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**SOME MATHEMATICAL CONSIDERATIONS REGARDING
THE CALCULATION OF PERMANENT DAMAGE OF
DEVICES DUE TO EMP PULSES**

July 1976

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Prepared for

DEPUTY FOR E-3A
HQ ELECTRONIC SYSTEMS DIVISION
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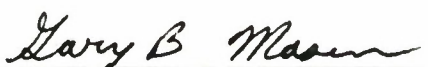
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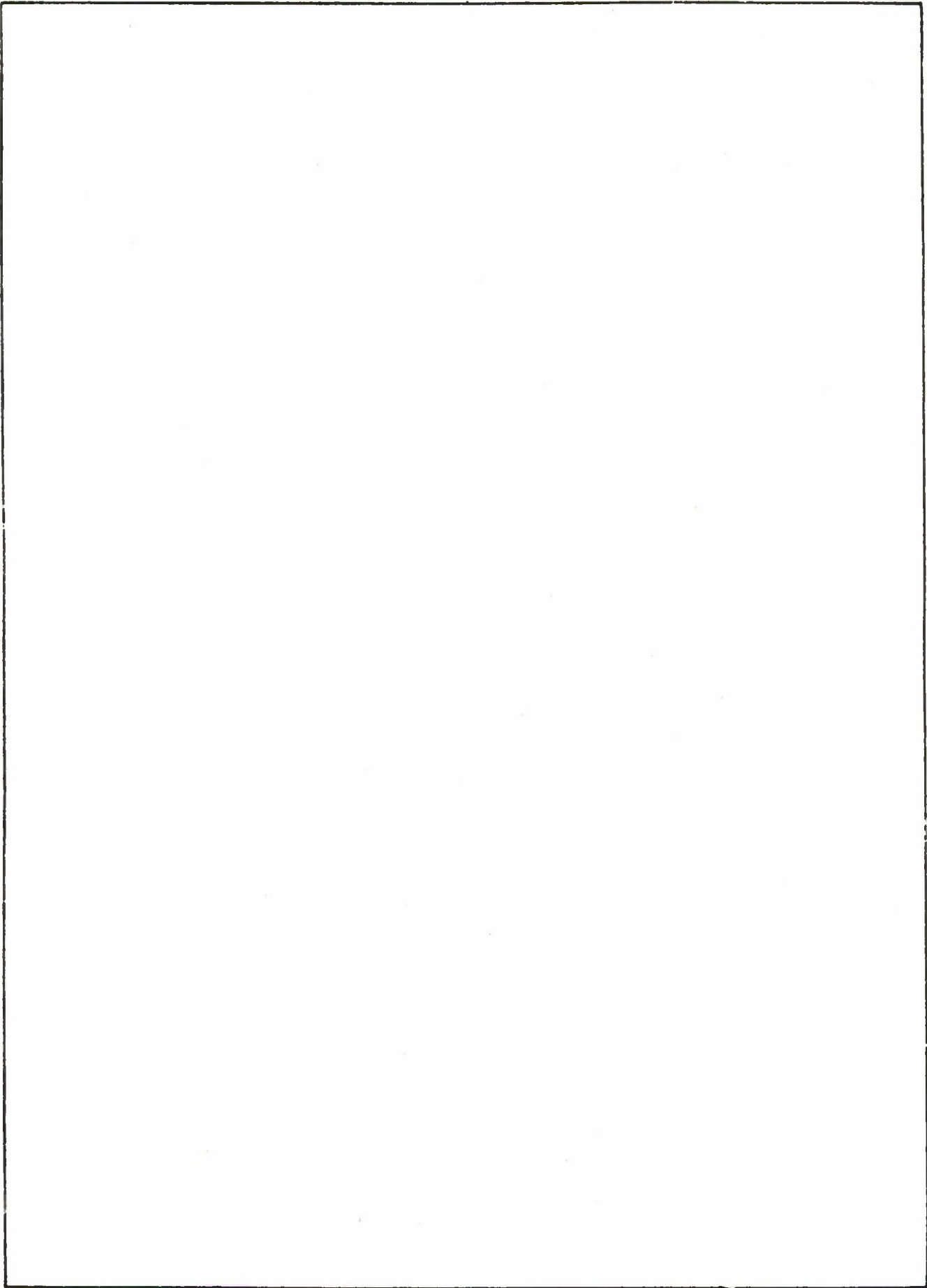

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ABSTRACT

A mathematical analysis of the failure of semiconductor junctions due to EMP induced pulses is presented. Included in this discussion are: the effects of finite-size p-n junctions on maximum temperature build up; an evaluation of interpulse cooling; and a determination of the response of these devices to damped periodic waveforms. Limitations of the results are presented. Such information will be generally useful for performing preliminary damage assessment and/or screening of devices.

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1.0 INTRODUCTION

The problem of damage assessment of interface circuits by EMP-generated pulses is of much interest to the E-3A Program, as well as other aircraft systems which must satisfy EMP nuclear requirements. From network analysis of the interface circuits, damage assessment is accomplished by evaluating whether the voltage pulses which appear across the susceptible components will damage the latter. References 1 and 2 collectively provide a good discussion of the damage mechanisms for various components.

The purpose of this investigation is to discuss some analytical techniques which can be used for evaluating permanent damage of semiconductor devices which are contained in interface circuits. The results are applicable to the study of failure of semiconductor junctions, as originally proposed by Wunsch⁽³⁾. In his model, the destruct mechanism is assumed to result from changes in the junction parameters due to the high temperatures produced locally within the junction area. These hot spots can be produced for both forward and reverse voltage conditions. Thus, the evaluation of component susceptibility reduces to determining whether the temperature increase due to power dissipation is sufficient to cause damage.

Damage assessment, therefore, depends upon the power delivered to a component, which in turn depends on the signal delivered to that component through a network of circuitry. The complexity of the calculation can vary considerably. Some cases are simple enough for hand analysis while others require computer codes⁽⁴⁾. Because computer analysis costs can be very high in a system containing a large number of components, the preliminary hand analysis/screening of interface circuits is an important step in the hardness evaluation process. An analytical assessment not only provides a reasonable assessment of circuit hardness, but also identifies those circuits requiring a more vigorous assessment.

In this study, we provide a discussion of some of the physical ideas and analytical concepts which are involved in hand analysis of component damage. It is not unlikely that a portion of what is presented here exists in other reports, but this information does not seem to have been conveniently coordinated. Therefore, this document also provides the reader with a general background of the problem.

The remainder of this report is organized as follows: Section 2.0 deals with the extension of Wunsch's analysis to finite size p-n junctions. The subsequent section discusses various aspects pertaining to the analytical behavior of junction temperature including the effect of interpulse temperature cooling; and the response of interface circuits to damped sine inputs. Concluding remarks are rendered in Section 4.0.

2.0 DERIVATION OF MAXIMUM TEMPERATURE IN P-N JUNCTION DIODE

Wunsch⁽³⁾ has derived a formula for the maximum temperature rise in a p-n junction in an infinite medium. The purpose of this section of the report is to extend the analysis to p-n junctions of finite size. Using the developed equations, one can readily assess the pulse length conditions for which the finite size of the junction should be taken into account even in a hand screening analysis.

Figure 1 shows a pictorial representation of a p-n junction diode. For analysis purposes, the p and n sides are taken to be of equal dimension, L/2, with the junction located at L/2.

When a voltage pulse is applied to a semiconductor device in the reverse direction, the principal voltage drop is across the junction. If we assume that the junction is infinitesimally thin, then the spatial distribution of volumetric heating (watts/cm³) is given by

$$q(x,t) = \frac{P(t)}{A} \delta(x - \frac{L}{2}) \left(\frac{\text{watts}}{\text{cm}^3} \right) \quad (2.1)$$

where P(t) is the time-dependent total power delivered to the device, A is the cross sectional area of the junction and $\delta(x)$ is the Dirac delta function. The general one-dimensional heat diffusion equation is given by⁽⁵⁾:

$$\frac{\partial}{\partial t}(\rho C T) = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + q(x,t) \quad (2.2)$$

where:

$$\rho = \text{density} \left(\frac{\text{gm}}{\text{cm}^3} \right)$$

$$C = \text{specific heat} \left(\frac{\text{joules}}{\text{gm} \cdot ^\circ\text{K}} \right)$$

T = temperature ($^{\circ}\text{K}$)

k = thermal conductivity ($\frac{\text{watts}}{\text{cm}\cdot^{\circ}\text{K}}$)

x = linear dimension (cm)

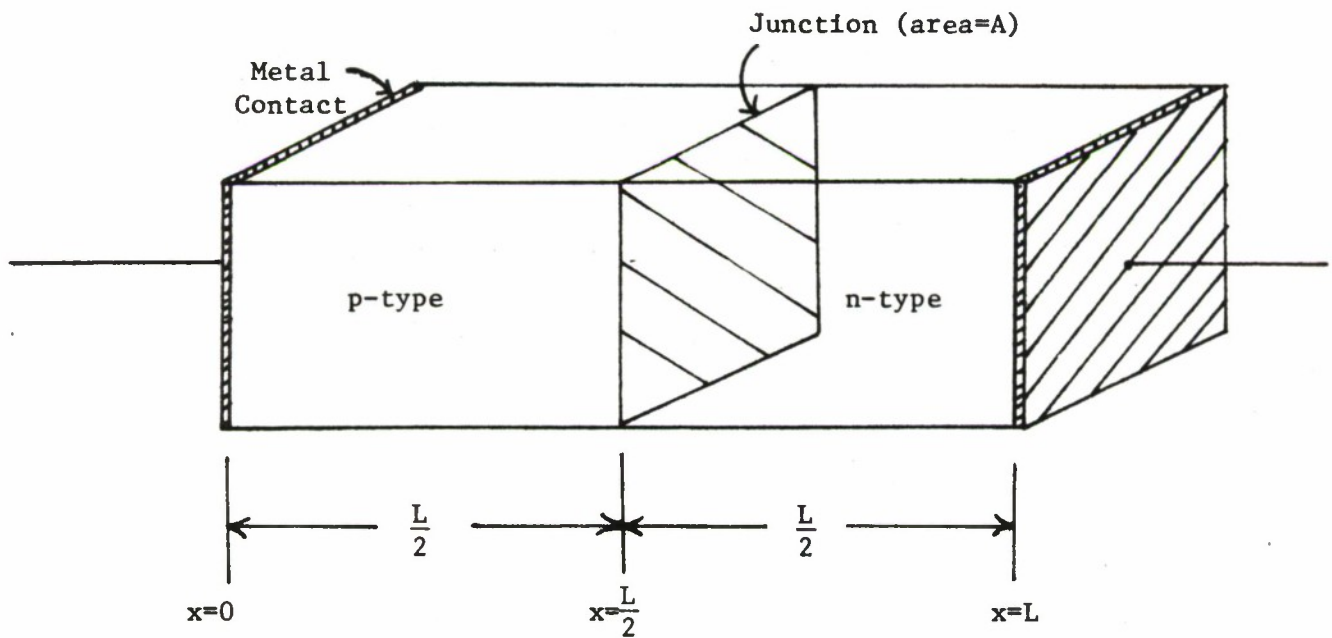


Figure 1. Diagram of p-n Junction Diode

Wunsch points out that k is a sensitive function of temperature, e.g., ranging from 1.56 watts/cm-°K at 300°K to 0.310 at 1000°K, so that strictly speaking its explicit dependence on T should be taken into account in Equation (2.2). However, for mathematical convenience, we shall approximate k by a suitable time-weighted average over the temperature range of interest⁽³⁾.

Equation (2.2) will be solved subject to the boundary conditions

$$\begin{aligned} T(x=0) &= T_a \\ T(x=L) &= T_a \end{aligned} \tag{2.3}$$

where T_a is the ambient temperature which may also be taken as the initial temperature. Without loss of generality, we arbitrarily measure the temperature with respect to T_a . We, thus, make the substitution $T \rightarrow T - T_a$ so that the new boundary conditions become $T(x=0) = T(x=L) = 0$. Using the aforementioned change of temperature reference, the solution of Equation (2.2) is obtained by using an orthogonal series expansion for $T(x,t)$. We let

$$T(x,t) = \sum_{n=1}^{\infty} B_n(t) \sin \frac{n\pi x}{L} \tag{2.4}$$

$$q(x,t) = \sum_{n=1}^{\infty} Q_n(t) \sin \frac{n\pi x}{L} \tag{2.5}$$

where the orthogonal functions $\sin \frac{n\pi x}{L}$ satisfy the condition:

$$\begin{aligned} \int_0^L \sin \frac{n\pi x}{L} \sin \frac{m\pi x}{L} dx &= \frac{L}{\pi} \int_0^{\pi} \sin nu \sin mu du \\ &= \frac{L}{2} \quad \text{if } m = n \quad ; \quad = 0 \quad \text{if } m \neq n \end{aligned} \tag{2.6}$$

The functions $Q_n(t)$ are determined from the equation:

$$\int_0^L \sin \frac{n\pi x}{L} q(x,t) dx = \frac{L}{2} Q_n(t) \quad (2.7)$$

which using Equation (2.1) yields:

$$Q_n(t) = \frac{2}{L} \left\{ \frac{P(t)}{A} \sin \frac{n\pi}{2} \right\} \quad (2.8)$$

For mathematical convenience, let us call

$$\begin{aligned} C_n &= \sin \frac{n\pi}{2} \\ C_n &= 0 \text{ for } n \text{ even} \\ C_n &= \pm 1 \text{ for } n \text{ odd} \end{aligned} \quad (2.9)$$

Substituting Equations (2.4) and (2.5) into Equation (2.1) and equating equal coefficients of $\sin \frac{n\pi x}{L}$ gives:

$$\rho C \frac{dB_n}{dt} = -k \left(\frac{n\pi}{L} \right)^2 B_n + Q_n(t) \quad (2.10)$$

Since $C_n = 0$ for n even, Equation (2.10) has meaning for odd n , a fact we will later use in summing up the infinite series. Dividing through by ρC gives the following simplified version of Equation (2.10):

$$\frac{dB_n}{dt} = -\lambda_n B_n + W_n(t) \quad (2.11)$$

where

$$\lambda_n = k \left(\frac{n\pi}{L} \right)^2 \quad (2.12)$$

$$W_n = \frac{Q_n(t)}{\rho C} = \frac{2P(t)}{\rho CAL} C_n \quad (2.13)$$

The initial conditions for all B_n are:

$$B_n(t=0) = 0, \quad (2.14)$$

since it is assumed initially that the junction is at uniform temperature T_a . The solution to Equation (2.11), subject to the condition of Equation (2.14) is given by:

$$B_n(t) = e^{-\lambda_n t} \int_0^t W_n(t') e^{\lambda_n t'} dt' \quad (2.15)$$

On physical grounds, it is apparent that the maximum temperature will occur at the junction, where $x = \frac{L}{2}$. Let us define this junction temperature as \hat{T} . We have:

$$\hat{T}(t) = T\left(x=\frac{L}{2}, t\right) = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} B_n(t) \sin \frac{n\pi}{L} \frac{L}{2} = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} B_n(t) C_n \quad (2.16)$$

where C_n is given by Equation (2.9). Substituting Equation (2.15) into Equation (2.16) gives:

$$\hat{T}(t) = \frac{2}{\rho CAL} \int_0^t P(t') \left(\sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} e^{-\lambda_n(t-t')} C_n^2 \right) dt' \quad (2.17)$$

Since according to Equation (2.9), $C_n = \pm 1$, it then follows that $C_n^2 = 1$ so that the infinite summation in Equation (2.17) becomes:

$$S \triangleq \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} e^{-\lambda_n(t-t')} C_n^2 = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} e^{-\lambda_1(t-t')n^2} = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} e^{-\alpha n^2} \quad (2.18)$$

where

$$\lambda_1 = \left(\frac{k}{\rho C}\right) \left(\frac{\pi}{L}\right)^2 \quad (2.19)$$

$$\alpha = \lambda_1 (t-t') \quad (2.20)$$

S is computed in the following way. We first convert Equation (2.18) to an unrestricted series through the substitution:

$$n = 2m + 1 \quad (2.21)$$

where the index m ranges from $m=0$ to $m=\infty$. S then becomes:

$$S = \sum_{m=0}^{\infty} e^{-\alpha(2m+1)^2} = \sum_{m=0}^{\infty} e^{-4\alpha(m+1/2)^2} \quad (2.22)$$

If α is always very small, as for example would be the case in an infinite medium for which $L \rightarrow \infty$ (cf. Equation 2.19), then it is clear that each succeeding term in the summation will only differ a small amount from its predecessor. In this case, the discrete summation can readily be approximated by $\int dm$. We have

$$\lim_{\alpha \rightarrow 0} S \approx \int_0^{\infty} e^{-4\alpha(m+1/2)^2} dm = \int_{1/2}^{\infty} e^{-4\alpha y^2} dy \quad (2.23)$$

Making the substitution

$$w^2 = 4\alpha y^2 \quad (2.24)$$

gives

$$\lim_{\alpha \rightarrow 0} S \approx \frac{1}{2\sqrt{\alpha}} \int_{\alpha}^{\infty} e^{-w^2} dw \approx \frac{1}{2\sqrt{\alpha}} \int_0^{\infty} e^{-w^2} dw \quad (2.25)$$

Since

$$\int_0^{\infty} e^{-w} dw = \frac{\sqrt{\pi}}{2} \quad (2.26)$$

we have

$$\lim_{\alpha \rightarrow 0} S = \frac{\sqrt{\pi}}{4\sqrt{\alpha}} = \frac{\sqrt{\pi} L \sqrt{\rho C}}{4 \sqrt{k} \pi \sqrt{t-t'}} = \frac{\sqrt{\rho C} L}{4 \sqrt{k} \pi (t-t')} \quad (2.27)$$

Substituting Equation (2.27) into Equation (2.17) then gives:

$$\hat{T}(t) = \frac{2}{\rho C A L} \cdot \frac{\sqrt{\rho C L}}{4 \sqrt{k} \pi} \int_0^t \frac{P(t') dt'}{\sqrt{t-t'}} = \frac{1}{2} \left(\frac{1}{\pi k \rho C} \right)^{1/2} \int_0^t \frac{P(t') dt'}{(t-t')^{1/2}} \quad (2.28)$$

By making the appropriate notational substitutions, and generalizing the results of reference 3 to the case of an arbitrary time history of $P(t)$, the reader can convince himself that Equation (2.28) is exactly equal to the result derived by Wunsch. For example, if we assume:

$$P(t) = P_0 : 0 \leq t \leq t_s \quad (2.29)$$

then Equation (2.28) yields the well-known formula:

$$\hat{T}(t) = \frac{P_o}{A} \left(\frac{1}{\pi k \rho C} \right)^{1/2} t^{1/2} \quad : \quad 0 \leq t \leq t_s \quad (a)$$

$$\hat{T}_{\max} = \frac{P_o}{A} \left(\frac{1}{\pi k \rho C} \right)^{1/2} t_s^{1/2} \quad (b)$$
(2.30)

Examination of Equations (2.19) and (2.20) show that in general α will be small whenever

$$\lambda_1 t \ll 1 \quad (2.31)$$

or equivalently

$$t \ll t_d = \frac{\rho C}{k} \left(\frac{L}{\pi} \right)^2 \quad (2.32)$$

where t_d is recognized as the diffusion time. If the pulse duration is t_p then Equation (2.32) shows that for $t_p \ll t_d$ the result given by Equation (2.28) provides an accurate estimate of the junction temperature.

On the other hand, when α becomes comparable to or greater than unity, the approximation of the infinite series by an integral becomes questionable and an alternate method must be found for evaluating $\hat{T}(t)$. Fortunately, this can be done by performing a term-by-term time integration in Equation (2.17). Making the substitution

$$\lambda_n = \lambda_1 n^2 \quad (2.33)$$

and using Equation (2.29) for $P(t)$ in Equation (2.17) for illustrative purposes, we obtain:

$$\hat{T}(t) = \left(\frac{2P_o}{\rho CAL} \right) \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{1}{\lambda_1 n^2} (1 - e^{-n^2 \lambda_1 t}) \quad (2.34)$$

$$\hat{T}(t) = \left(\frac{2P_o L}{\pi^2 AK} \right) \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{1}{n^2} (1 - e^{-n^2 \lambda_1 t})$$

Theoretically speaking, Equation (2.34) is correct for all times, but the infinite series has poor convergence properties in the limit of small $\lambda_1 t$. On the other hand, the series will converge rapidly when $\lambda_1 t \geq 1$ which corresponds to the regime where $t \geq t_d$. In this time regime, the exponential terms will decay rapidly with $\hat{T}(t)$ reaching an asymptotic value given by

$$\hat{T}_{\max}(t) = \left(\frac{2P_o L}{\pi^2 AK} \right) \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{1}{n^2} \quad (2.35)$$

From reference 6, we can deduce

$$\sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{1}{n^2} = \frac{3}{4} \sum_{m=1}^{\infty} \frac{1}{m^2} = \frac{3}{4} \frac{\pi^2}{6} = \frac{\pi^2}{8} \quad (2.36)$$

which when substituted into Equation (2.35) yields

$$\hat{T}_{\max}(t) = \frac{P_o L}{4AK} \quad (2.37)$$

$\hat{T}_{\max}(t)$ could have also been derived from a steady-state solution of Equation (2.2).

It is worthwhile to mention that, for long pulse lengths, as defined by the condition $t_p \gg t_d$, a steady-state solution is meaningful. In this case, the explicit temperature dependence of the conductivity can be taken into account in the computation, since in the p and n regions we deal with the equation

$$0 = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \quad (2.38)$$

This equation integrates to:

$$\int^T k(T') dT' = Cx \quad (2.39)$$

where C is a constant. By straightforward application of the boundary conditions, the spatial distribution of temperature can be determined.

In summary, Equation (2.34) provides a means of calculating $\hat{T}(t)$ for all times and finite size p-n junctions. For the limiting case in which $(t/t_d) \ll 1$, Wunsch's original result is valid. At the other extreme, when $(t/t_d) \gg 1$, the asymptotic bound on $\hat{T}(t)$ should be used. For intermediate times, a finite number of terms from Equation (2.34) would probably provide an adequate approximation.

3.0 ANALYTIC BEHAVIOR OF JUNCTION TEMPERATURE

The purpose of this section is to discuss various analytical properties of the time dependence of the junction temperature as a function of power pulse shape. The results may be useful in a screening assessment of damage susceptibility.

3.1 Alternate Form of Damage Criterion

For the purposes of this discussion, we shall assume that the junction temperature is given by Equation (2.28). For a square wave pulse of power P_o , and duration t_s , the maximum temperature is given by Equation (2.30b). If T_m is the temperature at which damage occurs, the following well-known relationship holds between T_m , P_o , and t_s :

$$P_o = K t_s^{-1/2} \quad (3.1)$$

where K is the damage constant defined by the equation

$$K = A \sqrt{\pi k \rho C} T_m \quad (3.2)$$

For this special case of a square pulse, the total energy delivered to the device is:

$$E_s = P_o t_s \quad (3.3)$$

which gives the equivalent relationship

$$E_s = K t_s^{1/2} \quad (3.4)$$

Using Equation (3.2), we can recast Equation (2.28) in the mathematically convenient form

$$\hat{T}(t) = \frac{T_m}{2K} G(t) \quad (3.5)$$

where

$$G(t) = \int_0^t \frac{P(t')}{(t-t')^{1/2}} dt' \quad (3.6)$$

Equations (3.5) and (3.6) provide an alternate way of evaluating damage. Since damage will occur when $\hat{T}(t)$ exceeds T_m , the condition for damage to occur is obtained from the inequality

$$G(t) \geq 2K \quad (3.7)$$

The foregoing expression for damage appears to be a new way of relating pulse shape to component susceptibility. If G_m is the maximum value of $G(t)$ in the interval $0 \leq t \leq t_p$, where t_p is the pulse duration, then damage will occur when

$$G_{\max} = \max \left\{ \int_0^t \frac{P(t')}{(t-t')^{1/2}} dt' \right\} \geq 2K \quad (3.8)$$

Even for pulses of finite duration, it is clear that the maximum of $G(t)$ need not occur at the end of the pulse interval, so that damage formulas which are based on total energy deposition

$$\int_0^t P(t') dt'$$

may lead to uncertainties in damage assessment. However, these uncertainties will not be very large for the commonly-assumed single cycle

waveforms (e.g., one cycle of a sine wave). For example, for a limited number of cases evaluated, it was found that about 75% of the pulse energy contributed to raising the junction temperature to its maximum value.

Equation (3.6) may be useful for determining the maximum temperature for complex waveforms, and also for more accurately assessing (cf. Section 3.3) the damped sine case than in reference 2. In addition, it provides an easy means of computing the interpulse cooling rate for analyses of repeated pulses (cf. Section 3.2).

3.2 Interpulse Temperature Cooling

The purpose of this section is to assess the temperature decrease between pulses. For mathematical simplicity, we consider the pulse train shown in Figure 2. Using the structure for $P(t)$ shown in this figure,

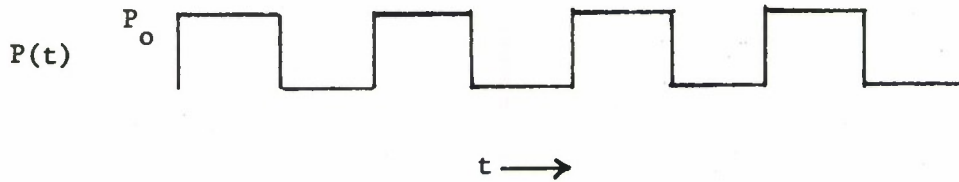


Figure 2. Power Pulse Train

the temperature is determined from the equation:

$$\hat{T}(t) = \frac{T_m}{2K} G(t) = \frac{T_m}{2K} \int_0^t \frac{P(t')}{(t-t')^{1/2}} dt' \quad (3.9)$$

In the range $0 \leq t \leq T$, the function $G(t)$ becomes:

$$0 \leq t \leq T: \quad G(t) = G_1(t) = 2P_0 t^{1/2} \quad (3.10)$$

while in the range $T \leq t \leq 2T$, we have:

$$T \leq t \leq 2T: \quad G(t) = G_2(t) = 2P_o [t^{1/2} - (t-T)^{1/2}] \quad (3.11)$$

The function $G_2(t)$ gives the temperature response following a square wave power impulse function and is actually valid for all times greater than T . For $t \gg T$, the function $G_2(t)$ can be approximated by the expansion:

$$\begin{aligned} G_2(t) &= 2P_o [t^{1/2} - t^{1/2} (1 - \frac{T}{t})^{1/2}] \\ &= 2P_o [t^{1/2} - t^{1/2} (1 - \frac{1}{2} \frac{T}{t} - \frac{1}{8} (\frac{T}{t})^2)] \\ &= (P_o \frac{T}{t}) + \frac{1}{4} P_o (\frac{T}{t})^2 \end{aligned} \quad (3.12)$$

The asymptotic expansion can, of course, be directly obtained from Equation (3.9) by neglecting t' with respect to t in the denominator of the integral. For times t long compared to the pulse width, and for any shape pulse, $\hat{T}(t)$ becomes:

$$\hat{T}(t) = \frac{T_m}{2K} \frac{1}{t^{1/2}} \int_0^{t_p} P(t') dt' = \frac{T_m}{2K} \frac{E_p}{t^{1/2}} \quad (3.13)$$

where

$$E_p = \int_0^{t_p} P(t') dt' = \text{energy delivered in pulse.} \quad (3.14)$$

Figure 3 shows a plot of $G(t)$ (normalized to $2P_o T = 1$) for the first pulse of the pulse train; the function $G_2(t)$ is shown for all time. An interesting feature of $G_2(t)$ is its relatively slow decay

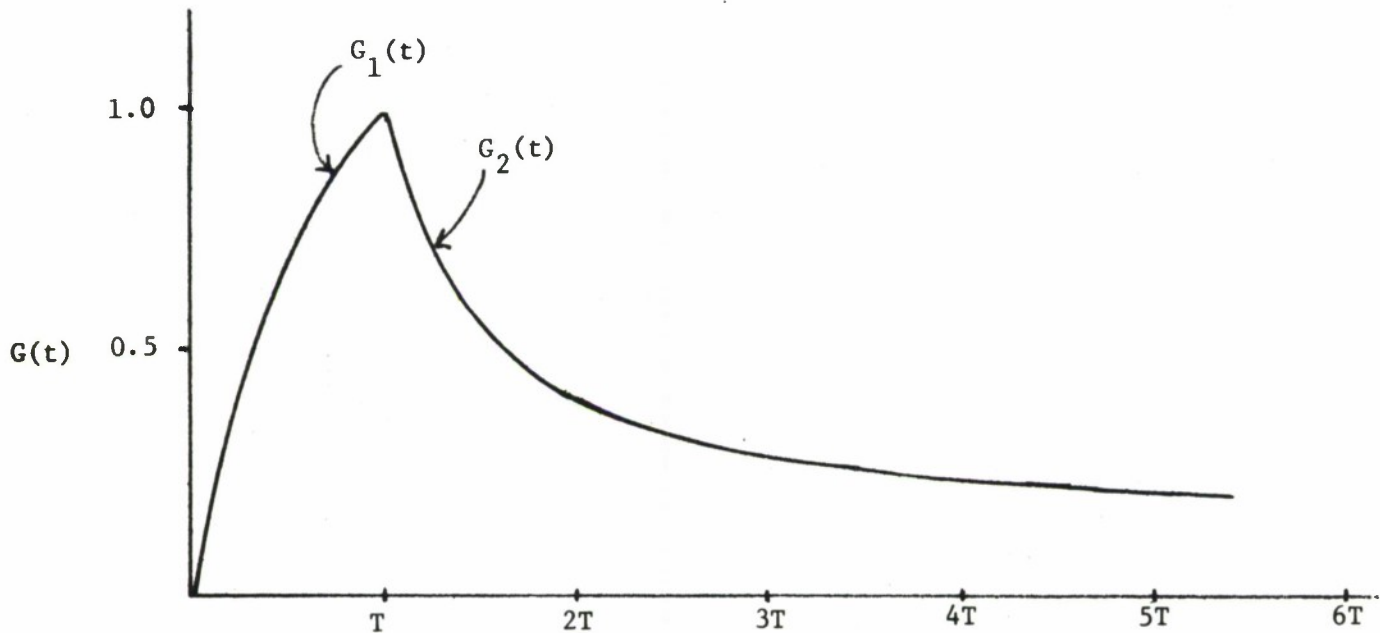


Figure 3. Plot of $G(t)$ for First Pulse

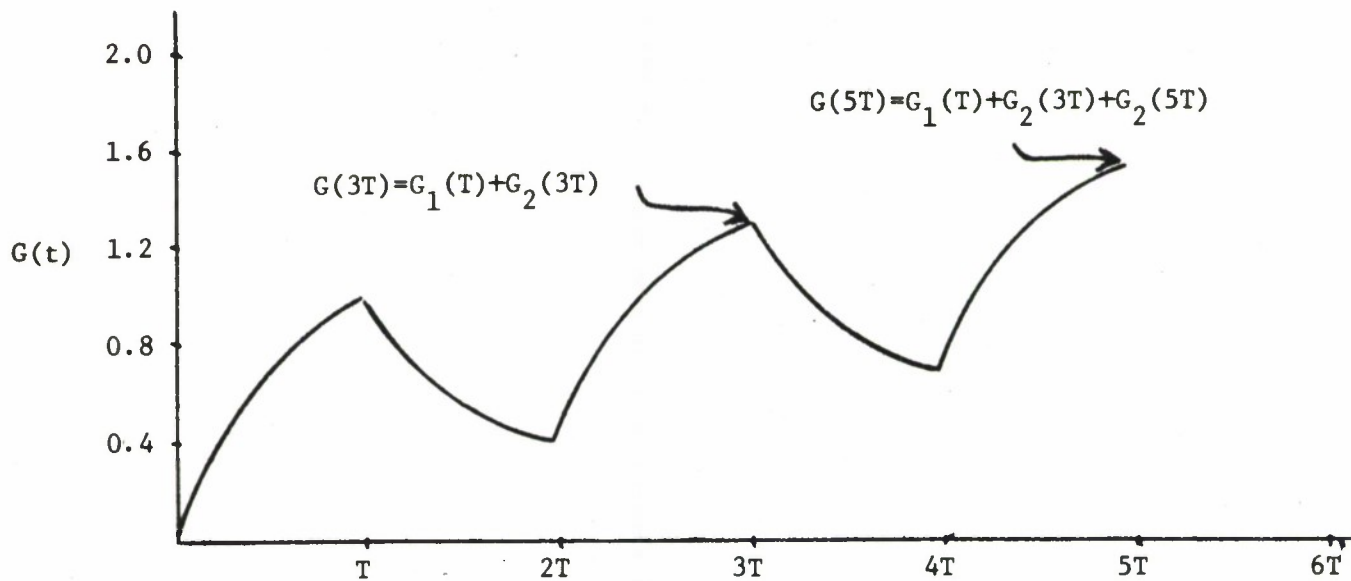


Figure 4. Plot of $G(t)$ for Three Pulses

with time. This is a reflection of Wunsch's physical model which limits the amount of cooling for an infinite system,

The function $G(t)$ at times $t = (2n+1)T$ is given by

$$G[(2n+1)T] = G_1(T) + \sum_{n'=1}^n G_2 [(2n'+1)T] \quad (3.15)$$

Figure 4 shows a plot of $G(t)$ for three pulses. For large n , the function $G_2(t)$ can be approximated by the first term in Equation (3.12) with the normalization $P_0 T = 1/2$. Thus, for large n we have:

$$G_2[(2n+1)T] = \frac{1}{2} \frac{T}{(2n+1)T} = \frac{1}{2(2n+1)} \quad (3.16)$$

which shows that the series of Equation (3.15) is logarithmically divergent. The upshot of this discussion is that it is precarious to make computations of damage assessment based upon one cycle of a series of pulses. For the Wunsch model, temperature quenching appears to be generally insufficient.

The foregoing discussion is useful for analysis of exponentially damped periodic pulses (which includes the damped sine case). Let T_c be the period of the oscillation, and γ the damping constant. The power waveform is expressed as

$$P(t) = Ae^{-\gamma t} W(t) \quad (3.17)$$

where $W(t)$ is defined as the periodic part which satisfies the condition

$$W(t+T_c) = W(t) \quad (3.18)$$

In order for Equation (3.17) to have any meaning, it is clear that

$\gamma T_c \ll 1$. If damping is now applied to the pulse waveform in Figure 2 ($T_c = 2T$), with γt being considered to be relatively constant between mT_c and $(m+1)T_c$, and approximated by the value mT during the interval, the the corresponding behavior of $G(t)$ at the odd intervals of T would be:

$$G[(2n+1)T] = e^{-2n\gamma T} G_1(T) + \sum_{n'=1}^n e^{-2(n-n')\gamma T} G_2[(2n'+1)T] \quad (3.19)$$

For this situation, it is not clear during what cycle the maximum temperature will occur. Equation (3.19) does, however, provide a means of readily evaluating $\hat{T}[(2n+1)T]$, and thereby determining the number of cycles which must be incorporated into the calculation.

3.3 Response of Interface Circuits to Damped Sine Inputs

It has been found experimentally and shown theoretically that the voltage input to interface circuits is frequently of the damped sine type. Such a voltage pulse is described by the form:

$$V(t) = V_o e^{-\gamma t} \sin \omega t \quad (3.20)$$

where in order for Equation (3.20) to have meaning we must have

$$\frac{\omega}{\gamma} \gg 1 \quad (3.21)$$

Voltages of the type given by Equation (3.20) may give rise to various forms of power dissipation in devices depending upon whether failure takes place in the forward or backward direction. Some of the typical power waveforms are discussed in references 2 and 4. Figure 5 shows some representative power waveforms resulting from damped sine inputs.

If the interface circuit is purely resistive, then the power dissipation in a particular device will be of the periodic type, not unlike those shown in Figure 5. On the other hand, if failure takes place in the reverse direction, the length of time for conduction will be voltage dependent⁽²⁾ and the power dissipation will not be strictly periodic. For a damped sine input, we will eventually reach the condition where the reverse voltage falls below the breakdown voltage and conduction in the negative direction ceases.

When inductive and capacitive effects have to be taken into account in the interface circuit model, it is not clear that power dissipation will be cyclic since the basic problem is a transient one. Circuit conditions in the second and subsequent cycles of the input voltage may be different from the first, depending on the values of the reactive elements. Therefore, in some cases, a time-dependent calculation would appear to be the only sensible method of solution.

However, there do exist several instances where effects arising from non-zero initial conditions quickly dampen out, thus permitting analyses of each cycle separately. The following comments provide a discussion of damage assessment under these conditions.

The first logical step in damage assessment is to approximate $V(t)$ by

$$V(t) = V_0 \sin \omega t \quad (3.22)$$

for the first few cycles subject to the condition $\gamma t < 1$. In view of the fact that $(\omega/\gamma) \gg 1$, this time regime may include many cycles. If damage is found to occur in the first few cycles, it is then unnecessary to include the damping. In performing such an analysis, it would of course be necessary to use the results of the previous section to account for temperature decay.

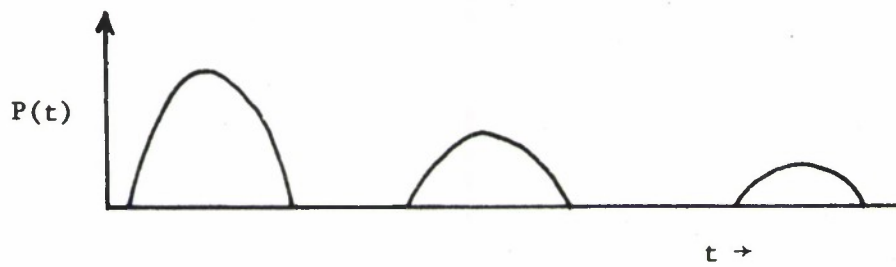
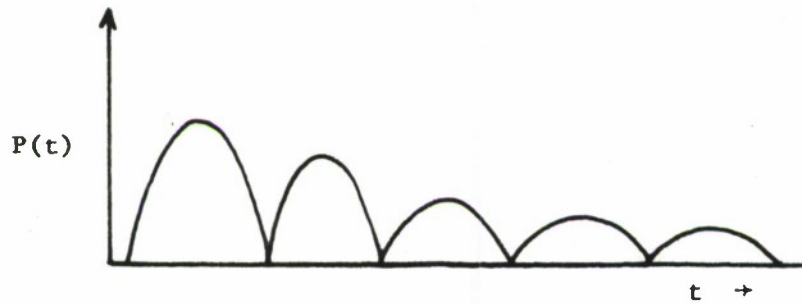


Figure 5. Representative Power Waveforms

However, additional insight into the problem can be found with use of the following observation: In the periodic response range (characterized by Equation 3.22), the power $P(t)$ across any device can be cast in the form:

$$P(t) = P_{ave} + \sum_{n=1}^{\infty} P_n \sin n\omega t \quad (3.23)$$

where P_{ave} is the average of $P(t)$ and P_n are the Fourier coefficients. Equation (3.23) would be valid in the regime $\gamma t < 1$ which, as previously indicated, could include the range $\omega t > 1$. If Equation (3.23) is inserted in the expression for $G(t)$, we obtain:

$$G(t) = \int_0^t \frac{P(t')}{(t-t')^{1/2}} dt' = 2P_{ave} t^{1/2} + \sum_{n=1}^{\infty} P_n \int_0^t \frac{\sin n\omega t'}{(t-t')^{1/2}} dt' \quad (3.24)$$

Working out the details for the integration in Equation (3.24), we obtain:

$$I_n(t) = \int_0^t \frac{\sin n\omega t'}{(t-t')^{1/2}} dt' = \left(\frac{1}{n\omega}\right)^{1/2} \left[\sin n\omega t f_{1n}(t) - \cos n\omega t f_{2n}(t) \right] \quad (3.25)$$

where

$$f_{1n}(t) = \int_0^{n\omega t} \frac{\cos \lambda}{\lambda^{1/2}} d\lambda \quad (3.26)$$

$$f_{2n}(t) = \int_0^{n\omega t} \frac{\sin \lambda}{\lambda^{1/2}} d\lambda$$

As per the previous discussion, the foregoing set of equations are of interest in the time regime encompassing many cycles, but not

long enough to include damping. In the regime where $\omega t \gg 1$ (e.g., $\omega t = 13$ for 2 cycles), it can be shown⁽⁷⁾ that

$$\int_0^{n\omega t} \frac{\sin \lambda}{\lambda^{1/2}} d\lambda \approx \int_0^{n\omega t} \frac{\cos \lambda}{\lambda^{1/2}} d\lambda = \sqrt{\frac{\pi}{2}} \quad (3.27)$$

in which case $I_n(t)$ becomes:

$$I_n(t) = \sqrt{\frac{\pi}{2}} \left(\frac{1}{n\omega}\right)^{1/2} [\sin n\omega t - \cos n\omega t] = \sqrt{\frac{\pi}{n\omega}} \left[\sin \left(n\omega t - \frac{\pi}{4}\right)\right] \quad (3.28)$$

Substituting Equation (3.28) into Equation (3.24) yields:

$$G(t) = 2P_{ave} t^{1/2} \left[1 + \sum_{n=1}^{\infty} \left(\frac{P_n}{2P_{ave}}\right) \sqrt{\frac{\pi}{n\omega}} \sin \left(n\omega t - \frac{\pi}{4}\right) \right] \quad (3.29)$$

The important feature of Equation (3.29) is the relative decrease in the contribution from harmonics as ωt becomes large. It is also evident that the dominant contribution to temperature increase comes from the average power contribution. Neglecting the harmonic terms gives:

$$G(t) = 2P_{ave} t^{1/2} \quad (3.30)$$

which is the response to a square wave pulse of power P_{ave} . If

$$2P_{ave} t^{1/2} \leq 2K \quad (3.31)$$

over many cycles, then damage must be assessed using the exponential decay factor. For cyclic responses, the power dissipation will be

given by the form:^{*}

$$\bar{P}(t) = e^{-\gamma t} \left(P_{ave} + \sum_{n=1}^{\infty} P_n \sin n\omega t \right) = e^{-\gamma t} P(t) \quad (3.32)$$

where $P(t)$ is given by Equation (3.23) Based on the analysis of the undamped case, we assume, without rigorous proof, that for times long enough so that damping becomes important, the dominant contribution to $G(t)$ will result from the first term in Equation (3.32), namely,

$$\bar{P}(t) = e^{-\gamma t} P_{ave} \quad (3.33)$$

Using the Equation (3.33), the expression for $G(t)$ becomes:

$$G(t) = P_{ave} \int_0^t \frac{e^{-\gamma t'}}{(t-t')^{1/2}} dt' \quad (3.34)$$

Making the substitution

$$x = \gamma t, \quad x' = \gamma t' \quad (3.35)$$

then yields

$$G(t) = \frac{P_{ave}}{\gamma^{1/2}} \int_0^x \frac{e^{-x'}}{(x-x')^{1/2}} dx' \quad (3.36)$$

Now letting

$$\mu = x'/x \quad (3.37)$$

* Depending on the relationship between the device voltage and the input voltage, the power damping rate for the device may be either γ or 2γ . For the purposes of this discussion, we use γ .

gives:

$$G(t) = P_{ave} t^{1/2} \int_0^1 \frac{e^{-\mu x}}{(1-\mu)^{1/2}} d\mu \quad (3.38)$$

$$= P_{ave} t^{1/2} 2 \left[1 - x \int_0^1 (1-\mu)^{1/2} e^{-\mu x} d\mu \right] \quad (3.39)$$

$$= \frac{2P_{ave}}{\gamma^{1/2}} x^{1/2} [1 - x\phi(x)] = \frac{2P_{ave}}{\gamma^{1/2}} \psi(x) \quad (3.40)$$

where

$$\phi(x) = \int_0^1 (1-\mu)^{1/2} e^{-\mu x} d\mu \quad (3.41)$$

$$\psi(x) = x^{1/2} [1 - x\phi(x)] \quad (3.42)$$

Figure 6 shows a plot of $\phi(x)$ vs. x . For small values of x , $\phi(x)$ is approximated as:

$$\phi(x) \approx \int_0^1 (1-\mu)^{1/2} d\mu = \frac{2}{3} \quad (3.43)$$

which gives for $x = \gamma t < 1$:

$$G(t) = \frac{2P_{ave}}{\gamma^{1/2}} (\gamma t)^{1/2} \left[1 - \frac{2}{3}(\gamma t) \right] = 2P_{ave} t^{1/2} \left[1 - \frac{2}{3}(\gamma t) \right] \quad (3.44)$$

On the other hand, for large values of x the $e^{-\mu x}$ term in the integrand will be the controlling factor in the integration. In the region where $e^{-\mu x}$ is non-negligible μ will be very small compared to unity and ϕ will be approximately given by:

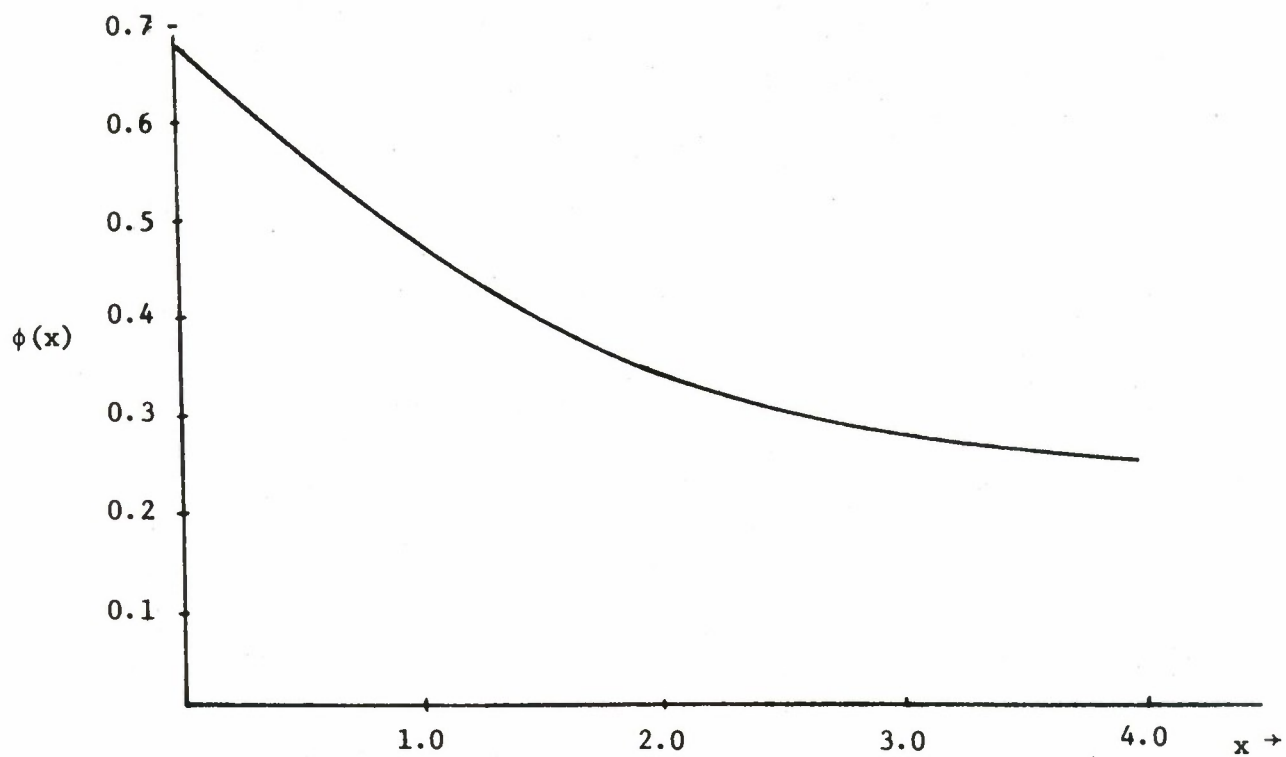


Figure 6. $\phi(x)$ vs. x

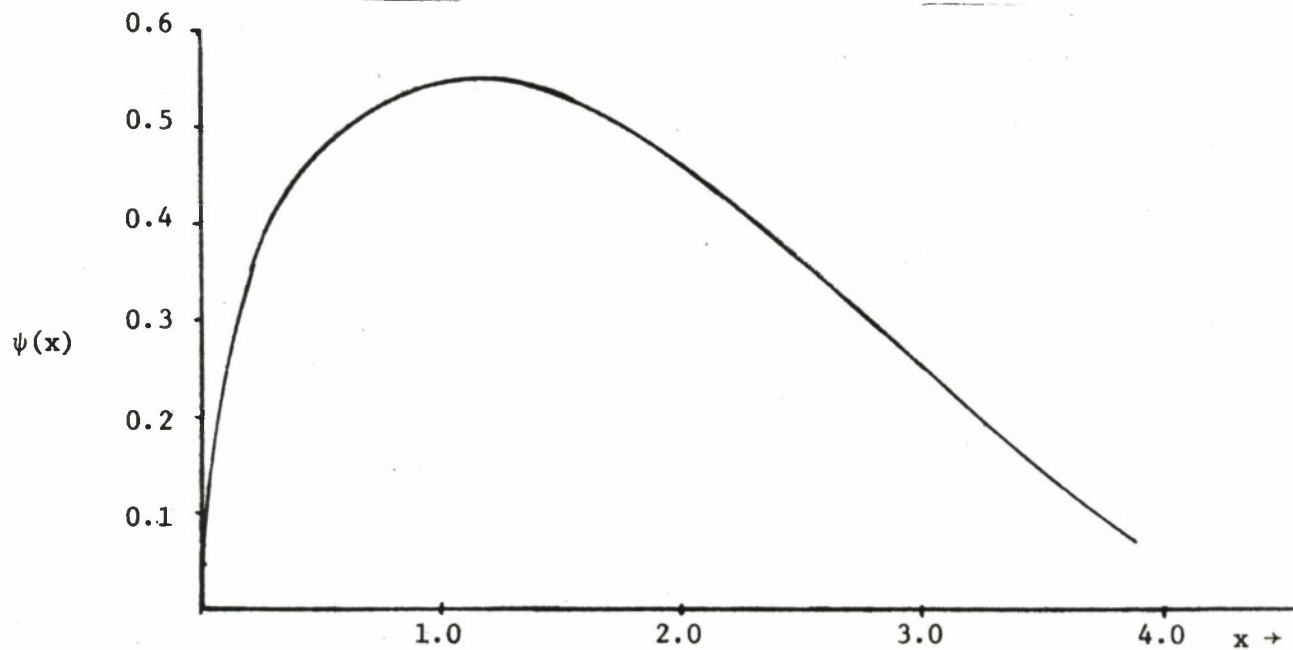


Figure 7. $\psi(x)$ vs. x

$$\phi(x) \approx \int_0^1 e^{-\mu x} d\mu = \frac{1}{x}(1 - e^{-x}) \quad (3.45)$$

Substituting Equation (3.45) in Equation (3.40) gives for $\gamma t > 1$,

$$G(t) = \frac{2P_{\text{ave}}}{\gamma^{1/2}} x^{1/2} [1 - (1 - e^{-x})] = \frac{2P_{\text{ave}}}{\gamma^{1/2}} x^{1/2} e^{-x} \quad (3.46)$$

Figure 7 shows a plot of the function $\psi(x)$, which is proportional to G . As observed, $\psi(x)$ reaches its maximum at the point

$$x \approx 1 \quad (3.47)$$

or equivalently at time

$$t \approx \frac{1}{\gamma} \quad (3.48)$$

The maximum value of $\psi(x)$ is 0.55 which then gives

$$G_{\text{max}} = \frac{2P_{\text{ave}}}{\gamma^{1/2}} (.55)^{1/2} = \frac{1.5P_{\text{ave}}}{\gamma^{1/2}} \quad (3.49)$$

4.0 CONCLUSION

In this report, we have developed several analytical techniques which can be used for evaluating permanent damage of semiconductor devices which are contained in interface circuits. These techniques are intended for use in a hand analysis and/or the screening phase of hardness evaluation.

Included in this discussion are: the effects of finite size p-n junctions on maximum temperature buildup; an evaluation of inter-pulse cooling; and a determination of the response of these devices to damped periodic waveforms. Limitations of the results are presented.

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