equal, the effect of a positive energy increment is minus the effect of a negative energy increment, and the effect in an offensive state is the negative of the effect in a defensive state. A function (K) reflecting these influences, as given in figure 7, modifies the figure-of-merit and yields performance index as follows:

$$PI = DA_N \, (1 - f_r) \, K \qquad (4)$$

OFFENSIVE:

$$K_{OFF} = 1 + \left[\frac{1}{1 + E_{dev} \, e^{-6.91 \frac{R}{R_0}}} + E_{dev} \, e^{-4 \left(\frac{2R - R_{MAX} - R_{OPT}}{R_{MAX} - R_{OPT}} \right)^2} - 1 \right] \frac{\Delta E_S}{\bar{E}_S}$$

DEFENSIVE:

$$K_{DEF} = \frac{1}{K_{OFF}}$$

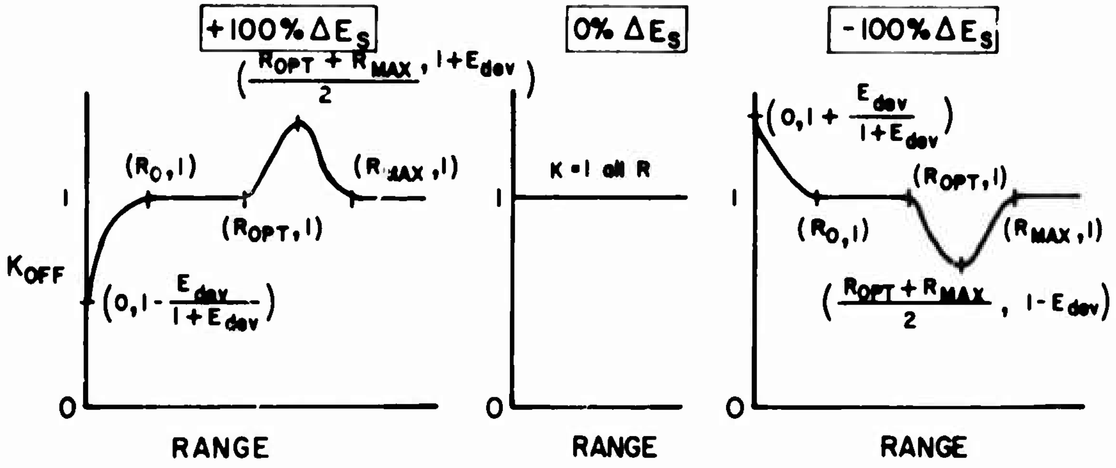


Figure 7
Energy Influence Function

PERFORMANCE INDEX COMPARATIVE ANALYSIS

Maneuver Conversion Model

21. Figure 8 is a comparison between the computed performance index (equation 4) and the maneuver conversion model (reference 9). The equations used for the maneuver conversion model were those of reference 1. The figure shows close agreement in engagement trend, but the performance index allows the analyst to determine the transition points much more accurately. For example, in figure 8, the point at which the fighter begins to show a negative engagement trend is around 60 seconds, and if the defensive state is established at PI -30, the fighter became defensive at 72 seconds into the engagement. The maneuver conversion model only indicates the point at which the fighter became defensive; figure 8 also illustrates the sensitivity of the discreet model to the accuracy of parameters. A small angular change (as little as 1 degree) can cause the state change that occurs at 81 seconds. No such influence is present in the performance index which has a small sensitivity to small changes in parameters and a large sensitivity to large changes in parameters.

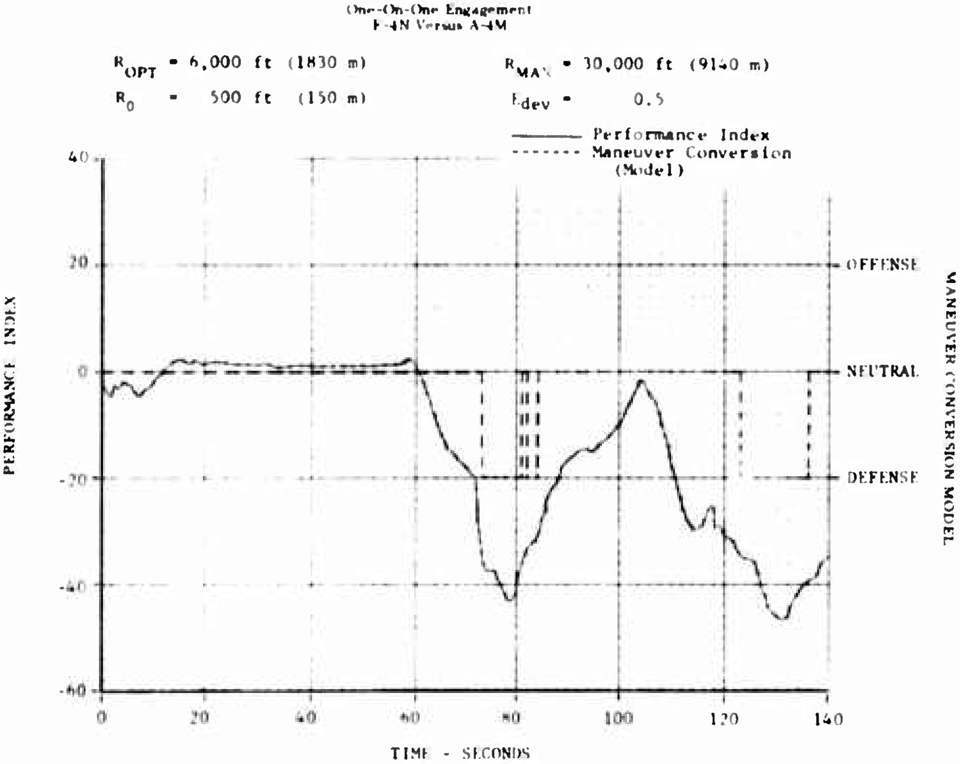


Figure 8
Comparison of the Performance Index with the Maneuver Conversion Model

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9) Center for Naval Analyses Report No. CRC 274, Air Combat Maneuver Conversion Model, of Nov 1974.

Directional Angle

22. Figure 9 shows the comparison between the computed performance index and the computed directional angle (reference 8) with the exception that the data were normalized (to the range) (+100 to -100) for purposes of comparison. The two analyses techniques give a marked difference in engagement trend during the first 60 seconds of the fight. During this portion of the engagement, the fighter is trading angles for range (up to 36 seconds), and the directional angle analysis method does not consider this. The actual separation became 24,000-30,000 feet (7 300-9 100 m) which is enough range to suppress the performance index to neutral with the constants used. The small differences that occur after 60 seconds are primarily due to the energy influence which favors the fighter and reduces his defensive posture slightly.

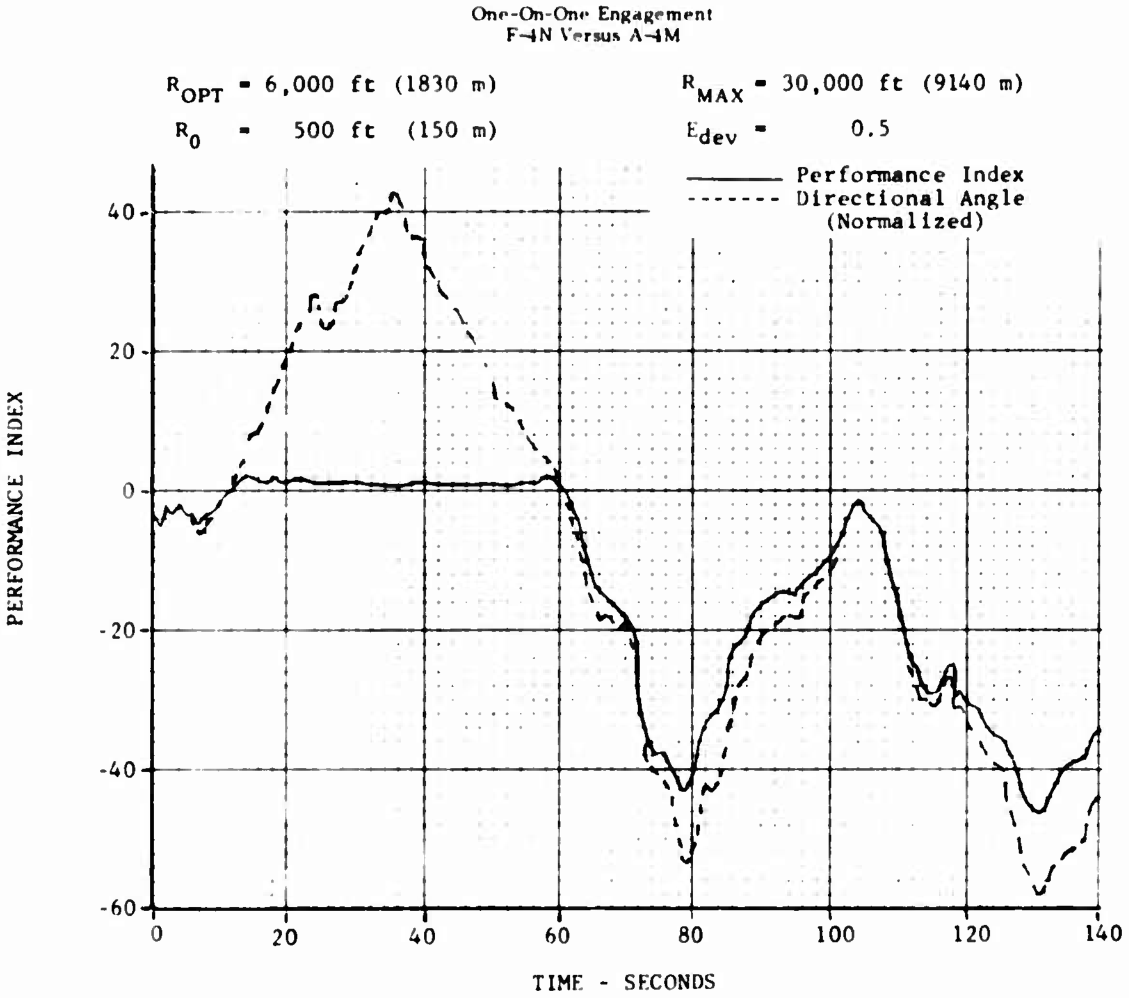


Figure 9
Comparison of the Performance Index
with the Directional Angle Criteria

PERFORMANCE INDEX PARAMETER STUDY

Range Effects

23. Figure 10 shows the effect of the maximum range parameter by computing the performance index (all else being equal) for two different values of R_{MAX}. As expected, the smaller value of R_{MAX} suppresses the fight to neutral with the most profound effect coming between 120 and 140 seconds (this is caused by ranges between 12,000 and 16,000 feet (3 600 and 4 900 m)).

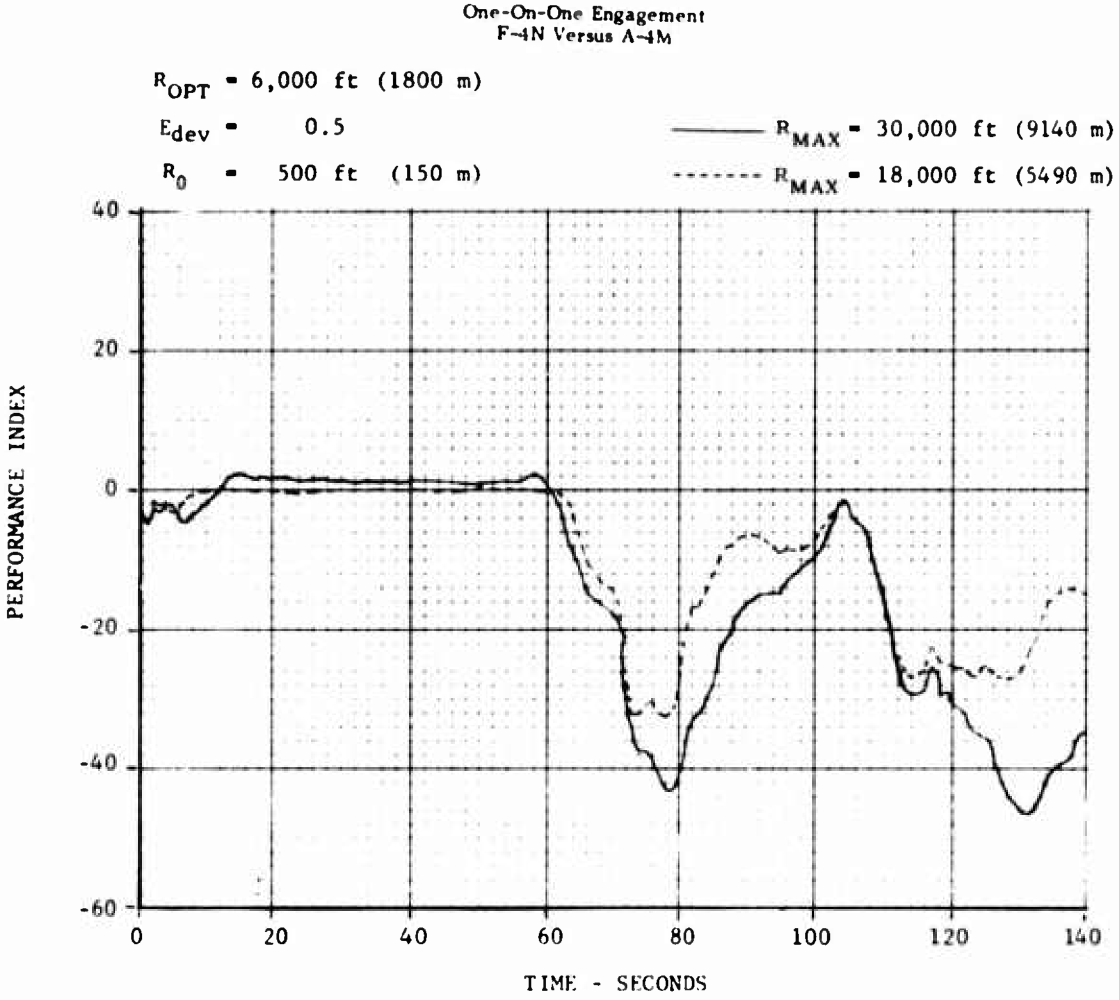


Figure 10
Influence of Maximum Range Parameter

24. In general, the R_{MAX} parameter represents the maximum interairplane range beyond which the fight is considered neutral. This is generally a sight type of parameter but could be interpreted to be seeker head or radar ranges for individual weapons analyses. The R_{MAX} values in table III are based upon using sight as the engagement criteria. These data are estimated from discussions with pilots at the Naval Fighter Weapons School, VF-111, VX-4, and others. In any given application, these values may change or may even become a function of engagement parameters such as altitude, meteorology, and background (sky, terrain, sun, etc.).

Table III

Estimated Value of R_{MAX} for Specific Aircraft Types

Aircraft	R_{MAX} (1)
F-4	30,000 ft (9 140 m)
F-14	27,000 ft (8 230 m)
F-15	27,000 ft (8 230 m)
F-5	18,000 ft (5 490 m)
A-4	24,000 ft (7 320 m)
F-8	24,000 ft (7 320 m)

NOTE: (1) Select R_{MAX} corresponding to defensive aircraft.

25. Figure 11 shows the influence of two different values of optimum range. The expected result of minimum influence for moderate change is desirable because the curve should generally be flat in the area of an optimum missile launch. This also allows some latitude in the choice of the optimum range parameter.

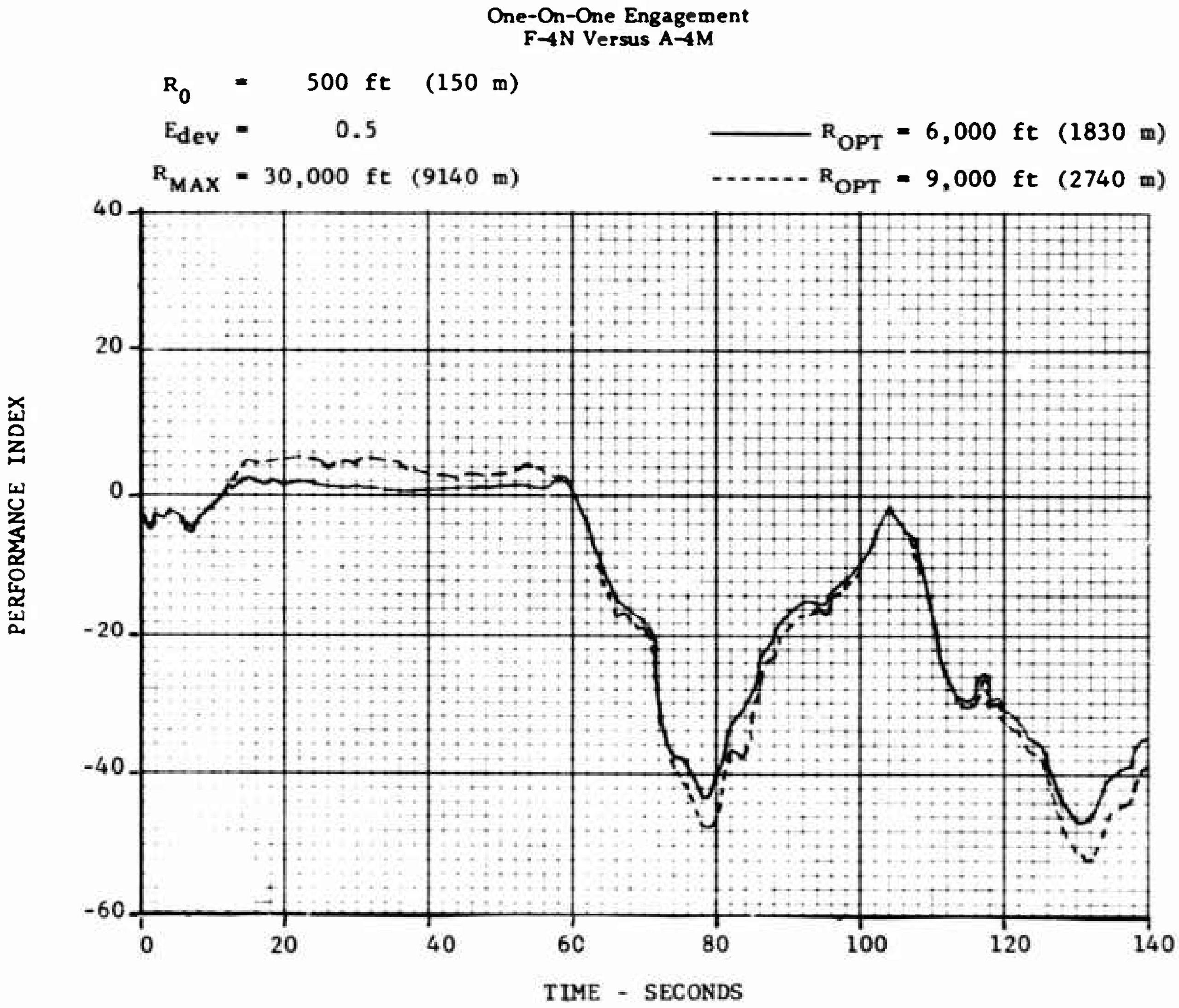


Figure 11
Influence of Optimum Range Parameter

Energy Effects

26. Figure 12 shows the computed value for two different values of energy coefficient (E_{dev}). The effects of this parameter are subtle with the F-4 enjoying about a 33% advantage in specific energy. A long-term energy advantage will be reflected in the index as a range or angle influence. The effect will be much more pronounced in a slashing type of fight (high energy) where the fighter may enjoy as much as 100% energy advantage. The coefficient E_{dev} may be chosen to yield the energy effect desired. Test cases used 0.5 for computation. In general, E_{dev} will be between 0 and 2.0 and is chosen to yield the desired magnitude of energy influence.

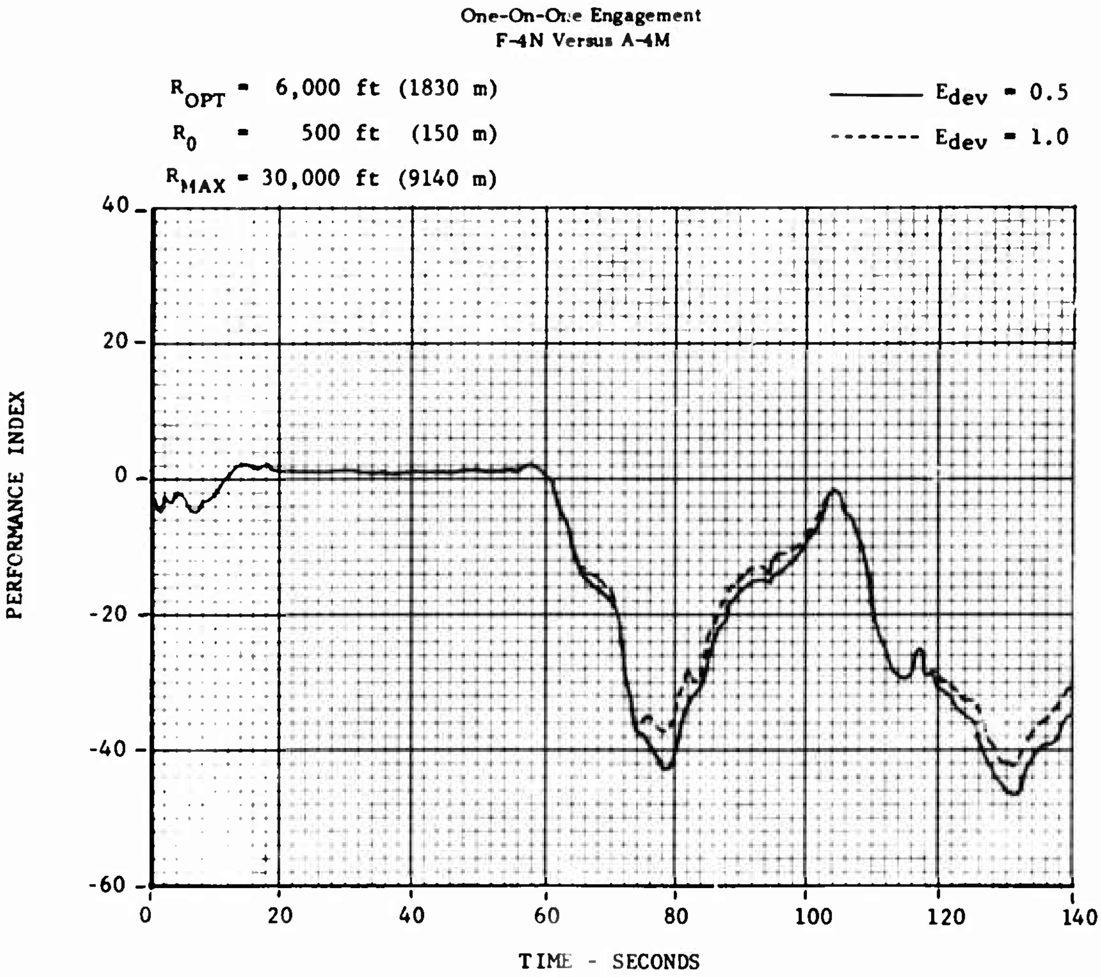


Figure 12
Influence of Energy Coefficient

Interenvelope Gun Penalty

27. The interenvelope gun penalty was structured into the model to include the case of an aircraft carrying long-range missiles and guns with no intermediate range missile. In cases where the gun and missile envelope overlap, the penalty would be zero. Test cases have used an arbitrary .025 value for f_{RG} as given in paragraph 17 and figure 5.

EXTENSION TO MULTIAIRCRAFT

28. One of the most difficult areas of ACM analysis is the extension to the multiaircraft situations. Each additional aircraft adds a multiplicity of complications to the problem both conceptually and mathematically. Few models attempt extension to this area even though an actual engagement has a much higher probability of being multiaircraft than one-on-one. Table IV is an extract of reference 9 and gives the logic in constructing a two-on-one maneuver conversion model. The table was constructed through extensive analysis of two-on-one engagements and pilot interviews and is both logical and intuitive. It does not, however, follow precise mathematical trends. For example, simultaneously having an offensive fighter and defensive fighter does not give a neutral section (condition 2 of table IV). The extension of the performance index will be in the same manner as the maneuver conversion model: i.e., a section performance index.

Table IV

Rules for State Evaluation of a Two-On-One Engagement

1. The section is OFFENSIVE WEAPON when at least one member is in offensive weapon state and the other is higher than a fatal defensive state.
2. The section is OFFENSIVE when at least one member has an offensive position and the other is higher than a fatal defensive state.
3. The section is NEUTRAL when both members are in neutral state.
4. The section is DEFENSIVE when at least one member is in defensive state and the other is either neutral or defensive.
5. The section is FATAL DEFENSIVE when at least one member is in fatal defensive state and the other has less than offensive weapon state.
6. The section is in a TRADEOFF state when one member of the section is in offensive weapon state and the other is in a fatal defensive state.

29. Figure 13 shows a two-on-one engagement, together with its maneuver conversion model, for both fighter-to-target pairs. This particular engagement is of specific interest because of the tradeoff situation between 50 and 60 seconds and the reversals of state present for the section. The individual indices follow well the engagement trends of the maneuver conversion model, but the combination must also follow for a section coefficient.

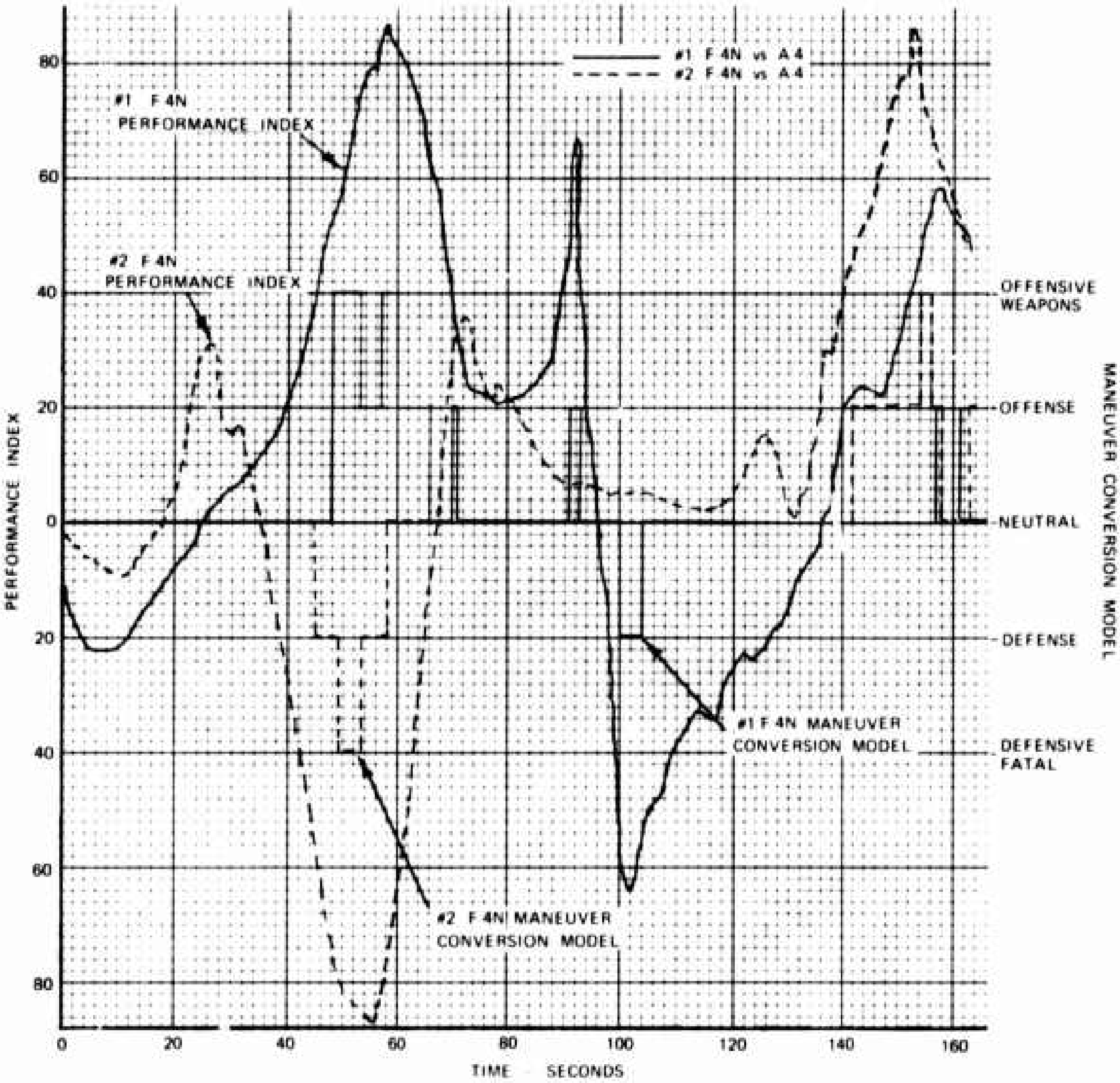


Figure 13
Individual Fighter Airplane Performance in a Two-On-One Engagement

30. After many trials, a mathematical form of section coefficient was derived which could follow the maneuver conversion extension as shown in figure 14. The specific calculational procedure termed vector sum is given by:

$$K^{*} = \frac{(PI_1)\,|PI_1| + (PI_2)\,|PI_2|}{\left|(PI_1)\,|PI_1| + (PI_2)\,|PI_2|\right|} = \pm 1 \qquad (5)$$

$$PI_{SECTION} = K^{*}\sqrt{\left|(PI_1)\,|PI_1| + (PI_2)\,|PI_2|\right|} \qquad (6)$$

The absolute value takes care of the sign of the individual engagement indices. Tradeoff was established by looking at the individual aircraft performance indices and appears to lag in time but this is not deemed significant because of the sensitivity of the maneuver conversion model to small parameter changes as discussed in paragraph 21. The section performance index is directly extendable to larger engagements (more aircraft) by:

$$K^{*} = C\,\frac{\sum_{i=1}^{n} (PI_i)\,|PI_i|}{\left|\sum_{i=1}^{n} (PI_i)\,|PI_i|\right|} = \pm C \qquad (7)$$

$$PI_S = K^{*}\sqrt{\left|\sum_{i=1}^{n} (PI_i)\,|PI_i|\right|} \qquad (8)$$

Where c is a proportionally constant to establish maximum and minimum values. For $PI_S = 100$, c is equal to the square root of the inverse of the number of pair possibilities.

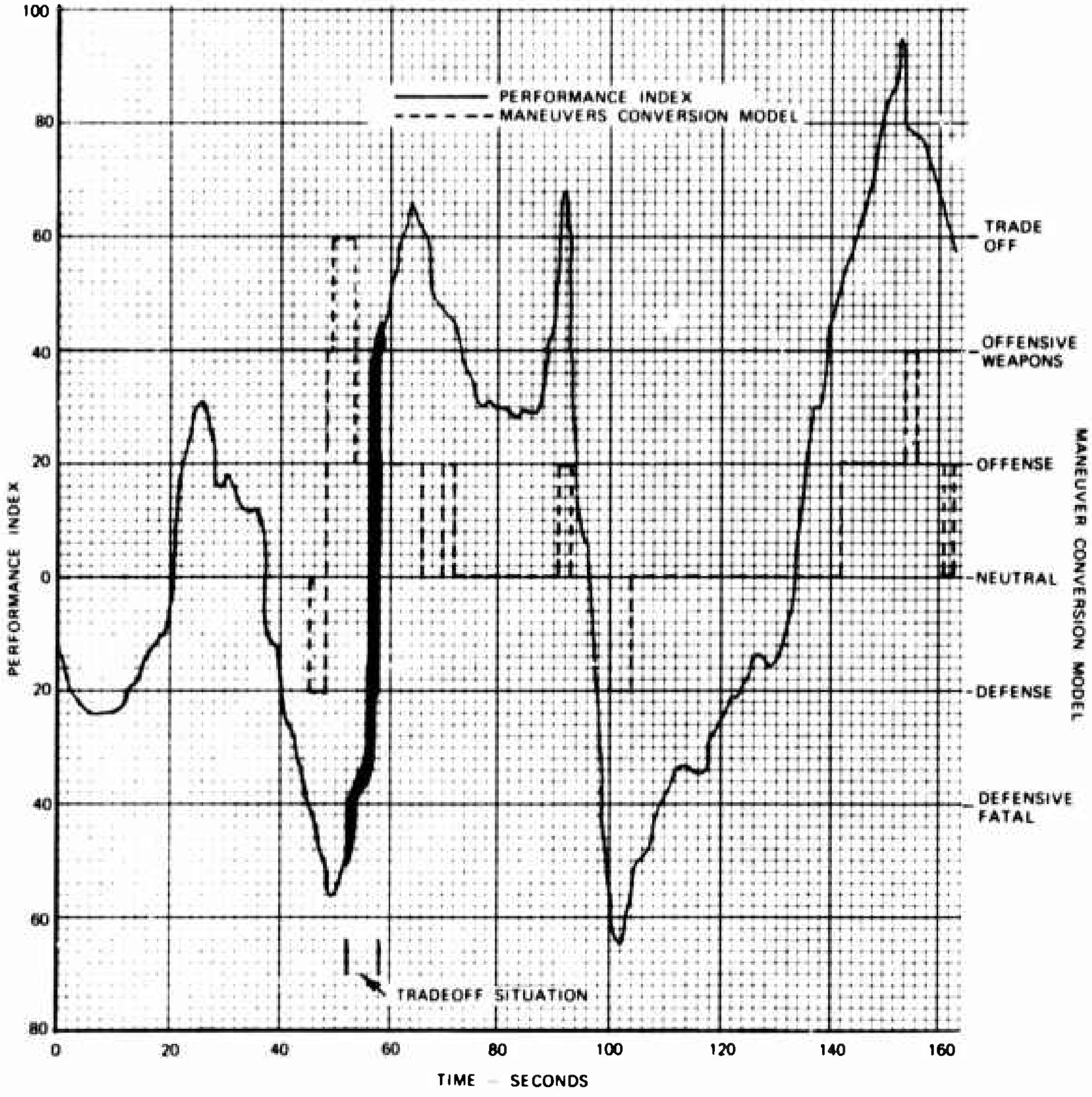


Figure 14
Fighter Section Performance Index
from Vector Sum Method

OTHER CONSIDERATION

31. Because of the mathematical extension of the multiaircraft situation, an opportunity is afforded to examine the relative contribution of each aircraft in the section and to quantify section coordination. A two-on-one example follows:

$$K_1 = \left| PI_1 - PI_S \right| \qquad\qquad K_2 = \left| PI_2 - PI_S \right| \qquad (9)$$

$$\phi(t) = \frac{K_j}{K_i} \quad \text{Where Ki is the greater of } K_1 \text{ or } K_2 \text{ and Kj is the lesser of } K_1 \text{ or } K_2 \qquad (10)$$

$$\phi \text{ total} = \frac{1}{t_{total}} \int_0^t \phi(t)\,dt \qquad (11)$$

A measure of the consistency of the coordination can be given by the difference between unity and the standard deviation of ϕ (t). For the engagement shown in figures 13 and 14, the coordination figure (ϕ total) is .2136, and the consistency is .7381. Insufficient data have been computed to date to assess these figures qualitatively, but it appears that good coordination may be above .20 and good consistency above .50.

FUTURE APPLICATIONS

32. The development of the ACM performance index has, thus far, been exploratory. The analysis of individual engagements, while important to the understanding of pilot actions and strategy, gives only a small amount of the information needed to understand air combat. The real complication comes when one considers that each engagement, whether it be one-on-one or ten-on-five, is unique and not repeatable. The pilot has a set of choices at each point of the engagement. He makes these choices on a combination of far too many variables to include in the problem, including anticipation, experience, and other intangibles. He has at his command an infinite number of actions which include the optimal maneuvers (both tactical and strategic) and nonoptimal maneuvers (even "bad" maneuvers). The combination of many engagement trials, as shown in figure 15, gives a stochastic or statistical view of the experiment. An adequate data base should allow a statistical description of the experiment, including means, standard deviations, and skewness as a function of time, which will describe the probable outcome of a large number of engagements.

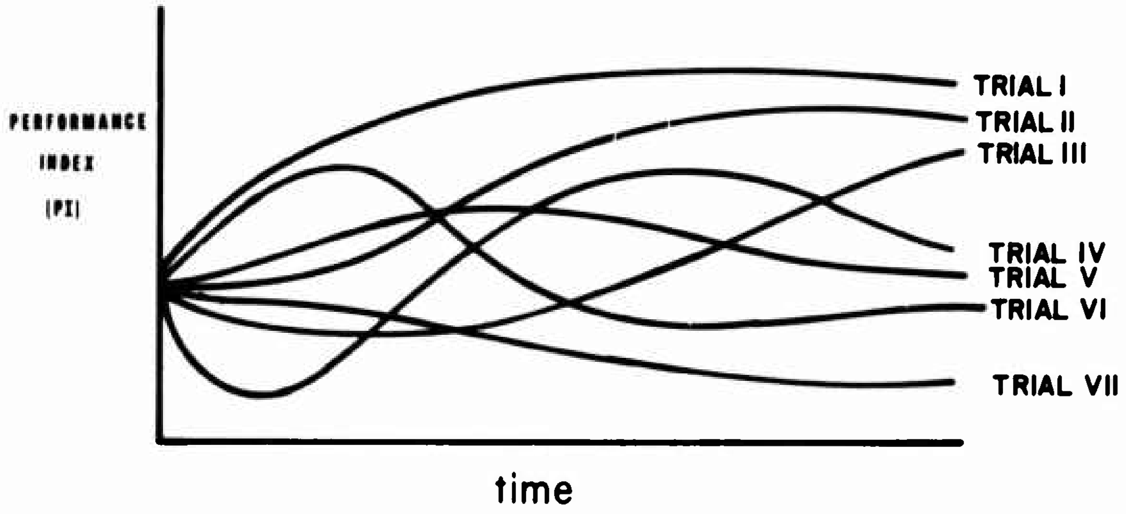
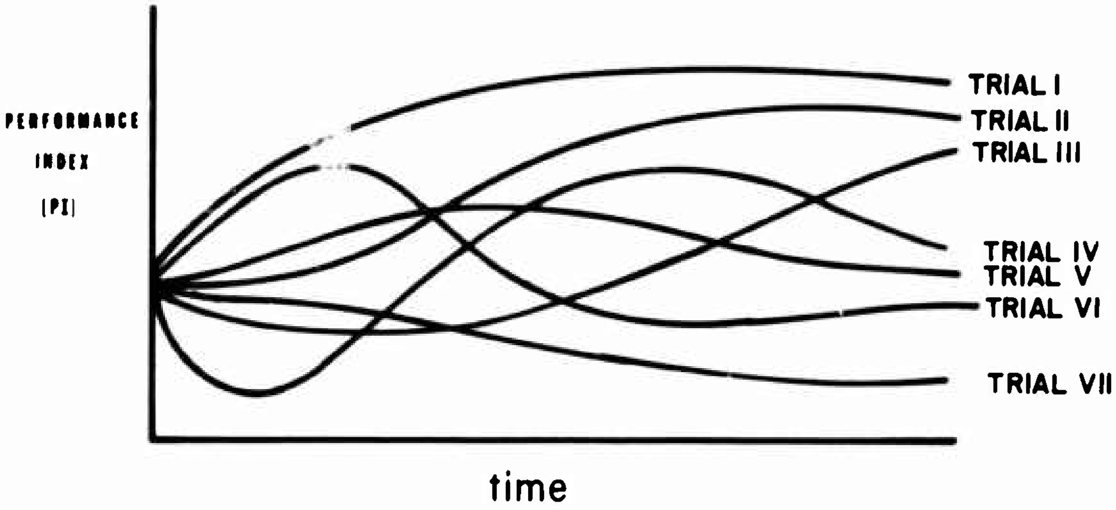


Figure 15
Nondeterministic Character of Several Engagement Trials

33. Finally, a statistical data base can be used to build an ACM predictor model which is "real-world" and involving multiaircraft air combat. One such approach is given in figure 16. The basic model assumes that some portions of the ACM encounters are controlled by performance aspects of the aircraft, weapons systems, and support facilities (such as GCI). The model will also allow for a definition of just what portions of multiple-airplane ACM are controlled by system performance and what parts are the result of pilot experience or other intangibles. Ultimately, the model could then be used to determine the value of increased airframe performance, better weapons, or radars, etc.



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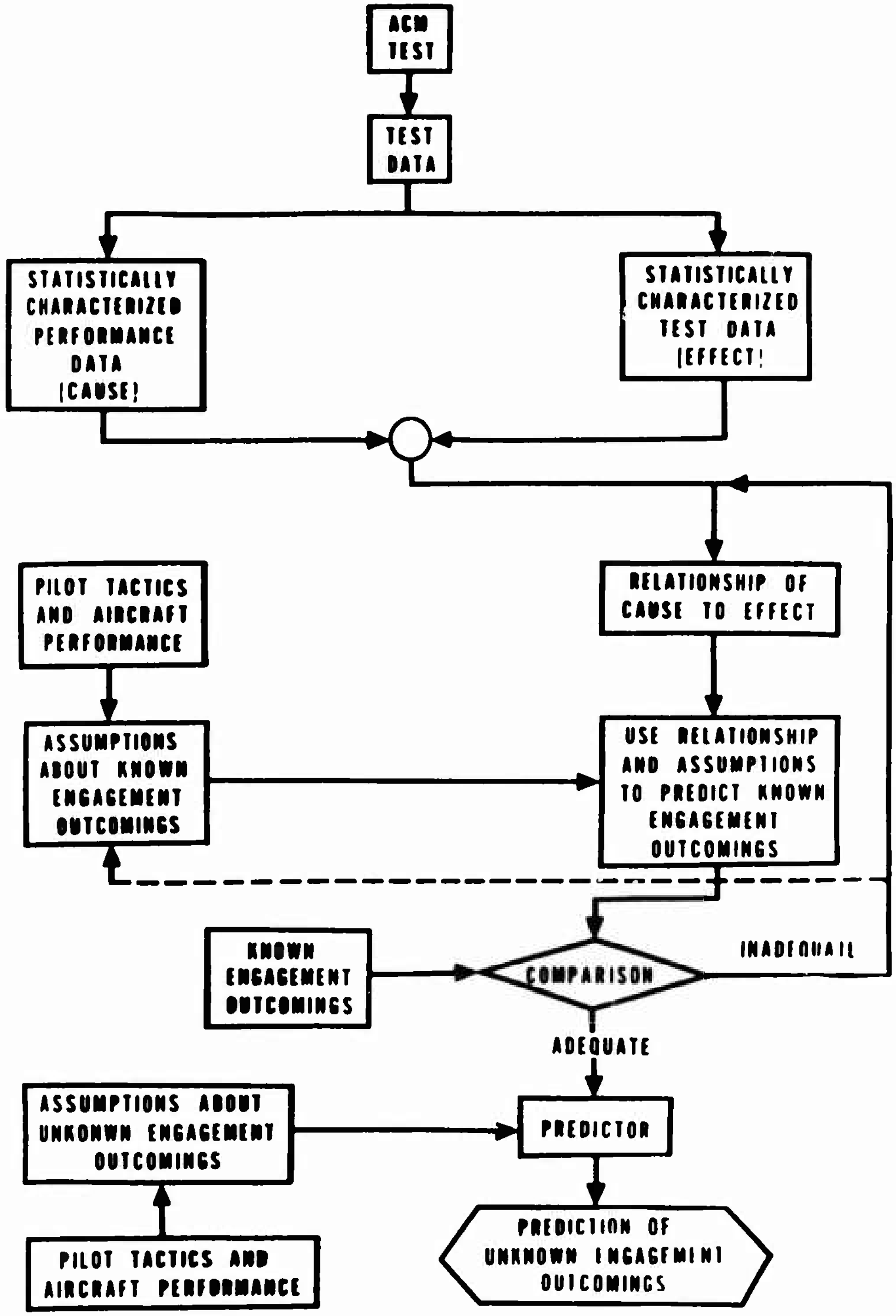

Figure 16
Stochastic Model ACM Performance Predictor

34. The performance index analysis has direct application to several areas of ACM, including aircraft design, test and evaluation, pilot training and proficiency, force strength predictions, etc. Some specific examples follow:

a. Pilot Training - Applications to training can be made in the area of multiaircraft fighter section coordination and consistency as given in paragraph 31. These figures can be used as measures of proficiency for pilot/wingman combinations or fighter squadron readiness as compared to an established criterion.

b. Aircraft and Armament Design - The direct application of performance index data to the design problem can be achieved by determining the sensitivity of engagement outcomes to variables under the control of the designer, through either a predictor model (paragraph 32), or a direct test of concepts.

c. Operational Planning - Estimated force strength requirements are a direct fallout of the predictor model as discussed in paragraph 32. Air superiority force strengths can be based on realistic projections of fighter aircraft attrition in air combat maneuvering.

d. Weapon System Effectiveness - The performance index method can be used as a measure of total system effectiveness in fighter aircraft test and evaluation. The mission systems effectiveness for the fighter mission would be given as the set of engagement outcomes.

e. Airplane System Test and Evaluation - Individual hardware (maneuvering flaps, thrust reversers, etc.) can be tested by its ability to change the performance index distribution in a given set of tests.

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LIST OF SYMBOLS AND ABBREVIATIONS

AA	Angular Advantage
ACEVAL	Air Combat Evaluation
ACM	Air Combat Maneuvering
ACMI	Air Combat Maneuvering Instrumentation
ACMR	Air Combat Maneuvering Range
AIS	Airborne Instrumentation Subsystem
ALT	Altitude
AOA	Angle of Attack
AOT	Angle Off Tail
ATA	Antenna Train Angle
c	Proportionality Constant
CCS	Computation and Control Subsystem
DA	Directional Angle
DDS	Display and Debriefing Subsystem
E_s	Differential Energy
e	Natural Constant (2.7183)
E_{dev}	Energy Coefficient
E_s	Specific Energy
E_s	Average Specific Energy
FM	Figure of Merit
f_r	Range Performance Penalty Function
f_{RG}	Interenvelope Gun Penalty

FRL	Fuselage Reference Line
f*	Interenvelope Gun Constant
GCI	Ground Control Intercept
IAS	Indicated Airspeed
$I_{\Delta E_s}$	Differential Energy Integral
K	Energy Influence Function
K*	Proportionality Constant
MT	Target Mach Number
N_z	Normal Acceleration (load factor)
ϕ	Section Coordination Coefficient
PI	Performance Index
R	Interairplane Range
R_G	Range at which gun tactics begin to control the engagement
R_{MAX}	Maximum Range
R_{MIN}	Minimum Range
R_{OPT}	Optimum Range
R_0	Zero Penalty Range
TIS	Tracking Instrumentation Subsystem
V	Velocity Vector
VC	Closing Velocity
VIFF	Vectoring in Forward Flight

Subscripts

1	Aircraft Pairing 1
2	Aircraft Pairing 2

DEF	Defensive
OFF	Offensive
s	Section
i	Aircraft Pairing i
j	Aircraft Pairing j
Total	Total (time)
N	Normalized