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FAR-FIELD, FINITE-AMPLITUDE RADIATION FROM DIRECTIVE SOURCES

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Introduction. The propagation of finite-amplitude waves in one dimension, that is, plane, spherical, and cylindrical waves and waves in horns, is relatively well understood.[1] Much less is known about two- and three-dimensional radiation. Westervelt's method, based on an integral solution of the inhomogeneous wave equation,[2,3] is useful for relatively weak multidimensional waves. Schindler[4] found a special class of solutions for multidimensional waves in the shock-free region. Zabolotskaya and Khokhlov[5] have studied axially symmetric beams of intense sound. The nonlinear wave equation they derived is similar to that for plane waves but contains terms to account for amplitude variation across the beam. Their solution shows that for single condensation pulses the beamwidth increases with propagation distance. In physical terms the variation in excess propagation speed across the beam causes the beam to diverge. Conversely, for single rarefaction pulses the beamwidth decreases, for the same reason. This phenomenon may be called "self-refraction."

In the work reported here directivity effects are accounted for in a very simple manner. We are concerned with the far field of directional sources, particularly for periodic waves. Suppose such a source emits a spherical wave whose small-signal directivity function is $D(\theta,\phi)$ (θ and ϕ are angle coordinates, and D represents the normalized amplitude response). Let the peak particle velocity Mach number be denoted by ϵ. Our method amounts to replacing ϵ by $\epsilon D(\theta,\phi)$ in any known solution for uniform spherical waves, for example, the weak-shock solution.[6] Self-refraction is not taken into account. It is neglected on the basis that beam broadening and narrowing occur alternately with the positive and negative phases of the wave, thus averaging out to zero. Beamwidth changes are predicted in our theory, but they arise from a different cause, namely variation across the beam of nonlinearly induced losses.

Theory. To second order the general nonlinear wave equation for lossless fluids is[7]

$$\nabla^2\phi - \frac{1}{c_o^2}\phi_{tt} = \frac{1}{c_o^2}\left[\nabla\phi\cdot\nabla\phi + \frac{\beta-1}{c_o^2}\phi_t^2\right]_t \quad , \qquad (1)$$

where ϕ is the velocity potential, c_o is the small-signal sound speed, t is time, and $\beta-1$ is a measure of the nonlinearity of the medium [for gases, for example, $\beta-1 = (\gamma-1)/2$, where γ is the ratio of specific heats]. If spherical coordinates r, θ, and ϕ are adopted and a new time variable $t' = t - r/c_o$ is chosen, the left-hand side of Eq. (1) becomes

$$\phi_{rr} - \frac{2}{c_o}\phi_{rt'} + \frac{2}{r}\phi_r - \frac{2}{rc_o}\phi_{t'} + \frac{1}{r^2\sin\theta}\left(\sin\theta\,\phi_\theta\right)_\theta + \frac{1}{r^2\sin^2\theta}\phi_{\phi\phi}$$

We now wish to assess the relative importance of these terms for far-field radiation. To this end the small-signal, far-field solution $\phi \approx r^{-1}D(\theta,\phi)\exp(j\omega t')$ is substituted in

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each term.[8] Provided $D(\theta,\phi)$ is reasonably well behaved, the most important terms (for large kr) are found to be the second and fourth ones. The same technique is used to find the most important nonlinear terms on the right-hand side of Eq. (1). When the unimportant terms are discarded from each side and the result is expressed in terms of the radial particle velocity $u = \phi_r - c_0^{-1}\phi_{t'} \doteq -c_0^{-1}\phi_{t'}$, the nonlinear wave equation obtained is

$$u_r + \frac{u}{r} - \frac{\beta}{c_0^2} u u_{t'} = 0 \quad . \tag{2}$$

This is the same equation as for <u>uniform</u> spherical waves.

A solution of Eq. (2) that includes effects of directivity is required. Since that equation contains no derivatives with respect to θ or ϕ, directivity must be accounted for in the boundary condition. For periodic wave motion we may use $u = u_0 D(\theta,\phi)\sin\omega t$ at $r=r_0$, where ω is the angular frequency and r_0 is a point in the far field. It will be be seen that $u_0 D(\theta,\phi)$ now plays the part of the (constant) amplitude in the solution for uniform spherical waves. To be explicit, let the solution be expressed as a Fourier series,

$$u = u_0 \frac{r_0}{r} D(\theta,\phi) \sum_n B_n(\sigma) \sin n\left[\omega t - k(r-r_0)\right] \quad , \tag{3}$$

where $k = \omega/c_0$ is the wave number and the distortion distance σ is defined by[6]

$$\sigma = \beta\epsilon D k r_0 \log_e(r/r_0) \tag{4}$$

$(\epsilon = u_0/c_0)$. For example, in the "Fubini solution" of this problem the harmonic amplitudes are given by

$$B_n = \frac{2}{n\sigma} J_n(n\sigma) \quad . \tag{5}$$

This expression is valid only in the shock-free region, i.e., in the region $r < \bar{r} = r_0 \exp[1/\beta\epsilon D k r_0]$. In the sawtooth region, which may be taken to begin at $\sigma = 3$, approximately,[6] the amplitudes are given by

$$B_n = \frac{2}{n(1+\sigma)} \quad . \tag{6}$$

The expression for B_n in the transition region, that is, $1 < \sigma < 3$, is relatively complicated but known.[6] Hence the problem of directive sources may be regarded as solved to the extent that the solution for uniform spherical waves is known.

A few limiting cases are of special interest. Let us compute the directivity D_n of the n^{th} harmonic. If the amplitude and distance are such that $\sigma < 1$ for all parts of the beam, the Fubini solution can be used for this purpose. From Eqs. (3) and (5) we obtain $D_n(\theta,\phi) = J_n(nD\sigma_1)/J_n(n\sigma_1)$, where σ_1 is the value of σ when $D = 1$. For small values of the arguments of the Bessel functions the result is particularly simple, namely

$$D_n(\theta,\phi) = D^n(\theta,\phi) \quad , \tag{7}$$

which result was obtained previously by Westervelt and Radue for the special case of the second harmonic.[3] The implication is that the higher the harmonic, the narrower the beam.

The situation changes drastically with distance, however, if a portion of the beam — say that defined by $\theta_0 < \theta < \theta_1$, $\phi_0 < \phi < \phi_1$ — is intense enough to form a sawtooth wave. In the sawtooth region the directivity is given by Eqs. (3) and (6) as

$$D_n(\theta,\phi) = D\,\frac{1 + \beta\epsilon k r_0 \log_e(r/r_0)}{1 + \beta\epsilon k r_0 D \log_e(r/r_0)}\ , \quad \left|\begin{array}{l}\theta_0 < \theta < \theta_1\\ \phi_0 < \phi < \phi_1\end{array}\right. . \qquad (8)$$

Not only is there no dependence on the harmonic number n, but also the beam is now broader. The broadening is due to extra attenuation caused by nonlinear effects. The intense part of the beam suffers much more attenuation than the edge. Furthermore, if there are minor lobes, they grow (with distance) relative to the major lobe, again because of the amplitude-dependent attenuation. Ultimately, at distances such that $\sigma >> 1$ Eq. (8) reduces to

$$D(\theta,\phi) = 1, \qquad \theta_0 < \theta < \theta_1, \quad \phi_0 < \phi < \phi_1\ , \qquad (9)$$

which means that this portion of the beam behaves like a uniform spherical wave. Beam broadening and growth of minor lobes have been observed experimentally.[9]

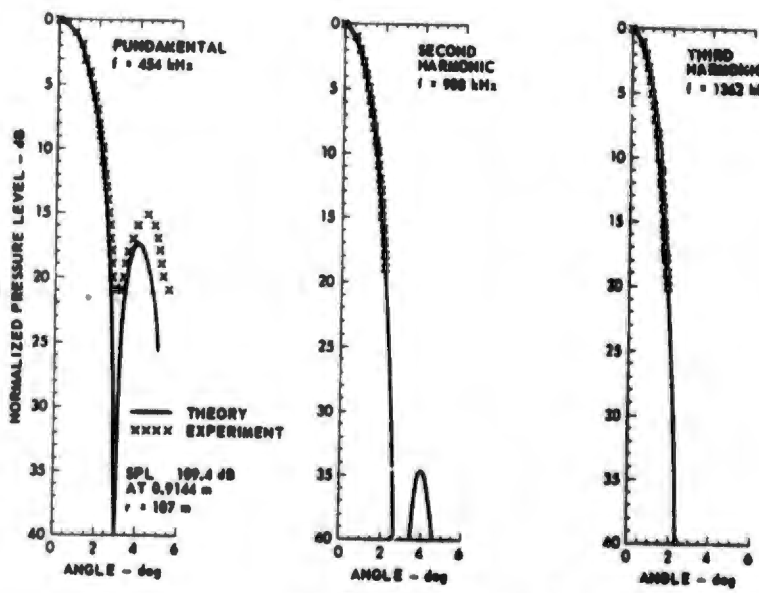

FIGURE 1
BEAM PATTERNS

Cylindrical waves from sources of infinite length can also be treated by this method. The wave equation obtained is just like that for uniform cylindrical waves[7] (namely, Eq. (2) with the middle term multiplied by the factor ½). The known solution[6] is again simply modified by replacing ϵ with $\epsilon D(\theta)$, where now θ is the polar angle in the cylindrical coordinate system. For sources of finite length cylindrical coordinates are not as convenient as spherical ones for finite-amplitude calculations. Spherical coordinates should thus be used.

Example. The circular piston mounted in an infinite baffle is a widely used source. Its directivity function is

$$D(\theta,\phi) = D(\theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}, \tag{10}$$

where a is the piston radius. Shown in Fig. 1 are some computed directivity patterns for relatively weak waves. Experimental measurements made in fresh water with a 3-inch piston operating at 454 kHz are also shown. At 0.9144 meter, which was taken to be the value of r_0, the axial pressure amplitude was 0.3 atm. ($\epsilon = 1.38 \times 10^{-5}$). The beam patterns were measured 107 meters from the source; at this distance $\sigma \doteq 0.4$ on the axis. The beam patterns are thus computed from the Fubini solution [Eq. (7) is a good approximation]. It will be seen that the agreement between theory and experiment is quite good for the major lobes. Agreement is not as good for the minor lobe of the fundamental, probably because our source did not radiate a pattern precisely like that given by Eq. (10). The difference is thus not associated with nonlinear effects. Notice the very high suppression of the minor lobes of the second- and third-harmonic signals. For example, if conditions are such that Eq. (7) holds, the suppression of the second-harmonic minor lobes will be twice (in dB) that for the fundamental.

Discussion. It will be seen that in effect we have divided up the space through which the wave propagates into ray tubes and associated a given starting amplitude with each ray tube. Since there is no interaction between ray tubes, there is no provision for self-refraction in this model. Nevertheless, the model does provide a useful and exceedingly simple generalization of finite-amplitude propagation theory. Experimental measurements made so far tend to confirm the analytical predictions.

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The theory of far-field radiation from intense spherical and cylindrical sources has been extended to include effects of directivity. Examples include the dipole, quadrupole, and baffled piston. Experimental measurements of the directivity of harmonics when the source is a piston are discussed to show the extent of validity of the theory.