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NAVORD OD 43251

1 JANUARY 1970

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AVAILABILITY EVALUATION PROGRAM MANUAL

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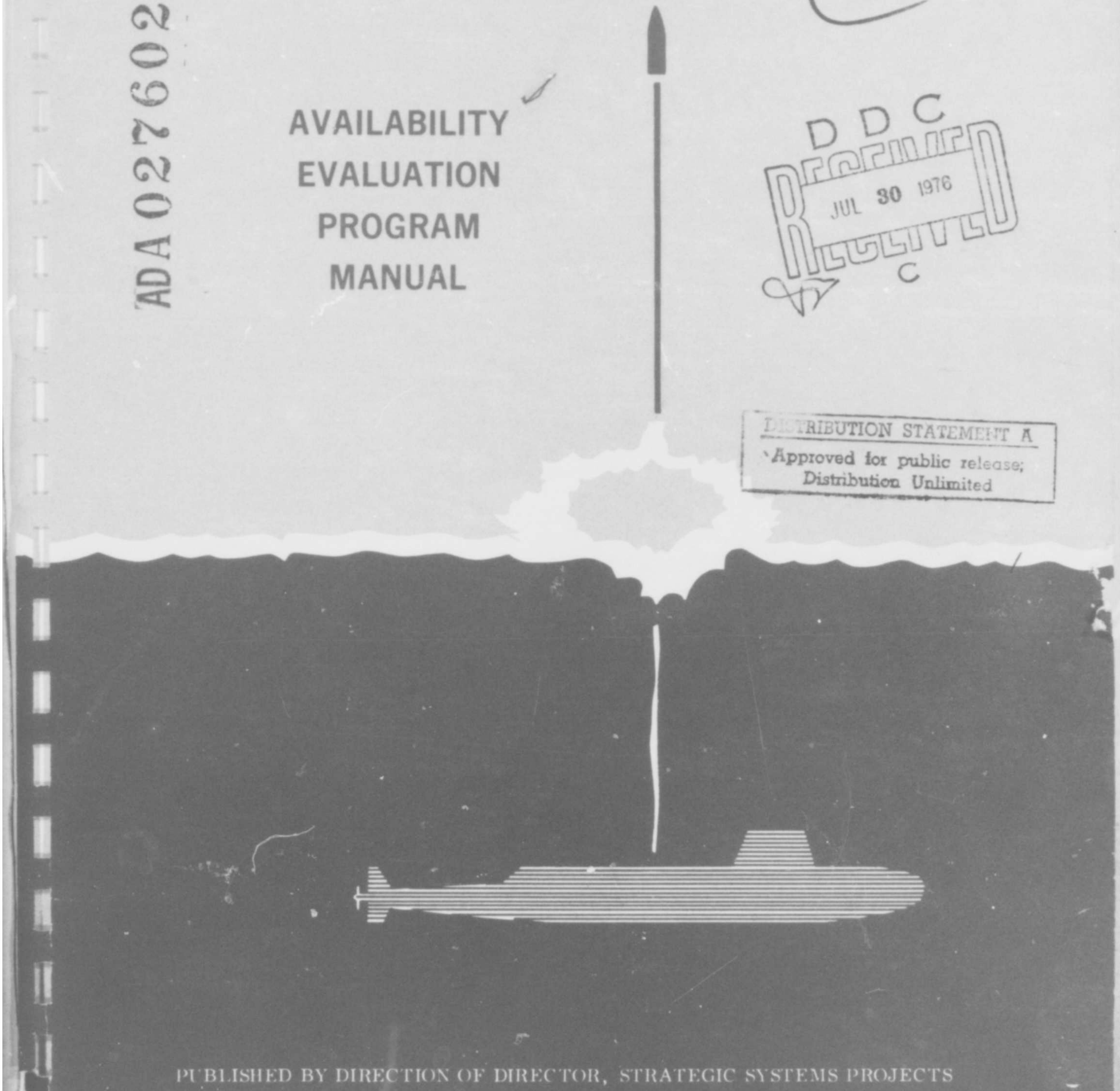

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ERRATA

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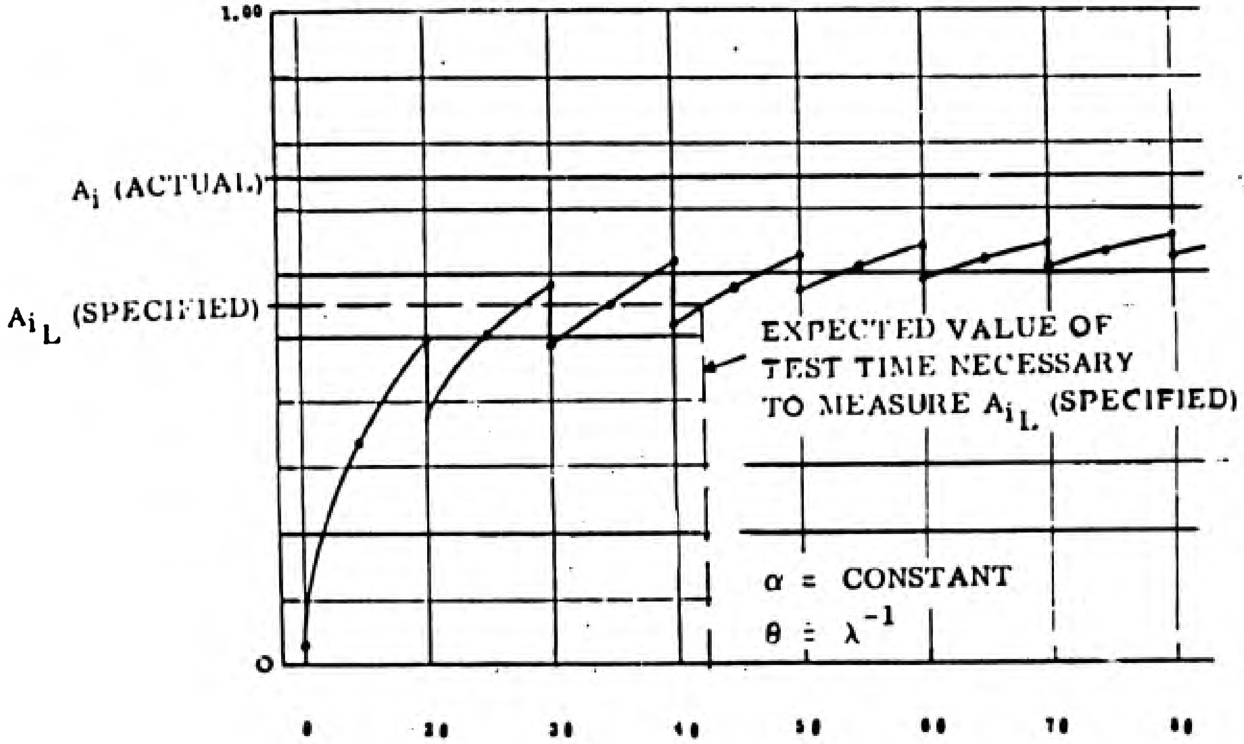

Figure 4-4. Most Probable Growth Curve

Page 4-7 Figure 4-5 curve is drawn incorrectly; table is incorrect. Substitute figure and table shown below.

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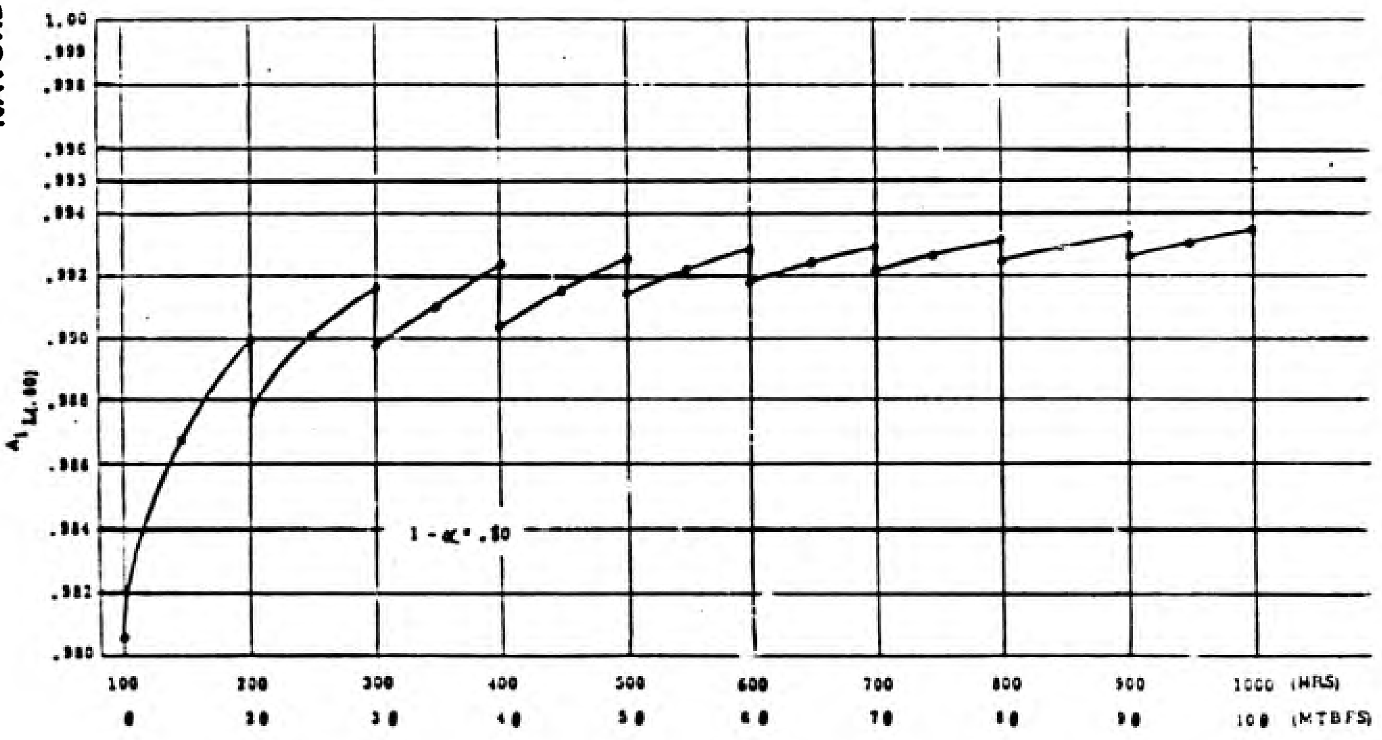

Figure 4-5. Subsystem Availability, Projected Growth Curve

t	n	$F_{.80}$	$\hat{\lambda}$	A_{i_L}
101	1	4.0000	.0099	.9806
150	1	4.0000	.0066	.9870
199	1	4.0000	.0050	.9901
201	2	2.4826	.0100	.9878
250	2	2.4826	.0080	.9902
299	2	2.4826	.0067	.9918
301	3	2.0619	.0100	.9898
350	3	2.0619	.0086	.9912
399	3	2.0619	.0075	.9923
401	4	1.8564	.0100	.9903
450	4	1.8564	.0089	.9918
499	4	1.8564	.0080	.9926
501	5	1.7316	.0100	.9914
550	5	1.7316	.0091	.9922
599	5	1.7316	.0083	.9929
601	6	1.6464	.0100	.9919
650	6	1.6464	.0092	.9925
699	6	1.6464	.0086	.9930
701	7	1.5840	.0100	.9921
750	7	1.5840	.0093	.9927
799	7	1.5840	.0088	.9931
801	8	1.5358	.0100	.9924
850	8	1.5358	.0094	.9928
899	8	1.5358	.0089	.9932
901	9	1.4970	.0100	.9926
950	9	1.4970	.0095	.9930
999	9	1.4970	.0090	.9933

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DEPARTMENT OF THE NAVY
STRATEGIC SYSTEMS PROJECT OFFICE
WASHINGTON, D. C. 20390

IN REPLY REFER TO

1 January 1970

NAVORD OD 43251
AVAILABILITY EVALUATION PROGRAM MANUAL

1. NAVORD OD 43251 is a comprehensive practical guide to evaluating the Availability of major weapon system or subsystems during development. This manual integrates various technological concepts and disciplines (viz. reliability, maintainability, engineering analysis, etc.) into a unified program with the ultimate objective of providing a feasible method for evaluating and assessing availability measures.

2. NAVORD OD 43251, "Availability Evaluation Program Manual" was developed as a companion document to NAVWEPS OD 29304, "Guide Manual for Reliability Measurement Program," for the implementation of MIL-Q-21549, "Product Quality Program Requirements for Fleet Ballistic Weapon System Contractors". Together they formulate a partial means for assessing System Effectiveness during development. The terminology and definitions utilized in this manual are in consonance with DOD policy and current military standards and specifications.

3. Comments and corrections regarding this manual should be forwarded to Strategic Systems Project Office, Washington, D. C. 20390.

4. Copies of NAVORD OD 43251 may be obtained from Commanding Officer (CTDO), Naval Ordnance Station (Code VI (c)) Louisville, Kentucky 40214.

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John B. Buescher
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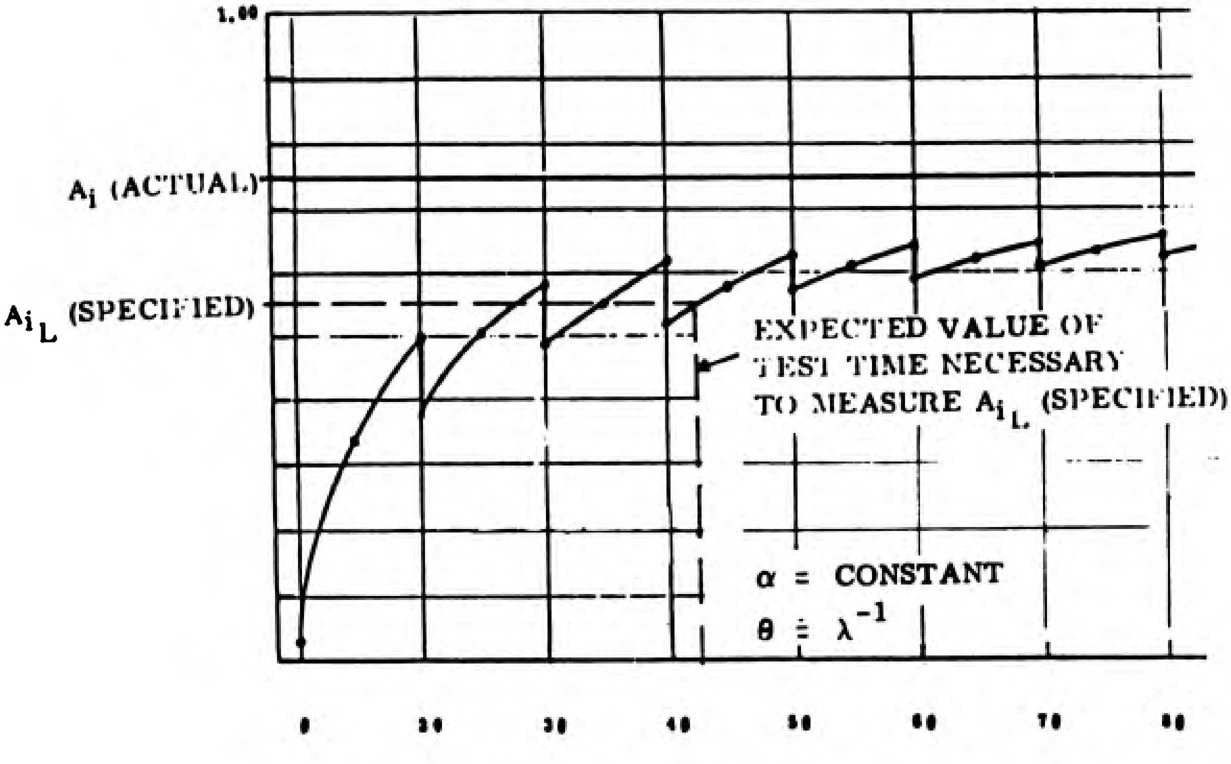

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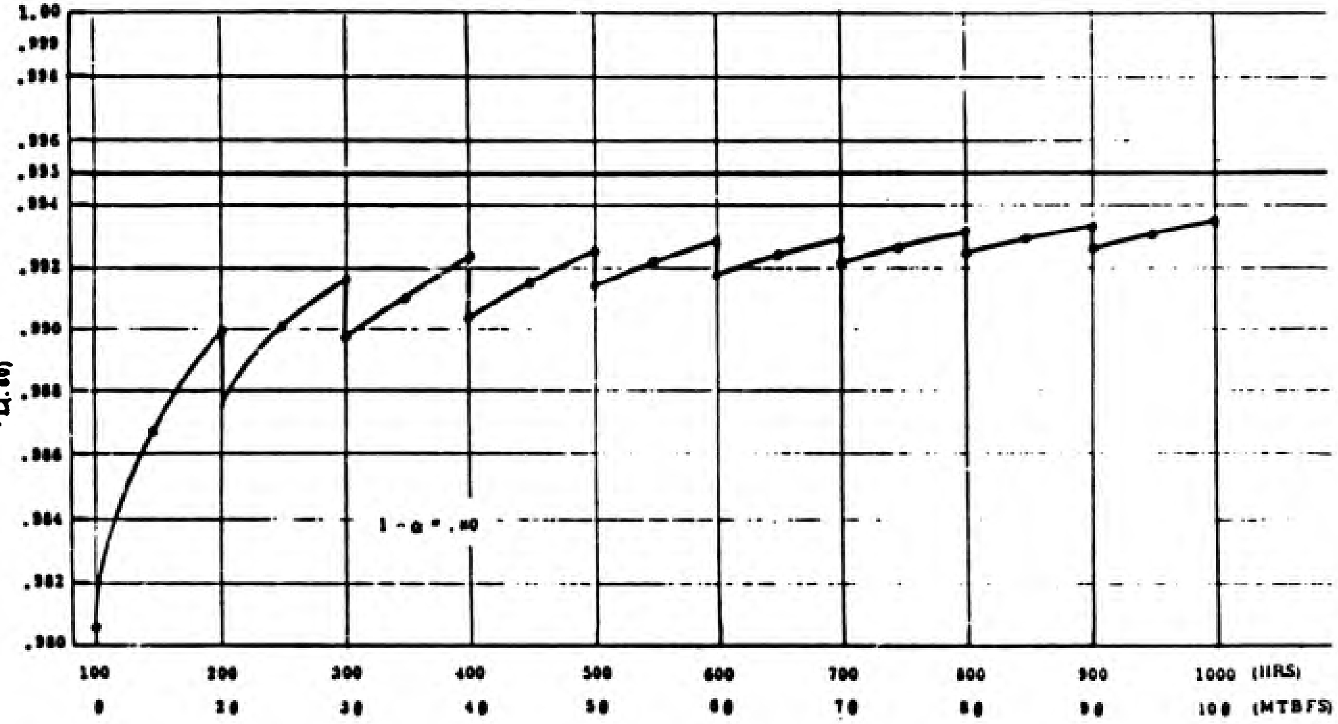

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FOREWORD

This manual was prepared under the direction of the Strategic Systems Project Office (SSPO), United States Navy. The methods presented were developed by the Re-entry and Environmental Systems Division of the General Electric Company.

Availability is a major requisite for system effectiveness. It is a system characteristic which involves the interaction of reliability and maintainability. There are at present no standardized procedures for evaluating availability during development and pilot production, for making meaningful projections to operational phases, or for measuring achieved availability against design goals or requirements. This manual has been written to provide procedures for evaluation of availability and related system characteristics for the FBM weapon system and its major subsystems, and to provide criteria for the interpretation of such measurements by the SSPO and its contractors. In addition, the uniformity provided by the manual will lead to the accumulation of a data pool useful for evaluating availability concepts in relation to future generations of systems. Although the frame of reference throughout this manual is the FBM weapon system, the procedures described can as readily be applied to other types of complex equipment.

The specific objectives of this manual are:

1. To provide contractors with a program for evaluating equipment and subsystem availability during development programs, which normally include pilot production programs.
2. To provide the SSPO with procedures for combining contractor-prepared subsystem availability reports into predictions of weapon system availability in the operational phase.
3. To provide the SSPO with a means for assessing weapon system availability in fleet service.

An effort has been made to achieve compatibility between the terminology and philosophy set forth in this manual and those of NAVWEPS OD 29304 "Guide Manual for Reliability Measurement". In particular, the designations of hardware indenture levels follow those of the previous manual. They are shown below:

System	Poseidon Weapon System
Subsystem	Fire Control Subsystem
Equipment	Digital Geo-Ballistic Computer
Component	Type 3 Module
Hardware/Item	General terms applicable to physical elements of a system

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GLOSSARY

APPORTIONMENT	A process of assigning goals or requirements to items in a system in accordance with some logical scheme.
AVAILABILITY	The probability that an item will be operable when needed.
AVAILABILITY, APPARENT	For an item checked out at intervals, the quotient of apparent uptime divided by apparent uptime plus apparent downtime. Apparent availability is greater than operational availability when failure detection is not immediate.
AVAILABILITY, ACHIEVED	Availability with respect to failure (corrective maintenance) and preventive maintenance jointly, under ideal support conditions; an intrinsic hardware characteristic. It is estimated by the ratio of total operating time to the sum of total operating time plus total maintenance time.
AVAILABILITY, INHERENT	Availability with respect to failure only, under ideal support conditions; an intrinsic hardware characteristic. It is estimated by the ratio of total operating time to the sum of total operating time plus total corrective maintenance time.
AVAILABILITY, INTERVAL	The time average of pointwise availability over intervals of stated length T.
AVAILABILITY,OPERATIONAL	Availability in the actual operating environment; a function of facility characteristics as well as hardware. It is estimated as the ratio of operating time plus alert time to total calendar time. Equivalent to operational readiness.
AVAILABILITY PHASE	Any phase of a mission when an availability figure-of-merit applies, i.e., failures are permissible if system is up when needed.
AVAILABILITY, POINTWISE	The probability that an item will be operable at a stated instant in time.

AVAILABILITY, STEADY-STATE	The asymptotic limit of interval availability as $T \to \infty$. Equivalent to the probability that an item will be operable at a randomly chosen instant in time. It is estimated by the uptime ratio.
CONFIDENCE	A measure of assurance that a statement based on statistical data is correct. The probability that an unknown parameter lies within a stated interval.
CONFIDENCE INTERVAL	A one-sided or two-sided region within which an unknown parameter is said to lie with stated probability.
CONFIDENCE LIMIT	A bound of a confidence interval.
DEMONSTRATION	Measurement of system characteristics with statistical confidence by testing or operation.
EQUIPMENT	The first indenture level below a subsystem.
EXPECTED VALUE	The first moment about the origin of a probability distribution. The arithmetic mean.
EXPONENTIAL DISTRIBUTION	A probability distribution having the density function $p(t) = \lambda e^{-\lambda t}$ where λ is a constant, called the failure rate. Under very general conditions it is the distribution of time between successive failures of complex systems.
FAILURE	Performance below a specified minimum level.
FBM WEAPON SYSTEM	The SSBN submarine, together with its supporting tactical subsystems - missile, fire control, navigation, launcher, and ship support.
FIGURE OF MERIT	An index or quantitative measure of merit used to characterize an item for analysis or comparison.
HARDWARE/ITEM	A general term denoting physical elements of a system.
MAINTAINABILITY	A measure of the ability of an item to be maintained. Mean preventive maintenance time and mean repair time are commonly used indices of maintainability. (The often-encountered definition of maintainability as the probability of repair within a stated time is not used because that probability is not used in availability expressions or computations in this manual.)

MAINTENANCE	All actions necessary for retaining an item in, or restoring it to, a specified condition.
MAINTENANCE CONSTRAINTS	Limitations on the quantity and/or quality of maintenance available to a system in use.
MAINTENANCE, CORRECTIVE	Actions performed, as a result of failure, to repair an item and restore it to a specified condition.
MAINTENANCE, PREVENTIVE	Actions performed on a scheduled or routine basis in an attempt to retain an item in a specified condition by providing systematic inspection, detection and prevention of incipient failure.
MEAN-TIME-BETWEEN-FAILURES (MTBF)	For a particular interval, the total functioning life of a population of an item divided by the total number of failures within the population during the measurement interval. The definition holds for time, cycles, miles, events, or other measure-of-life units.
MEAN-TIME-TO-REPAIR (MTTR)	The total repair time divided by the total number of repair actions during a given period of time.
MEASUREMENT	Evaluation of the characteristics of an item by observation of its performance in test or operational service. The term estimation as used herein is synonymous with measurement.
OPERATING MODE	A specific pattern of system operation in which a designated subset of the system's functions are realizable (e.g., standby mode, tracking mode, search mode).
OPERATIONAL READINESS	Operational availability of a weapon system in fleet service.
PERFORMANCE CAPABILITY	A measure of a system's ability to meet its mission requirements, given that it is both available and reliable.
POISSON DISTRIBUTION	A probability distribution applicable to situations where a large number of observations is involved and the probability of an event occurring in any specific observation is very small. It has the density equation $p(x) = e^{-m}m^{x}/x!$

POISSON EXPECTATION	The mean m of the Poisson distribution. In reliability problems the product λt is often a Poisson expectation.
PREDICTION	Judgement of the characteristics of an item by means of engineering analysis, using generic data and/or historical data obtained from antecedent items.
REDUNDANCY	The existence of more than one means for accomplishing a given function. All means of accomplishing the function need not necessarily be identical.
REDUNDANCY, ACTIVE	That redundancy wherein all redundant items are operating simultaneously rather than being switched on when needed.
REDUNDANCY, STANDBY	That redundancy wherein the alternative means of performing the function is inoperative until needed and is switched on upon failure of the primary means of performing the function.
RELIABILITY	The probability that an item will perform its intended function without failure for a specified interval under stated conditions, given that it is up at the beginning of the interval.
RELIABILITY PHASE	Any phase of a mission during which a reliability figure of merit applies (i.e., no failure is permissible). e.g., launch, flight.
SOFTWARE	Procedures and instructions including, in particular, computer programs.
SUBSYSTEM	The first indenture level below a system. For example, any of the subsystems comprising the FBM weapon system (e.g., navigation subsystem).
SUCCESS CRITERIA	The minimum functional performance required of an item for mission success.
SYSTEM	A collection of functionally related items, which together perform one or more useful functions. Also, the FBM Weapon System.

SYSTEM EFFECTIVENESS	A measure of the degree to which an item can be expected to achieve a set of specific mission requirements and which may be expressed in terms of availability, reliability and performance capability.
SYSTEM STATE	A designation of system status at a particular time with respect to operable and inoperable equipments. An n-equipment system can exist in 2^n states ranging from all equipments up to all equipments down.
TACTICAL	Pertaining to or necessary for the primary mission of the weapon system.
TIME	When used herein without a modifier the word time is interpreted to mean calendar time.
	Active Time – That time during which an item is in the operational inventory. For the FBM weapon system, the patrol period. It is the time base for availability calculations in this book.
	Adjustment-Calibration Time – That element of Repair Time in which needed adjustments and calibrations are made.
	Administrative Delay Time – Those elements of Delay Time attributable to administration of maintenance functions - includes meals, sleep, queueing resulting from maintenance demands in excess of shop capacity, management decisions to defer repairs, etc.
	Alert Time – That element of Uptime when an item is thought to be operable and is awaiting a command to perform its intended mission.
	Checkout Time – That element of Repair Time during which repair is confirmed and verified to be satisfactory.
	Cleanup Time – That element of Repair Time during which an item is enclosed and extraneous material not required for operation is removed.
	Corrective Maintenance Time (Repair Time) – That element of Downtime in which

TIME (Continued)

work is done to repair trouble or failure. Includes time to obtain tools, documents and spares from local stock rooms, set them up for repair, troubleshoot, test spares if necessary, effect repair, make necessary adjustments and calibrations, confirm the repair by test if necessary and close up the repaired item It specifically excludes time devoted to off-line repair of any item that was replaced. It also excludes elements of Delay Time such as meals, sleep, administrative delays including the postponement of repair by managerial decision, awaiting spares from off- site or remote locations, etc.

Delay Time – That element of Downtime during which no maintenance is accomplished on an item because of either supply delay or administrative reasons.

Downtime – Total time during which an item is not in condition to perform its intended function.

Environment Time – The duration of exposure to a specific environment during a mission.

Fault Correction Time – That element of Repair Time during which a failure is corrected by (a) repairing in place, (b) removing, repairing, and replacing, or (c) removing and replacing with a like serviceable item.

Fault Location Time – That element of Repair Time during which testing and analysis is performed on an item to isolate a failure.

Inactive Time – That time during which an item is not in active inventory, therefore, not expected to be operable. For the FBM weapon system, time not spent on patrol. Not included in availability calculations in this book.

Inspection Time – That element of Preventive Maintenance Time needed to inspect an item to determine its condition.

TIME (Continued)

Item Obtainment Time – That element of Repair Time during which the needed item or items are obtained from stockrooms within the facility. For the FBM weapon system, time to obtain items from the ship's stores.

Maintenance Time – That part of Downtime when maintenance work is actually being done.

Mission Time – That element of Uptime when an item is performing its designated mission.

Modification Time – That element of Downtime during which specific modifications or retrofits are made to an item to add to or improve its characteristics.

Operating Time – Cumulative Operating Time in testing or use.

Preparation Time – That element of Repair Time needed to obtain the necessary test equipment and maintenance manuals and set up the necessary equipment.

Reaction Time – That element of Uptime needed to initiate mission functions, measured from the time a command is received.

Servicing (Turn-around) Time – That element of Preventive Maintenance Time needed to perform routine or scheduled maintenance tasks (other than inspection) to retain an item in operable condition.

Supply (Logistic) Delay Time – That element of Delay Time during which a needed item is being obtained from other than designated facility stock rooms. For the FBM weapon system, time to obtain items not carried aboard the submarine.

Preventive Maintenance Time – That element of Downtime during which routine or scheduled tasks are performed to retain an item in an operable condition by systematic inspection and servicing.

TIME (Continued)	Uptime – That element of Active Time when an item is up, i.e., alert, reacting or performing mission functions.
	Uptime, Apparent – That element of Active Time when an item is thought to be up. Apparent Uptime may be greater than Uptime when failure detection is not immediate.
UPTIME RATIO	The quotient of Uptime divided by Uptime plus Downtime. Uptime ratio is a statistical estimate of steady-state availability.

GOVERNMENT DOCUMENTS REFERENCED IN TEXT

1. MIL-STD-756, Reliability Prediction
2. MIL-HDBK-217, Reliability Stress and Failure Rate Data for Electronic Equipment
3. MIL-STD-470, Maintainability Program Requirements
4. MIL-STD-471, Maintainability Demonstration
5. MIL-HDBK-472, Maintainability Prediction
6. NAVWEPS OD 29304, Guide Manual for Reliability Measurement Program
7. MIL-Q-21549, Product Quality Program Requirements for Fleet Ballistic Missile Weapon System Contractors
8. MIL-STD-721, Definitions of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety
9. SPINST 3100.1, Fleet Ballistic Missile Weapon System Trouble and Failure Report System
10. NAVORD OD 39223, Maintainability Engineering Handbook

LIST OF SYMBOLS

A,a	Steady-state availability.
a	Coefficient used to compute confidence interval.
A_a	Achieved availability. Availability with respect to failure (corrective maintenance) and preventive maintenance jointly.
A_i	Inherent availability. Availability with respect to failure (corrective maintenance).
A_o	Operational availability. Availability attained in actual use.
A_p	Availability with respect to preventive maintenance.
A(t)	Pointwise availability.
$\overline{A}_T$	Interval availability.
α	Level of significance; also Lagrangian multiplier.
C	Cost.
d	Observed Downtime in an interval.
D	Largest absolute deviation; also downtime.
Δ	A small interval or excursion from a nominal value.
δ	Delta function.
e	Base of natural logarithms.
ϵ	Limiting acceptable risk.
F	Fractile of the cumulative F-distribution.
f( )	Probability density function.
f	Frequency of preventive maintenance.
Γ	Gamma function.

G	Total cost function.
i	Subscript denoting failure mode, function, task, component, inherent characteristic.
k	Subscript denoting operating cycle; also fractional duty cycle.
L, U	Subscripts denoting lower and upper confidence limits
Λ, λ	Log normal distribution and density functions.
λ	Failure rate.
M, μ	Mean of a normally distributed random variable.
M_c	Corrective maintenance time; repair time.
$\overline{M}_c$	Mean corrective maintenance time; MTTR.
$\overline{M}_{c_G}$	Geometric mean corrective maintenance time.
M_p	Preventive maintenance time.
$\overline{M}_p$	Mean preventive maintenance time.
m	Number of observed repairs
μ	Repair rate; reciprocal of MTTR.
N, n	Normal distribution and density functions. Also item population.
n_i	Number of relevant failures.
n_o	Number of observed failures.
n_p	Number of preventive maintenance events.
ν	Reciprocal of $\overline{M}_{c_G}$.
o	Subscript denoting observed characteristic; also optimum value.
P, p	Probability; also fraction of real time.
q	Discrete probability of failure imposed by checkout or maintenance.
S	Measured cumulative frequency.
s	Standard deviation of a sample.

σ	Standard deviation of a normally distributed random variable.
T	Period of fixed or nominal length.
t	Time; also period of variable length; also the student's -t statistic.
θ	Mean time between failures, MTBF.
u	Observed uptime in an interval.
u, v	Chi-square variates.
U	Unavailability; also utility; also a chi-square variate; also uptime.
U_i	Unavailability with respect to failure; inherent unavailability.
U_p	Unavailability with respect to preventive maintenance.
V,v,Z,z	Log-normal variates.
W, w	Ratio of two random variables.
X, x	Normal variate.
$\overline{X}$	Mean of a sample.
χ^2	Fractile of the chi-square distribution.
$\hat{}$	Denotes estimate of a parameter.
$\sim$	Is distributed as.

SECTION 1

INTRODUCTION

1.1 NATURE OF AVAILABILITY AND PURPOSE OF EVALUATION PROGRAM

In this manual the term availability is defined as the probability that a system will be "up" or operable when called upon. It can extend to a variety of specific indices, each of which is separately defined as the probability that a system will be available if called upon at a time or in a manner limited by the definition. It is necessary that availability be thus broadly defined, since the functions and requirements of systems differ widely and no single index or figure-of-merit can meaningfully represent availability for all systems. Moreover, availability depends on the care and skill with which a system is transported, operated and maintained, as well as on the original system design. However, in this manual, use of the term is limited to measures which are primarily properties of system design. The manual is not concerned with measures of availability which are primarily reflections of logistic support, spares provisioning and similar factors external to systems.

Availability is of concern to the Navy because of the randomness of demands on systems in service. For example, a submarine on patrol is beyond the reach of support facilities for a period of months. Ideally, all of its missiles and their tactical supporting systems should be fully operable and ready for firing at any time during the patrol. But the maintenance of such capability at all times cannot be expected, because equipment failures may occur and require downtime for their repair. Typically, a complex system is able to function in a variety of modes, depending upon the numbers and types of its equipments that are operable. When the population of operable equipments is adequate to support the performance required in a particular operational mode, the system is up with respect to that mode; otherwise it is down.

The system effectiveness of a weapon system is of primary concern to the system manager. System effectiveness means the degree to which the weapon system is able to meet its mission requirements throughout the entire patrol period. The mission should be examined quite early during concept formulation and the required effectiveness should be specified quantitatively in a form from which availability requirements can be derived. Figure 1-1 illustrates the relationship of availability elements to mission success. If a score or rating of effectiveness is assigned to the weapon system's performance capability in each of its operational modes, its effectiveness can be expressed as a function of the fraction of time spent in each mode. There can then be specified a minimum acceptable availability or fraction of the total patrol time when the weapon system must equal or exceed the performance capability associated with each mode. The fractional uptime for any mode is a statistical estimate of the probability of finding the system operable in that mode at a randomly chosen instant during a patrol of sufficient length.

From the weapon system level, availability requirements can be apportioned to subordinate systems, so that if each subsystem meets its requirements the weapon system will achieve its required availability.

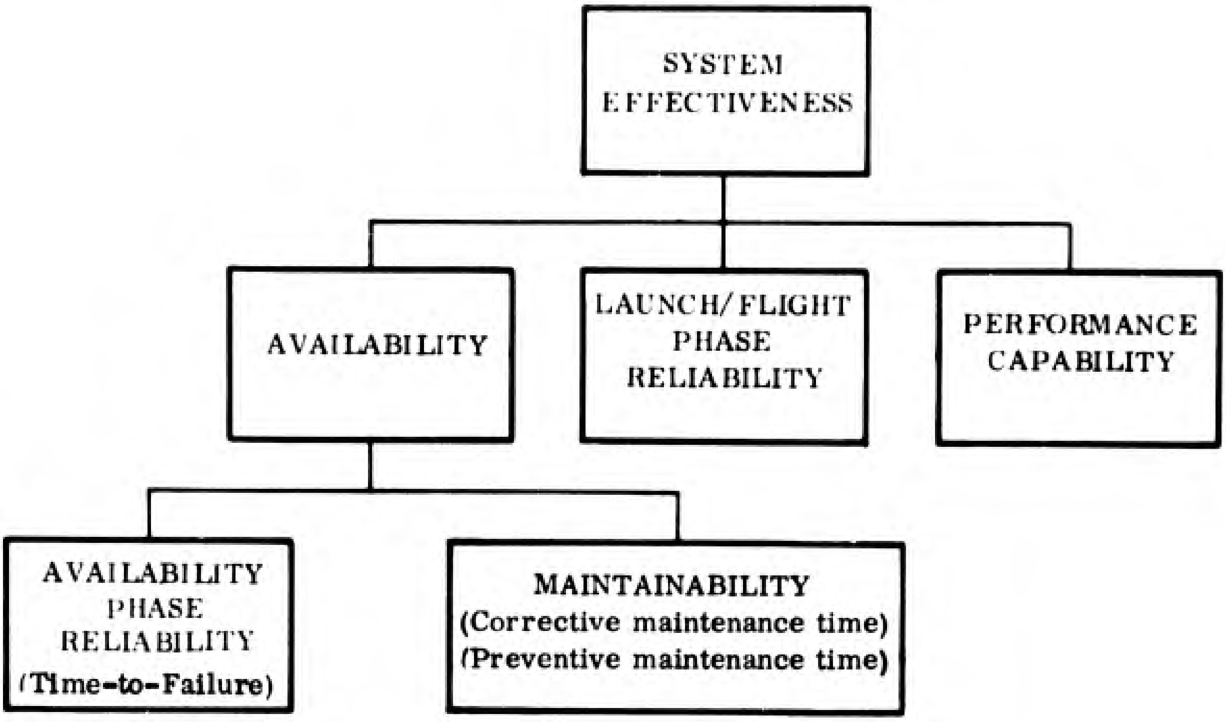


Figure 1-1. Relationship of Availability Elements to System Effectiveness

The availability exhibited by a system during periods of stated length is a statistically distributed variable. Its distribution is jointly determined by the system's reliability and maintainability, which in turn depend on the corresponding properties of the equipments that constitute the system. Reliability determines the frequency of unscheduled downtimes: maintainability determines their durations. Availability can be estimated quantitatively by means of reliability and maintainability prediction studies. Reliability prediction methods are presented in MIL-HDBK-217 "Reliability Stress and Failure Rate Data for Electronic Equipment" and MIL-STD-756 "Reliability Predictions". Maintainability prediction methods are set forth in MIL-HDBK-472 "Maintainability Prediction". While the validity of such "paper" predictions is obviously more open to question than estimates based on testing, handbook data often provide the only information available for availability evaluation early in a program.

An additional limitation of predictions is that they cannot be accompanied by statements of statistical confidence. (Note: Exceptions to this statement can occur when Bayesian formulations, encoding a priori information, are employed.) This manual makes specific provisions for the use of predictions in the early phases of the program to be described.

When hardware becomes available for testing, system availability can be evaluated quantitatively by observing the lengths of uptimes and downtimes in tests of the system or of its equipments. The probability of compliance with specified requirements can then be estimated and confidence statements can be associated with the estimates. This process is defined as availability measurement. It is necessary, however, that the operating and maintenance conditions of the tests be relatable to the mission environment if the tests are to serve as a valid basis for measurement. Availability measurement is a sequential process of comparing and evaluating test and operating experience against predicted and allocated performance standards. The measurement process continues during the system's service life with operating data supplanting test data.

The purpose of availability evaluation is to identify the need for corrective actions wherever the need exists and to furnish information to guide managers in implementing corrective actions and appraising the effectiveness of those actions.

If availability evaluations are to be useful for weapon system evaluation, the SSPO and its system contractors must cooperate to plan, organize and implement suitable programs. This manual provides procedures for each of the activities necessary to an availability evaluation program, beginning in the contractor's plant during the development phase, with the initial availability of one or more systems for testing and extending through pilot production, production, and into the operational phase when systems are in fleet service. The program is designed to enable contractors to evaluate the probabilities that their products will satisfy specified availability requirements when used in specified operational environments and configurations.

Section 1 of this manual identifies and discusses the rationale for each of the elements necessary to an availability evaluation program. Section 2 treats the analytical functions of mission analysis and system analysis and includes an example of their application to a typical subsystem. The specification of availability indices, and their representation as functions of system characteristics by means of mathematical models, are also discussed in Section 2. Section 3 includes statistical procedures for making availability estimates based on testing of full or fractional system configurations. The limited opportunities for testing that typically characterize a development program confer a premium on efficient collection and use of data by system contractors. Maximum utilization of suitable data from tests conducted for development, engineering evaluation, qualification, factory acceptance, production assessment and field evaluation, together with correct collection, screening, classification, management, processing and feedback of those data, are necessary to the task of measuring availability. Sections 4 and 5 present procedures for these functions. Finally, the recognition that availability numerics are inevitably more or less abstract characterizations of the behavior to be expected of a system in service leads to an obvious but often neglected need to interpret the results of availability measurement in terms meaningful for decision making by SSPO management and contractors alike. Thus, effective reporting, interpretation and use of the availability evaluation program outputs are discussed in Section 6. Derivations relevant to the statistical approach of this manual are presented in an appendix.

1.2 SPECIFICATION OF AVAILABILITY

Availability is a system characteristic to be measured against suitable specification requirements. It is a dimensionless number measured on the probability scale [0,1]. Availability must be specified in such a way that it can be measured unambiguously. Normally, availability is not specified alone. Both reliability and maintainability are usually specified together with availability. For durable, continuously operated hardware, reliability

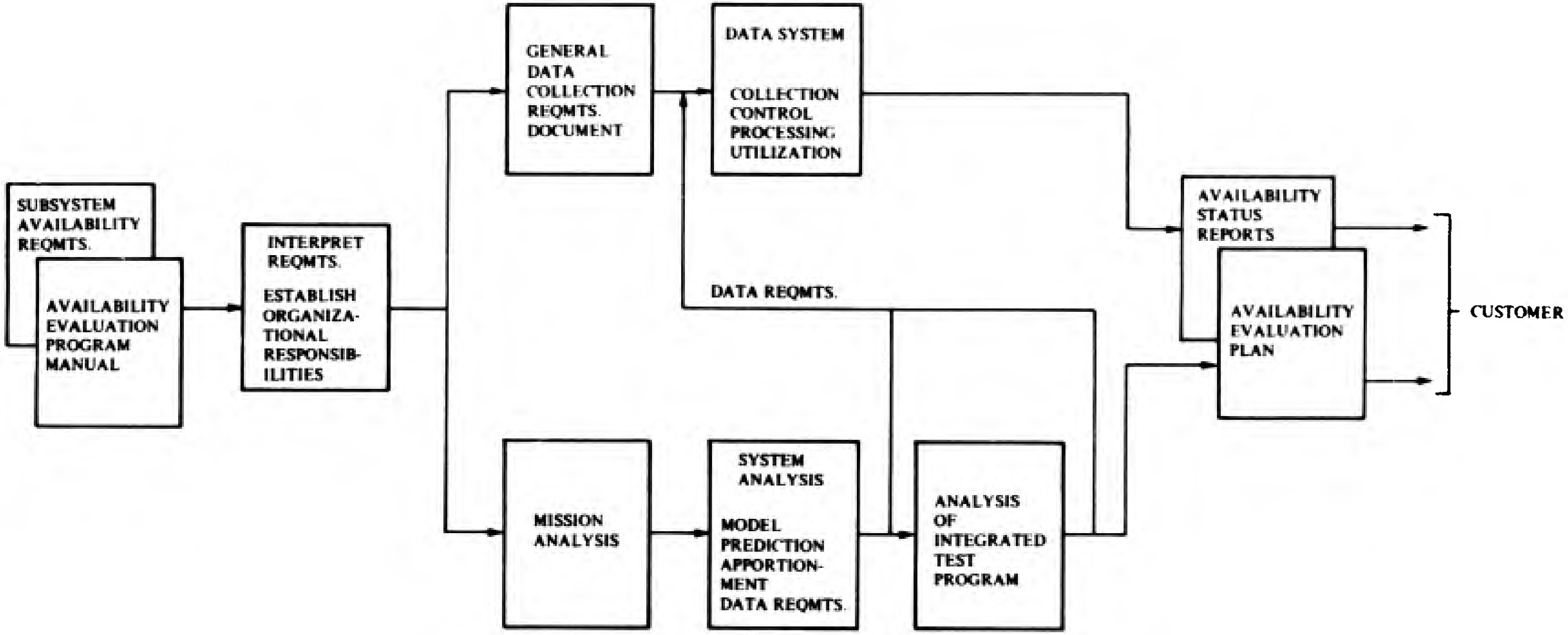


Figure 1-2. Availability Program Activities

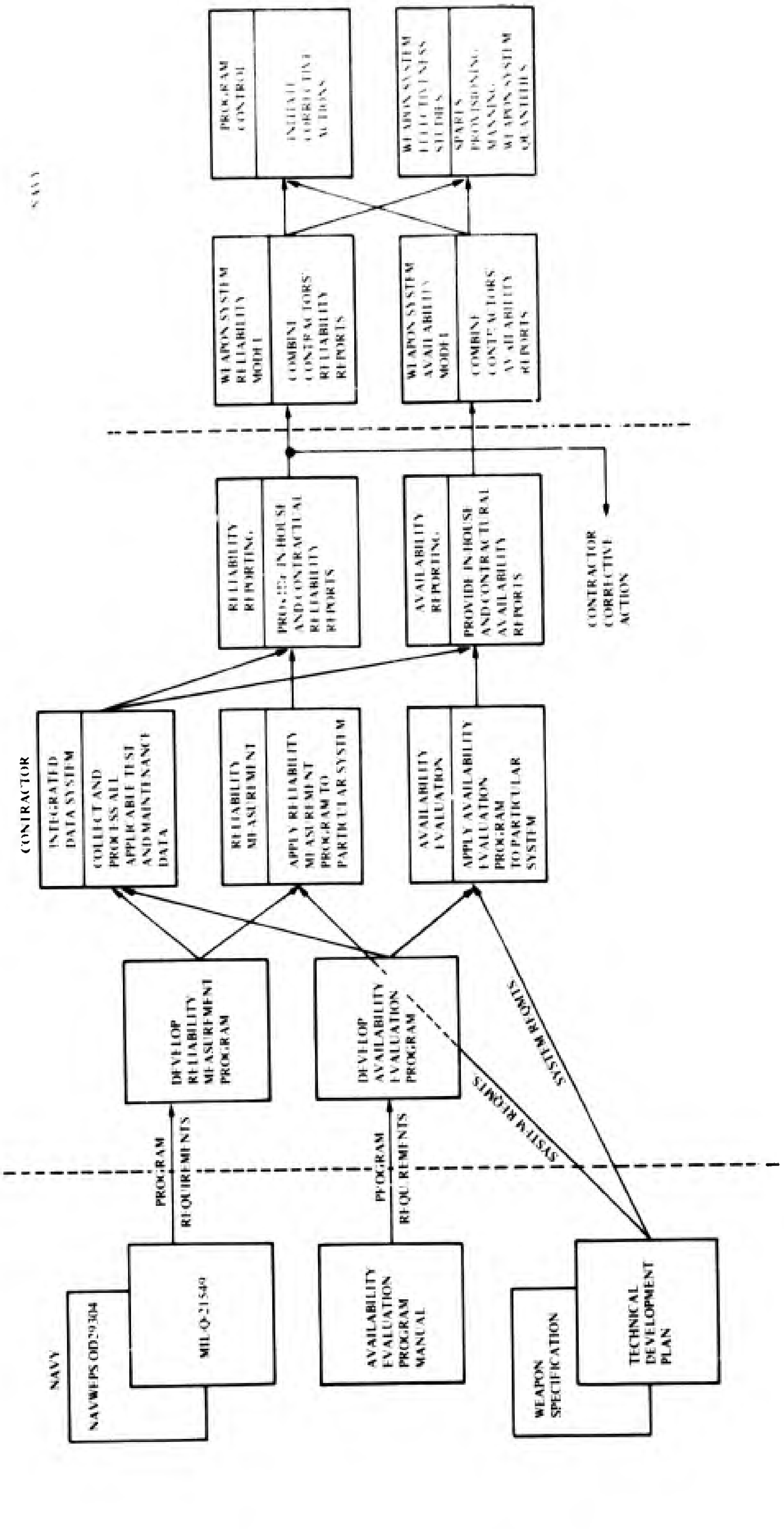


Figure 1-3. Relationship of Reliability and Availability Programs

can be specified by mean-time-between-failures (MTBF). Given the exponential form of time-to-failure distribution, which is applicable to most types of complex equipment, MTBF is a constant, the reciprocal of the failure rate. If the failure distribution is known to be exponential or is assumable as such, specification of MTBF is equivalent to specification of the complete reliability function.

Maintainability can be specified by mean-repair-time and mean-preventive-maintenance-time. Corrective maintenance times are often found to be well described by log-normal distributions, which are completely defined by two parameters, a mean and variance. It is customary, therefore, to specify in addition to the mean, a permissible variance for repair time or, more commonly, a maximum time to be exceeded in no more than one percent of all repair actions.

Enforcement of a specification is facilitated by providing suitable tests to measure or demonstrate compliance with its provisions. The actual reliability and maintainability achieved by a system cannot be measured as points on the probability scale, with positive statistical confidence during the system's life. However, on the basis of observed performance, it is possible to state that the true reliability and maintainability exceed stated minima and to accompany this with quantitative measures of confidence that the statements are true. Therefore, for demonstration purposes it is desirable to specify levels of reliability and maintainability lower than the system design goals and to require that these minimum levels be measured with specified statistical confidence.

1.3 ELEMENTS OF AN AVAILABILITY EVALUATION PROGRAM

The availability evaluation program described in this manual is comprised of analysis functions, reporting functions and data functions. All are necessary parts of the contractor's effort at subsystem level, and all are necessary by the SSPO at weapon system level. The functions are listed below and their precedence relationships are shown in Figure 1-2.

1. Mission Analysis
2. System Analysis
3. Analysis of Integrated Test Plan
4. Integrated Data System
5. Availability Reporting

1.4 RELATIONSHIPS TO NAVWEPS OD 29304

The functions necessary for availability evaluation parallel those required to evaluate reliability. Thus the activities presented herein correspond in part to the reliability measurement activities specified in NAVWEPS OD 29304 "Guide Manual for Reliability Measurement". However, statistical procedures for measuring availability differ from those employed for reliability measurement. Basic data for reliability measurement are total test time and number of failures. Data needed for availability measurement are the durations of uptimes and downtimes (as related to demands on the system). Nevertheless, the activities necessary to acquire availability data and to report the results of availability evaluations are largely extensions of the NAVWEPS OD 29304 Integrated Data Program, to include the accumulation and processing of maintainability data. Figure 1-3 shows the relationships of reliability and availability program activities.

1.5 DEVELOPMENT AND APPLICATION OF AN AVAILABILITY EVALUATION PROGRAM

The program in this manual is designed to make use of test data generated by the contractor in the course of system development and production. These data are the operating histories of the system and/or its constituent equipments obtained in development, engineering evaluation, qualification and acceptance tests, field tests or sea trials. The program also uses operating data from fleet service, which are fed back to the contractor by the FBM Trouble and Failure Report (TFR) program, service-logs and fleet work-logs. Much test and checkout data generated during a development program, while not applicable for estimating launch or flight reliability, can be of value for availability measurement. By systematically collecting and using data generated for these purposes, the contractor can eliminate or minimize the need for specific testing to measure availability. At the same time the contractor can evaluate the adequacy of a planned test program with respect to the objective of measuring the system's availability. The need for additional tests can be shown when the planned tests are inadequate, and tradeoffs can be made among the quantities of testing allocated for various portions of the system, so as to verify over-all system availability with a minimum number of tests.

In support of its decision-making functions, the SSPO requires availability evaluations beginning in the development phase before the first fabrication of hardware for testing and extending into the system's operational use phase. Development of a program to meet these needs is a joint activity of the weapon system manager and the system contractors. To this end, SSPO weapon system management specifies top-level system availability requirements at the outset of the development program. The SSPO also stipulates the need for availability evaluation as part of the Weapon System Requirements Specification. In support of contractor's analytical tasks, as described in Section 2 of this manual, the SSPO also provides information on the intended mission, system interfaces and the operational use, maintenance and logistic environments. Later, SSPO management will also function to integrate contractor's outputs for the evaluation of the availability of the complete weapon system.

The contractor's responsibilities include establishing availability evaluation as a program activity by means of policy directives and designation of organizational responsibilities for the analysis, data and reporting functions which comprise the program.

To implement the program for a particular system, the contractor must perform the mission and system analyses described herein, develop a mathematical model for availability and determine the type and quantity of data necessary to solve the model. Procedures, instructions and forms must be developed to collect, monitor and process data from the test sites. Computer programs for processing the data and for generating reports must also be prepared. More important, however, if the availability evaluation activity is to be of benefit to the development program, is the need for a suitable internal information loop to assure that contractor management is made aware of the current status of system availability, particularly as regards system elements that may require action to achieve satisfactory availability levels.

1.6 IMPLEMENTING AVAILABILITY EVALUATION

The program is initiated by establishing organizational responsibility for the functions listed in paragraph 1-3. The contractor can accomplish this by management direction interpreting the provisions of this manual. Existing test data collection requirements should be expanded where necessary to include categories of information necessary for availability assessment. A general test data requirements document should be prepared to delineate the required in-house data, to serve as a reference for designing or amending the necessary forms and procedures, and to ensure that commensurate requirements are invoked for subcontracted procurements. In addition to the test and failure data required for reliability evaluation, which are described in NAVWEPS OD 29304, the necessary information will normally include operating and alert time in each system mode and maintenance time for the preventive, corrective, modification and delay time categories.

These general requirements are interpreted and/or modified for particular equipments of the subsystem by means of the data collection requirements list prepared concurrently with the subsystem availability equation as part of the system analysis, using outputs of the mission analysis.

Application of the measurement program by the responsible elements of the contractor's organization can then go forward. The mission analysis and system analysis are performed, yielding a mission profile by phases, a listing of realizable subsystem states in each phase, and apportioned reliability and maintainability requirements for each equipment. From these, a mathematical equation of subsystem availability is prepared, together with a tabulation of specific test data requirements necessary to solve the equation. The general data collection requirements document is employed to pre-print the necessary forms for collecting operating and maintenance data.

The data requirements tabulation, subsystem apportionment and equation, together with prior predictions of the subsystem's reliability and maintainability, are inputs to evaluation of the Integrated Test Plan. A revised Integrated Test Plan, modified as necessary, is then prepared on the basis of the evaluation. The revised plan, in conjunction with the preceding analyses, becomes an input to an Availability Evaluation Plan, which summarizes the contractor's measurement program and projects the future growth of measured subsystem availability.

With the beginning of testing, the data system is applied to collect, control, process and utilize data for computerized measurement of availability status. This activity is of particular importance during the pilot production phase when the majority of data needed for availability measurement are normally obtained. Availability Status Reports are issued at intervals as required by the SSPO. The information loop is closed by feedback of the results of the evaluation program, through the contractor's management structure, to the engineering, production and assurance activities responsible for implementing corrective actions as needed.

SECTION 2

ANALYSES OF MISSION AND SYSTEM

2.1 MISSION ANALYSIS

Meaningful availability measures depend on precise definition of the mission. Thus, analysis must begin with a detailed description of the subsystem's mission. The contractor must understand the fundamental need that gave rise to the subsystem concept and must understand exactly what requirement or part of a requirement is to be fulfilled by the subsystem. Although it is relatively easy to define a number of performance capabilities a subsystem may possess, it is sometimes not obvious what combination of those capabilities determines how well the intended mission is achieved. Only after the mission has been pinpointed can the succeeding analytical steps be taken. Mission analysis serves to define the mission for which the subsystem's availability will be evaluated. Its inputs are the mission requirements and environmental information furnished by the SSPO in the subsystem specification and Technical Development Plan; its output is a mission profile for the subsystem over the availability phase of its mission. The analysis is done in two steps:

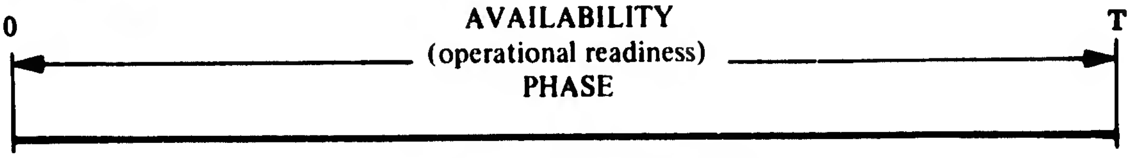
2.1.1 Definition of Tactical Mission Phases

The mission is represented as a sequence of events. When a range of alternate missions is possible, each is examined as a distinct event sequence. Each total mission is then separated into phases and the availability phase is defined. A reliability figure of merit is appropriate to any mission phase that begins with a demand on the subsystem, such that mission success does not permit subsystem failure at any time thereafter during the phase. (Reliability measurement procedures are given in NAVWEPS OD 29304.) An availability figure of merit applies to a mission phase preceding such a demand, where success is possible despite failures if the subsystem is operable at the time the demand is made. In this manual such a period is termed an availability phase.

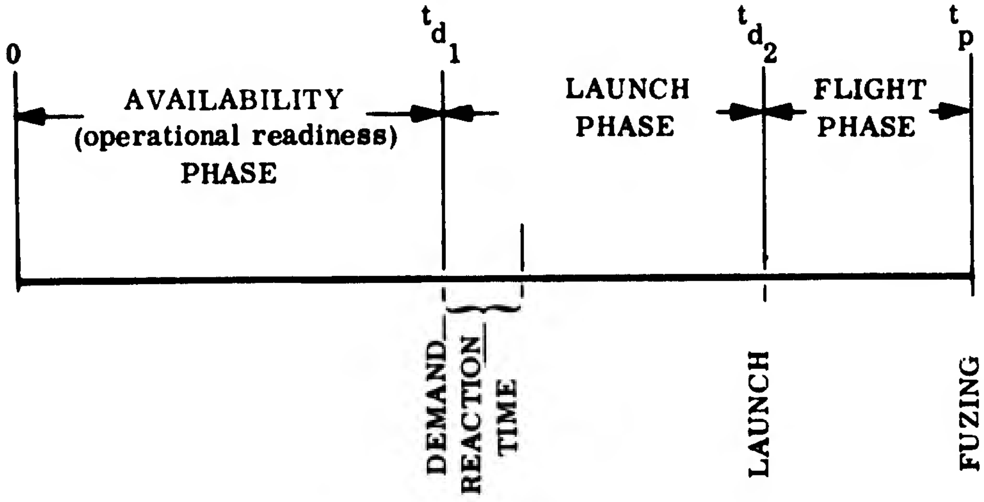
The most general sequence of mission phases is that shown in Figure 2-1.

The weapon system's mission is defined over a patrol period t_p, which in the absence of a demand for offensive action by the weapon system is of nominal duration T. For such a mission the entire patrol is the availability phase and there is no launch phase or flight phase. The weapon system remains on alert status, requiring a designated reaction time to commence firing.

Should a demand be made on the weapon system, that demand terminates the availability phase and initiates the ensuing launch phase and flight phase. During the launch phase the weapon system assumes a preparatory status, holds that status for a period of time up to a designated maximum, then initiates firing. The missile flight phase extends from launch through warhead fuzing over the target area.



a) Patrol without Launch



b) Patrol with Launch

Figure 2-1. Definition of Tactical Mission Phases

The availability and launch phase constitute the mission of the shipboard subsystems; the availability, launch and flight phases together comprise the mission of the missile subsystem. Principal variables influencing mission length are the demand time and hold time associated with each phase. For purposes of availability analysis, those times may generally be treated as random variables uniformly distributed within their respective ranges. This is equivalent to saying that a demand on the weapon system is equally likely to occur at any time during a patrol and that the holding period, if any, in the launch phase is equally likely to be of any length within the permissible range.

If failure is defined as any malfunction that necessitates downtime for repair, a successful mission normally precludes failure of any on-line subsystem during the launch phase, regardless of whether the subsystem is repairable. This is usually a valid assumption for purposes of hardware evaluation, even though it may be known that in a tactical situation the submarine's crew might effect one or more launches despite certain unrepaired failures. Thus, the effectiveness of a subsystem during the launch phase (and of missile subsystems during the flight phase) is measurable in terms of reliability or probability of failure-free operation. But success does not require failure-free operation throughout the availability phase if the subsystem is repairable. All that is required is that the subsystem be up at the time a demand is made. The measure of availability is the probability of that event.

For some subsystems the portion of the mission following initial demand may consist not of a single state but rather of an ordered sequence of states preparatory to firing. Holding times may apply to these states also, so that their durations may vary. If it is determined that availability measures apply to some of these states, it may be necessary to distinguish multiple availability phases. In summary, any phase of the mission where the success criteria permit the occurrence of failures is defined as an availability phase. Partition of a particular subsystem's mission is accomplished by defining the availability phase(s) and determining the range of time over which it may vary.

2.1.2 Development of Mission Profiles

When the mission phases have been defined, the operating modes of the subsystem which are applicable to each phase and the performance functions desired of the subsystem in each mode are listed and related to the relevant mission requirements.

Development of a mission profile begins with a definition of operational modes applicable to the subsystem. It may be desirable to omit from the analysis any modes that are not significant with respect to primary mission objectives (e.g., training mode). Each mode is tabulated against the mission phase(s) to which it applies.

Performance functions required of the subsystem in each mode and phase are then listed and associated with the constituent equipments necessary for their accomplishment. A form having the general information content of that illustrated Figure 2-2 is helpful in organizing this portion of the mission analysis. In general, not all of the subsystem's functions will be equally essential to the mission. Thus, it is necessary to define the minimum limits of successful performance, that is, of "up" status, for purposes of availability analysis. This is accomplished by listing that subset of the performance functions that are deemed essential to the primary mission.

Subsystem Mode	Function	Related Equipments	Success Criteria	Function Time	Environment	Environment Time	Maintenance Constraints

Figure 2-2. Development of Mission Profile

Performance times necessary for each of the essential functions are then listed. Where the times are random variables their maximum values may be used. Environmental levels that depart significantly from room ambient are listed, together with maximum times of subsystem performance in those environments. Finally, any constraints imposed by the mission on maintenance activities available to support the subsystem are listed. These include limitations on accessibility or time available for repairs imposed by interfacing systems, weather, enemy actions and so forth. The output of the analysis is a definitive statement of requirements on the subsystem as related to the planned mission.

2.2 SYSTEM ANALYSIS

System analysis defines the means by which the subsystem is able to respond to the requirements of the mission. Here an initial prediction of subsystem availability is made and compared against the subsystem requirement. If the prediction shows that the subsystem falls short of its required availability, specific goals are apportioned to elements of the subsystem against which their future availability progress can be evaluated. Seven steps constitute the analysis:

1. Definition of Subsystem Configuration: The subsystem is represented by means of block diagrams.

2. Definition of Significant Subsystem States: Up and down subsystem states are defined in terms of equipment failures.

3. Formulation of Subsystem Availability Equation: An equation is written for subsystem availability as a function of the availability of the subsystem's equipments.

4. Analysis of Corrective Maintenance Tasks and Prediction of Availability with Respect to Failure: A listing of failure repair functions is made, together with expected frequencies and times required. A_i is predicted.

5. Analysis of Preventive Maintenance Tasks and Prediction of Availability with Respect to Preventive Maintenance: A listing is made of tasks, frequencies, manning requirements and times required for scheduled maintenance. A_p is predicted.

6. Tradeoff Studies and Apportionment of Availability: Availability requirements are apportioned to equipment level.

7. Development of Measurement Data Requirements: A listing is made of test data necessary to solve the subsystem availability equation.

The major outputs of the analysis are a model and requirements against which subsystem availability can be evaluated in the following phases of the program.

2.2.1 Definition of Subsystem Configuration

The functional configuration of the subsystem in each of its operating modes is defined by means of block diagrams, normally one diagram for each mode. In the diagram each block represents an equipment or group of equipments. The directions of functional flows are labeled and inputs and outputs are identified. Thus a block diagram is a graphical representation of the dependence of subsystem performance on the operability of its hardware elements. In addition, an equipment "tree" diagram, based on packaging rather than functional relationships should be supplied, detailing hardware down to and including the component level. Figure 2-3 is a typical subsystem block diagram.

2.2.2 Definition of Significant Subsystem States

If the instantaneous state of an n-element subsystem is defined by the combination of "up" and "down" equipments, then 2^n states are normally realizable. For example, a subsystem consisting of two independent equipments can exist in $2^2 = 4$ states:

State	Condition
1	equipment 1 up, equipment 2 up
2	equipment 1 up, equipment 2 down
3	equipment 1 down, equipment 2 up
4	equipment 1 down, equipment 2 down

If the two equipments are used in series, state 1 is the only subsystem up-state; states 2, 3 and 4 are alike from the subsystem viewpoint in that they are all down-states. But if the equipments are used in parallel, with either equipment alone able to accomplish all subsystem functions, then states 1, 2 and 3 are alike in that they are all subsystem up-states, and state 4 is the only subsystem down-state. In either case there are only two subsystem states – up and down. However, in the more likely event that either equipment alone can perform part of the subsystem functions or can perform all the functions with reduced speed or efficiency, it may be meaningful to define states 2 and 3 as an intermediate subsystem state, which may be up or down depending on the success criteria used in the analysis.

For subsystems of realistic complexity, there may be too many states for convenience in analysis. Therefore, the next step in system analysis is to perform Failure Mode and Effects Analysis (FMEA) in accordance with the procedure set forth in NAVWEPS OD 29304, to define the states of the subsystem which have significant effects on its performance capability, grouping those states which are alike in that regard. For a series-configured subsystem, for example, only 1-failure states and the 0-failure state need be distinguished since any equipment failure fails the subsystem. With each state there is associated a level of functional subsystem capability. These are compared with the success criteria developed by the mission analysis. "Up" and "down" subsystem states are identified. The block diagram can often be simplified to reflect consolidation of analogous states (Figure 2-4).

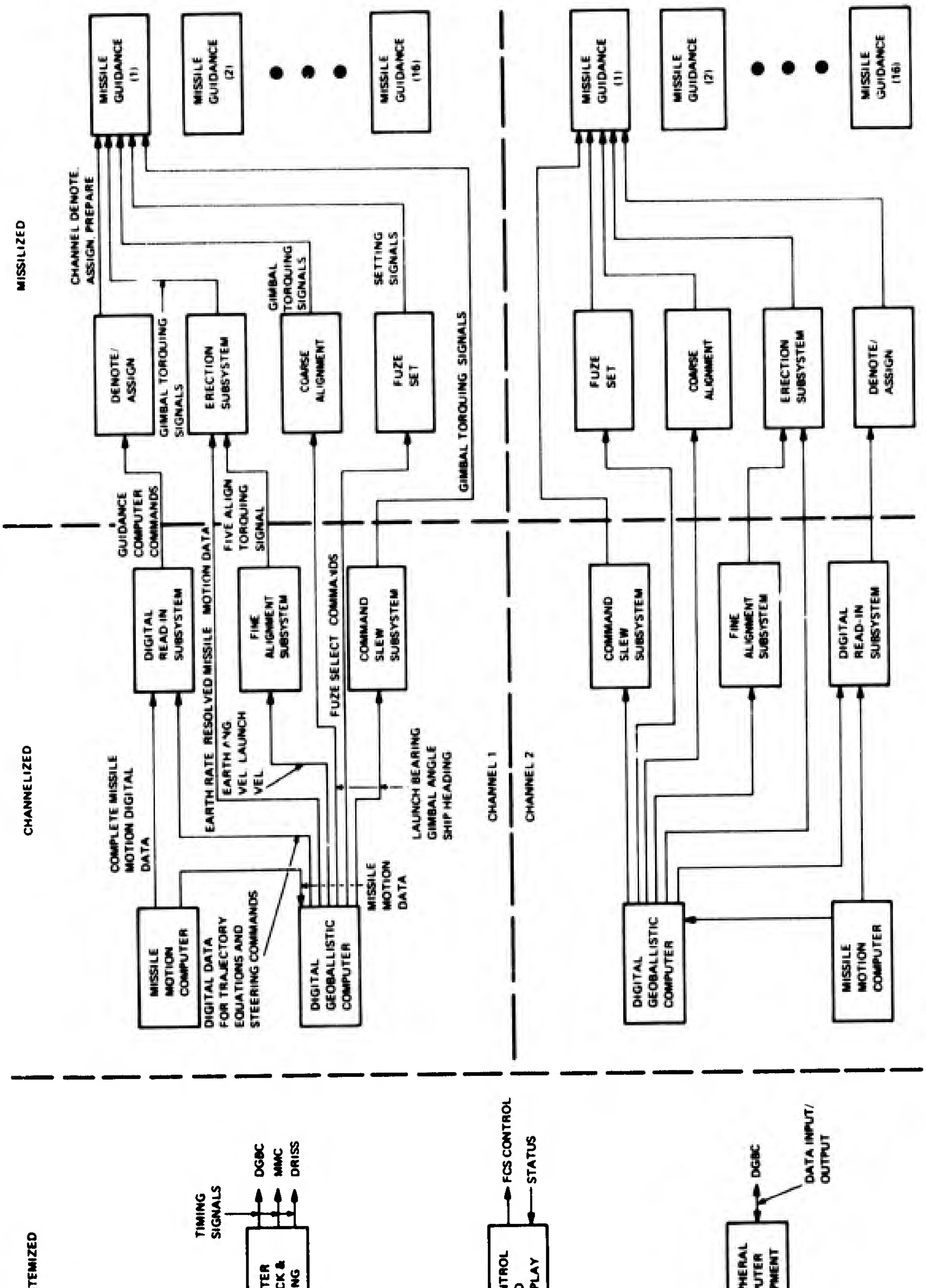


Figure 2-3. Fire Control Subsystem Block Diagram

Even with a consolidated block diagram it is usually not feasible to list all the significant subsystem states. The subsystem of Figure 2-4, for example, can assume 2^{19} states. Suppose, however, that a single level of functional capability is specified for the subsystem, so that the subsystem is up if the systemized equipment block is up, plus either or both of the channelized blocks, plus all 16 of the missilized blocks. Then the analysis need only be concerned with the 0-failure and 1-failure states of the systemized stage, the 0-, 1-, and 2-failure states of the channelized stage, and the 0- and > 0-failure states of the missilized stage. Most of the possible states are grouped in the > 0-failure state of the missilized stage.

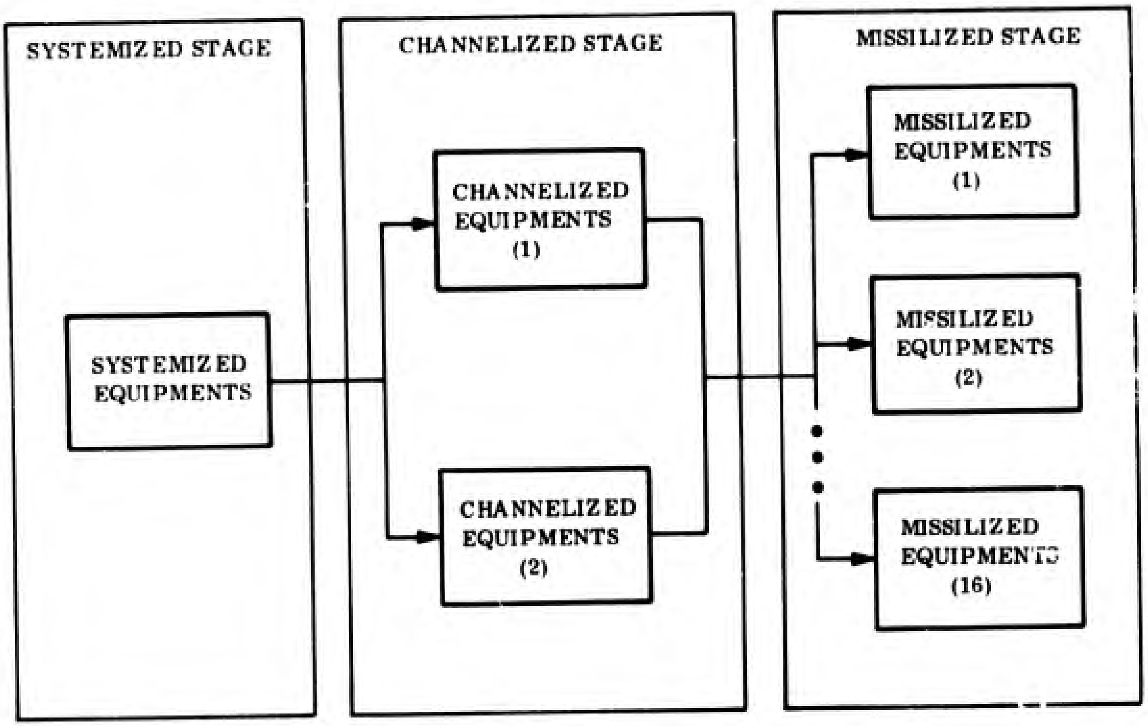


Figure 2-4. Fire Control Subsystem, Reduced Block Diagram

2.2.3 Formulation of Subsystem Availability Equation

From the information developed in the preceding steps, an equation is written expressing the subsystem availability, or probability of being in an up state, as a joint function of the availability of the subsystem's equipment blocks. In general terms it is

$$A = \sum_{k} \left[\prod_{j=1}^{n} a_j^{\delta_{jk}} (1 - a_j)^{(1 - \delta_{jk})} \right]$$

where $\delta_{jk} = 1$ if equipment block j is up in state k, which is a subsystem up-state, and 0 if it is down; a_j is the availability of equipment block j. The subsystem is down when it is undergoing repair or preventive maintenance, or both; it is available only when it is up with respect to both. Then when repair and preventive maintenance are independent, subsystem achieved availability is given by the product of availability with respect to failure A_i, and availability with respect to preventive maintenance A_p.

$$A_a = A_i A_p \tag{2-1}$$

Each of these, in turn, must be represented by an equation embodying appropriate assumptions regarding the nature of the failure and maintenance processes applicable to the subsystem under operational use conditions. For example, it will be shown in the example of paragraph 2.3 that the availability with respect to failure of the subsystem shown in Figure 2-4 can be represented by the equation

$$A_i = \left[\frac{\mu_S}{\lambda_S + \mu_S}\right] \left[\frac{\mu_C^2 + 2\mu_C\lambda_C}{(\lambda_C + \mu_C)^2}\right] \left[\left(\frac{\mu_M}{\lambda_M + \mu_M}\right)^{16}\right]$$

where λ is failure rate, μ is repair rate or reciprocal of MTTR, and subscripts refer to the systemized, channelized and missilized equipment blocks, respectively. An analogous equation can be written for availability with respect to preventive maintenancce A_p using the parameters f and $\overline{M}_p$.

The equation for A_i above embodies and is sensitive to the following conditions:

1. Failure and repair times are exponentially distributed.
2. Failure detection and repair are immediate.
3. Redundant blocks of the subsystem operate in parallel, with all equipments on continuously during the mission.

Any change in those assumptions would necessitate modifying the equation.

In the next two paragraphs (2.2.3.1 and 2.2.3.2), the general problems of writing mathematical models for inherent availability of complex devices are briefly discussed. For the sake of simplicity, A_p is neglected and availability is equated with A_i.

2.2.3.1 The Concept of Availability

It has already been noted that a subsystem is up when it is operating or on standby in a non-failed condition. It is down when it has failed or is undergoing repair, periodic checkout or other maintenance. Availability is defined as the probability that a subsystem will be up when called upon to perform. An appropriate index of availability must reflect the nature and duration of possible missions, whether repair is possible during a mission, the distribution of failure and repair times, of checkouts and preventive maintenance times, as well as the schedules by which these latter functions are regulated. Of many such indices that could be derived, steady-state availability is the one of widest general applicability. The concept can be defined in mathematical terms. Pointwise availability A(t) is defined as the instantaneous probability that a subsystem is up at time t. Interval availability $\bar{A}_T$ is the time average of A(t) during intervals of length T, and is readily obtained from the equation for the average value of a function.

$$\bar{A}_T = \frac{\int_0^T A(t)\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T A(t)\,dt \tag{2-2}$$

Alternately, it may be considered that there is some probability distribution h(t) on demand time. Then

$$\text{availability} = \int_0^\infty A(t)\,h(t)\,dt$$

In the special case where h(t) is uniform on the interval [0, T]

$$h(t) = \begin{cases} \frac{1}{T}, & 0 \le t \le T \\ 0, & t > T \end{cases}$$

and

$$\bar{A}_T = \frac{1}{T}\int_0^T A(t)\,dt$$

The same concept can serve to define A(t). In that case h(t) = δ(t), a spike of unit area at t.

Steady-state availability A, is the limiting interval availability as T → ∞.

$$A = \lim_{T\to\infty} \frac{1}{T}\int_0^T A(t)\,dt \tag{2-3}$$

If the subsystem does not undergo checkout or repair, its interval availability approaches zero. For systems with exponentially distributed failure times where:

$$A(t) = e^{-\lambda t} \tag{2-4}$$

equation (2-2) becomes

$$\bar{A}_T = \frac{1 - e^{-\lambda T}}{\lambda T} \tag{2-5}$$

and $\bar{A}_T \to A \to 0$ as $T \to \infty$. Thus, it can be seen that the steady-state availability or long-term uptime ratio of a subsystem which does not undergo repair approaches zero.

If a subsystem is subject to repair, its steady-state availability is not zero. For illustration, assume that failure detection and initiation of repair are immediate and that both failure and repair times are exponentially distributed with means $1/\lambda$ and $1/\mu$, respectively. The probability of a failure or completion of a repair in a small time interval h can be approximated very closely by λh and μh, respectively. Then

$$A(t+h) = A(t)\,(1-\lambda h) + (1 - A(t))\,\mu h \tag{2-6}$$

Equation (2-6) states that the instantaneous probability of the subsystem being up at t + h is the probability that it was up at time t and that no failure occurred during h, plus the probability that it was down at t and that a repair was effected during h. The equation can be put in differential form by subtracting A(t) from both sides and dividing by h. Then, as h → 0

$$A'(t) = -(\lambda + \mu)\,A(t) + \mu \tag{2-7}$$

This is a first order linear differential equation, having the general solution

$$A(t) = \frac{\mu}{\lambda + \mu}\left[1 - e^{-(\lambda+\mu)t}\right] + A(0)e^{-(\lambda+\mu)t} \tag{2-8}$$

where A(0) is the pointwise availability at some initial time t = 0. If t = 0 is the initiation of the availability phase of the mission, and if A(0) = 1, as is generally assumed, then the pointwise availability at time t is

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}\,e^{-(\lambda + \mu)t} \tag{2-9}$$

It can be seen that equation (2-9) consists of a steady-state term and a transient term, so that after a sufficient period of time has passed, the availability of a subsystem with immediate repair is no longer affected by whether or not the subsystem was operable at t =0.

The interval availability over a period, such as sea patrol of duration T, is given by equation (2-2) employing equation (2-9) for A(t).

$$\bar{A}_T = \frac{1}{T}\int_0^T \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}\right] dt$$

$$= \frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 T}\left[1 - e^{-(\lambda+\mu)T}\right] \qquad (2\text{-}10)$$

Again the expression includes a steady-state term and a transient term reflecting the initial state of the subsystem at t = 0. It can be seen that as T is increased, the interval availability approaches a constant value, the steady-state availability, which is written

$$A = \lim_{T\to\infty} \bar{A}_T = \frac{\mu}{\lambda+\mu} \qquad (2\text{-}11)$$

For missions of sufficient length, the transient term is ignored and steady-state availability is taken as the applicable measure. Throughout this manual it is assumed that the relevant mission is long enough to justify the use of steady-state availability. Any error stemming from this assumption will tend to render the analysis conservative; that is, interval availability is always greater than steady-state availability.

For many subsystems, immediate failure detection and repair are not achievable under operational use conditions. Subsystems checked out only at intervals can be repaired only at intervals. Thus, it is necessary to account for undetected downtime.

$$A = \frac{\text{(expected time non-failed in T)}}{\text{(duration of T) plus (time down in checkout and repair)}} \qquad (2\text{-}12)$$

A common situation is where the length of the interval between checkouts depends on the outcome of the preceding checkout. Each checkout is assumed to superimpose on the subsystem an additional probability of failure q, over that generated by the standing failure rate. If a checkout shows an up condition, the next interval is T. This happens with probability $(1 - q)\, e^{-\lambda T}$. But if the checkout shows that the system is down, the next interval is $T + 1/\mu$. This occurs with the complementary propability $1 - (1 - q)\, e^{-\lambda T}$. These factors can be manipulated to give the availability expression

$$A = \frac{1 - e^{-\lambda T}}{\lambda T + \frac{\lambda}{\mu}\left[1 - (1 - q)\, e^{-\lambda T}\right]} \tag{2-13}$$

which is analogous to equation (2-12), the denominator reflecting the average interval length. Equation (2-13) holds when checkout time is negligible compared to the checkout interval. It can be seen that for a periodically checked and maintained subsystem, A depends on the length of the checkout interval.

2.2.3.2 Availability of Redundant Subsystems

The availability of subsystems comprised of multiple equipments arranged in series, parallel or standby redundant configurations can be represented by relatively straightforward mathematics.

A subsystem consisting of n equipments in series is available when all n equipments are available. Thus, if

$$a_i = \frac{\mu_i}{\lambda_i + \mu_i} \tag{2-14}$$

is the availability of the ith equipment, and if all equipments fail and are repaired independently, the availability of the subsystem is

$$A = \prod_{i=1}^{n} a_i \tag{2-15}$$

The availability of a subsystem of n identical equipments in parallel, and r of which are required to be up for the subsystem to be up, is given by

$$A = \sum_{m=0}^{m=n-r} \binom{n}{m} a^{n-m} (1-a)^m \tag{2-16}$$

$$= \sum_{m=0}^{m=n-r} \binom{n}{m} \frac{\mu^{n-m} \lambda^m}{(\lambda + \mu)^n} \tag{2-16}$$

where

$$\binom{n}{m} = n!/[(n-m)!\, m!]$$

Equation (2-16) applies to the parallel case where all n equipments fail and are repaired independently, if they are repaired at all, without queueing. All units operate continuously while they are up.

It may happen that the functional load on a parallel redundant subsystem is a random variable with known or postulated probability distribution.* For example, the data processing capacity required to handle a variable-demand function, such as aerospace surveillance, or military command and control, might vary from 0.75 to 1.50 times the capacity of a single processor unit, with a distribution such as that shown in Figure 2-5.

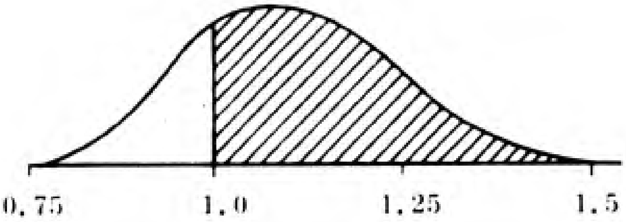


Figure 2-5. Distribution of System Loading

The shaded area represents the fraction of time when two units are necessary. The remaining time a single processor will suffice. Thus, the long-term availability, ignoring switching transients, is given by the Theorem of Total Probability:

A = (probability 2 units required) · (availability of at least 2 units)

plus (probability 1 unit required) · (availability of at least 1 unit)

The concept may be generalized by introducing the following notation.

A_i = availability of function (i.e., probability that the system can perform function i)

$P_i(a_j, b_k, c_l \ldots)$ = fraction of time that function i can be handled by j equipments of a type a plus k equipments of type b, plus l equipments of type c, ... (but no less)

$A(a_j, b_k, c_l \ldots)$ = availability of at least j equipments of type a, k equipments of type b, etc. (i.e., probability that at least the minimum required equipment configuration is available).

Note that

$$\sum_{j,k,l,\ldots} P_i\,(a_j,\ b_k,\ c_l\ \ldots) = 1.0 \qquad (2\text{-}17)$$

**This problem was studied by L. Greenberg, whose approach is followed here.*

and

$$A_i = \sum_{j,k,l,\ldots} P_i\,(a_j, b_k, c_l \ldots) \cdot A\,(a_j, b_k, c_l \ldots) \qquad (2\text{-}18)$$

For example, suppose a particular function can be handled by the subsystem configurations shown with the probabilities shown in Table 2-1.

TABLE 2-1. POSSIBLE CONFIGURATIONS

System Configuration			Probability that configuration is minimum necessary to handle function
Processors	**Memories**	**Input/Output**	
$1=a_1$	$1=b_1$	$1=c_1$	0
$1=a_1$	$2=b_2$	$1=c_1$	$0.5 = P(a_1, b_2, c_1)$
$1=a_1$	$3=b_3$	$1=c_1$	0
$2=a_2$	$1=b_1$	$1=c_1$	0
$2=a_2$	$2=b_2$	$1=c_1$	$0.4 = (a_2, b_2, c_1)$
$2=a_2$	$3=b_3$	$1=c_1$	0
$2=a_2$	$3=b_3$	$2=c_2$	$0.1 = P(a_2, b_3, c_2)$

The table shows that 50 percent of the time 1 processor, 2 memories, and 1 input/output unit are needed: 40 percent of the time 2 processors, 2 memories, and 1 input/output are needed; and the remaining 10 percent of the time 2 processors, 3 memories, and 2 input/outputs are needed. This last situation represents worst-case loading conditions.

Assume now that all equipments have an identical availability a = 0.99 and the sytem consists of 2 processors, 3 memories and 2 input/output units.

Then,

$$A(a_1, b_2, c_1) = \left(1-(1-a)^2\right) \cdot \left(a^3 + 3a^2\,(1-a)\right) \cdot \left(1-(1-a)^2\right)$$

$$= .9999 \times .9997 \times .9999 = .9995$$

$$A(a_2, b_2, c_1) = \left(a^2\right) \cdot \left(a^3 + 3a^2\,(1-a)\right) \cdot \left(1-(1-a)^2\right)$$

$$= .9801 \times .9997 \times .9999 = .9797$$

$$A(a_2, b_3, c_2) = (a^2) \cdot (a^3) \cdot (a^2)$$

$$= .9801 \times .9703 \times .9801 = .9321$$

and

$$A = .5\,(.9995) + .4\,(.9797) + .1\,(.9321) = .9848$$

Note that this is considerably higher than the estimate of .9321 that would be obtained if only the worst-case loading conditions were reflected in the model.

2.2.4 Analysis of Corrective Maintenance Tasks and Prediction of Availability with Respect to Failure

Upon completion of the modeling task, a listing is made of corrective maintenance functions that can arise out of failures in each of the equipment blocks, together with estimates of their relative frequencies and repair times. A form such as Figure 2-6 can be used to expedite the analysis. The sum of the partial failure rates is a prediction of the failure rate for the equipment block. The sum of the $\lambda \overline{M}_c$ column divided by the block failure rate is a prediction of the weighted average repair time. The final column is an approximation of the availability or fractional uptime of the equipment block with respect to failure (A_i).

The prediction may be based on any of the procedures of MIL-HDBK 217, MIL-STD-756 and/or MIL-HDBK-472. Source data may be based on historical experience, subjective evaluation, expert judgement or direct measurement of reliability and maintainability characteristics of elements of the subsystem. Or the contractor may elect to use a non-standard method specifically applicable to the type of hardware comprising the subsystem, subject to prior approval by the SSPO.

Combinational rules for developing subsystem parameters from those of lower indenture items depend on the usual assumptions of statistical independence and exponential behavior.

The failure rate of n items in series is given by

$$\lambda = \sum_{i=1}^{n} \lambda_i \tag{2-19}$$

The failure rate of n items in parallel is not constant but is a function of mission time t. If the items each have the same failure rate λ_i and the expectation $\lambda_i t$ is small, the approximation

$$e^{-\lambda t} \approx 1 - \lambda t, \qquad 0 < \lambda t < .1$$

Subsystem:								
Equipment Block	Component	Predicted Partial Failure Rate λ	Recovery Action		Predicted MTTR $\bar{M}_c$	$\lambda\ \bar{M}_c$	$U_i = \dfrac{\bar{M}_c}{(\Sigma\lambda_i)^{-1} + \bar{M}_c}$	$A_i = 1 - U_i$
			Repair	Replace				
equip A	Component 1	λ_{A_1}	X		$\bar{M}_{c_{A_1}}$	$\lambda_{A_1}\ \bar{M}_{c_{A_1}}$		
	Component 2	λ_{A_2}	X		$\bar{M}_{c_{A_2}}$	$\lambda_{A_2}\ \bar{M}_{c_{A_2}}$		
	Component 3	λA_3		X	$\bar{M}_{c_{A_3}}$	$\lambda_{A_3}\ \bar{M}_{c_{A_3}}$		
		$\Sigma\lambda_{A_i}$			$\bar{M}_{c_A} = \Sigma(\lambda_{A_i}\bar{M}_{c_{A_i}})/\Sigma\lambda_{A_i}$	$\Sigma(\lambda_{A_i}\bar{M}_{c_{A_i}})$		
equip B	Component 1	λ_{B_1}		X	$\bar{M}_{c_{B_1}}$	$\lambda_{B_1}\ \bar{M}_{c_{B_1}}$		
	Component 2	λ_{B_2}		X	$\bar{M}_{c_{B_2}}$	$\lambda_{B_2}\ \bar{M}_{c_{B_2}}$		
		$\Sigma\lambda_{B_i}$			$\bar{M}_{c_B} = \Sigma(\lambda_{B_i}\bar{M}_{c_{B_i}})/\Sigma\lambda_{B_i}$	$\Sigma(\lambda_{B_i}\bar{M}_{c_{B_i}})$		
		$\Sigma\lambda_i$			$\bar{M}_c = \Sigma(\lambda_i\bar{M}_{c_i})/\Sigma\lambda_i$	$\Sigma(\lambda_i\bar{M}_{c_i})$	.00XX	.XXXX

Figure 2-6. Analysis of Corrective Maintenance Tasks

can be employed to derive the approximate failure rate.

$$R = 1 - (1 - e^{-\lambda_i t})^n \approx 1 - \lambda t \approx 1 - (\lambda_i t)^n \qquad (2\text{-}20)$$

$$\lambda \approx \frac{(\lambda_i t)^n}{t}$$

If λ is normalized (expressed in units of failures per mission) per NAVWEPS OD 29304, equation (2-20) becomes simply

$$\lambda = \lambda_i^n \qquad (2\text{-}21)$$

Equations (2-20) and (2-21) hold for the situation where repair is made only after all redundant items have failed. MTTR ($\overline{M}_c$) is found as an average of mean repair times associated with the associated components, weighted for the relative failure rate of each component.

$$\overline{M}_c = \frac{\sum_{i=1}^{n} (\lambda_i \overline{M}_{c_i})}{\sum_{i=1}^{n} \lambda_i} \qquad (2\text{-}22)$$

2.2.5 Analysis of Preventive Maintenance Tasks and Prediction of Availability with Respect to Preventive Maintenance

A listing is made of preventive and other scheduled maintenance functions necessary during the availability phase, together with their frequencies and manning requirements. Only those functions that necessitate system downtime for their accomplishment need be included in the analysis. Where the performance times of preventive maintenance tasks are constant or nearly constant quantities, as is often the case, these times are listed if known. A form such as Figure 2-7 can be used for the listing. The right hand column of the form gives a prediction of fractional uptime with respect to preventive maintenance (A_p). The prediction procedure is identical to that for A_i.

When multiple preventive maintenance functions are performed together at the same intervals and during a single continuous period of downtime, they should always be considered as a single maintenance action for purposes of analysis. This is true whether the actions are performed serially or concurrently by the maintenance personnel. For example, if an equipment block requires three preventive maintenance tasks which require

Subsystem:

Equipment Block	Scheduled Maintenance Function	Frequency of Maintenance f $(\text{Hrs})^{-1}$	Predicted Mean Maint. Time $\bar{M}_p$	$f\,\bar{M}_p$	$U_p = \dfrac{\bar{M}_p}{(\Sigma f_i)^{-1} + \bar{M}_p}$	$A_p = 1 - U_p$
equip A	function 1	f_{A_1}	$\bar{M}_{p_{A_1}}$	$f_{A_1}\ \bar{M}_{p\,A_1}$		
	function 2	f_{A_2}	$\bar{M}_{p_{A_2}}$	$f_{A_2}\ \bar{M}_{p_{A_2}}$		
		$\Sigma\, f_{A_i}$	$\bar{M}_{p_A} = \Sigma(f_{A_i} \bar{M}_{p_{A_i}})/\Sigma f_{A_i}$	$\Sigma\,(f_{A_i} \bar{M}_{p_{A_i}})$		
equip B	function 1	f_{B_1}	$\bar{M}_{p_{B_1}}$	$f_{B_1}\ \bar{M}_{p_{B_1}}$		
	function 2	f_{B_2}	$\bar{M}_{p_{B_2}}$	$f_{B_2}\ \bar{M}_{p_{B_2}}$		
	function 3	f_{B_3}	$\bar{M}_{p_{B_3}}$	$f_{B_3}\ \bar{M}_{p_{B_3}}$		
		$\Sigma\, f_{B_i}$	$\bar{M}_{p_B} = \Sigma(f_{B_i} \bar{M}_{p_{B_i}})/\Sigma f_{B_i}$	$\Sigma\,(f_{B_i} \bar{M}_{p_{B_i}})$		
		$\Sigma\, f_i$	$\bar{M}_p = \Sigma\,(f_i \bar{M}_{p_i})/\Sigma\, f_i$	$\Sigma\, f_i\ \bar{M}_{p_i}$	.00XX	.XXXX

Figure 2-7. *Analysis of Preventive Maintenance Tasks*

maintenance downtimes of one hour, two hours and three hours respectively, and are performed together at 100-hour intervals during the mission, the tasks are considered as a single maintenance action requiring six hours if done serially or three hours if done concurrently.

2.2.6 Tradeoff Studies and Apportionment for Availability

Subsystem availability is a joint function of the availability of the subsystem's hardware items. Each of these, in turn, is a joint function of the reliability and maintainability of the item, which can combine in various proportions to yield a given level of element availability. In its initial configuration the subsystem will possess certain inherent levels of reliability and maintainability which together establish an upper limit on the availability attainable by the subsystem. Beyond this limit, additional effort can be applied to develop either more reliability or more maintainability or both, in order to increase availability. Figure 2-8 illustrates the dependence of availability on reliability and maintainability. Specification of an availability requirement defines an iso-availability curve on the response surface of Figure 2-8. All points on and above that curve comprise an admissible trade-off region between reliability and maintainability, either of which can be, in some degree, substituted for the other. A variety of criteria can form the basis for such a trade-off. One method considers the marginal costs of improving reliability and maintainability. The example in 2-3 includes an illustration of such a trade-off.

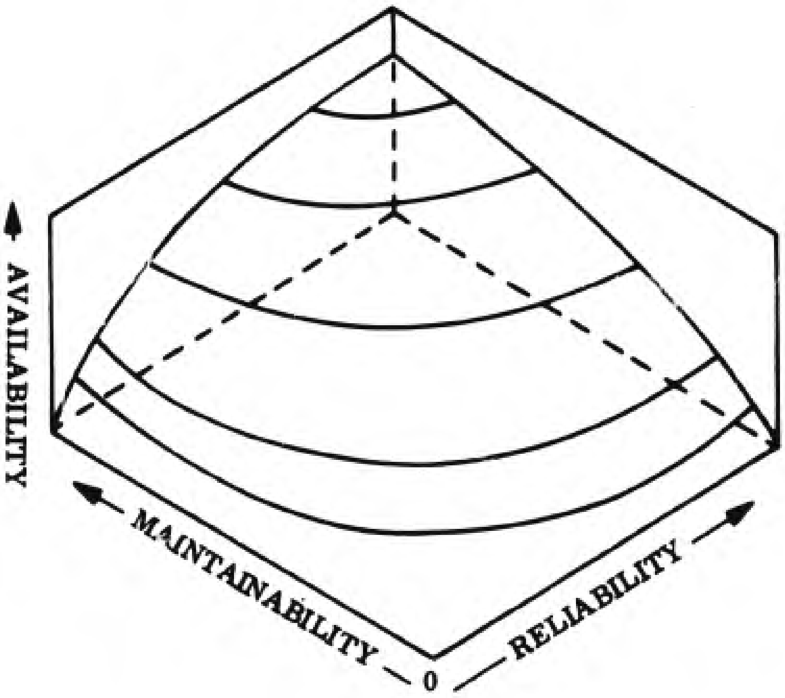


Figure 2-8. Availability Response Surface (after Goldman and Slattery)

When the trade-off has been made, as the final step in system analysis, availability requirements are apportioned to equipments of the subsystem in some consistent and logical manner. The objective of apportionment is to provide goals against which the availability growth of the subsystem's hardware elements can be measured, and to provide designers with goals for reliability and maintainability. A procedure for reliability apportionment, considering the factors of complexity, state-of-the-art, duty cycle, and criticality, is given in NAVWEPS OD 29304. Apportionment of maintainability can also be done in that manner, based on one or more of those same factors or based on considerations of location, packaging or physical configuration of the subsystem. It must be remembered, however, that, unlike reliability, maintainability requirements cannot logically be apportioned to hardware levels below the lowest indentured items specified as repairable on-site under the users' maintenance policies.

At the time trade-off studies are completed, specifications containing tentative reliability and maintainability requirements, determined subjectively, may already have been written for many equipments. Specifications prepared in that manner define a non-optimized subsystem because, by implementing the results of the trade-offs, the same availability may be achieved with reduced expenditure of effort or more availability may be gained for the same effort. Thus, when a contractor apportions reliability and maintainability goals, in effect he apportions resources and effort as well. The purpose of apportionment is to permit equipment requirements to be refined objectively, so that the subsystem requirements can be realized in a timely and economical manner. If prediction indicates that the subsystem will not meet its apportioned availability and/or reliability requirements, then reapportionment is necessary to guide the design effort.* Even if prediction indicates that the subsystem can be expected to meet or exceed the requirements, reapportionment may remain a desirable means toward optimization.**

Maintainability apportionment can be simplified in cases where it can be shown that peripheral conditions unique to the operational environment, such as access limitations or the availability of diagnostic equipment, are the principal factors acting to determine maintenance time. In many such instances, the variable element of maintenance time is tightly distributed about a central value fixed by factors such as those noted above which are outside the contractor's control. Where the dispersion is a negligible fraction of the central value, maintenance time may be treated as a constant for analytical purposes. If the quantity of preventive maintenance and the downtime necessitated by the need for preventive maintenance are significant functions of subsystem design, these factors contribute additional degrees of freedom to the apportionment task.

2.2.7 Development of Measurement Data Requirements

The availability equation implicitly defines the data necessary for its solution. An initial solution can be based on predictions, but before testing begins, it is necessary to define the

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**Blanchard, B. and Lowery, E., Maintainability Principles and Practices, McGraw-Hill (1969) pp 109-115.*

***NAVORD OD 39223, Maintainability Engineering Handbook, Naval Ordnance Systems Command, June 1969, Ch. 3.*

test and operating data that will be needed for availability measurement. These must be tabulated against the applicable equipments. In general terms, the data required for each equipment include:

Equipment Identification – nomenclature and serial number.

Operating Time – cumulative operating time in each relevant mode and condition.

Downtime – occurrence and performance time of all corrective and preventive maintenance tasks that entail downtime. Also, all modification and delay time.

Failure Information – indication of failure occurrence and failed item, operating time at failure, mode and cause of failure (to satisfy classification criteria).

Exceptions or additions to the general data requirements may be dictated by the structure of the equation. For example, non-operating time may be required in addition to operating time and downtime. The data collection requirements should reflect the need for data to evaluate each of the independent variables of the equation. The completed requirements list can serve as a basis for designing and preprinting data collection forms and preparing computer programs for data processing. Whenever possible, it is desirable that the data collection activity provide all components of each equipment's time history, e.g., fault correction time rather than merely corrective maintenance time. This objective is normally best accomplished by the use of operating logs and automatic elapsed time indicators (ETI). However, it is recognized that in certain industrial testing activities, satisfaction of a requirement for accurate operating and maintenance times may entail extensive modification or expansion of current procedures. When there are valid reasons why a contractor can not meet such a requirement, the SSPO may approve availability measurement based on less comprehensive data collection, or on approximation of operating times accumulated in a test or series of tests by analysis of test specifications, procedures, data sheets and reports. Subjective measurement of time entails many obvious risks of error, including the tendency of observers to round-off times to the nearest hour, half hour or quarter hour. This creates local modes at these points when times are plotted in histogram form. Observers may also tend to inflate estimates of active time intervals to cover elapsed calendar time, while reducing or eliminating reported delays and inactive periods.

2.2.7.1 Classification of Failures

Criteria for failure inclusion in reliability calculations depend on the cause of failure. In general, failures caused by design defects, manufacturing defects, or random part or material failure are relevant and should be included; failures caused by accidents, mishandling, test errors, test equipment failures, specification errors or exposure to non-mission environments are non-relevant and should be excluded. The essential test of relevance is whether a particular failure is the result of a hardware deficiency or the result of some extraneous influence.

Several types of failures warrant special consideration in determining relevance. Prior policies should be established covering classification of failures of minor consequence, such as blown fuzes, tripped circuit breakers, lamp failures and failures of elapsed-time indicators. This category might also include missing or wrong parts, where no tools, or only a simple hand tool such as a screwdriver, are required for installation.

The criteria of relevance for such failures may quite properly be made specific to the subsystem under test. A permissible general rule, however, is to exclude such failures when they do not cause additional failure effects in the subsystem.

Classification of failures discovered during periodic inspection or other scheduled maintenance can be based on whether the discovery is spontaneous, thus not referable to a failure during system operation, or represents corrective maintenance deferred to a scheduled maintenance period for reasons of convenience. The latter case is properly included as relevant; the former may or may not be included depending on the nature of the subsystem and its operating requirements.

Design deficiencies, thought to be corrected by hardware modification, require a subsequent period of satisfactory operation to verify the effectiveness of the modifications. The duration of the period should be sufficient to provide reasonable assurance that the failure mode has indeed been eliminated. If the failure mode does not recur during the verification period, failures in the subject mode may be excluded for measurement purposes. Should the failure mode recur later, those failures may again be included. The theoretical basis for such a policy is the assumption that subsystem failure rate is the sum of partial failure rates associated with specific failure modes (Ref. NAVWEPS OD 29304). Elimination of a mode reduces the subsystem failure rate by an amount equal to the partial rate for that mode. It should be noted that it is not correct to exclude operating time selectively from a system's history. If the modifications are so extensive that the hardware may be viewed as totally new, the entire prior history must be excluded and availability measurement begun anew.

Failures may occur for which a positive determination of cause cannot be made. In these cases certain assumptions may be justified and of value in making decisions of relevance or non-relevance. If a subsystem experiences repeated occurrences of the same failure mode within short intervals, a question may arise as to whether they are separate, randomly distributed, relevant failures, or whether previous diagnosis and corrective maintenance actions were ineffective, causing repeated iterations of the same failure. In the latter case, only the first occurrence should be considered as relevant, subsequent events being secondary non-relevant failures. In practice, there is often no definite answer to this problem. However, judgement indicates that the more closely similar failures are spaced, the more likely it is that the latter ones are secondary failures. A decision rule for such cases can be derived by computing the interval within which the probability of observing two or more failures is equal to or less than a chosen significance level. This is easily done using the subsystem's predicted or measured failure rate in the expression for Poisson expectation. Failures recurring within the limiting interval may then be excluded as secondary events with a risk of error equal to the significance level.

Broken wires, bent connector pins, and several similar types of failures result from human errors. The question of whether they were incipient in the hardware from its manufacture (relevant) or were introduced into it during maintenance (non-relevant) is often unanswerable. A useful guideline is to include such failures as relevant manufacturing defects until sufficient operating time has been accumulated to achieve a constant failure rate, the flat portion of the so-called "bathtub curve". In the flat region of the curve, the subsystem may be assumed to be fully debugged. Subsequent human errors can be excluded as maintenance induced, unless there is evidence to the contrary. Exceptions might include cold-solder and no-solder joints, which can be classified as manufacturing defects regardless of when they are discovered, unless there is evidence relating them to faulty maintenance. Other doubtful categories of malfunctions where classification policies are desirable include transient symptoms, which cannot be confirmed as failures, failures of undetermined cause that disappear without corrective action, and failures that are resolved by actions of a general nature with the specific causes not isolated.

Generally, it is not possible to write a set of totally objective classification criteria; thus provisions should be made for the application of informed judgements, where necessary, and for suitable resolution of contested decisions.

A typical set of failure classification categories might be shown below.

Code	Classification Category
RD	Relevant design defect
RM	Relevant manufacturing defect
RP	Relevant part defect
RU	Relevant unknown (corrective action attempted)
NH	Non-relevant human error (accident, mis-handling, installation error or maintenance induced)
NO	Non-relevant operator error
NS	Non-relevant specification procedure or other software error
NF	Non-relevant facility or test equipment failure
ND	Non-relevant dependent failure (due to failure of another part)
NM	Non-relevant missing part
NP	Non-relevant detected and repaired during preventive maintenance
NR	Non-relevant secondary (recurring) failure
NU	Non-relevant unknown (transient with no corrective action taken)

2.2.7.2 Classification of Time Elements

Figure 2-9, following MIL-STD-721, depicts a hierarchy of time categories applicable to hardware in operational use. But the time history of an item in testing seldom falls neatly into the same set of categories. Rather, the test record inevitably includes effects of the test environment as well as those of the hardware. Thus, it will include failures attributable to external factors such as accidents, mishandling, human errors and specification errors. Downtime experienced by a subsystem will include logistic and administrative delays not related to hardware design or manufacture, and not to be expected in fleet service. Also, the test phase may differ radically from fleet service in the amount of operating time accrued in partially active modes when some circuitry is inactive, or at a reduced functional capability due to partially-disabling failures. The test phase may give rise to some nonproductive operation of a subsystem, to integrate interfacing equipments, debug software, overcome test equipment problems, operator errors and similar problems associated with testing. Also, operating time necessary in the performance of preventive maintenance, retrofits and modifications, and to verify repairs is actually downtime even though the hardware involved may be operable.

Events such as those described above, occurring in the testing of a subsystem, tend to obscure the subsystem's inherent availability and performance qualities and render its evaluation more difficult. Appropriate classification criteria must therefore be applied to all time periods in the observed testing and maintenance history, so that effects unique to the testing and maintenance environment can be identified and the subsystem evaluated in a logical and consistent manner, dependent on hardware properties for which the contractor is responsible, and relatable to the use and maintenance environment expected in fleet service.

Confusion can easily develop over the classification of various periods of time in the testing and maintenance cycle. In some instances, it may be difficult to distinguish between preparation time or item obtainment time, which are included in maintenance time, and delay time which should not be included. It may be difficult to decide whether an item, operating at reduced capacity due to a partial failure, is up or down with respect to its minimum mission success criteria. Preventive maintenance performed during off-duty periods, or while the hardware remains up, is not normally considered in availability calculations, but exceptions to this can occur. The decision rules necessary to classify periods in the test history of a particular item should be based on the design mission. For each state realizable in testing, it must be decided whether the item is up or down with respect to the design mission, or whether it is in a category of use that has no corollary in the design mission and must therefore be excluded entirely from availability calculations. For example, an equipment is usually thought of as up or available when not in use. Thus, power-off time is normally included as uptime when it is not caused by maintenance, modification or delay. But, if the equipment has power on continuously when the submarine is on patrol, any off-duty periods logged during testing, which cannot be ascribed to one of the downtime categories, must be excluded from the availability time base. In such a case, non-use of the equipment is an inadmissible mode not related to the mission. Another example might be use of a subsystem embodying data processing hardware to debug software and operating procedures, a use that would not occur on patrol.

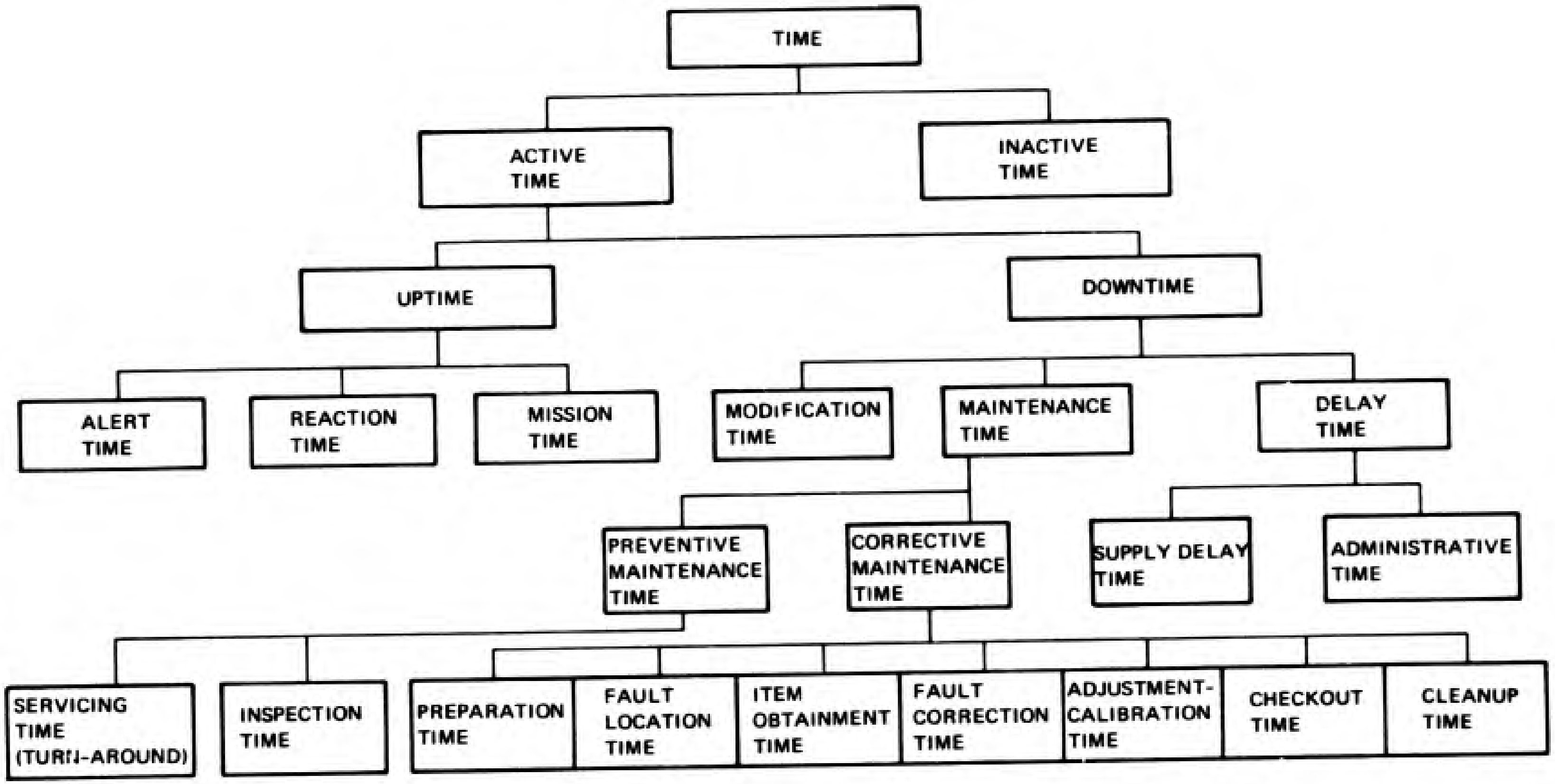


Figure 2-9. Time Relationships

The contractor must develop ground rules, as complete and comprehensive as possible, for classifying time elements observed in tests to be employed for availability evaluation. These rules should be consistent with the definitions given herein and with the needs of the availability equation.

2.2.7.3 Inherent, Achieved and Operational Availability

Inherent availability and achieved availability are intrinsic hardware characteristics. They are denoted by the symbols A_i and A_a, respectively. Inherent availability is estimated by the ratio of total operating time to the sum of total operating time plus total corrective maintenance time. Achieved availability is estimated by the ratio of total operating time to the sum of total operating time plus total maintenance time.

The ratio directly observed in testing or fleet use is termed operational availability and denoted by the symbol A_o. It is estimated by the ratio of total uptime to total active time. It depends on facility characteristics as well as the hardware and can never be greater than A_a.

The subscripts can be extended to the parameters that are used to estimate availability. Thus,

$$\hat{A}_i = \frac{\hat{\lambda}_i^{-1}}{\hat{\lambda}_i^{-1} + \hat{\bar{M}}_{c_i}} \qquad (2\text{-}23)$$

$$\hat{A}_a = \frac{\hat{\lambda}_i^{-1}}{\hat{\lambda}_i^{-1} + \hat{\bar{M}}_{c_i}} \quad \frac{f_i^{-1}}{f_i^{-1} + \hat{\bar{M}}_{p_i}} \qquad (2\text{-}24)$$

and

$$A_o = \frac{U}{U + D} \qquad (2\text{-}25)$$

where

U = total observed uptime

$\hat{\lambda}_i$ = estimated inherent failure rate, computed using relevant failures.

D = total observed downtime

$\hat{\bar{M}}_{c_i}$ = estimated mean repair time

$\hat{\bar{M}}_{p_i}$ = estimated mean preventive maintenance time

f_i = specified preventive maintenance frequency

Note that A_O is observed, not estimated. It is relevant only to the facility in which it is observed. The parameters are estimated during testing as shown below. Preventive maintenance frequency should be established by specification.

$$\lambda_i = \frac{\text{total relevant failures}}{\text{total operating time}} \tag{2-26}$$

$$\bar{M}_{p_i} = \frac{\text{total preventive maintenance time}}{\text{total number of preventive maintenance periods}} \tag{2-27}$$

$$\bar{M}_{c_i} = \frac{\text{total corrective maintenance time}}{\text{total number of failures}} \tag{2-28}$$

Whether any or all non-relevant failures should be excluded from the computation of $\bar{M}_{c_i}$ is another of the questions requiring decision rules specific to the hardware under evaluation. In general, non-relevant failures furnish valid repair time data provided the failure modes can realistically be expected in the operational phase.

2.3 EXAMPLE

This section presents an analysis of a Fire Control Subsystem such as the one described in Figure 2-4 as an example. The analysis is simplified for the sake of brevity and the parameters used are not those of the FBM Fire Control Subsystem. However, the procedures employed in each step are illustrated.

2.3.1 Mission Analysis

The mission consists of two availability phases, designated (a) and (b), and a launch phase. Subsystem functions in phase (a) are limited to monitoring and regulation of temperature and electrical power in each of 16 missile guidance subsystems. Maximum duration of phase (a) is 4000 hours. Phase (b) functions include those of phase (a) plus functions necessary to control the assignment and erection of the missiles. The subsystem is defined as fully up if it can perform the phase (b) functions and initiate the launch phase for all 16 of the missiles. Maximum duration of Phase (b) is 1500 hours. Functions during the launch phase are those required to control the preparation and firing of the missiles. The launch phase has a maximum length of 3 hours, during which no failures are permitted. Minimum acceptable availability is 0.85 in phase (a) and 0.99 in phase (b). A reliability requirement of 0.95 applies to the launch phase. The subsystem can function in several modes; however, the illustration will be limited to the tactical mode and to availability phase (b). Figure 2-10 shows the development of the mission profile.

2.3.2 System Analysis

The functional configuration is shown in Figure 2-4 and is repeated in Figure 2-11. All equipment blocks operate, fail and are repaired independently. The minimum subsystem configuration for functional capability is one S, one C and 16 M blocks.

Subsystem: Fire Control

Mission Phase	Mode	Function	Related Equipments	Success Criteria	Function Time	Env.	Env. Time	Maintenance Constraints
Availability (a)	Standby	1. Temperature regulation	Temp. monitoring power supplies	Repairable failures permitted	4000 hrs	Ambient	4000 hrs	Immediate failure detection and repair
		2. Power to guidance			4000 hrs	Ambient	4000 hrs	
Availability (b)	Tactical	1. Temperature regulation	Temp. monitoring power supplies	Repairable failures permitted	1500 hrs	Ambient	1500 hrs	DCC can be down for maintenance
		2. Power to guidance						
		3. Erect and coarse align all guidance systems	16 erection units 8 coarse alignment units Missile motion computers DGBC, control console					
		4. Denote and assign missiles to each channel	Missile select sw.					
		5. Self-check	Control console ITOP MTRE Module test set	Self-check not essential				
Launch	Tactical	1. Compute range and bearing data and fuze settings for each target	DGBC Fuze set MMC	No failures	3 hrs	Ambient	3 hrs	
		2. Supply digital trajectory parameters	DGBC Digital read-in SS					
		3. Fine alignment of missiles guidance systems	Optical alignment group Servo group SS					
		4. Transmit launch signals to launcher systems	Control and display DRISS Missile select sw.					
Flight	N/A	N/A						

Figure 2-10. Development of Mission Profile

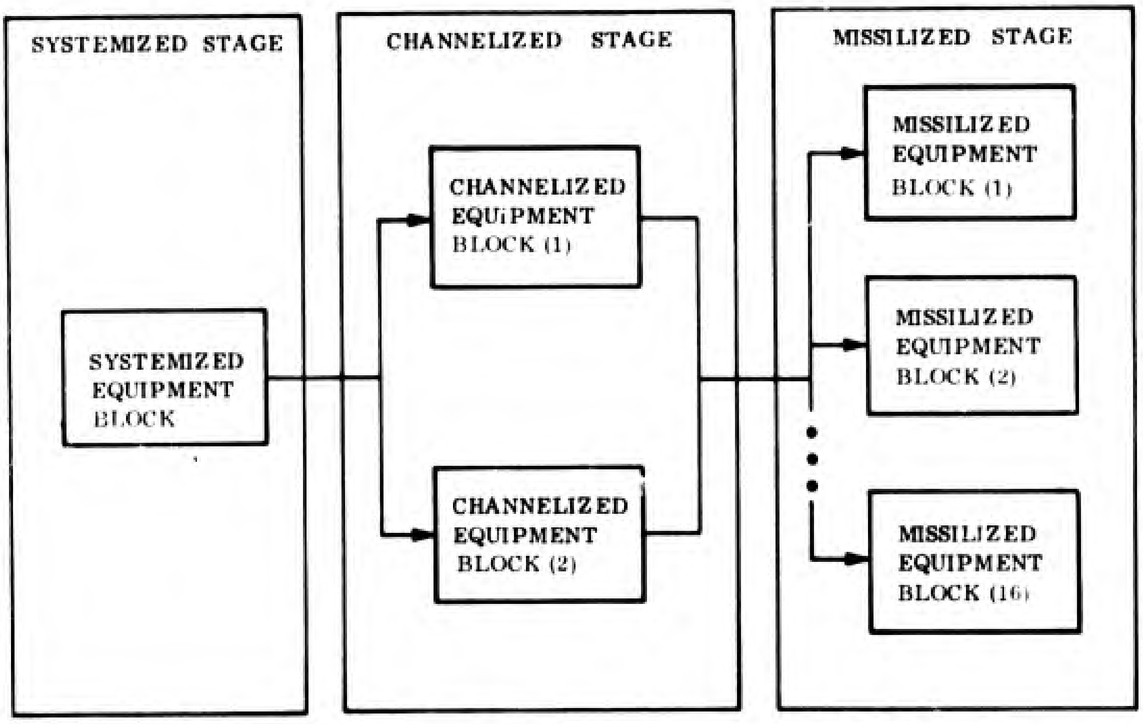


Figure 2-11. Subsystem Block Diagram

Corrective and preventive maintenance functions are analyzed in Figures 2-12 and 2-13. The 500-hour (f = .002) preventive maintenance tasks of Figure 2-13 are considered as a single task performed serially. If they were performed concurrently, the mean maintenance time of the S block would be 0.5 hour rather than 1.0 hour.

The equation for subsystem availability is derived in accordance with the principles previously set forth. The subsystem consists of three independent stages in series and is available when all three stages are up. Thus

$$A_i = \prod_{j=1}^{3} a_j$$

where a_j is the availability of the jth stage. The availability of a stage consisting of n equipment blocks in parallel, any r of which are required to be up for the stage to be up, is given by

$$a_j = \sum_{m=0}^{m=n-r} \binom{n}{m} a^{n-m} (1 - a)^m$$

$$= \sum_{m=0}^{m=n-r} \binom{n}{m} \frac{\mu^{n-m} \lambda^m}{(\lambda + \mu)^n}$$

where

$$\binom{n}{m} = n!/\left[(n-m)!\,m!\right]$$

Then stagewise,

$$A_i = A_{i_S} \cdot A_{i_C} \cdot A_{i_M}$$

Blockwise,

$$A_i = \left[\frac{\mu_S}{\lambda_S + \mu_S}\right] \left[\frac{\mu_C^2 + 2\mu_C\lambda_C}{(\lambda_C + \mu_C)^2}\right] \left[\left(\frac{\mu_M}{\lambda_M + \mu_M}\right)^{16}\right]$$

where

$$\mu = \overline{M}_c^{\,-1}$$

Availability with respect to preventive maintenance is unity for the M stage where no scheduled maintenance functions are required. It is also unity for the C stage, (though only .9955 for each block alone) because a sensible maintenance policy would stipulate that both equipment blocks not be down for scheduled maintenance simultaneously. Only the S stage requires downtime for preventive maintenance. Its availability A_{p_S} is estimated in the right hand column of Figure 2-13 by

$$A_{p_S} = \frac{f^{-1}}{f^{-1} + \overline{M}_p}$$

Thus, the subsystem achieved readiness is given by

$$A_a = A_{p_S} \cdot A_{i_S} \cdot A_{i_C} \cdot A_{i_M}$$

The equation is solved using the parameter predictions obtained in Figures 2-12 and 2-13, giving an initial prediction of achieved availability

$$A_a = (.998)(.9964)(.9999)(.8780) = .873$$

Subsystem: Fire Control

Hardware Block	Equipment	Predicted Partial Fail. Rate λ (Fail/Hr)	Recovery Action		Predicted MTTR $\bar{M}_c$ (Hrs)	$\lambda \bar{M}_c$	$U_i = \frac{\bar{M}_c}{\lambda^{-1} + \bar{M}_c}$	$A_i = 1 - U_i$
			Repair	Replace				
Systemized (S)	Control & Display	.0020	X		1.2	.0024		
	Peripheral Equipment	.0015	X		.8	.0012		
		$\lambda = .0035$			$\bar{M}_c = 1.03$	.0036	.0036	.9964
Channelized (C)	MMC	.0018	X		1.5	.0027		
	DGBC	.0030	X		1.0	.0030		
	DRISS	.0010	X		.5	.0005		
	FASS	.0008	X		2.0	.0016		
	CSS	.0020	X		1.1	.0022		
		$\lambda = .0086$			$\bar{M}_c = 1.16$	.0100	.0099	.9901
Missilized (M)	D/A	.0025	X		.8	.0020		
	ES	.0020	X		.8	.0016		
	CASS	.0016	X		1.5	.0024		
	FSS	.0022	X		1.0	.0022		
		$\lambda = .0083$			$\bar{M}_c = .99$	.0082	.0081	.9919

Figure 2-12. Analysis of Corrective Maintenance Tasks

Subsystem: Fire Control

Hardware Block	Scheduled Maintenance Task	Frequency of Maintenance f (Hrs^{-1})	Predicted Mean Maintenance Time $\overline{M}_p$	$f\overline{M}_p$	$U_p = \frac{\overline{M}_p}{f^{-1} + \overline{M}_p}$	$A_p = 1 - U_p$
Systemized (S)	Service printer	.002	0.5	.001	.001	
	Service tape drives	.002	0.5	.001	.001	
		f = .002	$\overline{M}_p = 1.0$	.002	.0020	.9980
Channelized (C)	Service DGBC peripheral equipment	.002	1.0	.002	.002	
	Clean filters in door-door-mounted equipments	.005	.5	.0025	.0025	
		f = .007	$\overline{M}_p = 0.63$	.0045	.0045	.9955
Missilized (M)	None					
(Note: All maintenance tasks performed serially)						

Figure 2-13. Analysis of Preventive Maintenance Tasks

Data required to support measurements are implicit in the parameters of the equation. They are the uptimes and downtimes of each equipment block from which statistical estimates of the failure rates and repair rates can be obtained. Preventive maintenance downtimes are also required for the S block.

Since the prediction does not indicate that the subsystem will initially meet its specified availability, (.99 in phase b), it is necessary to apportion requirements to elements of the subsystem. Before this is done, the subsystem should also be analyzed with respect to its reliability requirement since that requirement determines the upper limit of allowable failure rate.

Before proceeding with the example, it would be well to summarize the predicted parameters.

TABLE 2-2. SUMMARY OF PREDICTED PARAMETERS

		λ	$\overline{M}_c$	f	$\overline{M}_p$	A_i	A_p
S	BLOCK	.0035	1.03	.002	1.0	.9964	.9980
	STAGE	.0035	1.03	.002	1.0	.9964	.9980
C	BLOCK	.0086	1.16	.007	.63	.9901	1.0
	STAGE	*	1.16	0.0	0.0	.9999	1.0
M	BLOCK	.0083	0.99	0.0	0.0	.9919	1.0
	STAGE	.1328	0.99	0.0	0.0	.8780	1.0
SUBSYSTEM			0.998		1.0	.8750	.9980

**The failure rate of an n-block parallel stage is not constant but is a function of time. For small λ it is approximated by $(\lambda t)^n/t$.*

The reliability requirement for the subsystem in R(3 hours) ⩾ .95. Then, in terms of blocks (not stages),the subsystem model is

$$R_{subsys} = R_S \left[1 - \{1 - R_C\}^2 \right] R_M^{16}$$

$$= R_S R_C R_M^{16} (2 - R_C)$$

$$= 2e^{-(\lambda_S + \lambda_C + 16\lambda_M)3} - e^{-(\lambda_S + 2\lambda_C + 16\lambda_M)3}$$

$$= .95$$

If the prediction accurately reflects the relative complexities, stress levels, state-of-the-art factors, etc. that characterize each equipment block, it is reasonable to apportion failure rates among the blocks in the same ratios indicated by the prediction. Then the failure rates of the C and M blocks can be expressed in terms of the S failure rate.

$$\lambda_C = \frac{.0086}{.0035} \lambda_S = 2.46 \lambda_S$$

$$\lambda_M = \frac{.0083}{.0035} \lambda_S = 2.37 \lambda_S$$

Given these relative magnitudes of block failure rates, the subsystem model becomes

$$R(3) = 2e^{-41.38 \lambda_S (3)} - e^{-43.84 \lambda_S (3)} = .95$$

which is easily solved graphically to yield $\lambda_S = .00044$. Then, by the ratios previously stated, $\lambda_C = .00108$ and $\lambda_M = .00104$. These are the highest permissible block failure rates consistent with the subsystem reliability requirement. They correspond to MTBF's of 2280 hours, 930 hours and 960 hours for the S, C and M blocks respectively, and they represent bounds on the tradeoff regions available for meeting the subsystem availability requirement. The subsystem MTBF is found by integrating the reliability function. It should be noted that the reliability of electronic, mechanical and electro-mechanical devices can usually be characterized in terms of constant failure rates (implying exponentially distributed failure times), provided the devices are complex and consist of parts of varying ages or mixtures of part types having different mean lives and failure distributions*.

**For a brief discussion of the relevant theory, see Bazovsky, I., Reliability Theory and Practice, Prentice-Hall, Inc., 1961, pp. 52-58.*

$$\text{MTBF} = \theta = \int_0^\infty 2e^{-.01818t}\,dt - \int_0^\infty e^{-.01926t}\,dt$$

$$= \frac{2}{.01818} - \frac{1}{.01926} = 58 \text{ hrs.}$$

Apportionment of availability can be begun by considering the subsystem as a whole. If it is assumed A_p must remain fixed, then minimum acceptable A_i is

$$A_i = \frac{A}{A_p} = \frac{.99}{.998} = .992$$

An optimum combination of subsystem parameters θ (MTBF) and $\overline{M}_c$ (MTTR) is sought subject to the constraint

$$A_i = \frac{1}{1 + \frac{\overline{M}_c}{\theta}}$$

$$\frac{\overline{M}_c}{\theta} = \frac{1 - A_i}{A_i} = \frac{.008}{.992} = .00806$$

Several approaches to apportionment of goals for the improvement of reliability and/or maintainability are available and are discussed in the literature. One of the simplest is to determine the magnitude of improvement needed in each characteristic alone in order to satisfy the subsystem A_i requirement.

MTTR required at minimum MTBF:

$$\overline{M}_{c_\text{required}} = \theta\left(\frac{1 - A_i}{A_i}\right) = 58\left(\frac{.008}{.992}\right) = .464 \text{ hr.} = 28 \text{ minutes}$$

MTBF required at predicted MTTR:

$$\theta_\text{required} = \overline{M}_c\left(\frac{A_i}{1 - A_i}\right) = .998\left(\frac{.992}{.008}\right) = 124 \text{ hrs.}$$

$\overline{M}_c$ in the above equation is computed from equation 2-22. The frequency weighting factor for the C-stage is not really a failure rate but $2\lambda_C/3 = .0057$, the reciprocal of the stage MTBF**

***Bazovsky, op. cit. p. 100.*

Thus, it is apparent that the availability requirement can be met by improving the subsystem MTTR to 28 minutes while meeting the minimum MTBF consistent with the reliability requirement. Or, the availability requirement can be met with the predicted MTTR of one hour if the subsystem MTBF can be raised to 124 hours. Between these extremes there are an unlimited number of combinations of MTBF and MTTR that will also satisfy the requirement.

In order to apportion reliability and maintainability goals in an optimum manner, it is desirable to predict the relative difficulty of improving each. In this example, the predictions take the form of cost functions, although the actual resources involved may include engineering and manufacturing man-hours as the major or sole variables. It is not realistic to formulate reliability and maintainability as deterministic functions of the resources expended for their realization. At best, the analyst can invoke past experience to predict the functional relationships in a largely subjective manner. Feasible improvement actions may be listed, and engineers asked to make optimistic, expected and pessimistic predictions of the costs entailed in each and the degree of change each would produce in the reliability and maintainability of the system. The actions can then be listed as scaled sets and the distributions of the cost/improvement relationships estimated.*

For purposes of illustration, let it be assumed that such an analysis is performed for the Fire Control Subsytem and that the following expected cost functions are obtained over limited ranges of θ and $\bar{M}_C$ The unit of cost C is dollars x 10^6.

$$C(\bar{M}_c) = \bar{M}_c^{-2}$$

$$C(\theta) = \theta^2/3600$$

The method of Lagrange multipliers is employed to minimize the total cost function

$$G = C(\bar{M}_c) + C(\theta) + \alpha \left(\frac{\bar{M}_c}{\theta} - \frac{1 - A_i}{A_i} \right)$$

where α is a Lagrange multiplier. The partial derivatives of the cost function are set equal to zero and the resulting system solved simultaneously for the optimum values θ_o and $\bar{M}_{co}$.

$$\frac{\partial G}{\partial \bar{M}_c} = C'(\bar{M}_c) + \frac{\alpha}{\theta} = 0$$

$$\frac{\partial G}{\partial \theta} = C'(\theta) - \frac{\alpha M_c}{\theta^2} = 0$$

**For a thorough treatment of tradeoff techniques, see Goldman, A.S. and Slattery, T.B., Maintainability, John Wiley and Sons, New York, 1964. Their approach is followed here.*

$$\frac{\partial G}{\partial \alpha} = \frac{\overline{M}_c}{\theta} - \frac{(1 - A_i)}{A_i} = 0$$

from which

$$\theta_o = 86.6 \text{ hrs.}$$

$$\overline{M}_{c_o} = .693 \text{ hr.} = 42 \text{ minutes}$$

These optimum subsystem values can then be apportioned in a conventional manner back to block level and ultimately to equipment level. The procedures are identical to those employed for reliability apportionment. Since all of the blocks have roughly the same predicted maintainability, the apportioned MTTR would be close to 42 minutes for each type. Apportioned MTBF's would be about 3393 hours for the S block, 1379 hours for each C block, and 1432 hours for each M block. The apportionments are carried through below for purposes of illustration:

$$\theta_o = 86.6 = \int_0^{\infty} 2e^{-41.38\lambda_S t}\, dt - \int_0^{\infty} e^{-43.84\lambda_S t}\, dt = \frac{2}{41.38\lambda_S} - \frac{1}{43.84\lambda_S}$$

$$\lambda_S = .0002947, \quad \theta_S = 3393 \text{ hr.}$$

$$\lambda_C = 2.46\lambda_S = .0007250, \quad \theta_C = 1379 \text{ hrs.}$$

$$\lambda_M = 2.37\lambda_S = .0006984, \quad \theta_M = 1432 \text{ hrs.}$$

$$\overline{M}_{c_o} = .693 = \left(\lambda_S \overline{M}_{c_S} + \frac{2\lambda_C}{3} \overline{M}_{c_C} + 16\lambda_M \overline{M}_{c_M}\right) \Big/ \left(\lambda_S + \frac{2\lambda_C}{3} + 16\lambda_M\right)$$

$$= \frac{.0002947\overline{M}_{c_S} + \frac{2}{3}(.0007250)(1.13)\overline{M}_{c_S} + 16(.0006984)(0.96)\overline{M}_{c_S}}{.0002947 + \frac{2}{3}(.0007250) + 16(.0006984)}$$

$$\overline{M}_{c_S} = .6707 \text{ hr.} = 40 \text{ minutes}$$

$$\overline{M}_{c_C} = \left(\frac{1.16}{1.03}\right) \overline{M}_{c_S} = 1.13\, M_{c_S} = .7579 \text{ hr.} = 45 \text{ minutes}$$

$$\overline{M}_{c_M} = \left(\frac{0.99}{1.03}\right) \overline{M}_{c_S} = 0.96\, M_{c_S} = .6439 \text{ hr.} = 39 \text{ minutes}$$

SECTION 3

STATISTICAL ESTIMATION OF AVAILABILITY

3.1 BEST ESTIMATE OF AVAILABILITY

For a subsystem that is repaired, the measured uptime ratio

$$\hat{A} = \frac{(\text{total time up})}{(\text{total time up}) + (\text{total time down})} \tag{3-1}$$

can serve as a statistical estimate (expected value) of its operational availability. Achieved availability has been defined as the product $A_p A_i$. In general, the durations of uptimes and downtimes with respect to scheduled maintenance will not vary appreciably from their respective means. Thus, they need not be treated as random variables. Therefore, A_p will be essentially constant and easily measured. Whatever uncertainty attaches to measured availability will be associated with the estimate of A_i. Given conditions of continuous monitoring and immediate initiation of repair

$$\hat{A}_i = \frac{\hat{\mu}}{\hat{\lambda} + \hat{\mu}} \tag{3-2}$$

If the system is checked out only at intervals, operating untended over a period of length T between checkouts, its A_i is also estimated from the observed uptime ratio, in this case the ratio observed during n intervals of length T.

$$\hat{A}_i = \frac{\Sigma_{k=1}^{k=n} u_k}{nT} \tag{3-3}$$

where u_k is the uptime observed during the kth interval.

The expression $\Sigma_{k=1}^{k=n} u_k/n$ is an estimate of E [u], the expected value of u. Formally

$$E[u] = \lim_{n \to \infty} \left(\frac{1}{n}\right) \Sigma_{k=1}^{k=n} u_k \tag{3-4}$$

A best estimate of subsystem availability can be computed from data obtained at lower hardware levels, making use of the relationships given in equations (2-15) and (2-16); that is,

$$\hat{A}_i = \prod_{i=1}^{n} \frac{\hat{\mu}_i}{\hat{\lambda}_i + \hat{\mu}_i} \tag{3-5}$$

for serially configured stages and, for a parallel stage of n identical items,

$$\hat{A}_i = \sum_{0}^{m=n-r} \binom{n}{m} \frac{\hat{\mu}^{n-m}\hat{\lambda}^{m}}{(\hat{\lambda} + \hat{\mu})^{n}} \tag{3-6}$$

The Fire Control Subsystem modeled in the preceding section is an example of the process. Equation (3-2) is used to obtain an estimate for each element of the subsystem. The availability of a stage consisting of parallel blocks is then estimated by equation (3-6). Finally, subsystem availability is estimated by equation (3-5) as the product of the availability of series-related stages.

3.2 ESTIMATION OF CONFIDENCE LIMIT ON AVAILABILITY

The problem of finding a confidence limit for availability ultimately reduces to estimating the distribution of the quotient of two independently distributed random variables. For a subsystem considered as a single stage, the problem is tractable under rather general conditions. But for a multistage subsystem, where the measurement data are gathered at subordinate hardware levels, a solution is usually accessible only by the use of simulation methods. The analytical treatments that follow apply to the single stage case. Simulation methods for multistage subsystems are also discussed.

3.2.1 Determining the Distribution of Repair Times

The technique to be used in estimating a confidence limit on A_i is applicable whether the subsystem is viewed as a single stage or as multiple stages in series, since time to failure has the same distributional form in the latter case as in the former. However, it is necessary to treat repair times empirically, since the subsystem repair rate cannot be obtained by summing the rates of the series elements. The method used for confidence limit estimation depends on the distribution of repair times.

There is little theoretical reason to assume a particular functional form for the distribution of repair times. Indeed, a rigorous approach would require that the distribution for a given subsystem be viewed as a weighted combination of distributions, each associated with a particular equipment of the subsystem. In practice, however, subsystem repair times are usually well described by log normal or exponential frequency distributions. A log normal distribution may be found to fit the data most accurately, but the exponential form is mathematically more tractable and is to be preferred for that reason when it fits the observed data acceptably.* Or a goodness of fit test can be run for both exponential and log normal distributions and the distribution can be chosen which gives the better fit. The procedures given herein treat confidence limit estimation for three forms of repair time distribution – exponential, log normal and constant. The procedure begins with a determination of the goodness of fit of a sample of m measured repair times to the exponential form of distribution.

Mean repair rate is estimated from the sample

$$\hat{\mu} = \frac{m}{\sum_{i=1}^{i=m} M_{c_i}} \qquad (3\text{-}7)$$

The sample data are then ordered and plotted as a cumulative distribution of observed repair times. An expected distribution is also plotted from the relation

$$P(M_c) = 1 - e^{-\hat{\mu} M_c} \qquad (3\text{-}8)$$

as shown in Figure 3-1.

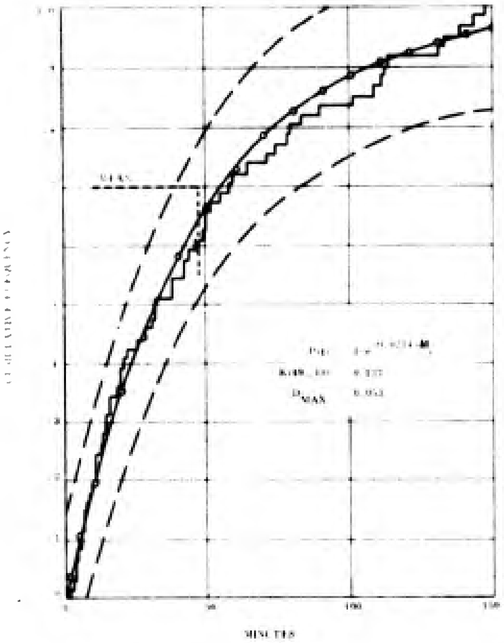


Figure 3-1. Cumulative Distribution of Repair Times with Kolmogorov-Smirnov Limits (Exponential)

**A strong case for exponential analysis of repair times is made by Creson, J.W., "Exponential Distribution Analysis of Corrective Maintenance Times", J. Electronics Div. ASQC, Vol. 2. No. 4, June 1964, pp. 19-29.*

The Kolmogorov-Smirnov test, as modified by Lilliefors,* is then applied to test the hypothesis that the data are from an exponential distribution with mean $\hat{\mu}$. The statistic evaluated is D, the largest absolute deviation between the observed and expected ordinates of the cumulative distribution.

$$D = \max \{ |P(M_c) - S(M_c)| \} \tag{3-9}$$

where

M_c = measured repair time

$P(M_c)$ = the computed cumulative frequency

$S(M_c)$ = the observed cumulative frequency

The sampling distribution of D is known (Table 3-1). Given a sample of m observed repair times and a significance level α the result $D > K(m, \alpha)$ supports rejection of the hypothesis with confidence $(1 - \alpha)$; otherwise the hypothesis is not rejected.

A contractor may elect to use other goodness-of-fit tests, rather than the Kolmogorov-Smirnov, if he can justify their use. However, the Kolmogorov-Smirnov test has several claims to preference over such tests as the chi-square and RMS. It treats individual observations instead of combining categories as does chi-square; it is applicable to small samples as chi-square is not. The RMS test distributes error uniformly, hence, is probably weaker than Kolmogorov-Smirnov in most cases.

If the exponential hypothesis is rejected, so that an exponential distribution cannot be fitted to the sample with satisfactory accuracy, it is necessary to fit a log normal distribution. All data points M_c are transformed into their natural logarithms $X = \ln M_c$ which are then treated as samples from a normal distribution with mean $\hat{M}$ and standard deviation $\hat{\sigma}$.

$$\hat{M} = \overline{X} = \frac{\sum_{i=1}^{m} X_i}{m} \tag{3-10}$$

$$\hat{\sigma} = s = \sqrt{\frac{\sum_{i=1}^{m} (X_i - \overline{X})^2}{m - 1}} \tag{3-11}$$

Lilliefors, H.W., "On the Kolmogorov-Smirnov Test for the Exponential Distribution with Mean Unknown", American Statistical Association Journal, March 1967, pp. 387-389.

A plot of the sample in cumulative form is again employed. The hypothesized cumulative normal distribution is plotted from the relation

$$P(X) = \frac{1}{\sqrt{2\pi}\,\hat{\sigma}} \int_{-\infty}^{X} e^{-(X-\hat{M})^2/2\sigma^2}\, dX \tag{3-12}$$

The Kolmogorov-Smirnov test, as modified by Lilliefors* for the normal distribution with estimated mean and variance (Table 3-2) is applied in the same manner as for the exponential.

3.2.2 Confidence Limit Estimation for Exponential Repair Times

When both failure and repair times are exponentially distributed, it can be shown (see Appendix) that the random variable $u = 2n\lambda/\hat{\lambda}$ is distributed as chi-square (χ^2) with 2n degrees of freedom, where n is the observed number of uptimes terminated by failure. Similarly, the random variable $v = 2m\mu/\hat{\mu}$ is distributed as χ^2 with 2m degrees of freedom, where m is the number of repair times observed. Since u and v are independently distributed as χ^2 with 2n and 2m degrees of freedom, the quantity

$$\frac{u/2n}{v/2m}$$

has the variance ratio density F. Thus, a $(1-\alpha)100$ percent upper confidence limit can be found for the quotient λ/μ by

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\mu}}\, F_{1-\alpha;\ 2n,\ 2m} \tag{3-13}$$

where F denotes the indicated fractile of the cumulative F density with 2n and 2m degrees of freedom. Then, a one-sided $(1-\alpha)100$ percent lower availability limit is

$$A_{i_L} = \frac{1}{1+\left(\frac{\lambda}{\mu}\right)_U} \tag{3-14}$$

**Lilliefors, H.W., "On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown". American Statistical Association Journal, June 1967, pp. 399-402.*

TABLE 3-1. KOLMOGOROV-SMIRNOV LIMIT FACTORS K (m,α) FOR EXPONENTIAL DISTRIBUTION WITH ESTIMATED MEAN

ABSOLUTE VALUES OF THE MAXIMUM DIFFERENCE D BETWEEN SAMPLE AND POPULATION CUMULATIVE FRACTIONS SIGNIFICANT AT THE 20, 15, 10, 5 AND 1 PERCENT LEVELS, m = sample size

Sample Size	Level of Significance α				
m	.20	.15	.10	.05	.01
3	.451	.479	.511	.551	.600
4	.396	.422	.449	.487	.548
5	.359	.382	.406	.442	.504
6	.331	.351	.375	.408	.470
7	.309	.327	.350	.382	.442
8	.291	.308	.329	.360	.419
9	.277	.291	.311	.341	.399
10	.263	.277	.295	.325	.380
11	.251	.264	.283	.311	.365
12	.241	.254	.271	.298	.351
13	.232	.245	.261	.287	.338
14	.224	.237	.252	.277	.326
15	.217	.229	.244	.269	.315
16	.211	.222	.236	.261	.30[illegible]
17	.204	.215	.229	.253	.297
18	.199	.210	.223	.246	.289
19	.193	.204	.218	.239	.283
20	.188	.199	.212	.234	.278
25	.170	.180	.191	.210	.247
30	.155	.164	.174	.192	.226
Over 30	$\frac{.86}{\sqrt{m}}$	$\frac{.91}{\sqrt{m}}$	$\frac{.96}{\sqrt{m}}$	$\frac{1.06}{\sqrt{m}}$	$\frac{1.25}{\sqrt{m}}$

TABLE 3-2. KOLMOGOROV-SMIRNOV LIMIT FACTORS K (m,α) FOR NORMAL DISTRIBUTION WITH ESTIMATED MEAN AND VARIANCE

ABSOLUTE VALUES OF THE MAXIMUM DIFFERENCE D BETWEEN SAMPLE AND POPULATION CUMULATIVE FRACTIONS SIGNIFICANT AT THE 20, 15, 10, 5 AND 1 PERCENT LEVELS, m = sample size

Sample Size	Level of Significance α				
m	.20	.15	.10	.05	.01
4	.300	.319	.352	.381	.417
5	.285	.299	.315	.337	.405
6	.265	.277	.294	.319	.364
7	.247	.258	.276	.300	.348
8	.233	.244	.261	.285	.331
9	.223	.233	.249	.271	.311
10	.215	.224	.239	.258	.294
11	.206	.217	.230	.249	.284
12	.199	.212	.223	.242	.275
13	.190	.202	.214	.234	.268
14	.183	.194	.207	.227	.261
15	.177	.187	.201	.220	.257
16	.173	.182	.195	.213	.250
17	.169	.177	.189	.206	.245
18	.166	.173	.184	.200	.239
19	.163	.169	.179	.195	.235
20	.160	.166	.174	.190	.231
25	.149	.153	.165	.180	.203
30	.131	.136	.144	.161	.187
over 30	$\frac{.736}{\sqrt{m}}$	$\frac{.768}{\sqrt{m}}$	$\frac{.805}{\sqrt{m}}$	$\frac{.886}{\sqrt{m}}$	$\frac{1.031}{\sqrt{m}}$

3.2.3 Confidence Limit Estimation for Log Normal Repair Times

When repair times are log normally distributed, the applicable measure of central tendency is the geometric mean

$$\hat{M}_{c_G} = \left(\prod_{i=1}^{m} M_{c_i} \right)^{\frac{1}{m}} \qquad (3\text{-}15)$$

and $\hat{\nu}$ is defined as $1/\hat{M}_{c_G}$. Then the statistic $1/\hat{\nu}e^{M}$ is distributed log normally.

$$\frac{1}{\hat{\nu}e^{M}} \sim \Lambda\left(0, \frac{\sigma^2}{m}\right) \qquad (3\text{-}16)$$

Failure times are distributed exponentially with 2n degrees of freedom. As previously noted

$$\frac{2n\lambda}{\hat{\lambda}} \sim \chi^2_{2n} \qquad (3\text{-}17)$$

Then, a (1 - α) 100 percent upper confidence limit for the ratio λ/μ can be found by

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\nu}} \left(\frac{e^{\frac{\hat{\sigma}^2}{2}}}{2n} \right) a_{1-\alpha;\ \frac{m}{\hat{\sigma}^2},\ n} \qquad (3\text{-}18)$$

The coefficient a is tabulated in the Appendix. Equation 3-18 is derived in the Appendix. The (1 - α) 100 percent lower confidence limit on availability is

$$A_L = \frac{1}{1 + \left(\frac{\lambda}{\mu}\right)_U} \qquad (3\text{-}19)$$

3.2.4 Confidence Limit Estimation for Constant Repair Times

When repair time is constant, the confidence limit on availability is of the form given by equation (3-19) but depends only on the distribution of λ. Limits are quickly found from the relation of equation (3-17) by noting that

$$\lambda \sim \frac{\hat{\lambda}\, \chi^2_{2n}}{2n} \tag{3-20}$$

Therefore,

$$\lambda_U = \hat{\lambda}\, \frac{\chi^2_{1-\alpha;\, 2n}}{2n} \tag{3-21}$$

Then, with μ constant, the lower $(1-\alpha)$ 100 percent limit of availability is

$$A_{i_L} = \frac{1}{1 + \frac{\lambda_U}{\mu}} \tag{3-22}$$

3.2.5 Confidence Limits for MTTR

Availability limits can be found by the procedures of 3.2.3 and 3.2.4 without need for confidence limits on μ or MTTR. Therefore, techniques for computing such limits are not treated in detail herein. If desired, however, confidence limits on μ can readily be computed for exponentially distributed repair times using the methods given in NAVWEPS OD 29304 for finding limits on failure rates. For log-normally distributed repair times, limits on MTTR are easily obtained using the standard error of the sample mean. A one-sided (100) α percent limit is

$$\text{MTTR}_{\alpha} = \text{antilog}_e \left(\overline{X} + t_{\alpha;\, m-1}\, s/\sqrt{m}\right) \tag{3-23}$$

where t is the student's -t statistic with m-1 degrees of freedom. Equations (3-10) and (3-11) are solved to obtain $\overline{X}$ and s. An advantage of equation (3-23) is that it is exactly correct for normally distributed X regardless of sample size and approximately correct with increasing sample size for virtually any distribution.

3.2.6 Monte Carlo Simulation of Multistage Subsystem Availability

Monte Carlo simulation has progressed during the past decade from a brute force method resorted to for problems intractable by other means, to its present stature as a primary tool of systems analysis and evaluation. Its value as an analogue of system experience would be difficult to overestimate. The mechanical aspects of computer simulation are now so well established that no discussion of them need be undertaken here. Instead the following paragraphs outline approaches to formulating the required distributions, given input data in the form of test results.

In general terms, the availability of a subsystem is a product of availabilities of subordinate hardware items. For example, the availability of a serial subsystem consisting of N components, all of which fail and are repaired independently, is the product of the individual availabilities.

$$\hat{A} = \prod_{j=1}^{N} \hat{A}_j$$

The true values of these availabilities are not known precisely but are estimated on the basis of test data. An estimate of subsystem availability without statistical confidence can be had by simple manipulation of the corresponding component estimates found by the methods already discussed. But valid subsystem confidence limits cannot be obtained by combining estimated component limits.

$$A_\alpha \neq \prod_{j=1}^{N} A_{j_\alpha}$$

An interval estimate of subsystem availability can be obtained by Monte Carlo simulation based on data taken at component level. Input data consist of $\hat{\lambda}$,n, m plus $\hat{\mu}$ in the case of exponential repair times or $\hat{\nu}$ and $\hat{\sigma}$ for log-normal repair times.

The core of the simulation technique is a Bayesian view which considers the true availability as a random variable and synthesizes its distribution $g\,(A|\hat{\lambda},\hat{\mu},\ n,\ m)$ conditioned on the estimate or, more correctly, on the data generating the estimate. It has been shown that

$$\frac{u/2n}{v/2m} = \frac{\lambda}{\hat{\lambda}} \cdot \frac{\hat{\mu}}{\mu} \sim F_{2n,\ 2m}$$

and

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\mu}} F$$

Since

$$A_L = \frac{1}{1+\left(\frac{\lambda}{\mu}\right)_U} = \frac{1}{1+\frac{\hat{\lambda}}{\hat{\mu}} F}$$

it is apparent that sampling from $F_{2n,2m}$ is equivalent to sampling from $g(A|\hat{\lambda}, \hat{\mu}, n, m)$ by the transformation shown above. It is a simple task to sample from any desired F distribution, beginning with random numbers distributed uniformly on the interval zero to one $R[0,1]$, or beginning with random numbers distributed normally with zero mean and unit variance $N[0,1]$. The transformations are

$$N = \sqrt{-2 \ln R \cos 2\pi R}$$

$$w = \sum_{i=1}^{k} N_i^2$$

where w is a $\chi^2(k)$ variate,

$$y = \frac{w/2n}{w/2m}$$

where y is a $F_{2n,\,2m}$ variate and the w are independent $\chi^2(2n)$ and $\chi^2(2m)$ variates*. Thus, 2n + 2m independent normal variates must be drawn to construct one F variate, which is then transformed to a sample value of A_i via

$$a_j = \frac{1}{1 + \frac{\hat{\lambda}_j}{\hat{\mu}_j} y}$$

The computer executes the above algorithm for each component, stores the results, then uses the stored values to solve the system model for A. That represents one pass through the simulation procedure. Repetition of the process builds up a histogram of the sample values of A, which approaches the shape of $g(A|\lambda, \mu, n, m)$ as the number of passes increases. The desired interval estimate is obtained simply by reading the appropriate percentile values.

When the repair time variable is log-normally distributed, the inverse function $p^{-1}(w) = R$ (see Appendix A.3) can be solved for the coefficient a in the denominator below by numerical integration on each pass, then substituted into

$$a_j = \frac{1}{1 + \frac{\hat{\lambda}_j}{\hat{\mu}_j} \left(\frac{a\, e^{\sigma/2^2}}{2n} \right)}$$

Note that a in the denominator is the coefficient of Table A-2, not availability.

Segal, R., "Generation of Random Numbers for Monte Carlo Simulations", General Electric Co. Report 65SD231, April 1965.

Or, much more simply, an interval can be formed by sampling the λ and μ variables independently and forming their quotient.

$$\lambda \sim \hat{\lambda}\ \chi^2/2n$$

$$\frac{1}{\nu e^{M}} \sim \Lambda\left(0, \frac{\sigma^2}{m}\right)$$

$$\ln\left(\frac{1}{\hat{\nu}}\right) - M = z \sim N\left(0, \frac{\sigma^2}{m}\right)$$

$$\mu \sim \exp\left[\ln\left(\frac{1}{\hat{\nu}}\right) - z + \frac{\sigma^2}{2}\right]$$

As in appendix A.3, $\sigma^2 = \hat{\sigma}^2$ is assumed. It is important to fit successive samples of λ and μ into the expression for A and build its histogram. Operating arithmetically with limits computed for each parameter separately will give a much larger "at least" type interval. For example, if A_L is computed using 80% limits on λ and μ, the resulting limit defines a 64% interval for A.

It is easy to read from the histogram a variety of relevant statistics with standard errors which are entirely under control, since they depend only on the number of passes n. Specifically, one can read the mean or expected value, the mode or maximum likelihood value, the median or fifty percent confidence limit, any desired precentiles in order to construct one-sided or two-sided interval estimates, and the range. The standard error of each of these estimates, except the mode, are computable by reference to Kendall.* Briefly, the standard errors are, for the mean $\hat{A}$,

$$\sigma_{\hat{A}} = \sigma_A / \sqrt{n}$$

where n is the number of passes, and for the pth and (100-p)th percentiles

$$\sigma_{A_p} = \phi\sigma_{\hat{A}}$$

**Kendall, M., The Advanced Theory of Statistics, Vol. 1, Chas. Griffith & Co., London, 1943-1946.*

Kendall tabulates a few values of the ratio ϕ, which is symmetrical about the median. When simulating is done often, it is useful to fit a smooth curve (Figure 3-2) and express ϕ as a function of the desired percentile.* The standard error of the mode is available with somewhat greater effort by use of Yasukawa's method.**

$$\phi = 1.93637 - 2.86403p + 2.86403p^2$$

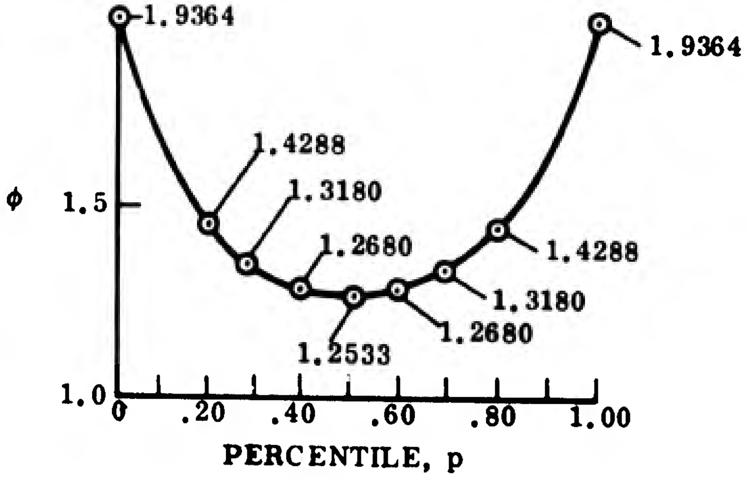


Figure 3-2. Standard Error of a Percentile as a Multiple of Standard Error of Mean

3.3 EXAMPLE

As an example, consider a single stage subsystem for which the following data have been observed and parameters estimated:

n = 10 failures
m = 10 repairs
r^1 = 143 hrs.
$\hat{\lambda}$ = .005 failures/hr.
$\hat{\mu}$ 10/28.9 = .346 repairs/hr.
$\overline{M}_p$ = 1.284 hrs.
M_{c_1} = 7.0 hrs.
M_{c_2} = 5.0 hrs.
M_{c_3} = 3.9 hrs.
M_{c_4} = 3.0 hrs.
M_{c_5} = 2.5 hrs.
M_{c_6} = 2.3 hrs.
M_{c_7} = 2.0 hrs.
M_{c_8} = 1.7 hrs.
M_{c_9} = 1.0 hrs.
$M_{c_{10}}$ = 0.5 hrs.

**Wilson, M. and Fagan T., "Monte Carlo Simulation of System Reliability", Proceedings Assoc. for Computing Machinery Natl. Conf. 1968. pp 289-293.*

***Yasukawa, K., "On the Probable Error of the Mode of Skew Frequency Distributions", Biometrika 18, 263-292.*

Availability with respect to scheduled maintenance is

$$A_p = \frac{f^{-1}}{f^{-1} + \overline{M}_p} = \frac{143}{143 + 1.284} = .9911$$

A best estimate of A_i is

$$\hat{A}_i = \frac{\hat{\mu}}{\hat{\lambda} + \hat{\mu}} = \frac{.346}{.005 + .346} = .9857$$

The repair time data are tested at the .10 significance level in Figure 3-3. The hypothesis of exponential distribution with mean 2.89 hours is not rejected. Then, a 1-sided 80 percent confidence limit on the ratio λ/μ is given by

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\mu}} F_{.80,20,20}$$

$$= \frac{.005}{.346} (1.4656) = .0211$$

and a 1-sided 80 percent limit on A_i is

$$A_{i_L} = \frac{1}{1.0211} = .979$$

The corresponding limit on achieved availability is obtained by multiplying bv A_p.

$$A_{a_L} = (.9911)(.979) = .970$$

If, instead of those given above, the observed repair times were as shown below, the hypothesis of exponentially distributed repair times would be rejected at the .10 level. This is seen in Figure 3-4.

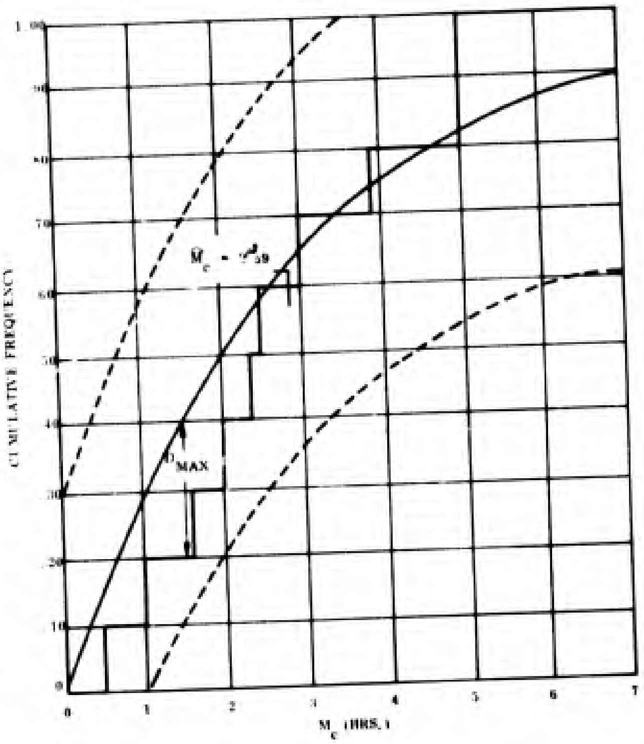


Figure 3-3. Cumulative Distribution of Repair Times with Kolmogorov-Smirnov Limits (Exponential)

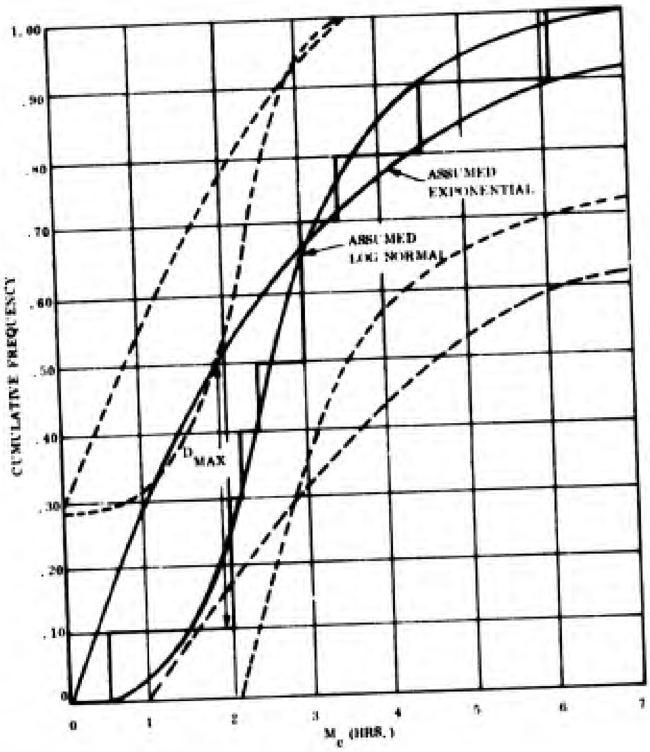


Figure 3-4. Cumulative Distribution of Repair Times with Kolmogorov-Smirnov Limits

$M_{c_1} = 6.1$

$M_{c_2} = 4.6$

$M_{c_3} = 3.5$

$M_{c_4} = 3.0$

$M_{c_5} = 3.0$

$M_{c_6} = 2.5$

$M_{c_7} = 2.2$

$M_{c_8} = 2.0$

$M_{c_9} = 2.0$

$M_{c_{10}} = 0.6$

Thus it becomes necessary to fit a log normal distribution to the observed data. The data points are transformed to their logarithms ($X_i = \ln M_{c_i}$). Then by equations (3-10) and (3-11):

$$\hat{M} = \bar{X} = \frac{\Sigma X_i}{m} = .9364$$

$$\hat{\sigma} = s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{m-1}} = .413$$

The geometric mean is $\hat{M}_{c_G} = e^{\hat{M}}$

$$\hat{M}_{c_G} = \left(M_{c_1} \cdot M_{c_2} \cdot M_{c_3} \cdot \ldots . \; M_{c_{10}}\right)^{0.1} = 2.55 \text{ hrs.}$$

and $\nu = .393$.

Then, by equation (3-18) and Table A-2 of the Appendix, an upper 80 percent confidence limit on λ/μ is:

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\nu}} \left(\frac{e^{\frac{\hat{\sigma}^2}{2}}}{2n}\right) a_{1-\alpha,\ \frac{m}{\hat{\sigma}^2},\ n}$$

$$= \frac{.005}{.393} \left(\frac{1.05}{20}\right) a_{.20,60,10}$$

$$= .01272\ (.0542)\ (14.049) = .010$$

The corresponding lower limits on A_i and A_a are

$$A_{i_L} = \frac{1}{1+\left(\frac{\lambda}{\mu}\right)_U} = \frac{1}{1.017} = .983$$

$$A_{a_L} = (.9911)(.983) = .974$$

Although the arithmetic mean repair time is nearly identical in both examples, the log normal data have a smaller variance than the exponential data. Thus, the 80 percent availability limit is slightly greater in the log normal case.

SECTION 4

ANALYSES OF TEST PROGRAMS

4.1 TESTS NECESSARY TO MEASURE AVAILABILITY

For many subsystems, reliability is defined over a mission characterized by combined stresses of variable durations which are not reproducible in combination during the development tests. In these circumstances, the mission reliability of the subsystem is not actually measured but is synthesized from single-environment tests on the assumption that failure rates and exposure times attributable to each of the mission environments are additive. This feature of the approach makes necessary the alpha values of NAVWEPS OD 29304, multipliers that convert environmental test time to equivalent missions.

Typically, however, the availability phase of a subsystem's mission is not characterized by an extreme or uncontrolled environment or by variable times of exposure to the environment that is experienced. Therefore, the concept employed in this manual is prediction of availability early in the program, followed by direct measurement of availability later in the testing program as data become available. It is normally desirable, therefore, to exclude for purposes of availability analysis tests conducted under extreme environments. Where mission analysis discloses a significant environmental factor in the expected availability mission, that factor should be applied during the tests. Failing this, the alpha-value concept can be invoked to synthesize estimates of mission failure and repair rates.

Economic considerations usually make it desirable to measure availability using only available data, i.e., data obtained by testing performed for other purposes or obtained by actual operation of subsystems in fleet service or training activities. Accordingly, the approach described herein is to evaluate the quantity of relevant test data the Integrated Test Plan (ITP) can be expected to provide. Usually, though not necessarily, these will be subsystem level tests. Then, using predictions of the availability the subsystem will exhibit, it is a routine matter to estimate the measurement precision attainable as a byproduct of the development and production programs, assuming no recourse to additional tests for the specific purpose of measuring availability. If the measurement program projected by analysis of the ITP appears inadequate, an estimate of the quantity of additional specific availability testing necessary to supplement the program is available.

As noted in Section 3, A_i is measured by observation of times between successive failures and times required to complete corrective maintenance tasks. An essential condition for such data to be valid is that the operation and maintenance of the tested hardware be carried out under conditions representative of the intended mission. It has been noted that failure rate is dependent on operating environment. In addition, the time required to repair a subsystem depends on skill levels, manning, diagnostic procedures, equipment and tools available to the maintenance crew. In a practical sense, there is usually no way to ensure absolute equivalence of usage and maintenance conditions in a test program with those that will prevail in fleet service, even when the service conditions are known – and they seldom are known with reasonable precision. But, it is necessary that the dependencies noted above be recognized during the process of test planning and that an effort be made to establish substantial equivalence of maintenance and use conditions with those expected in the mission.

Both the reliability and maintainability measurements necessary to measure availability are computed on the basis of assumptions of statistical regularity throughout the subsystem population – assumptions which make possible the direct substitution of test time for sample size. It is only necessary that cumulative operating and maintenance times be the sums of the times for all items tested. However, from both the statistical and engineering viewpoints, it is desirable to test a sample large enough to offer some assurance that the tested items are representative of the population. Therefore, whenever possible, availability measurements should be based on observation of more than a single specimen of the subsystem under development.

It should be noted that in this section, as in Section 3, preventive maintenance times and frequencies are treated as constants. Should analysis or experience indicate variable preventive maintenance times, their statistical properties can be evaluated in a manner identical to that described for corrective maintenance.

4.2 ANALYSIS OF INTEGRATED TEST PLAN

Throughout the development and pilot production phases of the subsystem program, the Integrated Test Program will provide the data for availability evaluation. An Integrated Test Plan (ITP), providing for central planning and control of all program testing activities is required by MIL-Q-21549. It must set forth the purposes and extent of the test program. The ITP is an essential input to the planning of availability evaluation. The value of the planned tests for availability measurement should be estimated by analysis of the ITP, which can then be expanded or amended as program requirements may dictate. Those tests that will contribute data for availability measurement must first be identified in accordance with the criteria discussed in Section 4.1, and their expected contribution indicated. This is accomplished by completing forms such as the Test Identification Form and Test Evaluation Form shown in Figures 4-1 and 4-2. These forms are similar to those specified by NAVWEPS OD 29304, for use in reliability analysis.

To complete the analysis, it will be necessary to resolve any questions that may arise as to whether tests of individual equipments or other portions of the subsystem will be employed for availability measurement, as well as tests of the full subsystem configuration. In general, tests of hardware at lower indenture levels should be accepted whenever the criteria of substantial mission equivalence can be satisfied with respect to operating environment, functional use and maintenance conditions. With these criteria as guidance, analysis of the ITP is made in the manner described in NAVWEPS OD 29304, except that times are expressed in minutes or hours rather than in equivalent missions.

Analysis of the ITP will yield an estimate of the total test time that will have accumulated for availability measurement at selected points in the development and pilot production

Type of Test	Level of Test	Purpose of Test	Hardware Involved	Test Duration	Operating Time	Pass/Fail Criteria	Maintenance Constraints

Figure 4-1. Test Identification Form

Hardware Name	Test Environment	Estimated Test Time			Time Required to Measure Availability	Difference
		Subsystem	Equipment	Total		

Figure 4-2. Test Evaluation Form

phases or at their conclusion. The following paragraphs describe how that information can be used to estimate the availability measurable during the program.

The test time required to measure or demonstrate a specified availability, at a specified confidence, is a random variable. It is quite possible, however, to predict its expected value or to predict the probability that a specified availability/confidence will be measured by the end of a program, once the program has been analyzed as described above. The technique is discussed here in terms of the subsystem as a unit, but it should be recognized that the same procedure is applicable at subordinate hardware levels.

Predictions of subsystem reliability and maintainability, embodied in the estimates $\widehat{\lambda}$ and $\widehat{\mu}$ will have been obtained as outputs of the system analysis function (Section 2). In analyzing the test plan, it is assumed that those predictions are accurate and that the quotient (λ/μ), commonly called the servicing factor, will determine the availability which the subsystem will exhibit during the test program. Further, exponentially distributed maintenance times are assumed, so as to avoid the need to predict their variance. It is also assumed that no data will be lost during testing, so the observed number of failures and repairs will always be equal. On the basis of the ITP analysis, an estimate is made of the number of failures (and repairs) that will have been experienced at selected points in the test program. Then, by reference to the tabulated F statistic, the contractor can plot the expected growth curve of measured availability vs. test time and can evaluate the adequacy of the test program by comparison of the growth curve with program requirements.

Substitution of equation (3-13) into equation (3-14) gives equation (4-1), which shows that the lower $(1 - \alpha)$ 100 percent confidence limit on A_i is a function of the observed number of failures (= repairs) n, which, in turn, is related probabilistically to cumulative test time t.

$$A_{i_L} = \frac{1}{1 + \left(\frac{\hat{\lambda}}{\hat{\mu}}\right) F_{1-\alpha;\, 2n,\, 2n}} \qquad (4\text{-}1)$$

Figure 4-3 represents equation (4-1), the measured A_{i_L} with n and α parameters. It can be seen that for a given n, A_{i_L} increases with time because $\hat{\lambda}$ continuously decreases.

It can be seen that the measured availability approaches the true availability asymptotically, more rapidly with increasing n, and that it is necessary to observe failures and repairs before a meaningful confidence statement can be made. A specified availability is demonstrated more quickly when it is well below the true availability achieved by the hardware. A_{i_L} also rises more rapidly when the required confidence $(1 - \alpha)$ is low.

It can be shown that given λ, the number of failures most likely to be observed in test time t is the integer part of the product λt.

$$n = [\lambda t] \qquad (4\text{-}2)$$

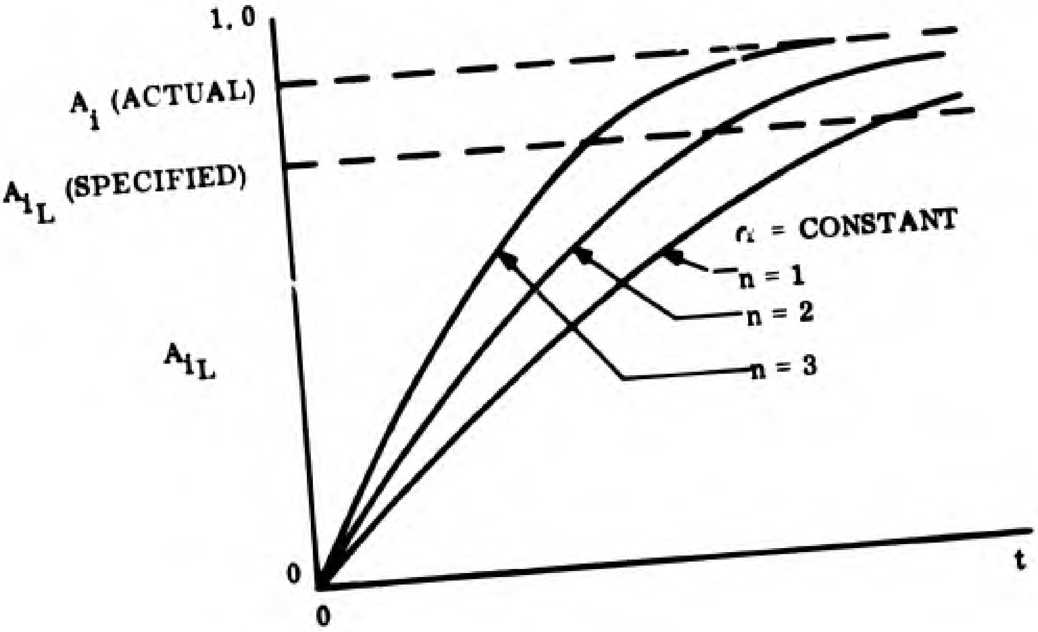


Figure 4-3. Growth of Measured Availability Limit

Thus, Figure 4-3 can be modified to show the most probable growth curve as a function of t. That curve is a saw-tooth function with periodic discontinuities, because only integer numbers of failures and repairs are possible. From a growth curve plotted in the manner of Figure 4-4, the expected test time necessary to measure a specified availability/confidence can be read directly. Or the curve can be entered on the abscissa at the maximum realizable test time projected by analysis of the ITP and an estimate can be read of the A_{i_L} that can be measured in that time. A simple example is given below to clarify the procedure.

Example

Predictions indicate that a subsystem to be evaluated will exhibit exponentially distributed failure and repair times with MTBF = 100 hours and MTTR = 0.5 hour. Therefore, $\hat{\lambda} = .01$, $\hat{\mu} = 2.0$, $\hat{\lambda}/\hat{\mu} = .005$ and $\hat{A}_i = .995$. Approximately 800 hours of subsystem level testing are planned. What inherent availability can be expected to be measured at 80 percent confidence? Is it reasonable to require that the subsystem demonstrate $A_{i_L} > .992$ at 80 percent confidence?

Figure 4-5 is plotted by first scaling off the abscissa in multiples of predicted MTBF. Then, recalling equation (4-2), the most likely value of A_{i_L} is computed for each region of the curve. It can be seen that .992 is a reasonable demonstration requirement and can be expected to be measured at somewhere near 700 hours of testing.

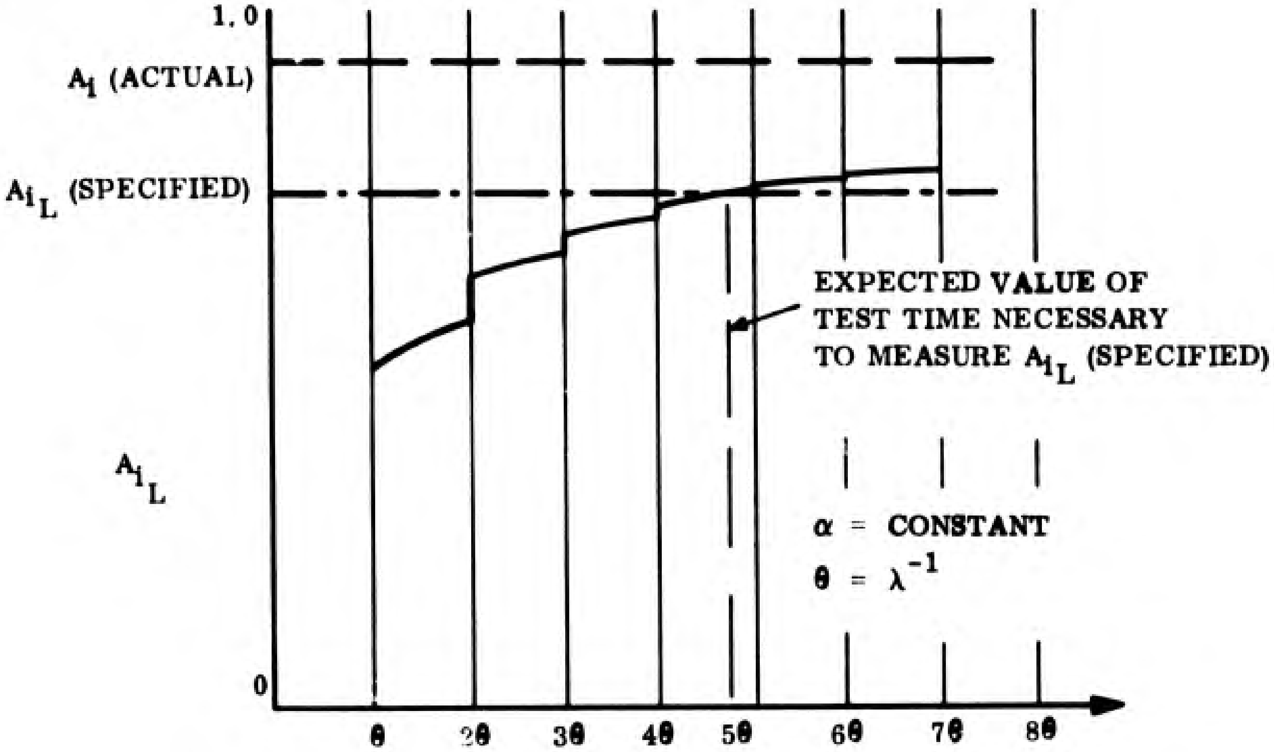


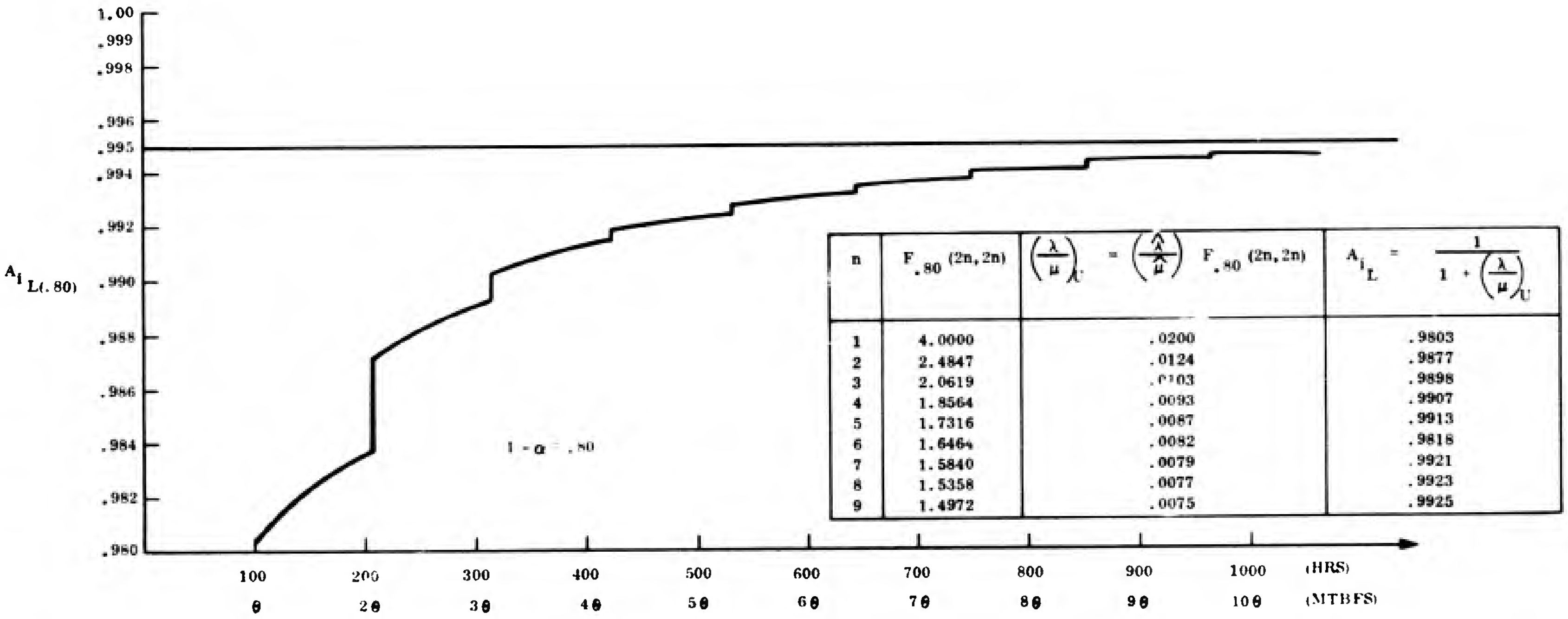
Figure 4-4. Most Probable Growth Curve

4.3 DETERMINING THE QUANTITY OF TESTS REQUIRED

By analysis such as that described above, the contractor can gain an insight, albeit a limited one and hedged by assumptions, into the adequacy of the Integrated Test Program as initially planned. If the test program is considered to be inadequate, the question of how much additional testing should be undertaken must be answered. A variety of criteria can be applied to structure a test program but, basically, a tradeoff must be made, objectively or subjectively, between the cost and the value of additional testing.

One objective approach to such a tradeoff is to write a utility or cost function, then try to optimize its expected value. The function must embody the cost of accruing test time versus the cost and risk of downtime during a mission. The true availability of a subsystem is the probability that it will be up when needed; the risk that it will be down is $U = 1 - A$. If the subsystem is down, the user incurs a loss, which may or may not be expressible explicitly. The actual asymptotic availability of a subsystem cannot be determined precisely while the subsystem is in service, but it may be estimated from test or operating data. Thus, at any time $\widehat{U} = 1 - \widehat{A}$ is the best estimate of the risk implicit in the use of the subsystem. The purpose of availability measurement testing is to assess $\widehat{U}$ and to confirm that it is below an acceptable level.

The difficulty of writing a utility function depends on the complexities of the mission and the circumstances of the development program. Under some conditions, the function may include items of positive utility (payoffs) as well as those of negative utility such as costs.



n	$F_{.80}(2n, 2n)$	$\left(\frac{\lambda}{\mu}\right)_U = \left(\frac{\hat{\lambda}}{\hat{\mu}}\right) F_{.80}(2n, 2n)$	$A_{I_L} = \frac{1}{1 + \left(\frac{\lambda}{\mu}\right)_U}$
1	4.0000	.0200	.9803
2	2.4847	.0124	.9877
3	2.0619	.0103	.9898
4	1.8564	.0093	.9907
5	1.7316	.0087	.9913
6	1.6464	.0082	.9818
7	1.5840	.0079	.9921
8	1.5358	.0077	.9923
9	1.4972	.0075	.9925

Figure 4-5. Subsystem Availability, Projected Growth Curve

Moreover, the costs and payoffs need not be expressed in monetary terms. Any meaningful unit of utility, dimensional or dimensionless, may be used provided all the terms of the function are expressible in that unit.

The function might take a very general form, such as

$$U = U_a - U_r - U_t \tag{4-3}$$

where

U = total utility associated with the availability measurement program

U_a = positive utility associated with the measured availability level

U_r = negative utility associated with residual risk level (unavailability) remaining after measurement

U_t = negative utility associated with the cost of testing

The planner would want to maximize a function such as equation (4-3). In general, it is sufficient to limit the elements of the utility function to costs, which usually can be thought of in conventional monetary terms. Thus, U_a can be neglected and the function written in terms of cost units C. Subsequent effort will be to minimize these since they have negative utility. The subscripts in equation (4-4) have the same meanings as in equation (4-3).

$$C = C_r + C_t \tag{4-4}$$

It can be shown that the best strategy for minimizing C is to minimize its expected value. First, it is necessary to establish clearly what is meant by expected value. If U (x) is the utility associated with a random variable x, the expected value of the utility is

$$E\{U(x)\} = \sum_{i=1}^{n} U(x_i) \cdot P(x_i) \tag{4-5}$$

where $P(x_i)$ is the probability of the variable taking the value x_i. If $U(z_i)$ is a step function of fixed value for each $x_{i-1} < z_i \leq x_i$ and

$$F(X) = \int_0^X f(x)\, dx \tag{4-6}$$

is the non-decreasing distribution function of x, then E [U (x)] is the sum of products

$$E\left[U(x)\right] = \sum_{i=1}^{n} U(z_i)\left[F(x_i) - F(x_{i-1})\right] \tag{4-7}$$

The definition above can be employed to write the expected value of the cost function. The total cost C is the sum of the testing cost C_t and a loss of magnitude C_r if the subsystem is down when needed and zero magnitude if the subsystem is up.

$$E\{C\} = C_r \hat{U} + (0)\hat{A} + C_t \tag{4-8}$$

Fixed costs of testing can be separated from incremental (time dependent) costs. In many cases, the latter increase approximately linearly with test time. Equation (4-8) can then be written as

$$E\{C\} = C_r \hat{U} + C_{to} + C_{ti}\, t \tag{4-9}$$

where:

C_{to} = fixed costs of testing

C_{ti} = incremental cost of one hour (or other unit) of test time

As pointed out in Section 2, it is meaningful to define up and down states of varying degrees for most subsystems. The subsystem's availability is usually different for each such state, as is the associated cost of having the subsystem in that state at the time a demand is made on it during the mission. Then the first term of equation (4-9) is replaced by a series, one term for each cost/risk level, and these are additive. Equation (4-10) illustrates this point. The cost of each subsystem state is multiplied by the estimated probability of occurrence of exactly that state.

$$E\{C\} = \sum_{i=1}^{n} C_{r_i}\left(\hat{U}_i - \hat{U}_{i-1}\right) + C_{ti}\, t + C_{to} \tag{4-10}$$

A function such as (4-10) can be minimized by analytical or graphic methods to estimate a test time t_0, which is optimum in that no other quantity of testing will yield a lower expectation of total cost. Figure 4-6 illustrates the tradeoff process conceptually. In some cases, it may not be reasonable to define the cost of mission downtime or failure in conventional terms. This is so, for example, if the social or political consequences of failure are the major concerns of the planner. Such a situation would dictate that a high level of availability be specified, together with a measurement test program of sufficient scope to verify achievement of satisfactory availability in the operational phase.

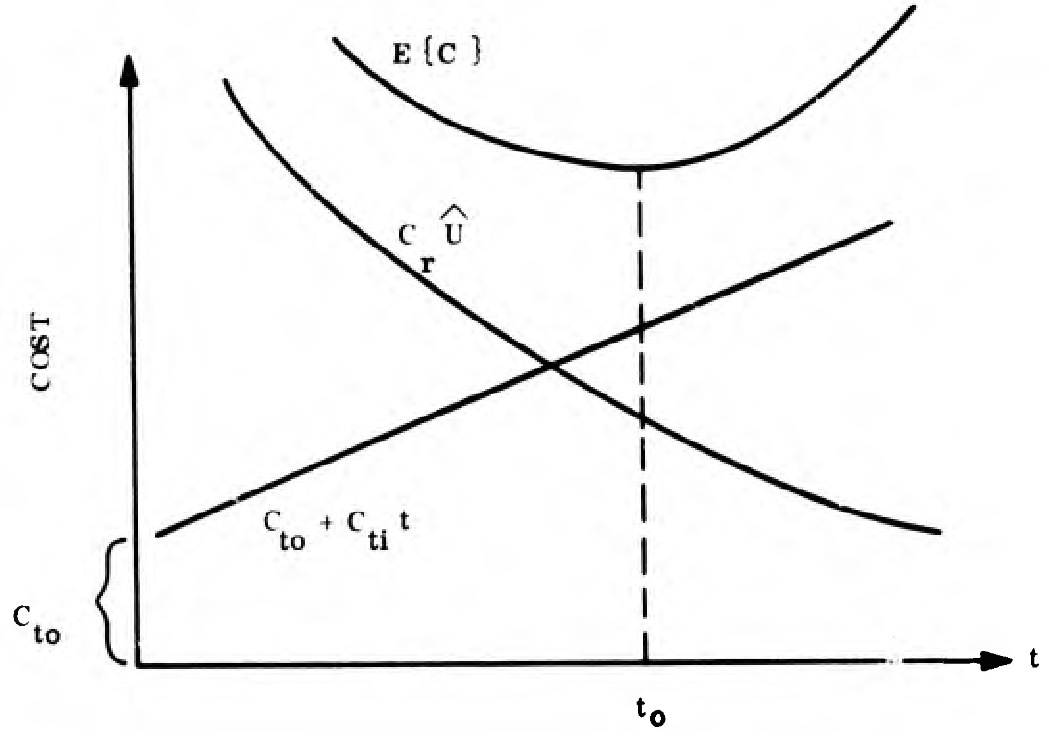


Figure 4-6. Cost Tradeoff Curve

The technique described above assumes that measurement data are gathered in real time, that is, by observing failures and repairs that occur naturally in the course of testing. In many programs, little useful maintainability data are realized prior to the pilot production phase. It should be noted that maintainability data can also be collected in "accelerated" time by simulating failures in the subsystem, in accordance with procedures such as those given in MIL-STD-471. As a result, maintainability demonstration is usually a much more economical undertaking than reliability demonstration. However, the value of maintainability demonstration in reducing the uncertainty interval associated with an estimate of A is generally much less than the value of reliability demonstration. This can be shown by applying the error propagation equation

$$\Delta A = \frac{\partial A}{\partial \mu} \Delta\mu + \frac{\partial A}{\partial \lambda} \Delta\lambda \qquad (4\text{-}11)$$

where the Δ symbols denote the respective uncertainty intervals.

Since

$$\frac{\partial A}{\partial \mu} = \frac{\lambda}{(\lambda + \mu)^2} \qquad (4\text{-}12)$$

and

$$\frac{\partial A}{\partial \lambda} = \frac{-\mu}{(\lambda + \mu)^2} \qquad (4\text{-}13)$$

it is clear that when $\lambda << \mu$, as is nearly always the case, the reliability influence is much the greater of the two.

SECTION 5

DATA SYSTEM

5.1 DATA SYSTEM REQUIREMENTS

MIL-Q-21549 establishes a requirement that the contractor maintain an Integrated Data System. NAVWEPS OD 29304 defines in detail a system acceptable for reliability measurement. Availability evaluation requires only straightforward extensions of that data system to encompass the collection, control, processing and utilization of usage mode, performance, and maintenance data, in addition to the operating time and failure data used for reliability measurement. In this manual, discussion of the integrated data system is limited to a summary of its functions and presentation of modifications necessary to implement availability measurement.

Four activities comprise the integrated data system: (1) Data Collection establishes the requirements and instructions to be used by test areas in reporting data; (2) Data Control monitors the reported data to ensure its completeness, accuracy and validity and keypunches it for computer input; (3) Data Processing performs the computer runs and checks to generate a historic test results file; and (4) Data Utilization prepares the required summaries and reports from the historic test results file. Application of the data system depends on the specific data requirements list, which is an output of the system analysis function. It is in that list that the information the data system must provide is defined. The following paragraphs discuss expansion of the four functional areas of the data system to meet the needs of availability measurement. Figure 5-10 outlines information flow in the integrated data system.

5.2 DATA COLLECTION

5.2.1 Factory Data Collection

The data collection activity issues procedures and instructions to test areas for recording test, failure, and maintenance data. It also develops the forms required to record the data and procedures for use of the forms. Information needed for reliability processing includes categories, A, B and C below. Category D gives typical additional information needed for availability measurement.

A. Test Description

1. Test Report Number
2. Test Level – Component Equipment or Subsystem
3. Test Type – Qualification, Acceptance, etc.
4. Test Site

5. Test Environment
6. Date of Test
7. Test Condition – Operating, Non-Operating or Cycling

B. Hardware Identification

1. Hardware Name
2. Hardware Drawing Number
3. Hardware Serial Number
4. Sub-hardware Actually Involved in Test
5. Hardware Level
6. Vendor
7. Project

C. Test Results

1. Operating Time or Cycles
2. Excludable (Non-productive) Time
3. Operating Mode
4. Test Environment (Availability Phase Ambient, Other)
5. Failures

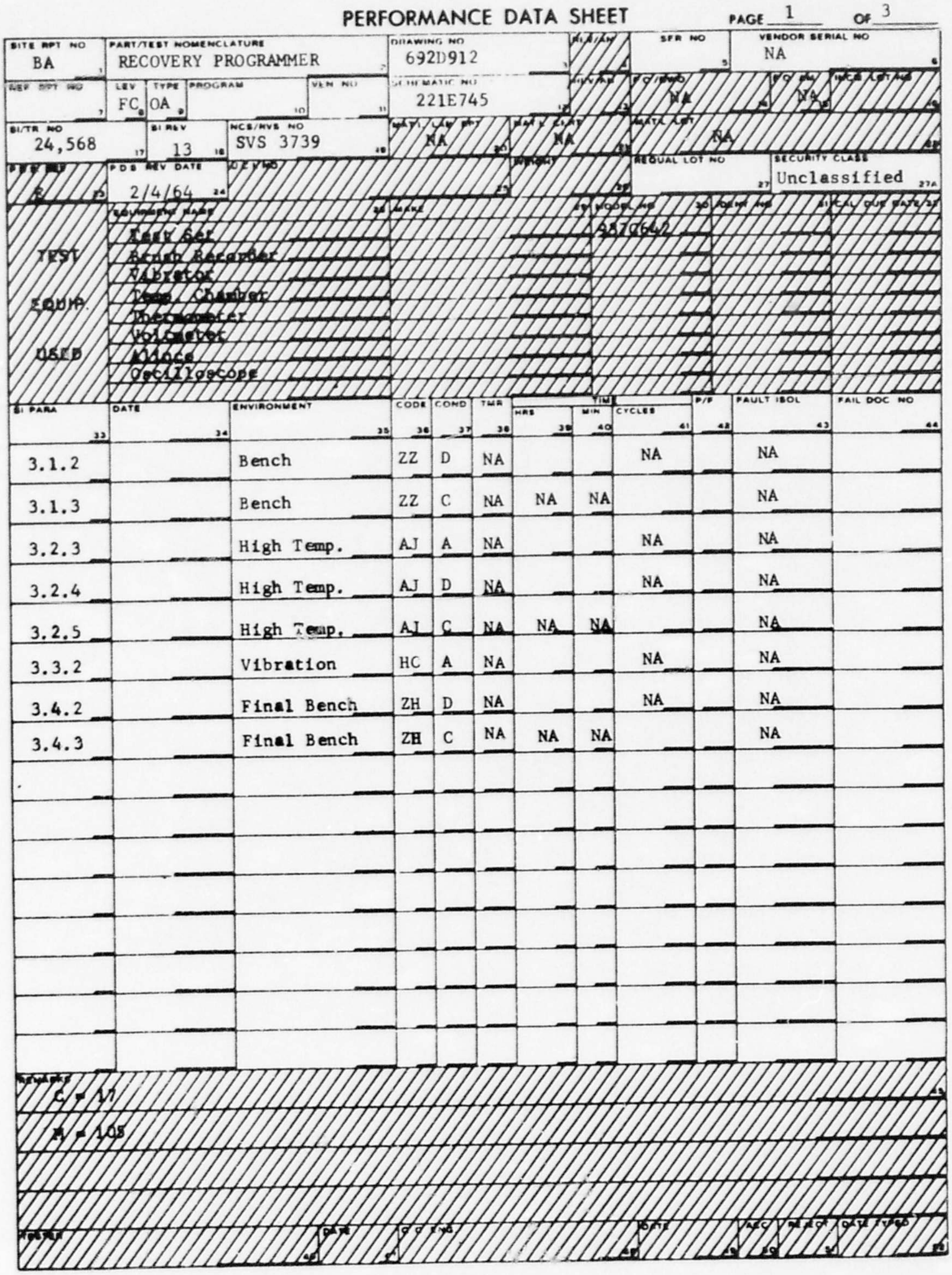
D. Maintenance Data

1. Preventive Maintenance Functions
2. Preventive Maintenance Time Components
3. Corrective Maintenance Functions
4. Corrective Maintenance Time Components
5. Delay Time Components
6. Modification Time
7. Time-to-Failure

Illustrations of typical data sheets applicable for use in test areas to record operating, maintenance, and failure data are given in Figures 5-1 through 5-6.

Figure 5-1 is typical of data collection forms for use in factory tests of equipments or lower indenture-level hardware. It identifies the hardware under test and gives the elapsed test time and test results in minutes and/or cycles as appropriate to the functions tested. Figures 5-2 and 5-3 present typical forms for reporting failure and corrective maintenance data from factory test areas.

Operating data at the subsystem level can be reported by means of utilization logs such as Figure 5-4, whether or not elapsed time indicators are included in the subsystem or in ancillary test equipment. The design and information content of utilization logs should be unique to the subsystem under evaluation. Figures 5-5 and 5-6 are examples. Typically, each one-line entry should fully describe a subsystem performance state and indicate the duration of the state. A new log entry should normally accompany any change of state, such as the failure of an equipment or a change of usage mode. Responsibility for maintenance of logs



PERFORMANCE DATA SHEET PAGE 1 OF 3

SITE RPT NO: BA
PART/TEST NOMENCLATURE: RECOVERY PROGRAMMER
DRAWING NO: 692D912
SFR NO:
VENDOR SERIAL NO: NA
REF RPT NO:
LEV: FC
TYPE: OA
PROGRAM:
VEN NO:
SCHEMATIC NO: 221E745
NA
NA
SI/TR NO: 24,568
SI REV: 13
NCS/RVS NO: SVS 3739
NA
NA
NA
B
PDS REV DATE: 2/4/64
WEIGHT
REQUAL LOT NO:
SECURITY CLASS: Unclassified

TEST EQUIP. USED

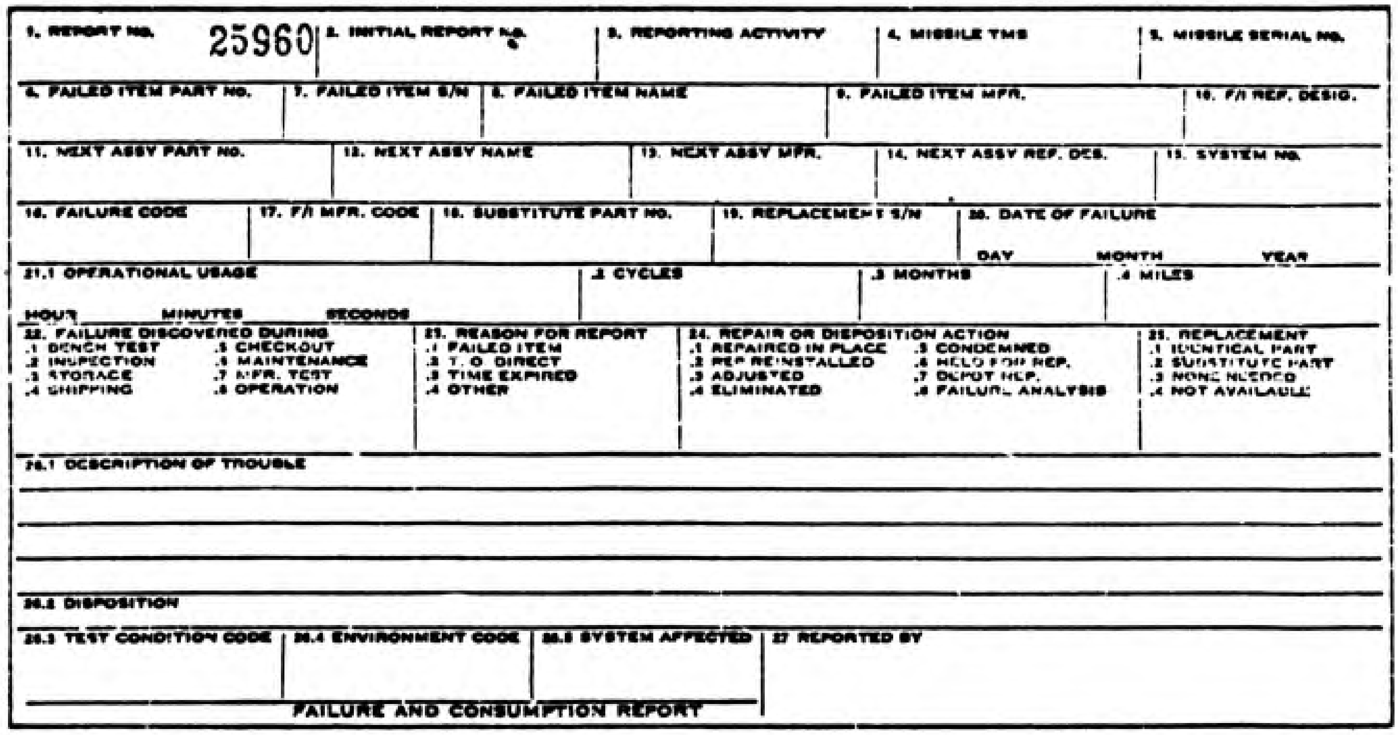
EQUIPMENT NAME	MAKE	MODEL NO
Test Set		9870642
Brush Recorder		
Vibrator		
Temp. Chamber		
Thermometer		
Voltmeter		
[illegible]		
Oscilloscope		

SI PARA	DATE	ENVIRONMENT	CODE	COND	TMR	TIME HRS	TIME MIN	CYCLES	P/F	FAULT ISOL	FAIL DOC NO
3.1.2		Bench	ZZ	D	NA			NA		NA	
3.1.3		Bench	ZZ	C	NA	NA	NA			NA	
3.2.3		High Temp.	AJ	A	NA			NA		NA	
3.2.4		High Temp.	AJ	D	NA			NA		NA	
3.2.5		High Temp.	AJ	C	NA	NA	NA			NA	
3.3.2		Vibration	HC	A	NA			NA		NA	
3.4.2		Final Bench	ZH	D	NA			NA		NA	
3.4.3		Final Bench	ZH	C	NA	NA	NA			NA	

REMARKS
C = 17
M = 105

TESTER | DATE | Q C ENG. | DATE | ACC | REJECT | DATE TYPED

Figure 5-1. Performance Data Sheet



1. REPORT NO. 25960

2. INITIAL REPORT NO.

3. REPORTING ACTIVITY

4. MISSILE TMS

5. MISSILE SERIAL NO.

6. FAILED ITEM PART NO.

7. FAILED ITEM S/N

8. FAILED ITEM NAME

9. FAILED ITEM MFR.

10. F/I REF. DESIG.

11. NEXT ASSY PART NO.

12. NEXT ASSY NAME

13. NEXT ASSY MFR.

14. NEXT ASSY REF. DES.

15. SYSTEM NO.

16. FAILURE CODE

17. F/I MFR. CODE

18. SUBSTITUTE PART NO.

19. REPLACEMENT S/N

20. DATE OF FAILURE
DAY MONTH YEAR

21.1 OPERATIONAL USAGE
HOUR MINUTES SECONDS

.2 CYCLES

.3 MONTHS

.4 MILES

22. FAILURE DISCOVERED DURING
.1 BENCH TEST
.2 INSPECTION
.3 STORAGE
.4 SHIPPING
.5 CHECKOUT
.6 MAINTENANCE
.7 MFR. TEST
.8 OPERATION

23. REASON FOR REPORT
.1 FAILED ITEM
.2 T. O. DIRECT
.3 TIME EXPIRED
.4 OTHER

24. REPAIR OR DISPOSITION ACTION
.1 REPAIRED IN PLACE
.2 REP REINSTALLED
.3 ADJUSTED
.4 ELIMINATED
.5 CONDEMNED
.6 HELD FOR REP.
.7 DEPOT REP.
.8 FAILURE ANALYSIS

25. REPLACEMENT
.1 IDENTICAL PART
.2 SUBSTITUTE PART
.3 NONE NEEDED
.4 NOT AVAILABLE

26.1 DESCRIPTION OF TROUBLE

26.2 DISPOSITION

26.3 TEST CONDITION CODE

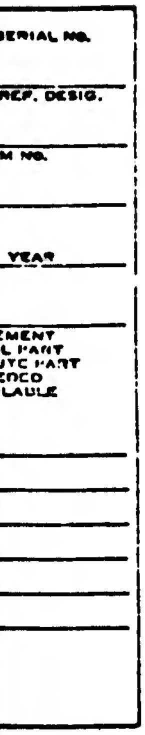
26.4 ENVIRONMENT CODE

26.5 SYSTEM AFFECTED

27 REPORTED BY

FAILURE AND CONSUMPTION REPORT

Figure 5-2a. Failure and Corrective Maintenance Report, Front



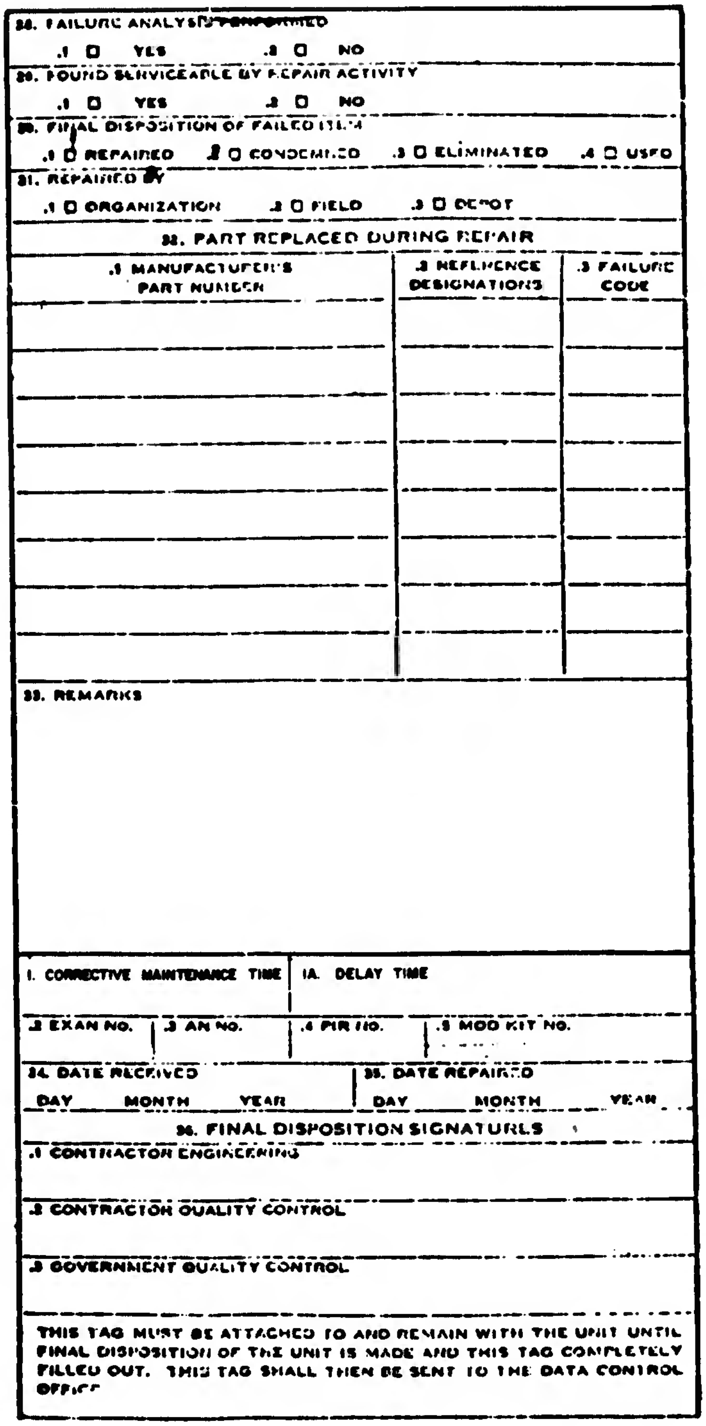
ERIAL NO.

REF. DESIG.

M NO.

YEAR

EMENT
L PART
TC PART
ENDED
LABLE



28. FAILURE ANALYSIS PERFORMED

.1 ☐ YES .2 ☐ NO

29. FOUND SERVICEABLE BY REPAIR ACTIVITY

.1 ☐ YES .2 ☐ NO

30. FINAL DISPOSITION OF FAILED ITEM

.1 ☐ REPAIRED .2 ☐ CONDEMNED .3 ☐ ELIMINATED .4 ☐ USED

31. REPAIRED BY

.1 ☐ ORGANIZATION .2 ☐ FIELD .3 ☐ DEPOT

32. PART REPLACED DURING REPAIR

.1 MANUFACTURER'S PART NUMBER	.2 REFERENCE DESIGNATIONS	.3 FAILURE CODE

33. REMARKS

1. CORRECTIVE MAINTENANCE TIME

1A. DELAY TIME

.2 EXAN NO.

.3 AN NO.

.4 PIR NO.

.5 MOD KIT NO.

34. DATE RECEIVED

DAY MONTH YEAR

35. DATE REPAIRED

DAY MONTH YEAR

36. FINAL DISPOSITION SIGNATURES

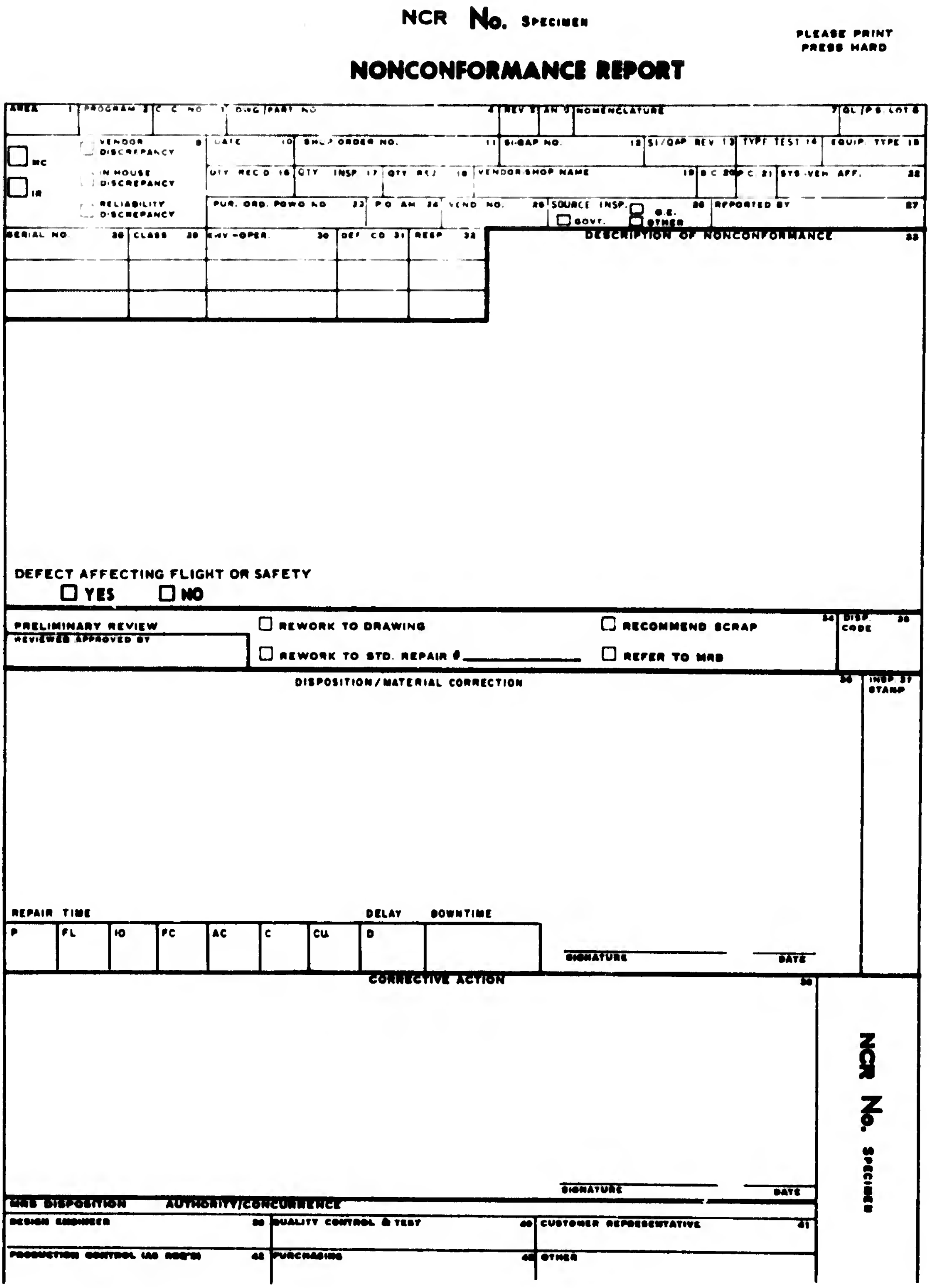
.1 CONTRACTOR ENGINEERING

.2 CONTRACTOR QUALITY CONTROL

.3 GOVERNMENT QUALITY CONTROL

THIS TAG MUST BE ATTACHED TO AND REMAIN WITH THE UNIT UNTIL FINAL DISPOSITION OF THE UNIT IS MADE AND THIS TAG COMPLETELY FILLED OUT. THIS TAG SHALL THEN BE SENT TO THE DATA CONTROL OFFICE

Figure 5-2b. Failure and Corrective Maintenance Report, Back



NCR **No.** SPECIMEN

PLEASE PRINT
PRESS HARD

NONCONFORMANCE REPORT

AREA 1	PROGRAM 2	C C NO 3	DWG/PART NO 4	REV 5	AN 6	NOMENCLATURE 7	QL/PS LOT 8

☐ NC

☐ IR

☐ VENDOR DISCREPANCY 9

☐ IN HOUSE DISCREPANCY

☐ RELIABILITY DISCREPANCY

DATE 10	SHOP ORDER NO. 11	SI-QAP NO. 12	SI/QAP REV 13
TYPE TEST 14	EQUIP. TYPE 15		

QTY REC'D 16	QTY INSP 17	QTY REJ 18	VENDOR-SHOP NAME 19	B C 20	P C 21	SYS-VEH AFF. 22

PUR. ORD. POWO NO 23	P O AM 24	VEND NO. 25	SOURCE INSP. 26 ☐ GOVT. ☐ G.E. ☐ OTHER	REPORTED BY 27

SERIAL NO. 28	CLASS 29	ENV-OPER. 30	DEF CD 31	RESP 32

DESCRIPTION OF NONCONFORMANCE 33

DEFECT AFFECTING FLIGHT OR SAFETY

☐ YES ☐ NO

PRELIMINARY REVIEW

REVIEWED APPROVED BY

☐ REWORK TO DRAWING

☐ REWORK TO STD. REPAIR # ____________

☐ RECOMMEND SCRAP

☐ REFER TO MRB 34

DISP. CODE 35

DISPOSITION/MATERIAL CORRECTION 36

INSP. STAMP 37

REPAIR TIME

DELAY DOWNTIME

P	FL	IO	FC	AC	C	CU	D	

SIGNATURE DATE

CORRECTIVE ACTION 38

SIGNATURE DATE

NCR **No.** SPECIMEN

MRB DISPOSITION AUTHORITY/CONCURRENCE

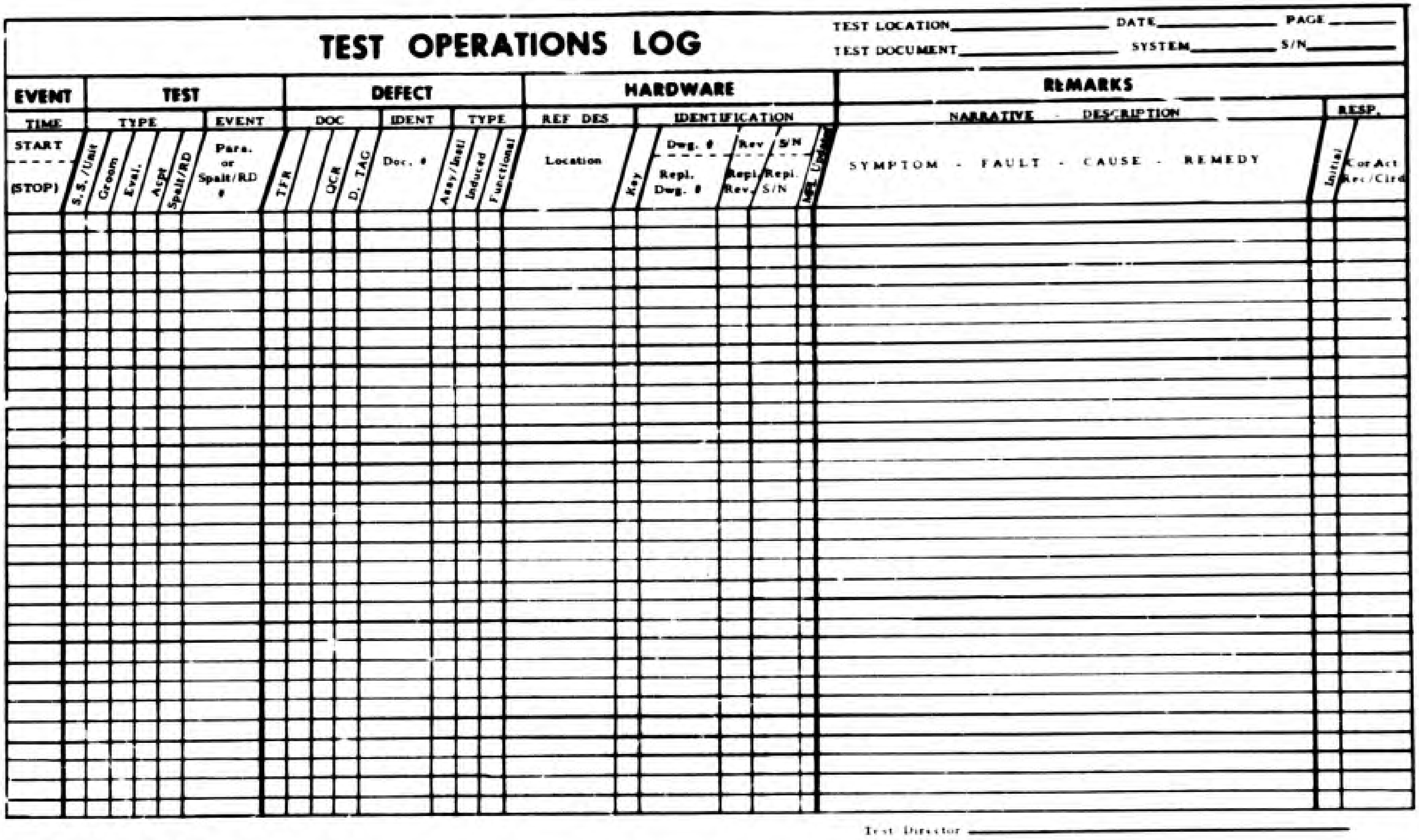
DESIGN ENGINEER 39	QUALITY CONTROL & TEST 40	CUSTOMER REPRESENTATIVE 41
PRODUCTION CONTROL (AS REQ'D) 42	PURCHASING 43	OTHER

Figure 5-3. Nonconformance Report

Subsystem | **SS S/N** | **Subsystem Utilization Log** | Page No. | Prec. Page No.

Date	**Clock Time**		**Elapsed Time This Oper.**	**Mode**	**Use Code**	**ETI Readings**																													Envir.	Oper. Init.	Remarks (Maint. Actions, Failure Report Nos., Etc.)
						Equipment Designators																																
M D Y	**Start**	**Stop**				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			

Figure 5-4. Subsystem Utilization Log



TEST OPERATIONS LOG

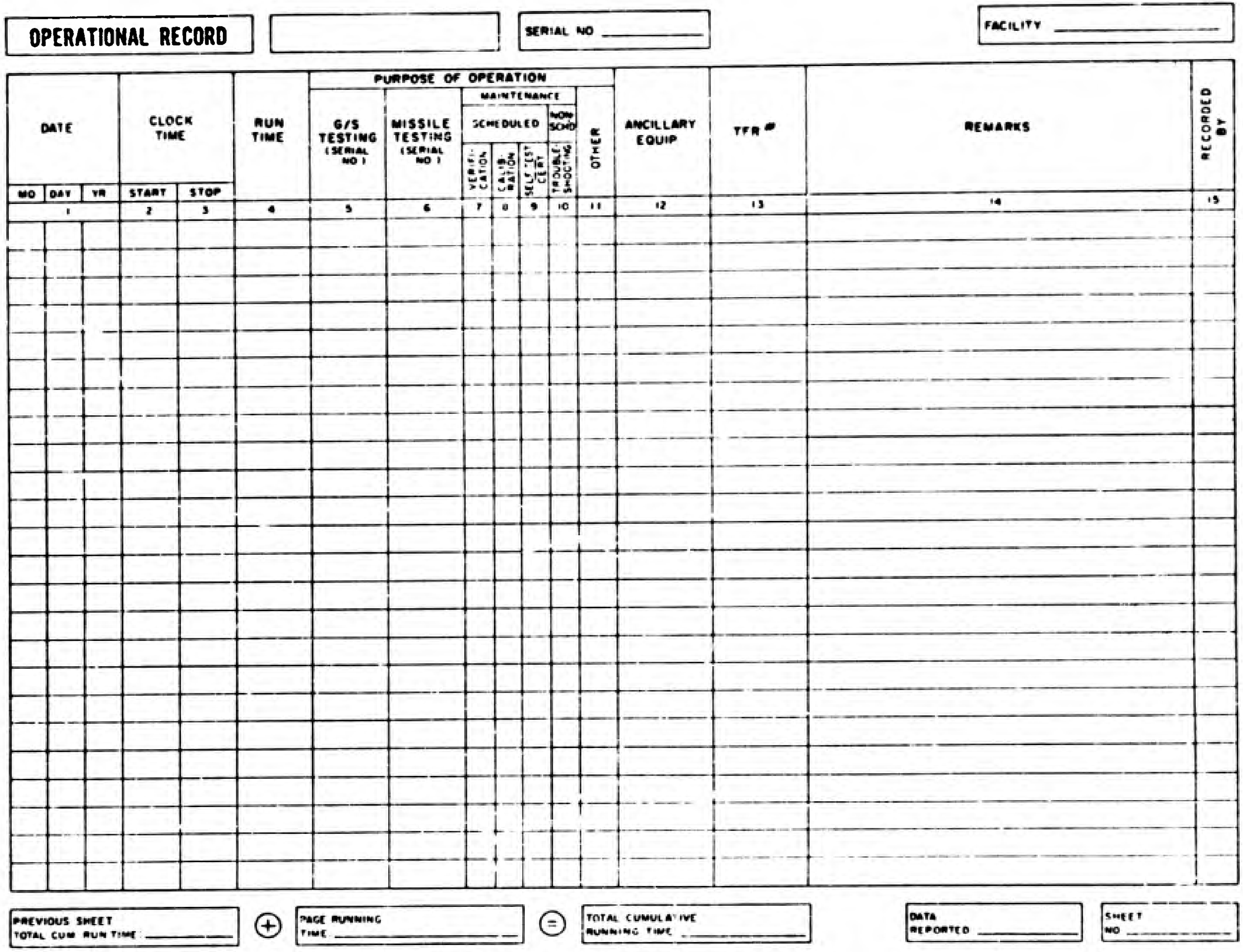
TEST LOCATION________ DATE________ PAGE____

TEST DOCUMENT________ SYSTEM________ S/N________

EVENT	TEST						DEFECT							HARDWARE						REMARKS	RESP.	
TIME	TYPE					EVENT	DOC			IDENT	TYPE			REF DES	IDENTIFICATION					NARRATIVE - DESCRIPTION		
START - - - - (STOP)	S.S./Unit	Groom	Eval.	Acpt	Spalt/RD	Para. or Spalt/RD #	TFR	QCR	D. TAG	Doc. #	Assy/Instl	Induced	Functional	Location	Key	Dwg. # Repl. Dwg. #	Rev Repl. Rev.	S/N Repl. S/N	MPL Update	SYMPTOM - FAULT - CAUSE - REMEDY	Initial	Cor Act Rec/Cird

Test Director ________________

Figure 5-5. Typical Lo Form



OPERATIONAL RECORD

SERIAL NO ____________

FACILITY ____________

DATE			CLOCK TIME		RUN TIME	PURPOSE OF OPERATION							ANCILLARY EQUIP	TFR #	REMARKS	RECORDED BY
						G/S TESTING (SERIAL NO)	MISSILE TESTING (SERIAL NO)	MAINTENANCE				OTHER				
								SCHEDULED			NON SCHD					
MO	DAY	YR	START	STOP				VERIFICATION	CALIBRATION	SELF TEST CERT	TROUBLE-SHOOTING					
1			2	3	4	5	6	7	8	9	10	11	12	13	14	15

PREVIOUS SHEET TOTAL CUM RUN TIME ____________ ⊕ PAGE RUNNING TIME ____________ ⊜ TOTAL CUMULATIVE RUNNING TIME ____________

DATA REPORTED ____________

SHEET NO ____________

Figure 5-6. Typical Log Form

should rest with the test supervisor during periods of testing, training, or experimentation, and with the subsystem maintenance supervisor at other times. Typical information coding for a log such as Figure 5-4 is shown below. It is intended only to guide the development of contractors' data systems.

Clock Time – Local time in military notation at beginning and end of state described.

Elapsed Time This Operation – Difference between start and stop times above.

Functions Tested – Type of test activity, programs run, etc.

Mode – Operating mode, subsystem configuration, etc.

Use Code

100 - Operating Time – Power on and subsystem operating.

101 - Reduced Capacity - Subsystem up at reduced capacity due to known malfunction. Identify failure and give failure report number in remarks column.

110 - Set-Up Time – Power on time used for setup, updating stored references, debugging software, integration of hardware and test equipment, etc.

111 - Lost Time – Time lost due to specification error, procedure error, software error, operator error, test equipment error, facility breakdown, etc.

112 - Idle Power On Time – Subsystem not in use.

120 - Power On Repair Time – Operating time necessary to effect repair. Subsystem down.

121 - Power On Modification Time – Operating time necessary to effect modification or retrofit. Subsystem down.

122 - Power On Preventive Maintenance Time – Operating time necessary to effect preventive maintenance. Subsystem down.

123 - Reduced Capacity Delay Time – Subsystem operating but down because of reduced capacity. Used when supervisor directs that a malfunction not be repaired immediately. Identify malfunction and failure report number in remarks column.

210 - Idle Power Off Time – Subsystem not in use.

220 - Power Off Repair Time – Subsystem down.

221 - Power Off Modification Time – Subsystem down for modification or retrofit.

222 - Power-Off Preventive Maintenance Time – Subsystem down. Approved scheduled maintenance time.

223 - Administrative Delay Time – Power off. Subsystem down.

224 - Supply Delay Time – Used when parts not available within the facility to repair a malfunction. Indicate failure report number and type of failure in remarks column.

Subsystem configuration – When the subsystem configuration is highly variable, the utilization log may be structured as shown in Figure 5-4, so that check marks (✓) in the correct columns can be used to designate the equipments comprising the subsystem in use at the time of an entry. When a line entry describes a failed state, a check can be used to denote the failed equipment.

Environment – Indicate environment if other than room ambient.

Operator – Identification of person making entry, by number or initials.

ETI Reading – Totals on all subsystem elapsed time indicators should be logged at the beginning of each log sheet, and preferably at more frequent intervals. Remarks column may also be used for ETI readings.

Data recorded at the level of detail indicated in the preceding paragraphs enable computation of a variety of availability statistics. For example, if decision rules implied in the use codes listed above and reflected in the time classification categories of Figure 5-7 were deemed applicable for a subsystem, the availability statistics might be estimated as shown below.

	power-on-time	Time logged under use codes 1xx.
U	uptime	Time logged under use codes 10x.
D	downtime	Time logged under use codes x2x.
	inactive time	Time logged under use codes x1x. Under different decision rules, all or part of this time might be classified as uptime and included in the time base for measuring A_O.
	active time	Calendar time less inactive time.
	delay time	Time logged under use codes 123, 223,224.
n_O	total malfunctions	The sum of all equipment failures.
n_i	relevant malfunctions	The sum of all relevant failures.

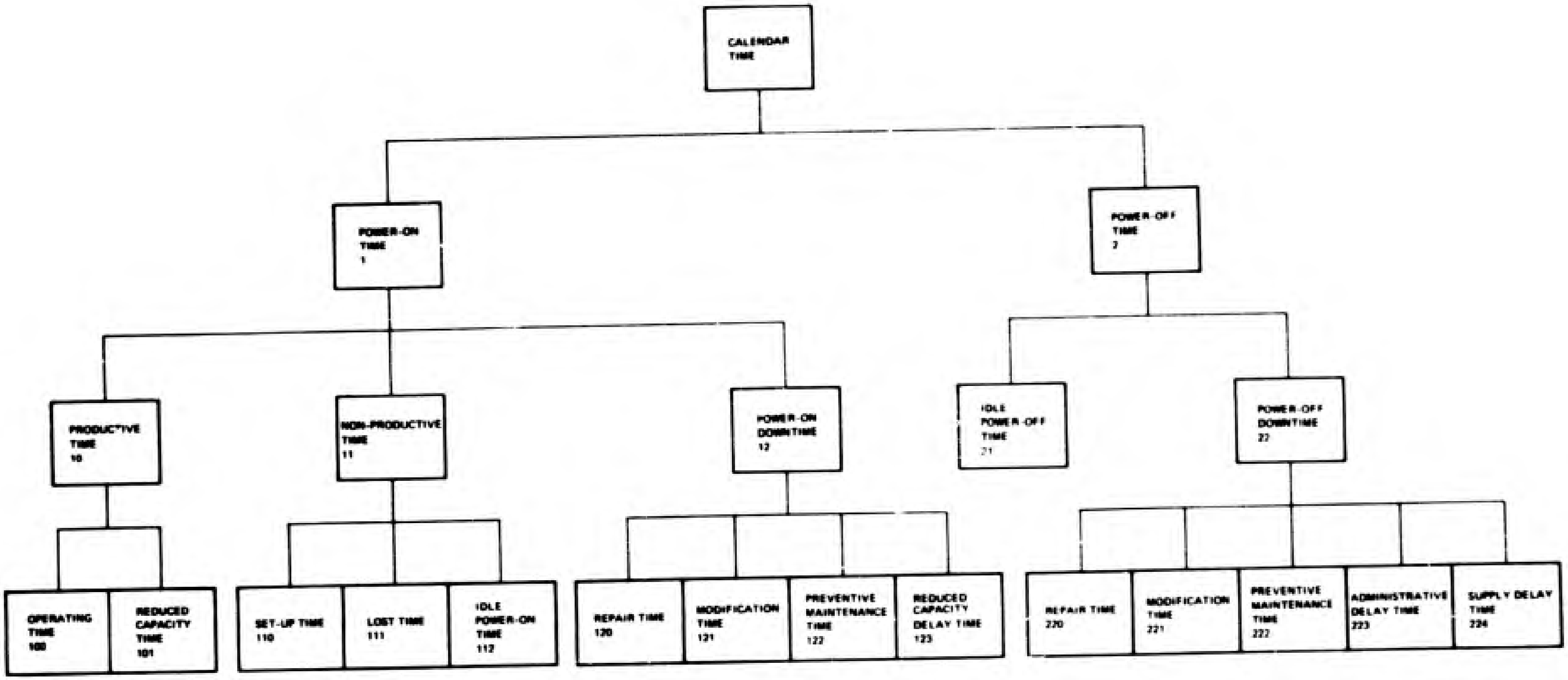
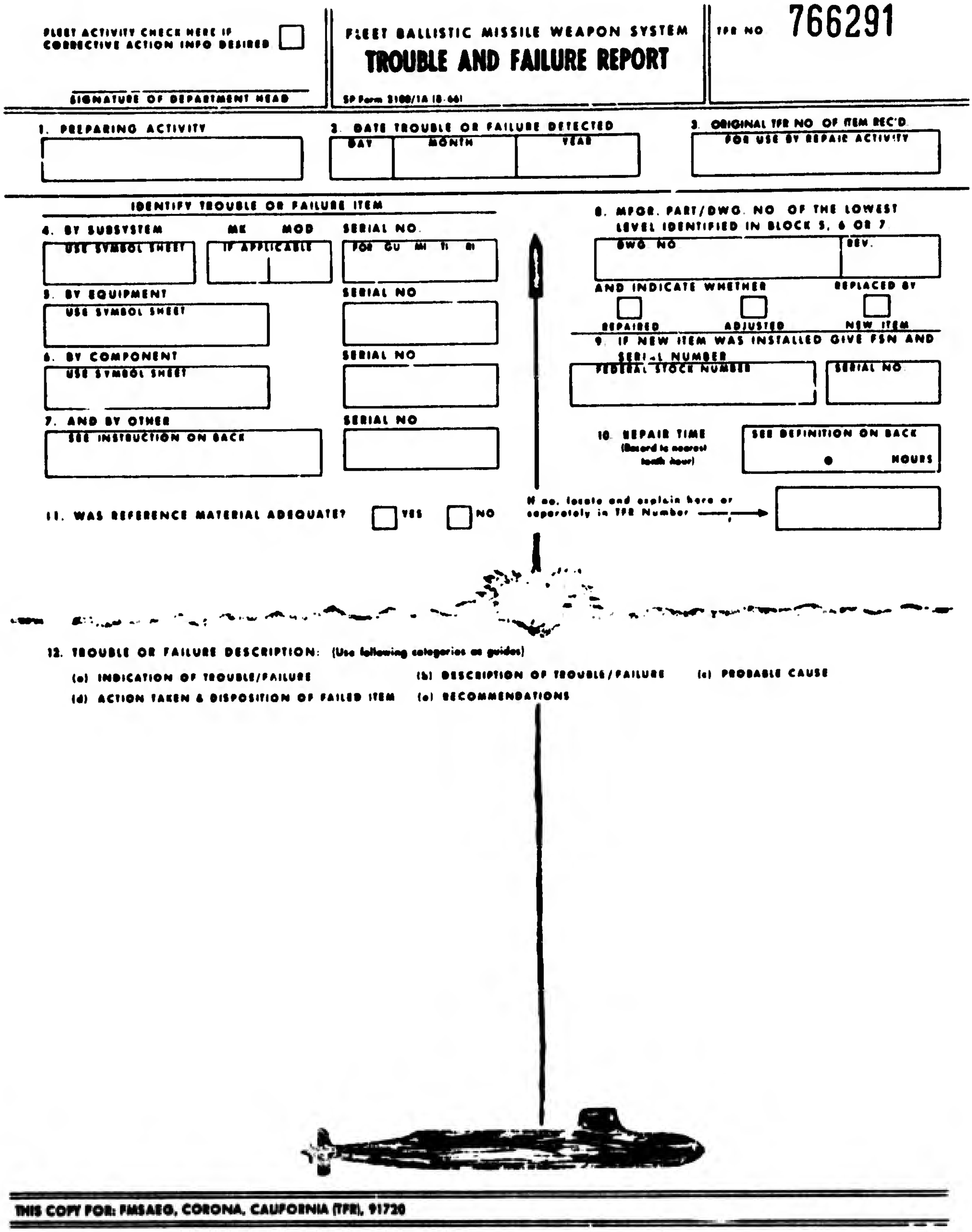


Figure 5-7. Typical Test Time Categories

	observed failure rate	Computed as n_o/power-on-time.
$\hat{\lambda}_i$	inherent failure rate	Computed as n_i/power-on-time.
M_c	repair time	Time logged under use codes x20.
$\hat{M}_c$	mean repair time	Computed as M_c/n_o.
	unscheduled downtime	Repair time plus delay time.
	mean downtime	Computed as D/total occurrences of use codes x2x.
n_p	preventive maintenance events	Total number of preventive maintenance actions.
M_p	preventive maintenance time	Time logged under use codes x22.
	modification time	Time logged under use codes x21.
$\hat{M}_p$	mean preventive maintenance time	Computed as M_p/n_p.
	observed preventive maintenance frequency	Computed as n_p/power-on-time.
f_i	specified preventive maintenance frequency	Set by specification.
$\hat{A}_i$	inherent availability (availability with respect to failure)	$\hat{\lambda}_i^{-1} \Big/ (\hat{\lambda}_i^{-1} + \hat{M}_{c_i})$
$\hat{A}_p$	availability with respect to preventive maintenance	$f_i^{-1} \Big/ (f_i^{-1} + \hat{M}_{p_i})$
$\hat{A}_a$	achieved availability	$\hat{A}_{i_i} \cdot \hat{A}_{p_i}$
$\hat{A}_o$	operational availability	$\frac{U}{U+D}$

5.2.2 Fleet Service Data Collection

Operational availability data is reported from fleet service by means of the FBMWS Trouble and Failure Reporting Program (SP Instruction 3100.1). The forms illustrated in Figures 5-8 and 5-9 exemplify those used to provide the minimum required information–hardware



FLEET ACTIVITY CHECK HERE IF CORRECTIVE ACTION INFO DESIRED ☐

SIGNATURE OF DEPARTMENT HEAD

FLEET BALLISTIC MISSILE WEAPON SYSTEM

TROUBLE AND FAILURE REPORT

SP Form 3100/1A (8-66)

TFR NO 766291

1. PREPARING ACTIVITY

2. DATE TROUBLE OR FAILURE DETECTED: DAY / MONTH / YEAR

3. ORIGINAL TFR NO OF ITEM REC'D: FOR USE BY REPAIR ACTIVITY

IDENTIFY TROUBLE OR FAILURE ITEM

4. BY SUBSYSTEM: USE SYMBOL SHEET — MK / MOD: IF APPLICABLE — SERIAL NO.: FOR GU MI TI RI

5. BY EQUIPMENT: USE SYMBOL SHEET — SERIAL NO

6. BY COMPONENT: USE SYMBOL SHEET — SERIAL NO

7. AND BY OTHER: SEE INSTRUCTION ON BACK — SERIAL NO

8. MFGR. PART/DWG. NO OF THE LOWEST LEVEL IDENTIFIED IN BLOCK 5, 6 OR 7: DWG NO / REV.

AND INDICATE WHETHER ☐ REPAIRED ☐ ADJUSTED REPLACED BY ☐ NEW ITEM

9. IF NEW ITEM WAS INSTALLED GIVE FSN AND SERIAL NUMBER: FEDERAL STOCK NUMBER / SERIAL NO

10. REPAIR TIME (Record to nearest tenth hour): SEE DEFINITION ON BACK ● HOURS

11. WAS REFERENCE MATERIAL ADEQUATE? ☐ YES ☐ NO

If no, locate and explain here or separately in TFR Number ⟶

12. TROUBLE OR FAILURE DESCRIPTION: (Use following categories as guides)

(a) INDICATION OF TROUBLE/FAILURE (b) DESCRIPTION OF TROUBLE/FAILURE (c) PROBABLE CAUSE

(d) ACTION TAKEN & DISPOSITION OF FAILED ITEM (e) RECOMMENDATIONS

THIS COPY FOR: FMSAEG, CORONA, CALIFORNIA (TFR), 91720

Figure 5-8. Trouble and Failure Report

FLEET BALLISTIC MISSILE WEAPON SYSTEM
ELAPSED TIME METER RECORD
SSB (N) 640 CLASS WEAPON DEPARTMENT

SP FORM 3100.1B15 (6-65)

HULL NO. AND PATROL NO. (or Activity Name)	DATE (Day, Month, Year)	TIME

INSTRUCTIONS

Record all indicated elapsed times at approximately the same time on Monday of each week. Be sure Hull No. and Patrol No. or activity name, date and time have been filled in. As soon as possible, air mail original to: Officer in Charge (TFR), U.S. Naval Fleet Missile Systems Analysis and Evaluation Group, Corona, California 91720.

METER	READING	METER	READING
DGBC CH 1		TPRS MK 133-9	
DGBC CH 2		TPRS MK 133-10	
F/C TACTICAL		TPRS MK 133-11	
F/C TEST/TNG		TPRS MK 133-12	
MOTS		TPRS MK 133-13	
CLOCKS		TPRS MK 133-14	
MTRE MK 6 CH 1		TPRS MK 133-15	
MTRE MK 6 CH 2		TPRS MK 133-16	
MTRE MK 7		TPRS MK 133-17	
ALIGNMENT CONTROLLER(PORT)		ULCER PREDICTOR P.S.	
ALIGNMENT CONTROLLER(STBD)		ULCER SS CONTROL AND SYNC	
AUTOCOLLIMATOR CONTROL UNIT		ULCER WV TRANSMITTER	
TPRS MK 133-1		MTRE MK 6 S/N	
TPRS MK 133-2		MTRE MK 6 S/N	
TPRS MK 133--3		MTRE MK 7	
TPRS MK 133-4			
TPRS MK 133-5			
TPRS MK 133-6			
TPRS MK 133-7			
TPRS MK 133-8			

DGBC CH 1, DGBC CH 2, F/C TACTICAL, F/C TEST/TNG, MOTS, CLOCKS, MTRE MK 6 CH 1, MTRE MK 6 CH 2, and MTRE MK 7 Meters are located on the 001A door of the Power Input Panel.

ALIGNMENT CONTROLLER (PORT and STBD) Meters are located on the Alignment Controller Timer and Switch Panel.

AUTOCOLLIMATOR CONTROL UNIT Meter is located on the Autocollimator Control Unit Relay and Indicator Module.

TPRS MK 133 Meters are located on the Display Panel on each unit.

ULCER Meters are located on each front panel.

MTRE MK 6 Record Serial Numbers of each unit from USN BUWEPS Assy Nameplate.

MTRE MK 7 Meter is located on the Power Distribution and Control module at Post 035.

Figure 5-9. Elapsed Time Meter Record

identification, operating time and maintenance time. Data from fleet service can be used in the availability measurement model given herein and can be analyzed and reported by the methods of this manual.

5.3 DATA CONTROL

The specific function of data control is to monitor data collection for timeliness and accuracy and to provide the necessary handling and preparation for computer processing. Data handling encompasses transmission of test data forms from test areas to a central Data Control area, where they are reproduced and distributed. Failure data forms are sent to Failure Analysis for review and classification, then reproduced and distributed.

Data monitoring includes checks of test areas to assure that all data are reported promptly. The data monitoring activity also checks all report forms, as received, for gross errors.

Preparation of test, failure and maintenance data for mechanized processing is accomplished by keypunching the information contained on the forms to standard electronic accounting machine (EAM) cards in formats suitable for computer usage. Definition of EAM formats must include:

a) One-to-one correspondence of data fields on original test or failure document with fields on EAM card.

b) Special handling instructions for any particular data form or information field.

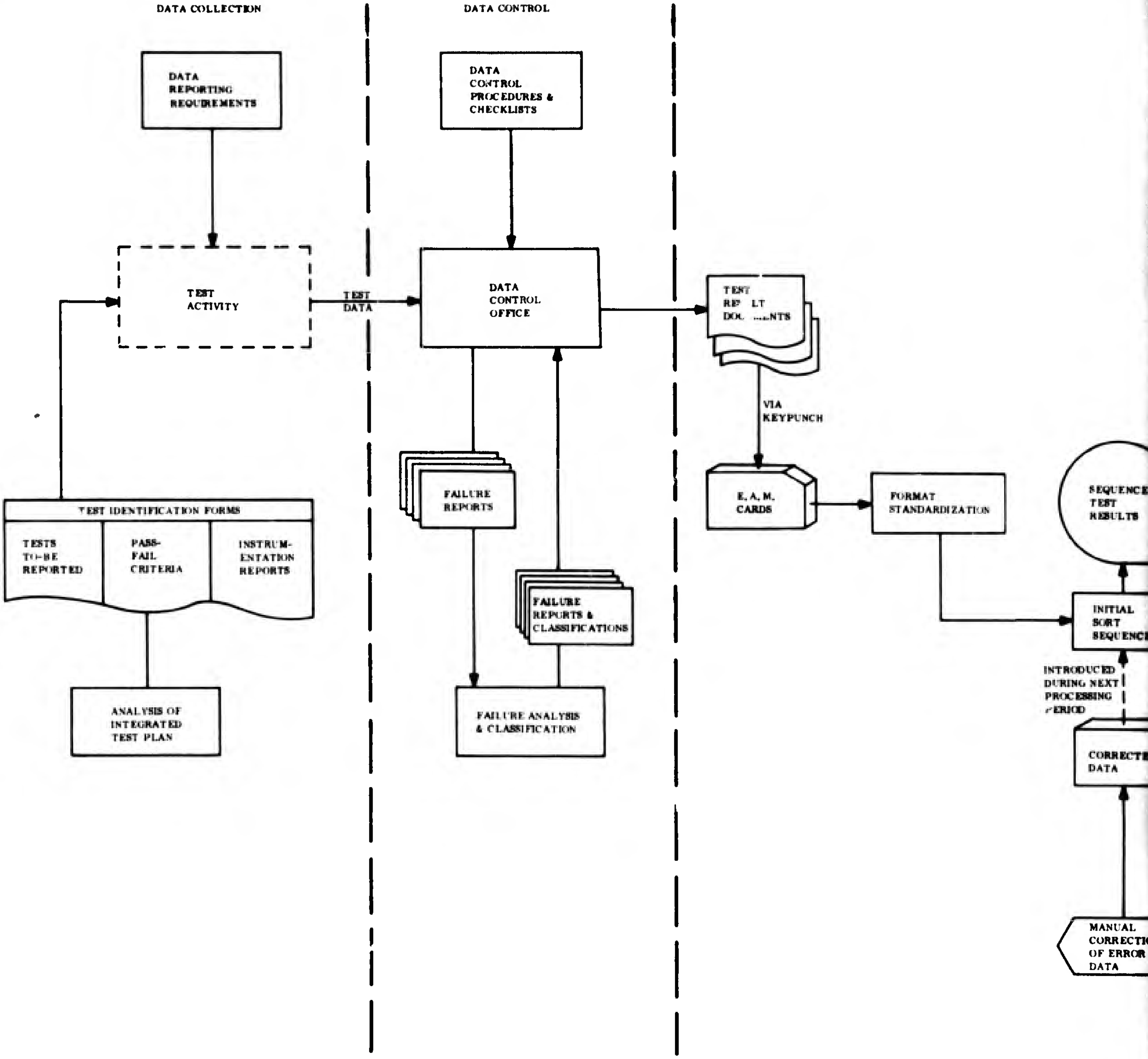
Detailed checking for data errors is accomplished in conjunction with the data processing activity by tabular analysis of the data prior to processing. This is done by the computer under software control. Comparing the data against previously established criteria of validity, the computer validates hardware identifications, test descriptions, definition of alphanumeric fields, ranges or dates, and the logic of the presence or absence of certain fields on the source documents. When errors are detected during this processing, listings defining the errors are sent to Data Control, which must then provide for correcting the error. Corrected data are again prepared for processing. A suspense file is provided to assure that all data errors have been corrected prior to final processing. It contains a list of all errors that have not been corrected.

5.4 DATA PROCESSING

The function of Data Processing is to generate and maintain a historic file of test, failure and maintenance data for each level of hardware, using information furnished by Data Control. In addition to the preliminary processing described above for error correction, several auxiliary processing functions are necessary to enable production of the historic file. Keypunched input formats are rearranged into a standard tape record format for further processing. This provides for standardization of the formats of data originated on different forms from the various test areas. Data are then segregated and grouped according to project. Finally, the historic file is compiled and updated by the addition of recently generated data. In addition to the composite status report, which is the primary output of the processing activity, provisions are made for generating optional data summaries within or across various parameters (serial number, environment, etc.) as desired. Updating of the historic data file is done monthly or more often as required by program reporting requirements.

5.5 DATA UTILIZATION

Data Utilization uses the historic data file to generate various summary reports and to compile an availability status report. Specifically, the utilization program employs the historic file for the solution of the availability model for each equipment and for the subsystem in relation to their defined mission. It also provides for the production of special reports from the historic file, as desired.


DATA COLLECTION
DATA CONTROL
DATA REPORTING REQUIREMENTS
DATA CONTROL PROCEDURES & CHECKLISTS
TEST ACTIVITY
TEST DATA
DATA CONTROL OFFICE
TEST RESULT DOCUMENTS
VIA KEYPUNCH
E. A. M. CARDS
FORMAT STANDARDIZATION
SEQUENCE TEST RESULTS
TEST IDENTIFICATION FORMS
TESTS TO-BE REPORTED
PASS-FAIL CRITERIA
INSTRUMENTATION REPORTS
FAILURE REPORTS
FAILURE REPORTS & CLASSIFICATIONS
INITIAL SORT SEQUENC
INTRODUCED DURING NEXT PROCESSING PERIOD
ANALYSIS OF INTEGRATED TEST PLAN
FAILURE ANALYSIS & CLASSIFICATION
CORRECTE DATA
MANUAL CORRECTI OF ERROR DATA

DATA PROCESSING

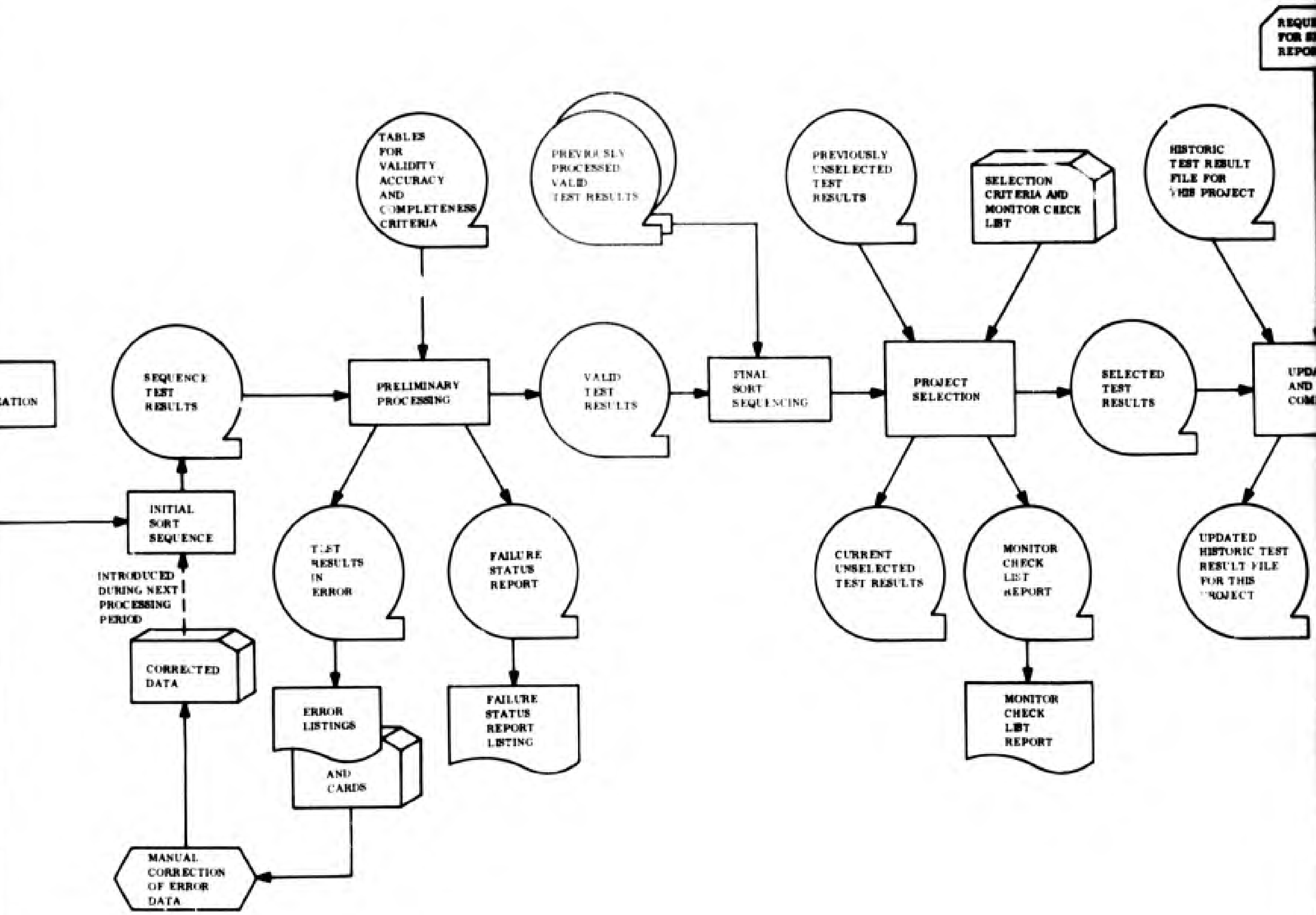


Figure 5-10

2

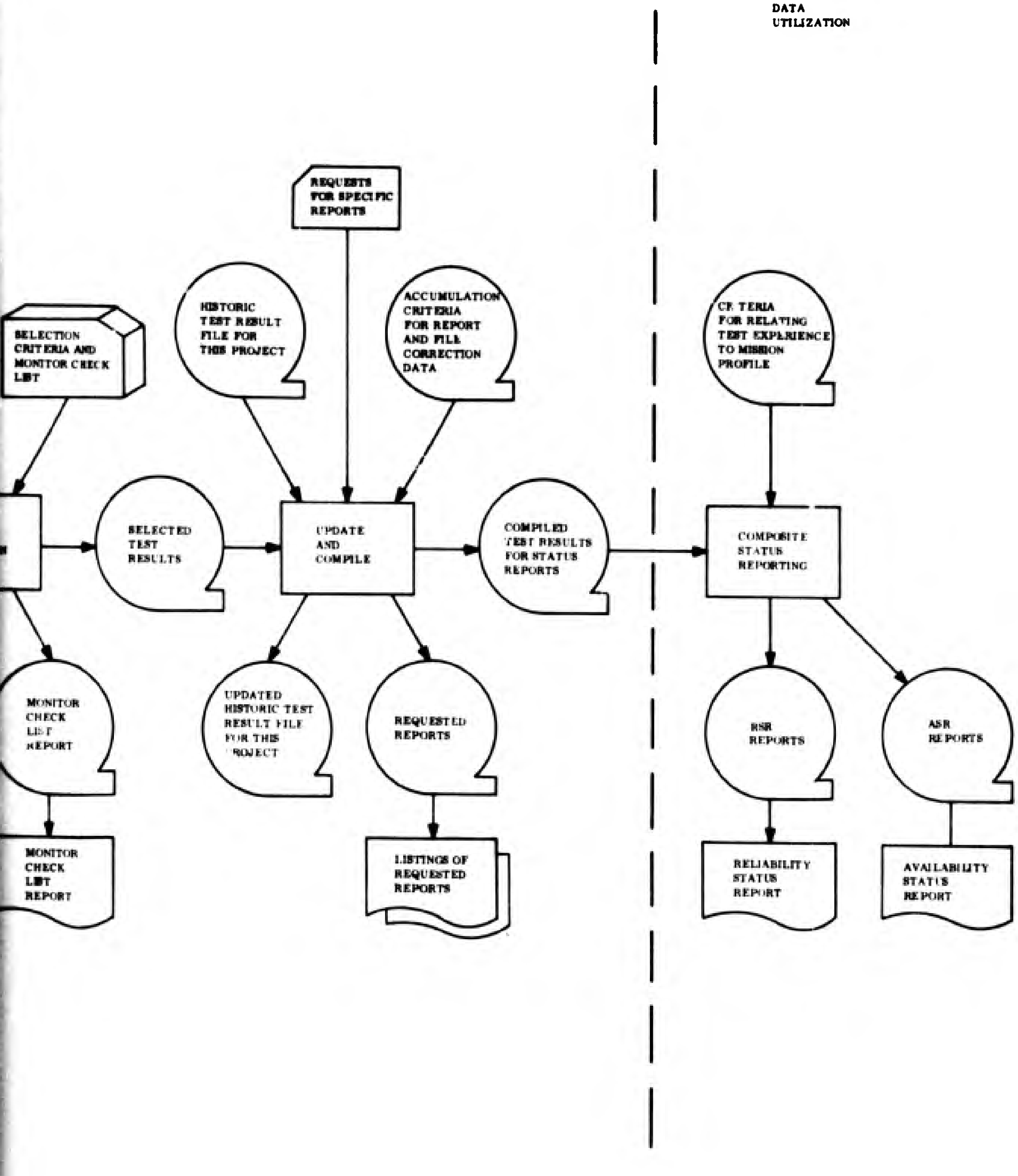


Figure 5-10. Integrated Data System Flow Chart

SECTION 6

OUTPUTS OF AVAILABILITY EVALUATION PROGRAM

6.1 AVAILABILITY EVALUATION PLAN

The evaluation plan is a contractual document prepared by the contractor to describe the scope and content of the evaluation program. First submission typically is required not later than 90 days following the start of development effort. A second submission is usually required 60 days after submission of the integrated test plan. The plan is then updated periodically as required by the SSPO on an annual or more frequent schedule. The evaluation plan contains, as a minimum, the basic analyses and planning necessary to develop the evaluation program. These are:

Analysis	Output
Mission analysis	Definition of tactical mission phases
	Mission profile
System analysis	Definition of subsystem configuration
	Definition of significant subsystem states
	Subsystem availability equation with derivation
	Analysis of preventive and corrective maintenance tasks
	Predictions of availability with respect to preventive maintenance and failure
	Identification of measurement data requirements
	Trade-off studies and apportionment to subsystem elements
Analysis of ITP	Evaluation of ITP
	Projected growth curve
Data system	Criteria for failure classification
	Criteria for classification of time elements
	Sample data collection forms

In addition, the evaluation plan describes the contractor's organization responsible for availability evaluation, the statistical procedures to be used in the evaluation program and the contents and schedules of planned reports.

6.2 AVAILABILITY STATUS REPORT

The Availability Status Report (ASR) is a quarterly contractual report, summarizing the achieved availability status of the subsystem and its equipments in relation to the goals established for them. The report includes at least the following sections:

Introduction – Identification of the contractual requirement for the report and reference to the Availability Evaluation Plan.

Summary and Conclusions -- Highlights of the report, significant events, achievements, etc.

Descriptive Statistics – Current measured system availability indices; best estimates and confidence limits; growth curves showing changes in measured availability during the course of the program; and statistics on measured repair times.

Failures – Summary of failures included and excluded in computing indices, corrective actions.

Problem Areas – Identification of problems and discussions of actions planned and taken.

Appendix – Block diagrams and availability model.

The information contained in the ASR does not obviate the need for the information reported in the Reliability Status Report (RSR) required by NAVWEPS OD 29304. The ASR contains maintainability and availability-phase reliability numerics; the RSR provides launch-phase and flight-phase reliability information. Nevertheless, it should be recognized that in some programs, economies will be realized by combining the two reports.

Preparation of the report is begun by selecting from the computer files of the integrated data system, listings of the relevant statistics describing the measured reliability and maintainability of the subsystem and of the equipments comprising the subsystem. The subsystem model number is employed as a selection criterion for this purpose. These statistics are substituted into the equipment and subsystem availability equations. These estimates, together with the input statistics are presented in tabular form in the report, the format of which appears as Figure 6-1. Availability growth curves are prepared for equipments and the subsystem showing cumulative progress of measured availability since the first report. Significant changes in design or manufacturing are noted on the curves. Equipments showing significantly lower availability than the apportioned levels are identified. Problem areas are discussed with emphasis on cause, nature and effective date of corrective actions. Subsequent quarterly reports should indicate effectiveness of the corrective actions based on more recent experience.

NOMENCLATURE	DWG. NO.	READINESS PHASE															
		A_i (apportioned)	A_i (predicted)	MEASURED INDICES													
				n_o	n_i	n_p	$\hat{\lambda}_i$	$\lambda_{i,.80}$	$\hat{\overline{M}}_c$	$\overline{M}_{c,.80}$	$\hat{\overline{M}}_p$	$\hat{\overline{M}}_{p,.80}$	A_i	$\hat{A}_p$	$\hat{A}_a$	$A_{a,.80}$	$\hat{A}_o$
												if applic-able					

Figure 6-1. Modified Availability Status Report

6.3 CONTRACTOR MANAGEMENT REPORTS

In addition to fulfilling customer reporting requirements, the data utilization function can be used to generate a variety of in-house reports useful to contractor management in preparing analyses and organizing material intended for contractually required reports. These reports and data files are also of value to availability-related functions such as spares provisioning, and for evaluating the limits of the "infant mortality", useful life and wearout phases of various types of hardware. The data base can also be used to evaluate the timeliness, completeness and accuracy of the data system and the effectiveness of the contractor's testing, review and inspection activities. Typical in-house reports, their contents and uses, are discussed in the following paragraphs. In general, the in-house reports described herein differ somewhat from those specified in NAVWEPS OD 29304 for reliability measurement. Thus, in the initial formulation of an integrated data system, the contractor must decide whether to generate independent in-house reports for the availability evaluation program or to combine them with the corresponding outputs of the reliability measurement program.

6.3.1 Historic Data File

This file contains one-line entries corresponding to each entry in the system utilization log and each failure report. Thus, it comprises a complete and accurate record of all test and utilization data in a form that enables the data to be sorted, analyzed and presented quickly and efficiently. The report contains:

A. Hardware Identification
    Subsystem nomenclature
    Subsystem serial number
    Constituent equipments
B. Use Description
    Mode
    Use code
    Environment
C. Subsystem State Data
    Cumulative operating time
    Elapsed time in state
    Failures
    Failure classification data
    Maintenance actions
    Maintenance times
D. Reference Information
    Date
    Site/location
    Log page number
    Failure report number

Beyond the preparation of contractually required reports, a variety of outputs are available to contractor management by manipulation of the data base embodied in the Historic Data

File. For example, the effectiveness of procedures employed by the contractor to eliminate potential defects from a subsystem can be evaluated using summaries of the experience represented in the data file.

A subsystem can be depicted, as in Figure 6-2, in its flow through successive analyses, reviews and tests intended to detect and divert defects from passing downstream to the operational use phase*.

Defects that are present in the subsystem and eligible for detection are shown entering the test block. Within the block some defects are generated in the course of the test. Flowing out of the block are those defects that are detected and diverted and those that escape. Defects that will enter the next screen downstream are the sum of the escapes plus any defects that may have completely by-passed the block for reasons of ineligibility (e.g., certain equipments containing defects had not been installed in time to be detected in the block) or decision (e.g., defects detectable by a test that could have been, but was not, run in the block). Effectiveness of a block can be characterized by a variety of indices such as

$$E = \frac{\text{defects detected}}{\text{total defects present}}$$

which, for Figure 6-2, is 254/(146 + 133) = .91. Such analyses are but examples of feasible applications of the Historic Data File. Figure 6-3 is a sample sheet from a file.

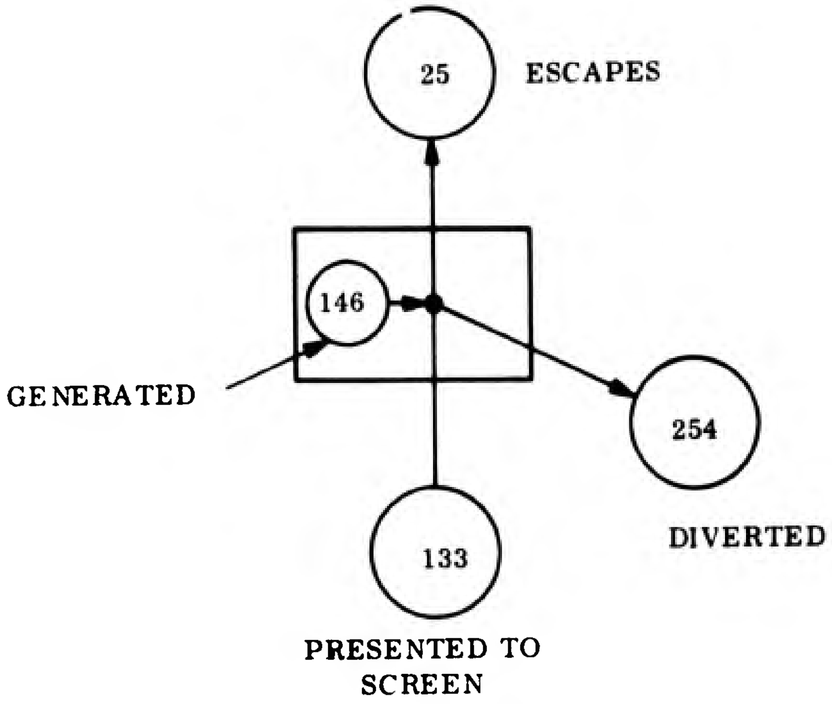


Figure 6-2. Test Screen Model

Smith, A.M. and Waltz, W.R., "Testing for Spacecraft Reliability – A Management Overview", Annals Assurance Sciences 1: No. 2, 1968, pp.214-220.

Project | Historic Data File | Current Date | Page

Nomenclature	Test Description							Operating Time Data								Downtime Data										Reference Data					
				Date							ETI		Elapsed Time					Corr. Maint		Prev. Maint		Delay Time		Mod Time					Entry Date		
	Type	Env.	Loc	Y	M	D	Mode	Use Code	Op. Eff. Code	Eq. Eff. Code	Start	Stop	Hrs	Min.	Log Page	Fail Doc	Fail Code	Hrs	Min.	Hrs	Min.	Hrs	Min.	Hrs	Min.	Op Init	Proj	Shop Order	Y	M	D
Instr. Contr. Grp. I	Dwg. No. 37D02347							Ser. No. 5227766								Vendor Ser. 152										Vendor Code 70321					
	S	A	ST	67	1	4	T	1			187.2	189.3	02	06	154											AC			67	1	8
	S	A	ST	67	1	4	T	1			189.3	191.5	02	12	154											AC			67	1	8
	S	A	ST	67	1	4		3			191.5	191.5	04	00	154					03	00	01	00			JT			67	1	8
	S	A	ST	67	1	5	I	4			191.5	194.5	03	00	154											AC			67	1	8
	S	A	ST	67	1	5	T	8			194.5	194.9	00	25	154											AC			67	1	8
	S	A	ST	67	1	5	T	5			194.9	195.2	00	18	154											AC			67	1	8
	S	A	ST	67	1	5	T	8		22	195.2	195.2	00	00	154	57421		02	12			00	24			JT			67	1	8
	S	A	ST	67	1	5	CO	8	17		195.2	195.7	00	30	155											AC			67	1	8
	S	A	ST	67	1	6	T	1			195.7	199.8	04	06	155											AC			67	1	8
								Ser. No. 5237199								Vendor Ser. 158										Vendor Code 70321					
	S	A	ST	67	3	12	T	1			014.4	017.4	03	00	171											AC			67	3	20
	S	A	ST	67	3	13		7			017.4	017.8	00	30	171									16	[illegible]	JT			67	3	20
	S	A	ST	67	3	14	CO	8	14		017.8	022.0	04	12	171							04	12			AC			67	3	20
	S	A	ST	67	3	15	T	1			022.0	029.3	07	18	171											AC			67	3	20
	S	A	ST	67	3	15	T	8		22	029.3	029.3	00	00	171	58101		01	36			01	12			JT			67	3	20
Instr. Contr. Grp. II	Dwg. No. 37D01980							Ser. No. 4329811								Vendor Ser. 181										Vendor Code 70321					
	S	A	ST	67	5	10	T	1			091.1	099.1	08	00	204											AC			67	5	25
	S	A	ST	67	5	10	T	4			099.1	103.4	04	17	204											AC			67	5	25
	S	A	ST	67	5	11	T	3			103.4	103.4	00	00	204					08	00					JT			67	5	25
	S	A	ST	67	5	11	T	1			103.4	106.4	03	00	204											AC			67	5	25
	S	A	ST	67	5	12	T	8		23	106.4	106.9	00	30	204	58336		00	30							JT			67	5	25
	S	A	ST	67	5	12	T	1		21	106.9	112.9	06	00	204	58337										AC			67	5	25
	S	A	ST	67	5	12		6								58337		04	12			00	50			JT			67	5	25
	S	A	ST	67	5	12	CO	5			112.9	113.2	00	18	204											AC			67	5	29
	S	A	ST	67	5	13	T	1			113.2	118.3	05	05	205											AC			67	5	29
								Ser. No. 4330273								Vendor Ser. 200										Vendor Code 70321					
	S	A	ST	67	7	5	T	1			041.1	042.1	01	00	212											AC			67	7	16
	S	A	ST	67	7	5	T	4			042.1	046.3	04	18	212											AC			67	7	16
	S	A	ST	67	7	6		3			046.3	046.3	00	00	212					02	00	01	15			JT			67	7	16

Figure 6-3. Historic Data File

```
RC#5-0931-05                 PRELIMINARY PROCESSING ERROR DIAGNOSTICS                            03/02           PAGE    1

        DRAWING          POS      CC      SER      VEND SER                                               ERROR  ERROR
                                                                                                          CODE   FIELD

     47C137280G0001   BA10219   01                                                                        09     47C137280
     47C137280G0001   BA10219   01                                                                        09     47C137280
     47C137280G0001   BA10219   01                                                                        09     47C137280
     47C137280G0001   BA10219   01                                                                        09     47C137280
     47C137280G0001   BA10219   01                                                                        09     47C137280
     47C137602P0004   LA10350   01   QM47900            1016                                              15     XL
     47C137602P0006   LA10133   01   00000000           1036                                              15     XL
     47C137602P0006   LA10134   01   QM48100            1037                                              15     XL
     47C137602P0006   LA10135   01   0C000000           1038                                              15     XL
     47C137602P0006   LA10136   01   00000000           1039                                              15     XL
     47C137602P0006   LA10137   01   QM48400            1040                                              15     XL
     47C142573P0003   BA10043   00   QM10600               1                                              27     TO650
     47C142573P0003   BA10044   00   QM10700               2                                              27     TO650
     47C142573P0003   BA10045   00   QM10800               3                                              27     TO650
     47C142573P0003   BA10046   00   QM10900               4                                              27     TO650
     47C142573P0003   BA10047   00   QM11000               5                                              27     TO650
     47C142573P0003   BB10048   00   QM11100               6                                              27     TO650
     47C142573P0003   BB10049   00   QM11200               7                                              27     TO650
     47C142573P0003   BB10050   00   QM11300               8                                              27     TO650
     47C142573P0003   BB10051   00   QM11500              10                                              27     TO650
     64E051281G0013   BA10003   00   MAV27A4                                                              02     MK17
     64E051281G0013   ST10011   00   MAV10A1                                                              02     MK17
     64E051282G0006   BA10025   00                                                                        08
     64E051282G0006   BA10026   01   MA026B2      0000000000                                              15     UZ
     64E051282G0006   BA10026   01   MA026B2      0500000000                                              15     UZ
     67A060761P0001   BA10117   01   QM41300            C218                                              15     NZ
     67A060761P0001   BA10118   01   QM41400            C219                                              15     NZ
```

Figure 6-4. Error List

6.3.2 Error List

An error list is generated as the result of programmed checks of accuracy, validity and completeness made on all inputs to the data processing functions. Errors are returned to the responsible personnel for correction, thereby closing the loop and tending to stabilize the reporting system (i.e., reduce error frequency). The basic tests are for correctness and completeness of data fields, correctness of formats, logical admissibility of reported data, and reconciliation of reported times with ETI readings and with calendar time. The computer can also be programmed to calculate the efficiency of the reporting function by summarizing the contents of the error list in terms of error rates (errors per unit of data reported), and mean reporting delays, etc. attributable to each reporting organization, location, etc. It has been shown that in addition to improving the numerical efficiency, conscientious feedback to the reporting function is necessary to maintain and improve the quality of the information reported*. Figure 6-4 is a specimen Error List.

6.3.3 Serial Number Summary

The Serial Number Summary contains a one-line entry for each piece of serialized component in the subsystem and for each serialized subsystem. It is prepared by the computer from the subsystem utilization logs and failure reports. It reports cumulative operating time on each equipment, number of failures, dates of failures, cumulative operating time at each failure, number of corrective maintenance actions, and the maintenance time of each. The latter four items of information are additions to the content of the Serial Number Report specified by NAVWEPS OD 29304 for reliability measurement.

The summary report is of value for determining the exponential failure regions for various equipment types, the so-called "green line" and "red-line" times as shown in Figure 6-5, and for verifying recovery time distributions, assessing learning effects, etc. A number of statistical procedures can be used to test the fit of observed data to various distributional forms. When computing failure rate as a function of operating time, data may be pooled across serial numbers only if normalized to a common cumulative operating time base. The Serial Number Summary Report provides the information necessary for such analysis. Figure 6-6 is a sample sheet.

6.3.4 Hardware Summary

The Hardware Summary Report (see Figure 6-7) contains a one-line entry for each type of equipment comprising the subsystem. This report contains a summation of total test time and number of failures. It is useful in logistic planning, particularly spares provisioning.

If it is decided that spares are to be provided for an item, and that usage may be expected to follow a Poisson distribution, an estimate of the failure rate (λ) can be used to predict the number of spares (n) necessary to support an item population (N) for a calendar time (T),

---

**Greenberg, G. et al, "Relative Frequency Analysis to Assess Communication Feedback in a Malfunction-Coding Scheme", IRE Trans. Human Factors in Electronics 3: No. 1, 1962, pp. 22-24.*

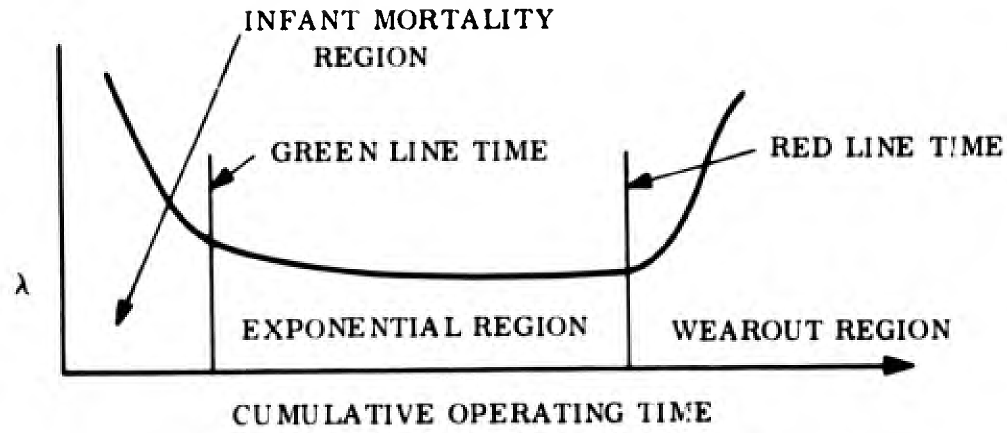


Figure 6-5. Green-line and Red-line Times

with a limiting acceptable risk (ϵ) of not having a spare of the required type at hand when needed. It is necessary to know or estimate the item's duty cycle or fractional operating time ($0 \leqslant k \leqslant 1$) in order to convert calendar time to operating time. The prediction equation is the partial Poisson sum

$$\sum_{i=0}^{n} \frac{e^{-N\lambda kT}(N\lambda kT)^{i}}{i!} \geq 1 - \epsilon \qquad (6\text{-}1)$$

evaluated for n. If spares must be provided for m types of item and the risks ϵ_j are very low, then the risks are approximately additive, so that $\Sigma_{j=i}^{j=m} \epsilon_j$ is roughly the probability that at least one type of spare will not be available when needed during the period T.

6.3.5 Composite Status Report

The Composite Status Report (see Figure 6-8) presents all test data and results by equipments comprising the subsystem, plus all computed reliability, maintainability and availability statistics, best estimates and confidence limits on all indices. It permits manual verification of ASR computations and is of value in plotting growth curves, and in searching for significant trends in the data.

Project Serial Number Summary Current Date Page

Dwg. No.	Ser. No.	Vendor Ser. No.	Cum. Op. Time		Date Last Test			No. of Failures	Date of Failure			Op. Time at Failure		No. Corr. Maint Actions	Cumulative Active Corr. Maint. Time	
			Hrs	Min	Y	M	D		Y	M	D	Hrs	Min		Hrs	Min
37D02347	5227768	152	199	48	67	1	6	6	66	8	10	18	40	6	18	21
									66	10	3	100	02			
									66	10	5	107	21			
									66	12	4	130	25			
									66	12	20	151	10			
									67	1	6	195	12			
	5237199	158	29	18	67	3	15	2	67	2	12	10	10	2	07	41
									67	3	15	29	18			
37D01980	4329811	181	118	18	67	5	13	6	67	1	14	02	25	6	16	50
									67	2	10	21	20			
									67	2	20	28	12			
									67	4	11	80	42			
									67	5	12	106	54			
									67	5	12	112	54			
	4330873	200	046	18	67	7	6	3	67	2	10	9	18	3	08	40
									67	4	4	15	15			
									67	6	20	37	22			
58E44122	0006123	020	212	00	67	6	6	12	66	12	2	06	14	12	33	15
									66	12	29	17	52			
									67	1	10	18	31			
									67	2	10	33	19			
									67	2	14	40	25			
									67	3	30	84	30			
									67	4	8	110	30			
									67	5	15	165	05			
									67	5	20	180	15			
									67	5	29	198	22			
									67	6	1	200	41			
									67	6	4	210	45			
	8106001	029	60	15	67	7	7	3	67	5	5	39	12	2	04	15
									67	6	23	53	19			
									67	6	29	58	48			

Figure 6-6. Serial Number Summary

Dwg. No.	Nomenclature	Vendor Code	Environment Amb.				Environment H. T.				Environment Vib.				Environment Hum.				Total		Date Last Test		
			Hr	Min	U	F	Hr	Min	U	F	Hr	Min	U	F	Hr	Min	U	F	U	F	Y	M	D
37D02347	MEM	65943	901	00	12	8									06	30	4	1	12	9	67	6	10
37C04139	CPU	44832	225	15	3	1													3	1	67	6	10
37C11921	IOD	65943	450	30	6	2													6	2	67	6	10
37B12322	STP	22122	118	20	3	1													3	1	67	6	10
37A11122	DC	16915	48	50	3	2													3	2	67	6	10
37C98764	IOP	44832	200	15	3	1													3	1	67	6	10
37D74213	CCP	65943	200	12	3	2													3	2	67	6	10
37D66881	MTC	16913	116	30	3	3													3	3	67	6	10
37D55411	ICG-1	70321	225	15	3	20													3	21	67	6	10
37C69872	PTR	44832	115	40	3	3													3	3	67	3	20
37C93211	PTRC	16915	72	15	3	1													3	1	67	3	20
37D65123	DCP	88188	72	15	3	2													3	2	67	3	20
37D77180	CC	65943	120	00	3	2													3	2	67	3	20
37D01982	K	22122	240	00	6	3													6	3	67	3	20
37D01980	ICG-2	70321	120	00	3	11													3	11	67	3	20

U = number of units tested
F = number of failures

Figure 6-7. Hardware Summary

Project Composite Status Report Current Date Page

Dwg. No: 37D02347 Equipment: Memory

History

Use Code	Op. E. Code	Eq. E. Code	Type Test	Ambient Hrs.	Ambient Min.	Temp. Hrs.	Temp. Min.	Vib. Hrs.	Vib. Min.	Hum. Hrs.	Hum. Min.	Total Hrs.	Total Min.	Units	Fails	Corr. Maint.	Prev. Maint.
100			ST	901	00					06	30	907	30	12			
122			ST	00	00							00	00				
222			ST	14	00							14	00	7			7
112			ST	02	40							02	40	1			
122			ST	00	45							00	45	1			
210			ST	03	00							03	00	2			
121			ST	10	30							10	30	1			
111			ST	20	25							20	25	1			
110			ST	03	05							03	05	1			
101			ST	06	30							06	30	2	7	7	
220			ST	00	42							00	42	2	2	2	

Indices

$\hat{\lambda}$	$\lambda_{.80}$	$\hat{\mu}$	$\mu_{.80}$	$\hat{A}_i$	$A_{i.80}$	$\hat{A}_p$	$A_{p.80}$	$\hat{A}_a$	$A_{a.80}$	$A_{apportioned}$	$A_{predicted}$
.0101	.0143	1.820	.800	.988	.982	.998	NA	.986	.980	.975	.990

Figure 6-8. Composite Status Report

APPENDIX A

DERIVATION OF MEASUREMENT STATISTICS

A.1 Confidence Limit for Exponential Failure and Recovery Times*

If the failure rate of a device is a constant λ, independent of time, the density of operating times between failures is

$$f(t) = \lambda \ e^{-\lambda t}$$

If test time is truncated by failure, the failure rate is estimated by

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i}$$

where n is the number of failures observed and t_i is the operating time between the ith and the (i - 1) th failures. Then the joint density of $(t_1\ t_2 \ldots.,\ t_n)$ can be written as

$$f(t_1,\ t_2,\ \ldots.\ t_n) = \lambda^n \ e^{-\lambda\left(\sum_{i=1}^{n} t_i\right)}$$

By transformation, the joint density of $(\hat{\lambda}, t_2, t_3, \ldots t_n)$ is obtained

$$g(\hat{\lambda},\ t_2,\ t_3,\ \ldots.\ t_n) = n\ \lambda^n \ e^{\left(-\frac{n\lambda}{\hat{\lambda}}\right)}$$

**The derivation presented here is due to Keesee, W. R., "A Method of Determining a Confidence Interval for Availability", Misc. Publ. No. NMC-MP-65-8, U.S. Naval Missile Center, Point Mugu, Calif. 9 July 1965.*

and the marginal density g (λ) is obtained by integrating over the variables $t_2, t_3, \ldots t_n$.

$$g(\hat{\lambda}) = \frac{\lambda^{n} \left(\frac{1}{\hat{\lambda}}\right)^{n-1} n^{n} e^{\left(-\frac{n\lambda}{\hat{\lambda}}\right)}}{(n-1)!}$$

Now let the random variable $u = 2n\,\lambda/\hat{\lambda}$. The density function g (u) is then a chi-square (χ^2) function with 2n degrees of freedom.

$$g\ (u) = \frac{u^{n-1} e^{\left(-\frac{u}{2}\right)}}{2^{n}\ (n-1)!}$$

If test time is truncated arbitrarily rather than by failure, the final operating time t_{n+1} does not end in a failure. Then the quantity $u = 2n\lambda/\hat{\lambda}$ is distributed as χ^2 with 2n + 2 degrees of freedom – twice the number of failures plus two.

If the repair rate of a device is also a constant μ, independent of time, the density function of repair times is also exponential,

$$f(M_c) = \mu e^{-\mu M_c}$$

μ being estimated by

$$\hat{\mu} = \frac{m}{\sum_{i=1}^{m} M_{c_i}}$$

where m is the number of repairs observed and M_{c_i} is the time required for the ith repair. The density function of $\hat{\mu}$ is obtained by a procedure identical to that for the density of $\hat{\lambda}$.

$$g\ (\hat{\mu}) = (\mu)^{m} \left(\frac{1}{\hat{\mu}}\right)^{m-1} m^{m} e^{\left(-\frac{m\mu}{\hat{\mu}}\right)}$$

Let the random variable $v = \frac{2m\mu}{\hat{\mu}}$. It is assumed that each repair time M_{c_i} is terminated by a repair. Then.

$$g(v) = \frac{v^{m-1} e^{\left(-\frac{v}{2}\right)}}{2^{m}(m-1)!}$$

is a χ^2 density with 2m degrees of freedom. Since u and v are independently distributed with 2n (or 2n + 2) and 2m degrees of freedom, the quantity $\frac{u/2n}{v/2m}$ has the variance ratio density F with 2n (or 2n + 2) and 2m degrees of freedom.

For the case where testing is truncated by failure, a 100 (1 - α) percent lower confidence limit on A_i is computed from the relation

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{\hat{\lambda}}{\hat{\mu}} F_{1-\alpha;\,2n,\,2m}$$

where $F_{1-\alpha;\,2n,\,2m}$ is the (1 - α) fractile of the cumulative F distribution with 2n and 2m degrees of freedom. Then

$$A_{i_L} = \frac{1}{1 + \left(\frac{\lambda}{\mu}\right)_U}$$

For testing truncated arbitrarily, the limit is obtained from

$$\left(\frac{\lambda}{\mu}\right)_U = \frac{n+1}{n} \left(\frac{\hat{\lambda}}{\hat{\mu}}\right) F_{1-\alpha;2n,\,2m}$$

Table A-1 gives $F_{.80;\,f_1,\,f_2}$ for $f_1 = 1\ (1)\ 80$, $f_2 = 1\ (1)\ 80$.

A.2 Estimation of the Parameters of a Log-Normal Distribution*

A random variable $z > 0$ has a log normal distribution if it has the density function

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**A complete discussion of the log-normal distribution can be found in Pangborn, C.E. and Arabadjis, C., "The Log-Normal Distribution", TIS R67SIPD12, General Electric Co., Special Information Products Dept., Syracuse, N.Y., 18 April 1967.*

f_1

f_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	9.4721	12.0000	13.0639	13.6443	14.0084	14.2577	14.4390	14.5766	14.6847	14.7718	14.8435	14.9035	14.9545	14.9983	15.0364	15.0698	15.0994	15.1257	15.1493	15.1706	15.1898
2	3.5556	4.0000	4.1563	4.2361	4.2844	4.3168	4.3401	4.3576	4.3712	4.3822	4.3911	4.3986	4.4049	4.4104	4.4151	4.4192	4.4229	4.4261	4.4290	4.4316	4.4340
3	2.6822	2.8860	2.9359	2.9555	2.9652	2.9707	2.9741	2.9763	2.9779	2.9791	2.9799	2.9806	2.9811	2.9816	2.9819	2.9822	2.9824	2.9826	2.9828	2.9830	2.9831
4	2.3507	2.4721	2.4847	2.4826	2.4780	2.4733	2.4691	2.4654	2.4623	2.4596	2.4572	2.4552	2.4533	2.4517	2.4503	2.4490	2.4478	2.4468	2.4458	2.4450	2.4442
5	2.1782	2.2591	2.2530	2.2397	2.2275	2.2174	2.2090	2.2021	2.1963	2.1914	2.1872	2.1835	2.1803	2.1776	2.1751	2.1729	2.1709	2.1691	2.1675	2.1660	2.1646
6	2.0729	2.1299	2.1126	2.0924	2.0755	2.0619	2.0508	2.0417	2.0342	2.0278	2.0224	2.0177	2.0136	2.0100	2.0068	2.0039	2.0014	1.9991	1.9970	1.9951	1.9933
7	2.0020	2.0434	2.0186	1.9937	1.9736	1.9575	1.9445	1.9339	1.9251	1.9176	1.9113	1.9058	1.9010	1.8968	1.8930	1.8897	1.8867	1.8840	1.8816	1.8793	1.8773
8	1.9511	1.9814	1.9513	1.9231	1.9005	1.8826	1.8682	1.8564	1.8466	1.8383	1.8312	1.8251	1.8198	1.8151	1.8109	1.8072	1.8039	1.8008	1.7981	1.7956	1.7933
9	1.9128	1.9349	1.9007	1.8700	1.8455	1.8262	1.8107	1.7980	1.7874	1.7784	1.7708	1.7642	1.7584	1.7533	1.7488	1.7447	1.7411	1.7378	1.7348	1.7321	1.7296
10	1.8829	1.8907	1.8614	1.8286	1.8027	1.7823	1.7658	1.7523	1.7411	1.7316	1.7235	1.7164	1.7103	1.7049	1.7000	1.6957	1.6918	1.6883	1.6852	1.6823	1.6796
11	1.8589	1.8697	1.8299	1.7955	1.7684	1.7470	1.7298	1.7157	1.7039	1.6940	1.6854	1.6780	1.6716	1.6658	1.6608	1.6562	1.6521	1.6484	1.6451	1.6420	1.6392
12	1.8393	1.8460	1.8042	1.7684	1.7403	1.7182	1.7003	1.6856	1.6734	1.6630	1.6542	1.6464	1.6397	1.6337	1.6284	1.6237	1.6194	1.6155	1.6120	1.6088	1.6059
13	1.8230	1.8262	1.7827	1.7458	1.7169	1.6941	1.6756	1.6605	1.6479	1.6372	1.6280	1.6200	1.6130	1.6068	1.6013	1.5964	1.5920	1.5879	1.5843	1.5809	1.5779
14	1.8091	1.8095	1.7646	1.7267	1.6971	1.6736	1.6547	1.6392	1.6262	1.6152	1.6058	1.5975	1.5903	1.5840	1.5783	1.5732	1.5686	1.5644	1.5607	1.5572	1.5540
15	1.7972	1.7952	1.7490	1.7103	1.6801	1.6561	1.6368	1.6209	1.6076	1.5964	1.5867	1.5782	1.5708	1.5643	1.5584	1.5532	1.5485	1.5442	1.5403	1.5367	1.5334
16	1.7869	1.7828	1.7356	1.6961	1.6653	1.6409	1.6212	1.6050	1.5915	1.5800	1.5700	1.5614	1.5538	1.5471	1.5411	1.5358	1.5309	1.5265	1.5225	1.5189	1.5155
17	1.7779	1.7719	1.7238	1.6837	1.6524	1.6276	1.6076	1.5911	1.5773	1.5656	1.5555	1.5467	1.5389	1.5321	1.5260	1.5205	1.5155	1.5110	1.5069	1.5031	1.4997
18	1.7699	1.7623	1.7134	1.6727	1.6410	1.6159	1.5956	1.5788	1.5648	1.5528	1.5426	1.5336	1.5257	1.5187	1.5125	1.5069	1.5018	1.4972	1.4931	1.4892	1.4857
19	1.7629	1.7538	1.7041	1.6630	1.6309	1.6054	1.5848	1.5679	1.5536	1.5415	1.5311	1.5220	1.5139	1.5068	1.5005	1.4948	1.4896	1.4850	1.4807	1.4768	1.4732
20	1.7565	1.7462	1.6958	1.6543	1.6218	1.5960	1.5752	1.5581	1.5436	1.5313	1.5208	1.5115	1.5034	1.4962	1.4897	1.4839	1.4787	1.4739	1.4696	1.4656	1.4619
21	1.7508	1.7393	1.6884	1.6464	1.6136	1.5876	1.5666	1.5492	1.5346	1.5222	1.5115	1.5021	1.4939	1.4865	1.4800	1.4741	1.4688	1.4640	1.4595	1.4555	1.4517
22	1.7457	1.7331	1.6817	1.6393	1.6062	1.5800	1.5587	1.5412	1.5265	1.5139	1.5030	1.4936	1.4852	1.4778	1.4712	1.4652	1.4598	1.4549	1.4504	1.4463	1.4425
23	1.7410	1.7275	1.6755	1.6328	1.5995	1.5730	1.5516	1.5339	1.5190	1.5063	1.4954	1.4858	1.4773	1.4698	1.4631	1.4571	1.4516	1.4466	1.4421	1.4379	1.4341
24	1.7367	1.7224	1.6699	1.6269	1.5933	1.5667	1.5451	1.5273	1.5122	1.4994	1.4883	1.4787	1.4701	1.4625	1.4558	1.4496	1.4441	1.4391	1.4345	1.4302	1.4263
25	1.7328	1.7176	1.6648	1.6215	1.5877	1.5609	1.5391	1.5211	1.5060	1.4931	1.4819	1.4721	1.4635	1.4558	1.4490	1.4428	1.4372	1.4321	1.4275	1.4232	1.4192
26	1.7292	1.7133	1.6601	1.6166	1.5825	1.5555	1.5336	1.5155	1.5003	1.4872	1.4759	1.4661	1.4574	1.4497	1.4427	1.4365	1.4309	1.4257	1.4210	1.4167	1.4127
27	1.7258	1.7093	1.6558	1.6120	1.5778	1.5506	1.5286	1.5103	1.4949	1.4818	1.4705	1.4605	1.4518	1.4440	1.4370	1.4307	1.4250	1.4198	1.4150	1.4106	1.4066
28	1.7227	1.7056	1.6517	1.6077	1.5733	1.5460	1.5238	1.5055	1.4900	1.4768	1.4654	1.4554	1.4465	1.4387	1.4316	1.4253	1.4195	1.4142	1.4094	1.4050	1.4010
29	1.7199	1.7022	1.6480	1.6038	1.5692	1.5418	1.5195	1.5010	1.4855	1.4722	1.4606	1.4506	1.4417	1.4337	1.4266	1.4202	1.4144	1.4091	1.4043	1.3998	1.3957
30	1.7172	1.6990	1.6445	1.6001	1.5654	1.5378	1.5154	1.4968	1.4812	1.4678	1.4562	1.44[illegible]	1.4371	1.4291	1.4220	1.4155	1.4097	1.4043	1.3994	1.3949	1.3908
31	1.7147	1.6960	1.6413	1.5967	1.5618	1.5341	1.5116	1.4930	1.4772	1.4638	1.4521	1.4419	1.4329	1.4248	1.4176	1.4111	1.4052	1.3998	1.3949	1.3904	1.3862
32	1.7124	1.6932	1.6382	1.5935	1.5585	1.5306	1.5080	1.4893	1.4735	1.4600	1.4482	1.4380	1.4289	1.4208	1.4136	1.4070	1.4011	1.3956	1.3907	1.3861	1.3819
33	1.7102	1.6905	1.6354	1.5905	1.5553	1.5274	1.5047	1.4859	1.4700	1.4564	1.4446	1.4343	1.4252	1.4170	1.4097	1.4031	1.3972	1.3917	1.3867	1.3821	1.3779
34	1.7081	1.6881	1.6327	1.5876	1.5524	1.5244	1.5016	1.4827	1.4667	1.4531	1.4412	1.4308	1.4217	1.4135	1.4061	1.3995	1.3935	1.3880	1.3830	1.3783	1.3740
35	1.7062	1.6858	1.6302	1.5850	1.5496	1.5215	1.4986	1.4796	1.4636	1.4499	1.4380	1.4276	1.4183	1.4101	1.4027	1.3961	1.3900	1.3845	1.3794	1.3748	1.3705
36	1.7044	1.6836	1.6278	1.5825	1.5470	1.5188	1.4958	1.4766	1.4607	1.4469	1.4350	1.4245	1.4152	1.4070	1.3995	1.3928	1.3867	1.3812	1.3761	1.3714	1.3671
37	1.7026	1.6815	1.6256	1.5801	1.5445	1.5162	1.4932	1.4741	1.4580	1.4441	1.4321	1.4216	1.4123	1.4040	1.3965	1.3898	1.3836	1.3780	1.3729	1.3682	1.3638
38	1.7010	1.6796	1.6235	1.5778	1.5422	1.5138	1.4907	1.4715	1.4553	1.4415	1.4294	1.4188	1.4095	1.4011	1.3936	1.3869	1.3807	1.3751	1.3699	1.3652	1.3608
39	1.6995	1.6777	1.6214	1.5757	1.5400	1.5115	1.4884	1.4691	1.4[illegible]	1.4389	1.4268	1.4162	1.4068	1.3984	1.3909	1.3841	1.3779	1.3723	1.3671	1.3623	1.3579
40	1.6980	1.6760	1.6195	1.5737	1.5379	1.5093	1.4861	1.4668	1.4505	1.4365	1.4244	1.4137	1.4043	1.3959	1.3883	1.3815	1.3753	1.3696	1.3644	1.3596	1.3552
41	1.6966	1.6743	1.6177	1.5718	1.5359	1.5073	1.4840	1.4646	1.4483	1.4343	1.4221	1.4114	1.4019	1.3935	1.3859	1.3790	1.3728	1.3671	1.3618	1.3570	1.3525
42	1.6953	1.6727	1.6160	1.5700	1.5340	1.5053	1.4820	1.4626	1.4462	1.4321	1.4199	1.4091	1.3996	1.3912	1.3835	1.3766	1.3704	1.3646	1.3594	1.3545	1.3501
43	1.6940	1.6712	1.6144	1.5682	1.5322	1.5034	1.4800	1.4606	1.4441	1.4300	1.4178	1.4070	1.3975	1.3889	1.3813	1.3744	1.3681	1.3623	1.3570	1.3522	1.3477
44	1.6928	1.6698	1.6128	1.5666	1.5304	1.5016	1.4782	1.4587	1.4422	1.4280	1.4158	1.4050	1.3954	1.3868	1.3792	1.3722	1.3659	1.3601	1.3548	1.3499	1.3454
45	1.6917	1.6684	1.6113	1.5650	1.5288	1.4999	1.4764	1.4569	1.4404	1.4262	1.4138	1.4030	1.3934	1.3848	1.3771	1.3702	1.3638	1.3580	1.3527	1.3478	1.3432
46	1.6906	1.6671	1.6099	1.5635	1.5272	1.4983	1.4748	1.4552	1.4386	1.4244	1.4120	1.4011	1.3915	1.3829	1.3752	1.3682	1.3618	1.3560	1.3506	1.3457	1.3412
47	1.6895	1.6658	1.6085	1.5621	1.5257	1.4967	1.4731	1.4535	1.4369	1.4226	1.4102	1.3994	1.3897	1.3811	1.3733	1.3663	1.3599	1.3541	1.3487	1.3438	1.3392
48	1.6885	1.6646	1.6072	1.5607	1.5243	1.4953	1.4716	1.4519	1.4353	1.4210	1.4086	1.3976	1.3880	1.3793	1.3715	1.3645	1.3581	1.3522	1.3468	1.3419	1.3373
49	1.6876	1.6635	1.6060	1.5594	1.5229	1.4938	1.4701	1.4504	1.4337	1.4194	1.4070	1.3960	1.3863	1.3776	1.3698	1.3627	1.3563	1.3504	1.3450	1.3401	1.3354
50	1.6867	1.6624	1.6048	1.5581	1.5216	1.4925	1.4687	1.4490	1.4323	1.4179	1.4054	1.3944	1.3847	1.3760	1.3682	1.3611	1.3546	1.3487	1.3433	1.3383	1.3337
51	1.6858	1.6613	1.6036	1.5569	1.5203	1.4911	1.4674	1.4476	1.4308	1.4164	1.4039	1.3929	1.3832	1.3744	1.3666	1.3595	1.3530	1.3471	1.3417	1.3366	1.3320
52	1.6849	1.6603	1.6025	1.5557	1.5191	1.4899	1.4661	1.4462	1.4295	1.4150	1.4025	1.3915	1.3817	1.3729	1.3651	1.3579	1.3515	1.3455	1.3401	1.3350	1.3304
53	1.6841	1.6593	1.6014	1.5546	1.5179	1.4886	1.4648	1.4450	1.4281	1.4137	1.4011	1.3901	1.3803	1.3715	1.3636	1.3565	1.3500	1.3440	1.3385	1.3335	1.3288
54	1.6833	1.6584	1.6004	1.5535	1.5168	1.4875	1.4636	1.4437	1.4269	1.4124	1.3998	1.3887	1.3789	1.3701	1.3622	1.3550	1.3485	1.3425	1.3371	1.3320	1.3273
55	1.6826	1.6575	1.599[illegible]	1.5524	1.5157	1.4863	1.4624	1.4425	1.4256	1.4111	1.3985	1.3874	1.3776	1.3688	1.3608	1.3537	1.3471	1.3411	1.3356	1.3306	1.3259
56	1.6818	1.6566	1.5985	1.5514	1.5146	1.4853	1.4613	1.4414	1.4245	1.4099	1.3973	1.3862	1.3763	1.3675	1.3595	1.3523	1.3458	1.3398	1.3343	1.3292	1.3245
57	1.6811	1.6558	1.5976	1.5505	1.5136	1.4842	1.4602	1.4403	1.4233	1.4088	1.3961	1.3850	1.3751	1.3662	1.3583	1.3511	1.3445	1.3385	1.3329	1.3278	1.3231
58	1.6804	1.6549	1.5967	1.5495	1.5126	1.4832	1.4592	1.4392	1.4222	1.4077	1.3950	1.3838	1.3739	1.3650	1.3571	1.3498	1.3432	1.3372	1.3317	1.3265	1.3218
59	1.6798	1.6542	1.5958	1.5486	1.5117	1.4822	1.4582	1.4381	1.4212	1.4066	1.3939	1.3827	1.3728	1.3639	1.3559	1.3486	1.3420	1.3360	1.3304	1.3253	1.3205
60	1.6792	1.6534	1.5950	1.5478	1.5108	1.4813	1.4572	1.4371	1.4201	1.4055	1.3928	1.3816	1.3717	1.3628	1.3547	1.3475	1.3409	1.3348	1.3292	1.3241	1.3193
61	1.6785	1.6527	1.5942	1.5469	1.5099	1.4804	1.4563	1.4362	1.4192	1.40458	1.3918	1.3805	1.3706	1.3617	1.3536	1.3464	1.3397	1.3336	1.3281	1.3229	1.3181
62	1.6780	1.6520	1.5934	1.5461	1.5090	1.4795	1.4554	1.4352	1.4182	1.4035	1.3908	1.3795	1.3695	1.3606	1.3526	1.3453	1.3386	1.3325	1.3269	1.3218	1.3170
63	1.6774	1.6513	1.5927	1.5453	1.5082	1.4786	1.4545	1.4343	1.4173	1.4026	1.3898	1.3785	1.3685	1.3596	1.3515	1.3442	1.3376	1.3315	1.3259	1.3207	1.3159
64	1.6768	1.6506	1.5920	1.5445	1.5074	1.4778	1.4536	1.4335	1.4164	1.4017	1.3889	1.3776	1.3676	1.3586	1.3505	1.3432	1.3365	1.3304	1.3248	1.3196	1.3148
65	1.6763	1.6500	1.5913	1.5438	1.5066	1.4770	1.4528	1.4326	1.4155	1.4008	1.3880	1.3767	1.3666	1.3577	1.3496	1.3422	1.3355	1.3294	1.3238	1.3186	1.3138
66	1.6758	1.6493	1.5906	1.5431	1.5059	1.4762	1.4520	1.4318	1.4147	1.3999	1.3871	1.3758	1.3657	1.3567	1.3486	1.3413	1.3346	1.3284	1.3228	1.3176	1.3128
67	1.6753	1.6487	1.5899	1.5424	1.5052	1.4755	1.4512	1.4310	1.4138	1.3991	1.3862	1.3749	1.3648	1.3558	1.3477	1.3404	1.3336	1.3275	1.3218	1.3166	1.3118
68	1.6748	1.6481	1.5893	1.5417	1.5045	1.4747	1.4505	1.4302	1.4130	1.3983	1.3854	1.3741	1.3640	1.3550	1.3468	1.3395	1.3327	1.3266	1.3209	1.3157	1.3108
69	1.6743	1.6476	1.5887	1.5411	1.5038	1.4740	1.4497	1.4295	1.4123	1.3975	1.3846	1.3732	1.3631	1.3541	1.3460	1.3386	1.3319	1.3257	1.3200	1.3148	1.3099
70	1.6738	1.6470	1.5881	1.5404	1.5031	1.4733	1.4490	1.4287	1.4115	1.3967	1.3838	1.3724	1.3623	1.3533	1.3451	1.3377	1.3310	1.3248	1.3191	1.3139	1.3090
71	1.6734	1.6465	1.5875	1.5398	1.5025	1.4727	1.4483	1.4280	1.4108	1.3960	1.3830	1.3717	1.3616	1.3525	1.3443	1.3369	1.3302	1.3240	1.3183	1.3130	1.3081
72	1.6729	1.6460	1.5869	1.5392	1.5018	1.4720	1.4476	1.4273	1.4101	1.3952	1.3823	1.3709	1.3608	1.3517	1.3435	1.3361	1.3294	1.3232	1.3174	1.3122	1.3073
73	1.6725	1.6455	1.5864	1.5386	1.5012	1.4714	1.4470	1.4267	1.4094	1.3945	1.3816	1.3702	1.3600	1.3510	1.3428	1.3353	1.3286	1.3224	1.3166	1.3114	1.3065
74	1.6721	1.6450	1.5858	1.5381	1.5006	1.4708	1.4464	1.4260	1.4087	1.3939	1.3809	1.3695	1.3593	1.3502	1.3420	1.3346	1.3278	1.3216	1.3159	1.3106	1.3057
75	1.6717	1.6445	1.5853	1.5375	1.5001	1.4702	1.4457	1.4254	1.4081	1.3932	1.3802	1.3688	1.3586	1.3495	1.3413	1.3338	1.3271	1.3208	1.3151	1.3098	1.3049
76	1.6713	1.6440	1.5848	1.5370	1.4995	1.4696	1.4451	1.4248	1.4074	1.3925	1.3796	1.3681	1.3579	1.3488	1.3406	1.3331	1.3263	1.3201	1.3143	1.3090	1.3041
77	1.6709	1.6436	1.5843	1.5364	1.4990	1.4690	1.4446	1.4241	1.4068	1.3919	1.3789	1.3674	1.3573	1.3481	1.3399	1.3324	1.3256	1.3194	1.3136	1.3083	1.3034
78	1.6706	1.6431	1.5838	1.5359	1.4984	1.4685	1.4440	1.4236	1.4062	1.3913	1.3783	1.3668	1.3566	1.3475	1.3392	1.3318	1.3249	1.3187	1.3129	1.3076	1.3026
79	1.6702	1.6427	1.5834	1.5354	1.4979	1.4679	1.4434	1.4230	1.4056	1.3907	1.3777	1.3662	1.3560	1.3468	1.3386	1.3311	1.3243	1.3180	1.3122	1.3069	1.3019
80	1.6698	1.6423	1.5829	1.5349	1.4974	1.4674	1.4429	1.4224	1.4051	1.3901	1.3771	1.3656	1.3554	1.3462	1.3379	1.3304	1.3236	1.3173	1.3115	1.3062	1.3013

TABLE A-1. F–DISTRIBUTION α = .20

f_1

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
.1257	15.1493	15.1706	15.1898	15.2074	15.2234	15.2381	15.2516	15.2641	15.2757	15.2865	15.2965	15.3059	15.3147	15.3229	15.3306	15.3379	15.3448	15.3512	15.3574	15.3632	15.3687	15.3740
.4261	4.4290	4.4316	4.4340	4.4361	4.4381	4.4399	4.4415	4.4431	4.4445	4.4458	4.4470	4.4482	4.4492	4.4502	4.4512	4.4521	4.4529	4.4537	4.4544	4.4552	4.4558	4.4565
.9826	2.9828	2.9830	2.9831	2.9832	2.9833	2.9834	2.9835	2.9835	2.9836	2.9837	2.9837	2.9838	2.9838	2.9838	2.9839	2.9839	2.9839	2.9840	2.9840	2.9840	2.9840	2.9841
.4468	2.4458	2.4450	2.4442	2.4434	2.4428	2.4421	2.4416	2.4410	2.4405	2.4400	2.4396	2.4392	2.4388	2.4384	2.4381	2.4377	2.4374	2.4371	2.4368	2.4366	2.4363	2.4361
.1691	2.1675	2.1660	2.1646	2.1634	2.1623	2.1612	2.1602	2.1593	2.1585	2.1577	2.1569	2.1562	2.1556	2.1550	2.1544	2.1538	2.1533	2.1528	2.1523	2.1519	2.1515	2.1511
.9991	1.9970	1.9951	1.9933	1.9917	1.9902	1.9889	1.9876	1.9864	1.9853	1.9843	1.9834	1.9825	1.9816	1.9808	1.9801	1.9794	1.9787	1.9781	1.9775	1.9769	1.9763	1.9758
.8840	1.8816	1.8793	1.8773	1.8754	1.8737	1.8721	1.8706	1.8692	1.8679	1.8667	1.8656	1.8646	1.8636	1.8626	1.8618	1.8609	1.8601	1.8594	1.8587	1.8580	1.8574	1.8567
.8008	1.7981	1.7956	1.7933	1.7912	1.7893	1.7875	1.7858	1.7843	1.7828	1.7815	1.7802	1.7791	1.7779	1.7769	1.7759	1.7750	1.7741	1.7732	1.7724	1.7717	1.7710	1.7703
.7378	1.7348	1.7321	1.7296	1.7273	1.7252	1.7233	1.7215	1.7198	1.7182	1.7167	1.7154	1.7141	1.7128	1.7117	1.7106	1.7096	1.7086	1.7077	1.7068	1.7060	1.7052	1.7044
.6883	1.6852	1.6823	1.6796	1.6771	1.6749	1.6728	1.6708	1.6690	1.6673	1.6658	1.6643	1.6629	1.6616	1.6604	1.6592	1.6581	1.6570	1.6561	1.6551	1.6542	1.6534	1.6525

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
.6484	1.6451	1.6420	1.6392	1.6366	1.6342	1.6320	1.6299	1.6280	1.6262	1.6245	1.6230	1.6215	1.6201	1.6188	1.6176	1.6164	1.6153	1.6142	1.6132	1.6123	1.6114	1.6105
.6155	1.6120	1.6088	1.6059	1.6031	1.6006	1.5983	1.5961	1.5941	1.5922	1.5905	1.5888	1.5873	1.5858	1.5844	1.5831	1.5819	1.5807	1.5796	1.5786	1.5776	1.5766	1.5757
.5879	1.5843	1.5809	1.5779	1.5750	1.5724	1.5700	1.5677	1.5656	1.5637	1.5618	1.5601	1.5585	1.5570	1.5555	1.5542	1.5529	1.5517	1.5505	1.5494	1.5483	1.5474	1.5464
.5644	1.5607	1.5572	1.5540	1.5511	1.5484	1.5459	1.5435	1.5413	1.5393	1.5374	1.5356	1.5339	1.5323	1.5308	1.5294	1.5281	1.5268	1.5256	1.5245	1.5234	1.5223	1.5213
.5442	1.5403	1.5367	1.5334	1.5304	1.5276	1.5250	1.5226	1.5204	1.5182	1.5163	1.5144	1.5127	1.5110	1.5095	1.5080	1.5066	1.5053	1.5041	1.5029	1.5018	1.5007	1.4996
.5265	1.5225	1.5189	1.5155	1.5124	1.5095	1.5068	1.5043	1.5020	1.4999	1.4978	1.4959	1.4941	1.4924	1.4908	1.4893	1.4879	1.4865	1.4853	1.4840	1.4829	1.4817	1.4807
.5110	1.5069	1.5031	1.4997	1.4965	1.4936	1.4908	1.4883	1.4859	1.4837	1.4816	1.4796	1.4778	1.4760	1.4744	1.4728	1.4714	1.4700	1.4686	1.4674	1.4662	1.4650	1.4639
.4972	1.4931	1.4892	1.4857	1.4824	1.4794	1.4766	1.4740	1.4716	1.4693	1.4671	1.4651	1.4632	1.4615	1.4598	1.4582	1.4567	1.4552	1.4539	1.4526	1.4514	1.4502	1.4491
.4850	1.4807	1.4768	1.4732	1.4698	1.4668	1.4639	1.4612	1.4587	1.4564	1.4542	1.4522	1.4502	1.4484	1.4467	1.4451	1.4435	1.4421	1.4407	1.4393	1.4381	1.4369	1.4357
.4739	1.4696	1.4656	1.4619	1.4585	1.4554	1.4525	1.4498	1.4472	1.4448	1.4426	1.4405	1.4385	1.4367	1.4349	1.4333	1.4317	1.4302	1.4288	1.4274	1.4261	1.4249	1.4237

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
.4640	1.4595	1.4555	1.4517	1.4483	1.4451	1.4421	1.4394	1.4368	1.4344	1.4321	1.4300	1.4280	1.4261	1.4243	1.4226	1.4210	1.4194	1.4180	1.4166	1.4153	1.4140	1.4128
.4549	1.4504	1.4463	1.4425	1.4390	1.4358	1.4327	1.4299	1.4273	1.4248	1.4225	1.4204	1.4183	1.4164	1.4146	1.4128	1.4112	1.4096	1.4081	1.4067	1.4054	1.4041	1.4029
1.4466	1.4421	1.4379	1.4341	1.4305	1.4272	1.4242	1.4213	1.4186	1.4161	1.4138	1.4116	1.4095	1.4075	1.4057	1.4039	1.4023	1.4007	1.3992	1.3977	1.3964	1.3951	1.3938
1.4391	1.4345	1.4302	1.4263	1.4228	1.4194	1.4163	1.4134	1.4107	1.4082	1.4058	1.4035	1.4014	1.3994	1.3975	1.3958	1.3941	1.3925	1.3909	1.3895	1.3881	1.3867	1.3855
1.4321	1.4275	1.4232	1.4192	1.4156	1.4122	1.4091	1.4061	1.4034	1.4008	1.3984	1.3961	1.3940	1.3920	1.3900	1.3882	1.3865	1.3849	1.3833	1.3818	1.3804	1.3791	1.3778
1.4257	1.4210	1.4167	1.4127	1.4090	1.4056	1.4024	1.3994	1.3966	1.3940	1.3916	1.3893	1.3871	1.3851	1.3831	1.3813	1.3795	1.3779	1.3763	1.3748	1.3734	1.3720	1.3707
1.4198	1.4150	1.4106	1.4066	1.4029	1.3994	1.3962	1.3932	1.3904	1.3877	1.3853	1.3829	1.3807	1.3787	1.3767	1.3748	1.3731	1.3714	1.3698	1.3683	1.3668	1.3654	1.3641
1.4142	1.4094	1.4050	1.4010	1.3972	1.3937	1.3904	1.3874	1.3846	1.3819	1.3794	1.3770	1.3748	1.3727	1.3707	1.3688	1.3670	1.3653	1.3637	1.3622	1.3607	1.3593	1.3579
1.4091	1.4043	1.3998	1.3957	1.3919	1.3884	1.3851	1.3820	1.3791	1.3764	1.3739	1.3715	1.3693	1.3672	1.3651	1.3632	1.3614	1.3597	1.3581	1.3565	1.3550	1.3536	1.3522
1.4043	1.3994	1.3949	1.3908	1.3870	1.3834	1.3801	1.3770	1.3741	1.3714	1.3688	1.3664	1.3641	1.3620	1.3599	1.3580	1.3562	1.3544	1.3528	1.3512	1.3497	1.3482	1.3469

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1.3998	1.3949	1.3904	1.3862	1.3823	1.3787	1.3754	1.3723	1.3693	1.3666	1.3640	1.3616	1.3593	1.3571	1.3550	1.3531	1.3512	1.3495	1.3478	1.3462	1.3447	1.3432	1.3418
1.3956	1.3907	1.3861	1.3819	1.3780	1.3744	1.3710	1.3678	1.3649	1.3621	1.3595	1.3571	1.3547	1.3525	1.3505	1.3485	1.3466	1.3449	1.3432	1.3415	1.3400	1.3385	1.3371
1.3917	1.3867	1.3821	1.3779	1.3739	1.3703	1.3669	1.3637	1.3607	1.3579	1.3553	1.3528	1.3505	1.3483	1.3462	1.3442	1.3423	1.3405	1.3388	1.3371	1.3356	1.3341	1.3327
1.3880	1.3830	1.3783	1.3740	1.3701	1.3664	1.3630	1.3598	1.3568	1.3539	1.3513	1.3488	1.3464	1.3442	1.3421	1.3401	1.3382	1.3364	1.3346	1.3330	1.3314	1.3299	1.3285
1.3845	1.3794	1.3748	1.3705	1.3665	1.3628	1.3593	1.3561	1.3530	1.3502	1.3475	1.3450	1.3426	1.3404	1.3383	1.3362	1.3343	1.3325	1.3307	1.3291	1.3275	1.3260	1.3245
1.3812	1.3761	1.3714	1.3671	1.3630	1.3593	1.3558	1.3526	1.3495	1.3467	1.3440	1.3414	1.3390	1.3368	1.3346	1.3326	1.3307	1.3288	1.3271	1.3254	1.3238	1.3223	1.3208
1.3780	1.3729	1.3682	1.3638	1.3598	1.3560	1.3525	1.3493	1.3462	1.3433	1.3406	1.3381	1.3356	1.3334	1.3312	1.3291	1.3272	1.3253	1.3236	1.3219	1.3203	1.3187	1.3172
1.3751	1.3699	1.3652	1.3608	1.3567	1.3530	1.3494	1.3461	1.3430	1.3401	1.3374	1.3348	1.3324	1.3301	1.3279	1.3259	1.3239	1.3220	1.3202	1.3185	1.3169	1.3154	1.3139
1.3723	1.3671	1.3623	1.3579	1.3538	1.3500	1.3465	1.3432	1.3400	1.3371	1.3344	1.3318	1.3293	1.3270	1.3248	1.3227	1.3208	1.3189	1.3171	1.3154	1.3137	1.3122	1.3107
1.3696	1.3644	1.3596	1.3552	1.3510	1.3472	1.3437	1.3403	1.3372	1.3343	1.3315	1.3289	1.3264	1.3241	1.3219	1.3198	1.3178	1.3159	1.3141	1.3124	1.3107	1.3091	1.3076

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1.3671	1.3618	1.3570	1.3525	1.3484	1.3446	1.3410	1.3376	1.3345	1.3315	1.3288	1.3261	1.3236	1.3213	1.3191	1.3170	1.3150	1.3130	1.3112	1.3095	1.3078	1.3062	1.3047
1.3646	1.3594	1.3545	1.3501	1.3459	1.3420	1.3384	1.3351	1.3319	1.3289	1.3261	1.3235	1.3210	1.3186	1.3164	1.3143	1.3123	1.3103	1.3085	1.3068	1.3051	1.3035	1.3019
1.3623	1.3570	1.3522	1.3477	1.3435	1.3396	1.3360	1.3326	1.3294	1.3264	1.3236	1.3210	1.3185	1.3161	1.3138	1.3117	1.3097	1.3077	1.3059	1.3041	1.3025	1.3008	1.2993
1.3601	1.3548	1.3499	1.3454	1.3412	1.3373	1.3337	1.3303	1.3271	1.3241	1.3212	1.3186	1.3161	1.3137	1.3114	1.3093	1.3072	1.3053	1.3034	1.3016	1.2999	1.2983	1.2968
1.3580	1.3527	1.3478	1.3432	1.3390	1.3351	1.3315	1.3280	1.3248	1.3218	1.3190	1.3163	1.3137	1.3113	1.3091	1.3069	1.3049	1.3029	1.3010	1.2992	1.2975	1.2959	1.2943
1.3560	1.3506	1.3457	1.3412	1.3369	1.3330	1.3293	1.3259	1.3227	1.3196	1.3168	1.3141	1.3115	1.3091	1.3068	1.3047	1.3026	1.3006	1.2987	1.2970	1.2952	1.2936	1.2920
1.3541	1.3487	1.3438	1.3392	1.3349	1.3310	1.3273	1.3238	1.3206	1.3175	1.3147	1.3120	1.3094	1.3070	1.3047	1.3025	1.3004	1.2984	1.2966	1.2948	1.2930	1.2914	1.2898
1.3522	1.3468	1.3419	1.3373	1.3330	1.3290	1.3253	1.3219	1.3186	1.3156	1.3127	1.3100	1.3074	1.3049	1.3026	1.3004	1.2983	1.2964	1.2945	1.2926	1.2909	1.2893	1.2877
1.3504	1.3450	1.3401	1.3354	1.3312	1.3272	1.3235	1.3200	1.3167	1.3136	1.3107	1.3080	1.3054	1.3030	1.3007	1.2984	1.2963	1.2944	1.2924	1.2906	1.2889	1.2872	1.2856
1.3487	1.3433	1.3383	1.3337	1.3294	1.3254	1.3217	1.3182	1.3149	1.3118	1.3089	1.3061	1.3035	1.3011	1.2988	1.2965	1.2944	1.2924	1.2905	1.2887	1.2869	1.2852	1.2836

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1.3471	1.3417	1.3366	1.3320	1.3277	1.3237	1.3199	1.3164	1.3131	1.3100	1.3071	1.3043	1.3017	1.2993	1.2969	1.2947	1.2926	1.2906	1.2886	1.2868	1.2850	1.2834	1.2817
1.3455	1.3401	1.3350	1.3304	1.3260	1.3220	1.3183	1.3147	1.3114	1.3083	1.3054	1.3026	1.3000	1.2975	1.2952	1.2929	1.2908	1.2888	1.2868	1.2850	1.2832	1.2815	1.2799
1.3440	1.3385	1.3335	1.3288	1.3245	1.3204	1.3167	1.3131	1.3098	1.3067	1.3037	1.3010	1.2983	1.2958	1.2935	1.2912	1.2891	1.2871	1.2851	1.2833	1.2815	1.2798	1.2782
1.3425	1.3371	1.3320	1.3273	1.3230	1.3189	1.3151	1.3116	1.3082	1.3051	1.3021	1.2994	1.2967	1.2942	1.2918	1.2896	1.2874	1.2854	1.2834	1.2816	1.2798	1.2781	1.2765
1.3411	1.3356	1.3306	1.3259	1.3215	1.3174	1.3136	1.3101	1.3067	1.3036	1.3006	1.2978	1.2952	1.2927	1.2903	1.2880	1.2859	1.2838	1.2818	1.2800	1.2782	1.2765	1.2748
1.3398	1.3343	1.3292	1.3245	1.3201	1.3160	1.3122	1.3086	1.3053	1.3021	1.2991	1.2963	1.2937	1.2911	1.2888	1.2865	1.2843	1.2823	1.2803	1.2784	1.2766	1.2749	1.2732
1.3385	1.3329	1.3278	1.3231	1.3187	1.3146	1.3108	1.3072	1.3038	1.3007	1.2977	1.2949	1.2922	1.2897	1.2873	1.2850	1.2828	1.2808	1.2788	1.2769	1.2751	1.2734	1.2717
1.3372	1.3317	1.3265	1.3218	1.3174	1.3133	1.3094	1.3059	1.3025	1.2993	1.2963	1.2935	1.2908	1.2883	1.2859	1.2836	1.2814	1.2793	1.2773	1.2754	1.2736	1.2719	1.2702
1.3360	1.3304	1.3253	1.3205	1.3161	1.3120	1.3082	1.3045	1.3012	1.2980	1.2950	1.2921	1.2895	1.2869	1.2845	1.2822	1.2800	1.2779	1.2759	1.2740	1.2722	1.2705	1.2688
1.3348	1.3292	1.3241	1.3193	1.3149	1.3108	1.3069	1.3033	1.2999	1.2967	1.2937	1.2908	1.2881	1.2856	1.2832	1.2809	1.2787	1.2766	1.2746	1.2727	1.2708	1.2691	1.2674

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1.3336	1.3281	1.3229	1.3181	1.3137	1.3096	1.3057	1.3021	1.2987	1.2955	1.2924	1.2896	1.2869	1.2843	1.2819	1.2796	1.2774	1.2753	1.2733	1.2714	1.2695	1.2678	1.2661
.3325	1.3269	1.3218	1.3170	1.3125	1.3084	1.3045	1.3009	1.2975	1.2943	1.2912	1.2884	1.2856	1.2831	1.2806	1.2783	1.2761	1.2740	1.2720	1.2701	1.2682	1.2665	1.2648
1.3315	1.3259	1.3207	1.3159	1.3114	1.3073	1.3034	1.2997	1.2963	1.2931	1.2900	1.2872	1.2845	1.2819	1.2794	1.2771	1.2749	1.2728	1.2708	1.2688	1.2670	1.2652	1.2635
1.3304	1.3248	1.3196	1.3148	1.3103	1.3062	1.3023	1.2986	1.2952	1.2920	1.2889	1.2860	1.2833	1.2807	1.2783	1.2759	1.2737	1.2716	1.2696	1.2676	1.2658	1.2640	1.2623
1.3294	1.3238	1.3186	1.3138	1.3093	1.3051	1.3012	1.2975	1.2941	1.2909	1.2878	1.2849	1.2822	1.2796	1.2771	1.2748	1.2726	1.2704	1.2684	1.2665	1.2646	1.2628	1.2611
1.3284	1.3228	1.3176	1.3128	1.3083	1.3041	1.3002	1.2965	1.2930	1.2898	1.2867	1.2838	1.2811	1.2785	1.2760	1.2737	1.2714	1.2693	1.2673	1.2653	1.2635	1.2617	1.2600
1.3275	1.3218	1.3166	1.3118	1.3073	1.3031	1.2992	1.2955	1.2920	1.2888	1.2857	1.2828	1.2800	1.2774	1.2750	1.2726	1.2704	1.2682	1.2662	1.2642	1.2624	1.2606	1.2589
1.3266	1.3209	1.3157	1.3108	1.3063	1.3021	1.2982	1.2945	1.2910	1.2878	1.2847	1.2818	1.2790	1.2764	1.2739	1.2716	1.2693	1.2672	1.2651	1.2632	1.2613	1.2595	1.2578
1.3257	1.3200	1.3148	1.3099	1.3054	1.3012	1.2972	1.2935	1.2901	1.2868	1.2837	1.2808	1.2780	1.2754	1.2729	1.2705	1.2683	1.2661	1.2641	1.2621	1.2602	1.2584	1.2567
1.3248	1.3191	1.3139	1.3090	1.3045	1.3003	1.2963	1.2926	1.2891	1.2858	1.2827	1.2798	1.2771	1.2744	1.2719	1.2696	1.2673	1.2651	1.2631	1.2611	1.2592	1.2574	1.2557

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1.3240	1.3183	1.3130	1.3081	1.3036	1.2994	1.2954	1.2917	1.2882	1.2849	1.2818	1.2789	1.2761	1.2735	1.2710	1.2686	1.2663	1.2642	1.2621	1.2601	1.2582	1.2564	1.2547
1.3232	1.3174	1.3122	1.3073	1.3027	1.2985	1.2945	1.2908	1.2873	1.2840	1.2809	1.2780	1.2752	1.2726	1.2701	1.2677	1.2654	1.2632	1.2612	1.2592	1.2573	1.2555	1.2537
1.3224	1.3166	1.3114	1.3065	1.3019	1.2977	1.2937	1.2900	1.2865	1.2832	1.2800	1.2771	1.2743	1.2717	1.2692	1.2668	1.2645	1.2623	1.2602	1.2582	1.2563	1.2545	1.2528
1.3216	1.3159	1.3106	1.3057	1.3011	1.2968	1.2929	1.2891	1.2856	1.2823	1.2792	1.2762	1.2734	1.2708	1.2683	1.2659	1.2636	1.2614	1.2593	1.2573	1.2554	1.2536	1.2519
1.3208	1.3151	1.3098	1.3049	1.3003	1.2960	1.2921	1.2883	1.2848	1.2815	1.2784	1.2754	1.2726	1.2699	1.2674	1.2650	1.2627	1.2605	1.2585	1.2565	1.2545	1.2527	1.2510
1.3201	1.3143	1.3090	1.3041	1.2995	1.2953	1.2913	1.2875	1.2840	1.2807	1.2775	1.2746	1.2718	1.2691	1.2666	1.2642	1.2619	1.2597	1.2576	1.2556	1.2537	1.2519	1.2501
1.3194	1.3136	1.3083	1.3034	1.2988	1.2945	1.2905	1.2867	1.2832	1.2799	1.2768	1.2738	1.2710	1.2683	1.2658	1.2634	1.2611	1.2589	1.2568	1.2548	1.2528	1.2510	1.2492
1.3187	1.3129	1.3076	1.3026	1.2981	1.2938	1.2898	1.2860	1.2825	1.2791	1.2760	1.2730	1.2702	1.2675	1.2650	1.2626	1.2603	1.2581	1.2560	1.2539	1.2520	1.2502	1.2484
1.3180	1.3122	1.3069	1.3019	1.2973	1.2930	1.2890	1.2853	1.2817	1.2784	1.2752	1.2723	1.2694	1.2668	1.2642	1.2618	1.2595	1.2573	1.2552	1.2531	1.2512	1.2494	1.2476
1.3173	1.3115	1.3062	1.3013	1.2966	1.2923	1.2883	1.2845	1.2810	1.2777	1.2745	1.2715	1.2687	1.2660	1.2635	1.2610	1.2587	1.2565	1.2544	1.2524	1.2504	1.2486	1.2468

f_1

f_2	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
1	15.3790	15.3837	15.3883	15.3926	15.3968	15.4006	15.4045	15.4081	15.4116	15.4150	15.4181	15.4213	15.4241	15.4271	15.4299	15.4326	15.4352	15.4376	15.4400	15.4423	15.4446	15.44
2	4.4571	4.4576	4.4582	4.4587	4.4592	4.4597	4.4602	4.4606	4.4610	4.4614	4.4618	4.4622	4.4626	4.4629	4.4633	4.4636	4.4639	4.4642	4.4645	4.4648	4.4650	4.46
3	2.9841	2.9841	2.9841	2.9841	2.9841	2.9842	2.9842	2.9842	2.9842	2.9842	2.9842	2.9842	2.9842	2.9842	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.98
4	2.4359	2.4356	2.4354	2.4352	2.4350	2.4348	2.4347	2.4345	2.4343	2.4342	2.4340	2.4339	2.4337	2.4336	2.4335	2.4333	2.4332	2.4331	2.4330	2.4329	2.4328	2.43
5	2.1507	2.1503	2.1500	2.1496	2.1493	2.1490	2.1487	2.1484	2.1481	2.1479	2.1476	2.1474	2.1472	2.1469	2.1467	2.1465	2.1463	2.1461	2.1459	2.1457	2.1456	2.14
6	1.9753	1.9749	1.9744	1.9740	1.9736	1.9732	1.9728	1.9724	1.9721	1.9717	1.9714	1.9711	1.9708	1.9705	1.9702	1.9699	1.9697	1.9694	1.9692	1.9689	1.9687	1.96
7	1.8562	1.8556	1.8551	1.8546	1.8541	1.8536	1.8531	1.8527	1.8523	1.8519	1.8515	1.8511	1.8508	1.8504	1.8501	1.8498	1.8495	1.8492	1.8489	1.8486	1.8483	1.84
8	1.7696	1.7690	1.7684	1.7678	1.7673	1.7667	1.7662	1.7657	1.7653	1.7648	1.7644	1.7640	1.7636	1.7632	1.7628	1.7624	1.7621	1.7617	1.7614	1.7611	1.7608	1.76
9	1.7037	1.7030	1.7024	1.7017	1.7011	1.7006	1.7000	1.6995	1.6990	1.6985	1.6980	1.6975	1.6971	1.6967	1.6963	1.6959	1.6955	1.6951	1.6947	1.6944	1.6941	1.69
10	1.6518	1.6510	1.6503	1.6496	1.6490	1.6484	1.6478	1.6472	1.6466	1.6461	1.6456	1.6451	1.6446	1.6442	1.6437	1.6433	1.6429	1.6425	1.6421	1.6417	1.6413	1.64
11	1.6097	1.6089	1.6081	1.6074	1.6067	1.6061	6054	1.6048	1.6042	1.6037	1.6031	1.6026	1.6021	1.6016	1.6011	1.6006	1.6002	1.5998	1.5994	1.5990	1.5986	1.59
12	1.5748	1.5740	1.5732	1.5725	1.5717	1.5710	1.5703	1.5697	1.5691	1.5685	1.5679	1.5673	1.5668	1.5663	1.5658	1.5653	1.5648	1.5644	1.5639	1.5635	1.5631	1.56
13	1.5455	1.5446	1.5438	1.5430	1.5422	1.5415	1.5408	1.5401	1.5394	1.5388	1.5382	1.5376	1.5371	1.5365	1.5360	1.5355	1.5350	1.5345	1.5341	1.5336	1.5332	1.53
14	1.5204	1.5195	1.5186	1.5178	1.5170	1.5162	1.5155	1.5148	1.5141	1.5134	1.5128	1.5122	1.5116	1.5110	1.5105	1.5100	1.5094	1.5090	1.5085	1.5080	1.5076	1.50
15	1.4987	1.4977	1.4968	1.4960	1.4951	1.4943	1.4936	1.4928	1.4921	1.4914	1.4908	1.4902	1.4896	1.4890	1.4884	1.4878	1.4873	1.4868	1.4863	1.4858	1.4854	1.48
16	1.4797	1.4787	1.4778	1.4769	1.4760	1.4752	1.4744	1.4736	1.4729	1.4722	1.4715	1.4709	1.4702	1.4696	1.4690	1.4685	1.4679	1.4674	1.4669	1.4664	1.4659	1.46
17	1.4629	1.4619	1.4609	1.4600	1.4591	1.4583	1.4575	1.4567	1.4559	1.4552	1.4545	1.4538	1.4532	1.4526	1.4520	1.4514	1.4508	1.4502	1.4497	1.4492	1.4487	1.44
18	1.4480	1.4469	1.4460	1.4450	1.4441	1.4432	1.4424	1.4416	1.4408	1.4401	1.4394	1.4387	1.4380	1.4374	1.4367	1.4361	1.4355	1.4350	1.4344	1.4339	1.4334	1.43
19	1.4346	1.4336	1.4325	1.4316	1.4307	1.4298	1.4289	1.4281	1.4273	1.4265	1.4258	1.4251	1.4244	1.4237	1.4231	1.4225	1.4219	1.4213	1.4207	1.4202	1.4196	1.41
20	1.4226	1.4215	1.4205	1.4195	1.4185	1.4176	1.4167	1.4159	1.4151	1.4143	1.4135	1.4128	1.4121	1.4114	1.4108	1.4101	1.4095	1.4089	1.4083	1.4078	1.4072	1.40
21	1.4117	1.4105	1.4095	1.4085	1.4075	1.4066	1.4057	1.4048	1.4040	1.4032	1.4024	1.4017	1.4009	1.4002	1.3996	1.3989	1.3983	1.3977	1.3971	1.3965	1.3960	1.39
22	1.4017	1.4006	1.3995	1.3985	1.3975	1.3965	1.3956	1.3947	1.3939	1.3931	1.3923	1.3915	1.3908	1.3901	1.3894	1.3887	1.3881	1.3875	1.3869	1.3863	1.3857	1.38
23	1.3926	1.3915	1.3904	1.3893	1.3883	1.3873	1.3864	1.3855	1.3846	1.3838	1.3830	1.3822	1.3815	1.3808	1.3801	1.3794	1.3787	1.3781	1.3775	1.3769	1.3763	1.37
24	1.3843	1.3831	1.3820	1.3809	1.3799	1.3789	1.3779	1.3770	1.3761	1.3753	1.3745	1.3737	1.3729	1.3722	1.3715	1.3708	1.3701	1.3695	1.3688	1.3682	1.3676	1.36
25	1.3766	1.3754	1.3742	1.3731	1.3721	1.3711	1.3701	1.3692	1.3683	1.3674	1.3666	1.3658	1.3650	1.3643	1.3636	1.3629	1.3622	1.3615	1.3609	1.3603	1.3597	1.35
26	1.3694	1.3682	1.3671	1.3660	1.3649	1.3639	1.3629	1.3620	1.3611	1.3602	1.3593	1.3585	1.3577	1.3570	1.3562	1.3555	1.3548	1.3541	1.3535	1.3529	1 3523	1.35
27	1.3628	1.3616	1.3604	1.3593	1.3582	1.3572	1.3562	1.3552	1.3543	1.3534	1.3526	1.3517	1.3509	1.3502	1.3494	1.3487	1.3480	1.3473	1.3466	1.3460	1.3454	1.34
28	1.3567	1.3554	1.3542	1.3531	1.3520	1.3510	1.3500	1.3490	1.3480	1.3471	1.3463	1.3454	1.3446	1.3438	1.3431	1.3423	1.3416	1.3409	1.3403	1.3396	1.3390	1.33
29	1.3509	1.3497	1.3485	1.3473	1.3462	1.3451	1.3441	1.3431	1.3422	1.3413	1.3404	1.3395	1.3387	1.3379	1.3371	1.3364	1.3357	1.3350	1.3343	1.3336	1.3330	1.33
30	1.3455	1.3443	1.3431	1.3419	1.3408	1.3397	1.3387	1.3377	1.3367	1.3358	1.3349	1.3340	1.3332	1.3324	1.3316	1.3308	1.3301	1.3294	1.3287	1.3280	1.3274	1.32
31	1.3405	1.3392	1.3380	1.3368	1.3357	1.3346	1.3335	1.3325	1.3316	1.3306	1.3297	1.3288	1.3280	1.3272	1.3264	1.3256	1.3249	1.3242	1.3235	1.3228	1.3221	1.32
32	1.3358	1.3345	1.3332	1.3320	1.3309	1.3298	1.3287	1.3277	1.3267	1.3258	1.3249	1.3240	1.3231	1.3223	1.3215	1.3207	1.3200	1.3193	1.3185	1.3179	1.3172	1.31
33	1.3313	1.3300	1.3288	1.3276	1.3264	1.3253	1.3242	1.3232	1.3222	1.3212	1.3203	1.3194	1.3185	1.3177	1.3169	1.3161	1.3153	1.3146	1.3139	1.3132	1.3125	1.31
34	1.3271	1.3258	1.3245	1.3233	1.3221	1.3210	1.3199	1.3189	1.3179	1.3169	1.3160	1.3151	1.3142	1.3134	1.3125	1.3117	1.3110	1.3102	1.3095	1.3088	1.3081	1.30
35	1.3231	1.3219	1.3205	1.3193	1.3181	1.3170	1.3159	1.3148	1.3138	1.3128	1.3119	1.3110	1.3101	1.3093	1.3084	1.3076	1.3069	1.3061	1.3054	1.3047	1.3040	1.30
36	1.3194	1.3180	1.3168	1.3155	1.3143	1.3132	1.3121	1.3110	1.3100	1.3090	1.3080	1.3071	1.3062	1.3054	1.3045	1.3037	1.3029	1.3022	1.3014	1.3007	1.3000	1.29
37	1.3158	1.3145	.3132	1.3119	1.3107	1.3096	1.3084	1.3074	1.3063	1.3053	1.3044	.3034	1.3025	1.3017	1.3008	1.3000	1.2992	1.2985	1.2977	1.2970	1.2963	1.29
38	1.3124	1.3111	1.3098	1.3085	1.3073	1.3061	1.3050	1.3039	1.3029	1.3019	1.3009	1.3000	1.2991	1.2982	1.2973	1.2965	1.2957	1.2949	1.2942	1.2934	1.2927	1.29
39	1.3092	1.3079	1.3065	1.3053	1.3040	1.3029	1.3017	1.3006	1.2996	1.2986	1.2976	1.2966	1.2957	1.2948	1.2940	1.2932	1.2923	1.2916	1.2908	1.2901	1.2894	1.28
40	1.3062	1.3048	1.3035	1.3022	1.3009	1.2998	1.2986	1.2975	1.2965	1.2954	1.2944	1.2935	1.2926	.2917	1.2908	1.2900	1.2892	1.2884	1.2876	1.2869	1.2861	1.28
41	1.3033	1.3019	1.3005	1.2992	1.2980	1.2968	1.2956	1.2945	1.2935	1.2924	1.2914	1.2905	1.2895	1.2886	1.2877	1.2869	1.2861	1.2853	1.2845	1.2838	1.2831	1.28
42	1.3005	1.2991	1.2977	1.2964	1.2952	1.2940	1.2928	1.2917	1.2906	1.2896	1.2886	1.2876	1.2867	1.2858	1.2849	1.2840	1.2832	1.2824	1.2816	1.2809	1.2801	1.27
43	1.2978	1.2964	1.2950	1.2937	1.2925	1.2913	1.2901	1.2890	1.2879	1.2868	1.2858	1.2849	1.2839	1.2830	1.2821	1.2812	1.2804	1.2796	1.2788	1.2781	1.2773	1.27
44	1.2953	1.2939	1.2925	1.2912	1.2899	1.2887	1.2875	1.2864	1.2853	1.2842	1.2832	1.2822	1.2813	1.2803	1.2795	1.2786	1.2778	1.2769	1.2761	1.2754	1.2746	1.27
45	1.2929	1.2914	1.2900	1.2887	1.2874	1.2862	1.2850	1.2839	1.2828	1.2817	1.2807	1.2797	1.2787	1.2778	1.2769	1.2760	1.2752	1.2744	1.2736	1.2728	1.2721	1.27
46	1.2905	1.2891	1.2877	1.2864	1.2851	1.2838	1.2827	1.2815	1.2804	1.2793	1.2783	1.2773	1.2763	1.2754	1.2745	1.2736	1.2728	1.2719	1.2711	1.2704	1.2696	1.26
47	1.2883	1.2868	1.2854	1.2841	1.2828	1.2816	1.2804	1.2792	1.2781	1.2770	1.2760	1.2750	1.2740	1.2731	1.2722	1.2713	1.2704	1.2696	1.2688	1.2680	1.2672	1.26
48	1.2861	1.2847	1.2833	1.2819	1.2806	1.2794	1.2782	1.2770	1.2759	1.2748	1.2738	1.2728	1.2718	1.2708	1.2699	1.2690	1.2682	1.2673	1.2665	1.2657	1.2650	1.26
49	1.2841	1.2826	1.2812	1.2798	1.2785	1.2773	1.2761	1.2749	1.2738	1.2727	1.2716	1.2706	1.2696	1.2687	1.2678	1.2669	1.2660	1.2652	1.2643	1.2635	1.2628	1.26
50	1.2821	1.2806	1.2792	1.2778	1.2765	1.2753	1.2740	1.2729	1.2717	1.2706	1.2696	1.2686	1.2676	1.2666	1.2657	1.2648	1.2639	1.2631	1.2622	1.2614	1.2607	1.25
51	1.2802	1.2787	1.2773	1.2759	1.2746	1.2733	1.2721	1.2709	1.2698	1.2687	1.2676	1.2666	1.2656	1.2646	1.2637	1.2628	1.2619	1.2611	1.2602	1.2594	1.2586	1.2
52	1.2784	1.2769	1.2754	1.2741	1.2727	1.2715	1.2702	1.2690	1.2679	1.2668	1.2657	1.2647	1.2637	1.2627	1.2618	1.2609	1.2600	1.2591	1.2583	1.2575	1.2567	1.2
53	1.2766	1.2751	1.2737	1.2723	1.2709	1.2697	1.2684	1.2672	1.2661	1.2650	1.2639	1.2628	1.2618	1.2609	1.2599	1.2590	1.2581	1.2572	1.2564	1.2556	1.2548	1.2
54	1.2749	1.2734	1.2719	1.2705	1.2692	1.2679	1.2667	1.2655	1.2643	1.2632	1.2621	1.2611	1.2601	1.2591	1.2581	1.2572	1.2563	1.2554	1.2546	1.2538	1.2530	1.2
55	1.2732	1.2717	1.2703	1.2689	1.2675	1.2662	1.2650	1.2638	1.2626	1.2615	1.2604	1.2594	1.2583	1.2574	1.2564	1.2555	1.2546	1.2537	1.2529	1.2520	1.2512	1.2
56	1.2717	1.2701	1.2687	1.2673	1.2659	1.2646	1.2634	1.2622	1.2610	1.2599	1.2588	1.2577	1.2567	1.2557	1.2547	1.2538	1.2529	1.2520	1.2512	1.2503	1.2495	1.2
57	1.2701	1.2686	1.2671	1.2657	1.2644	1.2631	1.2618	1.2606	1.2594	1.2583	1.2572	1.2561	1.2551	1.2541	1.2531	1.2522	1.2513	1.2504	1.2495	1.2487	1.2479	1.2
58	1.2686	1.2671	1.2656	1.2642	1.2629	1.2615	1.2603	1.2590	1.2579	1.2567	1.2556	1.2546	1.2535	1.2525	1.2516	1.2506	1.2497	1.2488	1.2480	1.2471	1.2463	1.2
59	1.2672	1.2657	1.2642	1.2628	1.2614	1.2601	1.2588	1.2576	1.2564	1.2552	1.2541	1.2531	1.2520	1.2510	1.2500	1.2491	1.2482	1.2473	1.2464	1.2456	1.2448	1.2
60	1.2658	1.2643	1.2628	1.2614	1.2600	1.2587	1.2574	1.2561	1.2550	1.2538	1.2527	1.2516	1.2506	1.2496	1.2486	1.2476	1.2467	1.2458	1.2450	1.2441	1.2433	1.2
61	1.2645	1.2629	1.2614	1.2600	1.2586	1.2573	1.2560	1.2548	1.2536	1.2524	1.2513	1.2502	1.2492	1.2482	1.2472	1.2462	1.2453	1.2444	1.2435	1.2427	1.2419	1.2
62	1.2632	1.2616	1.2601	1.2587	1.2573	1.2560	1.2547	1.2534	1.2522	1.2511	1.2499	1.2489	1.2478	1.2468	1.2458	1.2448	1.2439	1.2430	1.2421	1.2413	1.2405	1.2
63	1.2619	1.2603	1.2588	1.2574	1.2560	1.2547	1.2534	1.2521	1.2509	1.2497	1.2486	1.2475	1.2465	1.2455	1.2445	1.2435	1.2426	1.2417	1.2408	1.2399	1.2391	1.2
64	1.2607	1.2591	1.2576	1.2561	1.2548	1.2534	1.2521	1.2509	1.2496	1.2485	1.2473	1.2463	1.2452	1.2442	1.2432	1.2422	1.2413	1.2404	1.2395	1.2386	1.2378	1.2
65	1.2595	1.2579	1.2564	1.2549	1.2535	1.2522	1.2509	1.2496	1.2484	1.2472	1.2461	1.2450	1.2439	1.2429	1.2419	1.2410	1.2400	1.2391	1.2382	1.2374	1.2365	1.2
66	1.2583	1.2568	1.2552	1.2538	1.2524	1.2510	1.2497	1.2484	1.2472	1.2460	1.2449	1.2438	1.2427	1.2417	1.2407	1.2397	1.2388	1.2379	1.2370	1.2361	1.2353	1.2
67	1.2572	1.2556	1.2541	1.2526	1.2512	1.2499	1.2486	1.2473	1.2461	1.2449	1.2437	1.2426	1.2416	1.2405	1.2395	1.2385	1.2376	1.2367	1.2358	1.2349	1.2341	1.2
68	1.2561	1.2545	1.2530	1.2515	1.2501	1.2488	1.2474	1.2462	1.2449	1.2438	1.2426	1.2415	1.2404	1.2394	1.2384	1.2374	1.2364	1.2355	1.2346	1.2338	1.2329	1.2
69	1.2551	1.2535	1.2519	1.2505	1.2490	1.2477	1.2464	1.2451	1.2438	1.2427	1.2415	1.2404	1.2393	1.2383	1.2373	1.2363	1.2353	1.2344	1.2335	1.2326	1.2318	1.2
70	1.2540	1.2524	1.2509	1.2494	1.2480	1.2466	1.2453	1.2440	1.2428	1.2416	1.2404	1.2393	1.2382	1.2372	1.2362	1.2352	1.2342	1.2333	1.2324	1.2315	1.2307	1.2
71	1.2530	1.2514	1.2499	1.2484	1.2470	1.2456	1.2443	1.2430	1.2417	1.2405	1.2394	1.2383	1.2372	1.2261	1.2351	1.2341	1.2332	1.2322	1.2313	1.2304	1.2296	1.2
72	1.2521	1.2504	1.2489	1.2474	1.2460	1.2446	1.2433	1.2420	1.2407	1.2395	1.2384	1.2372	1.2362	1.2351	1.2341	1.2331	1.2321	1.2312	1.2303	1.2294	1.2285	1.2
73	1.2511	1.2495	1.2479	1.2464	1.2450	1.2436	1.2423	1.2410	1.2397	1.2385	1.2374	1.2362	1.2352	1.2341	1.2331	1.2321	1.2311	1.2302	1.2293	1.2284	1.2275	1.2
74	1.2502	1.2486	1.2470	1.2455	1.2441	1.2427	1.2413	1.2400	1.2388	1.2376	1.2364	1.2353	1.2342	1.2331	1.2321	1.2311	1.2301	1.2292	1.2283	1.2274	1.2265	1.2
75	1.2493	1.2477	1.2461	1.2446	1.2432	1.2418	1.2404	1.2391	1.2379	1.2366	1.2355	1.2343	1.2332	1.2322	1.2311	1.2301	1.2292	1.2282	1.2273	1.2264	1.2255	1.2
6	1.2484	1.2468	1.2452	1.2437	1.2423	1.2409	1.2395	1.2382	1.2370	1.2357	1.2346	1.2334	1.2323	1.2312	1.2302	1.2292	1.2282	1.2273	1.2264	1.2255	1.2246	1.2
77	1.2475	1.2459	1.2443	1.2428	1.2414	1.2400	1.2386	1.2373	1.2361	1.2348	1.2337	1.2325	1.2314	1.2303	1.2293	1.2283	1.2273	1.2264	1.2254	1.2245	1.2237	1.2
78	1.2467	1.2451	1.2435	1.2420	1.2405	1.2391	1.2378	1.2365	1.2352	1.2340	1.2328	1.2316	1.2305	1.2295	1.2284	1.2274	1.2264	1.2255	1.2245	1.2236	1.2228	1.2
79	1.2459	1.2443	1.2427	1.2412	1.2397	1.2383	1.2369	1.2356	1.2344	1.2331	1.2319	1.2308	1.2297	1.2286	1.2276	1.2265	1.2256	1.2246	1.2237	1.2228	1.2219	1.2
80	1.2451	1.2435	1.2419	1.2404	1.2389	1.2375	1.2361	1.2348	1.2335	1.2323	1.2311	1.2300	1.2288	1.2278	1.2267	1.2257	1.2247	1.2238	1.2228	1.2219	1.2210	1.2

TABLE A-1. F–DISTRIBUTION α = .20
(Continued)

f_1

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
[illegible]	[illegible]	15.4423	15.4446	15.4467	15.4489	15.4509	15.4529	15.4548	15.4567	15.4585	15.4602	15.4619	15.4635	15.4652	15.4668	15.4683	15.4698	15.4712	15.4726	15.4740	5.4754	15.4766
[illegible]	[illegible]	[illegible]	4.4650	4.4653	4.4656	4.4658	4.4661	4.4663	4.4665	4.4667	4.4669	4.4672	4.4673	4.4675	4.4677	4.4679	4.4681	4.4683	4.4685	4.4686	4.4688	4.4689
[illegible]	[illegible]	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.9843	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844	2.9844
[illegible]	[illegible]	2.4329	2.4328	2.4327	2.4326	2.4325	2.4324	2.4323	2.4322	2.4321	2.4320	2.4319	2.4318	2 4318	2.4317	2.4316	2.4315	2.4315	2.4314	2.4313	2.4313	2.4312
2.1461	2.1459	2.1457	2.1456	2.1454	2.1452	2.1450	2.1449	2.1447	2.1446	2.1444	2.1443	2.1442	2.1440	2.1439	2.1438	2.1436	2.1435	2.1434	2.1433	2.1432	2.1431	2.1430
[illegible]	[illegible]	1.9689	1.9687	1.9685	1.9683	1.9681	1.9678	1.9676	1.9675	1.9673	1.9671	1.9669	1.9667	1.9666	1.9664	1.9663	1.9661	1.9660	1.9658	1.9657	1.9655	1.9654
[illegible]	[illegible]	[illegible]	1.8483	1.8481	1.8478	1.8476	1.8473	1.8471	1.8469	1.8466	1.8464	1.8462	1.8460	1.8458	1.8456	1.8454	1.8453	1.8451	1.8449	1.8447	1.8446	1.8444
[illegible]	1.7614	1.7611	1.7608	1.7605	1.7602	1.7599	1.7597	1.7594	1.7591	1.7589	1.7587	1.7584	1.7582	1.7580	1.7577	1.7575	1.7573	1.7571	1.7569	1.7567	1.7566	1.7564
1.6951	1.6947	1.6944	1.6941	1.6937	1.6934	1.6931	1.6928	1.6925	1.6922	1.6920	1.6917	1.6914	1.6912	1.6909	1.6907	1.6905	1.6902	1.6900	1.6898	1.6896	1.6894	1.6892
[illegible]	[illegible]	1.6417	1.6414	1.6410	1.6406	1.6403	1.6400	1.6397	1.6394	1.6391	1.6388	1.6385	1.6382	1.6380	1.6377	1.6375	1.6372	1.6370	1.6368	1.6365	1.6363	1.6361

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
[illegible]	1.5994	1.5990	1.5986	1.5982	1.5978	1.5975	1.5971	1.5968	1.5965	1.5962	1.5959	1.5956	1.5953	1.5950	1.5947	1.5944	1.5942	1.5939	1.5937	1.5934	1.5932	1.5930
1.5644	1.5639	1.5635	1.5631	1.5627	1.5623	1.5620	1.5616	1.5612	1.5609	1.5606	1.5602	1.5599	1.5596	1.5593	1.5590	1.5587	1.5585	1.5582	1.5579	1.5577	1.5574	1.5572
1.5345	1.5341	1.5336	1.5332	1.5328	1.5324	1.5320	1.5316	1.5312	1.5309	1.5305	1.5302	1.5298	1.5295	1.5292	1.5289	1.5286	1.5283	1.5280	1.5277	1.5275	1.5272	1.5269
1.5090	1.5085	1.5080	1.5076	1.5071	1.5067	1.5063	1.5059	1.5055	1.5051	1.5048	1.5044	1.5041	1.5037	1.5034	1.5031	1.5028	1.5025	1.5022	1.5019	1.5016	1.5013	1.5010
1.4868	1.4863	1.4858	1.4854	1.4849	1.4845	1.4840	1.4836	1.4832	1.4828	1.4824	1.4821	1.4817	1.4814	1.4810	1.4807	1.4804	1.4800	1.4797	1.4794	1.4791	1.4789	1.4786
1.4674	1.4669	1.4664	1.4659	1.4654	1.4650	1.4645	1.4641	1.4637	1.4633	1.4629	1.4625	1.4621	1.4618	1.4614	1.4611	1.4607	1.4604	1.4601	1.4598	1.4595	1.4592	1.4589
[illegible]	[illegible]	1.4492	1.4487	1.4482	1.4477	1.4473	1.4468	1.4464	1.4460	1.4456	1.4452	1.4448	1.4444	1.4441	1.4437	1.4434	1.4430	1.4427	1.4424	1.4420	1.4417	1.4414
1.4350	1.4344	1.4339	1.4334	1.4329	1.4324	1.4319	1.4315	1.4310	1.4306	1.4302	1.4298	1.4294	1.4290	[illegible]	1.4282	1.4279	1.4275	1.4272	1.4269	1.4265	1.4262	1.4259
[illegible]	1.4207	1.4202	1.4196	1.4191	1.4186	1.4181	1.4177	1.4172	1.4168	1.4163	1.4159	1.4155	1.4151	1.4147	1.4143	1.4140	1.4136	1.4133	1.4129	1.4126	1.4123	1.4120
1.4089	1.4083	1.4078	1.4072	1.4067	1.4062	1.4057	1.4052	1.4047	1.4043	1.4038	1.4034	1.4030	1.4026	1.4022	1.4018	1.4014	1.4011	1.4007	1.4004	1.4000	1.3997	1.3994

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.3977	1.3971	1.3965	1.3960	1.3954	1.3949	1.3944	1.3939	1.3934	1.3930	1.3925	1.3921	1.3916	1.3912	1.3908	1.3904	1.3900	1.3897	1.3893	1.3889	1.3886	1.3882	1.3879
1.3875	1.3869	1.3863	1.3857	1.3852	1.3846	1.3841	1.3836	1.3831	1.3826	1.3822	1.3817	1.3813	1.3808	1.3804	1.3800	1.3796	1.3792	1.3789	1.3785	1.3781	1.3778	1.3775
1.3781	1.3775	1.3769	1.3763	1.3757	1.3752	1.3747	1.3741	1.3736	1.3732	1.3727	1.3722	1.3718	1.3713	1.3709	1.3705	1.3701	1.3697	1.3693	1.3689	1.3686	1.3682	1.3679
1.3695	1.3688	1.3682	1.3676	1.3671	1.3665	1.3660	1.3655	1.3649	1.3644	1.3640	1.3635	1.3630	1.3626	1.3622	1.3617	1.3613	1.3609	1.3605	1.3602	1.3598	1.3594	1.3591
1.3615	1.3609	1.3603	1.3597	1.3591	1.3585	1.3580	1.3574	1.3569	1.3564	1.3559	1.3554	1.3550	1.3545	1.3541	1.3536	1.3532	1.3528	1.3524	1.3520	1.3516	1.3513	1.3509
1.3541	1.3535	1.3529	1.3523	1.3517	1.3511	1.3505	1.3500	1.3495	1.3490	1.3485	1.3480	1.3475	1.3470	1.3466	1.3461	1.3457	1.3453	1.3449	1.3445	1.3441	1.3437	1.3434
1.3473	1.3466	1.3460	1.3454	1.3448	1.3442	1.3436	1.3431	1.3425	1.3420	1.3415	1.3410	1.3405	1.3401	1.3396	1.3392	1.3387	1.3383	1.3379	1.3375	1.3271	1.3367	1.3363
1.3409	1.3401	1.3396	1.3390	1.3384	1.2378	1.3372	1.3366	1.3361	1.3356	1.3351	1.3346	1.3341	1.3336	1.3331	1.3327	1.3322	1.3318	1.3314	1.3310	1.3306	1.3302	1.3298
1.3350	1.3343	1.3336	1.330	1.3324	1.3318	1.3312	1.3306	1.3301	1.3295	1.3290	1.3285	1.3280	1.3275	1.3271	1.3266	1.3261	1.3257	1.3253	1.3249	1.3245	1.3241	1.3237
1.3294	1.3287	1.3280	1.3274	1.3268	1.3262	1.3256	1.3250	1.3244	1.3239	1.3234	1.3228	1.3223	1.3218	1.3214	1.3209	1.3204	1.3200	1.3196	1.3192	1.3187	1.3183	1.3179

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.3242	1.3235	1.3228	1.3221	1.3215	1.3209	1.3203	1.3197	1.3191	1.3186	1.3180	1.3175	1.3170	1.3165	1.3160	1.3156	1.3151	[illegible]	1.3142	1.3138	1.3134	1.3130	1.3126
1.3193	1.3185	1.3179	1.3172	1.3166	1.3159	1.3153	1.3147	1.3142	1.3136	1.3130	1.3125	1.3120	1.3115	1.3110	1.3105	1.3101	1.3096	1.3092	1.3087	1.3083	1.3079	1.3075
1.3146	1.3139	1.3132	1.3125	1.3119	1.3113	1.3106	1.3100	1.3093	1.3089	1.3083	1.3078	1.3073	1.3068	1.3063	1.3058	1.3053	1.3049	1.3044	1.3040	1.3035	1.3031	1.3027
1.3102	1.3095	1.3088	1.3081	1.3075	1.3068	1.3062	1.3056	1.3050	1.3044	1.3039	1.3033	1.3028	1.3023	1.3018	1.3013	1.3008	1.3004	1.2999	1.2995	1.2990	1.2986	1.2982
1.3061	1.3054	1.3047	1.3040	1.3033	1.3027	1.3020	1.3014	1.3008	1.3002	1.2997	1.2991	1.2986	1.2981	1.2976	1.2971	1.2966	1.2961	1.2957	1.2952	1.2948	1.2943	1.2939
1.3022	1.3014	1.3007	1.3000	1.2994	1.2987	1.2981	1.2975	1.2968	1.2963	1.2957	1.2951	1.2946	1.2941	1.2936	1.2931	1.2926	1.2921	1.2916	1.2912	1.2907	1.2903	1.2899
1.2985	1.2977	1.2970	1.2963	1.2956	1.2950	1.2943	1.2937	1.2931	1.2925	1.2919	1.2913	1.2908	1.2903	1.2897	1.2892	1.2887	1.2883	1.2878	1.2873	1.2869	1.2864	1.2860
1.2949	1.2942	1.2934	1.2927	1.2921	1.2914	1 2907	1.2901	1.2895	1.2889	1.2883	1.2877	1.2872	1.2867	1.2861	1.2856	1.2851	1.2846	1.2842	1.2837	1.2832	1.2828	1.2824
1.2916	1.2908	1.2901	1.2894	1.2887	1.2880	1.2873	1.2867	1.2861	1.2855	1.2849	1.2843	1.2838	1.2832	1.2827	1.2822	1.2817	1.2812	1.2807	1.2802	1.2793	1.2793	1.2789
1.2884	1.2876	1.2869	1.2861	1.2854	1.2848	1.2841	1.2835	1.2828	1.2822	1.2816	1.2810	1.2805	1.2799	1.2794	1.2785	1.2784	1.2779	1.2774	1.2769	1.2765	1.2760	1.2756

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.2853	1.2845	1.2838	1.2831	1.2824	1.2817	1.2810	1.2804	1.2797	1.2791	1.2785	1.2779	1.2774	1.2768	1.2763	1.2757	1.2752	1.2747	1.2742	1.2738	1.2733	1.2728	1.2724
1.2824	1.2816	1.2809	1.2801	1.2794	1.2787	1.2781	1.2774	1.2768	1.2761	1.2755	1.2750	1.2744	1.2738	1.2733	1.2727	1.2722	1.2717	1.2712	1.2708	1.2703	1.2698	1.2694
1.2796	1.2788	1.2781	1.2773	1.2766	1.2759	1.2752	1.2746	1.2739	1.2733	1.2727	1.2721	1.2715	1.2710	1.2704	1.2699	1.2694	1.2689	1.2684	1.2679	1.2674	1.2669	1.2665
1.2769	1.2761	1.2754	1.2746	1.2739	1.2732	1.2725	1.2719	1.2712	1.2706	1.2700	1.2694	1.2688	1.2682	1.2677	1.2671	1.2666	1.2661	1.2656	1.2651	1.2646	1.2642	1.2637
1.2744	1.2736	1.2728	1.2721	1.2713	1.2706	1.2699	1.2693	1.2686	1.2680	1.2674	1.2668	1.2662	1.2656	1.2651	1.2645	1.2640	1.2635	1.2630	1.2625	1.2620	1.2615	1.2610
1.2719	1.2711	1.2704	1.2696	1.2689	1.2682	1.2675	1.2668	1.2661	1.2655	1.2649	1.2643	1.2637	1.2631	1.2625	1.2620	1.2615	1.2609	1.2604	1.2599	1.2594	1.2590	1.2585
1.2696	1.2688	1.2680	1.2672	1.2665	1.2658	1.2651	1.2644	1.2637	1.2631	1.2625	1.2619	1.2613	1.2607	1.2601	1.2596	1.2590	1.2585	1.2580	1.2575	1.2570	1.2565	1.2560
1.2673	1.2665	1.2657	1.2650	1.2642	1.2635	1.2628	1.2621	1.2614	1.2608	1.2602	1.2595	1.2590	1.2584	1.2578	1.2572	1.2567	1.2562	1.2557	1.2551	1.2547	1.2542	1.2537
1.2652	1.2643	1.2635	1.2628	1.2620	1.2613	1.2606	1.2599	1.2592	1.2586	1.2579	1.2573	1.2567	1.2561	1.2556	1.2550	1.2545	1.2539	1.2534	1.2529	1.2524	1.2519	1.2514
1.2631	1.2622	1.2614	1.2607	1.2599	1.2592	1.2585	1.2578	1.2571	1.2565	1.2558	1.2552	1.2546	1.2541	1.2534	1.2528	1.2523	1.2518	1.2512	1.2507	1.2502	1.2497	1.2493

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.2611	1.2602	1.2594	1.2586	1.2579	1.2572	1.2564	1.2557	1.2551	1.2544	1.2538	1.2531	1.2525	1.2519	1.2513	1.2508	1.2502	1.2497	1.2491	1.2486	1.2481	1.2476	1.2472
1.2591	1.2583	1.2575	1.2567	1.2559	1.2552	1.2545	1.2538	1.2531	1.2524	1.2518	1.2511	1.2505	1.2499	1.2493	1.2488	1.2482	1.2477	1.2471	1.2466	1.2461	1.2456	1.2451
1.2572	1.2564	1.2556	1.2548	1.2540	1.2533	1.2526	1.2519	1.2512	1.2505	1.2499	1.2492	1.2486	1.2480	1.2474	1.2468	1.2463	1.2457	1.2452	1.2447	1.2442	1.2437	1.2432
1.2554	1.2546	1.2538	1.2530	1.2522	1.2515	1.2507	1.2500	1.2493	1.2487	1.2480	1.2474	1.2468	1.2462	1.2456	1.2450	1.2444	1.2439	1.2433	1.2428	1.2423	1.2418	1.2413
1.2537	1.2529	1.2520	1.2512	1.2505	1.2497	1.2490	1.2483	1.2476	1.2469	1.2462	1.2456	1.2450	1.2444	1.2438	1.2432	1.2426	1.2421	1.2415	1.2410	1.2405	1.2400	1.2395
1.2520	1.2512	1.2503	1.2495	1.2488	1.2480	1.2473	1.2466	1.2459	1.2452	1.2445	1.2439	1.2432	1.2426	1.2420	1.2414	1.2409	1.2403	1.2398	1.2392	1.2387	1.2382	1.2377
1.2504	1.2495	1.2487	1.2479	1.2471	1.2464	1.2456	1.2449	1.2442	1.2435	1.2428	1.2422	1.2416	1.2410	1.2403	1.2398	1.2392	1.2386	1.2381	1.2375	1.2370	1.2365	1.2360
1.2488	1.2482	1.2471	1.2463	1.2455	1.2448	1.2440	1.2433	1.2426	1.2419	1.2412	1.2406	1.2399	1.2393	1.2387	1.2381	.2376	1.2370	1.2364	1.2359	1.2354	1.2349	1.2344
1.2473	1.2464	1.2456	1.2448	1.2440	1.2432	1.2425	1.2418	1.2410	1.2404	1.2397	1.2390	1.2384	1.2378	1.2371	1.2366	1.2360	1.2354	1.2349	1.2343	1.2338	1.2333	1.2328
1.2458	1.2450	1.2441	1.2433	1.2425	1.2417	1.2410	1.2403	1.2395	1.2388	1.2382	1.2375	1.2369	1.2362	1.2356	1.2350	1.2344	1.2339	1.2333	1.2328	1.2322	1.2317	1.2312

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.2444	1.2435	1.2427	1.2419	1.2411	1.2403	1.2395	1.2388	1.2381	1.2374	1.2367	1.2360	1.2354	1.2348	1.2341	1.2335	1.2330	1.2324	1.2318	1.2313	1.2307	1.2302	1.2297
1.2430	1.2421	1.2413	1.2405	1.2397	1.2389	1.2381	1.2374	1.2367	1.2360	1.2353	1.2346	1.2340	1.2333	1.2327	1.2321	1.2315	1.2310	1.2304	1.2298	1.2293	1.2288	1.2283
1.2417	1.2408	1.2399	1.2391	1.2383	1.2375	1.2368	1.2360	1.2353	1.2346	1.2339	1.2332	1.2326	1.2319	1.2313	1.2307	1.2301	1.2296	1.2290	1.2284	1.2279	1.2274	1.2268
1.2404	1.2395	1.2386	1.2378	1.2370	1.2362	1.2354	1.2347	1.2340	1.2333	1.2326	1.2319	1.2312	1.2306	1.2300	1.2294	1.2288	1.2282	1.2276	1.2271	1.2265	1.2260	1.2255
1.2391	1.2382	1.2374	1.2365	1.2357	1.2349	1.2342	1.2334	1.2327	1.2320	1.2313	1.2306	1.2299	1.2293	1.2287	1.2281	1.2275	1.2269	1.2263	1.2257	1.2252	1.2247	1.2241
1.2379	1.2370	1.2361	1.2353	1.2345	1.2337	1.2329	1.2322	1.2314	1.2307	1.2300	1.2293	1.2287	1.2280	1.2274	1.2268	1.2262	1.2256	1.2250	1.2245	1.2239	1.2234	1.2229
1.2367	1.2358	1.2349	1.2341	1.2333	1.2325	1.2317	1.2309	1.2302	1.2295	1.2288	1.2281	1.2274	1.2268	1.2262	1.2255	1.2249	1.2244	1.2238	1.2232	1.2227	1.2221	1.2216
1.2355	1.2346	1.2338	1.2329	1.2321	1.2313	1.2305	1.2298	1.2290	1.2283	1.2276	1.2269	1.2262	1.2256	1.2250	1.2243	1.2237	1.2231	1.2226	1.2220	1.2214	1.2209	1.2204
1.2344	1.2335	1.2326	1.2318	1.2309	1.2301	1.2294	1.2286	1.2279	1.2271	1.2264	1.2257	1.2251	1.2244	1.2238	1.2232	1.2226	1.2220	1.2214	1.2208	1.2203	1.2197	1.2192
1.2333	1.2324	1.2315	1.2307	1.2298	1.2290	1.2282	1.2275	1.2267	1.2260	1.2253	1.2246	1.2239	1.2233	1.2226	1.2220	1.2214	1.2208	1.2202	1.2197	1.2191	1.2186	1.2180

58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1.2322	1.2313	1.2304	1.2296	1.2268	1.2279	1.2272	1.2264	1.2256	1.2249	1.2242	1.2235	1.2228	1.2222	1.2215	1.2209	1.2203	1.2197	1.2191	1.2185	1.2180	1.2174	1.2169
1.2312	1.2303	1.2294	1.2285	1.2277	1.2269	1.2261	1.2253	1.2246	1.2238	1.2231	1.2224	1.2218	1.2211	1.2205	1.2198	1.2192	1.2186	1.2180	1.2175	1.2169	1.2163	1.2158
1.2302	1.2293	1.2284	1.2275	1.2267	1.2259	1.2251	1.2243	1.2235	1.2228	1.2221	1.2214	1.2207	1.2201	1.2194	1.2188	1.2182	1.2176	1.2170	1.2164	1.2158	1.2153	1.2147
1.2292	1.2283	1.2274	1.2265	1.2257	1.2249	1.2241	1.2233	1.2225	1.2218	1.2211	1.2204	1.2197	1.2190	1.2184	1.2177	1.2171	1.2165	1.2159	1.2154	1.2148	1.2142	1.2137
1.2282	1.2273	1.2264	1.2255	1.2247	1.2239	1.2231	1.2223	1.2215	1.2208	1.2201	1.2194	1.2187	1.2180	1.2174	1.2167	1.2161	1.2155	1.2149	1.2143	1.2138	1.2132	1.2127
1.2273	1.2264	1.2255	1.2246	1.2238	1.2229	1.2221	1.2213	1.2206	1.2198	1.2191	1.2184	1.2177	1.2171	1.2164	1.2158	1.2151	1.2145	1.2139	1.2134	1.2128	1.2122	1.2117
1.2264	1.2254	1.2245	1.2237	1.2228	1.2220	1.2212	1.2204	1.2196	1.2189	1.2182	1.2175	1.2168	1.2161	1.2155	1.2148	1.2142	1.2136	1.2130	1.2124	1.2118	1.2113	1.2107
1.2255	1.2245	1.2236	1.2228	1.2219	1.2211	1.2203	1.2195	1.2187	1.2180	1.2173	1.2166	1.2159	1.2152	1.2145	1.2139	1.2133	1.2126	1.2120	1.2115	1.2109	1.2103	1.2098
1.2246	1.2237	1.2228	1.2219	1.2210	1.2202	1.2194	1.2186	1.2178	1.2171	1.2164	1.2157	1.2150	1.2143	1.2136	1.2129	1.2123	1.2117	1.2111	1.2105	1.2100	1.2094	1.2088
1.2238	1.2228	1.2219	1.2210	1.2202	1.2193	1.2185	1.2177	1.2170	1.2162	1.2155	1.2148	1.2141	1.2134	1.2127	1.2121	1.2115	1.2108	1.2102	1.2096	1.2091	1.2085	1.2079

$$\lambda(z;\ \mu,\ \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}\,z} \exp\left[-\frac{1}{2}\left(\frac{\ln z - \mu}{\sigma}\right)^2\right]$$

Then its logarithm y = ln z has a normal distribution

$$n(y;\mu,\ \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma}\ e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

The cumulative log-normal distribution is denoted $\Lambda(z; \mu, \sigma^2)$.

An expression for it does not exist in terms of elementary functions but by the simple transformation y = l n z it is transformed into the normal integral.

$$\Lambda(z;\mu,\sigma^2) = N(\ln z;\mu,\sigma^2)$$

and by a further simple transformation to the standard normal variate $x = \frac{y-\mu}{\sigma}$. Cumulative probabilities of the log-normal can be found numerically from tabulated values of the standard normal N (x; 0, 1).

$$\Lambda(z;\ \mu,\ \sigma^2) = N\left(\frac{\ln z - \mu}{\sigma}\ ;\ 0,\ 1\right)$$

The parameters μ and σ can be estimated from log-normal data by means of the transformation

$$\hat{\mu} = \overline{\ln z} = \frac{\Sigma_{i=1}^{n} \ln z_i}{n}$$

$$\hat{\sigma} = s = \sqrt{\frac{\Sigma_{i-1}^{n} (\ln z_i - \overline{\ln z})^2}{n-1}}$$

A.3 Confidence Limit for Exponential Failure Times and Log-Normal Recovery Times

Gray and Lewis* have shown that if σ^2 (the variance of $\ln M_c$) is assumed to be known, then for a random sample of m repair times

$$\frac{\left(\prod_{i=1}^{m} M_{c_i}\right)^{1/m}}{e^{M}} = \frac{\hat{M}_{c_G}}{e^{M}} = \frac{1}{\hat{\nu} e^{M}} \sim \Lambda\left(0, \frac{\sigma^2}{m}\right)$$

where $\hat{M}_{c_G}$ is the geometric mean and $\hat{\nu}$ is $1/\hat{M}_{c_G}$. For purposes of analysis σ^2 is assumed to equal the variance estimated from the sample. For exponentially distributed failure times

$$\frac{2n\lambda}{\hat{\lambda}} \sim \chi^2 \, (2n)$$

Then a confidence interval for λ/μ can be found by finding the distribution of

$$\frac{\dfrac{2n\lambda}{\hat{\lambda}}}{\dfrac{1}{\hat{\nu} e^{M}}} = \hat{\nu} e^{M} \frac{2n\lambda}{\hat{\lambda}}$$

In general then, letting $U \sim \chi^2$ (k) and $V \sim \Lambda\left(0, \frac{\sigma^2}{m}\right)$

$$f(u, v) = \frac{u^{\frac{k}{2}-1} \epsilon^{-\frac{u}{2}} \sqrt{m}\, e^{-m/2\,(\ln v/\sigma)^2}}{\Gamma\left(\frac{k}{2}\right) 2^{k/2} \sigma \sqrt{2\pi}\, v} \qquad \begin{matrix} 0 < u < \infty \\ 0 < v < \infty \end{matrix}$$

$$= 0 \text{ elsewhere}$$

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*Gray, H.L. and Lewis T.O., "A Confidence Interval for the Availability Ratio, *Technometrics 9: 1967, pp. 465-71.*

Let W = U/V and Z = V; then

$$g(w) = c \int_0^\infty \exp\left[-\frac{m}{2\sigma^2} \ln^2 z - \frac{1}{2} wz\right] (wz)^{\frac{k}{2} - 1} dz, \quad 0 < w < \infty$$

$$= 0 \ \text{elsewhere}$$

where

$$c = \frac{\sqrt{\frac{m}{\sigma^2}}}{\Gamma\left(\frac{k}{2}\right) 2^{k/2} \sqrt{2\pi}}$$

Hence $P[a < w < b]$ is given by

$$P\left[a < w < b\right] = \int_a^b g(w)\, dw = p$$

Let

$$w = \frac{2\hat{\nu}\ e^{M} n\lambda}{\hat{\lambda}}$$

$$P\left[w < b\right] = P\left[\frac{\lambda}{\mu} < \frac{\hat{\lambda}}{\hat{\mu}} \cdot \frac{a e^{\sigma^2/2}}{2n}\right] = 1 - \alpha$$

Then a one-sided 100 (1 - α) percent confidence limit for A_i is

$$A_{i_L} = \frac{1}{1 + \frac{\hat{\lambda}}{\hat{\nu}} \left(\frac{a e^{\sigma^2/2}}{2n}\right)}$$

Table A-2 gives selected values of the coefficient $a_{1-\alpha;\ \frac{m}{\sigma^2},\ n}$, for

$$\alpha = \frac{\sqrt{\frac{m}{\sigma^2}}}{\Gamma(n)\, 2^n \sqrt{2\pi}} \int_0^a \int_0^\infty \exp\left[-\frac{m}{2\sigma^2} \ln^2 z - \frac{1}{2} wz \right] (wz)^{n-1}\, dz\, dw$$

$$= .20$$

TABLE A-2. a – DISTRIBUTION α = .20

n = number of failures

$\frac{m}{\sigma^2}$	3	4	5	6	7	8	9	10	11	12	13
2	2.278	3.340	4.461	5.523	6.595	7.697	8.802	9.895	10.985	12.083	13.188
3	2.409	3.660	4.900	6.047	7.237	8.473	9.703	10.912	12.120	13.341	14.572
4	2.460	3.893	5.162	6.348	7.628	8.958	10.263	11.541	12.823	14.127	15.442
5	2.503	4.086	5.319	6.539	7.901	9.300	10.649	11.969	13.307	14.674	16.051
6	2.590	4.228	5.404	6.671	8.115	9.559	10.928	12.279	13.663	15.081	16.506
7	2.767	4.305	5.441	6.778	8.297	9.760	11.136	12.512	13.939	15.402	16.862
8	3.006	4.319	5.453	6.880	8.457	9.915	11.291	12.694	14.163	15.663	17.150
9	3.170	4.286	5.458	6.991	8.595	10.030	11.407	12.841	14.351	15.883	17.388
10	3.211	4.226	5.475	7.114	8.709	10.111	11.495	12.967	14.516	16.071	17.586
12	3.069	4.096	5.597	7.363	8.843	10.195	11.625	13.186	14.800	16.374	17.890
14	2.840	4.060	5.862	7.516	8.862	10.222	11.743	13.394	15.036	16.595	18.100
16	2.670	4.226	6.113	7.522	8.813	10.249	11.887	13.596	15.218	16.743	18.243
18	2.664	4.579	6.171	7.427	8.757	10.321	12.065	13.770	15.337	16.831	18.342
20	2.921	4.782	6.069	7.305	8.749	10.462	12.251	13.889	15.395	16.878	18.421
22	3.302	4.725	5.902	7.217	8.825	10.655	12.394	13.940	15.405	16.905	18.501
24	3.374	4.545	5.749	7.212	8.996	10.840	12.464	13.931	15.388	16.933	18.594
26	3.221	4.349	5.666	7.319	9.215	10.955	12.457	13.884	15.370	16.981	18.709
28	3.010	4.205	5.689	7.530	9.396	10.975	12.395	13.824	15.369	17.058	18.843
30	2.832	4.153	5.839	7.770	9.471	10.916	12.305	13.777	15.402	17.169	18.984
35	2.763	4.537	6.397	7.921	9.270	10.637	12.138	13.826	15.666	17.521	19.263
40	3.276	4.920	6.284	7.580	8.956	10.504	12.263	14.165	16.039	17.736	19.308
45	3.354	4.617	5.898	7.303	8.906	10.731	12.672	14.512	16.160	17.681	19.171
50	2.997	4.276	5.710	7.361	9.237	11.181	12.962	14.540	16.014	17.480	19.012
60	2.792	4.455	6.328	8.151	9.754	11.201	12.605	14.049	15.591	17.269	19.090

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