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DYNAMICS OF LIQUID-FILLED PROJECTILES

GEORGE SCHLENKER

APRIL 1976

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US ARMY ARMAMENT COMMAND

SYSTEMS ANALYSIS DIRECTORATE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report presents a collection of analytic models which treat various aspects of the dynamics of liquid-filled, spin-stabilized projectiles. Several numerical examples are given applicable to eight-inch ammunition. Although idealized, these examples may provide understanding of the behavior of real systems such as the XM736 binary round. Chapter 1 examines the change in inertial characteristics with a change in liquid configuration. Chapter 2 treats the		

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dynamics of spinup of the liquid fill. Also considered are the range and deflection sensitivity to configuration change, and the question of "ballistic similitude" with a comparable solid-filled projectile. The distribution of energy in the liquid and an approximation for the frequency of a fundamental vibratory mode of the liquid are addressed in Chapter 3. The frequency of vibration is compared with precessional and nutational frequencies of the projectile during flight to assess the likelihood of stability problems.

Insofar as the approximations made here are applicable, one can conclude that the XM736 projectile is a reasonable ballistic match to the M509. Difference in mean point of impact can be corrected by small adjustments in aiming. The XM736 is judged to be ballistically stable but will likely have a somewhat larger ballistic dispersion than either the M509 or M106 projectiles.

PREFACE

The author gratefully acknowledges the assistance of his wife, Emily, in typing the manuscript.

The author is indebted to Bill D'Amico of the Ballistics Research Laboratory for reviewing a draft of this report and only regrets that time and project priorities did not permit him to pursue Bill's many interesting suggestions.

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INTRODUCTION

The initial objectives of this study were quite simple, namely, to analytically determine whether a liquid-filled projectile having a partial fill similar to the XM736, eight-inch binary projectile would be an adequate ballistic match to a comparable solid projectile, the M509. This study is an effort to respond to a portion of a study request of the XM736 projectile initiated by the U.S. Army Materiel Command. During our investigation the objectives and techniques were expanded. To a great extent the study assumed a methodological orientation, with the methods illustrated by simple examples.

In an effort to keep the report unclassified some specificity and fidelity to actual developmental configurations had to be sacrificed. For example, the effect of change in liquid position on inertial properties was examined via a hypothetical, simplified, liquid-filled, projectile configuration. Hopefully, this idealization does not preclude application to real systems. All numerical examples given here were chosen from the eight-inch family of ammunition. However, the methods used are considered to be applicable to other spin-stabilized, liquid-filled projectiles.

Insofar as the initial study objectives are concerned, one can conclude that the XM736 projectile is a reasonable ballistic match to the M509. Difference in mean point of impact can be corrected by small adjustments in aiming. The XM736 is judged to be ballistically stable but will likely have a somewhat larger ballistic dispersion than either the M509 or M106 projectiles.

These conclusions certainly can be challenged on the grounds that the methods and/or data inputs are inapplicable

to the XM736 projectile. Admittedly, the phenomena occurring during spinup and mixing of the chemical reagents in that system are not treated adequately in this study. In fact, limitations of the study are recognized and stated explicitly in the caveats below.

Although lacking physical rigor at several points, this report is offered in the hope that some of the methods and insights may be of interest and that some of the physical shortcomings may provoke further inquiry. The author suggests that a careful, finite-element approach to the analysis of mixing and spinup, considering pertinent physical phenomena, may be a useful avenue of approach.

Caveats

The following, numbered caveats are provided to identify restrictions in the scope and depth of the study:

1. Treatment of phenomena occurring during mixing of chemical reagents (for binary systems) is highly idealized.
 - a. Constant free volume within the shell cavity was assumed although it is known that prior to mixing the liquid ingredients in the XM736 occupy a smaller volume than that assumed, and that after mixing and reacting chemically, essentially the entire cavity is occupied by reaction products.
 - b. The value of kinematic viscosity was treated parametrically at two (constant) levels -- 10 and 1 centistoke. It is estimated from laboratory tests that the viscosity of the cold reactants is greater than ten centistokes whereas the hot reaction products have a viscosity of less than one centistoke.
2. Only certain kinds of liquid resonances were treated. It is known that pressure waves propagating in a liquid

confined within a cavity have certain characteristic- or eigen-frequencies. Although not treated in this report, a discussion of these resonant frequencies is found in AMCP-706-165, Liquid-Filled Projectile Design. The discussion of liquid resonances in this report is limited to the whole-body or sloshing motion of the liquid. Obviously, this discussion applies only to partially-filled shells.

3. An academic or idealized mathematical treatment of liquid spinup is contained herein. This development strictly applies only when liquid flow is laminar and, then, only for cavities which are quite long relative to their diameter. The diffusion of angular momentum within the liquid due to turbulence, generated either by a chemical reaction or by vorticity cells induced by the boundary conditions at the ends of the liquid cylinder, were not considered in this study. For most liquid-filled projectiles, the rapid achievement of homogeneous angular velocity is due mainly to the turbulent condition of the liquid during spinup which, effectively, creates a large apparent viscosity.

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CHAPTER 1
CHANGE IN INERTIAL CHARACTERISTICS OF
A LIQUID-FILLED PROJECTILE DURING FLIGHT

Inertial Characteristics in Two Limiting Configurations

Consider the following idealized model of a liquid-filled projectile during launch. Because of the large axial acceleration and small average angular velocity of the liquid, the liquid will be forced to the rear of the shell cavity so that its free surface will approximate a flat circular diaphragm. See Example 1 below. This liquid configuration, A, is shown below in Figure 1.1.

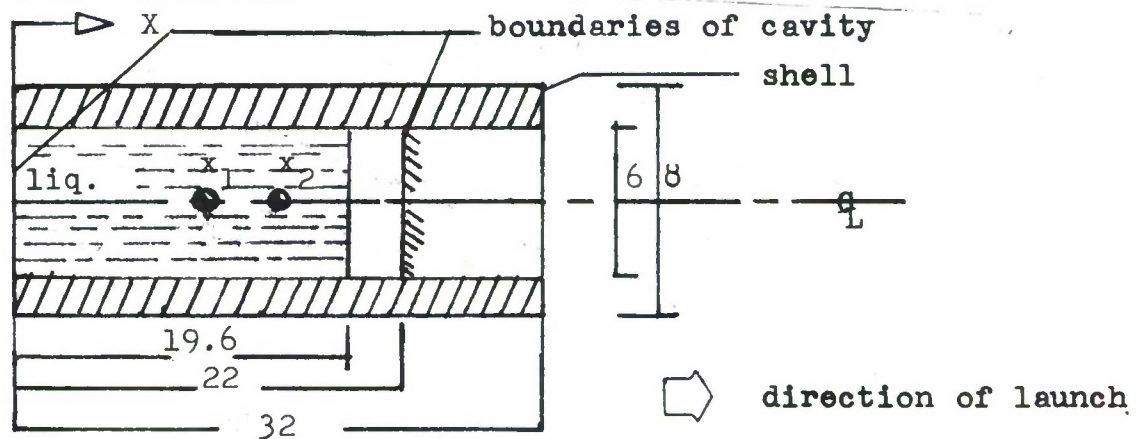


Figure 1.1. Configuration A

The solid shell body is approximated as a cylindrical sleeve 32 in. long, with a 1 in. wall thickness. Massless boundaries confine the liquid to a cylindrical cavity 22 in. long, having a 6 in. diameter. The liquid fills only 90% of the available volume. During launch the height of the column of liquid is 19.6 in. in the axial or X-direction. The average density of the solid shell body is assumed to be 0.2558 lb/in^3 , i.e., specific gravity 7.09. The density of the liquid is assumed to be 0.0361 lb/in^3 , i.e., specific gravity 1. See ref. [1]* for standard formulas and material values.

* Square-bracketed numbers refer to the bibliographic citations.

[1] Eshbach, C. Handbook of Engineering Fundamentals, c. 1952.

Other pertinent parameter values are shown in Table 1.1.

TABLE 1.1. INERTIAL CHARACTERISTICS OF
A HYPOTHETICAL LIQUID-FILLED PROJECTILE IN CONFIGURATION A

Characteristic	Value	Dimension
volume of solid	703.7	in ³
weight of solid	180	lb
volume of liquid	554.2	in ³
weight of liquid	20	lb
total weight of projectile	200	lb
center of gravity of liquid, x_1	9.8	in
center of gravity of solid, x_2	16	in
center of gravity of projectile	15.38	in
axial moment of inertia of projectile	0.50506	slug ft ²
axial moment of liquid	0.019426	slug ft ²
axial moment of solid	0.48564	slug ft ²
pitch moment of inertia of pro- jectile about its cg	3.8554	slug ft ²
pitch moment of liquid about x_1	0.14791	slug ft ²
pitch moment of solid about x_2	3.55813	slug ft ²

After exit from the cannon, the projectile will experience negligible axial acceleration, relative to its centripetal acceleration. Therefore, the free surface of the liquid will be a cylinder centered on the spin axis. This configuration, B, is shown in Figure 1.2.

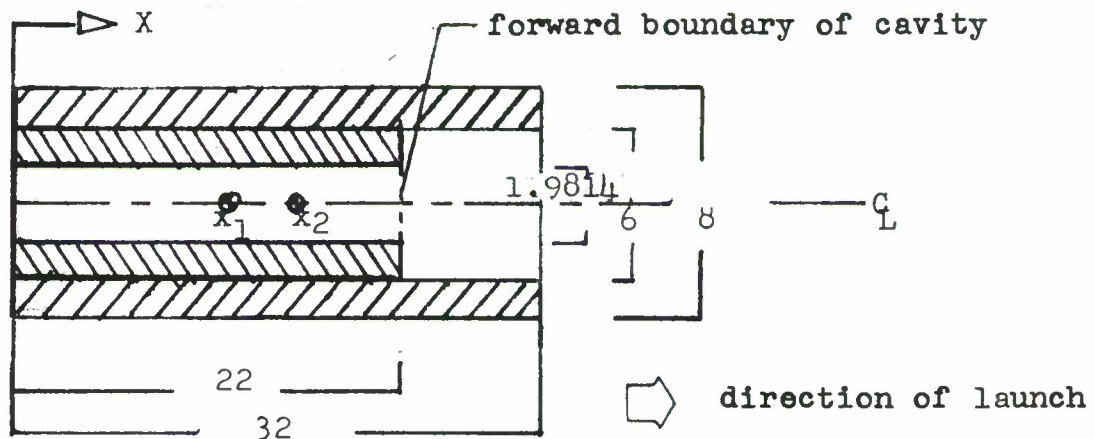


Figure 1.2. Configuration B

In Configuration B, the volumes and weights of solid and liquid are, of course, the same as in Configuration A. However, other inertial properties have changed from those shown in Table 1.1 to the values shown below:

center of gravity (cg) of liquid = 11.0 in

center of gravity (cg) of projectile = 15.5 in .

Thus, the projectile cg has moved forward by 0.12 in. In the M106 projectile, which has the same mass and caliber as this hypothetical projectile, the round-to-round standard deviation of cg position about the mean is 0.064 in., due to manufacturing tolerances [2]. Consequently, one may conclude that cg shifts due to liquid configuration changes in this

[2] Final Report of the HY-BRA Weapons System, Feb. 1970.

hypothetical projectile are less than two times the standard deviation associated with round-to-round og shifts in comparable solid projectiles. Also

axial moment of inertia of liquid
(Configuration B) = $0.02154 \text{ slug ft}^2$

axial moment of inertia of projectile
(Configuration B) = $0.50718 \text{ slug ft}^2$.

Further, comparisons of the tabular data show that the axial moment of inertia for Configuration B is $0.00212 \text{ slug ft}^2$ greater than for Configuration A. To put this difference in perspective, consider that the estimated round-to-round standard deviation in spin inertia for the M106 projectile, due to manufacturing variability, is $0.00252 \text{ slug ft}^2$ [2]. Thus, the change in spin inertia resulting from liquid configuration changes is less than one standard deviation of that of the comparable solid projectile. Using the deflection sensitivity to spin inertia given in Reference [2] for the M106 projectile, one expects a change in deflection of 0.26 mils due to change in inertial properties associated with the change in configuration.

Further,

pitch moment of inertia of liquid
(Configuration B) about x_1 = $0.18488 \text{ slug ft}^2$

pitch moment of inertia of projectile
(Configuration B) about its cg = 3.8401 slug ft^2 .

[2] Ibid.

Note that even though the pitch inertia of the liquid about its cg is greater in Configuration B than in Configuration A, the pitch moment of inertia of the whole projectile is smaller for Configuration B. This is due to the shift in cg of the liquid which brings it closer to the cg of the whole projectile. The difference in pitch inertia between the configurations is 0.0152 slug ft^2 .

The estimated [2] round-to-round standard deviation of pitch inertia in the M106 projectile due to typical manufacturing tolerances is 0.0252 slug ft^2 . Thus, liquid configuration change in the hypothetical projectile produces a change in pitch inertia only 0.6 of one standard deviation in pitch inertia of the M106.

One must conclude that the changes in inertial characteristics accompanying liquid configurational change in the hypothetical liquid-filled projectile would not be sufficient to produce significant changes in exterior ballistics, given only adequate ballistic stability of the liquid-filled projectile.

[2] Ibid.

Estimate of Shape of the Free Surface of the Liquid in a Liquid-Filled Projectile During Acceleration

Consider Figure 1.3 below in which a cross section of the liquid-filled cavity of a projectile is depicted. Acceleration of the projectile is assumed in the positive X-direction.

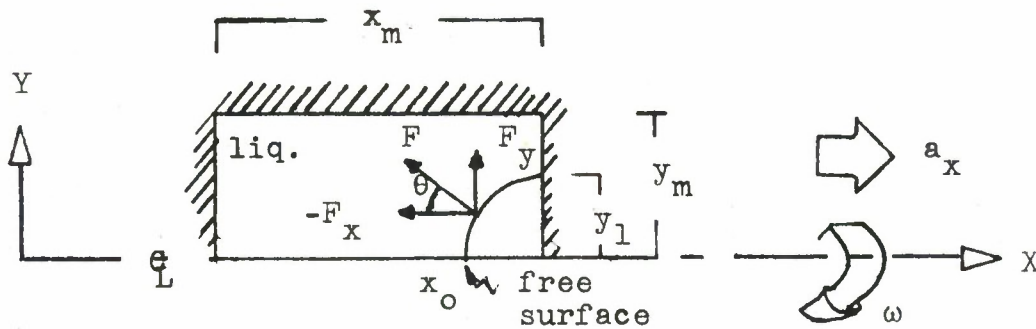


Figure 1.3. Liquid Free Surface During Acceleration

The free surface is defined by the function $y(x)$. The differential element at (x, y) , dv , experiences forces in the Y- and X-directions due to spin and setback, respectively. Neglecting gravity, which is small compared to the accelerations of interest, and assuming solid-body rotation, the forces on dv may be written as

$$F_x = - a_x \rho dv$$

$$F_y = \rho dv y \omega^2 \quad (1.1)$$

with

$$dv = 2 \pi y dy dx .$$

The angular velocity, ω , is given in terms of the velocity of projectile, v_p , caliber, D , and twist, T , as

$$\omega = \frac{2 \pi v_p}{D T} . \quad (1.2)$$

The vector sum of these forces on dv must be normal to the free liquid surface at equilibrium, since the liquid will not support shear stresses. Therefore, the slope of the free surface at (x, y) is given by

$$\frac{dy}{dx} = - (\tan \theta)^{-1} = \frac{-F_x}{F_y} = k y^{-1} , \quad (1.3)$$

with

$$k = a_x \omega^{-2} .$$

Equation (1.3) may be integrated to yield

$$y^2 = 2 k (x - x_0)$$

or

$$y = [2 k (x - x_0)]^{1/2} , \quad (1.4)$$

and

$$x = x_0 + \frac{y^2}{2 k} , \quad (1.5)$$

where the value of x_0 is determined by the volume of the cavity and volume of liquid. Since the volume, V , of liquid is assumed constant,

$$V = \pi (y_m^2 - y_1^2) x_m = \int_0^{y_1} 2 \pi y x(y) dy, \quad (1.6)$$

$$0 \leq y_1 \leq y_m , \quad 0 < x_0 \leq x_m ,$$

with

$$y_1 = \min\{y_m, y(x_m) = [2k(x_m - x_0)]^{1/2}\} \quad (1.7)$$

Then, from (1.5)

$$\frac{V}{2\pi} - (y_m^2 - y_1^2) \frac{x_m}{2} = \int_0^{y_1} \left(\frac{y^3}{2k} + x_0 y \right) dy$$

$$\frac{V}{2\pi} - \frac{y_m^2 x_m}{2} + (x_m - x_0) \frac{y_1^2}{2} - \frac{y_1^4}{8k} = 0$$

For $y_1 < y_m$,

$$y_1^4 = \frac{4k}{\pi} (V_c - V) \quad (1.8)$$

with

$$V_c = \pi x_m y_m^2$$

And for $y_1 = y_m$,

$$x_0 = \frac{V}{\pi y_m^2} - \frac{y_m^2}{4k} \quad (1.9)$$

Example 1.

Take the following conditions at peak projectile acceleration:

liquid volume, $V = 553.8 \text{ in}^3$ (0.3205 ft^3)

cavity volume, $V_c = 622 \text{ in}^3$ (0.36 ft^3)

projectile velocity, $v_p = 200 \text{ ft/s}$

gun tube twist, $T = 20 \text{ cal/rev}$

caliber, $D = 8 \text{ in}$ ($2/3 \text{ ft}$)

projectile acceleration, $a_x = 4.504 \cdot 10^5 \text{ ft/s}^2$

$x_m = 22 \text{ in}$ (1.833 ft)

$y_m = 3 \text{ in}$ (0.25 ft)

Then,

$$\omega = \frac{2 \pi v_p}{D T} = 94.248 \text{ rad/sec (15 hz) } ,$$

$$k = \frac{a_x}{\omega^2} = 50.70 \text{ ft} ,$$

$$y^2 = 101.4 (x - x_0) .$$

Assuming

$$y_1 = y_m = 0.25 \text{ ft} ,$$

$$x(y_1) - x_0 = 7 \cdot 10^{-3} \text{ in}$$

and

$$x_0 = 19.587 \text{ in.}$$

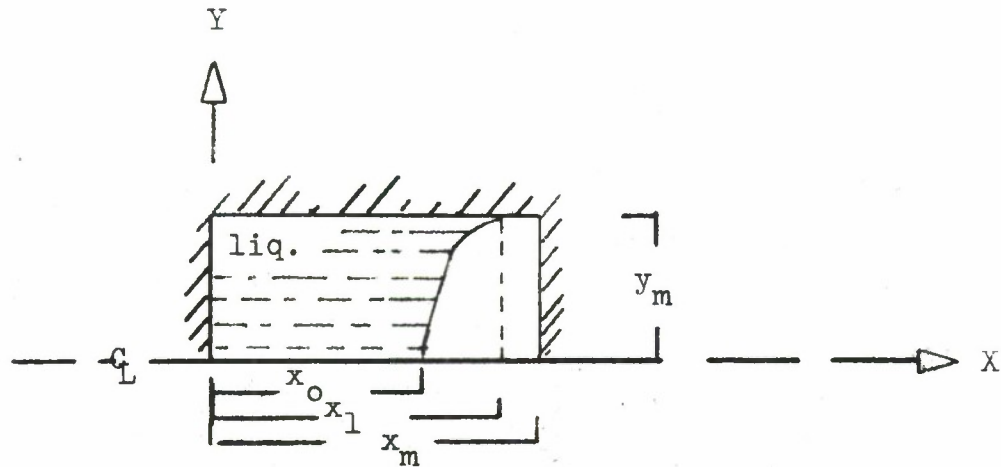


Figure 1.4. Liquid Free Surface During Acceleration

Figure 1.4 represents the position of the liquid surface where the unoccupied volume of the cavity is large. In this case the free (unoccupied) volume is

$$V_f = V_{\text{cavity}} - V_{\text{liq}} \quad (1.10)$$

From Figure 1.4,

$$V_f = \pi(x_m - x_1) y_m^2 + \int_{x_0}^{x_1} \pi y^2 dx, \quad 0 < x < x_m \quad (1.11)$$

Using (1.4, 1.11),

$$V_f = \pi(x_m - x_1) y_m^2 + \pi k(x_1 - x_0)^2 \quad (1.12)$$

But, from (1.5),

$$x_0 = x_1 - \frac{y_m^2}{2k} \quad (1.13)$$

Then (1.12, 1.13) yield an expression for x_1 .

$$x_1 = x_m + \frac{y_m^2}{4k} - \frac{V_f}{\pi y_m^2} \quad (1.14)$$

Given the cavity dimensions and liquid volume, Equation (1.14) can be solved for x_1 and with this value x_0 can be obtained from (1.13).

Example 2

Suppose that conditions at the muzzle of a gun are as follows:

projectile acceleration, $a_x = 8.086 \cdot 10^4 \text{ ft/s}$

projectile velocity, $v_p = 2000 \text{ ft/s}$

twist, $T = 20 \text{ cal/rev}$

caliber, $D = 2/3 \text{ ft (8 in)}$

cavity free volume, $V_f = 0.03935 \text{ ft}^3$

cavity length, $x_m = 1.8333 \text{ ft}$

cavity radius, $y_m = 0.25 \text{ ft}$

Using (1.2),

$$\omega = 942.48 \text{ rad sec}^{-1} \text{ (150 hz)}$$

From (1.3),

$$k = 0.09103 \text{ ft}$$

and

$$y^2 = 0.18206 (x - x_0) (\text{ft})^2 .$$

Then (1.14) yields

$$x_1 = 1.80457 \text{ ft} = 21.655 \text{ in} ,$$

and from (1.13),

$$x_0 = 1.46128 \text{ ft} = 17.535 \text{ in} ,$$

so that

$$x_1 - x_0 = 4.12 \text{ in.}$$

The radial acceleration at y_m in this example is a_y .

$$a_y(y_m) = y_m \omega^2 = .25(942.48)^2$$

$$a_y(y_m) = 22.2 \cdot 10^4 \text{ f/s}^2$$

$$\approx 0.69 \cdot 10^4 \text{ g} .$$

CHAPTER II
ANGULAR ACCELERATION OF THE LIQUID IN A
LIQUID-FILLED PROJECTILE DURING FLIGHT
Liquid "Spinup" in Configuration A

We treat the position of the liquid within the shell as in Configuration A (Figure 1.1). Let the tangential or circumferential velocity of the liquid at radius r and time t be denoted by v . We neglect axial velocity components.

Consider the annular volume element shown in Figure 2.1.

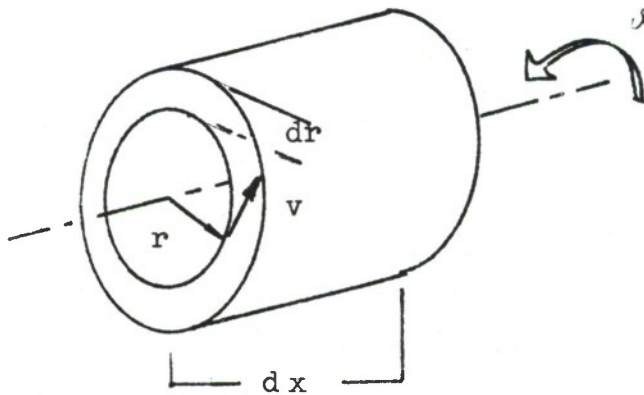


Figure 2.1. Volume Element Under Angular Acceleration

At position (r, x) in the liquid, the shear stress acting on the surface dA_- (below r) is

$$- \sigma_r(r, x)$$

where

$$dA_- = 2 \pi r dx .$$

At a radial position $r + dr$ the shear stress acting on the surface dA_+ is

$$\sigma_r(r+dr, x) ,$$

where

$$dA_+ = 2 \pi (r+dr) dx .$$

Similarly at the left of the element the shear stress acting on the differential area dA_0 is

$$- \sigma_x(r, x)$$

where

$$dA_0 = 2 \pi r dr .$$

And at axial position $x + dx$, the shear stress acting on dA_0 is

$$\sigma_x(r, x+dx) .$$

The axial moment of inertia of the differential element is

$$I = 2 \pi \rho r^3 dx dr . \quad (2.1)$$

The sum of all torques acting on the element is

$$\begin{aligned} M_j = & [\sigma_x(r, x+dx) - \sigma_x(r, x)] dA_0 r \\ & + [\sigma_r(r+dr, x) dA_+ - \sigma_r(r, x) dA_-] r . \end{aligned} \quad (2.2)$$

Defining the angular velocity of the liquid at (r, x) at time t as $\omega(r, x, t)$, Newton's law gives

$$\dot{\omega} = M_j / I , \quad (2.3)$$

where the functional dependence on r, x and time t has been suppressed notationally.

Then

$$\begin{aligned}\dot{\omega} = & (\rho r)^{-1} [\sigma_x(r, x+dx) - \sigma_x(r, x)] / dx \\ & + (\rho r^2)^{-1} [\sigma_r(r+dr, x)(r+dr) - \sigma_r(r, x) r] / dr\end{aligned}$$

Or in the limit as dx and dr approach zero,

$$\dot{\omega} = (\rho r)^{-1} \frac{\partial \sigma_x}{\partial x} + (\rho r^2)^{-1} \frac{\partial (r \sigma_r)}{\partial r} . \quad (2.4)$$

But the dynamic viscosity η is defined such that

$$\begin{aligned}\sigma_x &= \eta r \frac{\partial \omega}{\partial x} \\ \sigma_r &= \eta r \frac{\partial \omega}{\partial r}\end{aligned} \quad (2.5)$$

Then with η taken independent of x, r

$$\begin{aligned}\dot{\omega} &= (\rho r)^{-1} \eta r \frac{\partial^2 \omega}{\partial x^2} + (\rho r^2)^{-1} \eta \frac{\partial}{\partial r} (r^2 \frac{\partial \omega}{\partial r}) \\ \dot{\omega} &= \nu \frac{\partial^2 \omega}{\partial x^2} + \nu r^{-2} [2 r \frac{\partial \omega}{\partial r} + r^2 \frac{\partial^2 \omega}{\partial r^2}]\end{aligned}$$

where ν is the kinematic viscosity.

$$\nu = \eta \rho^{-1} \quad (2.6a)$$

$$\dot{\omega} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + 2 r^{-1} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2} \right] \quad (2.6b)$$

If the axial variability in ω can be neglected, the simplified result in one spacial dimension is

$$\dot{\omega} = \nu \left[2 r^{-1} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2} \right] . \quad (2.7)$$

The tangential velocity v in the liquid at radius r is

$$v = r \omega , \quad (2.8)$$

with v a function of r and time t ,

$$v = w(r, t) . \quad (2.9)$$

With (2.7, 2.8)

$$\dot{w} = \nu \frac{\partial^2 w}{\partial r^2}$$

or

$$w_t - \nu w_{rr} = 0 , \quad (2.10)$$

where the subscripts indicate partial differentiation with respect to that variable. Note that for stationary conditions, i.e., zero w_t , w_r is a constant c and $w(r) = c r$.

Notation can be simplified by defining a dimensionless radius ξ and dimensionless time τ .

Let

$$\xi = r/r_1$$

and

$$\tau = t \nu / r_1^2 \quad (2.11)$$

with r_1, ν constants.

Then Equation (2.10) becomes

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} \quad , \quad 0 < \xi < 1 \quad (2.12)$$
$$, \quad \tau > 0$$

where

$$u(\xi, \tau) = w(r, t) \quad . \quad (2.13)$$

The boundary conditions for (2.12) are

$$u(\xi, 0) = 0 \quad , \quad 0 \leq \xi \leq 1 \quad (2.14)$$

$$u(0, \tau) = 0 \quad , \quad \tau > 0 \quad (2.15)$$

$$u(1, \tau) = \varphi(\tau)$$

with

$$\varphi(\tau) = f(t) \quad , \quad t > 0 \quad (2.16)$$

$$f(t) = f_1(t) + f_2(t)$$

$$f_1(t) = a t \quad , \quad t \geq 0$$

$$f_2(t) = 0 \quad , \quad t \leq t_1$$

$$f_2(t) = - a(t - t_1) \quad , \quad t > t_1 \quad . \quad (2.17)$$

Thus

$$\varphi(\tau) = \varphi_1 + \varphi_2$$

$$\varphi_1(\tau) = \alpha \tau, \quad \tau \geq 0$$

$$\varphi_2(\tau) = 0, \quad \tau \leq \tau_1$$

$$\varphi_2(\tau) = -\alpha(\tau - \tau_1), \quad \tau > \tau_1, \quad (2.18)$$

with

$$\alpha = \frac{r_1^2}{\nu} a$$

$$\tau_1 = t_1 \nu / r_1^2. \quad (2.19)$$

Taking the Laplace transform of (2.12)

$$s u^*(\xi, s) - u(\xi, 0) = u_{\xi\xi}^*(\xi, s)$$

or from (2.14)

$$s u^*(\xi, s) = u_{\xi\xi}^*(\xi, s). \quad (2.20)$$

Also, from (2.15, 2.16),

$$u^*(0, s) = 0,$$

and

$$u^*(1, s) = \varphi^*(s), \quad (2.21)$$

or

$$\varphi^*(s) = \alpha s^{-2} - \alpha s^{-2} e^{-\tau_1 s}. \quad (2.22)$$

Since $u^*(\xi, s)$ is not a function of time, the partial derivatives in Equation (2.20) are actually total derivatives of u^* with respect to ξ .

Thus,

$$\frac{d^2 u^*}{d \xi^2} - s u^* = 0 \quad (2.23)$$

whose solution with initial conditions given by (2.21) is

$$u^*(\xi, s) = \varphi^*(s) \frac{\sinh \sqrt{s} \xi}{\sinh \sqrt{s}} \quad (2.24)$$

Using a series expansion of

$$\sinh \sqrt{s} \xi / \sinh \sqrt{s}$$

-- see p. 139, Churchill [3] -- and applying the convolution theorem

$$\begin{aligned} u(\xi, \tau) = & \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \left\{ \int_{\ell_1(n, \xi)}^{\infty} \varphi(\tau - (2n+1-\xi)^2 / (4\lambda^2)) e^{-\lambda^2} d\lambda \right. \\ & \left. - \int_{\ell_2(n, \xi)}^{\infty} \varphi(\tau - (2n+1+\xi)^2 / (4\lambda^2)) e^{-\lambda^2} d\lambda \right\}, \end{aligned}$$

with

$$\begin{aligned} \ell_1(n, \xi) &= \frac{2n+1-\xi}{2\sqrt{\tau}} \\ \ell_2(n, \xi) &= \frac{2n+1+\xi}{2\sqrt{\tau}} \end{aligned} \quad (2.25)$$

[3] Churchill, R.V. Operational Mathematics, 2nd Ed., McGraw-Hill, New York, c. 1958.

For the special case of $\varphi(\tau)$ a constant φ_0

$$u(\xi, \tau) = \varphi_0 \sum_0^{\infty} \left[\operatorname{erf}\left(\frac{2n+1+\xi}{2\sqrt{\tau}}\right) - \operatorname{erf}\left(\frac{2n+1-\xi}{2\sqrt{\tau}}\right) \right] . \quad (2.26)$$

Substitution of (2.22) into (2.25) appears to be too complex to pursue. A numerical approach starting with Equation (2.10) and boundary conditions given by (2.14, 2.15, 2.16) was judged more profitable.

For low viscosity liquids and rapid rise of driving angular velocity to a nearly constant level, the result in (2.26) may be a reasonable description. This expression was evaluated for unity φ_0 and plotted in Figure 2.2. Generally it was possible to truncate the series at four terms or less to achieve 0.1% accuracy.

The tangential velocity of the liquid in Configuration A is also shown as a function of τ for several radial positions in Figure 2.3.

Because angular acceleration of the liquid varies with position, it is useful to define an effective angular velocity as that value which would give the liquid angular momentum if distributed uniformly thruout the liquid.

Thus, for the nondimensional case,

$$\bar{\omega}_A = \frac{\int_0^1 \xi [u(\xi, \tau) \xi^{-1}] d\xi}{\int_0^1 \xi d\xi}$$

$$\bar{\omega}_A(\tau) = 2 \int_0^1 u(\xi, \tau) d\xi . \quad (2.27)$$

A plot of $\bar{\omega}_A$ versus τ is shown in Figure 2.4.

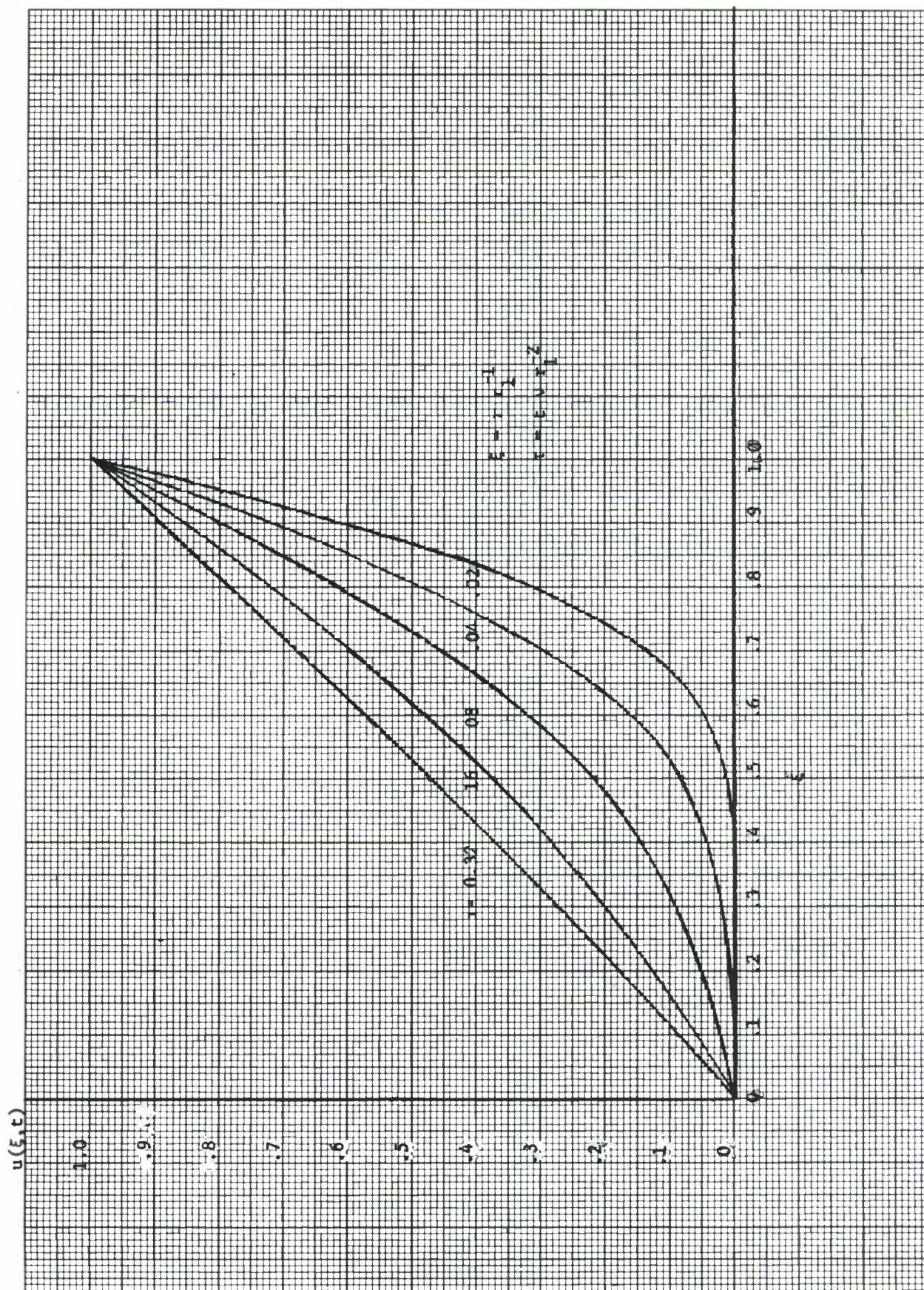


Figure 2.2. Tangential Velocity of Liquid Versus Radial Position at Several Values of Time (Liquid Configuration A)

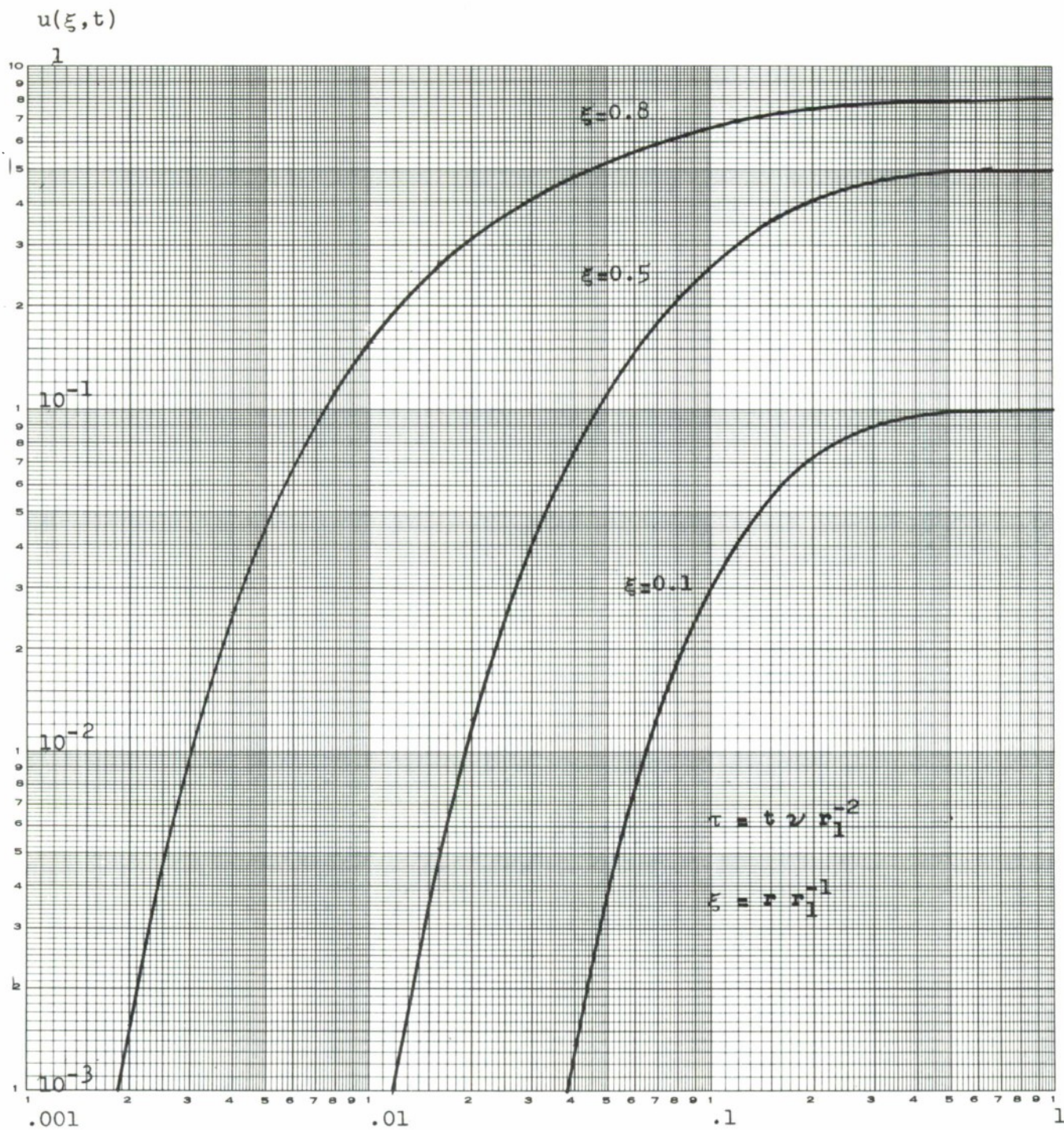


Figure 2.3. Tangential Velocity of Liquid Versus Time for Three Radial Positions

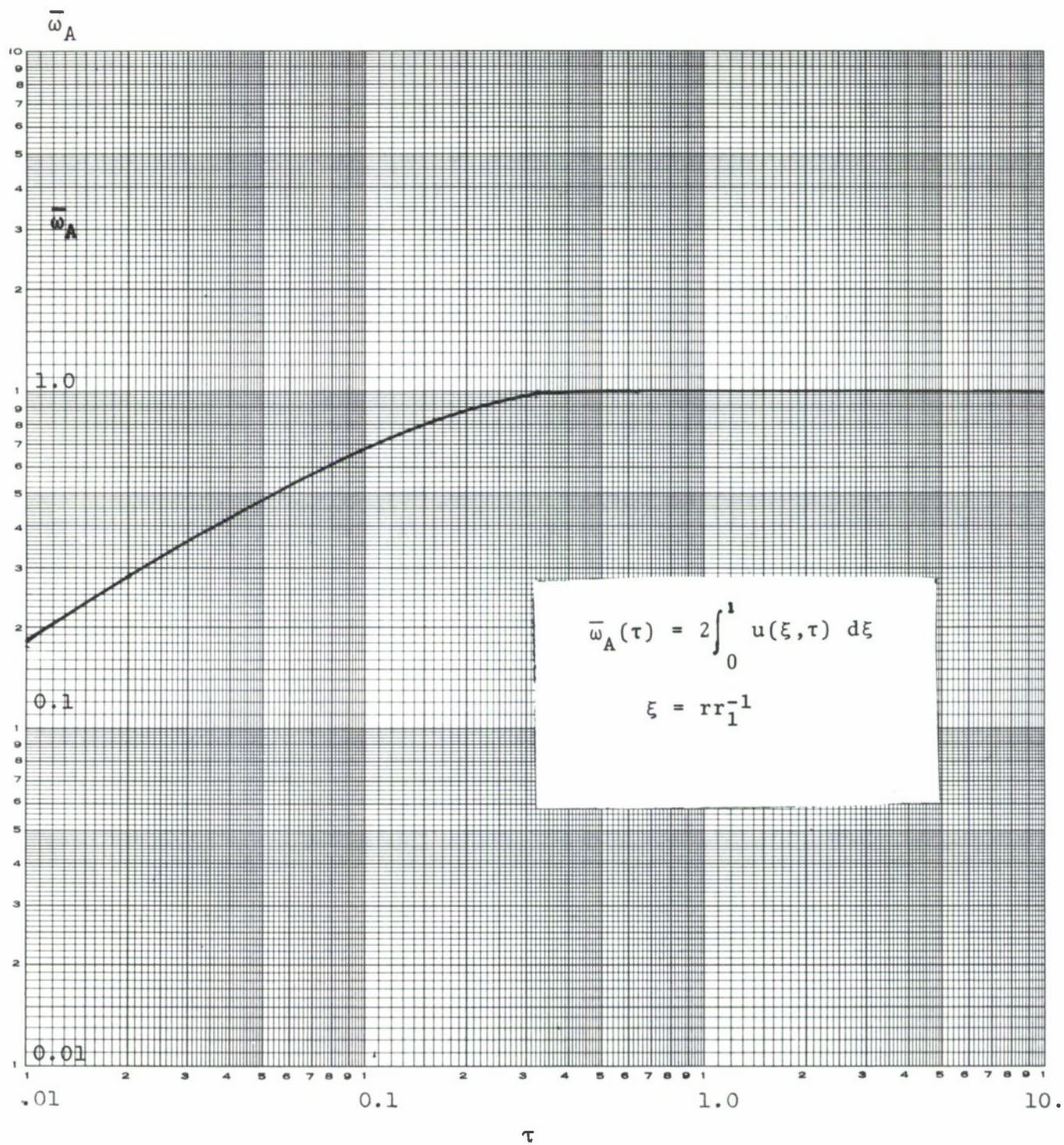


Figure 2.4. Effective Angular Velocity of Liquid in Configuration A Versus Time

Figure 2.3 shows that a nondimensional time τ of 0.1 is required to achieve 50% or more of the asymptotic velocity for $\xi > 0.5$. To apply these results to a specific example, suppose that

$$\nu = 10 \text{ centistokes (0.1 stoke)}$$

$$r_1 = 7.62 \text{ cm (3 in) .}$$

Then if "spinup" is arbitrarily defined as the time at which the angular kinetic energy reaches 70% of its asymptotic value*, a time corresponding to $\tau = 0.125$ is required. This time

$$\begin{aligned} t &= \tau r_1^2 \nu^{-1} \\ &= 0.125 (7.62)^2 10 \end{aligned}$$

$$t = 72.6 \text{ sec .}$$

Since this time exceeds the maximum time of flight, one would not expect spinup to occur in a liquid-filled projectile of this type having the above viscosity and remaining in Configuration A.**

Further during the inbore period, taken as 14 milliseconds, essentially no rotation of the liquid occurs since this time corresponds to τ of $2.4 \cdot 10^{-5}$ for the above example.

In order for the liquid to remain in a stable Configuration B after launch, the minimum radial acceleration must

* Approximately 75% of asymptotic angular momentum.

**As indicated in the Introduction, the conditions for this behavior do not obtain in the XM736 projectile, where experimental data indicate that spinup is complete within one to five seconds.

be at least one g. Thus, the limiting angular velocity of liquid for Configuration B is 19.74 rad/sec; and for a case spin of 942.5 rad/sec, implies a limiting value of $\bar{\omega}_A$ of approximately 0.02. From Figure 2.4 one notes that this value of $\bar{\omega}_A$ is obtained at

$$\tau = 0.012 ,$$

which, for this example, corresponds to a time

$$t = \tau r_1^2 \nu^{-1}$$

$$t = 0.012 (7.62)^2 10$$

$$t \cong 7 \text{ sec} .$$

Beyond the time at which Configuration B is attained, the spin dynamics of the liquid must be obtained from the solution of the following boundary-value problem.

Angular Acceleration of the Liquid in Configuration B

As previously developed, the governing differential equation is

$$\omega_t = \nu [\omega_{xx} + 2 r^{-1} \omega_r + \omega_{rr}] .$$

Letting

$$u(\xi, \lambda, \tau) = \omega(r, x, t) ,$$

with

$$\xi = r/r_1, \quad \xi_0 = r_0/r_1,$$

$$\lambda = x/r_1, \quad \lambda_1 = x_m/r_1, \quad \text{and}$$

$$\tau = t \nu / r_1^2, \quad (2.28)$$

$$u_\tau = u_{\lambda\lambda} + 2\xi^{-1} u_\xi + u_{\xi\xi}, \quad \xi_0 < \xi < 1 \quad (2.29)$$

$$0 < \lambda < \lambda_1$$

$$\tau > 0.$$

The initial and boundary conditions in this case are

$$u(\xi, \lambda, 0) = \psi(\xi, \lambda), \quad \xi_0 < \xi < 1$$

$$u_\xi(\xi_0, \lambda, \tau) = 0^* \quad 0 < \lambda < \lambda_1$$

$$u(1, \lambda, \tau) = \varphi(\tau) \quad 0 \leq \lambda \leq \lambda_1$$

$$u(\xi, 0, \tau) = \varphi(\tau)$$

$$u(\xi, \lambda_1, \tau) = \varphi(\tau), \quad \tau > 0. \quad (2.30)$$

Note τ equal to zero corresponds to the time at which Configuration B is realized.

We particularize the functions $\psi(\xi)$ and $\varphi(\tau)$ as follows:

* At a free surface the shear stress in the liquid must vanish.

$$\psi(\xi, \lambda) = \bar{\omega}_{\text{lim}} \quad (2.31)$$

$$\varphi(\tau) = 1 \quad (\text{Case 1}) \quad , \text{ or,}$$

$$\varphi(\tau) = (1.2 + c\tau)^{-0.2808} \quad (\text{Case 2}) \quad ,$$

with

$$\bar{\omega}_{\text{lim}} \text{ and } c \text{ constants.} \quad (2.32)$$

The limiting or minimum angular velocity for Configuration B is $\bar{\omega}_{\text{lim}}$. The form of $\varphi(\tau)$ for Case 2 is motivated by the despin characteristics of a typical eight-inch projectile. A generalization of this formula is developed in the Appendix.

In the case considered

$$\xi_0 = 0.33$$

$$\bar{\omega}_{\text{lim}} = 0.02$$

$$c = 0.0288 \, r_1^2 \, \nu^{-1} \quad (2.33)$$

$$r_1 = 7.62 \text{ cm}$$

$$\nu = 0.1 \text{ and } 0.01 \text{ stoke} \quad .$$

It is convenient to obtain a numerical solution to this problem thru spacial discretization.

The interval in ξ ($\xi_0, 1$) is divided into $n-1$ equal segments of length

$$\Delta\xi = (1 - \xi_0) / (n-1) \quad . \quad (2.34)$$

Similarly, the interval in λ $(0, \lambda_1)$ is divided into $m-1$ equal segments of length

$$\Delta\lambda = \lambda_1 / (m-1) \quad . \quad (2.35)$$

We define the value of u at the nodal points of this segmented space as follows

$$u_{ji} = u_{ji}(t) = u(\xi_j, \lambda_i, t) \quad , \quad 1 \leq j \leq n \quad (2.36)$$

$$1 \leq i \leq m \quad ,$$

with

$$\xi_j = \xi_0 + (j-1) \Delta\xi$$

$$\lambda_i = (i-1) \Delta\lambda \quad .$$

Division of the space is illustrated in Figure 2.5.

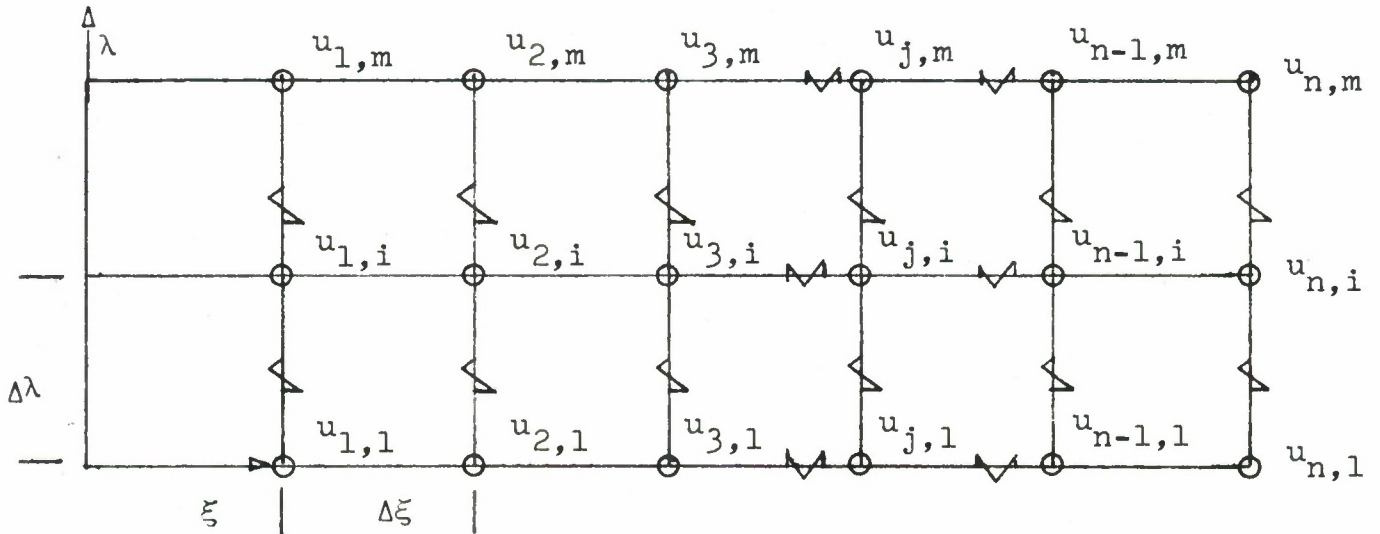


Figure 2.5. Nodal Points for Finite-Difference Method

The second central-difference approximation is used to approximate $u_{\xi\xi}$ and $u_{\lambda\lambda}$ as follows:

$$u_{\xi\xi}(\xi_j, \lambda_i) \cong (\Delta\xi)^{-2} (u_{j+1,i} - 2u_{j,i} + u_{j-1,i}) ,$$

$$u_{\lambda\lambda}(\xi_j, \lambda_i) \cong (\Delta\lambda)^{-2} (u_{j,i+1} - 2u_{j,i} + u_{j,i-1}) ,$$

$$1 < j < n$$

$$1 < i < m . \quad (2.37)$$

A central-difference approximation is also used for u_{ξ} .

$$u_{\xi}(\xi_j, \lambda_i) \cong \frac{u_{j+1,i} - u_{j-1,i}}{2 \Delta \xi} , \quad (2.38)$$

$$1 < j < n$$

$$1 < i < m .$$

With these approximations, Equation (2.29) is approximated as

$$\frac{d}{d\tau} u_{j,i} = (\Delta\xi)^{-2} [(1-\Delta\xi/\xi_j) u_{j-1,i} - 2u_{j,i}$$

$$+ (1+\Delta\xi/\xi_j) u_{j+1,i}] +$$

$$(\Delta\lambda)^{-2} [u_{j,i-1} - 2u_{j,i} + u_{j,i+1}] ,$$

$$1 < j < n$$

$$1 < i < m . \quad (2.39)$$

At the boundaries of the discrete space, from (2.31, 2.32)

$$\frac{d}{d\tau} u_{j,1} = \varphi_{\tau} = -0.2808 c (1.2 + c \tau)^{-1.2808} ,$$

$$\frac{d}{d\tau} u_{j,m} = \varphi_{\tau} , \quad 1 \leq j \leq n ,$$

and,

$$\frac{d}{d\tau} u_{n,i} = \varphi_{\tau} , \quad 1 \leq i \leq m . \quad (2.40)$$

For Case 1, c is set to zero.

$$\begin{aligned} \frac{d}{d\tau} u_{1,i} = (\Delta \xi)^{-2} \frac{[(\xi_2/\xi_1)^2 + 1]}{2} (u_{2,i} - u_{1,i}) \\ + (\Delta \lambda)^{-2} [u_{1,i-1} - 2 u_{1,i} + u_{1,i+1}] , \end{aligned}$$

$$1 < i < m . \quad (2.41)$$

The first term on the r.h.s. of (2.41) derives from the fact that the net torque due to radial gradients acting on the mass element within the annular segment ($r_1 \leq r < r_2$) produces an angular acceleration

$$\dot{\omega}_1 = (\rho r_1^2)^{-1} \frac{(r_2 \sigma_2 + r_1 \sigma_1)}{2 \Delta r} ,$$

and, for

$$\sigma_2 \approx \eta r_2 \frac{(\omega_2 - \omega_1)}{\Delta r} ,$$

$$\sigma_1 = \eta r_1 \frac{(\omega_2 - \omega_1)}{\Delta r} ,$$

$$\dot{\omega}_1 = \nu (\Delta r)^{-2} \frac{[1 + (r_2/r_1)^2]}{2} (\omega_2 - \omega_1) .$$

This expression is equivalent to the first term of the r.h.s. of (2.41).

The initial conditions for Cases 1 and 2 (2.31, 2.33) are:

$$u_{j,1} = u_{j,m} = 1, .944, 1 \leq j \leq n$$

$$u_{n,i} = 1, .944, 1 \leq i \leq m$$

$$u_{j,i} = \bar{\omega}_{lim} = 0.02 , \quad 1 < j < n \\ 1 < i < m . \quad (2.42)$$

Note that the initial projectile spin is normalized to unity. The actual liquid spin may be obtained from the relation

$$\omega(\tau) = \omega_0 u(\xi, \lambda, \tau) , \quad (2.43)$$

where ω_0 is the projectile spin at launch. At the time Configuration B is achieved, the normalized spin is taken as 0.944. This time is approximately 7 seconds after launch in the examples treated here.

Because of the relatively weak longitudinal gradients in the angular velocity relative to the radial gradients, the longitudinal (λ) dependence was neglected in the numerical results presented here.*

With no dependence on the index i , Equation (2.39) can be simplified as

* Check runs considering λ -dependence show essentially equivalent results.

$$\begin{aligned} \frac{d}{d\tau} u_j = (\Delta\xi)^{-2} [(1-\Delta\xi/\xi_j) u_{j-1} - 2 u_j \\ + (1+\Delta\xi/\xi_j) u_{j+1}] , \quad 1 < j < n . \end{aligned} \quad (2.44)$$

And, from (2.40, 2.41)

$$\begin{aligned} \frac{d u_n}{d\tau} &= 0 \quad (\text{Case 1}) \\ \frac{d u_1}{d\tau} &= (\Delta\xi)^{-2} \frac{[(\xi_2/\xi_1)^2 + 1]}{2} (u_2 - u_1) . \end{aligned} \quad (2.45)$$

Using values of

$$n = 11 ,$$

$$\xi_0 = 0.33 ,$$

$$\Delta\xi = 0.067 ,$$

and a finite time step $\Delta\tau = 10^{-4}$, (2.44, 2.45) were integrated using a fourth-order Runge-Kutta procedure. The numerical results are illustrated in **Figure 2.6**.

The equivalent angular velocity of the liquid can also be obtained for Configuration B. This is obtained numerically as follows:

$$\bar{\omega}_B = \frac{\sum_{j=1}^{n-1} (\xi_j + \xi_{j+1})(u_j + u_{j+1})}{2 \sum_{j=1}^{n-1} \xi_j + \xi_{j+1}} . \quad (2.46)$$

This result is plotted in **Figure 2.7**.

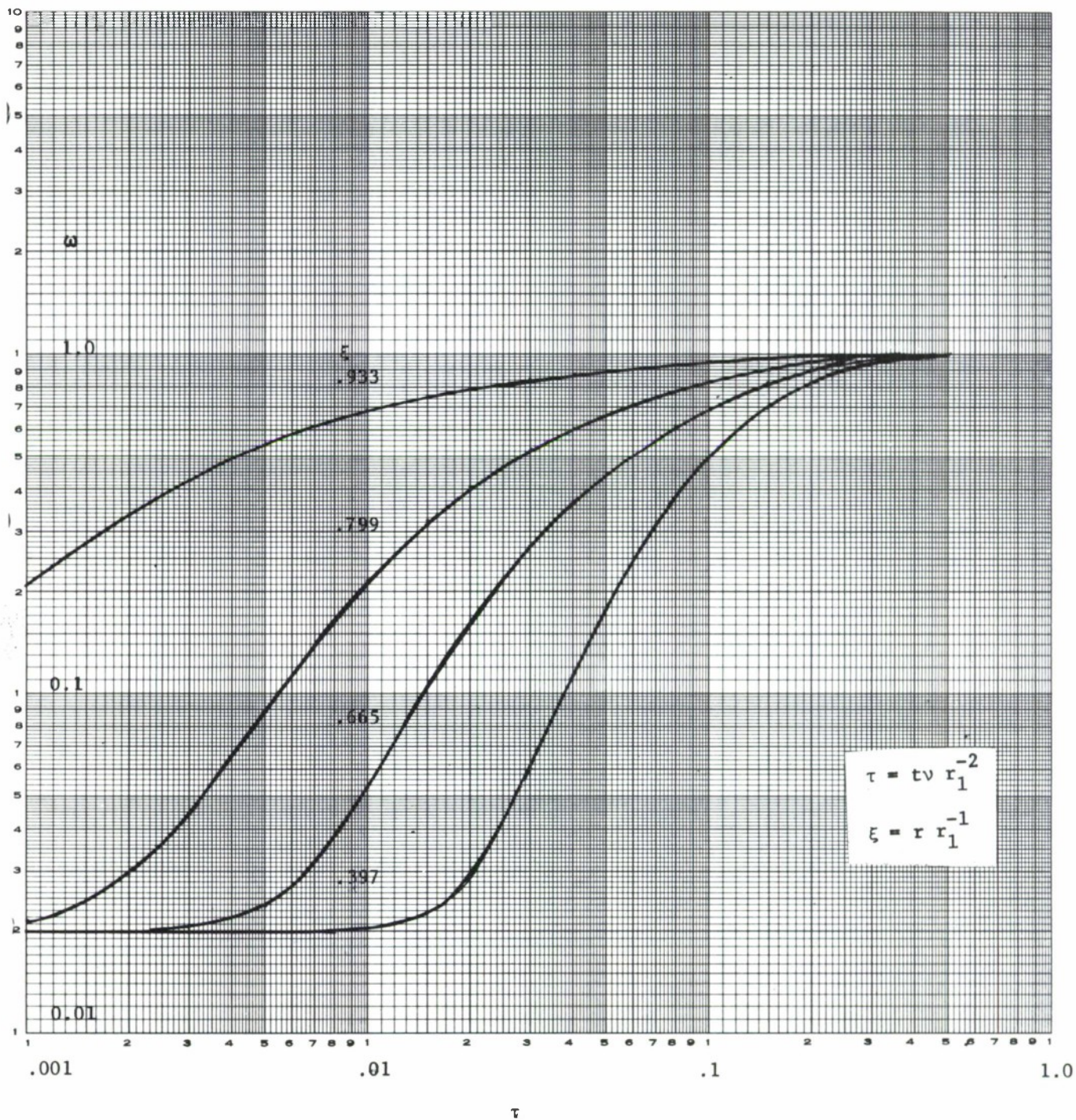


Figure 2.6. Angular Velocity of Liquid Versus Time for Several Radial Positions (Liquid Configuration B)

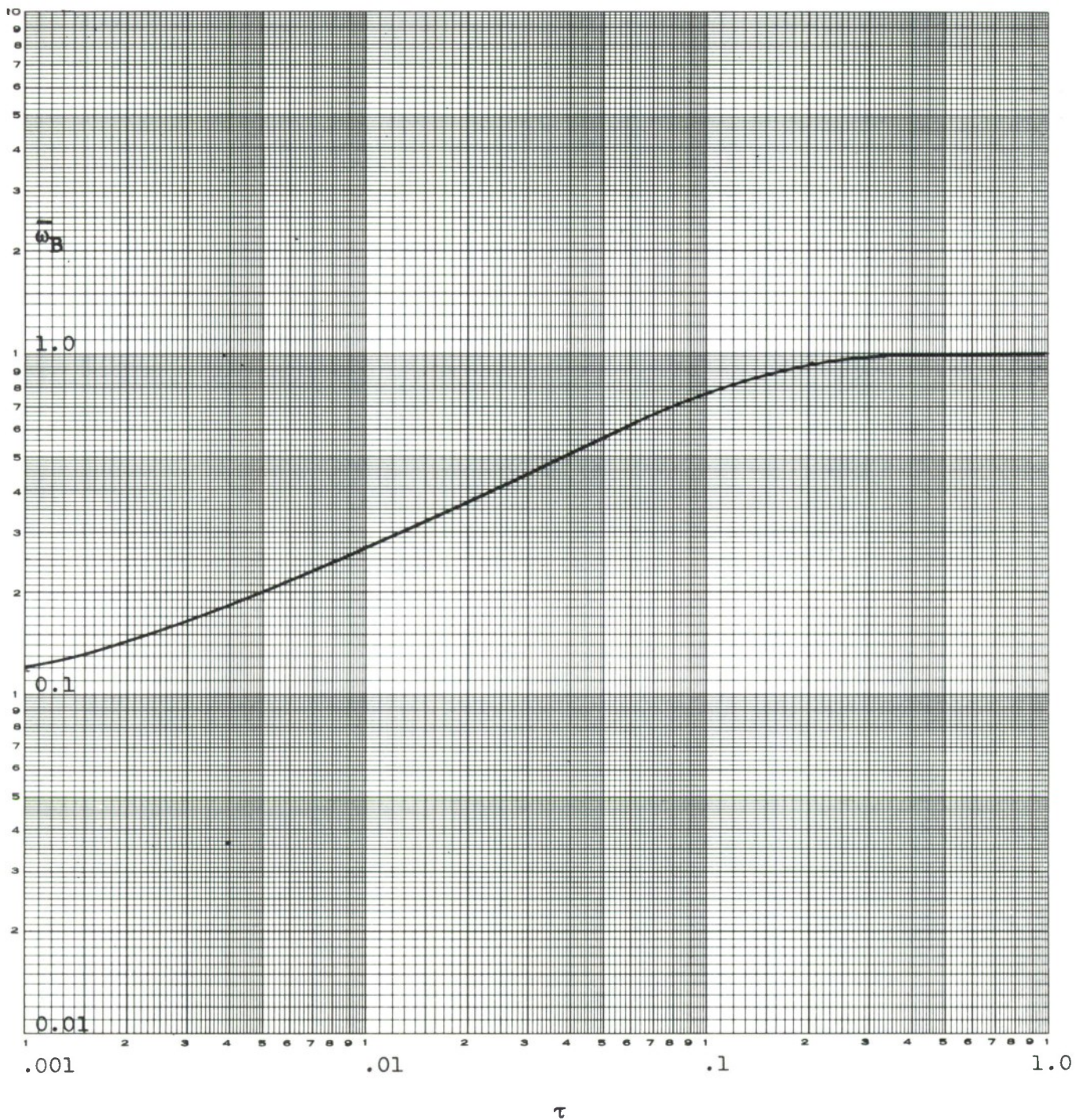


Figure 2.7. Effective Angular Velocity of Liquid in Configuration B Versus Time

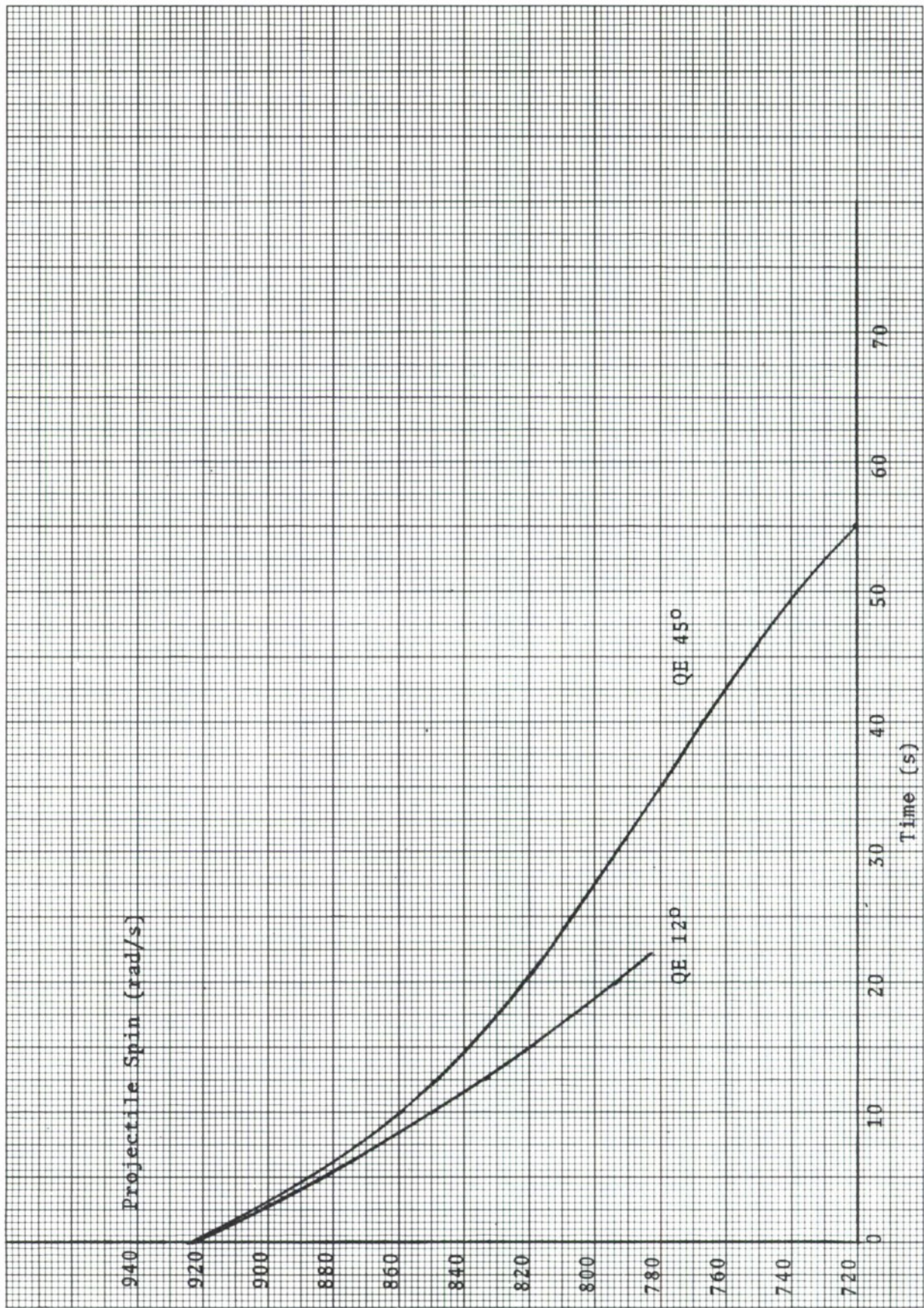


Figure 2.8. Spin Versus Time for a Modified M509 Projectile at Zone 7 from the XM201 Cannon (Inertial Characteristics of the XM736 Projectile)

Based upon the run down of spin over time for an M509 projectile type, a second numerical example was examined (Case 2). The spin versus time for a modified M509 projectile is shown in Figure 2.8. The boundary conditions at ξ_n for Case 2 -- Equation (2.40) -- closely match the spin characteristics of the M509. Consequently, this example is a reasonable simulation of the spin dynamics of the liquid in the XM736 projectile. As indicated in Equation (2.33), two values of liquid viscosity were used for Case 2 -- 1 and 10 centistokes. Because of the strong dependence of viscosity upon temperature, this viscosity range is necessary to encompass the expected range of liquid temperature. In this example a launch spin ω_0 of 923 rad/sec is assumed. Numerical results are shown in Figure 2.9.

At this point it is necessary to repeat the caveat that the above analysis of the dynamics of liquid spinup applies, strictly, only to liquid-filled projectiles having long, narrow cavities and quite viscous fills so that laminar flow obtains. In the XM736, these conditions do not hold (even at 10 centistokes). However, due to the brevity of the launch period, negligible liquid spinup is expected in the XM736 so that the angular momentum of the projectile is about 4% less than that of a comparable solid projectile having the same total mass and exterior configuration. Thus, in terms of solid-projectile behavior, the effective axial moment of inertia of the XM736 is approximately 4% less than its solid counterpart. Pitch inertia is negligibly affected by liquid rotation in this system, as previously shown.

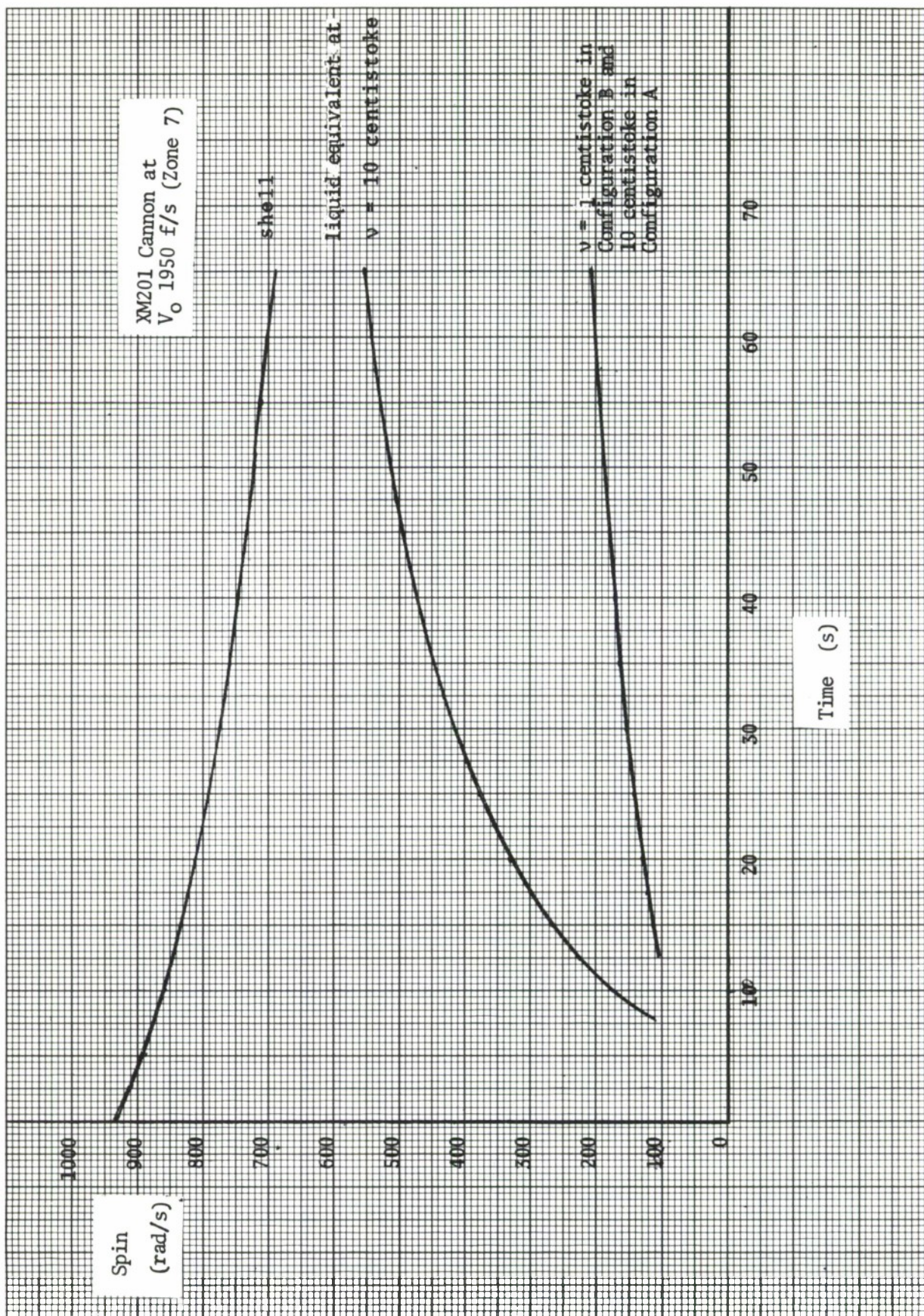


Figure 2.9. Estimated Spin Versus Time for the Liquid in a Type XM736 Projectile at Two Values of Viscosity

Exterior Ballistic Differential Effects

An attempt to assess the exterior ballistic effects of the indicated change in rotational inertia relative to a "ballistically matched" solid projectile was made in the following manner. A set of runs was made with a modified point-mass program [4] in which the inertial characteristics of a projectile having characteristics similar to the M106 projectile, and termed the standard eight-inch projectile, were changed incrementally as shown below in Table 2.1. Shifts in range and deflection relative to those of the standard projectile are noted. Projectile characteristics are shown in Table 2.4.

The combined effect of movement of the center of gravity toward the nose by 0.12 inch and reduction in the axial moment of inertia by 4% is to change the deflection by about 1.36 milliradians. This magnitude is less than one deflection probable error for the M106 system [2,5]. Change in range is negligible. Similar ballistic sensitivities for the M106 and M509 projectiles are shown in Tables 2.2, 2.3A, and 2.3B.

[2] Op. Cit.

[4] Lieske, R.F., and Reiter, M.L. Equations of Motion for a Modified Point Mass Trajectory, BRL Report No. 1314, Ballistics Research Labs, Aberdeen, Md., March 1966.

[5] Firing Tables for Cannon, 8-Inch Howitzer, M2, M2A1 and M47 Firing Projectile, HE, M106, FT 8-J-3, Hdqts Dept of Army, October 1960.

TABLE 2.1. BALLISTIC SENSITIVITY OF A STANDARD PROJECTILE
IN M2 8-INCH HOWITZER AT MAXIMUM RANGE TO CHANGES
IN CG POSITION AND AXIAL MOMENT OF INERTIA

Loc. CG (cal)	Axial MI (slug ft ²)	Range (m)	Δ Range (m)	Defl. (m)	Δ Defl. (m)
2.5	0.5	16,943	0	665	0
2.485	0.5	16,943	0	672	7
2.5	0.48	16,945	2	634	- 31
2.485	0.48	16,945	2	642	- 23

TABLE 2.2. BALLISTIC SENSITIVITY OF THE M106 PROJECTILE
IN M2 HOWITZER TO A JOINT CHANGE IN CG POSITION AND
AXIAL MOMENT OF INERTIA

QE 45°

V _o (f/s)	Loc. CG (cal)	Axial MI (kg m ²)	Range (m)	Defl. (m)	Δ Defl. (m)
1380	2.840	0.553	11751	210	0
(25)	2.825	0.530	11752	201	- 9
1950	2.840	0.553	16795	417	0
(27)	2.825	0.530	16796	400	- 17

TABLE 2.3A. BALLISTIC SENSITIVITY OF THE M509 PROJECTILE
IN M2 AND M110E2 HOWITZERS TO A JOINT CHANGE IN CG
POSITION AND AXIAL MOMENT OF INERTIA

QE 45°

V_o (f/s)	Loc. CG (cal)	Axial MI (kg m ²)	Range (m)	Defl. (m)	Δ Defl. (m)
M2 Howitzer					
1906	3.592	0.570	17059	298	0
(Z5)	3.577	0.547	17059	285	- 13
M110E2 Howitzer					
1040	3.592	0.570	8547	136	0
(Z3)	3.577	0.547	8548	130	- 6
1960	3.592	0.570	17622	398	0
(Z7)	3.577	0.547	17622	382	- 16
2440	3.592	0.570	22853	621	0
(Z9)	3.577	0.547	22853	595	- 26

TABLE 2.3B. BALLISTIC SENSITIVITY OF THE M509 PROJECTILE
IN THE M110E2 HOWITZER TO A JOINT CHANGE IN CG
POSITION AND AXIAL MOMENT OF INERTIA

QE (deg)	V _o /Zone (f/s)	Modif. No.*	Range (m)	Defl. (m)	Δ Defl. (m)	Correction (mils)
12	1040/Z3	0	3822	11	0	0
		1	3822	11	0	0.1
24		0	6627	42	0	0
		1	6627	41	- 1	0.3
45		0	8547	136	0	0
		1	8548	130	- 6	0.7
12	1960/Z7	0	9399	55	0	0
		1	9399	53	- 2	0.2
24		0	13967	158	0	0
		1	13966	152	- 6	0.4
45		0	17622	398	0	0
		1	17622	382	- 16	1.0
12	2440/Z9	0	12869	84	0	0
		1	12869	81	- 3	0.2
24		0	18369	247	0	0
		1	18368	237	- 10	0.5
45		0	22853	621	0	0
		1	22853	595	- 26	1.2

* For modification number n:

n	Loc. of CG (cal re nose)	Axial MI (kg m ²)
0	3.592	0.570
1	3.577	0.547

TABLE 2.4A. CHARACTERISTICS OF THE STANDARD PROJECTILE

caliber	203.2 mm
mass	200 lb
cg position	2.5 cal aft of nose
axial moment of inertia	0.5 slug ft ² (0.6779 kg m ²)
pitch moment of inertia	4.0 slug ft ² (5.4234 kg m ²)
center of pressure	1.73 cal at Mach 1.77
projectile length	4.3 cal (34.4 in)
muzzle velocity	1950 f/s at zone 7 in M2 howitzer
initial spin	735 rad sec ⁻¹

TABLE 2.4B. DRAG COEFFICIENT FOR THE STANDARD PROJECTILE

Mach No.	C _D (form)	C _D (skin)	C _D (total)
0.0	0.126	0.056	0.182
0.8	0.126	0.049	0.175
0.9	0.190	0.048	0.238
1.0	0.302	0.047	0.349
1.1	0.307	0.046	0.353
1.2	0.300	0.046	0.346
1.5	0.262	0.045	0.307
2.0	0.210	0.043	0.253

TABLE 2.5A. CHARACTERISTICS OF THE M106 PROJECTILE
FIRED FROM THE M2A2 CANNON

caliber	203.2 mm
mass	200 lb
cg position re nose	2.840 cal
projectile length	4.375 cal
axial moment of inertia	0.553 kg m ²
pitch moment of inertia	4.270 kg m ²
muzzle velocity	1950 f/s at zone 7 1380 f/s at zone 5
initial spin	735 rad/s at zone 7 520 rad/s at zone 5

TABLE 2.5B. DRAG COEFFICIENT* FOR THE M106 PROJECTILE

Mach No.	C _D
0.00	0.125
0.75	0.125
0.85	0.129
0.90	0.140
0.95	0.152
1.00	0.351
1.05	0.400
1.10	0.400
1.50	0.356
2.00	0.305
2.50	0.280

* BRL estimate [6]

[6] Dubin, J.A., et al. Ballistic Similitude: 8 Inch Ammunition, (SECRET), Technical Report 4165, Picatinny Arsenal, Dover, N.J., June, 1973.

TABLE 2.6A. CHARACTERISTICS OF THE M509 ICM PROJECTILE
USED WITH THE M2A2 AND THE XM201 CANNONS

caliber	203.2 mm
mass	205.9 lb
cg position re nose	3.592 cal
projectile length	5.674 cal
axial moment of inertia	0.570 kg m ²
pitch moment of inertia	4.7676 kg m ²
muzzle velocity (max)	1906 f/s in M2A2 2240 f/s in XM201
initial spin	718.5 rad/s in M2A2 1150 rad/s in XM201

TABLE 2.6B. DRAG COEFFICIENT* FOR THE M509 PROJECTILE

Mach No.	C _D
0.00	0.130
0.75	0.130
0.85	0.140
0.90	0.155
1.00	0.300
1.05	0.360
1.10	0.360
1.50	0.317
2.00	0.274
2.50	0.239

* BRL estimate [6]

[6] Dubin, J.A., et al. Op. Cit.

CHAPTER III
VIBRATION OF THE LIQUID IN A
SPINNING LIQUID-FILLED PROJECTILE

Introduction

In an effort to assess the consequences of vibration of the liquid surface while in Configuration B on the flight stability of the projectile, a simple methodology is proposed. First, one must examine the admissible shapes which the liquid surface can assume. For a set of vibrational modes of interest one can then estimate the associated natural vibrational frequencies for the liquid, treated as a conservative system. To be analytically tractable this treatment will assume the liquid to be rotating at a constant angular velocity ω . Actually, of course, the liquid does not have a constant ω everywhere during spinup and, further, at any point in the liquid ω depends upon time. Therefore, the degree of credibility of results derived from the above assumption will depend upon the relative magnitude of liquid acceleration due to spinup and the centrifugal acceleration due to spin. If the latter is much larger than the former, it is plausible to treat liquid vibration pseudostatically.

The ultimate goal of the present analysis is to compare the natural vibrational frequencies of the liquid with the frequencies of precession and nutation of the entire projectile. If any of the vibrational frequencies were found to remain close to the precessional or nutational frequency of the projectile during flight, a resonant condition could occur in which the system vibrational modes excited each other at their common frequency. Conceivably this could cause projectile flight instability if the liquid vibration was severe enough. At the very least a "mode lock" of this sort would increase projectile dispersion.

To motivate further developments, one should note that the precessional and nutational frequencies of concern are rather low, lying in the band from 0 to 20 hertz, approximately. For stable projectiles such as the M106 or the M509, the precessional frequency is typically about 1 to 3 hertz thruout flight. The Appendix includes derivations of the equations for precessional frequency and nutational frequency of a spin-stabilized projectile.

By contrast to the very low precessional frequency, nutational or yawing frequency is of greater concern relative to projectile stability* in liquid-filled projectiles since this frequency is such that a liquid vibrational frequency will cross it during spinup. To illustrate the band of nutational frequencies, Figure 3.1 displays the nutational frequency versus time, for several firing zones, during the flight of the M509 projectile from the XM201 cannon. Figure 3.2 shows a similar result for the M106 projectile from the M2A2 cannon.

* A stable projectile is one in which pitching or yawing motions are ultimately reduced in amplitude during flight without the projectile being completely overturned. In practice a condition of neutral, dynamic stability, such that amplitudes remain constant, is difficult to obtain. If the system starts to progressively increase its yaw, it does so quickly and ultimately overturns.

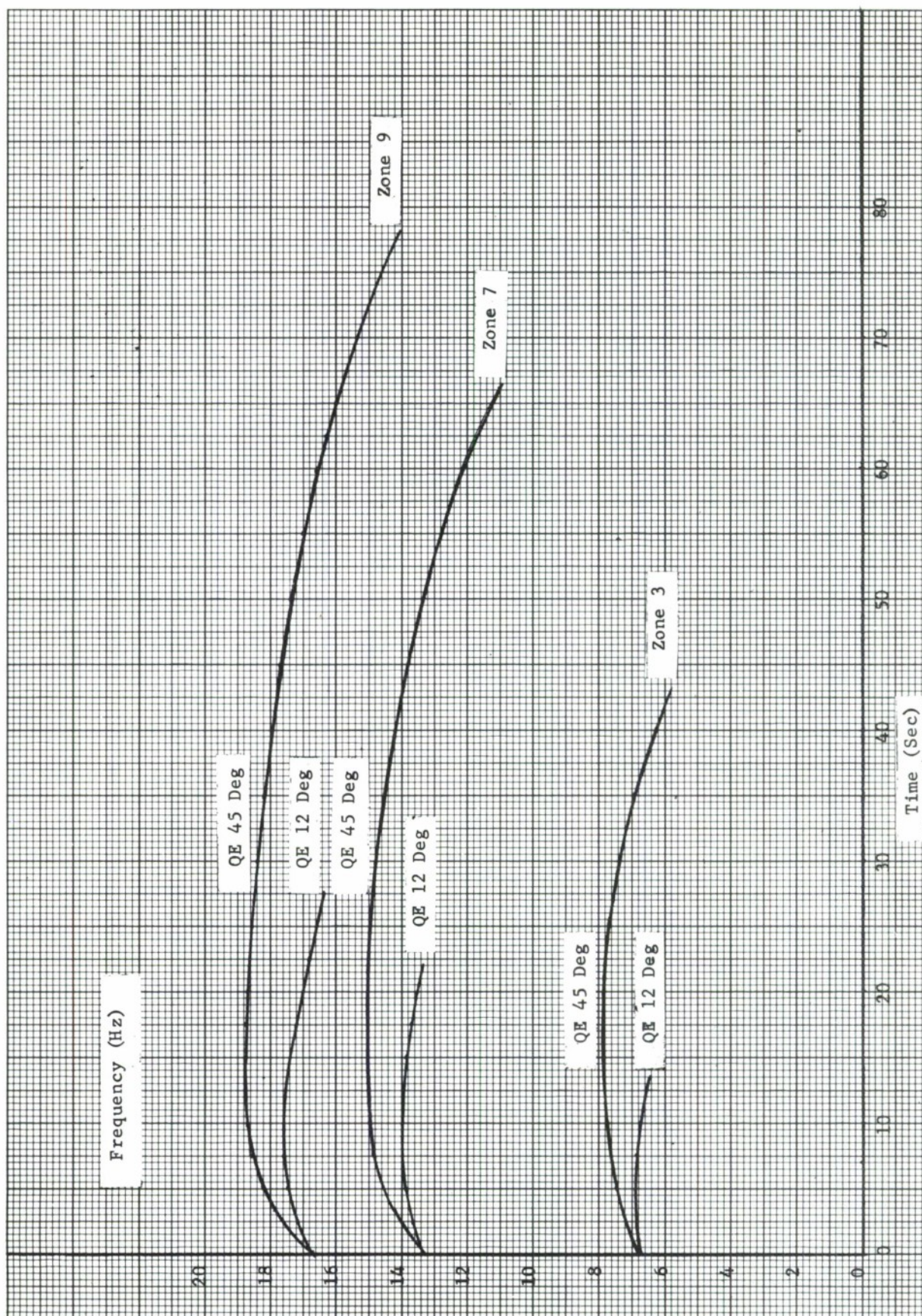


Figure 3.1. Nutational Frequency of the M509 Projectile at Several Zones in the XM201 Cannon



Figure 3.2. Nutational Frequency and Stability Factor During Flight of the M106 Projectile from the M2A2 Cannon

Vibrational Modes

We proceed with the analysis outlined above by considering perturbations to the liquid surface of Configuration B. In Figure 3.3 below, the equilibrium position of the free surface of the liquid at its inner radius is designated y_0 . The perturbation or deflection of this surface is δ where

$$\delta = f(x) \quad . \quad (3.1)$$

Thus the position of the surface, y , is given by

$$y = y_0 + \delta \quad . \quad (3.2)$$

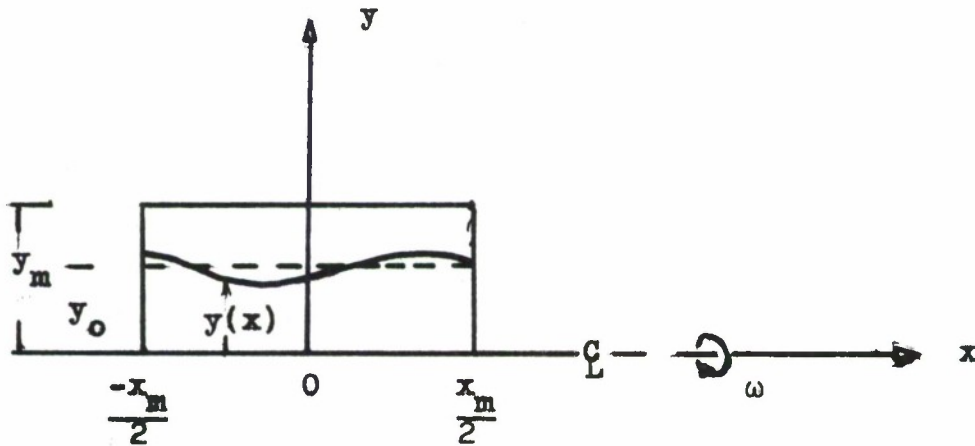


Figure 3.3. Surface of the Vibrating Liquid

The liquid volume v is given by

$$v = -\pi \int_{-x_m/2}^{x_m/2} y^2 dx + \pi y_m^2 x_m \quad (3.3)$$

$$v = \pi x_m (y_m^2 - y_0^2) \quad (3.4)$$

Then with a constant liquid volume given by (3.4) and with δ small, i.e.,

$$\delta \ll y_0,$$

$$\int_{-x_m/2}^{x_m/2} f(x) dx \approx 0 \quad (3.5)$$

Equation (3.5) places a constraint on the form of $f(x)$.

Now expand $f(x)$ in a Fourier series, i.e.,

$$f(x) = \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n x}{L} \quad (3.6)$$

Furthermore since we are interested in vibrational modes which might amplify yaw and destabilize the projectile, consider only odd components of $f(x)$ for the present. In this special case

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n x}{L} \quad (3.7)$$

Applying the constraint given by (3.5),

$$\int_{-x_m/2}^{x_m/2} dx \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n x}{L} = 0 \quad .$$

After exchanging the order of operations

$$\sum_{n=1}^{\infty} b_n \int_{-x_m/2}^{x_m/2} \sin \frac{2 \pi n x}{L} = 0$$

or

$$2 \sum_{n=1}^{\infty} b_n \cos \frac{\pi n x_m}{L} = 0, \quad (3.8)$$

for all integer n . For this expression to hold for an arbitrary set b_n , the argument of the cosine must be an odd integer multiple of $\pi/2$.

$$\frac{\pi n x_m}{L} = (2k - 1) \frac{\pi}{2}, \quad 1 \leq k$$

$$, \quad 1 \leq n$$

Therefore,

$$L = \frac{2 n x_m}{2 k - 1} \quad (3.9)$$

and with (3.7)

$$f(x) = \sum_{k=1}^{\infty} b_k \sin \frac{\pi (2k - 1) x}{x_m}. \quad (3.10)$$

Since we are principally interested in low frequency vibrations, we restrict the following development to the fundamental mode, i.e., to $k = 1$. Then

$$f(x) = b \sin \frac{\pi x}{x_m}, \quad \frac{-x_m}{2} < x < \frac{x_m}{2}. \quad (3.11)$$

The Fourier amplitude b will serve as the single degree of freedom characterizing the vibrations of the liquid surface.

Energy Considerations

At this point a brief excursion in the development is necessary to discuss the pressure in, and energy of, the rotating liquid.

The rotational kinetic energy of the liquid is

$$T_{\text{rot}} = \pi \rho \int_{-x_m/2}^{x_m/2} dx \int_y^{y_m} r^3 \omega^2 dr \quad (3.12)$$

where liquid density is ρ and where the angular velocity ω may depend upon both x and r . At dynamic equilibrium

$$y = y_0 ,$$

$$\omega = \omega_0 \text{ and}$$

$$T_{\text{rot}} = \frac{\pi}{4} \rho \omega_0^2 x_m y_m^4 (1 - q^4) \quad (3.13)$$

with

$$q = y_0/y_m .$$

The potential energy of the liquid in an equilibrium configuration is due only to the compression of the liquid under centrifugal force. The compression of the differential volume element dv ,

$$dv = 2 \pi r dr dx , \quad (3.14)$$

is due to the pressure needed to support the liquid below r , i.e., at a radius smaller than r . In a liquid rotating as a solid body with constant angular velocity ω_0

$$\frac{dp}{dr} = \rho \omega_0^2 r \quad (3.15)$$

which upon integration yields

$$p(r) = \frac{\omega_0^2 \rho}{2} (r^2 - y_0^2) \quad , \quad y_0 \leq r \leq y_m \quad (3.16)$$

If k is the volumetric compressibility of the liquid, i.e.,

$$k = \frac{1}{v} \frac{\partial v}{\partial p} \quad ,$$

then the compressional potential energy associated with the differential volume dv is

$$dV_c = \frac{k}{2} p^2 dv \quad (3.17)$$

Typically k is $10^{-10} \text{ cm}^2 \text{ dyne}^{-1}$ for liquids.

The total compressional potential energy

$$V_c = \int_v \frac{k}{2} p^2 dv \quad .$$

And, (3.14, 3.17) yield

$$V_c = \pi k \int_{-x_m/2}^{x_m/2} dx \int_y^{y_m} p^2 r dr \quad (3.18)$$

For a liquid in dynamic equilibrium, $p(r)$ is given by (3.16). Using this result

$$V_c = \pi k x_m \int_{y_0}^{y_m} \frac{\omega_0^4 \rho^2}{4} (r^2 - y_0^2)^2 r dr$$

or

$$V_c = \frac{\pi}{24} \omega_0^4 \rho^2 k x_m y_m^6 (1 - q^2)^3$$

with

$$q = y_0/y_m \quad . \quad (3.19)$$

To appreciate the significance of the value of liquid pressure, the rotational kinetic energy, and the compressive potential energy, we present the following numerical example.

Example

Using values used in previous examples,

$$\omega_0 = 942 \text{ rad sec}^{-1}$$

$$\rho = 1 \text{ gm cm}^{-3}$$

$$x_m = 55.88 \text{ cm (22 in)}$$

$$y_m = 7.62 \text{ cm (3 in)}$$

$$y_0 = 2.516 \text{ cm (0.991 in)}$$

and

$$k = 10^{-10} \text{ cm}^2 \text{ dyne}^{-1}$$

$$q = 0.3302$$

From (3.16), the pressure at radial position y_m is given by

$$\begin{aligned} p(y_m) &= \frac{\omega_o^2 \rho y_m^2}{2} (1 - q^2) \\ &= \frac{(942)^2 (7.62)^2}{2} (1 - 0.3302^2) \end{aligned}$$

$$p(y_m) = 2.295 \cdot 10^7 \text{ dyne cm}^{-2}$$

$$= 22.65 \text{ atm}$$

$$= 332.95 \text{ psi}$$

And from (3.19),

$$V_c = \frac{\pi}{24} (942)^2 (10^{-10}) (55.88) (7.62)^6 (1 - 0.3302^2)^3$$

$$V_c = 7.973 \cdot 10^7 \text{ dyne cm or } 7.973 \text{ joules}$$

Finally, from (3.13)

$$T_{\text{rot}} = \frac{\pi}{4} (942)^2 (55.88) (7.62)^4 (1 - 0.3302^4)$$

$$T_{\text{rot}} = 1.2974 \cdot 10^{11} \text{ dyne cm}$$

$$T_{\text{rot}} = 12,974 \text{ joules or } 9,569 \text{ ft lb}_f$$

The potential energy of the liquid at equilibrium is only 0.061% of the rotational kinetic energy under this condition. Therefore one would not expect the exchange of energy between kinetic and compressional potential forms to contribute significantly to liquid vibrations. Compressional potential energy is assumed negligible in subsequent calculations.

Estimate of a Fundamental Vibrational Frequency

At this point we return to the principal arguments associated with the derivation of an expression for the vibrational frequency of the liquid surface. To develop this expression, an estimate of the vibrational kinetic energy of the liquid will be required.

An estimate of the kinetic energy associated with a longitudinal vibrational mode of the liquid can be made simply by making these assumptions:

(1) The liquid surface during vibration remains axisymmetric; i.e., circumferential modes are not excited.

(2) The functional form describing the liquid surface changes with time only thru time-dependence of the coefficients in a Fourier transform of the function, i.e., only thru the Fourier amplitudes.

(3) The column length for flow of a liquid element at position x (relative to the center of the disturbance) is proportional to x .

By the first and second assumptions the first odd vibrational mode --

$$y = b \sin \frac{\pi x}{x_m}$$

has time derivative

$$\dot{y} = \dot{b} \sin \frac{\pi x}{x_m} . \quad (3.20)$$

At the surface element

$$2 \pi y_o dx ,$$

the path or column length of a control volume within which this surface element vibrates is

$$c x , \quad 0 < x < \frac{x_m}{2} , \quad (3.21)$$

by the third assumption, where c is a constant. This constant can be evaluated by requiring that the volume integral of all elemental control volumes produces the volume of the liquid v_ℓ . That is

$$v_\ell = \int_0^{x_m/2} 2 \pi y_o c x dx$$

or

$$c = \frac{4 v_\ell}{\pi y_o x_m^2} . \quad (3.22)$$

But

$$v_\ell = \pi y_m^2 x_o , \quad (3.23)$$

where x_o is the length of the liquid cylinder in Configuration A.

Then

$$c = \frac{4 x_o y_m^2}{x_m^2 y_o} . \quad (3.24)$$

In previous examples

$$x_0 = 19.6 \text{ in}$$

$$x_m = 22 \text{ in}$$

$$y_0 = 0.991 \text{ in}$$

$$y_m = 3 \text{ in}$$

so that

$$c = 1.471 \text{ .}$$

The kinetic energy of vibration of an elemental volume is

$$\pi c \rho y_0 x dx \dot{y}^2, \quad 0 < x \leq \frac{x_m}{2} \text{ .}$$

The total vibrational kinetic energy is obtained by volume integration of this expression with $\dot{y}$ given by (3.20).

$$T_{\text{vib}} = \pi c \rho y_0 \dot{b}^2 \int_0^{x_m/2} x \sin^2 \frac{\pi x}{x_m} dx \quad (3.25)$$

$$T_{\text{vib}} = \frac{(\pi^2 + 4)}{16 \pi} c \rho y_0 x_m^2 \dot{b}^2 \quad (3.26)$$

or

$$T_{\text{vib}} = K_t \dot{b}^2$$

with

$$K_t = \frac{(\pi^2 + 4)}{16 \pi} c \rho y_0 x_m^2 \text{ .} \quad (3.27)$$

Thus the vibrational kinetic energy is simply proportional to the square of the time derivative of the Fourier amplitude.

An expression for the vibrational potential energy associated with this mode is developed as follows. Let $p(r, y_0)$ be the pressure in the liquid at radial position r when the liquid is in Configuration B. An expression for $p(r, y_0)$ is given in (3.16). Then, providing the displacement δ of the surface from y_0 is small, the work performed to effect a change in liquid configuration from the equilibrium position y_0 to a terminal position y is

$$W = \int_{x=0}^{x_m/2} dx \int_{z=0}^b 2 \pi y_0 [p(y_0 + \delta, y_0 - \delta) d\delta]$$

with

$$\delta = z \sin \frac{\pi x}{x_m} . \quad (3.28)$$

$$W = \frac{\pi}{2} \rho \omega_0^2 y_0^2 x_m b^2 \quad (3.29)$$

But the vibrational potential energy is equal to the work done to change the configuration. Thus

$$V_{vib} = K_v b^2$$

with

$$K_v = \frac{\pi}{2} \rho \omega_0^2 y_0^2 x_m . \quad (3.30)$$

For b equal to 1 cm and the other parameters having the values given in the previous example, V_{vib} equals 49.3 joules or only 0.38% of the rotational kinetic energy.

Having obtained expressions for the potential and kinetic energy of vibration of the liquid in terms of the Fourier amplitude b and its first derivative, one can obtain the equation of motion of the liquid surface by direct application of Lagrange's equation for a conservative system.

$$\frac{d}{dt} \frac{\partial T_{vib}}{\partial \dot{b}} + \frac{\partial V_{vib}}{\partial b} = 0 \quad (3.31)$$

This result does, of course, neglect the effects of dissipative forces.

But from (3.27)

$$\frac{\partial T_{vib}}{\partial \dot{b}} = 2 K_t \dot{b}$$

and from (3.30)

$$\frac{\partial V_{vib}}{\partial b} = 2 K_v b$$

Therefore,

$$\ddot{b} + \frac{K_v}{K_t} b = 0 \quad (3.32)$$

The undamped angular frequency associated with this vibrational mode by inspection of (3.32) is

$$\Omega_{vib} = (K_v/K_t)^{1/2} \quad (3.33)$$

$$\frac{\Omega_{\text{vib}}}{\omega_0} = \left[\frac{8 \pi^2 y_0}{(\pi^2 + 4) c x_m} \right]^{\frac{1}{2}} \quad (3.34)$$

Since the natural vibratory frequency ν_{vib} is $\Omega_{\text{vib}}/2\pi$,

$$\nu_{\text{vib}} = \left[\frac{2 y_0}{(\pi^2 + 4) c x_m} \right]^{\frac{1}{2}} \omega_0 \quad (3.35)$$

With the parameter values used in previous examples,

$$y_0 = 0.991 \text{ in}$$

$$x_m = 22 \text{ in}$$

$$c = 1.471 \text{ ,}$$

$$\nu_{\text{vib}} = 0.06645 \omega_0 \quad (3.36)$$

The value of ω_0 in this expression is interpreted as the effective angular velocity of the liquid, i.e., that uniform angular velocity which produces the observed angular momentum when ω is not uniform thruout. At the muzzle spin for zone 7 of the M2 howitzer

$$\omega_0 = 735 \text{ rad sec}^{-1}$$

and

$$\nu_{\text{vib}} = 48.8 \text{ hz .}$$

For zone 7 in the M110E2 howitzer

$$\omega_0 = 923 \text{ rad sec}^{-1}$$

and

$$\nu_{\text{vib}} = 61.3 \text{ hz} .$$

Using (3.36) and the time-dependent, effective angular velocity for the XM736 projectile shown in Figure 2.9, the value of the function $\nu_{\text{vib}}(t)$ for this system has been computed. This result is displayed in Figure 3.4. Also shown here for comparison is the nutational frequency at zone 7 for the XM736 projectile.

It is noted that the spinup of a liquid with constant kinematic viscosity of 10 centistokes occurs rapidly enough to cause the vibrational frequency to quickly cross over the nutational frequency. In this case liquid-vibration-induced projectile instability is unlikely. With a liquid having lower viscosity while in Configuration B, the cross-over of frequencies is not so abrupt and may at least produce additional dispersion. Considering the magnitude of the perturbing effect of liquid vibration, complete projectile instability is unlikely.

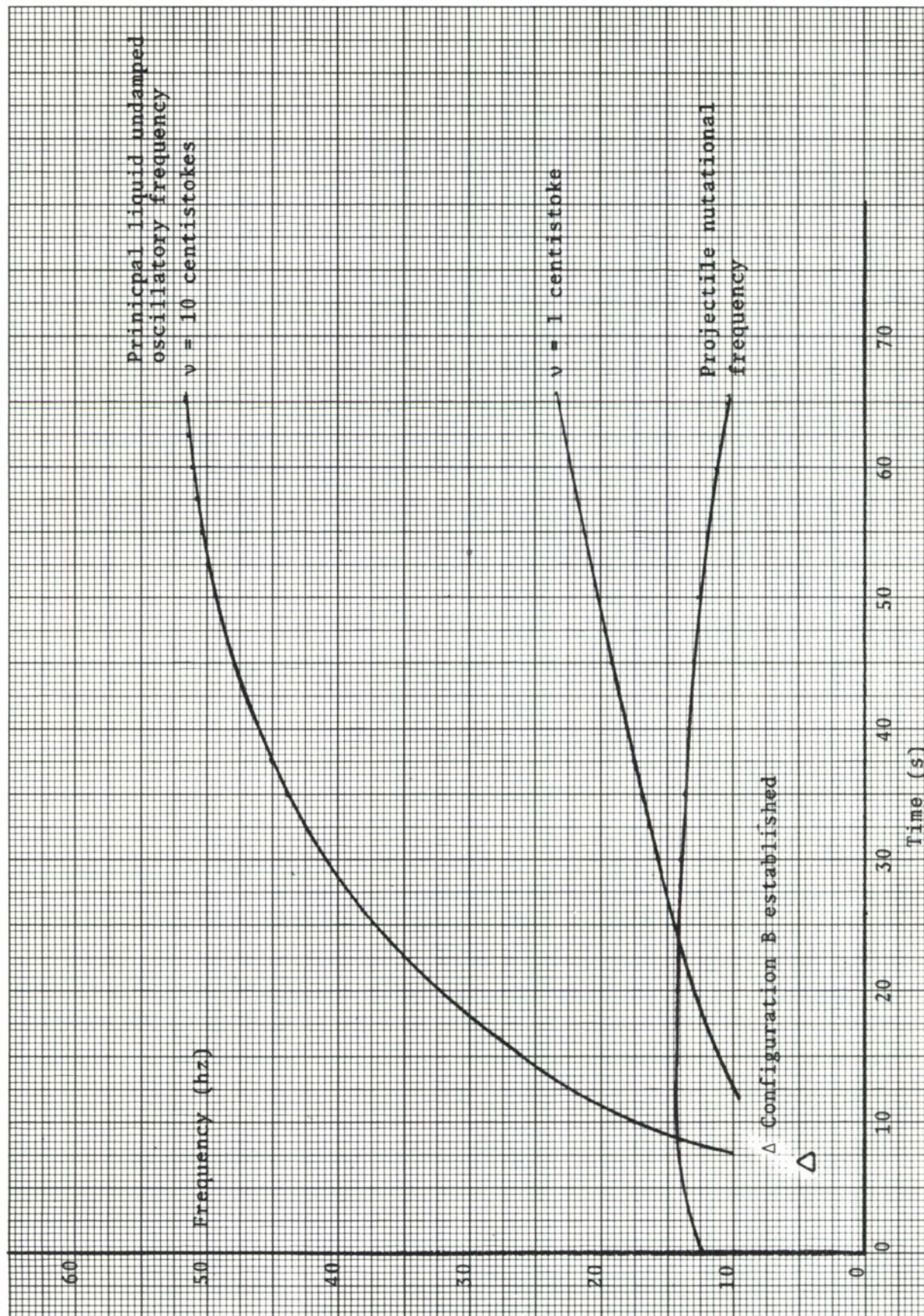


Figure 3.4. Comparison of Oscillatory Modes of the XM736 Liquid-Filled Projectile (Zone 7 at QE 450 in M110E2 Howitzer)

In view of the approximations required to perform this type of analysis, stronger conclusions are not warranted.* An experimental program to better define the values of the pertinent parameters characterizing the liquid certainly is suggested. Additionally, a program to define the ballistic dispersion of projectiles with low-viscosity liquid fills is indicated.

*Note [3.1]:

A more elaborate analysis such as a finite-element description of the liquid, embedded in a six-degree-of-freedom flight simulation appears to be a profitable direction to take analytically; however, this method will likely blur some of the insights afforded by the simpler procedure used here.

Some of the restrictive assumptions used in our analysis could be removed. As an example we note that our analysis in Chapter III neglects certain forces arising because of the non-Newtonian character of the coordinate system. A coordinate system which rotates with the projectile is assumed here. Since this is a non-Newtonian frame, a modified form of Euler's equations must be used. This formulation introduces the usual centrifugal and Coriolis forces as well as an "angular acceleration" force which arises from the angular acceleration of the projectile. The latter force is in the same direction as the Coriolis force and is proportional to the angular acceleration of the projectile.

For a liquid element of mass m in this rotating coordinate system, the apparent acceleration vector $\underline{a}_m$ in the moving frame due to an external force F is given by the equation

Note [3.1] (continued)

$$\underline{a}_m = \underline{F}/m - \ddot{\underline{r}}_O - 2 \underline{\omega} \times \dot{\underline{r}}_{ma} - \dot{\underline{\omega}} \times \underline{r}_m - \underline{\omega} \times (\underline{\omega} \times \underline{r}_m) ,$$

where

$\underline{r}_O$ is the origin of the moving frame relative to a Newtonian frame

$\underline{r}_m$ is the position of the mass element in the moving frame

$\dot{\underline{r}}_{ma}$ is the apparent velocity of the mass element in the moving frame

$\underline{\omega}$ is the angular velocity of the moving frame in a Newtonian frame

$\dot{\underline{\omega}}$ is the angular acceleration of the moving frame (projectile)

The term $-\underline{\omega} \times (\underline{\omega} \times \underline{r}_m)$ is the centrifugal acceleration which has been treated. The term $-2 \underline{\omega} \times \dot{\underline{r}}_{ma}$ is the Coriolis acceleration. In a right-handed frame with the x-axis along the spin axis of the projectile, this term reduces to

$$-2 \omega (\dot{y} \underline{k} - \dot{z} \underline{j})$$

with $\underline{j}$ and $\underline{k}$ unit vectors in the y- and z-directions. If the transverse components of velocity are small, this term is negligible and, in fact, was neglected. The term $-\dot{\underline{\omega}} \times \underline{r}_m$ is the angular acceleration component of acceleration and reduces to

$$-\underline{k} \dot{\omega} y + \underline{j} \dot{\omega} z .$$

This term has also been neglected due to the small value of $\dot{\omega}$.

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APPENDIX
DERIVATIONS AND COMPUTER PROGRAMS

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APPENDIX
DERIVATIONS AND COMPUTER PROGRAMS

Estimate of Precessional and Nutational Frequencies
of a Spin-Stabilized Projectile

For a gyroscopically stable spinning system with angular velocity ω and axial moment of inertia I_A , the precessional angular velocity Ω obtained when the gyro feels a torque τ is given by

$$\Omega = \frac{\tau}{I_A \omega} \quad (A1)$$

If the torque is produced by aerodynamic forces where the center of pressure is $-X_{sm}$ calibers in front of the cg, then

$$\tau = -X_{sm} D C_N A q = -X_{sm} D C_{N_\alpha} \alpha A q \quad (A2a)$$

where, notationally and by definition,

D = caliber

C_N = normal force coefficient

C_{N_α} = normal force derivative coefficient

α = projectile angle of attack

A = reference area

$$A = \frac{\pi}{4} D^2 \quad (A2b)$$

q = dynamic pressure

$$q = \frac{\rho V^2}{2} \quad (A2c)$$

Since the overturning-moment coefficient

$$C_M = -X_{sm} C_N = (X_{cg} - X_{cp}) C_N, \quad (A3)$$

equation (A2a) may be written

$$\tau = C_M D A q$$

$$\tau = C_{M_\alpha} \alpha D A q. \quad (A4)$$

The natural precessional frequency at constant α is

$$\begin{aligned} \nu_p &= \frac{\Omega}{2\pi} \\ \nu_p &= \frac{C_M D A q}{2\pi I_A \omega}. \end{aligned} \quad (A5)$$

Yaw Frequency

The solution of the linearized equations for the yawing motion of a projectile is given by McShane et al on p. 644 of [7]. The solution involves superposition of two sinusoids having different frequencies. The two frequencies are

$$\Omega_{1,2} = \frac{I_A \omega}{2 I_B} (1 \pm \sigma) \quad (\text{rad/s})$$

or

$$\nu_{1,2} = \frac{1}{2\pi} \Omega_{1,2} \quad (\text{hz}) \quad (A6)$$

with

$$\sigma = (1 - s^{-1})^{1/2}, \quad (A7)$$

where s is the gyroscopic stability factor given by

$$s = \frac{I_A^2 \omega^2}{8 I_B K_{M_\alpha} D^3 q}$$

[7] McShane, E., Kelly, J. and Reno, F. Exterior Ballistics, University of Denver Press, c. 1953.

or

$$s = \frac{I_A^2 \omega^2}{\pi I_B C_{M_A} D^3 q} \quad (A8)$$

The frequencies $\nu_{1,2}$ have been called the "nutational" and "precessional" frequency, respectively. However, it should be noted that ν_1 is not the reciprocal of the time between yaw maxima. Because of the superposition of the high- and low-frequency components of motion, the frequency with which maximum yaw occurs is the difference frequency:

$$\nu_y = \nu_1 - \nu_2 \quad (A9)$$

This is called the yaw or nutational frequency in this report. An alternative derivation for ν_y starts with an expression for the wavelength of yaw Λ , in calibers. Then,

$$\nu_y = \frac{V}{D \Lambda} \quad (\text{hz}) \quad (A10)$$

with projectile velocity V and caliber D .

An expression for Λ is obtained from p. 651 of [7]. Using our notation

$$\Lambda = \frac{2 \pi I_B V}{I_A D \omega \sigma} \quad (A11)$$

Then, (A10) and (A11) yield

$$\nu_y = \frac{\omega}{2 \pi} \frac{I_A \sigma}{I_B} \quad (A12)$$

This frequency has been computed as a function of time for several combinations of projectiles and cannons. Results for the M509 projectile in the XM201 cannon are shown in Figure 3.1. Comparable results for the M106 projectile in the M2A2 cannon are shown in Figure 3.2.

[7] McShane, E., Kelly, J. and Reno, F. Op. Cit.

Derivation of an Equation
for Spin Decay in Projectiles

Glossary of Terms

N = projectile spin (rad/sec)

N_0 = initial spin (rad/sec)

V = projectile velocity (m/sec)

V_0 = muzzle velocity (m/sec)

t = time since launch (sec)

ρ = air density (kg/m^3)

D = projectile caliber (m)

M = projectile mass (kg)

I_A = projectile axial moment of inertia (kg m^2)

K_A = spin damping moment coefficient

K_D = zero-lift drag coefficient

k = velocity decay parameter (m^{-1})

x = rangewise coordinate (m)

$$\dot{N} = - \frac{\rho D^4}{I_A} K_A N V \quad (\text{A13})$$

For flat trajectories at low QE, we assume negligible gravitational effects.

$$\dot{V} = - \frac{K_D \rho D^2}{M} V^2 \quad (\text{A14})$$

$$\frac{d(V)^{-1}}{dt} = \frac{K_D \rho D^2}{M} \quad (A15)$$

$$V^{-1} - V_0^{-1} = \frac{K_D \rho D^2}{M} t \quad (A16)$$

$$V = (V_0^{-1} + kt)^{-1} \quad (A17)$$

$$\text{with } k = \frac{K_D \rho D^2}{M} \quad (A18)$$

Then,

$$\dot{N} = -\lambda V N \quad \text{with} \quad (A19)$$

$$\lambda = \frac{\rho D^4 K_A}{I_A} \quad (A20)$$

$$\frac{\dot{N}}{N} = - \frac{\lambda}{V_0^{-1} + kt} \quad (A21)$$

$$d(\ln N) = - \frac{\lambda dt}{V_0^{-1} + kt} \quad (A22)$$

$$\ln N \Big|_{N_0}^N = - \frac{\lambda}{k} \ln (V_0^{-1} + kt) \Big|_0^t$$

$$\ln \frac{N}{N_0} = - \frac{\lambda}{k} \ln (1 + k V_0 t) \quad (A23)$$

$$\frac{N}{N_0} = (1 + k V_0 t)^\beta \quad (A24)$$

with

$$\beta = \frac{\lambda}{k} = \frac{K_A M D^2}{K_D I_A} \quad (A25)$$

$$\text{or } N = \frac{N_o}{(1 + k V_o t)^\beta} . \quad (\text{A26})$$

Integration of (A17) produces

$$V = V_o e^{-kx} \quad (\text{A27})$$

And with (A16),

$$t = k^{-1} (V^{-1} - V_o^{-1}) \quad (\text{A28})$$

or

$$t = k^{-1} (V_o^{-1} e^{kx} - V_o^{-1})$$

$$t = (e^{kx} - 1) / (k V_o) \quad (\text{A29})$$

Substitution of (A29) into (A26) yields

$$N = N_o e^{-\beta kx} . \quad (\text{A30})$$

An Example

For long trajectories the variation of drag coefficient and spin-damping moment coefficient with Mach number renders the above results quite approximate. However, in some instances this approximation may be adequate. In this example analytic results for spin damping versus time are compared with those produced by a computer simulation in which variation with Mach number is considered.

Take the M106, 8 inch HE projectile as fired from the M2 howitzer. The maximum range trajectory will be considered. In this case an average altitude ASL is about 10,000 ft. At this altitude air density is about 0.74 sea level standard. Thus

$$\rho = 0.74 \rho_0 = 0.9065 \text{ kg m}^{-3}$$

$$\rho_0 = 1.225 \text{ kg m}^{-3}$$

Other parameter values are

$$N(0) = N_0 = 735 \text{ rad sec}^{-1}$$

$$V_0 \cong 594.4 \text{ m sec}^{-1}$$

$$C_D \cong 0.30 \text{ (effective) or}$$

$$K_D \cong 0.1178 \text{ (effective)}$$

$$K_A = 0.006 \text{ rad}^{-1} \text{ (effective)}$$

$$D = 0.203 \text{ m}$$

$$M = 90.72 \text{ kg}$$

$$I_A = 0.678 \text{ kg m}^2$$

Then

$$k = K_D \rho D^2 M^{-1}$$

$$k = 4.8506 \cdot 10^{-5} \text{ m}^{-1}$$

and

$$\alpha = \rho D^4 K_A I_A^{-1}$$

$$\alpha = 1.3622 \cdot 10^{-5} \text{ m}^{-1}$$

$$\beta = \alpha/k$$

$$\beta = 0.2808$$

$$V_o k = 0.02883 \text{ sec}^{-1}$$

From (A26)

$$N = N_o (1 + k V_o t)^{-\beta}$$

$$N = 735 (1 + 0.02883 t)^{-0.2808} \quad (\text{A31})$$

The result in (A31) is plotted in the following graph with selected variables from a simulated trajectory. For some purposes the agreement shown between the analytical estimate and the more exact simulation may be satisfactory.

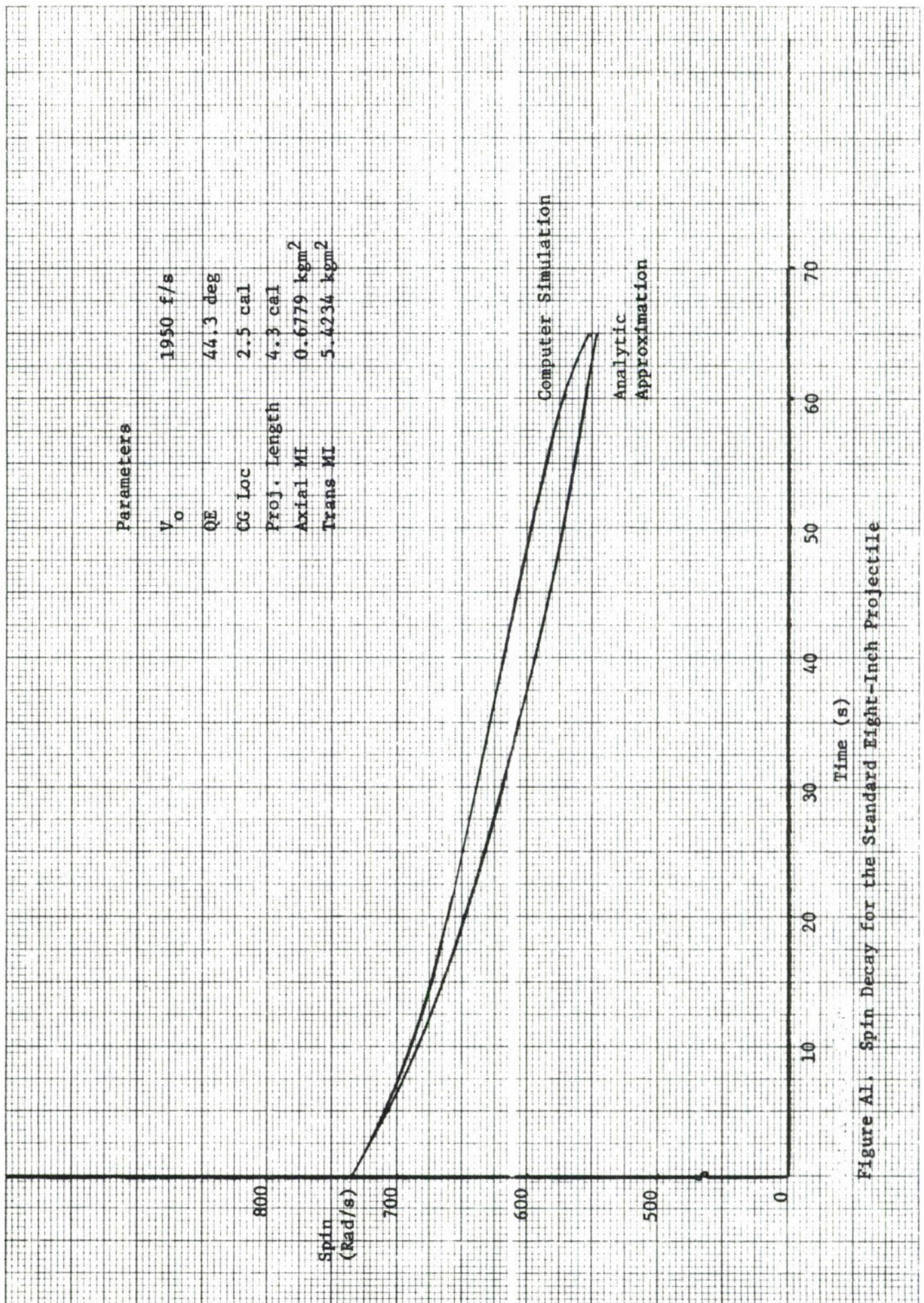


Figure A1. Spin Decay for the Standard Eight-Inch Projectile

Estimate of Stability of Spin-Stabilized Projectiles
Having Oscillating Inertial Properties

The objective of this section is to display and illustrate by examples a numerical technique for evaluating the stability of spin-stabilized projectiles which are subject to oscillations in the position of the center of gravity (X_{cg}) and simultaneous oscillations in the transverse moment of inertia (I_y). To preserve the greatest generality of approach, the equations of motion describing projectile pitch-yaw dynamics are solved numerically* in the time domain with cg position and transverse inertia given by the following functions:

$$X_{cg} = X_{cgo} + \delta X_{cg} \sin[2\pi t (\nu_0 + \dot{\nu} t) + \alpha]$$
$$I_y = I_{yo} + \delta I_y \sin[2\pi t (\nu_0 + \dot{\nu} t)] \quad , \quad (A32)$$

with X_{cgo} , δX_{cg} , I_{yo} , δI_y , α , ν_0 , $\dot{\nu}$ constants for $t \geq 0$.

The solution is carried far enough in time to determine whether the nutational amplitude is decreasing. While admittedly a brute-force approach such as this lacks the elegance of a frequency-domain approach to stability, there is no need to linearize or to assume the amplitudes δX_{cg} and δI_y are small or that the inertial properties oscillate at a constant driving frequency. Further, some insight is gained regarding the behavior of the projectile as the driving frequency matches the nutational frequency of the projectile.

* Using a fourth-order Runge-Kutta procedure with time step 0.002 sec.

Using principally the notation of AMCP 706-165, reference [8], the following auxiliary variables in the equations of motion are defined.

$$H = \frac{\rho_a A d}{2 m} [C_{N_x} - 2 C_D - k_y^{-2} (C_{M_q} + C_{M_x})]$$

$$M = \frac{\rho_a A d}{2 m} k_y^{-2} C_{M_x} = a^2 A q d C_{M_x} I_y^{-1}$$

$$q = \frac{1}{2} \rho_a v^2$$

$$T = \frac{\rho_a A d}{2 m} [C_{N_x} - C_D + k_x^{-2} C_{M_p}]$$

$$P = I_x I_y^{-1} \omega a$$

$$a = v^{-1} d$$

$$G = P g_{\perp} a v^{-1}$$

$$k_x^{-2} = m d^2 I_x^{-1}$$

$$k_y^{-2} = m d^2 I_y^{-1} \quad . \quad (A33)$$

In this notation:

A = reference area of the projectile

d = caliber of the projectile

m = mass of the projectile

[8] Engineering Design Handbook: Liquid-Filled Projectile Design, AMCP 706-165, April 1969.

I_x = longitudinal moment of inertia

I_y = transverse moment of inertia

k_x = longitudinal radius of gyration in calibers

k_y = transverse radius of gyration in calibers

v = velocity of the projectile

ω = spin of the projectile

ρ_a = air density

$g_{\perp}$ = the component of gravity normal to $\underline{v}$

C_D = the zero-lift drag coefficient

$C_{N_{\alpha}}$ = the normal force derivative coefficient

$C_{M_{\alpha}}$ = the overturning moment derivative coefficient

$C_{M_q} + C_{M_{\dot{\alpha}}}$ = the pitch damping moment coefficient

$C_{M_{p_{\alpha}}}$ = the Magnus moment derivative coefficient .

Using a right-handed coordinate system as in [8] with the x-axis coinciding with the form axis of the projectile, positive toward the nose, the pitching angular motion in a vertical plane (about the horizontal transverse axis) will be denoted by ϑ and the yaw about the mutually orthogonal axis will be denoted by ψ . With this notation, the equations of motion are:

[8] Op. Cit.

$$\begin{aligned}\ddot{j} &= a^{-2} M \dot{j} - a^{-2} P T \psi - a^{-1} H \dot{j} - a^{-1} P \dot{\psi} + a^{-2} G \\ \ddot{\psi} &= a^{-2} P T \dot{j} + a^{-2} M \dot{\psi} + a^{-1} P \dot{j} - a^{-1} H \dot{\psi} .\end{aligned}\quad (A34)$$

Adopting the systematic notation

$$\begin{aligned}x_1 &= \dot{j} \\ x_2 &= \psi \\ x_3 &= \dot{j} \\ x_4 &= \dot{\psi} ,\end{aligned}\quad (A35)$$

equations (A34) become

$$\dot{\underline{x}} = A \underline{x} + \underline{b} \quad (A36)$$

with

$$\begin{aligned}\underline{x} &= [x_1 \ x_2 \ x_3 \ x_4]' \\ \underline{b} &= [0 \ 0 \ b_3 \ 0]'\end{aligned}$$

The elements of the A matrix, $\{a_{ij}\}$, are given below in units of sec^{-2} .

$$a_{11} = a_{12} = a_{14} = 0$$

$$a_{13} = 1$$

$$a_{21} = a_{22} = a_{23} = 0$$

$$a_{24} = 1$$

$$a_{31} = a^{-2} M = A q d C_{M_{\alpha}} I_y^{-1}$$

$$a_{32} = - a^{-2} P T$$

$$= - \omega v^{-1} d^2 k_x^2 A q I_y^{-1} [C_{N_{\alpha}} - C_D + k_x^{-2} C_{M_{P_{\alpha}}}]$$

$$a_{33} = - a^{-1} H$$

$$= - A q v^{-1} m^{-1} [C_{N_{\alpha}} - 2 C_D - k_y^{-2} (C_{M_q} + C_{M_{\alpha}})]$$

$$a_{34} = - a^{-1} P = - I_x I_y^{-1} \omega$$

$$a_{41} = - a_{32}$$

$$a_{42} = a_{31}$$

$$a_{43} = - a_{34}$$

$$a_{44} = a_{33}$$

$$b_3 = a^{-2} G = I_x I_y^{-1} g_{\perp} v^{-1} \quad (A37)$$

Examples

Using the equations (A36) and (A37), two projectiles have been treated as numerical examples. Reference [8] provides data on the solid, WP loaded, 152 mm XM410 projectile. Under conditions in which a portion of the white phosphorous fill has liquified, both the effective transverse moment of inertia and center of gravity can be expected to oscillate in flight. Whereas the amplitude of these oscillations may be slight, a persistent resonance of the oscillations with the nutational frequency of the projectile can cause the

nutational amplitude to continuously increase. At a launch Mach number of 1.5, the nominal frequency of yaw maxima for this system is 17.5 hz. Accordingly several numerical experiments were performed in which ν_0 (in (A32)) was set to 17.5 and $\dot{\nu}$ was set to 0.0. During these experiments δX_{cg} and δI_y were varied systematically to determine the region of stability. The absolute stability criterion of diminishing yaw amplitude was used here. Preliminary experiments indicated that stability is adversely affected when δX_{cg} and δI_y are in phase. Thus, all experiments were run under the worst-case phase, namely for α in (A32) set to zero.

A second example was selected for comparative purposes. The numerical values of this example are best estimates of the parameters of the XM736 liquid-filled projectile at a launch Mach number of unity. Axial moment of inertia reflects only that of the metal parts, indicating that the angular momentum of the liquid at launch is treated as negligible. The amplitudes of the inertial increments were selected, somewhat arbitrarily, by taking values proportional to the differences observed in the properties of Configurations A and B of Chapter 1. The values of the parameters used in both examples are displayed in Table A1. Procedures for examining the stability region for the second example were identical to those employed for the first. Stability was examined at a forcing frequency equal to the nominal nutational frequency of 7.77 hz. Additionally, runs were made in which the frequency was swept linearly from 6.0 hz, at the rate of 0.25 hz/sec, to 8.5 hz at 10 sec. These runs clearly showed that, for certain values of δX_{cg} and δI_y , the projectile will remain quite stable under a condition of moving forcing frequency, whereas the projectile

will become unstable when forced at a constant, nutational frequency. The stability region for both examples is displayed in Figure A2. Under the assumed conditions, the projectiles for both examples are stable.

TABLE A1. PARAMETER VALUES FOR
GYROSCOPIC STABILITY ANALYSIS

parameter	symbol	value		dimension
		XM410	XM736	
caliber	d	0.5	0.6667	ft
proj. mass	m	1.313	6.31	slug
long. inertia	I_x	0.0446	0.4036	slug ft ²
trans. inertia	I_y	0.1548	3.5164	slug ft ²
amplitude of incr. in trans. inertia	δI_y	0.0006	0.0183	slug ft ²
muzzle velocity	v_o	1675.5	1117	ft s ⁻¹
proj. spin	ω	526.4	573.4	rad s ⁻¹
air density	ρ_a	2.3769	10 ⁻³	slug ft ⁻³
drag coef.	C_D	0.50	0.30	
normal force	C_{N_α}	2.90	2.10	rad ⁻¹
pitch damping	$C_{M_q} + C_{M_{\dot{\alpha}}}$	-5.00	-4.60	(rad sec ⁻¹) ⁻¹
magnus moment	$C_{M_{p_\alpha}}$	0.30	-0.10	rad ⁻¹
center of press.	X_{cp}	1.40	1.108	cal
proj. center of gravity	X_{cg}	1.85	3.472	cal
amplitude of incr. in c.g.	δX_{cg}	0.04	0.020	cal
gravitational component	$g_\perp$	0.0	0.0	ft s ⁻²

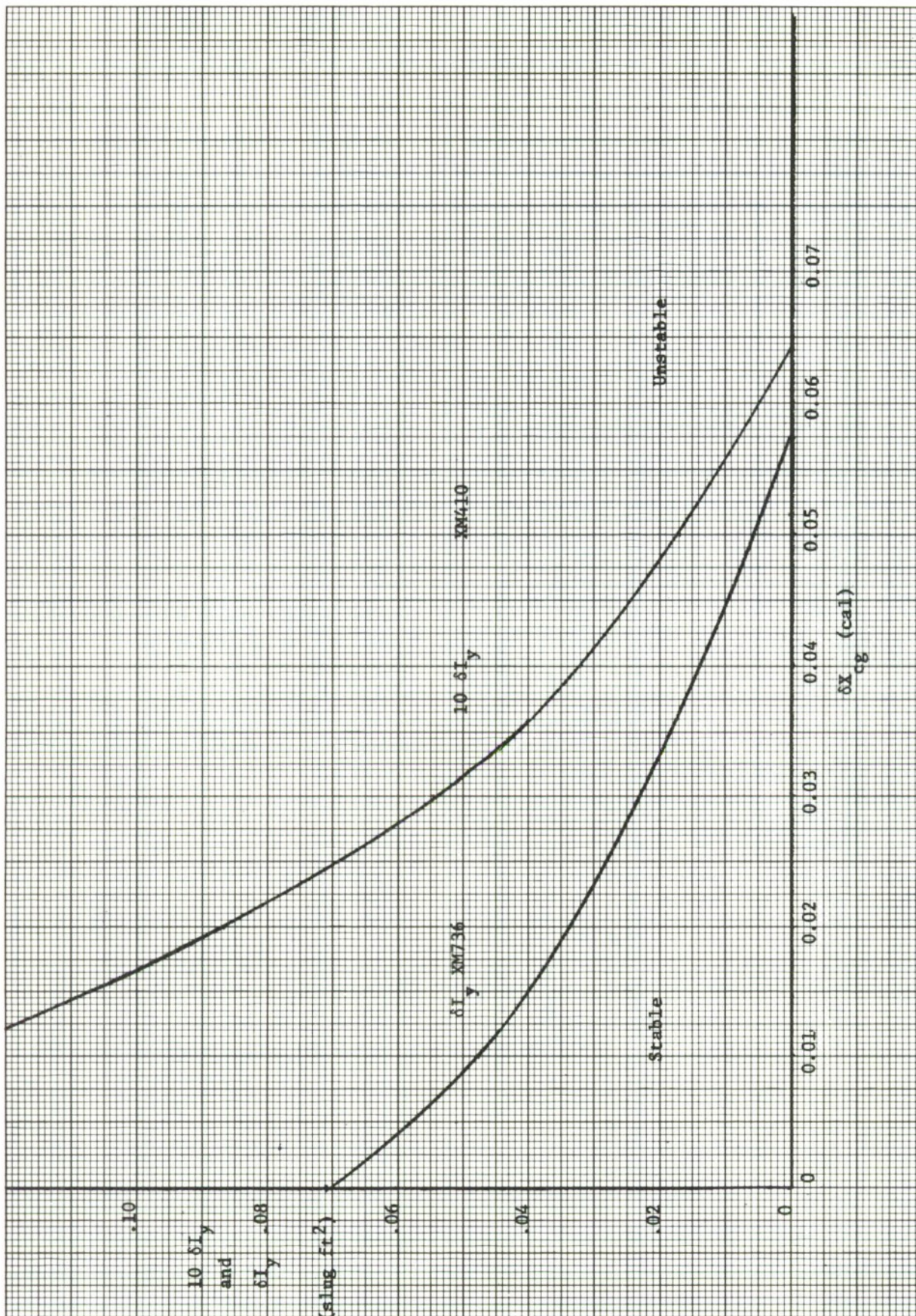


Figure A2. Stability Regions for Projectiles in Examples 1 and 2.

Computer Program for Numerical Solution
of One-Dimensional Equations for Spinup of Liquid

The source programs shown on the following pages were written in the FORTRAN IV language for the IBM 360-65 computer. Comments in the listings introduce each main program and subprogram describing its function and delineating the principal operations and variables.

Following each program is a sample of the output produced by the program.

Next page is blank.



```

$JDR 'MGEORGE',KP=29,LINES=560,TIME=360,PAGES=500
C**** DYNAMICS OF SPIN UP OF LIQUID-FILLED PROJECTILE
C
1  DIMENSION TITLE(20),U(22),WP(18),X(11)
2  COMMON AMRDA,N,X,DX,C1,C2,T0
C**** HEAD TITLE AND INPUT PARAMS
3  HEAD (5,100) TITLE,DT,T0,TCON,DX,X0,WLIM,N,NPRINT
4  I00 FORMAT(20A4/6F10.0,2I2)
5  WRITE (6,500) N,TCON
6  500 FORMAT(1H0,3HN =,I3,2X,6HTCON =,F10.2)
C**** PRINT COLUMN HEADINGS
7  WRITE (5,200) TITLE,DT,DX,X0,WLIM.
8  200 FORMAT(1H1,20A4/1H011HTIME STEP =,E15.5,14H SPACE STEP =,E15.5,
1  2X4HX0 =,F15.5,2X6HWLIM =,E15.5/1H0,9X,1H!,6X4HU(2),6X4HU(3),
2  6X4HU(4),6X4HU(5),6X4HU(6),6X4HU(7),6X4HU(8),6X4HU(9),
3  5X5HU(10),5X5HU(11),5X,5HOMEGA,10H LIQ SPIN)
C
C**** COMPUTE CONSTANTS
9  NM1=N-1
10 X(1)=X0
11 SUM=X0/2.
12 AMRDA=1./DX**2
13 DO 2 J=2,N
14 X(J)=DX*FLOAT(J-1)*X0
15 SUM=SUM+X(J)
16 2 CONTINUE
17 SUM=SUM*X(N)/2.
18 R=1./SUM
19 C2=0.0288*TCON
20 CI=-C2*0.2808
C
C**** INITIALIZE STATE VECTOR
21 U(1)=WLIM
22 DO 4 J=2,NM1
23 U(J)=WLIM
24 4 CONTINUE
25 U(N)=1./J
C
C**** INITIALIZE RUNGE-KUTTA SUBROUTINE
C**** SOLVE DIFF EQNS FOR DERIVATIVE 1CS
26 T=0.0
27 CALL DIFEQ(T,U,4)
28 I9 KOUNT=0
C
C**** START OF SOLUTION LOOP
29 20 CONTINUE
C
C**** MOVE STATE FROM T TO T+DT
30 CALL KUTTA(T,DT,U,WP,11,2,DIFEQ)
C
C**** COMPUTE EFFECTIVE ANGULAR VELOCITY
31 SUM=0.0
32 DO 30 J=1,NM1
33 SUM=SUM+(U(J)+U(J+1))*(X(J)+X(J+1))/4.
34 30 CONTINUE
35 OMEGA=R*SUM
36 FSPIN=923.*OMEGA
37 TP=TCON*T+T0
38 IF (TP.GT.66.) CALL EXIT
39 KOUNT=KOUNT+1

```

```

40      IF(KOUNT.EQ.NPRINT) GO TO 40
41      IF(T.GT. 1.0) CALL EXIT
42      GO TO 20
43      40 WRITE (6,300)TP,(U(I),J=2,11),OMEGA,FSPIN
44      300 FORMAT(1H,13F10.5)
45      IF(T.GT.1.0) CALL EXIT
46      GO TO 19
47      END

48      SUBROUTINE DIFEQ(TIME,U,KUTTA)
C
C**** DIFFERENTIAL EQUATIONS FOR ANGULAR VEL. IN CNFIG. A
49      DIMENSION U(22),X(11)
50      COMMON AMHDA,N,X,DX,C1,C2,T0
51      U(N+1)=AMHDA*(U(2)-U(1))*((X(2)/X(1))**2+1.)/2.
52      NM1=N-1
53      W0=1.
54      DO 10 I=2,NM1
55          WM1=(1.-DX/X(I))
56          WP1=1.+DX/X(I)
57          U(I+N)=AMHDA*(U(I-1)*WM1-2.*U(I)*W0+U(I+1)*WP1)
58      10 CONTINUE
59      U( *N)=0.0
60      RETURN
61      END

```

```

SUBROUTINE KUTTA(T,DT,V,W,NEQ,NORDP1,DIFFEQ)
DIMENSION V(NEQ,NORDP1),W(NEQ,NORDP1,4)
DT2=DT*0.5
DT6=DT/6.0
DO 1 I=1,NEQ
DO 1 J=1,NORDP1
1 W(I,J,1)=V(I,J)
DO 2 K=1,3
L=K+1
GO TO (3,3,4),K
3 DTW=DT2
GO TO 5
4 DTW=DT
5 TW=T+DTW
DO 6 I=1,NEQ
DO 6 J=2,NORDP1
J1=J-1
WP=W(I,J1,1)+W(I,J,K)*DTW
V(I,J1)=WP
6 W(I,J1,L)=WP
CALL DIFFEQ(TW,V,K)
DO 2 I=1,NEQ
2 W(I,NORDP1,L)=V(I,NORDP1)
DO 7 J=2,NORDP1
J1=J-1
DO 7 I=1,NEQ
7 V(I,J1)=W(I,J1,1)+DT6*(W(I,J,1)+2.0*(W(I,J,2)+W(I,J,3))+W(I,J,4))
T=TW
CALL DIFFEQ(T,V,4)
RETURN
END

```

```

00000100
00000200
00000300
00000400
00000500
00000600
00000700
00000800
00000900
00001000
00001100
00001200
00001300
00001400
00001500
00001600
00001700
00001800
00001900
00002000
00002100
00002200
00002300
00002400
00002500
00002600
00002700
00002800
00002900
00003000
00003100

```

DYNAMICS OF SPINUP OF LIQUID CONFIGURATION B, CASE 1

TIME STEP =	0.10000E-03	SPACE STEP =	0.67000E-01	X0 =	0.33000E 00	WLTM =	0.20000E-01												
E = .397																			
T	U(2)	U(3)	U(4)	U(5)	U(6)	U(7)	U(8)	U(9)	U(10)	U(11)	OMEGA	LIO SPIN							
0.00100	0.02000	0.02000	0.02000	0.02000	0.02000	0.02010	0.02164	0.04114	0.21021	1.00000	0.12086	111.55720							
0.00200	0.02000	0.02000	0.02000	0.02001	0.02011	0.02112	0.02967	0.08531	0.33796	1.00000	0.14563	134.41400							
0.00300	0.02000	0.02000	0.02001	0.02007	0.02058	0.02414	0.04462	0.13608	0.42788	1.00000	0.16704	154.17460							
0.00400	0.02000	0.02000	0.02004	0.02027	0.02176	0.02973	0.06482	0.18634	0.49396	1.00000	0.18604	171.71030							
0.00500	0.02000	0.02002	0.02013	0.02076	0.02397	0.03793	0.08339	0.23328	0.54439	1.00000	0.20323	187.58050							
0.00600	0.02001	0.02006	0.02033	0.02166	0.02737	0.04489	0.11375	0.27602	0.58415	1.00000	0.21902	202.15900							
0.00700	0.02003	0.02015	0.02071	0.02310	0.03204	0.05098	0.13980	0.31455	0.61637	1.00000	0.23740	215.70520							
0.00800	0.02007	0.02031	0.02133	0.02517	0.03796	0.07496	0.16577	0.34919	0.64308	1.00000	0.24746	228.40540							
0.00900	0.02014	0.02059	0.02226	0.02793	0.04503	0.09003	0.19118	0.38036	0.66565	1.00000	0.26045	240.39700							
0.01000	0.02027	0.02101	0.02354	0.03140	0.05313	0.10583	0.21576	0.40850	0.68503	1.00000	0.27279	251.78510							
0.01100	0.02046	0.02161	0.02523	0.03558	0.06212	0.12207	0.23935	0.43399	0.70189	1.00000	0.28456	262.65110							
0.01200	0.02075	0.02242	0.02734	0.04044	0.07185	0.13853	0.26187	0.45719	0.71674	1.00000	0.29584	273.05980							
0.01300	0.02116	0.02348	0.02989	0.04594	0.08219	0.15502	0.28332	0.47838	0.72993	1.00000	0.30668	283.06340							
0.01400	0.02170	0.02481	0.03290	0.05203	0.09301	0.17141	0.30371	0.49783	0.74176	1.00000	0.31712	292.70480							
0.01500	0.02240	0.02642	0.03634	0.05865	0.10419	0.18759	0.32307	0.51574	0.75244	1.00000	0.32722	302.01950							
0.01600	0.02328	0.02834	0.04022	0.06575	0.11564	0.20351	0.34146	0.53230	0.76214	1.00000	0.33699	311.03780							
0.01700	0.02436	0.03056	0.04452	0.07325	0.12727	0.21910	0.35893	0.54766	0.77101	1.00000	0.34646	319.78490							
0.01800	0.02564	0.03309	0.04920	0.08112	0.13901	0.23433	0.37554	0.56197	0.77916	1.00000	0.35567	328.28240							
0.01900	0.02716	0.03593	0.05425	0.08929	0.15080	0.24918	0.39134	0.57532	0.78668	1.00000	0.36463	336.54900							
0.02000	0.02890	0.03908	0.05964	0.09772	0.16260	0.26364	0.40638	0.58782	0.79364	1.00000	0.37335	344.60180							
0.02100	0.03089	0.04253	0.06534	0.10636	0.17435	0.27770	0.42071	0.59956	0.80012	1.00000	0.38186	352.45430							
0.02200	0.03312	0.04627	0.07133	0.11517	0.18603	0.29136	0.43438	0.61060	0.80617	1.00000	0.39016	360.11910							
0.02300	0.03561	0.05029	0.07757	0.12411	0.19761	0.30462	0.44744	0.62101	0.81182	1.00000	0.39828	367.60830							
0.02400	0.03834	0.05458	0.08405	0.13316	0.20906	0.31750	0.45992	0.63086	0.81713	1.00000	0.40621	374.93110							
0.02500	0.04133	0.05912	0.09073	0.14228	0.22037	0.33000	0.47186	0.64018	0.82213	1.00000	0.41397	382.09660							
0.02600	0.04456	0.06391	0.09760	0.15145	0.23153	0.34212	0.48330	0.64902	0.82684	1.00000	0.42157	389.11270							
0.02700	0.04803	0.06892	0.10463	0.16064	0.24252	0.35389	0.49427	0.65742	0.83129	1.00000	0.42902	395.98730							
0.02800	0.05174	0.07414	0.11180	0.16985	0.25334	0.36532	0.50480	0.66542	0.83551	1.00000	0.43632	402.72600							
0.02900	0.05568	0.07957	0.11909	0.17904	0.26398	0.37640	0.51491	0.67305	0.83951	1.00000	0.44348	409.33590							
0.03000	0.05984	0.08518	0.12649	0.18821	0.27444	0.38717	0.52463	0.68033	0.84331	1.00000	0.45051	415.82170							
0.03100	0.06422	0.09096	0.13397	0.19734	0.28472	0.39763	0.53399	0.68729	0.84693	1.00000	0.45741	422.18890							
0.03200	0.06880	0.09691	0.14153	0.20643	0.29481	0.40779	0.54301	0.69395	0.85039	1.00000	0.46418	428.44180							
0.03300	0.07358	0.10300	0.14915	0.21545	0.30471	0.41766	0.55170	0.70034	0.85369	1.00000	0.47084	434.58490							
0.03400	0.07855	0.10922	0.15682	0.22442	0.31444	0.42726	0.56008	0.70646	0.85684	1.00000	0.47738	440.62250							
0.03500	0.08370	0.11557	0.16453	0.23331	0.32398	0.43659	0.56817	0.71235	0.85986	1.00000	0.48381	446.55780							
0.03600	0.08902	0.12203	0.17227	0.24213	0.33334	0.44567	0.57600	0.71801	0.86276	1.00000	0.49014	452.39470							
0.03700	0.09450	0.12860	0.18002	0.25086	0.34252	0.45451	0.58356	0.72346	0.86554	1.00000	0.49636	458.13590							
0.03800	0.10013	0.13525	0.18779	0.25951	0.35154	0.46311	0.59088	0.72871	0.86821	1.00000	0.50248	463.78510							
0.03900	0.10590	0.14199	0.19556	0.26808	0.36038	0.47149	0.59797	0.73377	0.87078	1.00000	0.50850	469.34440							
0.04000	0.11181	0.14880	0.20332	0.27655	0.36905	0.47965	0.60483	0.73865	0.87326	1.00000	0.51443	474.81710							
0.04100	0.11783	0.15568	0.21107	0.28493	0.37756	0.48760	0.61149	0.74337	0.87564	1.00000	0.52027	480.20530							
0.04200	0.12397	0.16261	0.21881	0.29322	0.38591	0.49536	0.61795	0.74794	0.87794	1.00000	0.52602	485.51190							
0.04300	0.13022	0.16960	0.22653	0.30142	0.39410	0.50292	0.62422	0.75235	0.88017	1.00000	0.53168	490.73820							
0.04400	0.13656	0.17663	0.23422	0.30951	0.40214	0.51030	0.63031	0.75662	0.88231	1.00000	0.53726	495.88740							
0.04500	0.14300	0.18369	0.24188	0.31752	0.41004	0.51751	0.63623	0.76076	0.88439	1.00000	0.54275	500.96060							
0.04600	0.14951	0.19078	0.24951	0.32543	0.41778	0.52454	0.64199	0.76478	0.88640	1.00000	0.54817	505.96060							
0.04700	0.15610	0.19790	0.25711	0.33324	0.42539	0.53141	0.64759	0.76867	0.88834	1.00000	0.55351	510.88840							
0.04800	0.16275	0.20503	0.26466	0.34096	0.43286	0.53813	0.65304	0.77245	0.89023	1.00000	0.55877	515.74600							
0.04900	0.16947	0.21218	0.27218	0.34859	0.44020	0.54469	0.65835	0.77612	0.89206	1.00000	0.56396	520.53540							
0.05000	0.17623	0.21934	0.27964	0.35612	0.44740	0.55111	0.66352	0.77969	0.89383	1.00000	0.56908	525.25750							
0.05100	0.18305	0.22650	0.28707	0.36356	0.45448	0.55739	0.66856	0.78316	0.89555	1.00000	0.57412	529.91470							
0.05200	0.18990	0.23366	0.29444	0.37091	0.46144	0.56353	0.67348	0.78653	0.89723	1.00000	0.57910	534.50750							
0.05300	0.19679	0.24081	0.30177	0.37816	0.46828	0.56954	0.67828	0.78982	0.89885	1.00000	0.58401	539.03800							
0.05400	0.20370	0.24795	0.30904	0.38533	0.47500	0.57542	0.68296	0.79302	0.90044	1.00000	0.58885	543.50700							
0.05500	0.21064	0.25509	0.31626	0.39241	0.48160	0.58119	0.68753	0.79614	0.90198	1.00000	0.59363	547.91650							

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Computer Program for Modified
Point-Mass Exterior Ballistics

The program used to determine the ballistic differential effects of changes in inertial parameters is a general-purpose, exterior ballistics program capable of simulating the flight of gun-boosted rockets as well as purely ballistic systems.

The program employs two alternative ways of treating the aerodynamic forces normal to the body. The first option simply assumes trailing behavior, that is, that yaw is always zero. This option is inadequate for the purpose of this study. The second option computes an equilibrium yaw, or "yaw of repose," necessary to precess the velocity vector in the vertical plane at the proper rate. Using the yaw of repose, normal body forces are computed and resolved into components in an inertial frame of reference. These are added to the drag, gravitational, and Coriolis forces to complete a point-mass description. This procedure gives satisfactory agreement with experiment, providing accurate aerodynamic data exist and that the projectile displays adequate stability. This option is exercised by setting switch IOPTY equal to unity.

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ISI=02750448

C		00000100
C	EXTERIOR BALLISTICS OF BOOSTED ROCKETS	00000200
C	A THREE-DEGREE-OF-FREEDOM MODEL APPLICABLE WHERE	00000300
C	TRAILING OR FOLLOWING BEHAVIOR CAN BE ASSUMED	00000400
C		00000500
C		00000600
	REAL RS(401),TS(401),VS(401),CJRT(11),CJVT(11)	00000700
	DIMENSION TITLE(20),U(12),WP(48),XMTBL(11),CDTBL(11),COTBL(11)	00000800
	1, TMACH(11),TKA(11),TKOYAW(11),TKL(11),TKM(11),TKF(11),	00000900
	2 TKT(11),TKH(11),TKS(11),TCP(11)	00001000
	INTEGER *2 CHAR(1)/'*/	00001100
	DATA RE/6.378E67,OMEGA/0.72915E-4/	00001200
C	ASSIGN CONSTANTS NEEDED BY DIFFERENTIAL EQUATIONS TO COMMON	00001300
C		00001400
	COMMON EMO,EMB,SPI,FC,BRATE,OELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,	00001500
	1 VW,VCH,ALT,R,IEND,CNACH,REYNLD,RESIS,CAL,DLONG,IPTY,YAW,AMOM,	00001600
	2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIOE	00001700
	COMMON /SRCON/TM	00001800
	COMMON/COFCOM/XCG,SMARG,EM,THIO,PRNU,ALTRIM,CNATRM,GLIOE,EPSTHE	00001900
	1,STAFAC,YAWNU,TKA,TKDYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,THACH,NARTBL	00002000
	COMMON/WINCOM/RWC,XWC	00002100
	COMMON/DRGCOM/ XMTBL,CDTBL,COTBL,NTBL	00002200
	COMMON/SNOCOM/CAOENS	00002300
C		00002400
C****	TABLES OF AEROOYNAMIC COEFFICIENTS IS PARAM. SET INPUT SET -1.	00002500
C****	IF PARAMETERS NTBL AND NARTBL ARE BOTH ZERO, ENDOGENOUS	00002600
C****	FUNCTIONAL FITS TO THE AEROOYNAMIC TABLES(WITH THE T387 FORM)	00002700
C****	WILL BE USED. SEE SUBROUTINE ACOEFS.	00002800
C****	IF ONLY NARTBL IS ZERO, THE ZERO-LIFT DRAG TABLE IS REQUIRED	00002900
C****	WITHOUT REQUIRING TABLES FOR THE OTHER AERO COEFFICIENTS.	00003000
C****	IF CERTAIN AERO COEFFICIENTS ARE DEFINED (KNOWN), THESE	00003100
C****	CAN BE READ WITH THE OTHERS LEFT BLANK. THE PROGRAM WILL	00003200
C****	USE THE TABULATED COEFFICIENTS AND DEFAULT TO THE ENDOGENOUS	00003300
C****	FUNCTIONS FOR THOSE ENTERED AS ZERO.	00003400
C	O=PROJECTILE CALIBER, MILLIMETERS	PARAMETER INPUT 1 00003500
C	EMO=INITIAL PROJECTILE MASS, LBM	INPUT 2 00003600
C	EMB=BURNT MASS, LBM	INPUT 3 00003700
C	FC=NOMINAL THRUST LEVEL, LBF	INPUT 4 00003800
C	SPI=SPECIFIC IMPULSE OF ROCKET PROPELLANT, LBF/LBM/SEC	INPUT 5 00003900
C	BRATE=PROPELLANT BURNING RATE, LBM/SEC	ENDOGENOUS VARIABLE 00004000
C	OELT=THRUST RISE TIME, SEC	INPUT 6 00004100
C	TO=IGNITION TIME FOR ROCKET MOTOR	ENDOGENOUS VARIABLE 00004200
C	IN SUBROUTINE 'BURN' THE THRUST DECAY TIME IS ASSUMED	00004300
C	EQUAL TO THE THRUST RISE TIME. A TYPICAL VALUE = 0.1 SEC.	00004400
C	TB=EFFECTIVE BURNING INTERVAL, SEC	ENDOGENOUS VARIABLE 00004500
C	ISW= A SWITCH SIGNALING COMMENCEMENT OF BURNING	ENDO. VARIABLE 00004600
C	IENO=A SWITCH SIGNALING END OF BURNING	ENDO. VARIABLE 00004700
C	V=PROJECTILE VELOCITY, M/SEC	ENDO. VARIABLE 00004800
C	VO=MUZZLE VELOCITY OF THE PROJECTILE, FT/SEC.	INPUT 7 00004900
C	THETA=ATTITUDE OF PROJECTILE, DEG	ENDO. VARIABLE 00005000

C	CALSQ=CALIBER SQUARED, M**2	EN00. VARIABLE00005100
C	VW=VELOCITY OF HEADWIND, M/SEC (READ IN IN FT/SEC)	END0. VARIABLE00005200
C	VWF=VELOCITY OF HEADWIND IN FT/SEC	INPUT 8 00005300
C	HO=INITIAL ALTITUDE, FT.	INPUT 9 00005400
C	HTERM=TERMINAL ALTITUDE, FT.	INPUT 1000005500
C	FFCTR=FORM FACTOR RELATIVE TO PROGRAMMED DRAG FUNCTION.	INPUT 1100005600
C	SEE FUNCTION 'ORAG' FOR THE SPECIFIC ORAG FUNCTION USED.	00005700
C	QEO=INITIAL QUADRANT ELEVATION, DEG.	INPUT 1200005800
C	OQE=QUADRANT ELEVATION INCREMENT, DEG.	INPUT 1300005900
C	STEP=TIME STEP IN NUMERICAL INTEGRATION PROCEDURE, SEC.	INPUT 1400006000
C	TM=TIME AT WHICH BURNING OF ROCKET MOTOR SHOULD COMMENCE.	INPUT 1500006100
C	NQE=NUMBER OF INCREMENTS OF QUADRANT ELEVATION	INPUT 1600006200
C	NPRINT=NUMBER OF TIME STEPS EXECUTED BETWEEN PRINTS	INPUT 1700006300
C	CONTINUE	00006400
C	ALT=TRUE ALTITUDE ABOVE SEA LEVEL, M	EN00. VARIABLE00006500
C	R=RAIUS FROM CENTER OF EARTH TO PROJECTILE, M	END0. VARIABLE00006600
C	RE=NOMINAL RADIUS OF THE EARTH AT THE EQUATOR, M	CONSTANT00006700
C	OMEGA=ANGULAR VELOCITY OF THE EARTH, RAD/SEC	CONSTANT00006800
C	IOPTY= A SWITCH INDICATING CHOICE OF YAW OPTION.	INPUT 1800006900
C	IOPTY= 1 PRODUCES COMPUTATION OF YAW OF REPOSE FOR SPINNING PROJECTILE	00007000
C	IOPTY= 0 SIGNIFIES A TRAILING PROJECTILE WITHOUT SPIN. FOR	00007100
C	THIS OPTION THE FOLLOWING INPUTS ARE UNNECESSARY.	00007200
C	SPINO=INITIAL SPIN, RAD/SEC	INPUT 1900007300
C	XCG=POSITION OF CENTER OF GRAVITY AFT OF NOSE, CALIBERS	INPUT 2000007400
C	XCP=POSITION OF CENTER OF PRESSURE AFT OF NOSE, CAL	EN00. VARIABLE00007500
C	CLONG=PROJECTILE LENGTH IN CALIBERS.	INPUT 2100007600
C	AMOM=LONGITUDINAL MOMENT OF INERTIA OF THE PROJECTILE, KG*M**2	00007700
C		INPUT 2200007800
C	BMOM=TRANSVERSE MOMENT OF INERTIA OF THE PROJECTILE, KG*M**2	00007900
C		INPUT 2300008000
C	VCM=VELOCITY OF CROSSWIND FROM RIGHT LOOKING DOWNRANGE (READ IN FT/SEC)	INPUT 2400008200
C	WTAREA=WETTED AREA RATIO USED IN COMPUTING SKIN	00008300
C	FRICTION ORAG	INPUT 2500008400
C	ISEP=A SWITCH INDICATING CHOICE OF SEPARATE	00008500
C	COMPUTATION OF SKIN FRICTION DRAG.	INPUT 2600008600
C	= 1 IF FRICTION ORAG IS COMPUTED SEPARATELY AND ADDED TO FORM ORAG	00008700
C	= 0 IF FRICTION ORAG IS INCLUDED IN DRAG FUNCTION.	00008800
C	SMARG=PROJECTILE STATIC MARGIN, CAL.	EN00. VARIABLE00008900
C	PSI=ANGULAR ORIENTATION OF YAW VECTOR	END0. VARIABLE00009000
C	SRNG=SLANT RANGE TO PROJECTILE POSITION, M	EN00. VARIABLE00009100
C	VHXF=VELOCITY OF LAUNCHER IN RANGEWISE DIRECTION. (FT/SEC)	00009200
C	VHYF=VELOCITY OF LAUNCHER IN VERTICAL DIRECTION. (FT/SEC)	00009300
C	CONTINUE	00009400
C	RWC IS HEADWIND COEF.	00009500
C	XWC IS CROSSWIND COEF.	00009600
C	PSI MAY BE COMPUTED BY REMOVING 'C' S FROM COMMENT CARDS	00009700
C	IN SUBROUTINE FLIGHT.	00009800
C	CAOENS IS THE CORRECTION FACTOR FOR AIR DENSITY RELATIVE TO STANDARD	00009900
C		00010000
C	EXTERNAL FLIGHT	00010100
C	EQUIVALENCE (U(1),X),(U(2),Y),(U(3),Z),(U(4),SPIN),	00010200

1	(U(5),X0),(U(6),Y0),(U(7),Z0),(U(8),SPINO),	00010300
2	(U(9),XDD),(U(10),YDD),(U(11),ZDD),(U(12),SDD)	00010400
C	READ IN RUN DESCRIPTION, CONSTANTS IN FLIGHT EQUATIONS AND	00010500
C	INITIAL CONOITIONS.	00010600
C		00010700
	READ (5,256,END=30) TITLE,NTBL,NARTBL	00010800
256	FORMAT(20A4/2I2)	00010900
	IF(NTBL.EQ.0) GO TO 255	00011000
	READ (5,250) (XMTBL(I),I=1,NTBL)	00011100
	READ (5,250) (CDTBL(I),I=1,NTBL)	00011200
	READ (5,250) (CDTBL(I),I=1,NTBL)	00011300
250	FORMAT(8F10.0)	00011400
	WRITE (6,252) TITLE	00011500
252	FORMAT(1H1,20A4/1H0,10H MACH NO,10H COEF DRAG,10H DRAG INCR)	00011600
	00 253 I=1,NTBL	00011700
	WRITE (6,254) XMTBL(I),CDTBL(I),CDTBL(I)	00011800
253	CONTINUE	00011900
	IF(NARTBL.EQ.0) GO TO 255	00012000
	READ (5,250) (TMACH(I),I=1,NARTBL)	00012100
	READ (5,250) (TKA(I),I=1,NARTBL)	00012200
	READ (5,250) (TKDYAW(I),I=1,NARTBL)	00012300
	READ (5,250) (TKL(I),I=1,NARTBL)	00012400
	READ (5,250) (TKM(I),I=1,NARTBL)	00012500
	READ (5,250) (TCP(I),I=1,NARTBL)	00012600
	READ (5,250) (TKF(I),I=1,NARTBL)	00012700
	READ (5,250) (TKT(I),I=1,NARTBL)	00012800
	READ (5,250) (TKH(I),I=1,NARTBL)	00012900
	READ (5,250) (TKS(I),I=1,NARTBL)	00013000
254	FORMAT(1H ,3F10.4)	00013100
	WRITE (6,272)	00013200
272	FORMAT(1H0,10H MACH NO,8X,2HKA,5X,5HKDYAW,	00013300
	1 8X,2HKL,8X,2HKM,10H CP, CAL)	00013400
	00 257 I=1,NARTBL	00013500
	WRITE (6,270) TMACH(I),TKA(I),TKDYAW(I),TKL(I),TKM(I),TCP(I)	00013600
270	FORMAT(1H ,6F10.5)	00013700
257	CONTINUE	00013800
255	CONTINUE	00013900
C****	PROVISIONAL CONSTANT GLIDE ANGLE JAN 75	00014000
C****	SWITCH IGLIOE MUST BE SET TO 1 FOR CONST. GLIDE ANGLE TRAJECTORY.	00014100
	READ (5,264) IGLIDE,ALTRIM,CNATRM ,GLIDE,EPSTHE	00014200
264	FORMAT (11,9X,4F10.0)	00014300
	IF(IGLIDE.NE.1) GO TO 1	00014400
	WRITE (6,268)	00014500
268	FORMAT(1H0,15H TRIM ANGLE, R,6X,9HCNA(TRIM),	00014600
	1 15H GLIDE ANGLE, R,15H TOLERANCE, R)	00014700
	WRITE (6,266) ALTRIM,CNATRM,GLIDE,EPSTHE	00014800
266	FORMAT (1H ,4F15.5)	00014900
C****	PROVISIONAL CONSTANT GLIDE ANGLE JAN 75	00015000
	1 READ (5,2,ENO=30) TITLE,O,EMO,EMB,FC,SP1,OELT,VO,VWF,HO,HTERM,	00015100
	1 FFCTR,QEO,DQE,STEP,TM,NQE,NPRINT,IOPYT	00015200
	2 FORMAT(20A4/BF10.0/7F10.0,3I3)	00015300
C****	SWITCH NABLE IS SET FROM 0 TO 1 AT TIME TENABL	00015400

258	READ (5,260) CADENS,VCW,TENABL,THID	00015500
260	FORMAT(4F10.0)	00015600
	WRITE (6,262) TENABL,THID,CADENS	00015700
262	FORMAT(1H0,14HENABLE TIME = ,F10.4,5X,	00015800
1	21HTHRUST DRAG FACTOR = ,F10.4,5X,18HAIR DENS FACTOR = ,F10.4)	00015900
	IF(IOPY.NE.1) GO TO 25	00016000
	READ 26,SPINO,XCG,CLONG,AMOM,BMOM,WTAREA,ISEP	00016100
	READ 26,VHXF,VHYF,RWC,XWC	00016200
26	FORMAT(6F10.0,I2)	00016300
	GO TO 27	00016400
25	SPIND=0.0	00016500
	XCG=0.	00016600
	CLONG=0.	00016700
	AMOM=0.	00016800
	BMOM=0.	00016900
	WTAREA=0.	00017000
	ISEP=0	00017100
	VHXF=0.0	00017200
	VHYF=0.0	00017300
	RWC=0.0	00017400
	XWC=0.0	00017500
	SMARG=0.0	00017600
	STAFAC=0.0	00017700
	YAWNU=0.0	00017800
	PRNU=0.0	00017900
	PSI=0.0	00018000
	OMY=0.0	00018100
C	START QUAD-ELEV LOOP	00018200
27	QE=QE0-QQE	00018300
	SDO=0.0	00018400
	CAL=0*1.E-3	00018500
	DO 3 IQE=1,NQE	00018600
	QE=QE+DQE	00018700
	THETA=QE/57.29578	00018800
	TD=1.E10	00018900
	EM=EM0	00019000
C	IEND IS A SWITCH SIGNALLING END OF BURNING	00019100
	IEND=0	00019200
C	ASSIGN TIME INCREMENT FOR INTEGRATION.	00019300
C		00019400
	DT=STEP	00019500
C	NABLE IS A SWITCH SIGNALING CONTROLS DEPLOYED FOR GUIDED FLIGHT.	00019600
	NABLE=0	00019700
C	PRINT AND LABEL RUN DESCRIPTION, CONSTANTS, AND	00019800
C	INITIAL CONDITIONS	00019900
	PRINT 9,TITLE,FFCTR,VD,EMO,EMB,D,QE,OT,FC,SPI,VWF,HD,HTERM,VCW	00020000
	1,RWC,XWC	00020100
9	FORMAT(1H120A4/1H010X5HFFCTR13X2HV013X2HMO13X2HMB14X1HD/	00020200
1	1H 5F15.6/1H0,	00020300
2	6X9HQDAD ELEV8X7HTM STEP9X6HTHRUST5X10HSP IMPULSE9X6HV=WIND/	00020400
3	1H 5F15.6/1H04X11HINIT ALT,FT4X11HTERM ALT,FT15H VEL XWIND,FT/S,	00020500
4	12X,3HRWC,12X,3HXWC,/1H 5F15.6)	00020600

	PRINT 91,VHXF,VHYF	00020700
91	FORMAT(78H VHXF = ,F12.2,20H FT/SEC VHYF = ,F12.2,7H FT/SEC, /	00020800
	1/)	00020900
	PRINT 92,DELT,TM	00021000
92	FORMAT(1H 19HTHRUST RISE TIME = ,F12.4,5H SEC,8H TM = ,F12.4,	00021100
	1 5H SEC)	00021200
C		00021300
C		00021400
C	YO=INITIAL ALTITUDE, M	00021500
C	HO=ALTITUDE READ IN IN FT	00021600
C	YTERM=TERMINAL ALTITUDE, M	00021700
C	HTERM=TERMINAL ALTITUDE READ IN IN FT	00021800
	IF (IOPTY,NE.1) GO TO 29	00021900
	PRINT 28,XCG,CLONG,AMOM,BMOM	00022000
28	FORMAT(1H0,11HLOC OF CG =,E10.4,2X,3HCAL,14H PROJ LENGTH =,E10.4,	00022100
	1 2X,3HCAL,15H AXIAL M OF I =,E11.5,2X,7HKG M**2,	00022200
	2 15H TRANS M OF I =,E11.5,2X,7HKG M**2)	00022300
29	IF(SPI,EQ.0.0) GO TO 60	00022400
	BRATE=FC/SPI	00022500
	IF(BRATE,EQ.0.0) GO TO 60	00022600
	TB=(EMO-EMB)/BRATE	00022700
	PRINT 40, TB	00022800
40	FORMAT(1H0,22HEFFECTIVE BURN TIME = ,F10.4,4H SEC)	00022900
	GO TO 61	00023000
60	TB=0.	00023100
	BRATE=0.	00023200
C		00023300
C	COMPUTE AUXILLIARY CONSTANTS AND REDIMENSION INPUTS	00023400
61	CALSQ=0**2*1.E-6	00023500
	DLONG=D*1.0E-3*CLONG	00023600
	VELO=0.3048*VO	00023700
C	SUPVEL IS THE SUPREMIUM OF PROJECTILE VELOCITY.	00023800
	SUPVEL=VELO	00023900
C	SUPALT IS SUPREMIUM OF PROJECTILE ALTITUDE.	00024000
	SUPALT=0.0	00024100
	VW=0.3048*VWF	00024200
	VCW=0.3048*VCW	00024300
	YO=0.3048*HO	00024400
	YTERM=0.3048*HTERM	00024500
	HTERM=RE*YTERM	00024600
C	INITIALIZE TIME, X, X-DOT, Y AND Y-DOT.	00024700
C		00024800
	T=0.	00024900
	X=0.	00025000
	Y=YO	00025100
	Z=0.	00025200
	SPIN=SPINO	00025300
	VHX=.3048*VHXF	00025400
	VHY=.3048*VHYF	00025500
	XD=VELO*COS(THETA)+VHX	00025600
	YO=VELO*SIN(THETA)+VHY	00025700
	THETA=57.29578*ATAN(YD/XD)	00025800

V=SQRT(XD**2+YD**2)	00025900
ZD=0.	00026000
SPIND=0.0	00026100
XDD=0.0	00026200
YDD=0.0	00026300
ZDD=0.0	00026400
YAW=0.0	00026500
ALT=Y0	00026600
SRNG=0.0	00026700
ISW=IBURN(T,ALT,THETA,V)	00026800
IF (ISW.EQ.1) GO TO 70	00026900
GO TO 71	00027000
70 TO=0.0	00027100
DT=STEP/4.	00027200
C	00027300
C INITIALIZE RUNGE-KUTTA SUBROUTINE	00027400
71 CALL RUNGE1(U,WP,4,2,FLIGHT)	00027500
C SOLVE FLIGHT EQUATIONS FOR INITIAL CONDITIONS.	00027600
C	00027700
C CALL FLIGHT(T,U,4)	00027800
C	00027900
C INITIALIZE COUNTER FOR DETERMINING NUMBER OF POINTS TO BE PLOTTED	00028000
C AT END OF TRAJECTORY SOLUTION.	00028100
C	00028200
C NPL0T=0	00028300
C	00028400
C	00028500
C INITIALIZE COUNTER FOR COUNTING LINES PER PAGE.	00028600
C	00028700
C LINE=0	00028800
C	00028900
C IPRINT=-NPRINT	00029000
C YAWDEG=YAW*57.3	00029100
C GO TO PRINT OUT INITIAL CONDITIONS.	00029200
C	00029300
C GO TO 4	00029400
C	00029500
C START OF SOLUTION LOOP. SAVE LAST VALUES OF FLIGHT VARIABLES.	00029600
C	00029700
5 XP=X	00029800
RP=R	00029900
ZP=Z	00030000
XDP=XD	00030100
YDP=YD	00030200
VP=V	00030300
THETAP=THETA	00030400
CMACHP=CMACH	00030500
RESISP=RESIS	00030600
YAWP=YAWDEG	00030700
SPINP=SPIN	00030800
SMARGP=SMARG	00030900
STAFAP=STAFAC	00031000

	PSIP=PSI	00031100
	SRNGP=SRNG	00031200
C		00031300
C	CALL RUNGE-KUTTA SUBROUTINE TO SOLVE FLIGHT EQUATIONS FROM	00031400
C	T TO T+OT.	00031500
C		00031600
	CALL RUNGE2(T,OT)	00031700
C		00031800
C		00031900
C		00032000
C	SAVE POSITIONAL COORDINATES OF PROJECTILE FOR LATER PLOTTING OF	00032100
C	TRAJECTORY.	00032200
C		00032300
C	SAVE RANGE AND FLIGHT TIME IN ARRAYS FOR SUBSEQUENT	00032400
C	POLYNOMIAL FIT	00032500
C		00032600
	SRNG=SQRT(X*X+(Y-YO)**2+Z*Z)	00032700
	IF(T.LT.TENABL) GO TO 520	00032800
	IF(NABLE,EQ.1) GO TO 520	00032900
	PRINT 530,T	00033000
530	FORMAT(1H0,18HENABLEMENT TIME = ,F10.3)	00033100
	NABLE=1	00033200
C	*****	00033300
C	IF (NPL0T,EQ.400) GO TO 520	00033400
C	IF (SRNG.GT.SRNGEM) GO TO 520	00033500
C	NPL0T=NPL0T+1	00033600
C	TS(NPL0T)=T	00033700
C	RS(NPL0T)=SRNG	00033800
C	VS(NPL0T)=V	00033900
C	*****	00034000
520	CONTINUE	00034100
C		00034200
	YAWOEG=YAW*57.3	00034300
	IF(V.GT.SUPVEL) SUPVEL=V	00034400
	IF(ALT.GT.SUPALT) SUPALT=ALT	00034500
88	IF(ISW,EQ.0) GO TO 50	00034600
	IF(IENO,EQ.1) GO TO 51	00034700
	IF(T.GE.TO+TB*DELT) GO TO 49	00034800
	GO TO 51	00034900
49	DT=STEP	00035000
	IENO=1	00035100
	CALL BURN(T,XMASS,THRUST)	00035200
	PRINT 80, XMASS,THRUST,ALT,V,T	00035300
80	FORMAT(1H0,9H MASS = ,F10.4,11H THRUST = ,F10.4,	00035400
1	13H ALTITUOE = ,F10.2,10H SPEEDO = ,F10.2,10H TIME = ,F10.3)	00035500
	GO TO 51	00035600
50	ISW=IBURN(T,ALT,THETA,V)	00035700
	IF(EMO,EQ.EMB) ISW=0	00035800
	IF(ISW,EQ.0) GO TO 51	00035900
	TO=T	00036000
	PRINT 20, TO	00036100
20	FORMAT(1H0,15HBURN STARTS AT ,F10.4,4H SEC)	00036200

	DT=STEP/4.	00036300
51	IPRINT=IPRINT+1	00036400
	IF (IPRINT.EQ.0) GO TO 53	00036500
	GO TO 52	00036600
53	IPRINT=-NPRINT	00036700
C		00036800
C	ADVANCE LINES COUNTER AND CHECK IF TIME TO EJECT PAGE AND LABEL.	00036900
C		00037000
	4 LINE=LINE+1	00037100
	DMY=YAWNU	00037200
C***	OMY IS A DUMMY VARIABLE USED FOR OUTPUT OF CHOICE	00037300
	IF (LINE.LE.0) GO TO 6	00037400
	LINE=-50	00037500
	PRINT 7,TITLE	00037600
	7 FORMAT (1H120A4/1H0,9HTIME,SECS,7X3HX,M5X5HALT,M,7X,3HZ,M2X,	00037700
	1 8HXDDT,M/52X8HYDDT,M/55X5HV,M/51X8HMACH NO,2X7HORAG,LB,	00037800
	2 9H THETA,D,9H YAW,0,9H SPIN,R/S,9H STAFAC ,8H DUMMY V)	00037900
	6 PRINT 8,T,X,ALT,Z,XO,YD,V,CMACH,RESIS,THETA,YAWDEG,SPIN,STAFAC,OMY	00038000
	8 FORMAT (1H 1F9.3,3F10.1,3F10.1,F9.2,2F9.1,4F9.2)	00038100
	IF (T.GT.300.) GO TO 30	00038200
C	RULE FOR STOPPING SOLUTION - STOP WHEN PROJECTILE HITS GROUND.	00038300
C		00038400
	52 IF (.NOT. (R.LE.RTERM.AND.THETA.LT.0.0)) GO TO 5	00038500
C	INTERPOLATE SOLUTION VARIABLES FOR R=RTERM	00038600
C		00038700
	YE=YTERM	00038800
	TE=T-DT*(R-RTERM)/(R-RP)	00038900
	DEL=(T-TE)/DT	00039000
	XE=X-DEL*(X-XP)	00039100
	ZE=Z-DEL*(Z-ZP)	00039200
	XOE=XO-OEL*(XO-XOP)	00039300
	YDE=YD-DEL*(YD-YDP)	00039400
	VE=V-OEL*(V-VP)	00039500
	THETA=THETA-OEL*(THETA-THETAP)	00039600
	CMACHE=CMACH-OEL*(CMACH-CMACHP)	00039700
	RESISE=RESIS-DEL*(RESIS-RESISP)	00039800
	YAW=YAWOEG-DEL*(YAWOEG-YAWP)	00039900
	SPINE=SPIN-DEL*(SPIN-SPINP)	00040000
	STAFAE=STAFAC-OEL*(STAFAC-STAFAP)	00040100
	SMARGE=SMARG-DEL*(SMARG-SMARGP)	00040200
	PSIE=PSI-DEL*(PSI-PSIP)	00040300
	SRNGE=SRNG-DEL*(SRNG-SRNGP)	00040400
C	XPLOT(NPLOT)=XE	00040500
C	YPLOT(1,NPLOT)=YE	00040600
C		00040700
C	PRINT OUT SOLUTION VARIABLES FOR Y=YTERM.	00040800
C		00040900
	PRINT 8,TE,XE,YE,ZE,XDE,YDE,VE,CMACHE,RESISE,THETA,YAW,	00041000
	1 SPINE,STAFAE,SRNGE	00041100
C		00041200
	SUPVEL=SUPVEL/0.3048	00041300
	RANGE=SQRT(XE**2+ZE**2)	00041400

	RANGE=RANGE*(1.+(RANGE/RE)**2/6.)/1852.	00041500
C	PRINT MAXIMUM VELOCITY, RANGE, AND ALTITUDE.	00041600
	PRINT 90, SUPVEL, RANGE,SUPALT	00041700
90	FORMAT(1H0,20HMAX PROJ VELOCITY = ,F15.4,4H F/S,	00041800
1	3X,12HMAX RANGE = ,F15.4,11H NAUT MILES,3X,10HMAX ALT = ,	00041900
2	F15.4,8H METERS)	00042000
C		00042100
C	PLOT THE TRAJECTORY JUST COMPUTED.	00042200
C		00042300
C	LABEL PLOT WITH TITLE, QE AND VO.	00042400
C		00042500
C		00042600
	PRINT 10,TITLE,QE,VO	00042700
10	FORMAT (1H015X20A4/4H QE=F5.I,10H DEG, VO=F6.I,4H F/S)	00042800
3	CONTINUE	00042900
C	*****	00043000
C	CALL POLFIT(RS,TS,NPLOT,3,0,CJRT,G,G,,TRUE,,SDEV,	00043100
C	1 20HFLIGHT TIME VS RANGE)	00043200
C	CALL POLFIT(RS,VS,NPLOT,3,0,CJVT,H,HINV,,TRUE,,SOEV,	00043300
C	2 20HPROJ. VEL. VS. RANGE)	00043400
C	RETURN FOR ANOTHER CASE.	00043500
C	*****	00043600
	GO TO 1	00043700
30	CALL EXIT	00043800
	ENO	00043900
	SUBROUTINE SOUND(A,G,RHO,VISCO)	00044000
	COMMON EMO,EMB,SPI,FC,BRATE,DEL,TB,ISW,V,THETA,FFCTR,CALSQ,	00044100
1	VW,VCW,ALT,R,IEND,CMACH,REYNLD,RESIS,CAL,OLONG,IOPTY,YAW,AMOM,	00044200
2	BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE	00044300
	COMMON/SNDCOM/CADENS	00044400
	EQUIVALENCE (Y,ALT)	00044500
C		00044600
C	SUBROUTINE COMPUTES THE SPEED OF SOUND IN M/SEC	00044700
C	VERSUS ALTITUDE IN METERS. ALSO COMPUTED IS THE	00044800
C	ACCELERATION DUE TO GRAVITY IN M/SEC/SEC AND THE	00044900
C	AIR DENSITY IN KG/M**3 AND THE ABSOLUTE VISCOSITY	00045000
C	OF THE AIR IN KG/M/SEC. NOTE THAT REYNOLD'S NUMBER	00045100
C	PER METER IS GIVEN BY A*RHO*EMACH/VISCO.	00045200
C		00045300
	G=9.826*(6.378E6/(6.378E6+Y))**2	00045400
	O=6.356766E6+Y	00045500
	IF(Y.LE.11019.07) GO TO 1	00045600
	IF(Y.LE.20063.12) GO TO 2	00045700
	IF(Y.LE.32161.9) GO TO 3	00045800
	IF(Y.LE.47350.09) GO TO 4	00045900
	IF(Y.LE.52428.88) GO TO 5	00046000
	IF(Y.LE.61591.03) GO TO 6	00046100
	IF(Y.LE.79994.14) GO TO 7	00046200
	RHO=0.4636*EXP(-0.12207E-3*Y)	00046300
C	T=TEMPERATURE IN DEGREES KELVIN	00046400
	T=180.65	00046500
8	A=20.053*SQRT(T)	00046600

	RHO=RHO*CAOENS	00046700
	VISCO=0.00467*(T+110.)*(T/217.78)**1.5	00046800
C	THIS IS THE SUTHERLAND VISCDSTY LAW.	00046900
	RETURN	00047000
1	RHO=1.224999+Y*(-.1176033E-3+Y*(.433719E-8+Y*(-.7461659E-13	00047100
	1+Y*(.5537603E-18-.9572727E-24*Y)))	00047200
	T=(1.831702E9-4.103083E4*Y)/O	00047300
	GO TO 8	00047400
2	RHO=1.990142+Y*(-.2940114E-3+Y*(.1993974E-7+Y*(-.7637263E-12	00047500
	1+Y*(.1615921E-16-.1476764E-21*Y)))	00047600
	T=216.65	00047700
	GO TO 8	00047800
3	RHO=1.81561+Y*(-.235749E-3+Y*(.130807E-7+Y*(-.3819651E-12	00047900
	1+Y*(.5798729E-17-.3626654E-22*Y)))	00048000
	T=(1.250058E9+6.553416E3*Y)/O	00048100
	GO TO 8	00048200
4	RHO=1.10944+Y*(-.1140029E-3+Y*(.4817401E-8+Y*(-.1039241E-12	00048300
	1+Y*(.1138793E-17-.5052135E-23*Y)))	00048400
	T=(8.839083E8+1.7938E4*Y)/O	00048500
	GO TO 8	00048600
5	RHO=.8974979E-1+Y*(-.417905E-5+Y*(.3529753E-10+Y*(.1177144E-14	00048700
	1+Y*(-.2567072E-19+.1449113E-24*Y)))	00048800
	T=270.65	00048900
	GO TO 8	00049000
6	RHO=.1029082E-1+Y*(.1081853E-5+Y*(-.8523619E-10+Y*(.2075003E-14	00049100
	1+Y*(-.2184824E-19+.860425E-25*Y)))	00049200
	T=(2.381562E9-1.233888E4*Y)/O	00049300
	GO TO 8	00049400
7	RHO=0.4636*EXP(-0.12207E-3*Y)	00049500
	T=(3.157088E9-2.493041E4*Y)/O	00049600
	GO TO 8	00049700
	END	00049800
	SUBROUTINE FLIGHT(TIME,U,KUTTA)	00049900
	DIMENSION U(12)	00050000
	DIMENSION TMACH(11),TKA(11),TKDYAW(11),TKL(11),TKM(11).	00050100
1	TKF(11),TKT(11),TKH(11),TKS(11),TCP(11)	00050200
	OATA RE/6.378E6/,OMEGA/0.72915E-4/,PIOFOR/.7853981/,TWOG/19.58418/	00050300
C	RE=NOMINAL RADIUS OF THE EARTH AT THE EQUATOR IN METERS	00050400
C	OMEGA=ANGULAR VELOCITY OF THE EARTH IN RADIAN/SEC	00050500
C		00050600
C	TABLE OF EQUIVALENCES	00050700
C	U(1) = X	00050800
C	U(2) = Y	00050900
C	U(3) = Z	00051000
C	U(4) = SPIN	00051100
C	U(5) = XDOT	00051200
C	U(6) = YOOT	00051300
C	U(7) = ZOOT	00051400
C	U(8) = SPIND	00051500
C	U(9) = XDBL	00051600
C	U(10) = YOBL	00051700
C	U(11) = ZDBL	00051800

C	U(12) = DUMMY	00051900
	EXTERNAL CDRAE	00052000
	COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,	00052100
	1 VWO,VXW,ALT,R,IEND,CMACH,REYNLD,RESIS,CAL,DLONG,TOPTY,YAW,AMOM,	00052200
	2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE	00052300
	COMMON/COFCOM/XCG,SMARG,EM,THID,PRNU,ALTRIM,CNATRM,GLIDE,EPSTHE	00052400
	1,STAFAC,YAWNU,TKA,TKOYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,TMACH,NARTBL	00052500
	COMMON/WINCOM/RWC,XWC	00052600
	IF (ISW.EQ.0) GO TO 10	00052700
	IF (TIME.GT.TO+TB+DELT) GO TO 9	00052800
	CALL BURN(TIME,XMASS,THRUST)	00052900
	EM=XMASS	00053000
C	THRUST-INDUCEO DRAG	00053100
	FFC=FFCTR*THID	00053200
	H=4.44823*THRUST	00053300
	TERMX=H*U(5)	00053400
	TERMY=H*U(6)	00053500
12	VSQ=U(5)**2+U(6)**2+U(7)**2	00053600
	V=SQRT(VSQ)	00053700
	UP=RE*U(2)	00053800
	XSQ=U(1)**2	00053900
	YSQ=UP**2	00054000
	ZSQ=U(3)**2	00054100
	R=SQRT(XSQ+YSQ+ZSQ)	00054200
C		00054300
C	ALT=R-RE	00054400
	ALT=U(2)+XSQ/2./RE	00054500
	DRCOSX=U(1)/R	00054600
	DRCOSY=UP/R	00054700
	DRCOSZ=U(3)/R	00054800
	CALL SOUND(A,G,RHO,VISCO)	00054900
C		00055000
C	THIS GENERATES SPEED OF SOUND, GRAVITY, AIR DENSITY, AND VISCOSITY.	00055100
	IF (V.EQ.0.0) GO TO 13	00055200
	TERMX=TERMX/V	00055300
	TERMY=TERMY/V	00055400
C*****	COMPUTE VELOCITY OF WIND AS A FUNCTION OF ALTITUDE.	00055500
C	HARG=1.000*U(2)	00055600
C	VW=VWO+RWC*VWC(HARG)	00055700
C	VCW=VXW+XWC*VWC(HARG)	00055800
	VW=VWO	00055900
	VCW=VXW	00056000
	VRELSQ=((U(5)+VW)**2+U(6)**2+(U(7)+VCW)**2)	00056100
	VREL=SQRT(VRELSQ)	00056200
	CMACH=VREL/A	00056300
C	COMPUTE REYNOLD'S NUMBER AND SKIN FRICTION COEFFICIENT (SFC).	00056400
	FRICT=0.0	00056500
	IF (DLONG.EQ.0.0) GO TO 48	00056600
	REYNLD=DLONG*A*RHO*CMACH/VISCO	00056700
	ALR=ALOG10(REYNLD)	00056800
	PWR=0.05*ALR	00056900
	SFC=0.455/ALR**2.58/(1.+0.2*CMACH**2)**PWR	00057000

	IF (ISEP.EQ.0) GO TO 48	00057100
	FRICT=WTAREA*5FC	00057200
C	DENOM=MASS OF PROJECTILE IN KG.	00057300
48	DENDM=0.4536*EM	00057400
	OYNPRS=0.5*RHO*VRELSQ	00057500
	IF (IOPTY.EQ.1) GO TO 20	00057600
	YAW=0.0	00057700
	YAWSQ=0.0	00057800
	XKDYAW=0.0	00057900
	XTERMS=0.0	00058000
	YTERMS=0.0	00058100
	ZTERMS=0.0	00058200
	U(8)=0.0	00058300
21	CALL CDRA(CMACH,DRAG,CAD)	00058400
	COFDRG=FFC*DRAG+XKDYAW*YAWSQ+FRICT+CAO	00058500
C	THIS GENERATES THE CORRECTED COEFFICIENT OF DRAG	00058600
C		00058700
C	DIFFERENTIAL EQUATIONS	00058800
22	CONTINUE	00058900
	FORM=PIOFDR*CALSQ*OYNPRS	00059000
	DRG=-COFDRG*FORM	00059100
C		00059200
C	RESIS IS AIR RESISTANCE IN POUNDS.	00059300
	RESIS=DRG /4.44823	00059400
C****	PROVISIONAL CONSTANT GLIDE ANGLE JAN 75	00059500
	IF (IGLIDE.NE.1) GO TO 60	00059600
	THE=ATAN(U(6)/U(5))	00059700
	IF (NABLE.EQ.1) GO TO 62	00059800
	IF (THE.LE.GLIDE) NABLE=1	00059900
	HERR=ABS(THE-GLIDE)	00060000
	IF (HERR.LE.EPSTHE) NABLE=1	00060100
	IF (NABLE.NE.1) GO TO 60	00060200
62	SAVE=0.5*ARSIN(DENOM*TWOG*COS(GLIDE)/FORM/CNATRM)	00060300
	ALPH=AMINI(ALTRIM,SAVE)	00060400
	SAVE=CNATRM*FORM	00060500
	U(4)=SAVE*SIN(ALPH)/DENOM	00060600
C****	U(4) IS COMPUTED AS NORMAL ACCELERATION (M/S**2) INSTEAD OF SPIN.	00060700
	YAW=ALPH	00060800
	FL=0.5*SAVE*SIN(2.*ALPH)	00060900
	FDI=-SAVE*(SIN(ALPH))**2	00061000
	SINTH=U(6)/V	00061100
	CDSTH=U(5)/V	00061200
	TERMX=TERMX-FL*SINTH+FDI*CDSTH	00061300
	TERMY=TERMY-FL*CDSTH+FDI*SINTH	00061400
60	CONTINUE	00061500
C****	PROVISIONAL CONSTANT GLIDE ANGLE JAN 75	00061600
	U(10)=(DRG*U(6)/VREL+TERMY)/DENOM-G*DRCOSY+0.53166E-8*UP	00061700
1	+2.*OMEGA*U(7)+YTERMS	00061800
	U(9)=(DRG*(U(5)+VW)/VREL+TERMX)/DENOM-G*DRCOSX+XTERMS	00061900
	U(11)=-2.*OMEGA*U(6)-G*DRCOSZ+ZTERMS+DRG*(U(7)+VCW)/VREL/DENOM	00062000
	U(12)=0.0	00062100
C	AC IS THE ACCELERATION OF THE PROJECTILE ALONG THE TRAJECTORY.	00062200

C	AC=(U(5)*U(9)+U(6)*U(10)+U(7)*U(11))/V	00062300
	IF(IOPTY.EQ.0) GO TO 14	00062400
	U(8)=-RHO*CALSQ**2/AMOM*XKA*U(4)*V	00062500
14	IF(KUTTA.EQ.4) THETA=ARSIN(U(6)/V)*57.3	00062600
	RETURN	00062700
13	U(9)=0.	00062800
	U(10)=-G	00062900
	THETA=90.	00063000
	RETURN	00063100
9	EM=EMB	00063200
	GO TO 11	00063300
10	EM=EMO	00063400
11	TERMX=0.	00063500
	TERMY=0.	00063600
	FFC=FFCTR	00063700
	GO TO 12	00063800
20	CALL ACOEFS(CMACH,YAW,XKA,XKDYAW,XKL,XKM,XKF,XKT,	00063900
	1XKH,XKS,XCP)	00064000
	VXRL=U(5)+VW	00064100
	VZRL=U(7)+VCW	00064200
	VRLSQ=VXRL**2+U(6)**2+VZRL**2	00064300
	VRL=SQRT(VRLSQ)	00064400
C	TEST FOR DYNAMIC STABILITY.	00064500
	BOTTOM=8.0*BMOM*DYNPRS*CAL**3*XKM	00064600
	STAFAC=(U(4)*AMOM)**2/BOTTOM	00064700
	IF(STAFAC.LE.1.0) GO TO 25	00064800
C****	COMPUTE THE YAWING FREQUENCY	00064900
	YAWNU=AMOM/BMOM*SQRT(1.-1./STAFAC)*U(4)/6.2832	00065000
C****	COMPUTE THE OVERTURNING MOMENT AND PRECESSIONAL FREQUENCY	00065100
	OTNMOM=2.*XKM*CAL**3*DYNPRS	00065200
	PRNU=OTNMOM/AMOM/U(4)/6.2832	00065300
C	COMPUTE YAW OF REPOSE.	00065400
	ALPHAB=RHO*CAL*VRLSQ*(XKL*XKM*VRLSQ*CALSQ*XKF*XKT*U(4)**2)	00065500
	IF(ABS(ALPHAB).LT.1.E-20) GO TO 25	00065600
	ALPHAA=AMOM*XKL*U(4)/CALSQ/ALPHAB	00065700
	ALPHAB=DENOM*XKT*U(4)/ALPHAB	00065800
	AMB=ALPHAB-ALPHAA	00065900
	ALPHAX=AMB*(U(6)*U(11)-VZRL*U(10))-ALPHAB*VZRL*G	00066000
	ALPHAY=AMB*(VZRL*U(9)-VXRL*U(11))	00066100
	ALPHAZ=AMB*(VXRL*U(10)-U(6)*U(9))+ALPHAB*VXRL*G	00066200
	YAWSQ=ALPHAX**2+ALPHAY**2+ALPHAZ**2	00066300
	YAW=SQRT(YAWSQ)	00066400
	IF(YAW.GT.1.5708) GO TO 25	00066500
	ARG=(VXRL*ALPHAZ-VZRL*ALPHAX)*VRL	00066600
	ARG1=(VXRL*ALPHAY-U(6)*ALPHAX)*VXRL-(U(6)*ALPHAZ-VZRL*ALPHAY)*VZRL	00066700
	IF(ABS(ARG1).LE.1.0E-20) GO TO 50	00066800
	PSI=57.3*ATAN(ARG/ARG1)	00066900
	GO TO 53	00067000
50	IF(ARG*ARG1) 51,52,52	00067100
51	PSI=-90.	00067200
	GO TO 53	00067300
52	PSI=90.	00067400

53	CONTINUE	00067500
C		00067600
C	PSI=ORIENTATION OF YAW. THIS IS THE ANGLE BETWEEN THE PLANE	00067700
C	CONTAINING BOTH THE VELOCITY AND YAW VECTORS AND A VERTICAL	00067800
C	PLANE CONTAINING THE VELOCITY VECTOR. IT IS MEASURED	00067900
C	CLOCKWISE FROM THE VERTICAL PLANE.	00068000
C		00068100
C	END OF COMPUTATION OF YAW.	00068200
	OKFN=CAL*VKF*U(4)	00068300
	XKLVSQ=XKL*VRLSQ	00068400
	RDSQ=RHO*CALSQ/OENOM	00068500
	XTERMS=RDSQ*(XKLVSQ*ALPHAX*OKFN*(ALPHAY*VZRL-ALPHAZ*U(6)))	00068600
	YTERMS=RDSQ*(XKLVSQ*ALPHAY*OKFN*(ALPHAZ*VXRL-ALPHAX*VZRL))	00068700
	ZTERMS=RDSQ*(XKLVSQ*ALPHAZ*OKFN*(ALPHAX*U(6)-ALPHAY*VXRL))	00068800
	XKDYAW=2.54647*XKDYAW	00068900
C	*****	00069000
	IF(YAW.LT.0.69) GO TO 21	00069100
	PRINT 55,RHO,XTERMS,YTERMS,ZTERMS	00069200
55	FORMAT(1H,1P4E10.5)	00069300
C	*****	00069400
	GO TO 21	00069500
25	PRINT 26,STAFAC	00069600
26	FORMAT(1H0,40HUNSTABLE PROJECTILE STABILITY FACTOR = ,F10.4)	00069700
	CALL EXIT	00069800
	END	00069900
	SUBROUTINE CDRAG(EMACH,DAG,CAD)	00070000
C		00070100
C	PROGRAM COMPUTES THE COEFFICIENT OF DRAG VERSUS MACH NUMBER	00070200
C	AND THE THE COEFFICIENT INCREMENT DUE TO CANAROS.	00070300
C		00070400
	OMENSION XMTBL(11),COTBL(11),COTBL(11)	00070500
	COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,	00070600
1	VW,VCW,ALT,R,IENO,CMACH,REYNLO,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,	00070700
2	BMOH,PSI,WTAREA,ISEP,NABLE,IGLIDE	00070800
	COMMON/ORCOM/ XMTBL,CDTBL,COTBL,NTBL	00070900
	IF(NTBL.NE.0) GO TO 5	00071000
	IF(EMACH.LE.0.80) GO TO 1	00071100
	IF(EMACH.LE.1.10) GO TO 3	00071200
	IF(EMACH.LE.3.0) GO TO 4	00071300
	EM3=EMACH-3.0	00071400
	DAG=0.09+EM3*(-0.02+0.002*EM3)	00071500
	RETURN	00071600
1	ORAG=0.0589	00071700
	RETURN	00071800
3	C=10.*(EMACH-0.8)	00071900
	ORAG=0.07736*C**3*EXP(-C)+0.0589	00072000
	RETURN	00072100
4	DAG=0.21547*EMACH*(-0.05134+0.00317*EMACH)	00072200
5	OO 6 J=1,NTBL	00072300
	IF(EMACH.LT.XMTBL(J)) GO TO 8	00072400
6	CONTINUE	00072500
8	JL=J-1	00072600

FRAC=(EMACH-XMTBL(JL))/(XMTBL(J)-XMTBL(JL))	00072700
CD=CDTBL(JL)+(CDTBL(J)-CDTBL(JL))*FRAC	00072800
CAO=0.0	00072900
IF(NABLE.NE.1) GO TO 10	00073000
CAO=COTBL(JL)+(COTBL(J)-COTBL(JL))*FRAC	00073100
10 CONTINUE	00073200
ORAG=CO	00073300
CAO=CAO	00073400
RETURN	00073500
ENO	00073600
SUBROUTINE ACOEFS(EMACH,YAW,XKA,XKOYAW,XKL,XKM,XKF,XKT,XKH,XKS,XCP	00073700
1)	00073800
DIMENSION TMACH(11),TKA(11),TKDYAW(11),TKL(11),TKM(11),TKF(11),	00073900
1 TKT(11),TKH(11),TKS(11),TCP(11)	00074000
COMMON/COFCOM/XCG,SMARG,EM,THIO,PRNU,ALTRIM,CNATRM,OLIOE,EPSTHE	00074100
1,STAFAC,YAWNU,TKA,TKDYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,TMACH,NARTBL	00074200
C EMACH = MACH NUMBER	00074300
C XKA = SPIN DAMPING MOMENT COEFFICIENT	00074400
C XKDYAW = YAW DRAG COEFFICIENT	00074500
C XKL = LIFT FORCE COEFFICIENT	00074600
C XKM = OVERTURNING MOMENT COEFFICIENT	00074700
C XKF = MAGNUS FORCE COEFFICIENT	00074800
C XKT = MAGNUS MOMENT COEFFICIENT	00074900
C XKH = DAMPING MOMENT COEFFICIENT	00075000
C XKS = PITCHING FORCE COEFFICIENT	00075100
C XCP = CENTER OF PRESSURE AFT OF NOSE IN CALIBERS	00075200
C	00075300
C FOR DEPENDENCE OF ACOEFS UPON YAW SEE BRL MEMO. RPT. NO. 2023	00075400
C RELATIVE TO T387 TYPE PROJECTILE.	00075500
C XKT=-0.14+0.0576*(EMACH-1.25)**2	00075600
XKT=0.0	00075700
XKF=0.157	00075800
SYAW=SIN(YAW)**2	00075900
GO TO 50	00076000
51 CONTINUE	00076100
DO 60 J=1,NARTBL	00076200
IF(EMACH.LT.TMACH(J)) GO TO 70	00076300
60 CONTINUE	00076400
70 JL=J-1	00076500
FRAC=(EMACH-TMACH(JL))/(TMACH(J)-TMACH(JL))	00076600
IF(TKA(J).EQ.0.0) GO TO 52	00076700
XKA=TKA(JL)+(TKA(J)-TKA(JL))*FRAC	00076800
52 IF(TKOYAW(J).EQ.0.0) GO TO 53	00076900
XKDYAW=TKDYAW(JL)+(TKDYAW(J)-TKDYAW(JL))*FRAC	00077000
53 IF(TKL(J).EQ.0.0) GO TO 54	00077100
XKL=TKL(JL)+(TKL(J)-TKL(JL))*FRAC	00077200
54 IF(TKM(J).EQ.0.0) GO TO 55	00077300
XKM=TKM(JL)+(TKM(J)-TKM(JL))*FRAC	00077400
55 IF(TKF(J).EQ.0.0) GO TO 56	00077500
XKF=TKF(JL)+(TKF(J)-TKF(JL))*FRAC	00077600
56 IF(TKT(J).EQ.0.0) GO TO 57	00077700
XKT=TKT(JL)+(TKT(J)-TKT(JL))*FRAC	00077800

57	IF(TKH(J).EQ.0.0) GO TO 58	00077900
	XKH=TKH(JL)*(TKH(J)-TKH(JL))*FRAC	00078000
58	IF(TKS(J).EQ.0.0) GO TO 59	00078100
	XKS=TKS(JL)*(TKS(J)-TKS(JL))*FRAC	00078200
59	IF(TKM(J).EQ.0.0) GO TO 62	00078300
	IF(XKL.EQ.0.0) CALL EXIT	00078400
	SMARG=-XKM/XKL	00078500
	XCP=XCG+SMARG	00078600
	RETURN	00078700
62	IF(TCP(J).EQ.0.0) GO TO 63	00078800
	XCP=TCP(JL)*(TCP(J)-TCP(JL))*FRAC	00078900
63	SMARG=XCP-XCG	00079000
	XKM=-XKL*SMARG	00079100
	RETURN	00079200
50	CONTINUE	00079300
	IF(EMACH.LE.0.8) GO TO 10	00079400
	IF(EMACH.LE.0.9) GO TO 20	00079500
	IF(EMACH.LE.1.0) GO TO 30	00079600
	IF(EMACH.LE.1.1) GO TO 35	00079700
	IF(EMACH.LE.1.30) GO TO 40	00079800
	IF(EMACH.GT.1.5) GO TO 45	00079900
C	VALID FOR EMACH GTR 0.8 AND LT 1.5	00080000
5	XKA=0.0038+0.002*EXP(-1.5*(EMACH-0.8))	00080100
C	VALID FOR EMACH GTR 0.9	00080200
6	EM9=EMACH-0.9	00080300
	HOLD=1.-EXP(-5.*EM9)	00080400
	XKL=0.5507+0.4*HOLD	00080500
	XKL=XKL+6.6*SYAW	00080600
	XCP=0.237+1.57*HOLD	00080700
C	VALID FOR EMACH GTR 1.0	00080800
7	XKS=-4.0+1.78*(EMACH-1.)	00080900
C	VALID FOR EMACH GTR 1.1	00081000
	XKDYAW=1.5+2.38*EXP(-2.72*(EMACH-1.1))	00081100
C	VALID FOR EMACH GTR 1.3	00081200
	XKH=3.7	00081300
C	VALID FOR ALL EMACH	00081400
9	SMARG=XCP-XCG	00081500
	XKM=-XKL*SMARG	00081600
	IF(NARTBL.NE.0) GO TO 51	00081700
	RETURN	00081800
10	XKA=0.0058	00081900
	XKDYAW=1.5	00082000
11	XKL=0.62-0.077*EMACH	00082100
	XKL=XKL+4.3*SYAW	00082200
	XCP=1.2-1.07*EMACH	00082300
12	XKS=-4.0	00082400
13	XKH=0.71+2.3*EMACH	00082500
	GO TO 9	00082600
20	EM8=EMACH-0.8	00082700
	XKA=0.0038+0.002*EXP(-1.5*EM8)	00082800
	XKDYAW=1.5+2.5*SIN(6.283*EM8)	00082900
	GO TO 11	00083000

30	EM8=EMACH-0.8	00083100
	XKA=0.0038*0.002*EXP(-1.5*EM8)	00083200
	XKDYAW=1.5+2.5*SIN(6.283*EM8)	00083300
	EM9=EMACH-0.9	00083400
	HOLD=1.-EXP(-5.*EM9)	00083500
	XKL=0.5507*0.4*HOLD	00083600
	XKL=XKL+5.5*SYAW	00083700
	XCP=0.237+1.57*HOLD	00083800
	GO TO 12	00083900
35	EM8=EMACH-0.8	00084000
	XKA=0.0038*0.002*EXP(-1.5*EM8)	00084100
	XKDYAW=1.5+2.5*SIN(6.283*EM8)	00084200
	EM9=EMACH-0.9	00084300
	HOLD=1.-EXP(-5.*EM9)	00084400
	XKL=0.5507*0.4*HOLD	00084500
	XKL=XKL+5.5*SYAW	00084600
	XCP=0.237+1.57*HOLD	00084700
	XKS=-5.78+1.78*EMACH	00084800
	GO TO 13	00084900
40	EM8=EMACH-0.8	00085000
	XKA=0.0038*0.002*EXP(-1.5*EM8)	00085100
	XKDYAW=1.5+2.38*EXP(-2.72*(EMACH-1.1))	00085200
	EM9=EMACH-0.9	00085300
	HOLD=1.-EXP(-5.*EM9)	00085400
	XKL=0.5507*0.4*HOLD	00085500
	XKL=XKL+6.6*SYAW	00085600
	XCP=0.237+1.57*HOLD	00085700
	XKS=-5.78+1.78*EMACH	00085800
	GO TO 13	00085900
45	XKA=0.0038*0.002*EXP(-1.5*(EMACH-0.8))	00086000
	GO TO 6	00086100
	END	00086200
	SUBROUTINE RUNGE1(V,W,NEQ,NORD,DIFEQ)	00086300
	DIMENSION V(12),W(48)	00086400
	NV=NEQ*NORD	00086500
	N=N*NEQ	00086600
	RETURN	00086700
C		00086800
	ENTRY RUNGE2(T,DT)	00086900
	DT2=DT*.5	00087000
	DT6=DT/6.	00087100
	DO 1 I=1,N	00087200
1	W(I)=V(I)	00087300
	DO 2 J=1,3	00087400
	NJM=NEQ+N*(J-1)	00087500
	JDECK=N*J	00087600
	IF (J-3)3,4,4	00087700
3	DTW=DT2	00087800
	GO TO 5	00087900
4	DTW=DT	00088000
5	TW=T+DTW	00088100
	DO 6 I=1,NV	00088200

	K=I+JDECK	00088300
	L=I+NIJH	00088400
	W(K)=W(I)+W(L)*DTW	00088500
6	V(I)=W(K)	00088600
	CALL DIFEQ(TW,V,J)	00088700
	DO 2 I=1,N	00088800
	K=I+JOECK	00088900
2	W(K)=V(I)	00089000
	DO 7 I=1,NV	00089100
	K1=I+NEQ	00089200
	K2=K1+N	00089300
	K3=K2+N	00089400
	K4=K3+N	00089500
7	V(I)=W(I)+DTW*(W(K1)+2.*(W(K2)+W(K3))+W(K4))	00089600
	T=TW	00089700
	CALL DIFEQ(T,V,4)	00089800
	RETURN	00089900
	END	00090000
	FUNCTION VWC(Y)	00090100
	IF(Y.GE.6700.)GOTO3	00090200
	VWC=5.1816+4.972E-5*Y+1.3494E-7*Y*Y	00090300
	RETURN	00090400
3	VWC=10.058+3.9624*COS(.42946E-3*(Y-9448.8))	00090500
	IF(Y.GT.13700.)WRITE(6,1)	00090600
	RETURN	00090700
1	FORMAT(28H ALTITUDE ABOVE 13700 METERS)	00090800
	END	00090900
	SUBROUTINE BURN(TIME,XMASS,THRUST)	00091000
C	SUBROUTINE COMPUTES PROJECTILE MASS IN POUNDS MASS AND	00091100
C	ROCKET THRUST IN POUNDS FORCE	00091200
C	TO=TIME AT WHICH BURNING COMMENCES	00091300
C	EMO=INITIAL MASS, LBM	00091400
C	EMB=BURNT MASS, LBM	00091500
C	TIME=TIME AFTER LAUNCH, SEC	00091600
C	SPI=SPECIFIC IMPULSE, LBF/LBM/SEC	00091700
C	FC=CONSTANT NOMINAL THRUST LEVEL, LBF	00091800
C	IBURN= INDICATOR OF COMMENCEMENT OF BURNING(1BURN=1)	00091900
C	DELT=RISE TIME OF THRUST--ASSUMED EQUAL TO DECAY TIME, SEC	00092000
C	BRATE=FC/SPI=BURNING RATE, LBM/SEC	00092100
C	TB=(EMO-EMB)/BRATE=EFFECTIVE BURNING TIME, SEC	00092200
C		00092300
	COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,	00092400
1	VW,VCH,ALT,R,LEND,CMACH,REYNLO,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,	00092500
2	BMOM,PSI,WTAREA,ISEP,NABLE,IGLYDE	00092600
	IF(TIME.LE.TO) GO TO 1	00092700
	IF(TIME.LE.TO+OELT) GO TO 2	00092800
	T2=TO+TB	00092900
	IF(TIME.LE.T2) GO TO 3	00093000
	IF(TIME.LT.T2+OELT) GO TO 4	00093100
	THRUST=0.	00093200
	XMASS=EMB	00093300
	RETURN	00093400

1	XMASS=EMO	00093500
	THRUST=0.	00093600
	RETURN	00093700
2	XMASS=EMO-BRATE*(TIME-T0)**2/(2.*DELT)	00093800
	THRUST=(TIME-T0)/DELT*FC	00093900
	RETURN	00094000
3	XMASS=EMO-BRATE*(TIME-T0-DELT/2.)	00094100
	THRUST=FC	00094200
	RETURN	00094300
4	XMASS=EMO-BRATE*((TB-DELT/2.)+(TIME-T2)*(1.-(TIME-T2)/DELT/2.))	00094400
	THRUST=FC*(1.-(TIME-T2)/DELT)	00094500
	RETURN	00094600
	END	00094700
	FUNCTION IBURN(TIME,ALT,QE,VELO)	00094800
C		00094900
C	FUNCTION PRODUCES INDICATION OF COMMENCEMENT OF	00095000
C	BURNING--IBURN=1.	00095100
C	IBURN=0 UNTIL BURN BEGINS	00095200
C	USER CHOOSES TO FROM CURRENT TIME OR ALT (ALTITUDE) OR	00095300
C	QE (LOCAL QUADRANT ELEVATION) OR VELO (VELOCITY)	00095400
	COMMON /SRCOM/TM	00095500
	DATA ALTMAX/30000.,VMIN/0.0/	00095600
	IF (TIME.GE.TM) GO TO 3	00095700
	IF (ALT.GE.ALTMAX) GO TO 3	00095800
	IF (VELO.LE.VMIN) GO TO 3	00095900
C	IF (QE.GT.45.0) GO TO 2	00096000
	IBURN=0	00096100
	RETURN	00096200
C	2 IF (QE.LT.46.0) GO TO 3	00096300
C	4 IBURN=0	00096400
C	RETURN	00096500
	3 IBURN=1	00096600
	RETURN	00096700
	END	00096800

IN NM DIRECTORY. TTR IS NOW ALTERED.
0000000

AERO DATA (BRL CALC) FOR M509 8 IN ICM PROJECTILE

MACH NO	COEF DRAG	DRAG INCR
0.0	0.1300	0.0
0.7500	0.1300	0.0
0.8500	0.1400	0.0
0.9000	0.1550	0.0
1.0000	0.3000	0.0
1.0500	0.3600	0.0
1.1000	0.3600	0.0
1.5000	0.3170	0.0
2.0000	0.2740	0.0
2.5000	0.2390	0.0

ENABLE TIME = 100.0000 THRUST DRAG FACTOR = 1.0000 AIR DENS FACTOR = 1.0000

EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

FFCTR	VO	MO	MB	D
1.00000	10+0.0000	205.899994	205.899994	203.199997
QUAD ELEV	TM STEP	THRUST	SP IMPULSE	V-WIND
12.00000	0.10000	0.0	0.0	0.0
INIT ALT, FT	TERM ALT, FT	VEL XWIND, FT/S	RWC	XWC
0.0	0.0	0.0	0.0	0.0
VHXF =	0.0 FT/SEC	VHYF =	0.0 FT/SEC	
THRUST RISE TIME =	0.0	SEC	TM =	100.0000 SEC
LOC OF CG = 0.3577E+01 CAL PROJ LENGTH = 0.5673E+01 CAL AXIAL M OF I = 0.54700E+00 KG M**2 TRANS M OF I = 0.47676E+01 KG M**2				

EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

TIME, SECS	X, M	ALT, M	Z, M	XDOT, M/S	YDOT, M/S	V, M/S	MACH NO.	DRA, LB	THETA, D	YAW, D	SPIN, R/S	STA	FAC	DUMMY V
0.0	0.0	0.0	0.0	310.1	65.9	317.0	0.93	-89.9	12.0	0.0	490.00	1.93		6.20
1.000	308.1	60.6	0.1	306.2	55.4	311.2	0.91	-75.9	10.2	0.26	486.81	2.01		6.30
2.000	612.6	110.8	0.2	302.9	45.0	306.2	0.90	-64.7	8.5	0.27	483.69	2.09		6.38
3.000	914.0	150.8	0.6	299.9	34.9	302.0	0.89	-60.9	6.6	0.29	480.63	2.14		6.40
4.000	1212.5	180.7	1.0	297.1	24.8	298.2	0.88	-57.9	4.8	0.30	477.62	2.18		6.41
5.000	1508.3	200.6	1.6	294.4	14.9	294.8	0.87	-55.4	2.9	0.31	474.66	2.21		6.41
6.000	1801.4	210.6	2.2	291.8	5.0	291.9	0.86	-53.3	1.0	0.32	471.74	2.23		6.40
7.000	2092.0	210.8	3.1	289.3	-4.8	289.4	0.85	-51.6	-0.9	0.32	468.86	2.24		6.37
8.000	2380.1	201.2	4.0	286.9	-14.5	287.3	0.85	-50.5	-2.9	0.33	466.00	2.25		6.34
9.000	2665.8	182.0	5.1	284.5	-24.1	285.5	0.84	-49.8	-4.8	0.33	463.17	2.25		6.30
10.000	2949.1	153.3	6.3	282.2	-33.6	284.2	0.84	-49.3	-6.8	0.33	460.36	2.24		6.25
11.000	3230.1	115.1	7.7	279.8	-43.1	283.1	0.83	-49.0	-8.8	0.33	457.56	2.22		6.19
12.000	3508.8	67.4	9.2	277.5	-52.5	282.5	0.83	-48.9	-10.7	0.33	454.77	2.19		6.12
13.000	3785.2	10.4	10.8	275.3	-61.8	282.1	0.83	-49.0	-12.7	0.32	451.99	2.16		6.05
13.133	3821.9	0.0	11.0	274.9	-63.0	282.1	0.83	-49.0	-12.9	0.32	451.62	2.16		3821.91

MAX PROJ VELOCITY = 1039.9998 F/S MAX RANGE = 2.0637 NAUT MILES MAX ALT = 211.8979 METERS

EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

QE= 12.0 DEG, V0=1040.0 F/S

Computer Program for the Gyroscopic Dynamics
of Projectiles Having Oscillating Inertial Properties

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C      GYRO DYNAMICS
C**** THIS PROGRAM PRODUCES A NUMERICAL INTEGRATION OF THE
C**** DIFFERENTIAL EQUATIONS DESCRIBING THE DYNAMICS OF A SPIN-
C**** STABILIZED PROJECTILE HAVING OSCILLATING CENTER OF PRESSURE
C**** AND TRANSVERSE MOMENT OF INERTIA. THE PROGRAM EMPLOYS A
C**** DOUBLE-PRECISION VERSION OF THE FOURTH-ORDER RUNGE-KUTTA
C**** METHOD IN SUBROUTINE DKUTTA. THE DIFFERENTIAL EQUATIONS
C**** ARE LOCATED IN SUBROUTINE DIFEQ.
C****
C**** IMPLICIT REAL *8 (A-H,O-Z)
C**** DIMENSION TITLE(20),UT(8),WP(32),X(4,2)
C**** COMMON CALIB,CNA,CMQ,CD,OCG,OJ,DCG,DJ,C31,C32,C33,C34,CP,
C**** 1 OMEGA,DOMEGA,PMASS
C**** READ TITLE AND INPUT PARAMS
C****
C**** INPUT VARIABLES:
C**** TITLE = ALPHAMERIC DESCRIPTION OF SYSTEM BEING STUDIED
C**** DT = INTEGRATION TIME STEP (SEC)
C**** TMAX = MAXIMUM TIME PERMITTED FOR SOLUTION (SEC)
C**** THETA0 = INITIAL VALUE OF PITCH (RAD)
C**** PSIO = INITIAL VALUE OF YAW (RAD)
C**** THED0 = INITIAL VALUE OF PITCH RATE (RAD/SEC)
C**** PSID0 = INITIAL VALUE OF YAW RATE (RAD/SEC)
C**** TORQUE = EXTERNAL TORQUE APPLIED IN PITCH (DYNE CM)
C**** NPINT = NUMBER OF TIME STEPS BETWEEN PRINTOUTS
C**** ISW = SWITCH FOR OPTION NOT EXERCISED IN PRESENT VERSION
C**** FREQ = THE CONSTANT PART OF THE FORCING FREQUENCY (HZ)
C**** DFREQ = THE SWEEP RATE OR FIRST TIME-DERIVATIVE OF THE
C**** FORCING FREQUENCY (HZ/SEC)
C**** TS = SAMPLING INTERVAL USED IN THE SEMIANALYTIC MODEL (SEC)
C**** SEE NOMENCLATURE BELOW FOR PROJECTILE PARAMETERS.
C**** ALL DIMENSIONS MUST BE IN A RATIONALIZED SET OF UNITS SUCH AS:
C**** LENGTH IN FT, MASS IN SLUGS, FORCE IN POUNDS, ANG TIME IN SEC.
C**** READ (5,100) TITLE,DT,TMAX,THETA0,PSIO,THED0,PSID0,TORQUE,NPINT,
C**** 1 ISW
C**** 100 FORMAT (20A4/7F10.0,2I2)
C**** READ (5,110) FREQ,DFREQ,TS
C**** 110 FORMAT(3F10.0)
C**** 200 FOKMAT (IMI,20A4/IM0,11TIME STEP =,E15.5,2X,3HSEC,10M TORQUE =,
C**** 1 E15.5,2X,7HDYNE CM)
C**** 115 FOKMAT(1H,5X,7HFREQ =,F10.2,6H HZ ,7HDFREQ =,F10.2,6H HZ/S ,
C**** 1 IUSAMP INT =,F10.3,2X,3HSEC)
C**** 400 FOKMAT(1H,6F15.6)
C**** 121 FOKMAT(8F10.0)
C**** READ (5,121) CALIB,PMASS,AXMI,OJ,DJ,V0,SPIN,RHO,
C**** 1 CU,CNA,CMQ,CMPA,CP,OCG,DCG
C**** WRITE (6,122) CALIB,PMASS,AXMI,OJ,DJ,V0,SPIN,RHO,
C**** 1 CU,CNA,CMQ,CMPA,CP,OCG,DCG
C**** 122 FOKMAT(1H0,5X,5HICALIB,5X,5HPMASS,5X,5HAX MI,2X,9H V0 MI,2X,
C**** 1 4HINC T MI,8X,2H V0,6X,4HSPIN,7X,3H RHO,7F10.4,1F10.6/
C**** 2 8X,2HCD,7X,3HCNA,1X,3HCMQ,6X,4HCMCPA,8X,2HCP,4X,6H V0 CG,

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3 4X,6HINC CG/7F10.4)
C*** COMPUTE CONSTANTS IN DIFFERENTIAL EQUATIONS
0017 AREA=0.7853982*CALIB**2
0018 OYNPRS=0.5*RHO*V0**2
0019 C31=AREA*OYNPRS*CALIB
0020 GYRXS=AXMI/PMASS*CALIB/CALIR
0021 C32=-SPIN/V0*CALIB*GYRXS/C31*(CNA-CO*CMPA/GYRXS)
0022 C33=-AREA*OYNPRS/V0/PMASS
0023 C34=-AXMI*SPIN
0024 CMA=CNA*(UCG-CP)
0025 STAFAC=C34**2/4./OJ/C31/CMA
C*** THE HIGH- AND LOW OSCILLATORY MODES OF THE PROJECTILE ARE
C*** TERMED FNUI AND FPREC, RESPECTIVELY.
C*** THE FREQUENCY WITH WHICH YAW MAXIMA OCCUR IS DENOTED FOIF.
0026 SIGMA=DSQRT(1.-I./STAFAC)
0027 SAVE=0.0795775*AXMI/OJ*SPIN
0028 FNUT=SAVE*(1.+SIGMA)
0029 FPREC=SAVE*(1.-SIGMA)
0030 FOIF=FNUT-FPREC
0031 WHITE (6,124) STAFAC,FNUT,FPREC,FOIF
0032 I 13HUTAT FREQ = ,F10.3/3X,
I 13HUTAT FREQ = ,F10.3,17H HZ PREC FREQ = ,F10.3,
2 16H HZ DIFF FREQ = ,F10.3,3H HZ)
C*** NOMENCLATURE:
C*** CALIB IS REFERENCE DIAMETER OF THE PROJECTILE
C*** AREA IS THE PROJECTILE REFERENCE AREA
C*** PMASS IS THE MASS OF THE PROJECTILE
C*** AXMI IS THE AXIAL MOMENT OF INERTIA
C*** OJ IS THE NOMINAL TRANSVERSE MOMENT OF INERTIA
C*** DJ IS THE AMPLITUDE OF THE EXCURSION IN THE TRANS. MOMENT OF INERTIA
C*** VO IS THE MUZZLE VELOCITY OF THE PROJECTILE
C*** SPIN IS THE PROJECTILE SPIN IN RAD/SEC
C*** RHO IS THE AIR DENSITY.
C*** CD IS THE DRAG COEFFICIENT
C*** CNA IS THE NORMAL FORCE DERIVATIVE COEFFICIENT (PER PAO)
C*** CMU IS THE COMPOSITE DAMPING COEFFICIENT (PER RAD/SEC)
C*** CMPA IS THE MAGNUS MOMENT DERIVATIVE COEFFICIENT
C*** CP IS THE CENTER OF PRESSURE AFT OF THE NOSE IN CALIBERS
C*** CU IS THE POSITION OF THE CENTER OF GRAVITY AFT OF THE NOSE (CAL)
C*** CG IS THE AVERAGE POSITION OF THE CG
C*** OCG IS THE AMPLITUDE OF THE EXCURSION OF THE CG (CAL)
C*** PRINT COLUMN HEADINGS
300 FORMAT (1H0,11X4TIME,10X5HPITCH,12X3HYAW,5X10HPITCH RATE,7X,
I 8HYAW RATE,5X,10HSSQ ANGLE,11X,4HFREQ)
WHITE (6,300)
0034 OMEGA=6.2832*FREQ
0035 OOMEGA=6.2832*DFREQ
0036 TAMAX=U.0
0037 AMAX=0.0
C*** INITIALIZE STATE VECTOR
0039 U(1)=THETA0
0040 U(2)=PSI0
0041 U(3)=THED0

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FORTRAN IV 6 LEVEL 21	MAIN	DATE = 75301	13/47/12	PAGE 0003
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0042      U(4)=PSIDU
        C**** INITIALIZE RUNGE-KUTTA SUBROUTINE
        C**** SOLVE DIFF EQNS FOR DERIVATIVE ICS
        T=0,
        CALL OIFEQ(T,U,*)
        19 KOUNT=0
        C
        C**** START OF SOLUTION LOOP
        20 CONTINUE
        C**** MOVE STATE FROM T TO T+DT
        CALL DKUTIA(T,DT,U,WP,4,2,DIFEQ)
        SAVE=U(1)**2+U(2)**2
        IF (SAVE.GT.AMAX) GO TO 22
        21 CONTINUE
        IF (T.GT.TMAX) GO TO 42
        KUUNT=KOUNT+1
        IF (KUUNT.EQ.NPRINT) GO TO 40
        GO TO 20
        22 AMAX=SAVE
        TAMAX=T
        GO TO 21
        40 TP=T
        ATHT=57.2958*U(1)
        APSI=57.2958*U(2)
        DATHT=57.2958*U(3)
        DAPSI=57.2958*U(4)
        RSSX=DSQRT(U(1)**2+U(2)**2) *57.2958
        FFREQ=FREQ+T*DFREQ
        C
        C      AWRTDV=57.2958*DATAN2(-DSIN(U(2))*DCOS(U(1)),DSIN(U(1))*
        C      1 OCOS(U(2)))
        WRITE (6,900) TP,ATHT,APSI,DATHT,DAPSI,RSSX,FFREQ
        900 FORMAT(1H,7F15.6)
        IF (RSSX.GE.20.00) CALL EXIT
        GO TO 19
        42 AMAX=57.2958*DSQRT(AMAX)
        WRITE (6,44) TAMAX,AMAX
        44 FORMAT(1H,3HAT ,F12.6,20H SEC THE MAX YAW DF ,F12.6,
        1 11H DEG EXISTS)
        CALL EXIT
        0072      END
        0073

```

FORTRAN IV G LEVEL 21	DIFEQ	DATE = 75301	13/47/12	PAGE 0001
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0001 SUBROUTINE DIFEQ(TIME,U,KUITA)
C**** DIFFERENTIAL EQUATIONS FOR A SPIN-STABILIZED PROJECTILE
0002 IMPLICIT REAL * 8 (A-H,D-Z)
0003 DIMENSION U(8)
0004 COMMON CALIB,CNA,CMQ,CD,OCG,OJ,DCG,DJ,C31,C32,C33,C34,CP,
1 OMEGA,DOMEGA,PMASS
C**** U(1) IS THE PITCH ANGLE AND U(2) IS THE YAW ANGLE
SINWT=DSIN((OMEGA+DOMEGA*TIME)*TIME)
0005 XJ=OJ+DJ*SINWT
0006 CG=OCG+DCG*SINWT
0007 CMA=CNA*(CG-CP)
0008 A31=C31*CMA/XJ
0009 GYRYS=XJ/PMASS/CALIB/CALIB
0010 A32=C32/XJ
0011 A33=C33*(CNA-2.*CD-CMQ/GYRYS)
0012 A34=C34/XJ
0013 A41=-A32
0014 A42=A31
0015 A43=-A34
0016 A44=A33
0017 U(5)=U(3)
0018 U(6)=U(4)
0019 U(7)=A31*U(1)+A32*U(2)+A33*U(3)+A34*U(4)
0020 U(8)=A41*U(1)+A42*U(2)+A43*U(3)+A44*U(4)
0021 RETURN
0022 END
0023

```

STABILITY ANALYSIS OF THE XM410 PROJECTILE AT MACH 1.5

TIME STEP = 0.200001-03 SEC TORQUE = 0.0 DYNE CM
 FREQ = 17.50 HZ DFREQ = 0.0 HZ/S SAMP INT = 0.0 SEC

CALIB	PMASS	AX MI	AVG T MI	INC T MI	VO	SPIN	RHO
0.5000	1.3130	0.0446	0.1548	0.0055	1675.5000	526.4000	0.002377
CD	CNA	CMQ	CMFA	CP	AVG CG	INC CG	
0.5000	2.9000	-5.0000	0.3000	1.4000	1.8500	0.0300	

NOM STAB FAC = 2.083
 NUTAT FREQ = 20.770 HZ PREC FREQ = 3.367 HZ DIFF FREQ = 17.403 HZ

TIME	PITCH	YAW	PITCH RATE	YAW RATE	RSSQ ANGLE	FREQ
0.010000	1.192863	1.356268	-1.027796	42.675685	1.806208	17.500000
0.020000	0.957394	1.890347	-50.060366	54.234828	2.118966	17.500000
0.030000	0.239986	2.228883	-84.614976	5.339857	2.241766	17.500000
0.040000	-0.492864	1.973189	-50.700330	-49.380244	2.033812	17.500000
0.050000	-0.694811	1.489465	5.058644	-34.613641	1.643554	17.500000
0.060000	-0.618095	1.401928	-2.361505	14.591277	1.532137	17.500000
0.070000	-0.897323	1.603213	-54.294940	13.707221	1.837248	17.500000
0.080000	-1.581653	1.477499	-71.123824	-43.925565	2.164401	17.500000
0.090000	-2.069888	0.774237	-16.892781	-86.567611	2.209951	17.500000
0.100000	-1.893577	0.025862	43.715615	-50.411481	1.893754	17.500000
0.110000	-1.477902	-0.152208	25.449952	8.592103	1.485719	17.500000
0.120000	-1.512374	-0.052227	-29.483298	-2.287692	1.513275	17.500000
0.130000	-1.873062	-0.368487	-29.432447	-62.118844	1.908964	17.500000
0.140000	-1.874365	-1.163676	35.308607	-84.166517	2.206215	17.500000
0.150000	-1.217303	-1.768965	84.686855	-26.050138	2.147339	17.500000
0.160000	-0.492184	-1.660700	46.622776	38.017171	1.732100	17.500000
0.170000	-0.365501	-1.316810	-14.336825	16.035492	1.366594	17.500000
0.180000	-0.513031	-1.471696	-0.713600	-43.972127	1.558554	17.500000
0.190000	-0.194805	-1.990007	65.987096	-45.376288	1.999519	17.500000
0.200000	0.670290	-2.126323	93.124219	25.236950	2.229470	17.500000
0.210000	1.356254	-1.537915	32.325100	79.816929	2.050514	17.500000
0.220000	1.298367	-0.867651	-33.122251	39.893667	1.561594	17.500000
0.230000	1.019907	-0.817269	-7.435601	-22.347273	1.306958	17.500000
0.240000	1.284240	-1.038363	57.245424	-6.797090	1.651506	17.500000
0.250000	1.950134	-0.753341	60.628556	66.003093	2.090585	17.500000
0.260000	2.219404	0.141354	-14.749194	97.957373	2.223900	17.500000
0.270000	1.712551	0.869971	-72.828548	35.436722	1.920854	17.500000
0.280000	1.120675	0.839879	-30.916079	-29.793277	1.400467	17.500000
0.290000	1.167075	0.611037	32.417425	-0.652554	1.317358	17.500000
0.300000	1.486159	0.964775	15.748947	68.476550	1.771851	17.500000
0.310000	1.266275	1.596334	-62.551215	74.255431	2.167894	17.500000
0.320000	0.379282	2.149969	-98.838959	-4.818308	2.183167	17.500000
0.330000	-0.354460	1.728425	-35.299549	-64.616829	1.764397	17.500000
0.340000	-0.326088	1.229501	28.682381	-20.518778	1.272009	17.500000
0.350000	-0.122935	1.386720	-3.464059	44.057694	1.392158	17.500000
0.360000	-0.538266	1.823246	-76.938666	27.000426	1.901041	17.500000
0.370000	-1.435763	1.694084	-85.414628	-56.270775	2.220661	17.500000
0.380000	-1.926615	0.846941	-3.711678	-96.143228	2.104555	17.500000
0.390000	-1.585644	0.143063	56.099162	-32.014443	1.592085	17.500000
0.400000	-1.184461	0.197647	9.653842	30.266245	1.200838	17.500000
0.410000	-1.458205	0.404930	-56.518457	-4.316499	1.513383	17.500000
0.420000	-2.023744	-0.038766	-39.690626	-82.114200	2.024115	17.500000
0.430000	-2.002077	-1.006783	47.990996	-93.451212	2.240965	17.500000
0.440000	-1.220098	-1.570842	90.420416	-10.165361	1.989016	17.500000
0.450000	-0.576637	-1.298023	25.867896	48.202006	1.420343	17.500000
0.460000	-0.685194	-0.989116	-34.769166	-0.647269	1.203263	17.500000

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