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A COMPUTER-AIDED DESIGN ANALYSIS FOR THE STATIC AND DYNAMIC PORT CHARACTERISTICS OF LAMINAR PROPORTIONAL AMPLIFIERS

HARRY DIAMOND LABORATORIES

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This paper presents, in a computer program form, the analysis of laminar proportional amplifiers (LPA's) that Manion and Drzewiecki presented at the Harry Diamond Laboratories (HDL) Fluidic State-of-the-Art Symposium in October 1974. The equations have been programmed for a 16K-core minicomputer with typewriter graphics capability. Results of the numerical computations for the amplifier port characteristics are presented on a single sheet of computer output in both tabular and graphical form. Tabulated data

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE represented the input characteristic (flow and pressure with simultaneous values of the channel resistance as it varies with the flow), the output characteristic (flow, pressure, and resistance), the self-staged operating conditions and gain and the blocked gain (jet-edge pressure and deflection resistance as a function of bias pressure). Graphical outputs are the input and output characteristics--blocked gain versus bias pressure and the Bode diagrams for self-staged, self-loaded, or block-loaded conditions. Typical calculations are presented for standard types of amplifier configurations, and experimental data plotted on the computer output show that the computer analysis is within ±10 percent of the measured values.

The availability of such an analysis program now makes it possible to design new amplifiers on paper, check their performance on the computer, and only then build elements that perform the desired function. The savings in fabrication alone should be considerable, not to mention the elimination of trial-and-error testing that normally would have preceded a design. As an example of the design capability of the program the maximization of self-staged gain simultaneously with bandwidth is shown.

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NOMENCLATURE

a_1	net entrainment coefficent
A	cross-sectional area, m^2
$A_{0,1,2}$	coefficients of second-order response equation
A_4	deflection impedance coefficient
b	width, m
B	normalized width, b/b_s
C'	capacitance $m^4 s^2/kg$
c	viscous loss coefficient for outlet channel flow
c_d	discharge coefficient
c_θ	momentum flux coefficient
$c_{d_{b_s}}$	two-dimensional discharge coefficient
f	frequency, Hz
F	normalized frequency, $f\, b_s/\left[c_d \left(2\left(P_s' - P_v'\right)/\rho\right)^{1/2}\right]$
G	time-independent terms for G_p
G_p	pressure gain
h	amplifier height (depth), m
j	$(-1)^{1/2}$
J'	momentum flux, $kg\text{-}m/s^2$
k, k_2	empirical coefficient for channel resistance
k_1	velocity distribution parameter
k_3, k_{SV}	coefficients for flow-dependent term in R_{OC} and R_{SV}
$k_{4,5}$	entrainment flow distribution coefficients

L'	inductance, kg/m^4
N_R	Reynolds number, $\left(b_s/\nu\right)\left(2\left(P'_s - P'_v\right)/\rho\right)^{1/2}$
N'_R	modified Reynolds number, $N_R/\left(\left(X_{th} + 1\right)\left(1 + 1/\sigma\right)^2\right)$
P'	pressure, Pa
Q'	flow, m^3/s
R'	resistance, kg/m^4s
s	Laplace variable, 1/s
U'	velocity, m/s
V'	center line or Bernoulli velocity, m/s
x	length, m
X	normalized length, x/b_s
y	lateral dimension, m
Y	admittance, 1/Z
Z'	impedance, kg/m^4-s
δ	jet deflection, m
μ	dynamic viscosity, kg/m-s
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
σ	aspect ratio, h/b_s
ϕ	phase shift, degrees
ψ	normalized lateral dimension, y/b_s
ω	radian frequency, rad/s

Subscripts

add	additional
B	additional momentum flux
b_s	supply nozzle width
c	control
CD	center dump
d	driving
e	entrainment
eq	equivalent
i	input, deflection
j	jet edge
J	jet-to-control edge
L	linear
0	(zero) basic condition
o	output
oc	outlet channel
p	pressure
r	recovered
s	supply
SB	spill-back
sp	splitter
ss	self-staged
SV	side vent
t	amplifier throat (spacing between downstream control edges)

th	supply nozzle throat (straight section)
v	vent
1	distance from splitter leading edge to receiver (in x_1)
2	static head equals dynamic head

Superscripts

(')	prime indicates dimensional flow quantity
(-)	bar indicates average value

1. INTRODUCTION

Design of fluidic proportional amplifiers has traditionally been relegated to "cut-and-try" techniques that are essentially inefficient and quite unreliable. However, with the publication of an analysis for predicting the port characteristics of laminar proportional amplifiers (LPA's) by Manion and Drzewiecki,[1] such devices can now be designed entirely on paper with computer-aided techniques. In one such computer technique, given the amplifier geometry and an operating Reynolds number, a set of static and dynamic port characteristics is computed and presented in hard copy. While not a direct design tool (where specifications result in an amplifier design), it does allow the engineer to choose, check, and change the design, observe differences, and approach the desired solution in a few iterations. This process may only take a few hours as opposed to weeks or even months if each design is fabricated and tested in the laboratory. With computer aid, only those designs that are most promising need be critically examined. It is hoped that in the near future, with expanded computational capabilities, such a program can be used as part of an iterative procedure for a design to meet given specifications. The user would enter a certain number of specifications, not to exceed the degrees of freedom of the problem, and the computer would then search through sets of geometries until one was found that satisfied the specifications. A plotting routine would then be called upon to trace an outline of the planview of the preferred design for direct use by either an optical-tracing milling machine, a step-and-repeat pattern generator, or a draftsman.

This report presents a synopsis of the equations used in the computer program. Most derivations refer to the report that analyzed LPA's.[1] Where the original equations have been updated, corrected, or changed, or where additional equations have been incorporated, the derivations will be presented in full. Some experimental verification in addition to that already presented by Manion and Drzewiecki[1] and Drzewiecki, Wormley and Manion[2] will be presented directly on the computer printouts. A design example of optimization of self-staged gain and bandwidth will be presented.

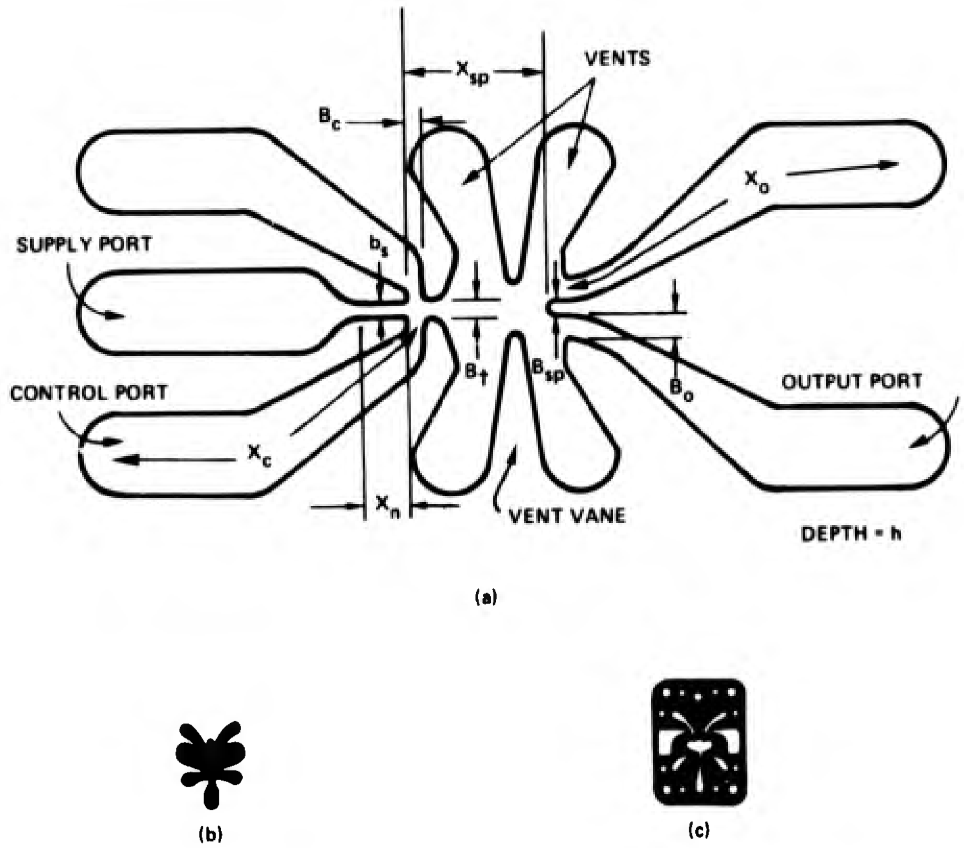
---

[1]*F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

[2]*T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).*

2. THE GOVERNING EQUATIONS

The analysis and assumptions have been formulated for LPA's of the general configuration shown in figure 1a. This type of configuration is the one commonly used at the Harry Diamond Laboratories (HDL), but the formulation is found to hold for other amplifiers that may incorporate center vents (fig. 1b), side-vent outputs (fig. 1c), as well as those without a vent vane (both 1b and c). The vent vane prevents recirculating flow (due to receiver spillage) from impinging on the jet in the sensitive region near the controls. The vent-vane geometry *per se* does not appear in the program. The assumption is made that the vent is at a constant ambient pressure with no spill-back



AMPLIFIERS

Figure 1. (a) Schematic of an HDL LPA and (b), (c) silhouettes of typical LPA's.

effects. The assumption, then, is valid for well-vented devices without the vane. Examples of agreement for devices with and without the vanes are well documented.[2]

The flows into and out of the amplifier are through channels that offer fluid impedance. The vents, however, are assumed to be perfect grounds and as such are assumed to not offer any impedance. The impedances of the amplifier with the jet centered are presented first. The rest of the equations follow in logical order.

2.1 Supply Resistance

The resistance of the supply nozzle is used to normalize all the impedances presented in the following sections. The pressure-flow relationship for flow through a nozzle is obtained simply from the Bernoulli equation and modified with a discharge coefficient, c_d, as given by Manion and Drzewiecki,[1] and here as equation (1). Prime over pressure, P, and flow Q (for example, P', Q') refers to dimensional quantities.

$$Q'_s = c_d\, b_s^2\, \sigma \left[2\left(P'_s - P'_v\right)/\rho\right]^{1/2} \tag{1}$$

where

Q'_s = supply flow, (m^3/s)

c_d = nozzle discharge coefficient, dimensionless

b_s = supply nozzle width, (m)

σ = nozzle aspect ratio, h_s/b_s, dimensionless

P'_s = supply pressure, (Pa)

P'_v = vent pressure, (Pa)

ρ = fluid density, (kg/m^3)

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[1]*F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

[2]*T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).*

The discharge coefficient for flow through a planar nozzle has been determined analytically,[3] but not in closed form. The discharge coefficient, c_d, has been found to be a function of a modified Reynolds number, N_R', that includes the nozzle design in the characteristic length dimension rather than just its width. A sixth-order polynomial in N_R' has been fitted to the numerical solution for $5 \le N_R' \le 100$, and a straight line for $100 \le N_R' \le 200$, where

$$c_d = -0.03422 + 0.0569445\, N_R' - 2.305652 \times 10^{-3}\, N_R'^2$$

$$+ 5.13000 \times 10^{-5}\, N_R'^3 - 6.2507 \times 10^{-7}\, N_R'^4$$

$$- 3.93475 \times 10^{-9}\, N_R'^5 + 1.004255 \times 10^{-11}\, N_R'^6 \text{ for } 5 \le N_R' \le 100 \qquad (2a)$$

$$c_d = 7.8 \times 10^{-3}\, N_R' + 0.624 \text{ for } 100 \le N_R' \le 200 \qquad (2b)$$

where the modified Reynolds number, N_R', and Reynolds number, N_R, are defined as

$$N_R' = N_R \Big/ \left(\left(X_{th} + 1\right) \left(1 + 1/\sigma\right)^2\right)$$

and

$$N_R = \left(b_s/\nu\right) \left[2\left(P_s' - P_v'\right)/\rho\right]^{1/2}$$

where

ν = kinematic viscosity (m^2/s), and

X_{th} = normalized supply nozzle throat length $\left(x_{th}/b_s\right)$.

The supply resistance R_s' is defined as the ratio of the nozzle total pressure drop to flow through it.

$$R_s' \equiv \frac{P_s' - P_v'}{Q_s'} = \left[\rho\left(P_s' - P_v'\right)/2\right]^{1/2} \Big/ \left(c_d\, \sigma\, b_s^2\right) \qquad (3a)$$

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[3] *T. M. Drzewiecki, Fluerics 37: A General Planar Nozzle Discharge Coefficient Representation, Harry Diamond Laboratories TM-74-5 (August 1974).*

or rearranging

$$R'_s = \mu N_R / 2\left(c_d b_s^3 \sigma\right) \tag{3b}$$

where μ = fluid viscosity (kg/m-s). Note that the prime over R, the ratio of a P and Q, indicates that it is dimensional.

In order to operate an amplifier in the laminar range, but near its maximum gain, a typical supply operating point can be defined by the expression $N'_R X_{sp}/8 = 85$. This limits the Reynolds number to a value that is about 20 percent below that of laminar-to-turbulent transition. (Note: All capitalized and Greek-letter geometric dimensions are normalized to the supply nozzle width, b_s. Lowercase letter dimensions are not normalized.)

2.2 Control Channel Impedance

The flow in a control channel (fig. 1a) is very slow; hence, it is assumed to always have a fully developed section with an entrance development effect appearing as a flow dependent term. (N.B.: Unless specifically indicated, all quantities are from now on dimensionless and referenced to the supply conditions, as $P = P'/P'_s$, $R = R'/R'_s$ and $Q = Q'/Q'_s$.) The control channel resistance, R_c, is defined in equation (8) of Manion and Drzewiecki[1] as:

$$R_c = \frac{P_c - P_j}{Q_c} = 24\, X_c\, c_d \left(\sigma/\bar{B}_c + \bar{B}_c/\sigma + k\right)/\left(\bar{B}_c^2 \sigma N_R\right)$$

$$+ 0.95 \left(c_d/B_{c_{min}}\right)^2 Q_c \tag{4}$$

where

P_c = control port pressure

P_j = pressure at the jet edge at end of control channel

Q_c = control flow

X_c = channel length

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[1] *F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

$\overline{B}_c$ = inverse square average channel width, $\overline{B}_c = 1/\left[\left(1/X_c\right)\int_0^{X_c} \left(1/B_c(x)\right)^2 dx\right]^{1/2}$ (Note: Among other parameters, viscous resistance is dependent on $1/B^2$. In the case of nonconstant cross-sections, an average along the streamwise direction of $1/B^2$, $\left(\overline{1/B_c^2}\right)$ gives good results.)

$B_{c_{min}}$ = minimum channel width

k = empirical correction for corner effects to match linear closed-form solution to the exact series solution, where $0.35 \leq k \leq 0.5$ for $1 \leq \sigma/\overline{B}_c \leq 2$

and

k = 0.5 for $\sigma/\overline{B}_c > 2$. (See Drzewiecki[4] for complete details of the determination of the value of k.)

The frequency-dependent component of the channel impedance has been assumed to be purely inertive, resulting in $Z_c' = R_c' + j\omega L_c'$. The fluid compliance is negligible even in high-frequency gaseous applications, when it is compared with the large compliance (capacitance) associated with the volume swept out by the deflecting jet. Fluid inertance (inductance) is defined as the product of the fluid density and the channel length divided by the channel average cross-sectional area. For the control channel the inertance then is $L_c' = \rho x_c/\left(\overline{b}_c h\right)$ where $\overline{b}$ is a linear average width rather than the inverse square average used in the resistance. However, since the two are close and since the resistance is the more sensitive parameter, use of the already available inverse square average will not produce much error.

The channel impedance is therefore

$$Z_c' = R_c' + j\omega' L_c' = R_c' + j\omega' s\, x_c/\left(\overline{b}_c h\right).$$

If this equation is now normalized by the supply operating resistance, R_s', as defined by equation (3a), where it is noted that $\left(2\left(P_s' - P_v'\right)/\rho\right)^{1/2} = V_s'$, the Bernoulli velocity, then the expression for the impedance becomes

---

[4]*T. M. Drzewiecki, The Interpretation of Surface Static Pressure Distributions in Fluid Amplifier Applications, Harry Diamond Laboratories TR-1627 (July 1973).*

$$Z_c = Z_c'/R_s' = R_c + \left(\frac{\omega' \rho X_c}{\bar{b}_c}\right)\left(\frac{2c_d \sigma b_s^2}{\rho V_s}\right) j \ .$$

If now the radian frequency, ω', is normalized by the reciprocal of the average jet transport time, $c_d V_s'/b_s$, the above equation simply becomes

$$Z_c = R_c + j\omega\left(2x_c c_d^2/\bar{b}_c\right) ,$$

or if one notes that $x_c/b_c \equiv X_c/\bar{B}_c$, then

$$Z_c = R_c + j\omega\left(2X_c c_d^2/\bar{B}_c\right) \ . \ . \ . \qquad (5)$$

The normalized inertance is therefore defined from equation (5) as the term that multiplies the normalized frequency, or

$$L_c \equiv 2c_d^2 X_c/\bar{B}_c \ldots \qquad (6)$$

2.3 Resistance Due to the Restriction Between Jet and Control Edge

The derivation of the resistance, R_J, offered to flow by the space between the jet and the downstream control edge (fig. 2) is presented by Manion and Drzewiecki.[1] This resistance is called "jet-edge resistance.*" Since R_J changes only slightly with jet-edge pressure, an average value of R_J for jet-edge pressures from $P_j = 0.1$ to 0.5 is used. This eliminates calculating R_J for different bias conditions. The jet-edge resistance is obtained by averaging the value from equation (20) of Manion and Drzewiecki over the representative bias range of 0.1 and 0.5, and is

$$R_J = \frac{1}{0.4}\int_{0.1}^{0.5} c_{d_{b_s}} \ P_j / \left\{ \int_{\psi_2}^{B_t/2} \left[\left(\operatorname{sech}^4 k_1\psi + P_j\right)^{1/2} - \operatorname{sech}^2 k_1\psi\right] d\psi \right\} dP_j \qquad (7)$$

where

$$c_{d_{b_s}} = (1 - \sigma)/2 \left[\left(\sigma + 1\right)^2/4 - \sigma\left(1 - c_d\right)\right]^{1/2} \qquad (8)$$

= effective discharge coefficient of lateral nozzle boundary layers only

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[1] *F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

**Due to restriction between jet and control edge.*

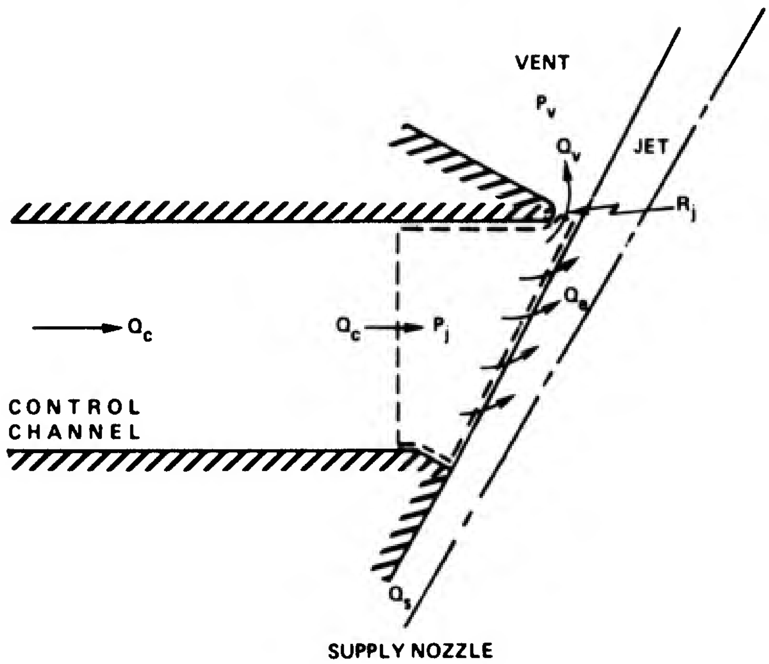

Figure 2. Sketch showing the clearance between the jet and the downstream control edge.

$$k_1 = 1/\left(c_d\, Q_e + c_{d_{b_s}}/2\right) \tag{9}$$

$$Q_e = 1.651\left(0.021\, c_d\, c_\theta + c_\theta\, B_c/N_R\right)^{1/3}/c_d - 0.5 \tag{10}$$

= entrained flow over the distance B_c

$$\psi_2 = \left[\text{sech}^1\, P_j^{1/4}\right]/k_1 \tag{11}$$

= lateral location of streamline where velocity head equals static head

$$c_\theta = 1.32\, c_d^2 = \text{momentum flux discharge coefficient.} \tag{12}$$

There is no inertive reactance attributed to the space between the jet and control edges. For the solution at hand, R_J is constant at any given supply operating point.

2.4 Outlet Channel Impedance

The impedance of the outlet channel is calculated the same way as the control channel impedance (eq 4). The channel resistance R_{oc} is composed of fully developed and entrance losses and is represented by equation (13).

$$R_{oc} = \frac{P_d - P_o}{Q_o} = 24\, X_o\, c_d \left(\sigma/\bar{B}_o + \bar{B}_o/\sigma + k_2\right)\Big/\left(\bar{B}_o^2\, \sigma\, N_R\right)$$

$$+ 0.95 \left(c_d/B_{o_{min}}\right)^2 Q_o \qquad (13)$$

Notice that equation (13) may be written as a constant plus a flow-dependent term so that

$$R_{oc} = R_{oL} + k_3\, Q_o \qquad (13a)$$

where

P_d = total pressure at channel entrance

P_o = output port static pressure

Q_o = output flow

X_o = outlet channel length

$\bar{B}_o$ = inverse square average width of outlet channel

$\bar{B}_{o_{min}}$ = outlet channel minimum width (usually equal to receiver width, B_o)

k_2 = empirical correction for corner effects

k_3 = coefficient of flow-dependent term in R_{oc} (eq 13a)

R_{oL} = constant (linear) part of R_{oc} (eq 13a)

$$0.35 \le k_2 \le 0.5; \text{ for } 1 \le \sigma/\bar{B}_o \le 2$$

$$k_2 = 0.5; \text{ for } \sigma/\bar{B}_o \ge 2.$$

The inertance of the fluid in the channel is similar to that of equation (6):

$$L_{oc} = 2\ c_d^2\ X_o/\overline{B}_o \tag{14}$$

Again, when one assumes that the compliant reactance is negligible, the output channel impedance is

$$Z_{oc} = R_{oc} + j\omega L_{oc}\ . \tag{15}$$

2.5 Output Impedance

The output impedance, Z_o, differs from the outlet channel impedance, Z_{oc}, in that Z_o is the change of output port pressure, P_o, with output flow, Q_o, $\partial P_o/\partial Q_o$; hence, it includes the variation of the total pressure at the receivers (channel entrance).

In Manion and Drzewiecki[1] the driving (total) pressure, P_d, at the entrance to the output channel is given as a function of output flow so that

$$P_d = P_r\left(1 - Q_o^2/2\ Q_{or}^2\right) \tag{16}$$

where

$$Q_{or} = Q_o\left(P_o = 0\right) = -R_{oL}/\left(2\ k_3\right) + \left[\left(R_{oL}/2\ k_3\right)^2 + P_r/\left(2\ k_3\right)\right]^{1/2} \tag{17}$$

and the blocked pressure recovery, P_r, is given as

$$P_r = c_\theta\left(1 - 8\left(X_{sp} + X_1\right)/\sigma^2\ N_R\ c_\theta\right)\left(1 - 1.1\ B_{sp}/\left[c_d\ N_R\ B_{sp}/2\right]^{1/2}\right)/B_o \tag{18}$$

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[1] *F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

where

X_{sp} = distance between supply nozzle exit and splitter tip

X_1 = additional run distance along splitter prior to flow turning

B_{sp} = splitter end diameter (width).

While this representation is adequate, it does not take into account the entire physics of the jet/output channel interactions. As a result the output flow tends to be overestimated at low output pressure and the output impedance underestimated near blocked conditions. If one examines the flow as it impinges on the output channel a more precise model can be made.

Consider the following sketch of the flow entering the output channel and flow being spilled out (fig. 3).

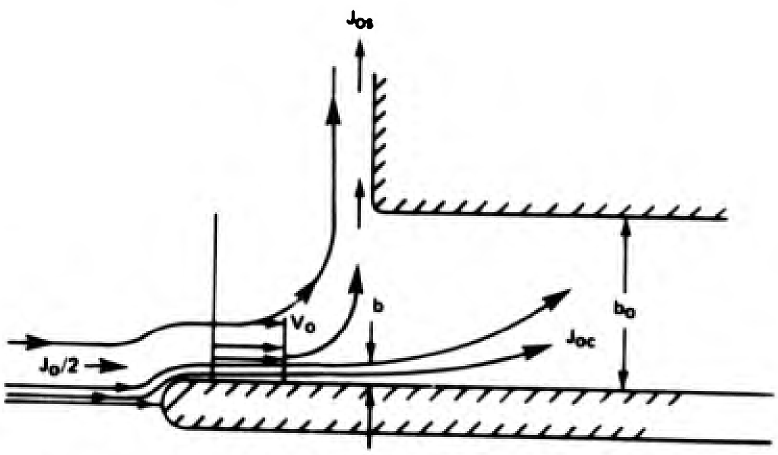

Figure 3. Flow model for impinging jet on output channel.

Essentially, the static pressure developed by the turning of the flow is that due to the spilled momentum, J_{os}, divided by the area $(b_o - b)h$, since the flow is not turned over the entire receiver face. In addition, however, there is a dynamic head due to the high-velocity stream entering the channel. This dynamic head is averaged over the receiver face. The spilled momentum is determined by the amount of flow removed from the profile, Q_o, which then specifies the width, b, at which this flow enters the output channel.

$$b = Q_o'/(U_o'h) \tag{19}$$

where U'_o is the jet velocity impinging on the output. Note that the momentum flux impinging on the output is then $\rho b_s h U'^2_o/2 = J_o/2$.

The total (or driving) pressure impinging on the output channel is the sum of the static and dynamic pressures available. The static pressure is assumed uniform over the width and is determined from the spilled momentum over the area of turning, and the dynamic head of the flow entering the outlet channel is determined by averaging the velocity head over the channel width.

$$P'_d = \frac{J_{os}}{\left(b_o - b\right)h} + \tfrac{1}{2}\rho U'^2_o \frac{b}{b_o} = \frac{J'_o/2\left(1 - 2b/b_s\right)}{\left(b_o - b\right)h} + \tfrac{1}{2}\rho U'^2_o \frac{b}{b_o} .$$

When the expression for b from equation (19) is substituted into the above equation and if one notes that $J_o/2 = \rho b_s h U'^2_o/2$ and if it is normalized by $\left(P'_s - P'_v\right)$ the result is

$$\frac{P'_d}{P'_s - P'_v} = P_d = \frac{\dfrac{\rho b_s h U'^2_o}{2}\left(1 - \dfrac{2Q'_o}{U'_o h b_s}\right)}{\left(P'_s - P'_v\right)\left(b_o - \dfrac{Q'_o}{U'_o h}\right)h} + \frac{\rho U'^2_o b}{P'_s - P'_v\, b_o}$$

The normalized recovery pressure is $P'_r/P'_s = \rho U'^2_o b_s h/ P'^2_s b_o h$. This can be then rearranged to yield $P_r B_o = \rho U'^2_o/2P_s$. When this relation is subsituted for U' and the output flow is normalized by $Q'_s = b_s h c_d\left(2\ P'_s - P'_v\ /\rho\right)^{1/2}$ the result is:

$$P_d = \frac{P_r\left(1 - 2Q_o c_d/ \left[P_r B_o\right]^{1/2}\right)}{1 - Q_o c_d/\ B_o\left[P_r B_o\right]^{1/2}} + c_d\left(P_r/B_o\right)^{1/2}\ Q_o .$$

As can be seen, the above equation reduces to the identity $P_d = P_r$ when $Q_o = 0$.

The above inviscid analysis has neglected all viscous effects in the receiver region. The momentum turning part of the analysis has been shown to be accurate by the good agreement between data on blocked pressure recovery and theory,[1,2] but one may argue that the inviscid average dynamic head term may not reduce the recovered pressure enough. Experimental data on the output characteristics have indicated little curvature for P_o versus Q_o, except at a knee. The above equation will lead to a fairly straight curve, but it overestimates the flow. The amount of overestimation is a function of aspect ratio when aspect ratio is less than two. An empirical correction to the dynamic head term should be sufficient to adjust it for adequate agreement with data. The mere presence of top and bottom plate boundary layers is assumed to give a loss coefficient of 0.5. This appears to suffice for high aspect ratios, $\sigma > 2$. A linear relationship $c = \sigma/2$, for $\sigma < 2$, is then used as an additional loss coefficient due to the decreased plate spacing. (When $\sigma = 2$, the loss coefficient is unity.) Equation (20) results from the inclusion of these loss coefficients and is the relation for driving pressure on the outlet channel.

$$P_d = \frac{P_r\left(1 - 2Q_o c_d/\left[P_r B_o\right]^{1/2}\right)}{1 - Q_o c_d/\left(B_o\left[P_r B_o\right]^{1/2}\right)} + \begin{cases} \dfrac{c_d \sigma}{4}\left(P_r/B_o\right)^{1/2} Q_o, & \text{for } \sigma < 2 \\ \dfrac{c_d}{2}\left(P_r/B_o\right)^{1/2} Q_o, & \text{for } \sigma \leq 2 \end{cases} \tag{20}$$

Manion and Drzewiecki[1] did not consider the reduction in jet momentum flux due to flow spillage by the downstream edges of the controls. An additional loss coefficient must, therefore, be incorporated in the pressure recovery (eq (18)). The following analysis will develop such a coefficient.

The entrained jet flow at the downstream control edge can be considered, in a simplified way, as having an exponentially decreasing velocity distribution from the entrainment streamline, ψ_e to infinity. If B_v as derived[1] is the distance from the control edge to that streamline

$$B_v = 0.5\left(B_t - 1 - B_c/0.021\ c_d\ N_R\right), \tag{21}$$

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[1] *F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

[2] *T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).*

the entrainment streamline location with respect to the jet center line is

$$\psi_e = B_t/2 - B_v$$

where

B_t = lateral distance across the amplifier between control edges.

Note that as N_R decreases B_v decreases or, in other words, the jet spreads out more at low Reynolds numbers. The exponential velocity* distribution of the entrained flow can be written as

$$U = U'/U'_s = k_5 e^{-k_4\left(\psi - \psi_e\right)} . \tag{22}$$

The coefficients k_4 and k_5 may be evaluated by noting that the integral of equation (22) must be the entrained flow and that when $\psi = \psi_e$ the velocity is the entrainment streamline velocity. Hence

$$\int_{\psi e}^{\infty} U \, d\psi = c_d \, Q_e . \tag{23}$$

And substituting equation (22) into (23) results in

$$k_5/k_4 = +c_d \, Q_e \tag{24}$$

where ψ - lateral coordinate - y/b_s. The velocity at the entrainment streamline U_e is estimated as the velocity on the bell-shaped jet velocity profile (eq (11)) at the dividing point between the supply flow and the entrained flow, or

$$U_e = \text{sech}\left(\tanh^{-1} c_{d_{b_s}} \; k_1/2\right) , \tag{25}$$

so that when equation (22) is evaluated at $\psi = \psi_e$, $k_5 = U_e$. When this result is substituted into equation (24),

$$k = U_e/\left(c_d \, Q_e\right) . \tag{26}$$

**An exponential velocity distribution for the entrained flow is chosen because it has been observed that the profile is more distended than the normally expected hyperbolic secant distribution.*

An estimate of the flow Q_{SB}, spilled back by the control edge, can be made by the integration of equation (22) from $B_t/2$ to infinity and by reducing the result by two to include (at least superficially) plate effects.

$$Q_{SB} = \frac{1}{2} Q_e e^{-k_4 B_v} . \tag{27}$$

The net entrained flow $Q_{e_{net}}$ is

$$Q_{e_{net}} = Q_e - Q_{SB}. \tag{28}$$

The amount of momentum lost due to this spilled-back flow is determined by integrating the square of the jet velocity (eq (22)) from $B_t/2$ to infinity. The pressure recovered, P_r, is directly proportional to the impinging momentum; hence, the jet momentum is decremented by this spilled-back momentum. The net momentum flux at the control edges, $J_e - J_{SB}/J_e$, is

$$\frac{J_e - J_{SB}}{J_e} = \left(1 - J_{SB}/J_e\right) = 1 - 3.0303\ k_4\ Q_{SB}^2 \tag{29}$$

where $J_e = c_\theta$ is the normalized exit momentum flux. The recovered pressure, equation (18), must be multiplied by equation (29). The equation relating the output pressure, P_o, to output flow Q_o is obtained by solving for P_o from equation (13).

$$P_o = P_d - R_{oL}\ Q_o - k_3 Q_o^2 \tag{30}$$

where P_d, the driving jet head, is defined by equation (20).

It was mentioned earlier that, except for a knee, in some output characteristics the curve was fairly staight. Indeed, equation (30) has this form; however, it has no "knee," or abrupt change between two virtually constant slopes. After considerable lack of success in finding a suitable explanation for such a knee, this physical description was reached: During operation of a device, when the impedances of the output, $\partial P_o/\partial Q_o$, and the load, P_o/Q_o, are great enough to spill flow at the receivers, the model explained earlier in this section holds. However, when the load and the output demand more flow than is available (a sucking or jet pumping condition) the jet can only supply as much flow as exists in the profile for the given pressure. In other words, when the load plus output impedance is less than the jet impedance, the output impedance becomes the jet impedance, since there is not enough potential to drive out any more flow. The pressure and flow will change linearly from that point up to the condition when all the flow into the amplifier (Q_s, Q_c) comes out. This argument will sometimes produce an output characteristic that has a "knee," because the slope will be constant below a certain pressure.

For a center-vented device, flow and momentum are extracted from the center of the jet. If one assumes that the pressure impinging on the vent is the center-line velocity head, and that the vent can be described by a constant cross-sectional rectangular channel, the amount of center-vent flow can be readily determined. This extracted flow establishes a net width reduction of a constant velocity (square profile) jet, that reduces the net momentum flux impinging on the receiver. The flow through the center dump, Q'_{CD}, is simply the ratio of the stagnation pressure, P'_{CD}, to the dump resistance, R'_{CD}:

$$Q'_{CD} = P'_{CD}/R'_{CD} \tag{31}$$

This flow can be written as the product of an equivalent area and the average jet center-line velocity at the splitter, V'_{SP},

$$Q'_{CD} = b_{CDeq}\ h\ V'_{SP} \tag{32}$$

where b_{CDeq} is the equivalent jet width that, with the depth h, forms the equivalent area. If Q'_{CD} is normalized by $Q'_s = b_s h c_d \left[2\left(P'_s - P'_v\right)/\rho\right]^{\frac{1}{2}}$, and if it is noted that $V'^2_{SP} = J'_{SP}/\rho b_s h = 2P'_{CD}/\rho$, then solving for the equivalent width,

$$B_{CDeq} = c_d \left(P_{CD}\right)^{\frac{1}{2}}/R_{CD} \quad . \tag{33}$$

The center dump resistance is considered to be linear and is similar to the linear part of the control resistance (eq (4)).

$$R_{CD} = 24\ X_{CD}\ c_d \left(\sigma/B_{CD} + B_{CD}/\sigma + k_6\right)/\left(N_R\ \sigma\ B^2_{CD}\right) \tag{34}$$

where X_{CD} and B_{CD} are, respectively, the length and width of the vent and k_6 (like k_2) is an empirical correction for corner effects.

The pressure developed at the face of the center dump is $P'_{cd} = \frac{1}{2} \rho V'^2_{SP}$, but since $J'_{SP} = \rho b_s h V'^2_{sp}$ then $P'_{CD} = J'_{SP}/2b_s h$. Normalizing with respect to P'_s yields $P_{CD} = J_{SP}$. J_{SP} is the momentum flux available in the jet at the face of the center dump (splitter). This is simply the initial momentum flux reduced by the nozzle, control edge, and plate effects. From equations (18) and (29), then,

$$P_{CD} = J_{SP} = c_\theta \left(1 - 8\left(X_{sp} + X_1\right)/\left(\sigma^2 N_R \; c_\theta\right)\right)\left(1 - 3.03 \; k_4 Q_{SP}{}^2\right). \tag{35}$$

The jet momentum available to the receivers is the total jet momentum at the splitter reduced by the quantity $\left(1 - B_{CDeq}\right)$.

The recovered pressure with a center-vented device, $P_{r_{CD}}$, is therefore P_r, the recovered pressure with no center vent, reduced by the same factor as the momentum in the jet, or

$$P_{r_{CD}} = P_r \left(1 - B_{CDeq}\right) . \tag{36}$$

In addition, the recovered pressure may be augmented by applying a bias control pressure, which introduces additional flow that fills out the jet profile. If one notes that the additional momentum flux J'_B can be represented as the flux added to both sides of the jet or twice the flux on one side, then

$$J'_B = 2\rho' V'^2_j / A'_j \tag{37}$$

where

A'_j = effective area of flow between jet and control edge

V'_j = average velocity of flow, Q'_j, between jet and control edge.

Since $V'_j = Q'_j/A'_j$, then

$$J'_B = 2\rho' V'^2_j / A'_j \tag{38}$$

If the space between the jet and control edge is assumed to behave as an orifice, the effective flow area can be calculated from the Bernoulli equation

$$Q'_j = A'_j \left[2\left(P'_j - P'_v\right)/\rho'\right]^{1/2}$$

or

$$A'_j = Q'_j/\left[2\left(P'_j - P'_v\right)/\rho'\right]^{1/2}. \quad (39)$$

If equation (39) is substituted into equation (38) the additional momentum flux becomes

$$J'_B = 2\, Q'_j \left[2\left(P'_j - P'_v\right)\rho'\right]^{1/2}. \quad (40)$$

When J'_B is normalized by $J'_s = 2\, b_s\, h\, P'_s$, and equation (40) is rearranged in the form $1 + J_B/J_e$, where J_e is the momentum flux at the nozzle exit $(J_e = c_\theta = J'_e/J'_s)$, and if it is noted that $Q_j = \left(P_j - P_v\right)/R_j$, then

$$1 + J_B/J_e = 1 + 2\, c_d \left(P_j - P_v\right)^{3/2}/\left(c_\theta\, R_j\right). \quad (41)$$

One should note that when J_e is multiplied by equation (41) the result is $J_e + J_B$, the net flux at the end of the controls. The complete representation of the pressure recovery, P_r, can be obtained by combining equations (18), (29), (36), and (41). Equation (42) is the result.

$$P_r = c_\theta \left[1 - 8\left(X_{sp} + X_1\right)/\sigma^2 N_R\, c_\theta\right]\left[1 - 1.1\, B_{SP}/\left(c_d\, N_R\, B_{SP}/2\right)^{1/2}\right]\left[1 - 3.03 \times k_4 Q_{SB}^{2}\right] \times \left[1 - B_{CDeq}\right] \times \left[1 + 2\, c_d \left(P_j - P_v\right)^{3/2}/c_\theta\, R_j\right]/B_o. \quad (42)$$

The pressure recovery P_r in equation (42) includes losses due to

(1) supply nozzle, (2) control edge spill-back, (3) top and bottom plates, (4) splitter interaction, and (5) center venting. It also includes an increase due to added axial flow introduced by a bias control pressure.

Some commercial amplifiers, originally designed to operate in the turbulent regime, have been operated fairly successfully in the laminar regime. These devices may include an outlet channel arrangement that includes a side vent (illustrated, fig. 4).

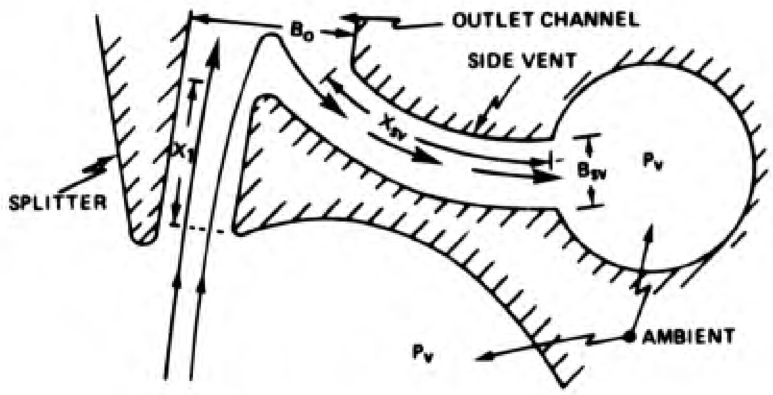

Figure 4. Schematic diagram of a typical output side vent.

The pressure recovery may still be determined by using equation (42), however, recovery is augmented by an increase in static pressure caused by the forcing of the jet flow through the side vent.

Under blocked conditions and for relatively low values of vent resistance (less than the jet impedance), the narrow laminar jet passes through the receiver area unhampered except for the additional plate loss over the distance X_1. The momentum is turned, as before, but now an additional static pressure is developed by the flow passing through the resistance of the vent channel. If one assumes that the jet is essentially a flow source and that the flow out of the vent is roughly equal to the flow recovery of the amplifier (normally about half the jet flow), and that as the flow is drawn out of the output port the vent flow is reduced by that output flow, the output port pressure P_o is augmented simply by the static head developed across the vent resistance

$$P_{add} = \left(Q_{or} - Q_o\right) R_{SV} \tag{43}$$

and the output pressure then becomes

$$P_o = P_d - R_{oL} Q_o - k_3 Q_o^2 + \left(Q_{or} - Q_o\right) R_{SV} \tag{44}$$

where

Q_{or} = recovered flow = Q_o (P_o = 0) from equation (30), or Q_{or} = 0.5 + Q_c, whichever is smaller,

R_{SV} = side vent resistance

$$R_{SV} = 24\ X_{SV}\ c_d\left(\sigma/B_{SV} + B_{SV}/\sigma + k_2\right)/B_{SV}^2\ \sigma N_R$$

$$+ 0.95\ \left(c_D/B_{SV}\right)^2\ \left(Q_{or} - Q_o\right) \tag{45}$$

or

$$R_{SV} = R_{SVL} + k_{SV}\left(Q_{or} - Q_o\right)\ . \tag{46}$$

The output resistance R_o is obtained by differentiating either equation (30) or (44) with respect to Q_o.

$$R_o = \partial P_o/\partial Q_o\ . \tag{47}$$

For the case without the side vents the complex impedance is

$$Z_o = R_o + j\omega L_o \tag{48}$$

where

$$L_o = 2\ c_d^2\ X_o/\bar{B}_o\ .$$

2.6 Deflection Impedance

While the amplifier is operated differentially, the jet moves from its centered position, affording easier passage of flow through a larger space between the jet and the control edge on one side. This has been reported,[1] and the ratio of the control pressure to flow change in this mode is called the small-signal complex deflection impedance Z_i. It is given as

$$Z_i = Z_c\left(1 + A_4\right)/A_4 \tag{49}$$

where

$$A_4 = Z_c\left[a_1 B_c^2/4c_\theta + 1/R_v + 2\ Y_i + B_c^2\left(P_j - P_v\right)/\left(2\ c_d\ c_\theta\left[\left|P_j - P_v\right|\right]^{\frac{1}{2}}\right)\right] \tag{50}$$

[1]*F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium,* 1, *Harry Diamond Laboratories, Adelphi, MD (October 1974).*

or

$$A_4 = Z_c A_5$$

and

$$A_5 = a_1 B_c^2/(4c_\theta) + 1/R_v + 2Y_i + B_c^2(P_j - P_v)/\left(2c_d c_\theta\left[|P_j - P_v|\right]^{\frac{1}{2}}\right)$$

a_1 = net entrainment coefficient $d\Delta Q_{net}/d\delta$

Y_j = jet compliant admittance--C_j $(j\omega)$

C_j = jet capacitance--$B_c^3/12\ c_\theta$

δ = jet deflection fron the center line at downstream control edge.

The net entrainment coefficient is obtained by differentiation of equation (27) with respect to δ and the evaluation of the derivative at $\sigma = 0$, and then by noting that addition of σ to B_v decreases Q_{SB} and vice versa. Thus, in equation (50)

$$a_1 = 2\,\frac{dQ_{SB}}{d\delta} = 2\,\frac{d}{d\delta}\left(\tfrac{1}{2}Q_e e^{-k_4\ B-\delta)}\right) = 2\ k_4 Q_{SB} \quad .$$

Note that deflection impedance is a function of bias pressure in that it depends on the level of P_j. Even though Z_i is the small-signal value, the large linear range of most typical amplifiers allows the use of this value for deflections as large as 70 percent of saturation.

2.7 Laminar Proportional Amplifier Pressure Gain

The blocked output pressure gain G_p, given by Manion and Drzewiecki (eq(60)),[1] must be decremented by the control spill-back and the center-vent loss coefficients. This results in

$$G_p = B_c^2\left[1 - 8(X_{sp} + X_1)/(C_\theta \sigma^2 N_R)\right]\left[1 - 1.1\ B_{sp}/(c_d N_R B_{sp}/2)^{\frac{1}{2}}\right]\left[2X_{sp}/B_c - 1\right]$$

$$\times \left[1 - 3.03\ k_4\ Q_{SB}\right]\left[1 - B_{CDeq}\right]/\left[B_o\ (1 + A_4)\right]$$

$$= \frac{G}{1 + A_4} = \frac{G}{1 + Z_c A_5} \qquad (52)$$

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[1] *F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

where G represents the product of all the frequency independent terms.

The added momentum (due to control bias) is not considered when computing the pressure gain. On one hand, it is felt that the addition of momentum equally on either side of the jet degrades the gain, because when the jet deflects to one side, blocking off one control reduces the added momentum on one side and increases it on the other in an opposite sense to the increase in output pressure difference. On the other hand the increased pressure recovery increases gain, hence the effects are offset.

The pressure gain, G_p (eq (52)), is a function of frequency because A_4 is a function of frequency. Equation (52) may be rewritten as a second-order dynamic system (as in Manion and Drzewiecki[1]), so that

$$G_p = G_{po} / \left[\left(A_2/A_o \right) \left(j\omega \right)^2 + \left(A_1/A_o \right) j\omega + 1 \right] \qquad (53)$$

where G_{po} is the blocked gain at DC, and the A_n coefficients are

$$A_o = 1 + R_c \left[a_1 B_c^2 / \left(4c_\theta \right) + 1/R_v + B_c^2 \left(P_j - P_v \right) / \left(2c_d \; c_\theta \left[\left| P_j - P_v \right| \right]^{1/2} \right) \right] \qquad (53a)$$

$$A_1 = L_c \left[a_1 B_c^2 / \left(4c_\theta \right) + 1/R_v + B_c^2 \left(P_j - P_v \right) / \left(2c_d \; c_\theta \left[\left| P_j - P_v \right| \right]^{1/2} \right) \right] \qquad (53b)$$

$$A_2 = 2 \; L_c \; C_j \qquad (53c)$$

From equations (53a-c) the magnitude of G_p and the phase angle as a function of frequency can be obtained. The pure time delay contribution due to the transport time of a fluid particle from the nozzle to the splitter must be included in the phase calculation.

2.7.1 Self-Staged Gain

The self-staged gain of an amplifier is the gain of a device driven by and driving into identical devices with the identical supply operating conditions. It is the blocked gain at self-bias multiplied by the ratio of its (succeeding stage) input impedance to its sum of input and output impedance.

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[1]*F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

$$G_{p_{ss}} = G_p \Big|_{P_c = P_o} \frac{Z_i}{Z_i + Z_o} \tag{54}$$

where $G_p \Big|_{P_c = P_o}$ = blocked pressure gain where the control bias pressure equals the output pressure at the self-loaded condition.

If equation (54) is rewritten by substituting for Z_i from equation (49) and for G_p from equation (52), we obtain

$$G_{p_{ss}} = \left[\frac{G}{1 + A_4}\right]\left[\frac{1}{1 + \frac{Z_o A_4}{Z_c(1 + A_4)}}\right] \tag{55}$$

where G again represents all the time-independent terms in equation (52). After some manipulation the terms can be rearranged to read as

$$G_{p_{ss}} = G/\left[1 + \frac{1}{Z_c}\left(Z_c + Z_o\right)A_4\right] = G/\left[1 + \left(Z_c + Z_o\right)A_5\right] . \tag{56}$$

If equation (56) is compared with equation (52) one notes that the term in the denominator of equation (56) is the same as the last term in equation (52), except that $Z_o + Z_c$ is substituted for Z_c. In other words, the self-staged gain dynamics are the same as the blocked output dynamics of an amplifier identical in all aspects except that the input impedance is greater by the device's own output impedance Z_o. In this manner the response of both loaded and blocked devices is treated very simply.

2.7.2 Frequency Response

The variation of gain and phase angle as a function of frequency is usually presented in a Bode diagram. The closed-form relations for gain and phase may be obtained from equation (53). The magnitude of the gain (absolute value of G_p) may be written as

$$\left|\left[G_p(\omega)\right]\right| = G_{po}\, A_o/\left(\left[A_o - A_2(2\pi F)^2\right]^2 + A_1{}^2(2\pi F)^2\right)^{\frac{1}{2}} . \tag{57}$$

The frequency, F, is a normalized frequency in cycles-per-unit transport time, $F = \omega/2\pi$. The phase shift (the ratio of real to imaginary parts of G_p, plus the phase shift due to transport time) is

$$\phi(F) = \tan^{-1}\left[A_1(2\pi F)/\left(A_o - A_2(2\pi F)^2\right)\right] + 360\, X_{sp}\, F\,. \tag{58}$$

3. THE COMPUTER PROGRAM

The computer programming language used is BASIC as adapted for a Wang 2200B programmable calculator. Due to the extensive length of the program, at least 16K bytes of core space are required.

Symbols allowed in BASIC are limited to a maximum of two characters, the first of which must be alphabetic and the second numeric (from 0 to 9, or a blank). Thus, the user is limited to 286 variable names. Each variable, however, may be subscripted to allow for large arrays. There are 286 additional names allowed for string variables, or those variables having alphanumeric values, such as answers to questions like YES and NO, or plot symbols.

The reader is directed to the basic primers[5-7] for the mechanics of BASIC programming and Wang plotting.

The block diagram/flow chart of figure 5 describes the programming steps in detail. A listing of the program in its entirety can be found in appendix A.

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[5] *The 2200A/B Reference Manual, Wang Laboratories, Tewksbury, MA (1967).*

[6] *The 2200 A/B Programming Manual, Wang Laboratories, Tewksbury, MA (1974).*

[7] *The 2202 Plotting Output Writer Reference Manual, Wang Laboratories, Tewksbury, MA (1973).*

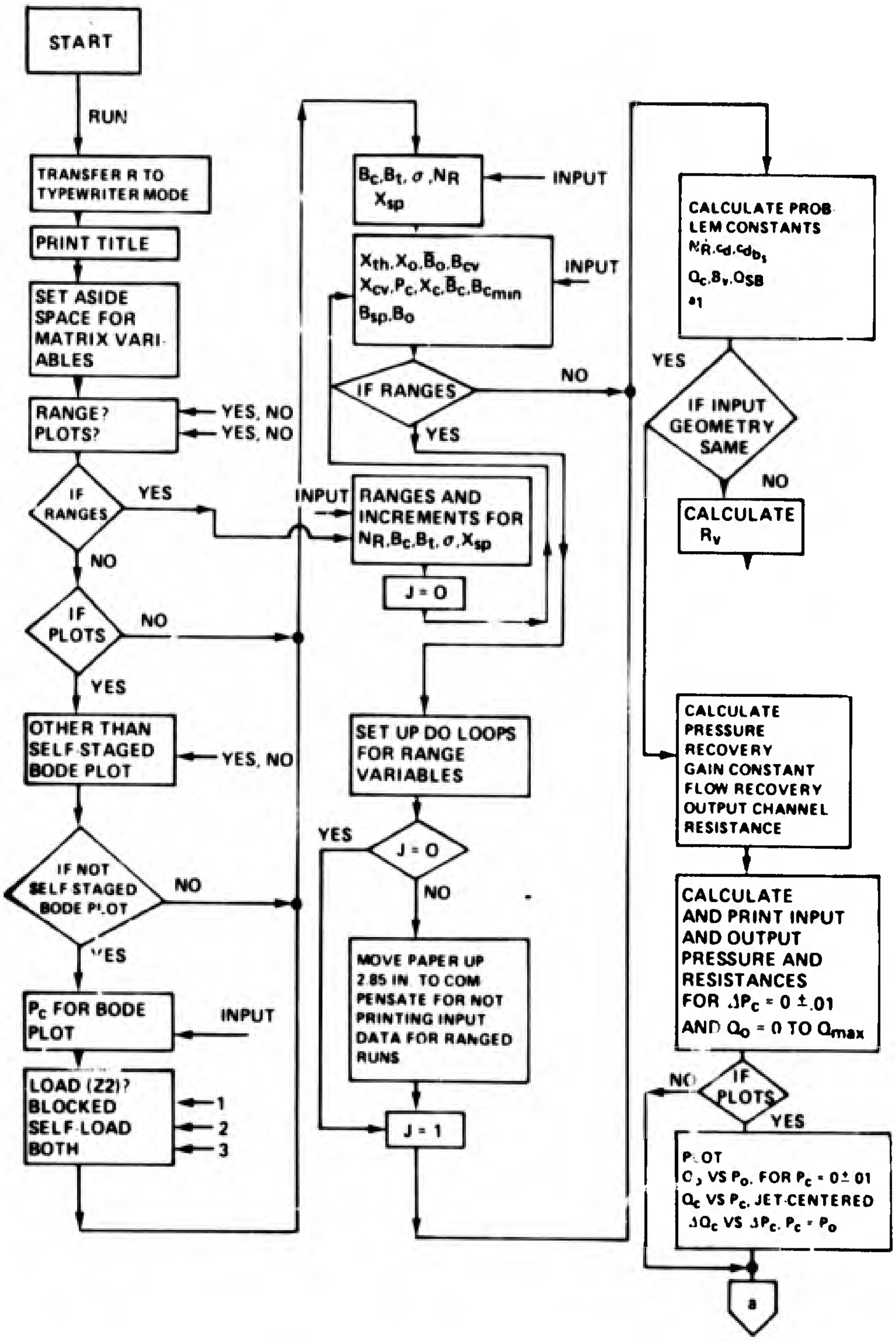


Figure 5. Flow chart for LPA program (top half).

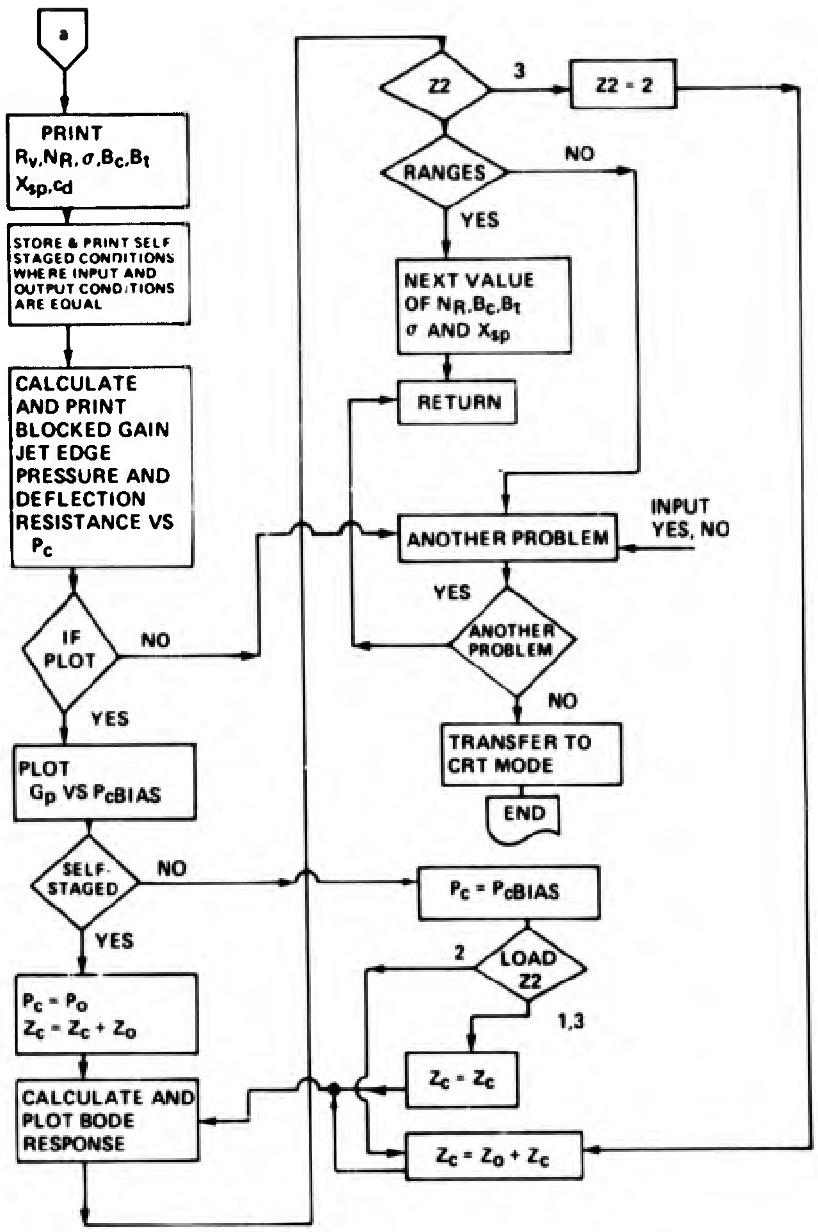


Figure 5 cont'd. Flow chart for LPA program (bottom half).

4. EXPERIMENTAL VERIFICATION OF RESULTS

Considerable experimental data have been presented elsewhere[1,2] that verify the analysis. Several different examples of port characteristics are reproduced here. The experimental data are plotted directly on the computer printout, and experimentally calculated values are given in brackets next to the computer value. The configurations used in these tests are those used at HDL. They are all basically of the type shown in figure 1. Silhouettes of each device are given on the printout sheet. The printout sheets (shown in fig. 6a through 9a) are composed of a number of important sections of computed data:*

(a) A table of the jet-centered output and input pressures and flows, (PO, PC, Q), the output power, PO*QO, and output slope resistance, RO, and the input channel resistance, RC. The self-staged operating point is printed as the operating flow (located on the left, No. 1).

(b) A graph of the table with self-staged deflection resistance added at the operating point. The output P,Q is represented by ***; the input, P,Q jet centered is represented by +++; and the deflection resistance is shown by .... The upper and lower output curves are the output characteristics for a ±1-percent differential control input pressure, at a bias level determined by the programmer (located on the right, No. 2).

(c) Miscellaneous values; resistance between jet and control edge, RV; Reynolds number, NR; aspect ratio, AR; control width, BC; distance between control edges, BT; nozzle-to-splitter distance XSP; and discharge coefficient, CD (located at center, No. 3).

(d) Self-staged operating conditions (No. 4).

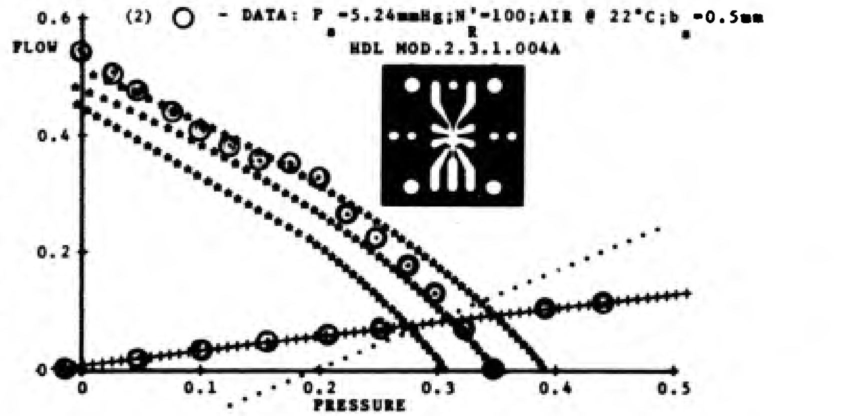
**N. B. The computer only prints capital letters in this program hence some confusion may occur if this is not remembered. For example CD refers to c_d, BT to B_t etc. The second letter is usually a subscript. All quantities are dimensionless.*

[1]*F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proc HDL Fluidic State-of-the-Art Symposium, 1, Harry Diamond Laboratories, Adelphi, MD (October 1974).*

[2]*T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).*

(1) CENTERED INPUT AND OUTPUT CHARACTERISTICS

Q	PO	PO*QO	RO	PC	RC
0.000	0.347	0.000	-0.418	-0.022	0.163
0.050	0.325	0.016	-0.497	0.181	0.182
SELF-STAGED OPERATING FLOW = 0.090					
0.100	0.300	0.030	-0.584	0.387	0.201
0.150	0.273	0.041	-0.683	0.594	0.220
0.200	0.242	0.048	-0.796	0.803	0.239
0.250	0.208	0.052	-0.926	1.014	0.258
0.300	0.170	0.051	-1.077	1.227	0.277
0.350	0.128	0.045	-1.254	1.442	0.296
0.400	0.081	0.032	-1.464	1.659	0.315
0.450	0.028	0.012	-1.716	1.878	0.334



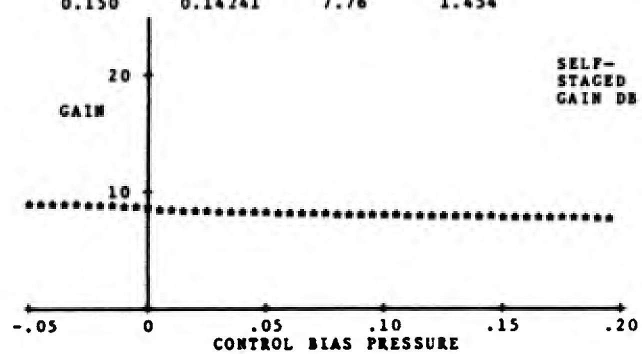
(3) RV= 3.889,NR= 1200,AR= 0.99,BC= 1.11,BT= 1.13,XSP= 8.0,CD= .70

(4) SELF-STAGED DEFL. RESIST.= 1.198
SELF-STAGED OUTPUT RESIST.= -0.566
BLOCKED GAIN AT SELF-BIAS = 7.393
SELF-STAGED BIAS = 0.306
SELF-STAGED GAIN = 5.021

(5)

BIAS PRESS.	P JET	GAIN	DEFL.RES.
-0.050	-0.04888	8.88	-45.178
0.000	-0.00089	8.52	4.414
0.050	0.04698	8.07	1.931
0.100	0.09474	7.89	1.623
0.150	0.14241	7.76	1.454

(6)



(7)

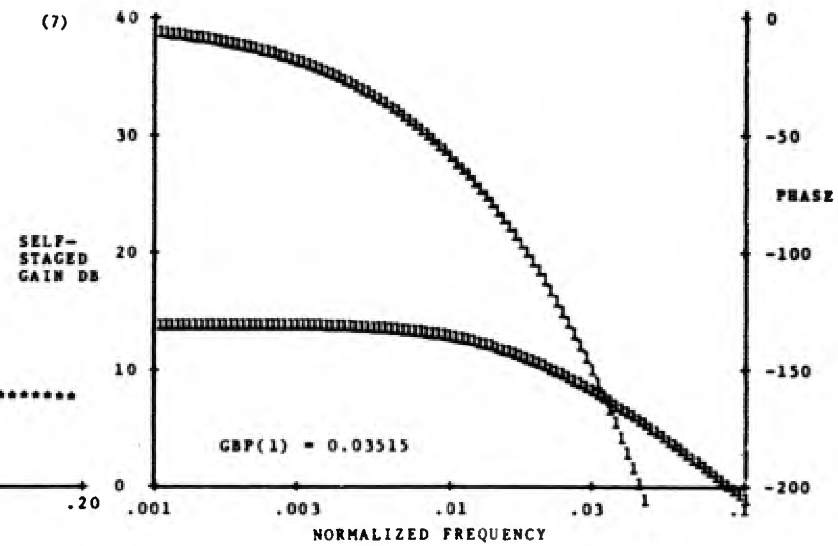


Figure 6(a). Computer printout and data for HDL Mod 2.3.1.004A LPA, $\sigma = 1$.

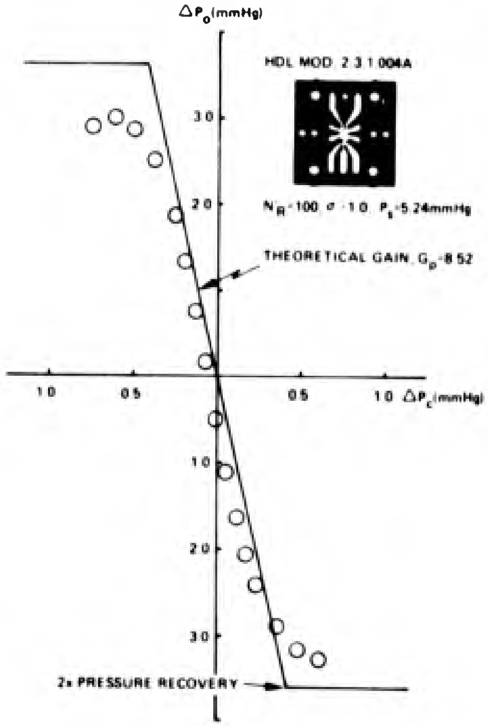


Figure 6(b). Experimentally measured transfer characteristic compared with theory for HDL Mod 2.3.1.004A LPA, $\sigma = 1$.

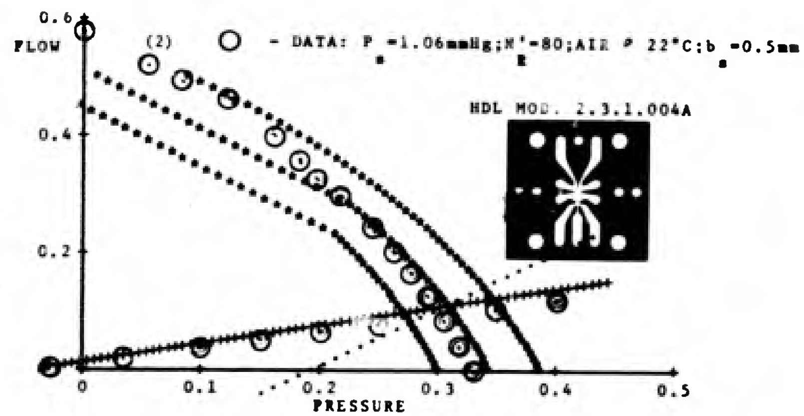
(e) Table of blocked pressure gain, deflection resistance, and jet-edge pressure as a function of control bias pressure (No. 5).

(f) Plot of a blocked pressure gain as a function of control bias pressure (No. 6).

(g) Bode diagram (in standard Bode dimensions 10 dB/in. and 50 deg/in. versus 2 in./decade) of gain and phase angle as a function of normalized frequency, F. Plot symbols "1" and "2" are used. If both "1" and "2" appear they refer to self-load and blocked-load output conditions, respectively. If only "1" appears it refers to the self-staged condition, and the gain axis will be so labelled. The gain-bandwidth product is given in the lower left of the graph. It is the product of the gain and the frequency when the phase shift is 45 deg (No. 7).

(1) CENTERED INPUT AND OUTPUT CHARACTERISTICS

Q	PO	PO*QO	RO	PC	RC
0.000	0.342	0.000	-0.269	-0.032	0.129
0.050	0.327	0.016	-0.345	0.124	0.148
0.100	0.310	0.031	-0.430	0.283	0.166
SELF-STAGED OPERATING FLOW = 0.110					
0.150	0.290	0.043	-0.526	0.444	0.184
0.200	0.268	0.053	-0.636	0.606	0.207
0.250	0.242	0.060	-0.763	0.770	0.221
0.300	0.205	0.061	-0.975	0.936	0.239
0.350	0.156	0.054	-0.975	1.104	0.257
0.400	0.107	0.043	-0.975	1.274	0.275
0.450	0.058	0.026	-0.975	1.446	0.294
0.500	0.010	0.005	-0.975	1.619	0.312



(3) RV= 2.991,NR= 540,AR= 1.98,BC= 1.11,BT= 1.13,XSP= 8.0,CD= .68

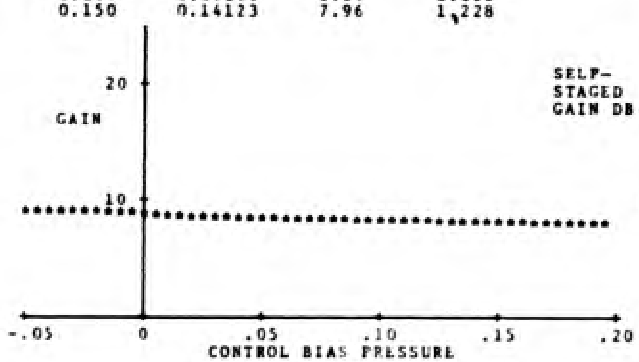
(4)

SELF-STAGED DEFL. RESIST.=	1.033
SELF-STAGED OUTPUT RESIST.=	-0.448
BLOCKED GAIN AT SELF-BIAS =	7.589
SELF-STAGED BIAS =	0.307
SELF-STAGED GAIN =	5.291

(5)

BIAS PRESS.	P JET	GAIN	DEFL.RES.
-0.050	-0.04927	9.00	14.133
0.000	-0.00138	8.69	3.108
0.050	0.04633	8.27	1.572
0.100	0.09386	8.10	1.352
0.150	0.14123	7.96	1.228

(6)



(7)

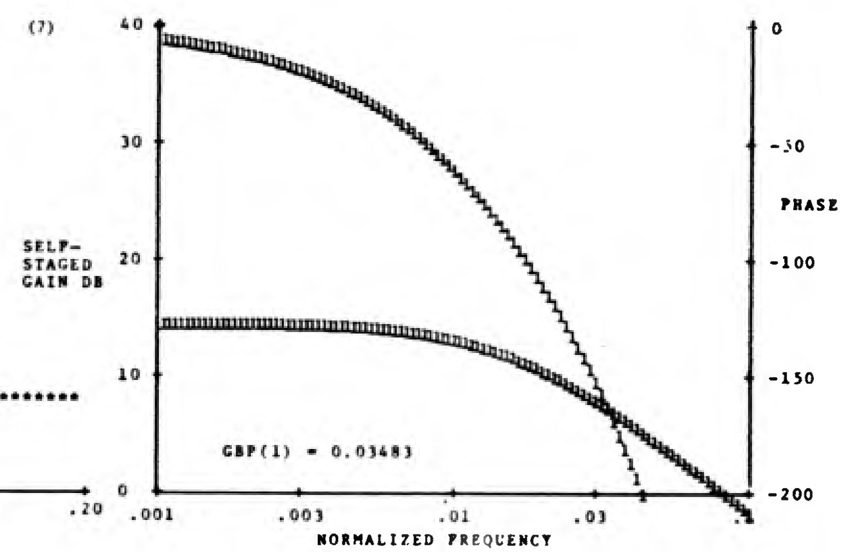


Figure 7(a). Computer printout and data for HDL Mod 2.3.1.004A LPA, $\sigma = 2$.

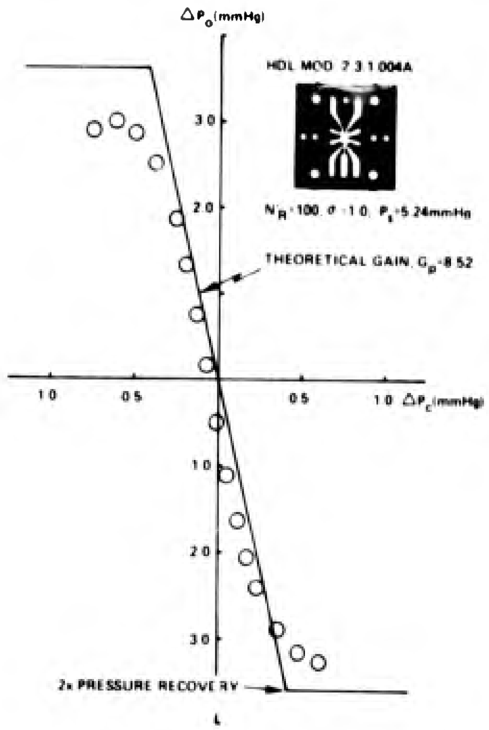


Figure 6(b). Experimentally measured transfer characteristic compared with theory for HDL Mod 2.3.1.004A LPA, $\sigma = 1$.

(e) Table of blocked pressure gain, deflection resistance, and jet-edge pressure as a function of control bias pressure (No. 5).

(f) Plot of a blocked pressure gain as a function of control bias pressure (No. 6).

(g) Bode diagram (in standard Bode dimensions 10 dB/in. and 50 deg/in. versus 2 in./decade) of gain and phase angle as a function of normalized frequency, F. Plot symbols "1" and "2" are used. If both "1" and "2" appear they refer to self-load and blocked-load output conditions, respectively. If only "1" appears it refers to the self-staged condition, and the gain axis will be so labelled. The gain-bandwidth product is given in the lower left of the graph. It is the product of the gain and the frequency when the phase shift is 45 deg (No. 7).

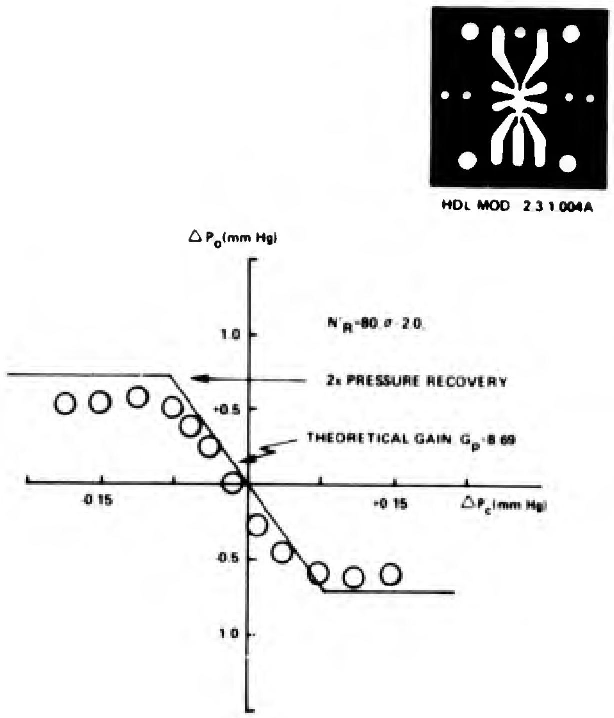


Figure 7(b). Experimentally measured transfer characteristic compared with theory for HDL Mod 2.3.1.004A LPA, $\sigma = 2$.

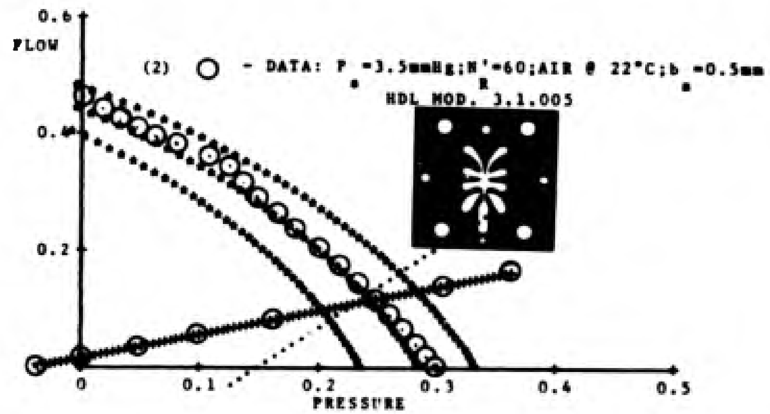
Input data, including the pertinent geometric dimensions, the operating point Reynolds number, and the bias for which the output characteristic is to be plotted, are typed in a block located at the upper left corner of the sheet (shown in fig. 10). This block does not appear in figures 6 to 9, since it was suppressed in a parametric situation. Figures 6 to 9 are excerpts from a set of computations done for various Reynolds numbers. The input block only appears on the sheet containing the first Reynolds number.

As can be seen in figures 6 through 9, the agreement between data and theory is quite good for an engineering model.

Each figure has two parts, a and b. Part a is the computer printout for the amplifier with data superimposed on the input and output characteristics. Part b is a plot of the experimentally measured transfer characteristic, differential output pressure versus differential control pressure, and superimposed straight lines representing the computed gain at zero control bias, and the level of twice the pressure recovery. When the jet is deflected completely into one output, theory states that the recovered pressure is merely twice

(1) CENTERED INPUT AND OUTPUT CHARACTERISTICS

Q	PO	PO*QO	RO	PC	RC
0.000	0.283	0.000	-0.314	-0.029	0.105
0.050	0.266	0.013	-0.403	0.091	0.117
0.100	0.246	0.024	-0.508	[illegible]	0.128
SELF-STAGED OPERATING FLOW = 0.120					
0.150	0.224	0.033	-0.635	0.338	0.140
0.200	0.197	0.039	-0.792	0.463	0.152
0.250	0.167	0.041	-0.988	0.589	0.163
0.300	0.132	0.039	-1.242	0.716	0.175
0.350	0.090	0.031	-1.576	0.845	0.187
0.400	0.041	0.016	-2.031	0.974	0.198



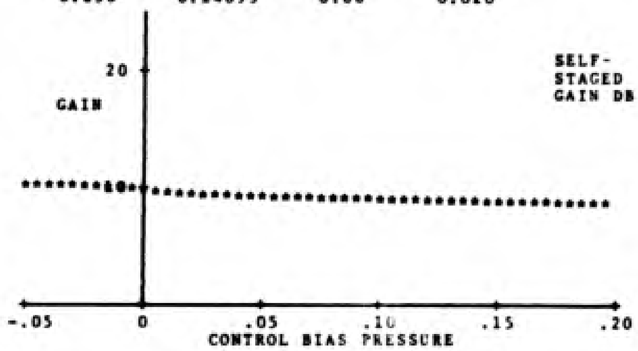
(3) RV= 2.312,NR= 720,AR= 0.98,BC= 1.27,BT= 1.24,XSP= 8.0,CD= .62

(4) SELF-STAGED DEFL. RESIST.= 0.738
SELF-STAGED OUTPUT RESIST.= -0.556
BLOCKED GAIN AT SELF-BIAS = 8.335
SELF-STAGED BIAS = 0.238
SELF-STAGED GAIN = 4.754

(5)

BIAS PRESS.	P JET	GAIN	DEFL.RES.
-0.050	-0.04913	10.23	-18.830
0.000	-0.00133	9.74	2.590
0.050	0.04627	9.11	1.091
0.100	0.09370	8.87	0.920
0.150	0.14093	8.66	0.828

(6)



(7)

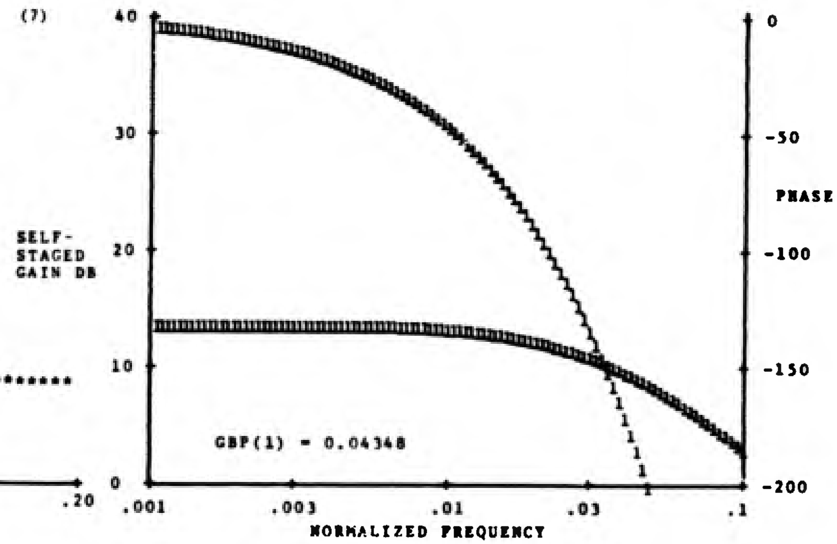


Figure 8(a). Computer printout and data for HDL Mod 3.1.005A LPA, $N'_R = 60$.

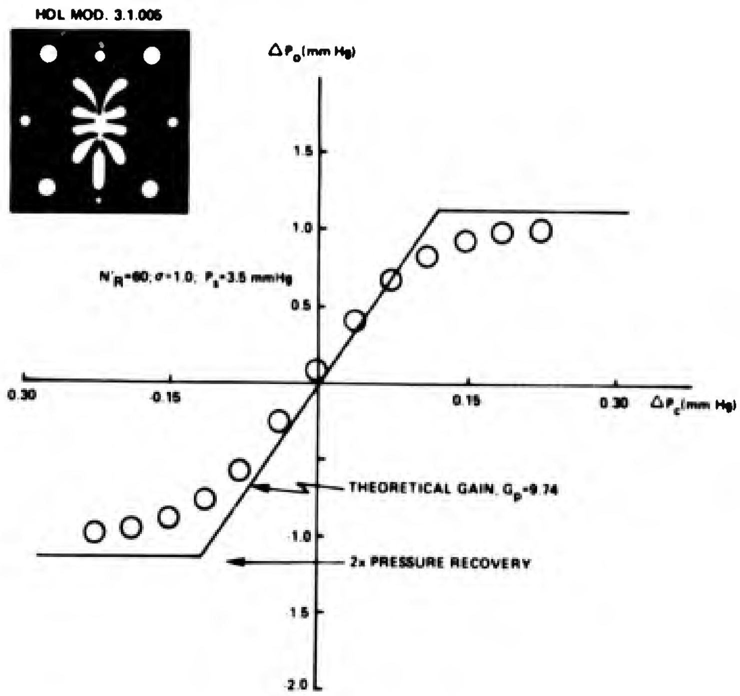
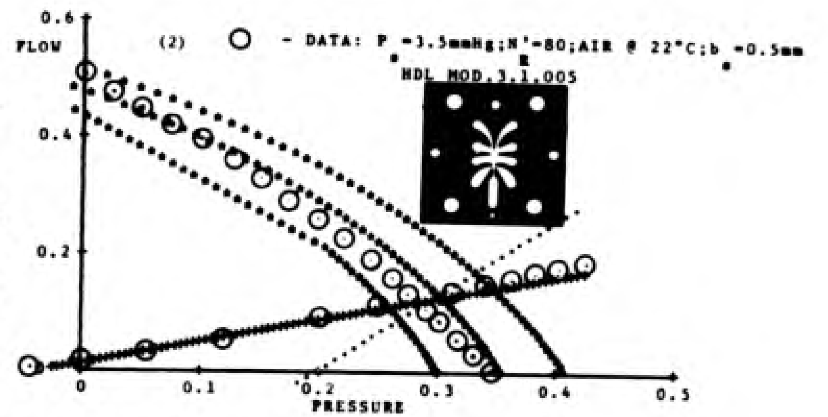


Figure 8(b). Experimentally measured transfer characteristic compared with theory for HDL Mod 3.1.005A LPA, $N'_R = 60$.

that when the jet is centered, since twice the momentum impinges on the output. Figures 6 and 7 show the same amplifier operated at different aspect ratios. Figures 8 and 9 show another amplifier at different modified Reynolds numbers. In each case the agreement with data is satisfactory, as long as small deviations around the centered position are chosen. The saturation pressures are close; however, work may have to be done on the transfer characteristic to include the nonlinearity. The output characteristics are all quite well represented in the region near the blocked pressure recovery. Some investigation will have to be done at high output flow. This is in the partial channel flow area and probably will involve extensive investigation.

(1) CENTERED INPUT AND OUTPUT CHARACTERISTICS

Q	PO	PO*QO	RO	PC	RC
0.000	0.353	0.000	-0.336	-0.024	0.084
0.050	0.335	0.016	-0.433	0.106	0.097
0.100	0.313	0.031	-0.546	0.237	0.110
SELF-STAGED OPERATING FLOW = 0.130					
0.150	0.289	0.043	-0.680	0.370	0.123
0.200	0.260	0.052	-0.840	0.504	0.136
0.250	0.227	0.056	-1.034	0.640	0.150
0.300	0.189	0.056	-1.275	0.777	0.163
0.350	0.145	0.050	-1.579	0.915	0.176
0.400	0.093	0.037	-1.970	1.054	0.189
0.450	0.032	0.014	-2.488	1.195	0.202



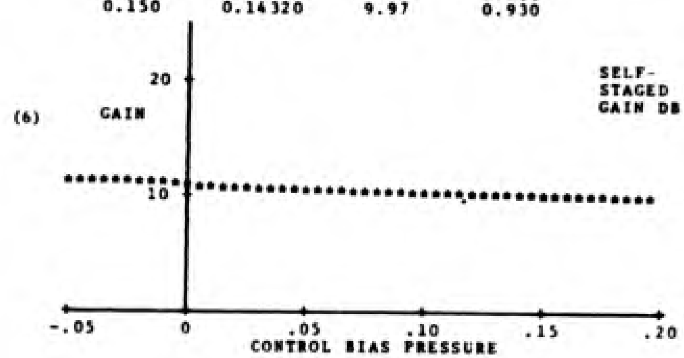
(3) RV= 2.507,NR= 960,AR= 0.98,BC= 1.27,BT= 1.24,XSP= 8.0,CD= .66

(4) SELF-STAGED DEFL. RESIST.= 0.775
SELF-STAGED OUTPUT RESIST.= -0.624
BLOCKED GAIN AT SELF-BIAS = 9.489
SELF-STAGED BIAS = 0.299
SELF-STAGED GAIN = 5.257

(5)

BIAS PRESS.	P JET	GAIN	DEFL.RES.
-0.050	-0.04918	11.22	-48.641
0.000	-0.00080	10.84	2.755
0.050	0.04738	10.36	1.231
0.100	0.09538	10.15	1.036
0.150	0.14320	9.97	0.930

(6)



(7)

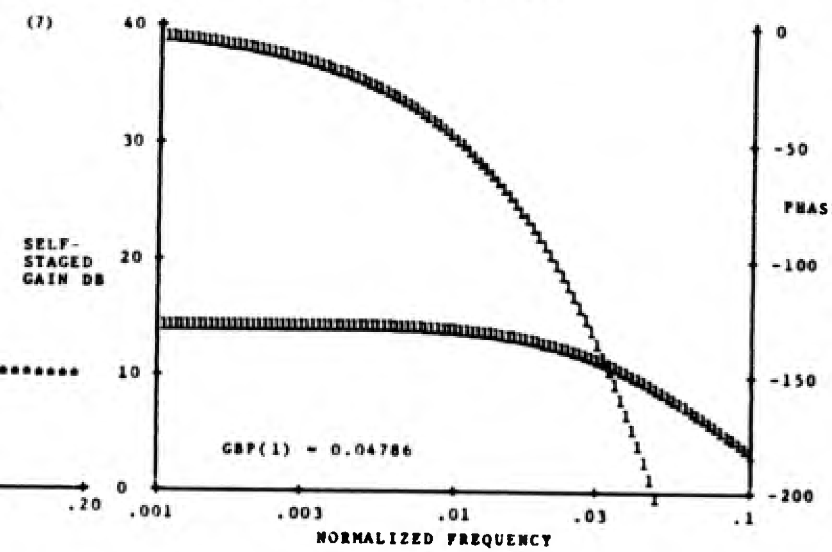


Figure 9(a). Computer printout and data for HDL Mod 3.1.005 LPA, $N'_R = 80$.

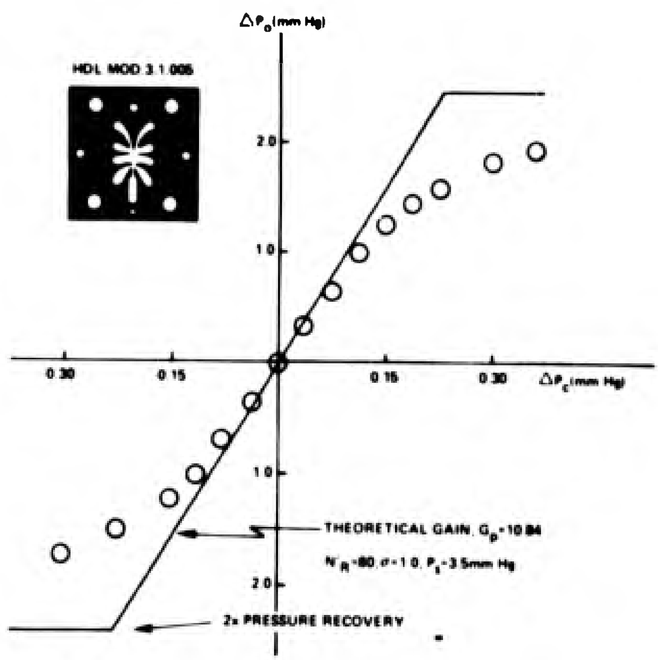
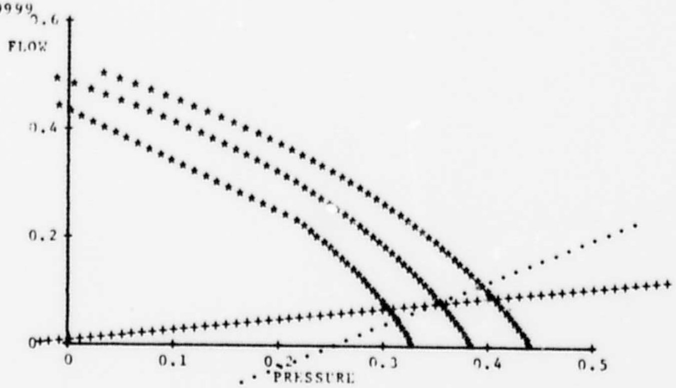


Figure 9(b). Experimentally measured transfer characteristic compared with theory for HDL Mod 3.1.005 LPA, N'_R = 80.

```
DO U WANT RANGES,DO U WANT PLOTS? NO,YES
DO U WANT RESP.OTHER THAN SS? NO
CONTROL,THROAT WIDTH? 1,1.08
ASPECT RATIO,REYNOLDS NO.? 1,1000
SPLITTER DISTANCE? 10
SUPPLY NOZZLE THROAT LENGTH? 2
OUTPUT LENGTH,AVG.WIDTH? 10,2.35
CENTER DUMP WIDTH,LENGTH? .000000001,999999999
BIAS CONTROL PRESSURE? 0
XC,UCAVG,WCMIN? 10,2.5,1
SPLITTER WIDTH,OUTPUT WIDTH? .5,1.35
```

CENTRED INPUT AND OUTPUT CHARACTERISTICS

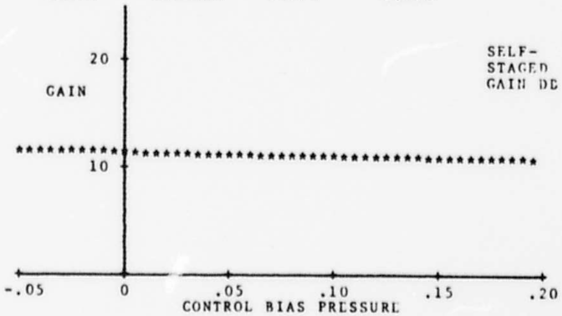
Q	PO	PO*QO	RO	PC	RC
0.000	0.384	0.000	-0.335	-0.026	0.090
0.050	0.365	0.018	-0.440	0.220	0.113
SELF-STAGED OPERATING FLOW = 0.050					
0.100	0.344	0.034	-0.563	0.471	0.136
0.150	0.318	0.047	-0.706	0.723	0.158
0.200	0.289	0.057	-0.877	0.978	0.181
0.250	0.254	0.063	-1.084	1.235	0.204
0.300	0.215	0.064	-1.337	1.494	0.227
0.350	0.168	0.058	-1.655	1.756	0.250
0.400	0.113	0.045	-2.062	2.020	0.272
0.450	0.049	0.022	-2.594	2.286	0.295



RV= 4.345,NR= 1000,AR= 1.00,BC= 1.00,BT= 1.03,XSP=10.0,CD= .69

```
SELF-STAGED DEFL. RESIST.=      1.246
SELF-STAGED OUTPUT RESIST.=    -0.511
BLOCKED GAIN AT SELF-BIAS =    10.349
          SELF-STAGED BIAS =     0.353
          SELF-STAGED GAIN =     7.336
```

BIAS PRESS.	P JET	GAIN	DEFL.RES.
-0.050	-0.04958	11.55	-35.855
0.000	-0.00050	11.31	5.087
0.050	0.04848	11.01	2.194
0.100	0.09737	10.87	1.821
0.150	0.14618	10.76	1.618



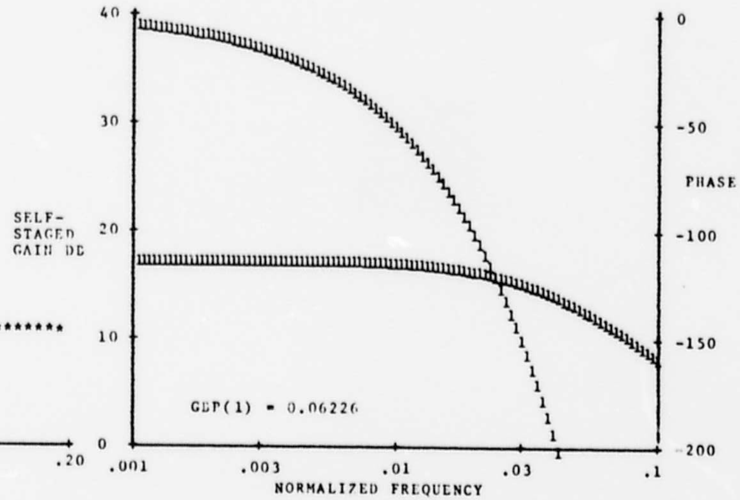


Figure 10. Computer printout for near optimal LPA design.

5. EXAMPLE OF PROGRAM USE FOR DESIGN

Two amplifier configurations were used in the previous examples. One had long input and output channels, and the other had short channels. In each case, however, the control-edge spacing was such that when the jet was fully directed at one outlet (δ_{sp} = 0.5 at X_{sp}) there was still some clearance.

A high input impedance (deflection impedance) and a low output impedance are often desirable. One way for obtaining a device with these characteristics is to make the output channel half its length, and to place the control edge so that there is no clearance when the jet is fully deflected into the output (e.g., deflection at the splitter is half a nozzle width). Shortening the input channel reduces the input signal attenuation and the temperature sensitivity. In the latter, the viscous resistance term is decreased and, since the clearance is like an orifice (temperature-insensitive), the higher the orifice resistance the less temperature-dependent the whole amplifier. Figure 10 shows the computer results for such geometry. This design shows a significant increase in self-staged gain over that of the experimental examples, due in part to the higher input-to-output impedance, and in part to the slightly reduced output receiver width and slightly increased splitter distance. Even though the receiver entrance was narrowed slightly in this device, the channel as a whole still had less output impedance than the other devices. The self-staged gain-bandwidth product was improved.

If one uses the computer model in this manner, amplifiers can be "built" and "tested" without fabricating any but those that appear to be best-suited for the desired task. Using the characteristics, one can also design staged assemblies, and programs are being compiled that will solve the staged and packaged conditions. The turnaround time for this kind of amplifier design is thus considerably reduced when compared to the time required to fabricate an amplifier and then test it in the laboratory.

6. SUMMARY AND CONCLUSIONS

A set of equations governing LPA operation has been presented and incorporated into a computer program. The validity of the analytical model, although previously verified, has been further strengthened by additional data. A brief example of how the program can be used as a design tool has been presented.

The program itself is a starting place for more extensive design procedures, not only for individual devices, but also for staged assemblies and complete circuits. In its simplest form a design procedure would involve an iteration that would take the engineer's specifications--for example, gain, bandwidth, and input-to-output impedance ratio--and move through various geometries until the specifications were met, or until all feasible geometric configurations were exhausted. Ideally, one would want to establish an inverse algorithm that would work backwards from the specifications to the initial geometry, so that iteration would not be required. Such an algorithm is not yet available, and considering the difficulties involved, probably will not be available in the foreseeable future.

It has been frequently observed that the gain of an amplifier is a continuous, monotonically increasing function of Reynolds number in the laminar region and a constant in the turbulent region. These observations can be used in determination of the turbulent characteristics, because all one must do is establish the point of transition and then calculate the laminar solution, which then would hold for turbulent operation. Transition-to-turbulence is estimated to lie in the range 100 N_R' $(X_{sp}/8$ 125. Further experimenta and theory on the determination of the maximum laminar operating Reynolds number are presently going on. More in-depth analysis should be conducted on the transition phenomenon.

Other laminar jet-deflection devices, such as the laminar-jet angular rate sensor (LJARS) can be evaluated by using this program. When the output characteristics of such a sensor are matched to amplifier packages the port characteristics of the devices can readily be used as an amplifier. This technique has already been used in the design and evaluation of rate-sensor/preamplifier subsystems.[2]

In conclusion, the program presented in this paper can furnish the fluidic component or system designer with the accurate amplifier port characteristics necessary for design, to include both static and dynamic amplifier performance, in the form of input and output impedances, gain or transfer characteristics, and a frequency response in a Bode diagram.

ACKNOWLEDGEMENT

I would like to thank our untiring summer student, Charles Paras, for taking the experimental data on the amplifier.

---

[2] *T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).*

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(2) T. M. Drzewiecki, D. N. Wormley, and F. M. Manion, Computer-Aided Design Procedure for Laminar Fluidic Systems, Journal of Dynamic Systems, Measurement and Control, Trans ASME, 97, 4, series G (December 1975).

(3) T. M. Drzewiecki, Fluerics 37: A General Planar Nozzle Discharge Coefficient Representation, Harry Diamond Laboratories TM-74-5 (August 1974).

(4) T. M. Drzewiecki, The Interpretation of Surface Static Pressure Distributions in Fluid Amplifier Applications, Harry Diamond Laboratories TR-1627 (July 1973).

(5) The 2200A/B Reference Manual, Wang Laboratories, Tewksbury, MA (1967).

(6) The 2200 A/B Programming Manual, Wang Laboratories, Tewksbury, MA (1974).

(7) The 2202 Plotting Output Writer Reference Manual, Wang Laboratories, Tewksbury, MA (1973).

APPENDIX A.--PROGRAM FOR COMPUTER DESIGN OF LAMINAR PROPORTIONAL AMPLIFIERS

This appendix presents the computer program for design of laminar proportional amplifiers (LPA's) in its entirety, including all plotting and control statements. The program follows exactly the flow chart presented in the text, although the chart is an obvious abridgement of the mechanics of the program.

```
10SELECT PLOT 413,PRINT 213:PLOT [,,]:SELECT CO 213
20PRINT "THE LPA PORT CHARACTERISTICS"
30DIM V5(56),W5(56),V6(56),W6(56),V7(56),W7(56),G7(55),G1(100),P
4(100)
40INPUT "DO U WANT RANGES,DO U WANT PLOTS",Z3$,Z9$
50IF Z3$="YES"THEN 180:IF Z9$="NO"THEN 70
60INPUT "DO U WANT RESP.OTHER THAN SS",Z5$:IF Z5$="NO"THEN 70:IN
PUT "PC FOR RESP.",J6:INPUT "SELF-LOAD(1),BLCKD(2),BOTH(3)",Z2
70INPUT "CONTROL,THROAT WIDTH",W,B
80INPUT "ASPECT RATIO,REYNOLDS NO.",S,N
90INPUT "SPLITTER DISTANCE",B6
100INPUT "SUPPLY NOZZLE THROAT LENGTH",L9
110INPUT "OUTPUT LENGTH,AVG.WIDTH",X3,F8
120INPUT "CENTER DUMP WIDTH,LENGTH",V1,V2
130INPUT "BIAS CONTROL PRESSURE",H3
140INPUT "XC,WCAVG,WCMIN",X1,W0,W3
150INPUT "SPLITTER WIDTH,OUTPUT WIDTH",B5,B7
160IF Z3$="YES"THEN 210
170GOTO 290
180INPUT "RANGE AND STEP OF NR,PC,BT,AR,XSP",N1,N2,N3,Z4,Z5,Z6,Z
7,Z8,Z9,S3,S4,S5,X4,X5,X6
190J=0
200GOTO 100
210FOR N=N1TO N2STEP N3
220FOR S=S3TO S4STEP S5
230FOR B6=X4TO X5STEP X6
240FOR W=Z4TO Z5STEP Z6
250FOR B=Z7TO Z8STEP Z9
260IF J=0THEN 280
270SELECT PLOT 413:PLOT [,-395,]:SELECT PRINT 213
280J=1
290N9=N/((L9+1)*(1+1/S)!2)
300GOSUB 4020
310D=(1-S)/2+SQR((S+1)!2/4-S*(1-C))
320Q=1.651*(.02772+1.32*W/(C*N))!(1/3)-.5
330K=1/(C*Q+D/2)
340Y=K*B/2
350B8=.5*(B-1-W/(.021*C*N))
360T1=LOG((1+.5*D*K)/(1-.5*D*K))/2
370K3=4/(EXP(T1)+EXP(-T1))!2
380K4=K3/Q/C
390Q4=Q*EXP(-K4*B8)/2:A=K4*Q4*2:C3=1-3.0303*K4*Q4!2
```


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APPENDIX A

```
400R4=0
410IF W[ ]M5THEN 460
420IF B[ ]M6THEN 460
430IF N[ ]M7THEN 460
440IF S[ ]M8THEN 460
450GOTO 640
460FOR P1=.1TO .5STEP .05
480P=P1!(.25)
490S1=LOG(1/P+SQR(1/P!2-1))
500Y0=S1/K
510D1=(B/2-Y0)/10
520I=0
530FOR Y1=Y0TO B/2STEP D1
540S2=2/(EXP(Y1*K)+EXP(-Y1*K))
550A1=(SQR(S2!4+P1)-S2!2)*D1
560I=I+A1
570NEXT Y1
580R1=D*P1/I
590%RV= #.###,NR= ####,AR= #.##,BC= ##.##,BT=##.##,XSP=##.#,CD=
.##
600R4=R4+R1
610NEXT P1
620R1=R4/10
640M5=W:M6=B:M7=N:M8=S
680IF W0/S]2THEN 690:K2=.35:GOTO 700
690K2=.5
700T4=24*X1*C*(S/W0+W0/S+K2)/(W0!2*S*N):T5=.95*(C/W3)!2
770C2=1.32*C*C
790IF V1/S]2THEN 830
800IF S/V1]2THEN 830
810K9=.35
820GOTO 850
830K9=.5
850IF C].5THEN 880
860J1=16*C*(1+1.5/B6)
870GOTO 885
880J1=8*(1+1.5/B6)
885P2=H3:GOSUB 4180:GOSUB 4110:A3=Q2+.5
890P6=(1-J1*B6/(C2*N*S*S))*(1-1.1*B5/(SQR(C*N*B5/2)))*C2/B7*(1-S
QR(C2*(1-J1*B6/(C2*N*S!2)))*V1!2*S*N/(24*V2*(S/V1+V1/S+K9)))*(1+
2*C*P1*SQR(ABS(P1))/(R1*C2))*C3
895D6=(1-J1*B6/(C2*N*S*S))*(1-1.1*B5/(SQR(C*N*B5/2)))*W*W/B7*(2*
B6/W-1)*(1-SQR(C2*(1-J1*B6/(C2*N*S*S)))*V1!2*S*N/(24*V2*(S/V1+V1
/S+K9)))*C3
900IF P6]0THEN 960
910PRINT "P0[0 GO TO NEXT PROBLEM"
920SELECT PLOT 413
930PLOT [,-730,]
940SELECT PRINT 213
950GOTO 2840
960GOSUB 4180:GOSUB 4060
965E=1:02=1:03=1
970FOR M=-.01TO .01STEP .01
980IF Z9$="YES"THEN 990:M=0
990P5=P6+G*M/2:A5=A3*P5/P6
1000R5=24*X3*C*(S/F8+F8/S+K2)/(F8!2*S*N)
1010K5=.95*C!2/B7!2
1030IF M[ ]0THEN 1060
1035FOR I=1TO 5:PRINT HEX(0A):NEXT I
1040PRINT " CENTERED INPUT AND OUTPUT CHARACTERISTICS"
1045FOR I=1TO 3:PRINT HEX(0A):NEXT I
1050PRINT "    Q       P0      P0*Q0      R0       PC       RC"
1060H=0
```

```
1080E2=0:E3=0:E4=0:E6=0:E7=0:E8=0:T7=0:A6=0:Z3=1
1090FOR Q0=0TO .5STEP .010:IF A6=1THEN 1112
1095IF S[2THEN 1100:Z3=2/S
1100P0=P5*(1-2*Q0*C/SQR(P5*B7))/(1-Q0/B7/SQR(B7*P5))-R5*Q0-K5*Q0
!2+C*SQR(P5)*Q0*.25*S*Z3
1110R6=SQR(P5/B7)/(1-Q0/B7/SQR(P5*B7))*(-2*C+(1-2*Q0*C/SQR(P5*B7
))/B7)/(1-Q0/B7/SQR(P5*B7))-R5-2*K5*Q0+C*SQR(P5)*.25*S*Z3
1111Z=R5+K5*Q0+P0/(Q0+.000001):A4=.5/A5:IF A4[ZTHEN 1120:IF P0/(
A5-Q0)[-R6THEN 1120:A6=1:A7=Q0:A8=P0
1112P0=A8/(A5-A7)*(A5-Q0):R6=-A8/(A5-A7)
1120E1=(P0-E2)*1000+E3
1130E2=P0
1140E3=E1-INT(ABS(E1))*SGN(E1)
1150E4=E4+INT(ABS(E1))*SGN(E1)
1160E5=(Q0-E6)*500+E7
1170E6=Q0
1180E7=E5-INT(ABS(E5))*SGN(E5)
1190E8=E8+INT(ABS(E5))*SGN(E5)
1200IF M]-.01THEN 1210:V5(E)=E1:W5(E)=E5:L5=E4:Y5=E8:E=E+1:GOTO
1230
1210IF M]0THEN 1220:V6(O2)=E1:W6(O2)=E5:L6=E4:Y6=E8:O2=O2+1:GOTO
1230
1220V7(O3)=E1:W7(O3)=E5:L7=E4:Y7=E8:O3=O3+1
1230IF M[]0THEN 1380
1240Q2=Q0
1250GOSUB 4000
1260GOSUB 4090
1270F=P0-P2
1280IF F]0THEN 1350
1290IF T7=1THEN 1350:T7=1:R7=R6
1300P7=P0
1310R8=R0
1320Q7=Q2
1330 PRINTUSING 1340,Q0
1340%   SELF-STAGED OPERATING FLOW = -#.###
1350IF (H-INT(H))]0THEN 1380
1360PRINTUSING 1370,Q0,P0,Q0*P0,R6,P2,R0
1370%-#.### -#.### -#.### -##.### -#.### -#.###
1380H=H+.2
1390IF P0[0THEN 1410
1400NEXT Q0
1410IF Z9$="YES"THEN 1420:M=.01
1420NEXT M
1430IF Z9$="NO"THEN 1720
1440SELECT PLOT 413
1450PLOT [500,300,],60[,-5,HEX(27)],100[5,,HEX(2D)],[,,"+"],5[-1
00,,"+"],3[,100,"+"],[,-300,"+"]
1520FOR E9=1TO E-1
1530PLOT [V5(E9),W5(E9),"*"]
1540NEXT E9
1550PLOT [-L5,-Y5,"+"]
1560FOR E9=1TO O2-1:PLOT [V6(E9),W6(E9),"*"]:NEXT E9
1570PLOT [-L6,-Y6,"+"]
1580FOR E9=1TO O3-1:PLOT [V7(E9),W7(E9),"*"]:NEXT E9
1590PLOT [-L7,-Y7,"+"]
1600O4=0:O2=0:O3=0
1610C4=INT(100*Q7)/100+.04
1620FOR Q2=0TO C4STEP .002
1630GOSUB 4000
1640GOSUB 4090
1650O1=(P2-O2)*1000+O3
1660O2=P2
1670O3=O1-INT(ABS(O1))*SGN(O1)
1680O4=O4+INT(ABS(O1))*SGN(O1)
```

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```
1690PLOT [O1,1,"+"]
1700NEXT Q2
1710PLOT [-O4,-(C4/.002+1),"+"]
1720R0=R8
1730P2=P7
1740Q2=Q7
1750P1=P2-R0*Q2
1760GOSUB 4060
1770GOSUB 4040
1780G2=G*R3/(R3-R7)
1790IF Z9$="NO"THEN 1890
1800PLOT [P7*1000-R3*150,Q7*500-75,HEX(FB)]:I4=0
1810FOR I1=1TO 30:PLOT [R3*10+I3,5,HEX(FB)]:I2=R3*10+I3:I3=I2-IN
T(I2):I4=I4+INT(I2)
1820NEXT I1:PLOT [-P7*1000+R3*150-I4,-Q7*500-75,"+"]
1830PLOT [0,-15,"0"],[80,0,"0.1"],[70,0,"0.2"]
1840PLOT [70,0,"0.3"],[70,0,"0.4"],[70,0,"0.5"]
1850PLOT [-545,15,"0"],[-10,100,"0.2"]
1860PLOT [-30,100,"0.4"],[-30,100,"0.6"],[205,-330,"PRESSURE"],[
-335,305,"FLOW"],[-200,-325,]
1890SELECT PRINT 213:PRINT HEX(0A):PRINTUSING 590,R1,N,S,W,B,B6,
C
1930PRINT " "
1940PRINTUSING 1990," SELF-STAGED DEFL. RESIST.=",R3
1950PRINTUSING 1990,"SELF-STAGED OUTPUT RESIST.=",R7
1960PRINTUSING 1990,"BLOCKED GAIN AT SELF-BIAS =",G
1970PRINTUSING 1990,"           SELF-STAGED BIAS =",P2
1980PRINTUSING 1990,"           SELF-STAGED GAIN =",G2
1990%########################### -###.###
2000PRINT " "
2010PRINT " BIAS PRESS.   P JET     GAIN    DEFL.RES."
2020G6=0
2040G4=0:G8=1:FOR P2=-.05TO .195STEP .005:GOSUB 4180:GOSUB 4110:
GOSUB 4060:G7(G8)=G:G8=G8+1:IF ABS(G)[99THEN 2050:G6=100
2050NEXT P2
2060FOR G8=1TO 49STEP 10:P2=(G8-11)*.005:GOSUB 4180:GOSUB 4110:G
=G7(G8):GOSUB 4040:GOSUB 4150:NEXT G8
2080IF G6[100THEN 2090:SELECT PLOT 413:PLOT [80,-125,]:SELECT PR
INT 213:GOTO 2160
2090IF Z9$="NO"THEN 2840
2100SELECT PLOT 413:PLOT [120,0,],50[0,-5,HEX(27)]
2110PLOT 80[5,0,HEX(2D)],[,,"+"],5[-100,0,"+"],20[5,0,HEX(2D)]
2120PLOT [0,200,"+"],2[0,-100,"+"],[-110,,]
2130G9=0:G4=0:FOR G8=1TO 50:J7=(G7(G8)-G4)*10+G5:G5=J7-INT(ABS(J
7))*SGN(J7):G9=G9+INT(ABS(J7))*SGN(J7):PLOT [10,J7,"*"]:G4=G7(G8
)
2140NEXT G8:PLOT [-390,-G9,"+"],[-120,-15,"-.05"],[80,0,"0"],[80
,0,".05"],[70,0,".10"],[70,0,".15"],[70,0,".20"]
2150PLOT [-450,115,"10"],[-20,100,"20"],[70,-230,"CONTROL BIAS P
RESSURE"],[-340,200,"GAIN"]
2160IF Z3$[]"YES"THEN 2170:Z5$="NO"
2170SELECT PLOT 413:T6=0:Z6$="1"
2180IF Z5$="NO"THEN 2190:P2=J6:GOSUB 4180:Q7=Q2:P7=J6:IF Z2=1THE
N 2190:IF Z2=3THEN 2190:X3=0:R7=0
2190C5=W!3/(12*C2)
2200L1=2*C*C*(X1/W0+X3/F8)
2210Q2=Q7
2220GOSUB 4000
2230R9=R0-R7
2240K6=W*W/(4*C2):P1=P7-R0*Q2
2250K1=(1+R9*(A*K6+1/R1+2*K6*P1/(SQR(ABS(P1))*C)))/(2*L1*C5)
2260A0=K1*2*L1*C5
2270A3=(A0-1)*L1/R9
2280A2=2*L1*C5
```

```
2290K0=1+R0*(A*K6+1/R1+2*K6*P1/(SQR(ABS(P1))*C))
2300G3=1:P3=0
2310Z3=0
2320GOSUB 4060
2330T8=0
2340FOR L=1TO 100
2350F2=10!(L/50-3)
2360W9=2*#PI*F2
2370G0=G*K0/SQR((A0-A2*W9!2)!2+(A3*W9)!2)
2380F1=SQR(K1)/(2*#PI)
2390G1(L)=20*.432945*(LOG(G0)-LOG(G3))
2400G3=G0
2410P8=ARCTAN(A3/SQR(A2*A0)*F2/F1/(1-(F2/F1)!2))
2420IF P8]0THEN 2440
2430Z3=180
2440P9=P8*180/#PI+B6*F2*360+Z3
2450P4(L)=P9-P3:P3=P9
2460IF P9-45[0THEN 2480
2470IF T8=1THEN 2480:T8=1:F3=F2:G2=G0
2480NEXT L
2490IF T6=0THEN 2500:Z1=G2*F3:GOTO 2550
2500Z=G2*F3
2510SELECT PLOT 413
2520PLOT [500,230,],80[0,-5,HEX(27)],100[5,0,HEX(2D)]
2530PLOT 80[0,5,HEX(27)]
2540PLOT [0,0,"+"],4[0,-100,"+"],2[-250,0,"+"],[120,,"+"],[250,,
"+"],[-370,,],4[0,100,"+"]
2550PLOT [0,-400,"+"]:U2=0:U3=0
2560FOR L=1TO 100
2570U1=G1(L)*10+U2
2580U2=U1-INT(ABS(U1))*SGN(U1)
2590U3=U3+INT(ABS(U1))*SGN(U1)
2600PLOT [5,U1,Z6$]
2610NEXT L
2620PLOT [-500,-U3,"+"]:U5=0:U6=0:PLOT [0,400,"+"]
2630FOR L=1TO 100
2640U4=P4(L)*2+U5
2650U5=U4-INT(ABS(U4))*SGN(U4)
2660IF U6[400THEN 2680
2670GOTO 2710
2680U6=U6+INT(ABS(U4))*SGN(U4)
2690PLOT [5,-U4,Z6$]
2700NEXT L
2710PLOT [-5*(L-1),U6,"+"]:IF Z2=3THEN 2720:PLOT [-40,0,"40"],[-
20,-100,"30"]:GOTO 2730
2720Z2=2:T6=1:Z6$="2":GOTO 2180
2730PLOT [-20,-100,"20"],[-20,-100,"10"],[-10,-100,"0"]
2740PLOT [0,-20,".001"],[85,,".003"],[95,0,".01"],[85,,".03"],[1
05,0,".1"]
2750PLOT [10,20,"-200"],[-40,100,"-150"],[-40,100,"-100"]
2760PLOT [-40,100,"-50"],[-20,100,"0"],[,-150,"PHASE"]
2770PLOT [-440,-290,"NORMALIZED FREQUENCY"]
2780IF Z5$="YES"THEN 2790:PLOT [-450,250,"SELF"],[-40,-15,"STAGE
D"],[-60,-15,"GAIN DB"]:GOTO 2800
2790PLOT [-450,250,"DB GAIN"],[-30,-30,],[100,-145,]:SELECT PRIN
T 213:GOTO 2810
2800PLOT [100,-145,]:SELECT PRINT 213
2810IF Z=0THEN 2820:T7=1:PRINTUSING 2830,T7,Z
2820IF Z1=0THEN 2840:T7=2:SELECT PLOT 413:PLOT [600,,]:SELECT PR
INT 213:PRINTUSING 2830,T7,Z1
2830%GBP(#) =-#.#####
2840IF Z3$="YES"THEN 2900
```

APPENDIX A

```
2850PRINT " ":PRINT " ":PRINT " ":PRINT " ":PRINT " "
2860INPUT "DO YOU WANT TO DO ANOTHER PROBLEM",Z4$
2870IF Z4$="NO"THEN 2920
2880SELECT PLOT 413:PLOT [,-95,]:SELECT PRINT 213
2890GOTO 30
2900NEXT B:NEXT W:NEXT B6:NEXT S:NEXT N
2910PRINT " ":PRINT " ":PRINT " "
2920SELECT CO 005,PRINT 005:END
4000R0=T4+T5*Q2
4010RETURN
4020IF N9[100THEN 4025:C=.00078*N9+.624:GOTO 4030
4025C=-.03422+.0569445*N9-.002305652*N9!2+5.13E-5*N9!3-6.2507E-7
*N9!4+3.93475E-9*N9!5-1.004225E-11*N9!6
4030RETURN
4040R3=(1+R0*(A*W*W/(4*C2)+1/R1+W*W*P1/(2*C2*C*SQR(ABS(P1)))))/(
A*W!2/(4*C2)+1/R1+W!2*P1/(2*C2*C*SQR(ABS(P1))))
4050RETURN
4060G=D6*(1/(1+R0*(A*W*W/(4*C2)+1/R1+W*W*P1/(2*C2*C*SQR(ABS(P1))
))))
4080RETURN
4090P2=-(Q-Q4)*R1+(R1+R0)*Q2
4100RETURN
4110P1=(P2/R0-(Q-Q4))*R0*R1/(R0+R1)
4120RETURN
4150PRINTUSING 4160,P2,P1,G,R3
4160%   -#.###     -#.#####    -###.##     -###.###
4170RETURN
4180Q2=-.5*(T4+R1)/T5+SQR(.25*((T4+R1)/T5)!2+(P2+R1*(Q-Q4))/T5):
GOSUB 4000
4190RETURN
```

APPENDIX A

TABLE A-1. COMPUTER VARIABLE NAME DEFINITIONS

A	net entrainment coefficient, a_1
A0	A_o
A1	term used in calculation of R_v
A2	A_2
A3	During output computation A3 is half the sum of the supply and control flows. During dynamic computations A3=A_1.
A4	jet impedance
A5	total flow available at output when jet is deflected
A6	switch, 1 or 0, for output resistance calculation past "knee"
A7	output flow at "knee"
A8	output pressure at "knee"
B	B_t
B5	B_{sp}
B6	X_{sp}
B7	B_o
B8	B_v
C	c_d
C2	c_θ
C3	control edge spill-back effect pressure recovery coefficient
C4	limit of flow for Q_c vs P_c plot
C5	C_j
D	$c_{d_{b_s}}$
D1	Δy for R_v integration

APPENDIX A

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

D6	gain constant
E	subscript value for (-) deflected jet
E1	plot increment for P_o
E2	last value of P_o
E3	truncation error for P_o plot
E4	running total of P_o plot increments
E5	plot increment of Q_o
E6	last value of Q_o
E7	truncation error for Q_o plot
E8	running total of Q_o plot increments
E9	counter for Q_o vs P_o plots
F	$(P_o - P_c)$ for changing $Q_o = Q_c$
F1	normalized hertz bandwidth
F2	normalized hertz frequency
F3	normalized frequency at 45-deg phase
F8	$\bar{B}_o$
G	G_p (blocked), dc
GO	G_p (frequency)
G1	dB gain plot increment
G2	self-staged gain, $G_{p_{ss}}$
G3	last value of GO
G4	last value of G
G5	truncation error in G_p vs P_c plot

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

G6	gain limiter = 100
G7	blocked gain matrix variable
G8	gain loop counter
G9	running total of gain plot increments
H	counter in print cycle of Q_o vs P_o
H3	bias control pressure for Q_o vs P_o
I	counter
I1	counter in deflection resistance plot
I2	plot increment for ΔQ_c deflection
I3	$\Delta Q_{c_{defl.}}$ truncation error
I4	running total of ΔQ_c deflection increments
J	counter
J1	plate loss coefficient
J6	control bias pressure for dynamics
J7	gain plot increment
J9	increase in gain due to side vent configuration
K	K_1
K0	$(1 + A_4)_{DC}$
K1	A_o/A_2
K2	control channel empirical coefficient
K3	K_5
K4	K_6
K5	flow dependent term of R_{oc}

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

K6	deflection coefficient $B_c^2/4c_\theta$
L	log-frequency counter
L1	inductance, $L_c + L_o$ or L_c
L5	running total of V5
L6	running total of V6
L7	running total of V7
L8	side vent width, B_{sv}
L9	side vent length, X_{sv}
M	control pressure differential for P_o vs Q_o
M5	last value of B_c
M6	value of B_t
M7	last value of N_R
M8	last value of σ
N	N_R
N1	starting value of N_R
N2	ending value of N_R
N3	step of N_R
N9	N'_R
O1	plot increment for P_c
O2	during output computation counter for centered jet; last value of P_c for plot
O3	during output computation counter for + deflected jet; P_c truncation error
O4	running total of P_c plot increments

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

P	$(P_j)^{1/4}$
P0	P_o
P1	P_j
P2	P_c
P3	last value of phase angle
P4	phase plot increment
P5	$P_r(\delta)$
P6	$P_r\ (\delta = o)$
P7	self-staged output pressure
P8	phase in radians
P9	phase in degrees
Q	Q_e
Q0	Q_o
Q2	A_{Qc}
Q4	Q_{SB}
Q5	flow recovery
Q7	self-staged output flow
R	side vent resistance, R_{sv}
R0	R_c
R1	R_v
R3	R_i
R4	$R_v\ (P_j)$
R5	R_{ocL}

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

R6	R_o
R7	self-staged output resistance
R8	self-staged control resistance
R9	$R_o + R_c$
S	σ
S1	sech^{-1} P
S2	sech^{-2} K*Y1
S3	starting σ
S4	ending σ
S5	step of σ
T1	$\tanh^{-1}$ D*K/2
T4	R_{cL} - linear term of R_c
T5	nonlinear term of R_c
T6	counter
T7	counter
T8	counter
U1	dynamic gain plot increment
U2	dynamic gain increment truncation error
U3	running total of U1
U4	phase plot increment
U5	U4 truncation error
U6	running total of U4
V1	B_{cd}

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

V2	X_{cd}
V5	P_o $(\Delta P_c = -0.01)$ matrix
V6	P_o $(P_c = 0)$ matrix
V7	P_o $(P_c = +0.01)$ matrix
W	B_c
W0	$\bar{B}_c$
W3	$B_{c_{min}}$
W5	Q_o $(P_c = -0.01)$ matrix
W6	Q_o $(P_c = 0)$ matrix
W7	$Q_o(P_c = +0.01)$ matrix
W9	normalized radian frequency
X1	X_c
X3	X_o
X4	starting value of X_{sp}
X5	ending value of X_{sp}
X6	step of X_{sp}
Y	entrainment streamline, ψ
Y0	streamline location
Y1	lateral distance, Y
Y5	running total of W5
Y6	running total of W6
Y7	running total of W7

APPENDIX A

TABLE A-I. COMPUTER VARIABLE NAME DEFINITIONS (CONT'D)

Z	During output computation Z = sum of output and load resistance; during dynamic computation Z = gain-bandwidth product.
Z2	loading condition marker, 1, 2, 3
Z3	During output computation Z3 = loss coefficient; during correction of 180 deg for quadrant change of $\tan^{-1}$.
Z4	starting value of B_c
Z5	ending value of B_c
Z6	step of B_c
Z7	starting value of B_t
Z8	ending value of B_t
Z9	step of B_t
Z3$	"YES/NO", "ranges?"
Z4$	"YES/NO", Another problem?"
Z5$	"YES/NO", "Response other than self-staged?"
Z6$	"1" or "2" plot symbol
Z9$	"YES/NO", "Plots?"