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CELEST COMPUTER PROGRAM FOR COMPUTING SATELLITE ORBITS

by
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OCTOBER 1976

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The Celest Computer Program uses raw Doppler data to determine satellite orbits. It provides diagnostic information on the quality of the orbits and the data used in producing those orbits. The basic technique employed is one of weighted least squares where the data is edited and weighted within the program. An iterative capability exists for nonlinear problems. Trajectories are formed by directly integrating the equations of motion in an inertial frame. The force equation has components due to earth, sun and moon gravity, solar radiation, thrust, atmospheric drag, solar and lunar tidal distortion. A satellite frequency offset error can be determined, and the program has the facility for determining unknown receiver locations. The computer program occupies 130 octal units of memory, is structured as nine major overlays, and is completely written in Fortran. The program is primarily operated on a sixty bit CDC 6700 computer.

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FOREWORD

The Celest Computer Program uses raw Doppler data to determine satellite orbits. It provides diagnostic information on the quality of the orbits and the data used in producing those orbits. The basic technique employed is one of weighted least squares where the data is edited and weighted within the program. An iterative capability exists for nonlinear problems. Trajectories are formed by directly integrating the equations of motion in an inertial frame. The force equation has components due to earth, sun and moon gravity, solar radiation, thrust, atmospheric drag, solar and lunar tidal distortion. A satellite frequency offset error can be determined, and the program has the facility for determining unknown receiver locations. The computer program occupies 130 octal units of memory, is structured as nine major overlays, and is completely written in Fortran. The program is primarily operated on a sixty bit CDC 6700 computer.

This report has been compiled by James W. O'Toole with major technical contributions by Larry K. Beuglass, Alfred F. Buonaguro, James W. O'Toole and Mark G. Tanenbaum. Conversations with Robert W. Hill were very useful in the design of the program. The computer program structure was designed by Patrick E. Beveridge and Treva D. Burgess who also directed the programming carried out by Donald Clark, John B. Ellis, Allen E. Fisher, Joseph M. Futcher and Louise P. Gordon. The production of this report was carried out by Clair D. Cousins and supervised by Jo Ellen Wilcox of Offset Composition Services. This report was reviewed by Richard J. Anderle and produced under Defense Mapping Agency authorization.

Released by:

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CONTENTS

CONTENTS – (Continued)

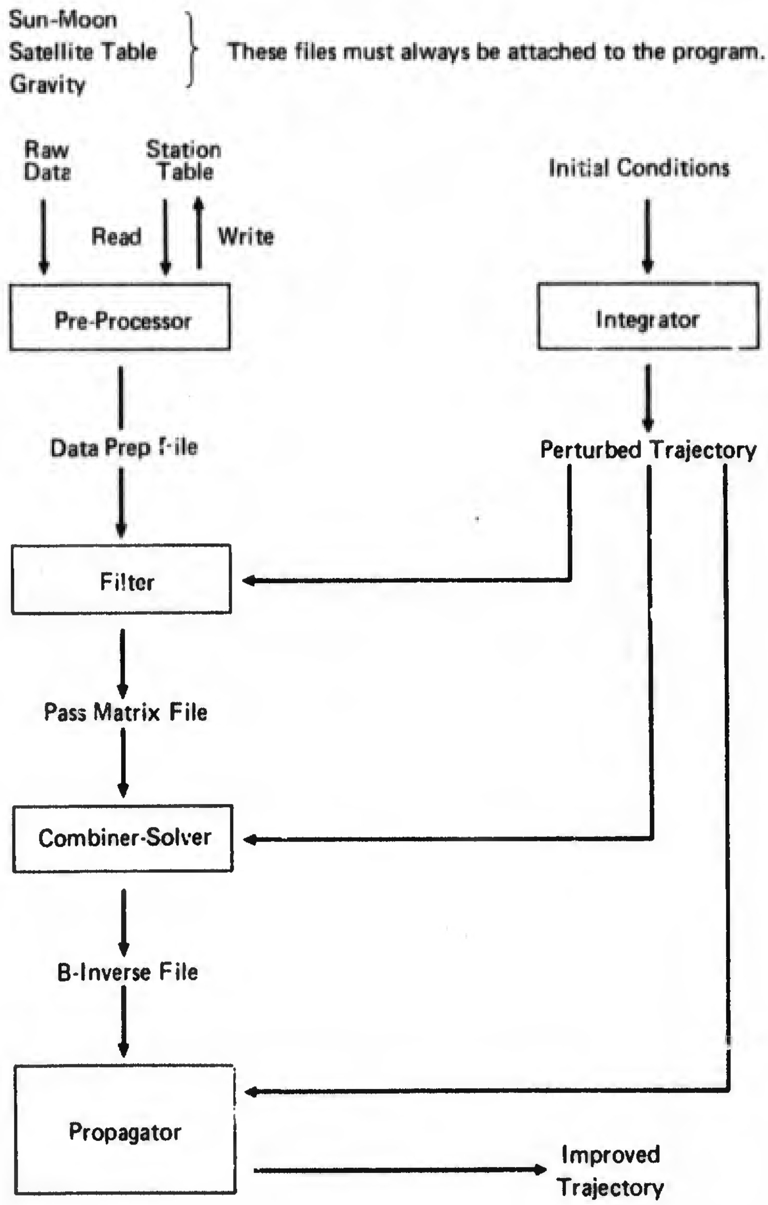
INTRODUCTION

The Celest Computer Program takes raw Doppler data as input and reduces it to a precise ephemeris and associated diagnostic information. This process requires

1. Editing and time correcting raw observations and their time tags
2. Applying ionospheric corrections when required
3. Determining proper weights to be used in the least square procedure
4. Forming pass normal equations to be used in building a total normal matrix for some fit span
5. Building and inverting the total normal matrix
6. Computing the refined ephemeris and associated covariance information
7. Producing reports and diagnostics

Tasks One and Two take place in the Pre-Processor, Three and Four in the Filter, Five in the Matrix Solver, Six in the Propagator, and Seven in the Orbit Computation Report segment.

One of the primary features of this program is the pass matrix concept whereby data is handled largely on a pass basis. Once weights have been determined and pass normal equations formed, a file is written containing this information. New data can now be merged or old data deleted from this file. Solutions can be obtained over varying spans using any desired data contribution in an efficient and inexpensive manner. The diagrams on pages 2-5 illustrate basic program flow, major files, and the pass matrix data bank concept.



Sun-Moon
Satellite Table
Gravity
These files must always be attached to the program.
Raw Data
Station Table
Read
Write
Initial Conditions
Pre-Processor
Integrator
Data Prep File
Perturbed Trajectory
Filter
Pass Matrix File
Combiner-Solver
B-Inverse File
Propagator
Improved Trajectory

Diagram 1

LONG-ARC-FLOW

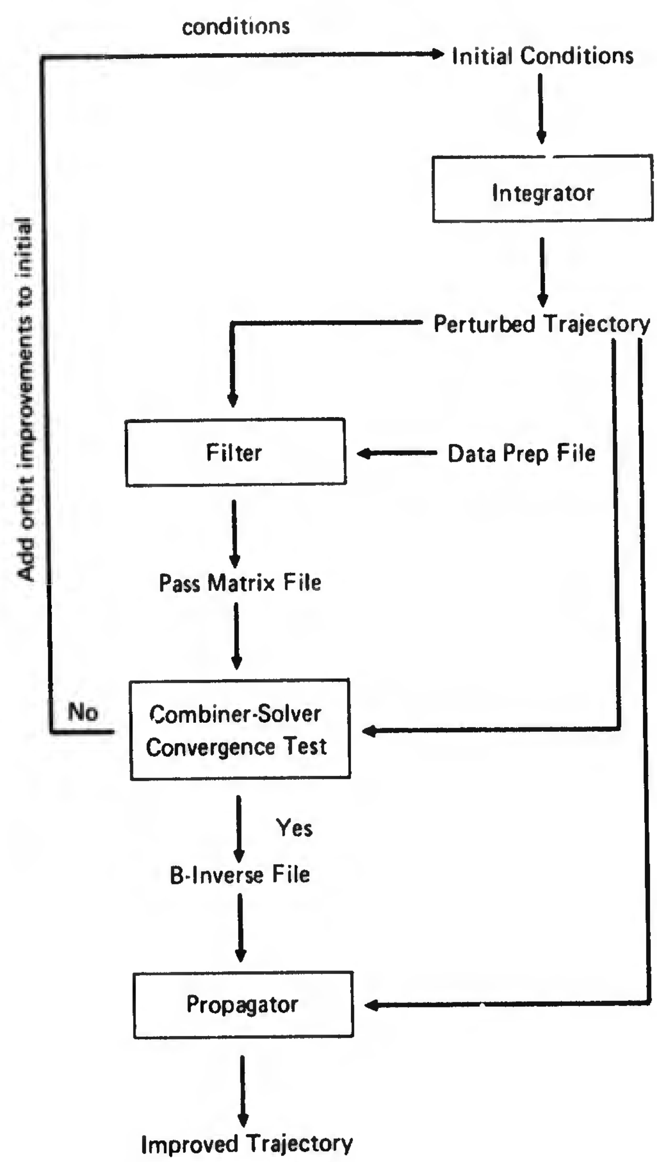


Diagram 2

SHORT-ARC-FLOW

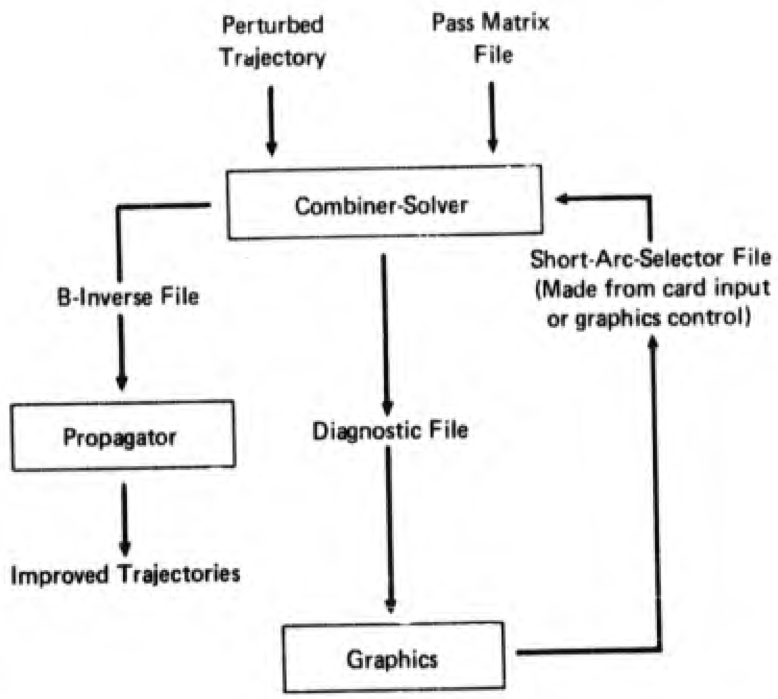


Diagram 3

DATA BANK CONCEPT

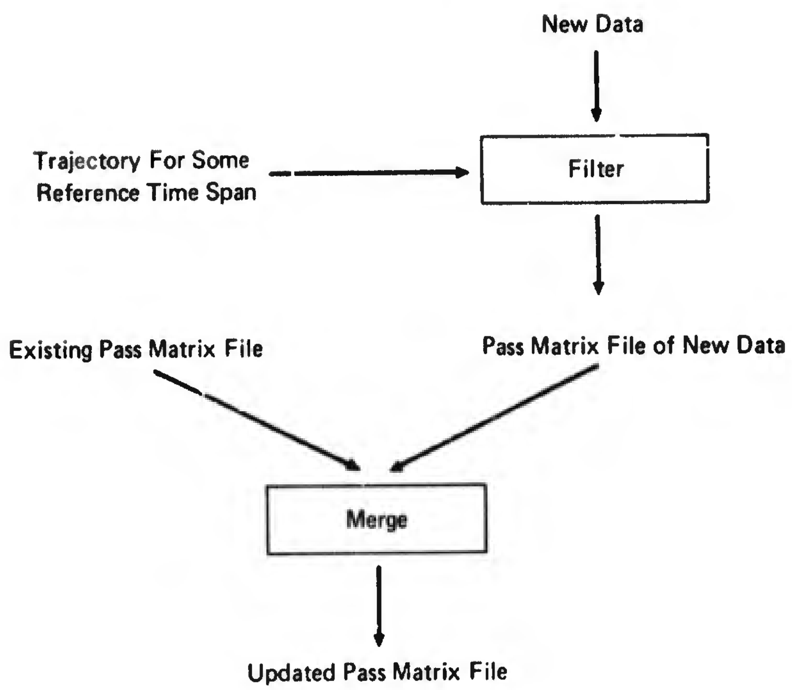


Diagram 4

PRE-PROCESSOR

Doppler data of two basic kinds are processed by the pre-processor. Doppler frequency (Class 7) consists of instantaneous Doppler measurements, and integrated Doppler (Class 9) consists of range-difference measurements. In addition, the Class 9 data comes from various equipment types. This results in the program assigning both a class number and type number to each form of data.

The major purposes of the pre-processor are as follows:

1. Convert the raw data to frequency or range-difference measurements
2. Perform preliminary filtering by deleting garbled data records and tagging questionable data points
3. Compute and apply ionospheric refraction corrections when necessary
4. Compute and apply time corrections to data point observation times
5. Assign average time corrections to each station on a daily basis. Store these values on a retrievable file
6. Output an unfiltered, time corrected raw data file (Data Prep File). This file is used in the filter module where the data is edited and weights are assigned.

FILTER AND FORMAT DATA

1. Obtain time spans for data processing from input.
2. Obtain data, on a pass by pass basis, from the raw data tape. Check data type, time, and satellite number. Data point times are checked against the input time span, satellite number is checked against values on the satellite table and data type against values 4, 7, 8, and 9. The pass is rejected if any check fails.
3. Depending on type, decode the data, i.e., transform from character to numeric value.
4. Type 7 and 9 data from a navigation satellite contains time correction points. Tag these points. Such a point, time value = (whole part, fractional part), is tagged by setting the fractional part to minus (-) itself.
5. Filter by data type, described below, then write the results to mass storage.

Type 4

a. Convert the data value to range difference.

$$\Delta R_i = \frac{c}{f_s} [D_i - D_{i-1} + (T_i - T_{i-1}) N_q \Delta V_s] \quad i \neq 1 \tag{1}$$

$$\Delta R_1 = 0$$

$$f_s = N_q (10^6 + \Delta V_s)$$

D_i = refraction corrected Doppler count

T_i = header initial time + idt + 1bit + dt_{add}

dt_{add} = 0 unless i = multiple of 26, in which case the value is given below by satellite.

Navigation Satellite	*Geodetic Satellite*
N_q = 400	N_q = 324
1bit = .01966246018	1bit = .01966243567
dt_{add} = .373586761	dt_{add} = .373586278
FAC = 24/55	FAC = 8/9
dt = 4.601015893	dt = 4.601009948

The refraction corrected Doppler count D_i is given by

$$D_i = D_i\,(\text{not corrected}) - (\text{Refraction count} - 2000)FAC \tag{2}$$

b. Check the Doppler count for size. All 9's imply a bad point and all 9's in the refraction count imply a bad point. Check that the Doppler count is monotone in each group of 26 values. If a point is bad, tag it but do not throw it away. Tags are numbers identifying the character of data points. Due to the fact that the Doppler counter is reset at every 26th point, the 26th point and every multiple of 26 points is tagged.

c. If the pass ends up with less than 4 points, reject the pass.

d. If the alert rise time does not match the first data point time to within 30 minutes, reject the pass.

e. If the pass has not been rejected, write the range-difference values to mass storage.

Types 7 and 9

a. Count the time correction points.

b. Reject the pass if all points are time points.

c. Tag points with zero Doppler count.

d. Recover higher order time digit by data type.

Type 9—Recover the higher order time digit for time correction points by looking at the time for the data point just prior to the time point. Time point values look like 9XXXX, where the 9 indicates that the value is a time correction value. The 9 must then be replaced by the correct higher order digit.

Type 7—Recover the higher order time digit for data points from the rise time in the header. The new time value for a data point is

$$\bar{t}_c + 1000 \text{ Integer part of } \left[\frac{10\,000\, X_1 + X_2 - \bar{t}_c + 5000}{10\,000} \right]$$

where X_1 is the higher order time digit from the header rise time and $\bar{t}_c$ is the time value reported for the data point.

e. Decode the data points and put in the decimal point.

f. Remove the time correction points and put them in the header.

g. Perform a monotone filter check on the data.

h. Check the time correction points in the header for the proper gap. The gap test tolerances are

minimum gap = 119 sec.
maximum gap = 1500 sec.

Any two consecutive time points must satisfy the gap test.

(1 2 3 4 5 6 7)

If the test fails at 4 5, then point 5 is removed from the header and the test is restarted. The test continues until the remaining points pass the test or all points are removed.

i. Reject the pass if there are less than

31 data points (Type 7)
4 data points (Type 9)

j. If the pass was not rejected, write the raw data Doppler counts to mass storage. The data has not yet been converted to frequency or range difference values.

Type 8

This data comes in Mode 3 and non-Mode 3 format. Mode 3 data looks just like non-Mode 3 or geoceiver data but contains time corrections only. This data is immediately written to mass storage.

Geoceiver Data

a. Convert the time to seconds from (hr, min, sec, fractional sec).

b. Check for day cross-over as this data flips to zero at cross-over.

c. Perform a monotone filter test on the data point times.

d. Normalize the Doppler count for test purposes.

$$\text{Normalized count} = D_n(i) = \frac{D_i}{T_i - T_{i-1}}$$

e. Perform a monotone filter test on the normalized counts. If $D_n(i)$ or T_i fails the test, tag the i and $i+1$ data points.

f. Check the raw Doppler count against a maximum and minimum value. These values are obtained from the Satellite Table. If the count fails, tag the point.

g. If the absolute value at refraction is not less than 500, tag the point.

h. If the pass ends up with less than three points, reject it.

i. If there is a ΔT_i with $2 \leq i \leq n-1$ such that $\Delta T_i > 361$, then reject the pass. $\Delta T_i = T_{i+1} - T_i$.

j. If the pass has not been rejected, write it to mass storage. The data points have not yet been converted to range-difference values.

6. At this point three arrays are formed

$$\text{Array TIME} = \left\{\begin{array}{l}\text{1st time point and}\\ \text{1st data point of}\\ \text{each pass}\end{array}\right\}$$

$$\text{Array IREJ} = \left\{\begin{array}{l}\text{For each pass set a tag of}\\ \text{blank for good and non-blank}\\ \text{for rejected.}\end{array}\right\}$$

$$\text{Array ISUB} = \{1,2,3,\ldots \text{number of passes}\}$$

7. Sort array TIME into ascending order reordering ISUB.

$$\text{TIME} \xrightarrow{\text{1 to 1}} \text{IREJ} \xrightarrow{\text{1 to 1}} \text{ISUB}$$

8. Retrieve the good passes from mass storage in time order.

9. Duplicate passes, consecutive passes with the same rise time and station number, are rejected. The first of a duplicate pair is retained.

10. Computation of observation values is made at this time. This work depends on data type and with the exception of geoceiver data assumes ionospheric corrections have been made at the receiving station.

Ionospheric Corrections and Data Formulae

Starting with a base frequency f_0 (5 MHZ) and some offset ΔV_s (–80 ppm) the satellite multiplies the frequency up to two frequencies f_1 and f_2 (400 and 150 with offsets).

$$f_1 = \alpha f_0 10^6 + \alpha f_0 \Delta V_s$$

$$f_2 = \beta f_0 10^6 + \beta f_0 \Delta V_s$$

$$\lambda = \frac{f_2}{f_1} = \frac{\beta}{\alpha} < 1$$

The received frequencies, assuming only first order ionospheric effect, are

$$f_{R_1} = f_1 - \frac{f_1}{c}\dot{\rho} + \frac{a_1}{f_1}$$

$$f_{R_2} = f_2 - \frac{f_2}{c}\dot{\rho} + \frac{a_1}{f_2}$$

Thus

$$f_{R_1} - \lambda f_{R_2} = f_1 - \lambda f_2 - \frac{f_1}{c}\dot{\rho} + \lambda\frac{f_2}{c}\dot{\rho}$$

$$f_{R_1} - \lambda f_{R_2} = \left(f_1 - \lambda f_2\right)\left(1 - \frac{\dot{\rho}}{c}\right)$$

and

$$M(f_{R_1} - \lambda f_{R_2}) = M\left(f_1 - \lambda f_2\right)\left(1 - \frac{\dot{\rho}}{c}\right) \quad M \neq 0$$

$$M(f_{R_1} - \lambda f_{R_2}) = Mf_1\left(1 - \lambda^2\right)\left(1 - \frac{\dot{\rho}}{c}\right) \quad M \neq 0$$

Define

$$f_s = Mf_1(1 - \lambda^2) \tag{3}$$

Then

$$f_s(\text{Received}) = f_s\left(1 - \frac{\dot{\rho}}{c}\right)$$

Since

$$f_1 = \alpha f_0 10^6 + \alpha f_0 \Delta V_s$$

$$f_s = M(1 - \lambda^2)\alpha f_0 10^6 + M(1 - \lambda^2)\alpha f_0 \Delta V_s$$

Define

$$N_q = M(1 - \lambda^2)\alpha f_0 \qquad (4)$$

Store N_q on the Q table and ΔV_s on the satellite table. We can now write

$$f_s = N_q(10^6 + \Delta V_s) \qquad (5)$$

The first order ionospheric corrected Doppler frequency can now be given by

$$\text{Doppler frequency} = \frac{dN_c}{dt} = f_r - f_s\,(\text{Received})$$

where f_r is the station reference frequency $N_q\ (10^6 + \Delta V_0)$ and ΔV_0 is the deviation of the reference frequency from *WWV*. The value ΔV_0 is recorded by the station and placed on the raw data tape.

$$\frac{dN_c}{dt} = f_r - f_s\left(1 - \frac{\dot{\rho}}{c}\right)$$

The Doppler frequency (Type 7) observation value is taken to be

$$\frac{-f_s\dot{\rho}}{c} = f_r - f_s - \frac{dN_c}{dt} \qquad (6)$$

Integrating this expression gives

$$\frac{-f_s}{c}(\rho_i - \rho_{i-1}) = (f_r - f_s)(T_i - T_{i-1}) - N_c$$

or

$$(\rho_i - \rho_{i-1}) = \frac{c}{f_s}\left[N_c - (f_r - f_s)(T_i - T_{i-1})\right] \qquad (7)$$

This last expression is taken as the observed value of range difference for data types 4, 8 and 9. Computations can now be carried out for the observed time, value, and sigma according to data type.

Type 7 (Frequency)

a. Retrieve the Q number from the Q Table. Reject the pass if there is no number on the table.

b. Compute the frequency value for each data point. Type 7 signals are sampled at four-second intervals by measuring the time to count a fixed number of cycles. The data consists of

(time of day, time to count a fixed number of cycles)

$$\text{Observation time} = T_i = t_{0_i} + \frac{t_i}{2} + \Delta t_c \qquad (8)$$

where

t_{0_i} = ith observation time from the raw data tape

t_i = time to count the fixed number of cycles

Δt_c = station clock error from the header of the raw data tape = difference between WWV and the station clock value.

$$\text{Observation Value} = F_i = \left[f_r - \frac{N_c}{t_i} \right] - f_s \qquad (9)$$

where

$f_s = N_q\,(10^6 + \Delta V_s)$

$f_r = N_q\,(10^6 + \Delta V_0)$

N_q = frequency from Q Table

N_c = number of cycles counted

t_i = time to count N_c

ΔV_s = deviation of the satellite frequency from the Q Table value. This deviation value is on the satellite table.

ΔV_0 = deviation of station frequency from WWV. This value is obtained from the raw data tape.

$$\text{Obs. Sigma} = \text{Sigma}\,(F_i) = (.2)^2 \frac{N_c}{(t_i)^{1/2}} \qquad (10)$$

c. Preliminary time of closest approach (TCA) is computed by

$$\text{TCA} = \frac{T_n - T_1}{2} \qquad T_n = \text{last point in pass}, \quad T_1 = \text{first point in pass} \qquad (11)$$

d. First order ionospheric corrections were made at the station. No corrections are performed here.

e. Temperature, pressure, and humidity values are set to zero.

f. Obtain station coordinates from the station table and reject the pass if no coordinates are available.

g. Write the data on Tape 8, the non-time corrected data file.

Type 9 (CCID)

a. Retrieve the Q number from the Q Table. Reject the pass if no number is available.

b. Type 9 signals are obtained by counting a fixed number of cycles and recording the time for the count. Signals are sampled every 20 seconds. A data point consists of

(time, time to count a fixed number of cycles)

The observation value and time are given by

$$T_i = t_{0_i} - \Delta t_c \tag{12}$$

$$\Delta R_i = \frac{c}{f_s} [N_c - (f_r - f_s)(T_i - T_{i-1})] \tag{13}$$

where

t_{0_i} = ith observed time
Δt_c = station time correction from the header
N_c = number of cycles counted
$f_r = N_q (10^6 + \Delta V_0)$
$f_s = N_q (10^6 + \Delta V_s)$
N_q = frequency from the Q Table
ΔV_s = deviation of the satellite frequency from the Q Table value. This value is on the satellite table.
ΔV_0 = deviation of the station frequency from WWV. This value is on the raw data tape.

The observation sigma is given by

$$\text{Sigma}(\Delta R_i) = 10^{-4}(T_i - T_{i-1})^{1/2} \tag{14}$$

Continuous count integral Doppler (CCID) data has been converted to range-difference (ΔR) measurements. The conversion formula used above was derived from the relationship

$$\int_{T_{i-1}}^{T_i} \left[f_r - f_s\left(1 - \frac{\dot{\rho}}{c}\right)\right] dt = N_c$$

c. A preliminary time of closest approach (TCA) is obtained by sensing a change of sign in the ΔR_i values. If no change of sign occurs, TCA is set to the time of the second observation.

d. First order ionospheric corrections have been made at the receiving station. No corrections are made here

e. Temperature, pressure, and humidity values are set to zero.

f. Station coordinates are obtained from the station table, and the pass is rejected if none are available.

g. The data is now written to Tape 8, the non-time corrected data file.

Type 4 (ITT)

a. Compute a sigma value by

$$\text{Sigma}\,(\Delta R_i) = 10^{-4}(T_i - T_{i-1})^{1/2}$$

b. Retrieve station coordinates from the station table and reject the pass if none are available.

c. Write the data to Tape 8.

Type 8 (Geoceiver)

Recall that this data type comes in two modes. One mode consists of time corrections only and is called Mode 3 date, the other mode is called Geoceiver data.

Mode 3

a. Check for a duplicate pass. The check is made on (day, station number, rise time).

b. Compute equipment delays from Mode 3 passes and print out the results. These values are monitored and compared with the value used by the program, located on a STAB card. The value on the STAB card is an average value. Thus, equipment delays are averaged, and when this average wanders from the card value a new value is placed on the card. The mean and RMS are computed by

$$\text{Mean} = \sum_{i=1}^{N} \frac{\text{delay}\,(i)}{N}$$

$$\text{RMS} = \left[\sum_{i=1}^{N} \frac{(\text{delay}\,(i) - \text{Mean})^2}{N} \right]^{1/2}$$

The print line for these results is

<u>Station No.</u> <u>Day</u> <u>Rise time</u> <u>No. of delays</u> <u>Mean</u> <u>RMS</u>

c. Write the Mode 3 data on Tape 8. The data is (time, correction value, drift).

Geoceiver Data

a. Geoceiver equipment continuously counts offset Doppler and refraction cycles at 30-second intervals. A data point consists of

(Time the count ended, Doppler count, refraction count)

The ionospheric refraction corrected count is given by

$$D_i = D_i\,(\text{uncorrected}) - \left(\frac{IR_i}{C_j}\right) \qquad (15)$$

where

IR_i = ith ionospheric retraction count
C_j = 55/6 or 9 for j = 1 or 2 respectively
j = 1 Navy Navigation Satellite (transmits at 150/400 MHZ)
j = 2 Geodetic Satellite (transmits at 162/324 MHZ)

The observation time, value and sigma is given by

$$T_i = t_{0_i} \qquad (16)$$

$$\Delta R_i = \frac{c}{f_s}\left[D_i + N_q \Delta V_s (T_i - T_{i-1})\right] \quad i \neq 1 \qquad (17)$$

$$\Delta R_1 = 0$$

$$\text{Sigma}\,(\Delta R_i) = 10^{-4}(T_i - T_{i-1})^{1/2} \qquad (18)$$

where

t_{0_i} = time from the raw data tape
N_q = 400 or 324
f_s = $N_q\,(10^6 + \Delta V_s)$
ΔV_s = deviation of the satellite frequency from the Q Table value. This value comes from the satellite table.

b. A preliminary time of closest approach is obtained by sensing a change of sign in the ΔR_i values. If no change of sign is sensed, TCA is set to the time of the second data point.

c. Station coordinates are obtained from the station table, and the pass is rejected if none are available.

d. Write the data on Tape 8. the non-time corrected data file.

11. After all passes have been written to Tape 8, a print line is provided for each pass.

Pass No.–Station No.–Yr.–Day–Rise–TCA–TC Pts.–Pts.–Reject Code

TC Pts. stands for time correction points, Pts. stands for total number of points and Reject codes are as follows:

Reject Code	*Meaning*
0	No data record
1	Rise time from the alert does not match the first point time within a tolerance of ±30 minutes.
2	This pass was rejected because it was a duplicate.
4	Time corrections only appeared in the data record.
5	This was a geoceiver pass which had more than the first and last point fail the test: $\|t_{i+1} - t_i\| < 361$ sec. If the tolerance is violated only at the first and last points, then only those points are deleted.
6	The pass had less than P points. $P = \begin{cases} 31 \text{ Doppler (Type 7)} \\ 4 \text{ ITT (Type 4) or CCID (Type 9)} \\ 3 \text{ Geoceiver (Type 8)} \end{cases}$
10	The Q number was not in the Q Table.
11	The station either did not have coordinates on the station table or did not have a STAB card in the input deck.
16	The monotone filter removed either all points or enough to cause the pass to fail the test in 6 above.

12. At this point the Time Correction Overlay is called.

TIME CORRECTIONS

The time correction overlay has two options. Option C applies average time corrections, stored on a station table, to data that does not have its own time correction signals. These average corrections for each station were computed on a daily basis in Option B and stored on the table.

Option C

1. Obtain the year from Tape 8.
2. Obtain the year from the Station Table.

3. Compare 1 and 2. Stop if they do not match.
4. Read the first pass from Tape 8.
5. If the pass is a Mode 3 pass go to the next pass.
6. Retrieve the time corrections from the Station Table.
7. Correct the data point time by subtracting the table value from the data point value.

$$\text{new } T_i = T_i - c_1 - c_3(T_i - c_2) \tag{19}$$

where

T_i = ith observation time in seconds

c_1 = clock error at a given epoch

c_2 = epoch of the clock error

c_3 = clock rate error at the given epoch

8. Write the corrected time value, data value, and sigma on the time corrected observation file (Tape 14).

Option B

This option computes individual time corrections for each pass of data and averages all pass corrections for a given station to store on the station table.

1. Obtain the year and day from Tape 8.

2. Obtain the year and day for time correction processing from the second input SPAN card.

3. Obtain Spasur elements for the satellite from the Spasur element cards. If the available elements do not have an epoch within a tolerance of the (yr, day) in One above, then the program stops.

4. Use the Spasur elements in the Brower orbit computation routine. This routine uses elements at a given epoch to compute elements at any later time. The new elements are converted to position and velocity, giving the time correction overlay an ephemeris generation capability.

5. At this point the data from certain reference stations is used to determine a satellite clock error. Reference stations have very stable clocks and can therefore be used to determine the error in the satellite clock.

a. Obtain the first reference station pass.

b. Obtain the equipment delay and one-bit delay from the STAB card.

c. Obtain the time correction points from the header of Tape 8. If there are less than four points, skip the pass.

d. Transform the station coordinates from geodetic to inertial.

e. For each time correction point determine its distance in seconds from the epoch of the Spasur elements.

f. Use the time from (e) to determine the position of the satellite at the time-correction point time.

g. Compute the range to the satellite at the time correction point time.

h. Check the zenith angle and if the satellite is below the horizon do not use the time correction point.

i. Compute a time transmission value.

j. Use the time transmission value to compute the time at which the time signal left the satellite. Store these values in an array:

$$\text{Array TKE} = \left\{ \begin{matrix} \text{all time correction} \\ \text{points from all} \\ \text{passes} \end{matrix} \right\} = \left\{ T_K \right\}$$

r = satellite position at $t = t_K + \Delta t_K$

r_s = inertial station position at $t = t_K + \Delta t_K$

$$r_s = \left(\frac{a}{(1 - e^2 \sin^2 \varphi_s)^{1/2}} + h_s \right) \begin{pmatrix} \cos \varphi_s \cos (\lambda_0 + \tilde{\omega}(t - t_{ve})) \\ \cos \varphi_s \sin (\lambda_0 + \tilde{\omega}(t - t_{ve})) \\ \sin \varphi_s \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ \dfrac{ae^2 \sin \varphi_s}{(1 - e^2 \sin^2 \varphi_s)^{1/2}} \end{pmatrix} \tag{20}$$

t_{tr} = time transmission error = $|r - r_s|/c$
a = semi-major axis of the reference ellipsoid
e = eccentricity = $f(2 - f)$ = $[2 - (1/\text{EL})]/\text{EL}$
f = flattening
EL = oblateness
φ_s = station geodetic latitude (radians)
λ_0 = station geodetic longitude (radians)
h_s = station geodetic height (km)
$\tilde{\omega}$ = earth rotation rate
t = $t_K + \Delta t_K$ in seconds from the beginning of the year
t_{ve} = time of vernal equinox in seconds, for the first day of the year. The t_{ve} and Julian day numbers are coded into the program in the form of a table. This table is updated periodically to insure sufficient values for t_{ve} and Spasur element requirements.

Define

$$T_K = t_K + \Delta t_K - t_{tr} - t_{sd} - t_{ed} - \text{one bit} \tag{21}$$

where

T_K = time the kth time signal was triggered from the satellite

t_K = time the satellite time mark is received

Δt_K = fractional part of t_n in seconds to the nearest microsecond

t_{tr} = time transmission error

t_{sd} = satellite clock delay

t_{ed} = station equipment delay

one bit = one bit delay = .0196624 seconds

k. Compute the satellite clock error (TSO).

For this routine to operate there must be a minimum of three reference station passes. There must be at least 40,000 seconds between each pass. All reference station passes are used collectively to compute TSO. In the event TSO cannot be computed the program provides an input default value TSO2. In addition, the program can be commanded to use an input value TSO3. For each reference station pass define

$$X_K = \frac{fso}{\text{NCS}} (T_K - \text{TSO}) \tag{22}$$

where

T_K = time the kth signal left the satellite

TSO = satellite clock error initialized at an input value TSO1. This value is usually zero.

fso = standard satellite frequency from input. This value is usually .10799136 E + 09 cycles/second.

NCS = number of cycles counted between time signals. This value is initialized at an input value NCS1. NCS1 is usually .129589632 E + 11.

Define

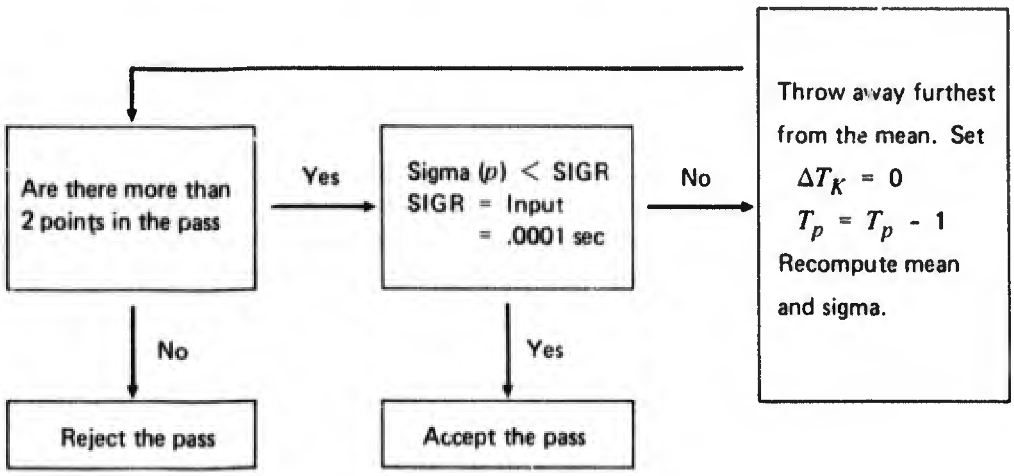
$$\Delta T_K = T_K - \left(\text{TSO} + \frac{\text{NCS}}{fso} [X_K]\right) \tag{23}$$

where ΔT_K is the observed minus computed residual for the kth time point and $[X_K]$ is the integer nearest to X_K. Compute a mean and sigma by

$$\overline{\Delta T}_p = \sum_{k=1}^{T_p} \frac{\Delta T_K}{T_p} \qquad T_p = \text{number of time points in the pass}$$

$$\text{Sigma}\ (p) = \left[\sum_{k=1}^{T_p} \frac{(\Delta T_K - \overline{\Delta T}_p)^2}{(T_p - 1)} \right]^{1/2}$$

Filter the reference station passes as indicated in the diagram below.



After filtering all reference station passes use all the good points from all accepted passes in a least square process to determine TSO and NCS. The necessary partials are

$$\frac{\partial \Delta T_K}{\partial \text{TSO}} = -1$$

$$\frac{\partial \Delta T_K}{\partial \text{NCS}} = \frac{-[X_K]}{fso}$$

The process iterates until $|\Delta \text{TSO}| < .1\ \text{E} - 05$ and the number of data points used remain constant for two iterations. Each iteration of the least square process goes back through the reference station filtering. After convergence a print line is produced.

TSO = initial value NCS = initial value

Iteration No.	TSO	ΔTSO	NCS	ΔNCS
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Final Value

TSO = NCS =

A table of print is produced titled Reference Station Analysis. The table is

Station No.	Time	Points Available	Points Rejected	Mean (micro sec)	Sigma (micro sec)
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

6. Determine time corrections for all passes.

Mode 3

a. Compute the clock error (CE) and drift rate (DR) from the Mode 3 data. The Mode 3 header is

Temperature	Pressure	Humidity	Comment

(Temperature, Pressure) = clock offset (microsec)

$$\text{Humidity} = \text{clock drift} \left(\frac{\text{microsec}}{\text{hour}}\right)$$

Comment = a code to interpret the clock drift

Comment code

0 = positive drift between 0 and 99
1 = negative drift between 0 and 99
2 = positive drift between 100 and 199
3 = negative drift between 100 and 199
.
.
.
9 = negative drift between 400 and 499

Example

If the clock drift value is 67 and the comment is 3 then the actual drift is −167 microseconds/hour. The clock error is the clock offset plus or minus the equipment delay. The sign of the clock offset determines the plus or the minus option. The actual computations are

$$C\,E_{sec} = \text{Sign of Temp.} * \left[\frac{(\text{Abs (Temp)} * 1000 + \text{Press.})}{1.E-06} + \text{Equip. Delay}\right] \tag{24}$$

$$DR_{sec/sec} = \begin{bmatrix} +1 \text{ if comment is 0 or even} \\ -1 \text{ if comment is odd} \end{bmatrix} * \left[\left[\frac{\text{interger part of comment}}{2}\right] * 100 + \text{Hum}\right] * \frac{1.E-06}{3600} \tag{25}$$

b. These computations from the Mode 3 pass are recorded on the Station Table as follows.

Retrieve the number of segments (max. of 3) from the table for the given station and day. NavSat segments are time corrections determined from navigation satellite time correction computations. Mode 3 segments are those computed as described above. NavSat segments are labeled 1, 2, 3; and Mode 3 segments are labeled 31, 32, 33.

If the segmentation is from a NavSat skip writing the Mode 3 corrections on the table.

If the segmentation is from Mode 3 and there are 3 segments, skip writing the corrections on the table.

If segmentation is from Mode 3 and there are not 3 segments, then, providing the Mode 3 is not a duplicate (same yr, day, sec), extrapolate the previous clock error, using the previous drift rate, to the new epoch. Compare with the new clock error (compute difference). If the $|\text{difference}| < .005$ sec., continue; if not, change the sign of the new clock error and compare again. If this gives a satisfactory comparison, continue; if not, change the sign back to the original sign and continue.

Continue = Store the new Mode 3 results on the Station Table. If the Mode 3 being written is the first segment, then the drift rate is used to back up the clock error from the given epoch to zero seconds of the day.

Type 8 (Geoceiver)

In the event time corrections are computed for a geoceiver station from NavSat data the following procedure is used.

a. Use all time points in the geoceiver pass as time corrections since geoceiver data is taken at equally spaced intervals.

b. If the pass being processed is a Mode 9 (station clock reset pass) the station table is segmented at this point.

c. *Define*

$$TC_i = \bar{T}_i + t_{tr_i} + t_{ed} + S_i + \text{TSO} \tag{26}$$

where

$\bar{T}_i$ = nearest half minute of the ith observation time in seconds

t_{tr_i} = ith transmission time

t_{ed} = equipment delay

TSO = previously determined TSO value of satellite clock error

S_i = constant readout delay as furnished by Magnavox

$$S_i = \begin{cases} 0.0 \text{ for even 2 min. time mark} \\ 2.071100 \text{ for even 2 min. time plus 30 sec} \\ -0.186793 \text{ for even 2 min. time plus 60 sec} \\ 2.020318 \text{ for even 2 min. time plus 90 sec} \end{cases}$$

d. Compute individual corrections, the average correction and sigma for the pass.

$$\Delta T_i = T_i - TC_i$$

$$\overline{\Delta T}_p = \sum_{i=1}^{N_p} \frac{\Delta T_i}{N_p}$$

$$\text{Sigma } (p) = \left[\sum_{i=1}^{N_p} \frac{(\Delta T_i - \Delta T_p)^2}{N_p - 1} \right]^{1/2}$$

e. If $|\Delta T_i - \overline{\Delta T}_p| > 2.5$ Sigma (p) or if $|\Delta T_i| > .9$ sec, delete the point and recompute $\overline{\Delta T}_p$ and Sigma (p). Continue this process until no more points are deleted. If after no more points are deleted there are less than two points remaining or Sigma (p) > .0001, delete the pass and put a flag in arrays DCB and STD. If two or more points remain and Sigma (p) ⩽ .0001, then write $\overline{\Delta T}_p$ in array DCB and Sigma (p) in array STD. These arrays will be used later to obtain daily average time corrections for the Station Table.

Types 7 and 9 (Frequency and CCID)

For each time point determine if the satellite was above the horizon at that time. If not, delete the point. For all remaining points compute

$$T_K = t_K - (t_{tr_K} + t_{sd} + t_{ed} + \text{one bit}) \tag{28}$$

where

t_K = time of reception of the timing signal

t_{tr_K} = kth transmission time

t_{sd} = satellite clock delay

t_{ed} = equipment delay

one bit = one bit delay

T_K = time of emission of timing signal

$$X_K = \frac{fso}{\text{NCS}} (T_K - \text{TSO}) \tag{29}$$

where NCS and TSO are the previously determined values

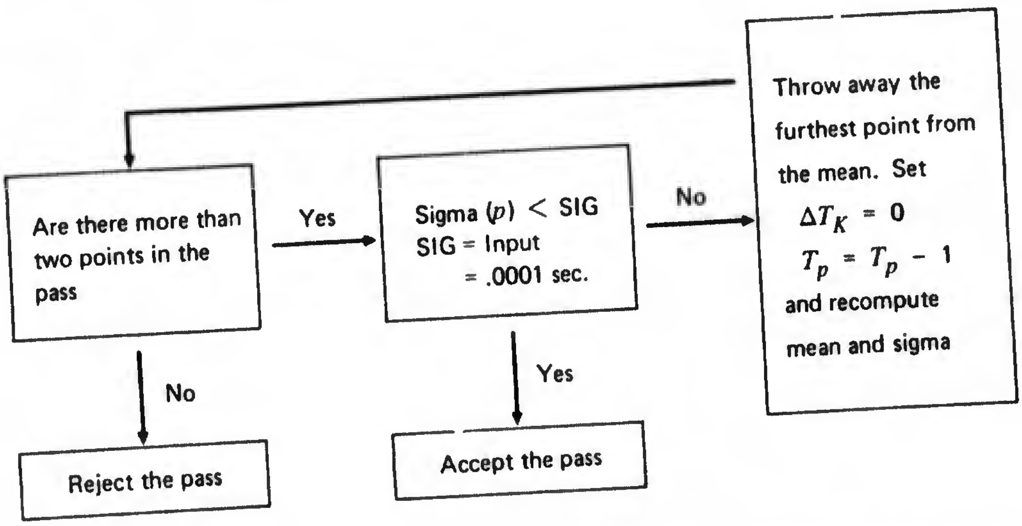
$$\Delta T_K = T_K - \left(\text{TSO} + \frac{\text{NCS}}{fso} [X_K] \right) \tag{30}$$

Delete all ΔT_K with $|\Delta T_K| > T_1$ an input tolerance usually set at .0005 sec. If fewer than three points remain use T_2 = .0200 sec and retest all ΔT_K against this tolerance. If again less than three points remain, delete this pass from the time correction process. Compute mean and sigma by

$$\overline{\Delta T_p} = \sum_{k=1}^{T_p} \frac{\Delta T_K}{T_p} \qquad T_p = \text{number of timing points}$$

$$\text{Sigma } (p) = \left[\sum_{k=1}^{T_p} \frac{(\Delta T_K - \overline{\Delta T_p})^2}{(T_p - 1)} \right]^{1/2}$$

Filter passes by the diagram below.



If a pass is rejected place a flag in DCB. If a pass is accepted write $\overline{\Delta T_p}$ in DCB and Sigma (p) in STD.

At this point all time corrections have been determined and placed in Array DCB. Each non-Mode 3 pass on Tape 8 is individually time corrected based on the values in DCB. The time corrected data is written on Tape 14. Flags in DCB indicate that the data for that pass should not be corrected yet.

A print line is produced for each pass.

Mode 3

Station No.	Day	Time	Clock Error	Drift Rate
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Non-Mode 3

Station No.	Day	TCA	Correction	Sigma
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

All time corrections are now sorted by time within station and printed under *Station Analysis.*

Station No.	Time	No. of Points Available	No. of Points Rejected	Time Correction	Sigma
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

7. Determine average time corrections per station.

a. Type 7 and 9 stations obtain their average corrections by averaging the average corrections per pass over the given stations. These values are written on the Station Table over any other values that may already be there. Type 7 and 9 passes which were flagged in DCB are now corrected using the Station Table value and placed on Tape 14. A print line is produced for each such pass under Average Time Corrections Applied to These Passes.

Station No.	Time	Time Correction
.	.	.
.	.	.
.	.	.

b. Type 8 passes flagged in DCB are not written on Tape 14.

c. Mode 3 passes have already been written on the station table.

d. Type 8 stations can have their corrections written on the station table as soon ar proper drift rates are computed and the day is properly segmented. Recall that Mode 3 data is time correction data and segments the day of a geoceiver station. Geoceiver data (Type 8) can also be used to segment the day and will write over Mode 3 corrections.

Using all non-flagged geoceiver passes from DCB whose sigma from STD is less than .000075 sec we will compute drift rates by

Drift 2

Pass 1, Pass 2, Pass 3, . . . Pass i, Pass $(i + 1)$, . . .

Drift 1 Drift (i)

where the passes have been sorted to time within station.

If any $|\text{Drift } (i)| > .9\text{ E} - 07$, the pass is set to a Mode 9. A Mode 9 is a clock reset pass and can either be so indicated in the pass header or determined in the above fashion. The Mode 9 segments the day with a maximum of 3 segments permitted.

Process the passes in each segment independently to compute an average drift per segment.

$$\text{Average Drift}_{\text{segment}} = \begin{cases} \text{average of drifts computed in the segment if the number of drifts is greater than 1.} \\ \text{Drift from the previous segment if number of drifts not greater than 1.} \end{cases}$$

If there are no passes in the segment (a possibility in Segment 1), extrapolate the previous clock error to the start of the day using the previous drift rate. Write this result on the station table only if there is nothing already written there.

If there are passes in the segment there are two cases to consider.

Segment 1

Back up the clock error of the first pass in Segment 1 to zero sec of the day using the average drift for Segment 1. The time associated with a clock error $\overline{\Delta T_p}$ in DCB is the pass rise time. Write the resulting clock error, average drift for Segment 1 and zero sec on the station table.

Segments 2 and 3

If the segmentation was determined by a Mode 9 defined in the header of a pass, use the clock error of that pass, the rise time of that pass, and the average drift of the segment as the values to be written on the station table.

If the segmentation was determined by $|\text{Drift } (i)| > .9\text{ E} - 07$, use the clock error of Pass $(i + 1)$, rise time of Pass $(i + 1)$, and the average drift of the segment as values to be written on the station table.

8. Print under Stored Time Corrections

Segment Time	Segment	Station	Time Correction	Avg. Drift
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

FILTER

The basic input to the filter is an inertial perturbed trajectory and an observation file. The trajectory contains necessary satellite positions and their associated partial derivatives with respect to possible fit parameters. The observation file contains station coordinates, observation values, and estimates of the standard error for each observation.

The basic output is a Pass Matrix File consisting of pass normal equations for each satellite pass and an Observation File consisting of good observations and best estimates for their standard errors.

The process of point filtering is carried out on a pass by pass basis. The basic concept is to remove model error by fitting the orbit to the given pass of data and using the result to compute a standard error for the data points. Residuals are then adjusted for the solution of the fit. The adjusted residuals are tested against a sigma tolerance.

$$\text{Adjusted Residual} < \text{Tolerance} * \text{Sigma} * \text{Sigma Multiplier}$$

If the adjusted residual of the observation fails the test, then the point is labeled bad for the next iteration through the filter. The adjusted residuals of all points are tested, even those which were labeled bad for the present fit. The next iteration will use the newly determined good points, the new sigma values, and the original reference orbit. The tolerance used above is either computed as a function of the good points or taken as an input constant. The sigma for the observation is continually recomputed by initializing it at a value from the pre-processor and adjusting it after each iteration through filter by multiplying the initial value with a sigma multiplier. The sigma multiplier is that number which when used will cause the RWS of adjusted residuals to equal one.

$$\left[\sum_{\text{Present Fit}} \left[\frac{\text{Adjusted Residual}}{\text{Sigma}\cdot\text{Sigma Multiplier}}\right]^2 \Big/ N\right]^{1/2} = 1$$

N = number of good points + number of bias parameters used in filtering.

The fit used at each iteration of filtering solves for a subset (best determined parameters) of refraction scaling, frequency bias, frequency drift, and the six position and velocity parameters obtained by resolving position and velocity in the local frame at time of closest approach (TCA), Diagram 5. The solution attempts to determine which of the above parameters are well determined and carry out the solution for those parameters only. To do this the pass normal matrix, with the present set of good values, is transformed to the local frame at TCA and relabeled so that the parameters appear in the above order. The first bias parameter is accepted as being well determined. This parameter is eliminated and the next bias parameter is examined as follows:

Check the main diagonal of the second bias parameter from the eliminated normal matrix against the same term in the original normal matrix.

$$\frac{B^{\text{Eliminated}}_{\text{bias,bias}}}{B_{\text{bias,bias}}} > 10^{-4}$$

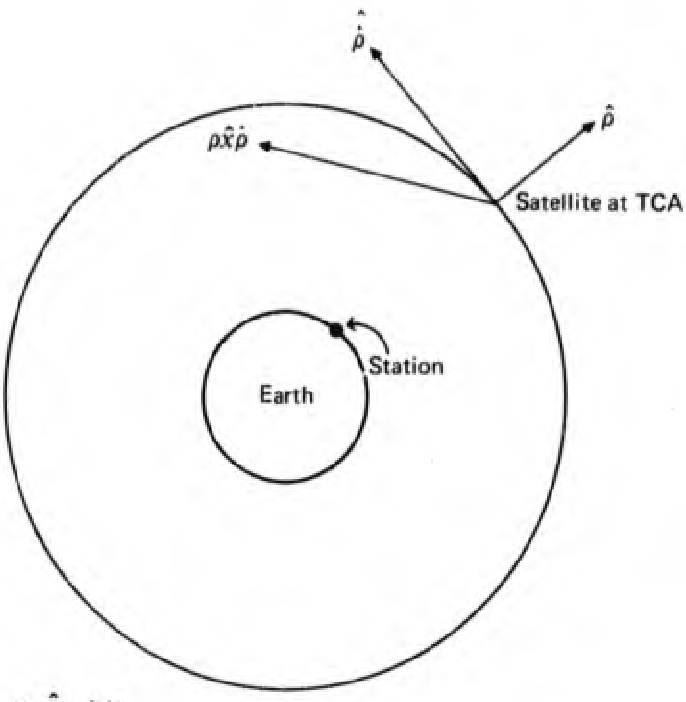


Diagram 5

If this test fails the parameter is dropped and the next bias is examined. If the test succeeds the parameter is eliminated and the next bias is examined using the newly eliminated matrix.

When the position and velocity parameters are reached, the same procedure is used, but the test is

$$\frac{B^{\text{Eliminated}}_{x_i x_i}}{\sum_{i=1}^{3} B_{x_i x_i}} > 10^{-4}$$

where x_i is a position parameter and

$$\frac{B^{\text{Eliminated}}_{\dot{x}_i \dot{x}_i}}{\sum_{i=1}^{3} B_{\dot{x}_i \dot{x}_i}} > 10^{-4}$$

where $\dot{x}_i$ is a velocity parameter.

The iteration process terminates when either the same number of points are labeled good three successive times or a maximum iteration count is reached. Should the process terminate without reaching maximum iteration the normal matrix formed will have the best estimate of good points and associated weights. This matrix is placed on the Pass Matrix File.

POINT FILTERING PROCEDURE

1. Read the observation file for the next (or first) good pass.

2. Convert the station coordinates (from the Observation File) from geodetic to rectangular earth fixed components (31).

3. Calculate the terms (N_1, N_2, R_0, R_1, R_2) used in the refraction model (37).

4. Determine the time of closest approach (TCA) of the pass by

   a. Using the value from the Observation File.
   b. Using the value from the Observation File for an initial estimate and iterating on $\rho * \dot{\rho}$ for a maximum input number of iterations (34).

5. Compute the slant range (ρ) at TCA (32).

6. Check the zenith angle at TCA against the input zenith angle tolerance. Reject the pass if the zenith angle at TCA is greater than the tolerance (35).

7. Count the observations before and after TCA. Check the difference against the input unbalanced pass tolerance. If the difference is less than the tolerance reject the pass.

8. Interpolate for the satellite and station position, velocity, and acceleration at TCA. Interpolate for the satellite perturbations at TCA. Save these values and the zenith angle at TCA for later use (36).

9. Compute the slant range or range rate at an observation time (32), (33).

10. Check the zenith angle at the observation time against an input tolerance. If the zenith angle is greater than the tolerance then tag the point.

11. Compute the refraction effect on range or range rate for a good point (46), (47).

12. Interpolate for trajectory perturbations at the observation time (36).

13. Form the data partials for station, orbit, and bias parameters (48)–(61).

14. Compute residual values (48)–(61).

15. Compute the contribution to the condition equation (64).

16. Form the pass normal equations (65). This matrix is given in Diagram 6.

Canonical Pass Matrix

$$\begin{bmatrix} B_{ee} & B_{ec_D} & B_{eA} & B_{ekr} & B_{eb} & E_e \\ & B_{c_Dc_D} & B_{c_DA} & B_{c_Dkr} & B_{c_Db} & E_{c_D} \\ & & B_{AA} & B_{Akr} & B_{Ab} & E_A \\ & * & & B_{krkr} & B_{krb} & E_{kr} \\ & & & & B_{bb} & E_b \end{bmatrix}$$

e = orbital elements or coordinates-velocity at some epoch t_s
c_D = drag parameter
A = three thrust parameters
kr = radiation pressure parameter
b = bias parameter set consisting of three earth fixed station coordinates, refraction scaling, frequency bias and frequency drift.

Diagram 6

17. Pass initialization:

Set $\Delta p_{\text{navigate}} = 0$

$\sigma_{\text{multiplier}} = 1$

Tags = as defined by Observation File

18. Adjust the residuals, $(0 - D)_i$, due to the navigation solution, (68) by

$$(0 - D)_i \text{ adjusted} = (0 - D)_i - \frac{\partial D_i}{\partial p} \Delta p_{\text{navigate}}$$

19. Tag $(0 - D)_i$ bad if

$|(0 - D)_i \text{ adjusted}| >$ Absolute Tolerance or

$|(0 - D)_i \text{ adjusted}|^2 > \text{Sigtoi}^2 \times \sigma_i^2 \times \sigma_{\text{multiplier}}^2$

20. Test the number and percentage of points remaining against input tolerances. If violated delete the pass.

21. Reform the pass normal equations with the present set of good points.

22. Transform the orbit and bias section of the normal equations to the local TCA frame using the values saved at step 8 (68).

23. Solve the transformed equations (68).

24. Resolve the solution in the original frame obtaining $\Delta p_{navigate}$.

25. Perform filter test as follows:

If the same number of points have been tagged on this iteration as the last two or if the iteration count is exhausted, then we say that this is the last iteration. In this case print all necessary statistics, write the residual file as directed by input, and write the normal equations on the Pass Matrix File. On option, a filtered Observation File can also be written consisting of good observations and the latest estimate of their standard errors.

If this is not the last iteration then

a. Tag all points good except the zenith angle rejects.
b. Set $\sigma^2_{multiplier} = (S/N)^2_{predicted} * \sigma^2_{multiplier}$.

c. Set Sigtol $= \left(2 + \frac{N}{N + 50}\right) * \frac{N}{N_R}$ or input constant

where N is the total number of points in the pass less zenith angle rejects, N_R is the number of points remaining at the last iteration, and $(S/N)_{predicted}$ is the predicted signal to noise value discussed under *O*ut put *S*tatistics. The sigma tolerance (Sigtol) for the first iteration is an input with default equal to $E + 50$. The absolute tolerance is constant and also comes from input with default values

5 km for class 9 data

$(E - 06)*F_s$ for class 7 data

d. Go to Step 18.

STATION COORDINATE CONVERSION

$$r_{s0} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = (A + h_s)\begin{pmatrix} \cos\varphi_s \cos\lambda_0 \\ \cos\varphi_s \sin\lambda_0 \\ \sin\varphi_s \end{pmatrix} - Ae^2\begin{pmatrix} 0 \\ 0 \\ \sin\varphi_s \end{pmatrix} \qquad (31)$$

where

r_{s0} = earth fixed station coordinates

h_s = station geodetic height above a reference ellipsoid

e = eccentricity of the reference ellipsoid

$$= [(2 - f)f]^{1/2} = \left[\frac{2 - \frac{1}{EL}}{EL}\right]^{1/2}$$

f = flattening

EL = oblateness

a = semi-major axis of the reference ellipsoid

φ_s = station geodetic latitude

λ_0 = longitude from Greenwich meridian

$A = a/(1 - e^2 \sin^2 \varphi_s)^{1/2}$

RANGE AND RANGE RATE COMPUTATION

$$\rho = \rho(t_R) = r(te) - r_s(t_R) \tag{32}$$

where

$te = t_R - (\rho * \rho)^{1/2}/c$ $\quad$ t_R = reception time

te to be determined by iteration starting with $te = t_R$

$$\dot{\rho} \underset{\text{order}}{\overset{1^{\text{st.}}}{=}} \dot{r}(t_e) - \dot{r}_s(t_R) - \hat{\rho}^*/c(\dot{r}(t_e) - \dot{r}_s(t_R))\dot{r}(t_e) \tag{33}$$

where

$\dot{r}_s = \Omega r_s = \overline{\omega} \times r_s$ from *Tropospheric Refraction Model*

$\ddot{r}_s = \Omega \dot{r}_s = \Omega^2 r_s$

COMPUTATION OF TIME OF CLOSEST APPROACH (TCA)

Define

$$t_{i+1} = t_i - (\rho * \dot{\rho})_{t_i} / (\dot{\rho} * \dot{\rho} + \rho * \ddot{\rho})_{t_i} \tag{34}$$

In the event TCA is to be searched for, set t_1 = value from the observation file and iterate until $|t_{i+1} - t_i| <$ .05 sec or maximum iteration count is reached.

COMPUTATION OF ZENITH ANGLE

$$Z_v = \tan^{-1}([1 - (\hat{\rho} * \hat{u}_s)^2]^{1/2} / \hat{\rho} * \hat{u}_s) \tag{35}$$

$$u_s = (\text{ABCD})* \begin{pmatrix} s_1 \\ s_2 \\ s_3/(1 - e^2) \end{pmatrix}$$

INTERPOLATION PROCEDURE

The Lagrange interpolation formula comes from setting

$$y(t) = \sum_{i=1}^{n} \ell_i(t) f(t_i)$$

and finding polynomials ℓ_i such that this expression is an identity when y is itself a polynomial. This results in

$$\ell_i(t) = \pi(t)/(t - t_i)\pi'(t_i) \qquad t \neq t_1, t_2, \ldots t_n$$

$$\ell_i(t_j) = \delta_{ij}$$

$$\pi(t) = (t - t_1)(t - t_2) \ldots (t - t_n)$$

$$\pi'(t) = \sum_{i=1}^{n} \frac{\pi(t)}{(t - t_i)}$$

Letting $t = t_1 + sh$ where h is the data storage step for equally spaced data gives

$$\ell_i(t) = \frac{\underset{j \neq i}{\pi(t - t_j)}}{\underset{j \neq i}{\pi(t_i - t_j)}} = \ell_i(t_1 + sh) = \frac{\underset{j \neq i}{\pi(s - j + 1)}}{\underset{j \neq i}{\pi(i - j)}} = \ell_i(s)$$

Thus

$$y(t) = y(t_1 + sh) = \sum_{i=1}^{n} \ell_i(s) f_i$$

$$\dot{y}(t) = \dot{y}(t_1 + sh) = \sum_{i=1}^{n} \frac{\ell_i'(s) f_i}{h}$$

$$\ddot{y}(t) = \ddot{y}(t_1 + sh) = \sum_{i=1}^{n} \frac{\ell_i''(s)}{h^2} f_i \qquad (36)$$

where

$$\ell_i(s) = \pi_{j \neq i}(s - j + 1) \Big/ \pi_{j \neq i}(i - j)$$

$$\ell_i'(s) = \ell_i(s) \sum_{j \neq i} \frac{1}{s - j + 1} = \ell_i(s) g_i$$

$$\ell_i''(s) = \ell_i(s) \sum_{j \neq i} \frac{1}{s - j + 1} \left(g_i - \frac{1}{s - j + 1} \right)$$

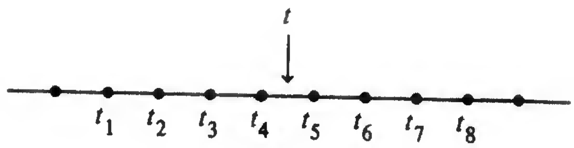
$$\ell_{i \neq 4}'(3) = \pi_{\substack{j \neq i \\ j \neq 4}}(s - j + 1) \Big/ \pi_{j \neq i}(i - j)$$

$$\ell_4'(3) = -1/4$$

$$\ell_{i \neq 4}''(3) = 2 \sum_{\substack{k \neq 4 \\ k \neq i}} \prod_{\substack{j=1 \\ j \neq i \\ j \neq 4 \\ j \neq k}}^{8} (4 - j) \Big/ \pi_{j \neq i}(i - j)$$

$$\ell_4'' 3 = 2 \sum_{\substack{k, \ell = 1 \\ k \neq \ell \neq 4}}^{8} \frac{1}{(4 - k)(4 - \ell)} = -49/18$$

This is an eight point Lagrange central interpolation procedure. If asked to interpolate for a value at time t, the interpolation routine locates t in $[t_4, t_5)$ and uses the above computations. First and second derivative values are obtained by using the above derivative capabilities of the routine.



TROPOSPHERIC REFRACTION MODEL (HOPFIELD)

N_{T_1} = dry team

N_{T_2} = wet term

$R_0 = \|r_{s0}\|$

$R_1 = R_0 + 40.1 + .149T$

$R_2 = R_0 + 12.0$

$N_1 = [(.776)*E-04]^{P/Tk}$

$N_2 = [(.373)*E-02]^{E/Tk^2}$

E = water vapor pressure

H = humidity (percent)

P = pressure (millibars)

T_k = temperature (Kelvin)

$= T + 273$

H = 50% (default value)

P = 980 (default value)

T = 15 (default value) (37)

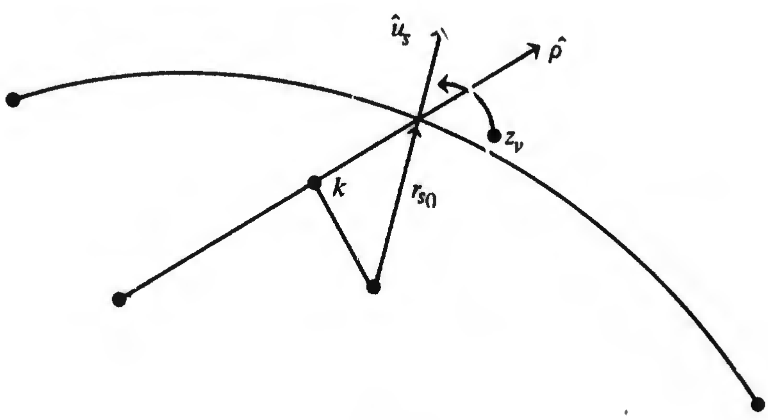
$E = H \exp(-37.2465 + .213166\, T_k - .000256908\, T_k^2)$

n = index of refraction

$n - 1 = N_{T_1} + N_{T_2}$

$$N_{T_i} = \begin{cases} N_i \left(\dfrac{r^2 - R_i^2}{R_0^2 - R_i^2} \right)^4 & \text{if } r \text{ is in } [R_0, R_i] \\ 0 & \text{if } r \text{ is not in } [R_0, R_i] \end{cases} \quad (38)$$

Let $\hat{u}_s$ be the unit normal to the ellipsoid at the station.



$$u_s = (\mathrm{ABCD})^* \begin{pmatrix} s_1 \\ s_2 \\ s_3/(1 - e^2) \end{pmatrix}$$

$$k = \|r_{s0}\| \sin z_v = R_0 \sin z_v$$

$$\cos z_v = \hat{u}_s^* \hat{\rho}$$

The refraction model is

$$(1 + C_R)\Delta f_R \tag{39}$$

$$\Delta f_R = \begin{cases} -\dfrac{f_s}{C} \Delta \dot{R} & \text{(range rate)} \\ \Delta R & \text{(range)} \\ \Delta R(t_n) - \Delta R(t_{n-1}) & \text{(range difference)} \end{cases} \tag{40}$$

$$\Delta R = \int_{r_{\text{station}}}^{r_{\text{satellite}}} (n - 1) \frac{r^* dr}{(r^* r - k^2)^{1/2}} \tag{41}$$

$$\Delta \dot{R} = k\dot{k} \int_{r_{\text{station}}}^{r_{\text{satellite}}} (n - 1) \frac{r^* dr}{(r^* r - k^2)^{3/2}} \tag{42}$$

Computation of $k\dot{k}$ in Earch Fixed and Inertial Frames

Earth Fixed

$$k^2 = r_{s0}^* r_{s0} - (r_{s0}^* \hat{\rho})^2$$

$$k\dot{k} = -(r_{s0}^* \hat{\rho})(r_{s0}^* \dot{\hat{\rho}})$$

$$\dot{\hat{\rho}} = (\rho/|\rho|) = \frac{|\rho|\dot{\rho} - \rho|\dot{\rho}|}{|\rho|^2}$$

$$= \frac{\dot{\rho}}{|\rho|} - \frac{\rho|\dot{\rho}|}{|\rho|^2} = \frac{\dot{\rho}}{|\rho|} - \frac{\hat{\rho}}{|\rho|}|\dot{\rho}|$$

$$= \frac{\dot{\rho}}{|\rho|} - \frac{\hat{\rho}}{|\rho|}\frac{\rho^* \dot{\rho}}{|\rho|} = \frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^* \dot{\rho})\hat{\rho}}{|\rho|}$$

$$k\dot{k} = -(r_{s_0}^* \hat{\rho})\left(r_{s_0}^* \left[\frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^*\dot{\rho})}{|\rho|} \hat{\rho}\right]\right)$$

$$= -\frac{R_0^2}{|\rho|} \cos z_v \dot{\rho}^* [I - \hat{\rho}\hat{\rho}^*] \hat{u}_s$$

$$= -\frac{R_0^2}{|\rho|} \cos z_v \dot{r}^* [I - \hat{\rho}\hat{\rho}^*] \hat{u}_s$$

Inertial

$$k^2 = r_s^* r_s - (\hat{\rho}^* r_s)^2$$

$$k\dot{k} = r_s^* \dot{r}_s - (\hat{\rho}^* r_s)\dot{(\hat{\rho}^* r_s)}$$

$$= r_s^* \dot{r}_s - [\hat{\rho}^* \dot{r}_s + \dot{\hat{\rho}}^* r_s](\hat{\rho}^* r_s)$$

$$= r_s^* \dot{r}_s - \left[\hat{\rho}^* \dot{r}_s + r_s^* \left(\frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^*\dot{\rho})}{|\rho|} \hat{\rho}\right)\right](\hat{\rho}^* r_s)$$

$$r_s = (ABCD)^* r_{s_0}$$

$$\dot{r}_s = (A\dot{B}CD)^* r_{s_0}$$

$$= (A\dot{B}CD)^* (ABCD) r_s$$

$$= (CD)^* \dot{B}^* B (CD) r_s$$

$$= (CD)^* \begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD) r_s$$

$$= \Omega r_s \tag{43}$$

$$\dot{r}_s = \Omega r_s = \bar{\omega} \times r_s \tag{44}$$

where

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \cos(\tilde{\omega}t) & \sin(\tilde{\omega}t) & 0 \\ -\sin(\tilde{\omega}t) & \cos(\tilde{\omega}t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\omega} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

Remarks

$$\begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} r_s = \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \times r_s$$

$$\begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD) r_s = \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \times (CD) r_s$$

Since (CD) is orthogonal it preserves orthogonal frames, i.e.,

$$(CD)^* \begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD) r_s = (CD)^* \left[\begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \times (CD) r_s \right]$$

$$= (CD)^* \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \times (CD)^* (CD) r_s$$

$$= (CD)^* \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \times r_s$$

Thus

$$\dot{r}_s = \bar{\omega} \times r_s$$

where

$$\bar{\omega} = (CD)^* \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix} \tag{45}$$

Continuing with the computation of $k\dot{k}$ gives

$$k\dot{k} = -\left[\frac{\rho^* \dot{r}_s}{|\rho|} + r_s^* \left(\frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^* \dot{\rho})\hat{\rho}}{|\rho|} \right) \right] (\hat{\rho}^* r_s)$$

$$= -\left[\rho^* \frac{(\bar{\omega} \times r_s)}{|\rho|} + r_s^* \left(\frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^* \dot{\rho})}{|\rho|} \right) \hat{\rho} \right] (\hat{\rho}^* r_s)$$

$$= r_s^* \left[\frac{(\bar{\omega} \times \rho)}{|\rho|} - \left(\frac{\dot{\rho}}{|\rho|} - \frac{(\hat{\rho}^* \dot{\rho})}{|\rho|} \hat{\rho} \right) \right] (\hat{\rho}^* r_s)$$

$$k\dot{k} = -\frac{R_0^2}{|\rho|} \cos z_v [(I - \hat{\rho}\hat{\rho}^*)\dot{\rho} - \bar{\omega} \times \rho]^* \hat{u}_s$$

Computation of ΔR and $\Delta \dot{R}$

$$I_i = \int_{R_0}^{R_i} \frac{(r^*r - R_i^2)^n}{(r^*r - k^2)^{m/2}} r^*dr$$

$$= \int_{R_0}^{R_i} \frac{[(r^*r - k^2) + (k^2 - R_i^2)]^n}{(r^*r - k^2)^{m/2}} r^*dr$$

$$= \int_{R_0}^{R_i} \sum_{j=0}^{n} \binom{n}{j} (r^*r - k^2)^{j - m/2 + 1} (k^2 - R_i^2)^{n-j} r^*dr$$

$$= \sum_{j=0}^{n} \binom{n}{j} \frac{(r^*r - k^2)}{2j - m + 2} (k^2 - R_i^2)^{n-j} \Bigg|_{R_0}^{R_i}$$

Letting $R_0 = r_{\text{station}}$ and $R_i = r_{\text{satellite}}$ we can compute range and range rate corrections by

$$\Delta R = \int_{R_0}^{R_i} \frac{(n-1)r^*dr}{(r^*r - k^2)^{1/2}} = \sum_{i=1}^{2} \frac{N_i}{(R_0^2 - R_i^2)^4} \sum_{j=0}^{4} \frac{1}{2j+1} \binom{4}{j} (r^*r - k^2)^{j+1/2} (k^2 - R_i^2)^{4-j} \Bigg|_{R_0}^{R_i} \tag{46}$$

$$\frac{\Delta \dot{R}}{k\dot{k}} = \int_{R_0}^{R_i} \frac{(n-1)r^*dr}{(r^*r - k^2)^{3/2}} = \sum_{i=1}^{2} \frac{N_i}{(R_0^2 - R_i^2)^4} \sum_{j=0}^{4} \frac{1}{2j-1} \binom{4}{j} (r^*r - k^2)^{j-1/2} (k^2 - R_i^2)^{4-j} \Bigg|_{R_0}^{R_i} \tag{47}$$

DATA CLASSES

$$D_9 = \left[|\rho| - \frac{c}{f_{00}} f_b(t - \text{TCA}) - \frac{c}{2f_{00}} \dot{f}_b(t - \text{TCA})^2 + (1 + c_R)\Delta f_R \right]_{t_{i-1}}^{t_i} \tag{48}$$

where f_{00} is an input quantity usually taken as $E + 06$ so that the bias solution will be in parts per million

$$\frac{\partial D_9}{\partial q} = \frac{\partial [\,]_i}{\partial q} - \frac{\partial [\,]_{i-1}}{\partial q} \qquad q = p_k \text{ (orbit parameter)},\; x_{s0} \text{ (station parameter)},\; c_R, f_b, \dot{f}_b \tag{49}$$

$$\frac{\partial [\,]}{\partial p_k} = \hat{\rho}^* \frac{\partial x}{\partial p_k} = \rho^* \psi_{p_k} \text{ (column of } \psi \text{ matrix)} \tag{50}$$

$$\frac{\partial[\,]}{\partial x_{s0}} = -\hat{\rho}^*(ABCD)^* \tag{51}$$

$$\frac{\partial[\,]}{\partial c_R} = \Delta f_R \tag{52}$$

$$\frac{\partial[\,]}{\partial f_b} = -\frac{c}{f_{00}}(t - \mathrm{TCA}) \tag{53}$$

$$\frac{\partial[\,]}{\partial \dot{f}_b} = -\frac{c}{2f_{00}}(t - \mathrm{TCA})^2 \tag{54}$$

D_7 = (Range rate) Frequency

The vacuum received frequency using a transmitted frequency of f_s is given by

$$f_R = f_s \left(\frac{1 + \dfrac{\hat{\rho}^* \dot{r}_s(t_R)}{c}}{1 + \dfrac{\hat{\rho}^* \dot{r}(t_e)}{c}} \right)$$

where $\rho(t_R) = r(t_e) - r_s(t_R)$

$$t_e = t_R - \frac{(\rho^*\rho)^{1/2}}{c} = \text{emitted time}$$

t_R = received time

Computing $\dot{\rho}$ from the expression for ρ and taking into account the change in t_e as a function of t_R gives

$$\dot{\rho} = \frac{\dot{r}(t_e) - \dot{r}_s(t_R)}{\left(1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\right)} + \frac{\dfrac{\hat{\rho}^*\dot{r}_s(t_R)}{c}\dot{r}(t_e) - \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\dot{r}_s(t_R)}{\left(1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\right)}$$

$$1 - \frac{\hat{\rho}^*\dot{\rho}}{c} = \frac{1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c} - \left(\dfrac{\hat{\rho}^*\dot{r}(t_e)}{c} - \dfrac{\hat{\rho}^*\dot{r}_s(t_R)}{c}\right)}{\left(1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\right)}$$

$$= \frac{\left(1 + \dfrac{\hat{\rho}^*\dot{r}_s(t_R)}{c}\right)}{\left(1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\right)}$$

$$= \frac{f_R}{f_s}$$

Thus if $\dot{\rho}$ is computed to first order in $1/c$, f_R/f_s will be given to second order in $1/c$.

$$\dot{\rho} \underset{\text{order}}{\overset{1^{\text{st.}}}{=}} \dot{r}(t_e) - \dot{r}_s(t_R) - \hat{\rho}^*/c(\dot{r}(t_e) - \dot{r}_s(t_R))\dot{r}(t_e)$$

Adding in the frequency and refraction bias terms gives

$$D_7 = f_s\left(1 - \frac{\hat{\rho}^*\dot{\rho}}{c}\right) + \frac{f_s}{f_{00}}\left(1 - \frac{\hat{\rho}^*\dot{\rho}}{c}\right)f_b + \frac{f_s}{f_{00}}\left(1 - \frac{\hat{\rho}^*\dot{\rho}}{c}\right)\dot{f}_b(t - \text{TCA}) + (1 + c_R)\Delta f_R \tag{55}$$

The above formula is accurate to second order in $1/c$, whereas the corresponding formula for class 9 data is accurate to all orders in $1/c$.

$$\frac{\partial D_7}{\partial p_k} = -\frac{f_s}{c}\left[\frac{\dot{\rho}^*}{|\rho|}(I - \hat{\rho}\hat{\rho}^*)\psi_{p_k}(1,2,3) + \hat{\rho}^*\psi_{p_k}(4,5,6)\right] \tag{56}$$

where $\psi(1,2,3)$ is the position part of the perturbation matrix and $\psi(4,5,6)$ is the velocity part.

$$\frac{\partial D_7}{\partial x_{s0}} = \frac{f_s}{c}\left[\frac{\dot{\rho}^*}{|\rho|}(I - \hat{\rho}\hat{\rho}^*) + \hat{\rho}^*(CD)^*\begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}(CD)\right](ABCD)^* \tag{57}$$

$$\frac{\partial D_7}{\partial c_R} = \Delta f_R \tag{58}$$

$$\frac{\partial D_7}{\partial f_b} = \frac{f_s}{f_{00}}\left(1 - \frac{\hat{\rho}^*\dot{\rho}}{c}\right) \tag{59}$$

$$\frac{\partial D_7}{\partial \dot{f}_b} = \frac{f_s}{f_{00}}\left(1 - \frac{\hat{\rho}^*\dot{\rho}}{c}\right)(t - \text{TCA}) \tag{60}$$

D_9 is the formula used for all range difference data types, i.e.

Type 4 = ITT
5 = SGLS
8 = Geoceiver
9 = CCID

A similar formula is used for Type zero (range data)

$$D_{9,0} = |\rho| + R_b + \dot{R}_b(t - t_{\text{epoch}}) \tag{61}$$

D_7 is the formula used for all frequency data types, i.e.

Type 5 = SGLS
7 = Doppler

Residual values for all of the above data types can now be computed by differing the observed value (from Pre-Processor) from the computed value given above.

FORMATION OF CONDITION EQUATIONS

P = (6 orbit parameters (elements or coordinates), 1 drag parameter if drag is to be improved, 3 thrust parameters if thrust is to be improved, 1 radiation parameter if radiation is to be improved, refraction, frequency, frequency drift)

N = Number of data points in the pass

$$\sigma_p = (\sigma_1, \sigma_2, \ldots \sigma_{|p|}) \tag{62}$$

$$W = \text{Weight Matrix} = \begin{bmatrix} 1/\sigma_1^2 & & 0 \\ & 1/\sigma_2^2 \ddots & \\ 0 & & 1/\sigma_{|p|}^2 \end{bmatrix} \tag{63}$$

$$A = \frac{\partial D}{\partial p} = \begin{bmatrix} \dfrac{\partial D_{t_1}}{\partial p_1} & \cdots\cdots & \dfrac{\partial D_{t_1}}{\partial p_{|p|}} \\ \dfrac{\partial D_{t_N}}{\partial p_1} & \cdots\cdots & \dfrac{\partial D_{t_N}}{\partial p_{|p|}} \end{bmatrix} \tag{64}$$

FORMATION OF PASS NORMAL EQUATIONS

$$B_{\text{pass}} = A^*WA \qquad E_{\text{pass}} = A^*W(0 - D) \tag{65}$$

Each time pass normal equations are formed only the untagged points are used. The 0 in the above equation represents the observed data values from the Pre-Processor and the D represents the associated computed values.

SOLUTION OF PASS NORMAL EQUATIONS

The pass normal equations are given by

$$\begin{bmatrix} B(t_0)_{\text{orbit, orbit}} & B(t_0)_{\text{orbit, model}} & B(t_0)_{\text{orbit, bias}} & E(t_0)_{\text{orbit}} \\ B(t_0)_{\text{model, orbit}} & B(t_0)_{\text{model, model}} & B(t_0)_{\text{model, bias}} & E(t_0)_{\text{model}} \\ B(t_0)_{\text{bias, orbit}} & B(t_0)_{\text{bias, model}} & B(t_0)_{\text{bias, bias}} & E(t_0)_{\text{bias}} \end{bmatrix} \tag{66}$$

Transforming the (orbit,bias) sections to the TCA epoch gives

$$B_{0,0}(\text{TCA}) = \psi^{-1*}(\text{TCA})B_{0,0}(t_0)\psi^{-1}(\text{TCA})$$

$$B_{0,b}(\text{TCA}) = \psi^{-1*}(\text{TCA})B_{0,b}(t_0)$$

$$B_{b,b}(\text{TCA}) = B_{b,b}(t_0)$$

$$E_0(\text{TCA}) = \psi^{-1*}(\text{TCA})E_0(t_0)$$

$$E_b(\text{TCA}) = E_b(t_0)$$

Defining

$$R = [\hat{\rho}(\text{TCA}), \hat{\dot{\rho}}(\text{TCA}), \widehat{\rho x \dot{\rho}}(\text{TCA})] \tag{67}$$

$$\bar{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

we can transform the normal equations to the local R frame at TCA obtaining

$$\begin{bmatrix} \bar{R}^*B_{00}(\text{TCA})\bar{R} & \bar{R}^*B_{0b}(\text{TCA}) \\ B_{b0}(\text{TCA})\bar{R} & B_{bb}(\text{TCA}) \end{bmatrix} \begin{pmatrix} \Delta N \\ \Delta b \end{pmatrix} = \begin{bmatrix} \bar{R}^*E_0(\text{TCA}) \\ E_b(\text{TCA}) \end{bmatrix} \tag{68}$$

where ΔN represents parameter corrections in the R frame. The actual process for carrying out the solution in this frame is described below under *Singular Solution of Pass Normal Equations.*

Singular Solution of Pass Normal Equations

This solution will determine which parameters are well-determined and solve for those parameters. Generally these parameters come out to be position and velocity resolved in the radial and along track directions.

The pass normal equations are:

$$B\Delta p = E$$

or

$$\begin{bmatrix} B_{bb} & B_{b0} \\ B_{0b} & B_{00} \end{bmatrix} \begin{pmatrix} \Delta p_b \\ \Delta p_0 \end{pmatrix} = \begin{bmatrix} E_b \\ E_0 \end{bmatrix} \qquad \begin{matrix} b = \text{bias} \\ 0 = \text{orbit} \end{matrix}$$

$B\Delta p = E$ is equivalent to $R^*R\Delta p = R^*E_R$ where R is a triangular matrix with the property that $R^*R = B$. We can now solve $R\Delta p = E_R$. Since R is triangular R^{-1} can be easily determined. The solution is $\Delta p = R^{-1}E_R$, the predicted variance Vpred. = Variance - $E^*\Delta p$ = Variance - $E^*_R E_R$, and the covariance matrix $B^{-1} = (R^*R)^{-1} = R^{-1}R^{-1*}$. The algorithm for this work follows:

$$S_i = B_{ii} \text{ for biases (if any)}$$

$$S_i = \sum_{rca} B_{ii} \text{ for } \Delta R_{ca} \text{ components}$$

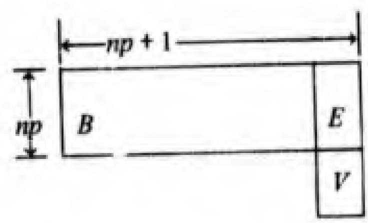
$$S_i = \sum_{\dot{r}ca} B_{ii} \text{ for } \Delta \dot{R}_{ca} \text{ components}$$

$$S_i = 0 \text{ for the variance}$$

$$dpr(B_{ij}, B_{k\ell}, m) = \sum_{t=1}^{m} B_{i+t-1,j} B_{k+t-1,\ell}$$

$$V_{\text{pred}} = B(np+1, np+1) - dpr(B_{1,np+1}, B_{1,np+1}, np)$$

Arrange the equations as follows:



Reduction of B to R

```
        ε = 10^-4
        NS = Np = # of parameters to solve for
┌────── do i = 1, np
│       t = Bii - dpr. (B1i, B1i, i - 1)
│       if (t - Si*ε) -, 0, +
│  0    ns = ns - 1
│     ┌ do j = i, np + 1
│     └ Bij = 0
│       Go To End of Loop
│  +    t = 1/√t
│     ┌ do j = i + 1, np + 1
│     └ Bij = (Bij - dpr.(B1i, B1j, i - 1))*t
│       Bii = t
└────── End loop
```

Determine R^{-1}

```
   ┌    do i = 1, np
   │ ┌  do j = i, np
   └─└  Bji = (Sji - dpr(Bii, Bij, j - i))*Bjj
```

Determine $\Delta p_i, \sigma_{pi}$

```
     ┌  do i = 1, np
     │  Δpi = dpr.(Bii, Bnp+1,i, np - i + 1)
     └  σpi = √dpr(Bii, Bii, np - i + 1)
```

OUTPUT STATISTICS

SNP = Signal to Noise Predicted

$$= \left[\frac{\text{Variance} - E^{*}\Delta p}{\text{No. good points} + \text{No. bias parameters}}\right]^{1/2}$$

SNPO = Signal to Noise Predicted of Observation

$$= \left[\frac{\text{Variance} - E^{*}\Delta p - \sum_{\text{bias}} \Delta\,\text{bias}^{*}\,\Delta\,\text{bias}/\sigma^2_{\text{bias}}}{\text{No. good points}}\right]^{1/2}$$

SigTol = Number used for the sigma test in filtering out points.

Filtered Noise = RMS of residuals after modeling error is removed. This value is scaled to 1 MHZ for Doppler data. Fuller description given under *Station Analysis Report in Combiner – Solver Section.*

Sigma Multiplier = Total multiplicative adjustment to the sigma value read from the Observation File

Variance = Sum of squares of weighted residuals, where the sum is over all good points and the weights are taken as the reciprocal of the data point sigma.

S/N = Signal to Noise

$$= \left[\frac{\text{Variance}}{ndf}\right]^{1/2}$$

ndf = Number of good points plus number of bias parameters

MATRIX COMBINER SOLVER

OVERVIEW

The function of this section is to produce an arc normal equation using the pass normal equations formed in Filter, solve the equation and produce associated diagnostic information. The primary input for this task is a perturbed trajectory from Integrator and a pass matrix file from Filter.

The main task in forming the arc normal matrix is that of expanding pass matrices from Filter to include parameters not already present and updating the pass matrices to reflect solution at an epoch not presently referenced. Diagram 6 shows what an input pass matrix looks like. This matrix comes from Filter designed to improve orbital elements at the epoch t_s.

$$B_{pass} = \frac{\partial D^*}{\partial e_{t_s}} \frac{\partial D}{\partial e_{t_s}} \qquad E_{pass} = \frac{\partial D^*}{\partial e_{t_s}}(0 - D) \tag{69}$$

If we desire to improve elements at epoch $t_s(1)$ we must transport the normal matrix to $t_s(1)$ using the map $\psi^{-1}(t_s(1))T(t_s(1))$.

$$\psi(t) = \frac{\partial x(t)}{\partial e(t_s)} \qquad T(t) = \frac{\partial x(t)}{\partial e(t)} \tag{70}$$

The matrix from Filter may not include certain necessary parameters. This can sometimes be corrected by using the fact that the required parameters have matrix contributions which are functionally related to the available matrix elements. After referencing the matrices to the proper epoch and including the necessary parameters, the bias parameter portion of the pass matrix is eliminated and saved. Matrices are added together paying attention to station coordinate entries for different stations. When the addition terminates, the station portions of the resulting matrix are eliminated and saved. This resulting arc matrix of dynamic parameters only is inverted to obtain the dynamic solution. All other solutions are obtained by back substitution.

DIAGNOSTIC PROCEDURES

Navigation

One of the primary quality checks for an ephemeris is the navigation solution. This solution gives that station movement which minimizes the residuals of an individual pass while keeping the orbit fixed. Since in general the station coordinates are well-known, any "improvement" may be attributed to ephemeris error. To use this procedure we will have to generate a set of pass normal equations for the ephemeris we desire to check. In the case of the basic reference trajectory the associated pass matrices are already properly referenced. It remains only to use the station-bias section of the matrix to carry out the navigation. If an improvement has been obtained for the initial conditions of the reference trajectory, then the associated pass matrices must be adjusted (up to first order) so they will be referenced to the new trajectory. In either case the station bias section of the matrix is then rotated so it will reflect a solution in the special TCA system $\{\hat{\rho}, \dot{\hat{\rho}}, \hat{\rho}x\dot{\hat{\rho}}\}$. In this system $\hat{\rho}$ is a unit vector along the topocentric satellite position vector at time of closest approach, $\dot{\hat{\rho}}$ is the associated velocity, and $\hat{\rho}x\dot{\hat{\rho}}$ completes the right handed orthonormal system (Diagram 7).

The solution is generally carried out in the ρ and $\dot{\rho}$ directions only. The resulting solution vector is

$$\Delta p_{\text{Nav.}} = \begin{pmatrix} \Delta \text{ Radial} \\ \Delta \text{ Tangential} \\ \Delta \text{ Bias} \end{pmatrix}$$

The standard errors are obtained from the parameter covariance matrix. The signal-to-noise of a parameter is defined by

$$S/N(p) = \Delta p/(\sigma_p + \sigma_{\text{orbit}})$$

where σ_{orbit} is an input estimate of orbit error.

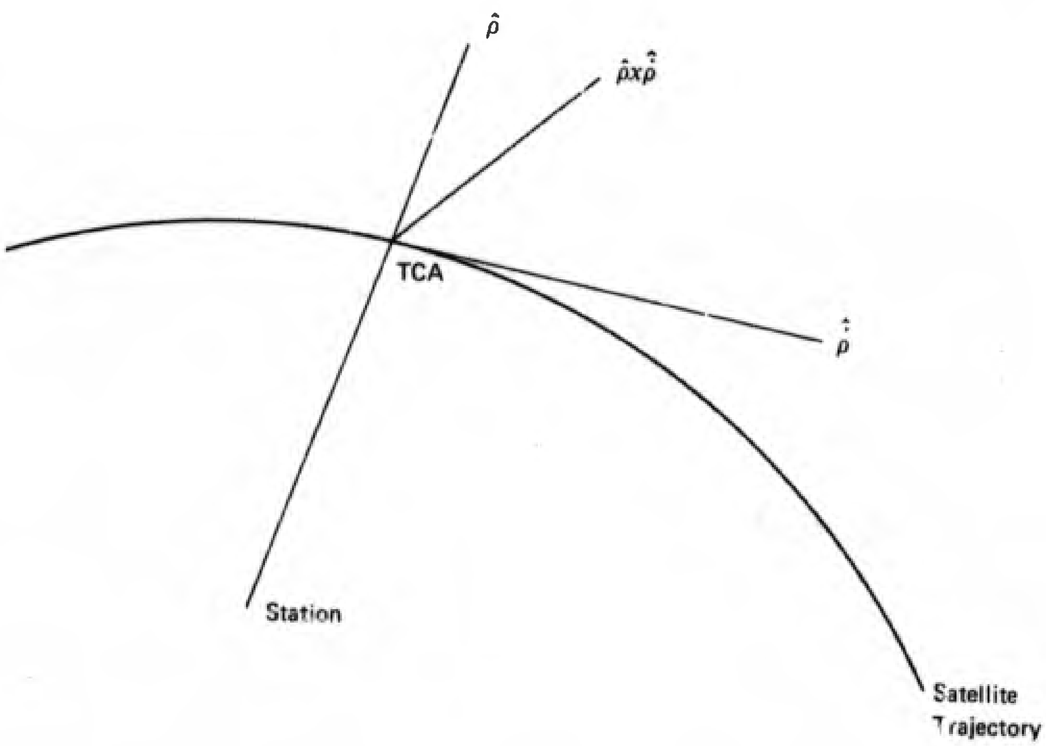


Diagram 7

Cross-Pass-Filtering

This is a process for filtering out passes on the basis of navigation solutions. After obtaining a navigation solution for each pass the individual $|\Delta R|$ and $|\Delta T|$ values corresponding to passes used in the recent arc solution (if no arc solution, use all values) are tested against absolute tolerances. If a value fails its test then the pass is tagged. Straight lines are then fit to the passes tested but not failing. The dispersions about these lines are determined. Each navigation residual, including those for passes which did not enter into the previous arc solution, is then tested against the proper sigma:

$$|\Delta R - \text{straight line value}| < F \text{ sigma } R$$

$$|\Delta T - \text{straight line value}| < F \text{ sigma } T$$

If either test fails the pass is tagged. Those passes tagged during cross-pass filter are subtracted out of the arc matrix prior to station elimination. Passes not tagged but previously tagged are added in to the arc matrix prior to station elimination. Station elimination and a new arc solution can now be performed. This procedure can iterate to a maximum iteration number or until the same passes are tagged on two successive iterations. The last solution obtained in cross-pass filtering is the solution for the given cycle, where cycle number refers to the number of times a new reference trajectory has been integrated.

Covariance Check

This process is used only in Short-Arc Mode. In this mode orbit improvement is carried out on a revolution by revolution basis. "Revolution" is defined by inputting a value for satellite period and time of first revolution. A span of fit encompassing the revolution is chosen in such a way as to minimize the covariance over the revolution itself. The procedure for choosing the span is for Short-Arc-Selector to assign a nominal span and let Covariance Check determine the optimal span by looking at certain extensions of the nominal. Diagram 15 illustrates the concept. The nominal fit span depicted would be that chosen by Short-Arc-Selector. In Diagram 15 the nominal fit is one fourth revolution on each side of the revolution of interest. Covariance-Check determines the covariance matrix for the nominal span and several other spans as shown. The covariance matrix is propagated to the start and end times of drag segments within the revolution of interest, and the positional covariance is averaged over these points. The quantity obtained is checked against an input tolerance and if successful that span is chosen. If no span passes the test then the one with the minimum average positional covariance is chosen. Covariance-Check is performed only on the first cycle. On any succeeding cycle the original choice of fit span is respected.

Signal To Noise

As part of the diagnostic information computed for a given solution we have:

Old Signal to Noise. This quantity is a measure of the current raw residuals.

Bias Reduced Signal to Noise. This quantity is the above measure of residuals when the bias solution has been applied.

Predicted Signal to Noise. This quantity is the above measure of residuals with the bias and orbit solution applied. This is the criterion for program convergence, i.e., criterion for continuing program flow.

Expected Percent Change in Variance. This quantity is the criterion for recycling the program. Recycling means re-integrating a trajectory. Variance is the quantity being minimized by the program, and a sufficiently small change terminates recycling.

DETAILED DESCRIPTION

Perturbation Equations

The satellite equation of motion can be expressed in the form

$$\ddot{x} = G_1(x) + C_D(t)D(x, \dot{x}) + L(x, \dot{x})A(t) + krR(x) \tag{71}$$

$$= G(x, \dot{x}, C_D, A, kr)$$

where G_1 is the acceleration due to earth, sun, and moon gravity plus lunar and solar tidal bulge acceleration; D is acceleration due to atmospheric drag; A is acceleration due to satellite thrusting motor and R is radiation pressure acceleration. A has three components, and L is either the identity or the rotation matrix which converts A to the inertial system.

The drag acceleration is adjusted with a drag coefficient C_D. The description of which values C_D will acquire during a fixed period constitutes the drag profile for that period. In a similar fashion a thrust profile will be required to describe A. The proper time to use the radiation pressure parameter, kr, is determined by a test during integration. The test is one of deciding if the satellite is in sunlight or shadow. The non-zero value of kr is used during sunlight.

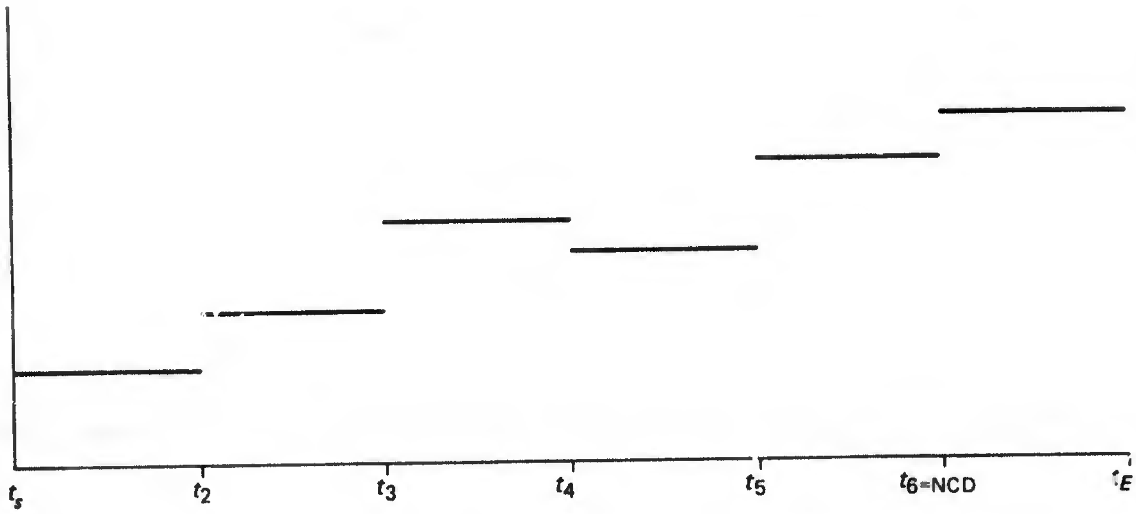
From (71) it can be seen that the solution will depend on the drag profile, thrust profile, radiation pressure value, and six orbit parameter values. The six orbit parameters can be either state components or an equivalent set of osculating elements. In Celest the osculating elements are

a	where	a = semi-major axis	
$e \sin(\omega)$		e = eccentricity	
$e \cos(\omega)$		i = inclination	(72)
i		ω = argument of perigee	
$\ell + \omega$		ℓ = mean anomaly	
Ω		Ω = right ascention	

The inertial system may be the mean equator and equinox system of 1950.0 or the mean equator and equinox system of zero sec of the day of epoch t_s.

For simplicity we will write $e(t_s)$ for either the initial state vector components or osculating element values. Again for simplicity we write x rather than $(x, \dot{x})$. Using this notation we can express the fact that the solution of (71) depends on its initial condition set by

$$x = x(t, e(t_s), C_{D_1} \cdots C_{D_{NCD}}, A^1 \ldots A^{NT}, kr) \tag{73}$$



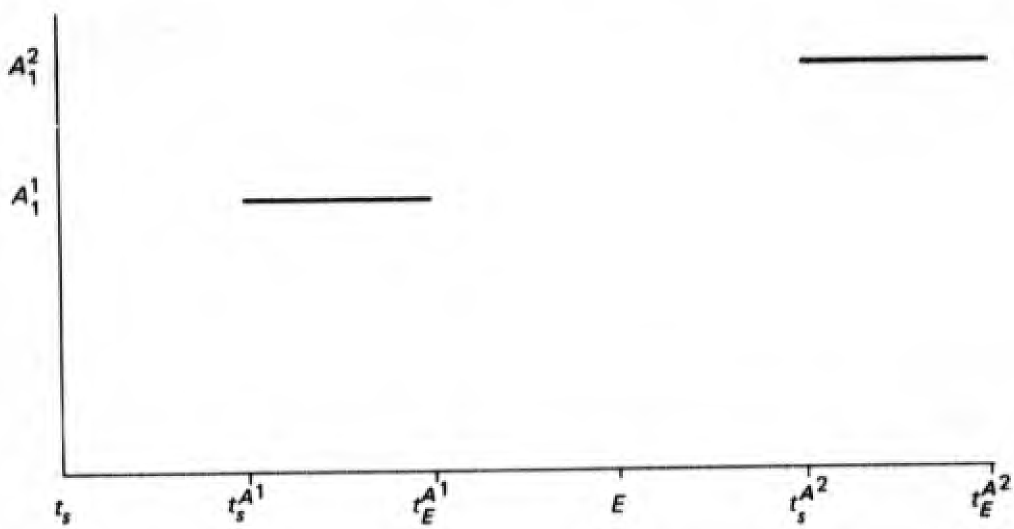
The parameter C_D takes on the values C_{D_i} during time period $(t_i, t_i + 1]$. When $t = t_s$ initialize $C_D = C_{D_1}$

The time period $(t_i, t_i + 1]$ will be referred to as the domain of C_{D_i}. $C_{D_i} = 0$ outside its domain.

t_s = trajectory start time

t_E = trajectory end time

Drag Profile (Example NCD = 6)



The time period $(t_s^{A^i}, t_E^{A^i}]$ will be referred to as the domain of A^i.

The thrust parameters A^i have the values (A_1^i, A_2^i, A_3^i) during domain of A^i and zero outside the domain.

Thrust Profile (Example *NT* = 2)

Diagram 8.

The perturbation equations for the parameter set $p = (e(t_s), C_{D_i}, A^i, kr)$ are

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h} \qquad \text{for } p = e(t_s) \tag{74}$$

= orbit set

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h} + \frac{\partial G}{\partial p} \qquad \text{for } p = \{C_{D_i}, A^i, kr\} \tag{75}$$

= model set

The solution of (71), (74), and (75) will be referred to as the perturbed or reference trajectory for the parameter set p. The solutions of (71), (74), and (75) are labeled $x(t)$, $\psi(t)$, and $\psi_p(t)$, respectively. $x(t) = (x_1(t), x_2(t), x_3(t), \dot{x}_1(t), \dot{x}_2(t), \dot{x}_3(t))$ is the state vector values at time t.

$$\psi(t) = [\psi_{e_1}(t), \ldots \psi_{e_6}(t)] = \left[\frac{\partial x_i(t)}{\partial e_j(t_s)}\right]$$

is the matrix of partials representing changes in the state vector at time t with respect to unit changes in the element values at epoch t_s.

$$\psi_p(t) = \left[\frac{\partial x_i(t)}{\partial p}\right]$$

is the column matrix of partials representing changes in the state vector at time t with respect to unit changes in the model parameter p. The independent time parameter t is assumed to be in a domain $[t_s, t_E]$ and the initial conditions for (71), (74), and (75) are

p_0 = (values for $e(t_s)$, drag profile, thrust profile and radiation value)

$$\psi(t_s) = \frac{\partial x(t_s)}{\partial e(t_s)} \quad \text{reducing to } I \text{ if } e(t_s) = x(t_s).$$

$$\psi_{C_{D_i}}(t_i) = 0 \tag{76}$$

$$\psi_{A^i}(t_s^{A^i}) = 0$$

$$\psi_{kr}(t_s) = 0$$

We have referred to the set (74) and (75) as perturbation equations since their solutions $\psi_p(t)$ are $\partial x(t)/\partial p$ and therefore give perturbations about the reference trajectory $x(t, p)$.

Rather than integrate (75) directly for each C_{D_i} or A^i in the drag and thrust profile, we will introduce canonical drag and thrust parameters, C_D and A, whose perturbation solutions will be used to obtain the perturbation solutions of C_{D_i} and A^i. The canonical drag and thrust are introduced by adding $C_D D$ and LA to (71). The parameter domain for both C_D and A is taken to be the entire trajectory and the parameter values are initialized to zero.

As an example we will examine drag. The perturbation solution for the canonical parameter C_D is related to the solutions for C_{D_i} by

$$\psi_{CD} = \psi_{CD_1} + \psi_{CD_2} + \dots + \psi_{CD_{NCD}} \tag{77}$$

Using

$$\frac{\partial G}{\partial C_{D_i}} = \begin{cases} D & \text{if } t \text{ is in } (t_i, t_{i+1}] \\ 0 & \text{if } t \text{ is not in } (t_i, t_{i+1}] \end{cases}$$

we know that ψ_{CD_i} is the solution to (75) having $\psi_{CD_i}(t_i) = 0$. However, ψ_{CD} is the solution to the equation

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h} + D$$

where $\psi_{CD}(t_s) = 0$.

It is clear that during the time that t is in $(t_i, t_{i+1}]$ the differential equations are the same. It is known that any two solutions of an equation of this type differ by a solution to the associated homogeneous equation

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h}$$

We recognize this equation as (74).

Since we have six independent solutions of (74), it follows that during the interval $(t_i, t_{i+1}]$ the solutions ψ_{CD_i} and ψ_{CD} will differ by a linear combination of the six solutions $\psi_{e_1}, \dots \psi_{e_6}$ of (74).

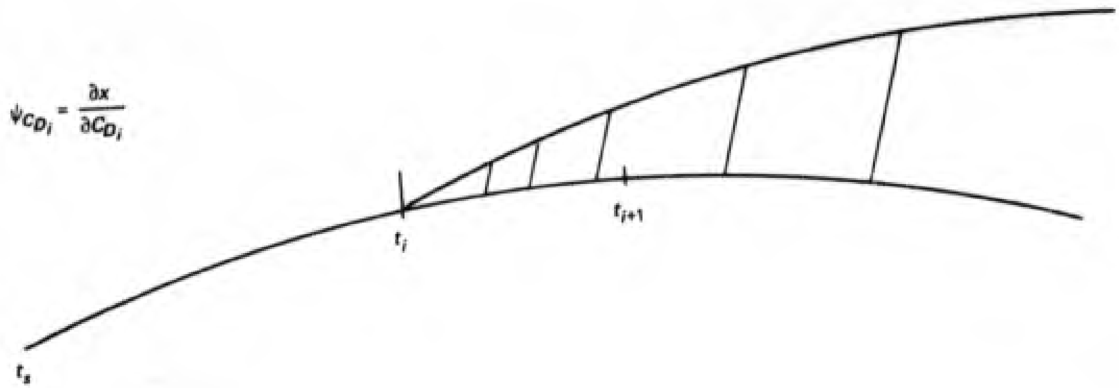
Looking at the drag perturbation equation outside the region $(t_i, t_{i+1}]$ shows $\partial G/\partial_{C_{D_i}} = 0$ and ψ_{CD_i} is itself a solution to (74). Again it must be a linear combination of our six solutions $\psi = (\psi_{e_1}, \dots \psi_{e_6})$.

We will adopt this procedure with thrust as well and refer to the solutions $x(t)$, $\psi(t)$, $\psi_{CD}(t)$, $\psi_A(t)$ and $\psi_{kr}(t)$ as the canonical perturbed trajectory.

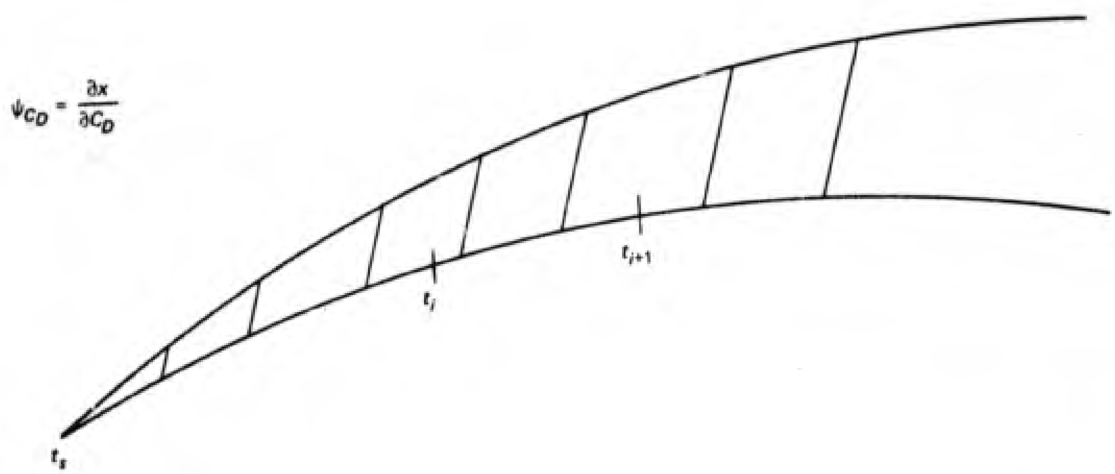
Diagram 9 illustrates the difference between ψ_{CD_i} and ψ_{CD}.

We can now describe in some detail the relation between the reference perturbed trajectory and the canonical perturbed trajectory.

Assuming that we require a reference trajectory over a region $[t_s(1), t_E(1)]$, we will describe how this can be obtained from the canonical trajectory over a larger region $[t_s, t_E]$. We will associate a domain with each parameter and assign an associated canonical parameter with domain the entire trajectory.



$\psi_{C_{D_i}} = \frac{\partial x}{\partial C_{D_i}}$
t_i
t_{i+1}
t_s



$\psi_{C_D} = \frac{\partial x}{\partial C_D}$
t_i
t_{i+1}
t_s

Diagram 9.

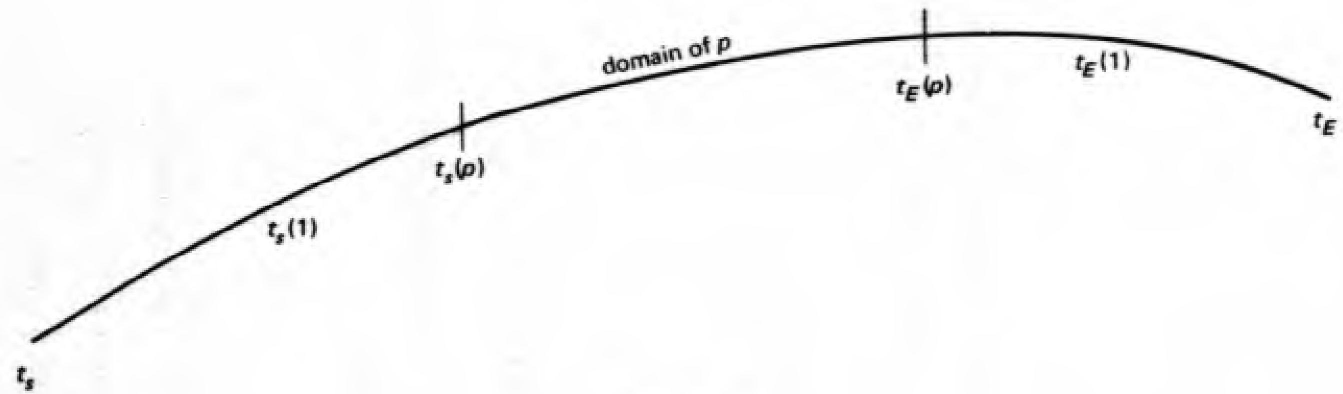


Diagram 10

Definition. Let p be a parameter from the dynamic parameter set $\{e_1, e_2, \ldots e_6, C_{D1}, \ldots C_{DNCD}, A^1, \ldots A^{NT}, kr\}$ and denote its associated canonical parameter by $\tilde{p}$.

$$
\begin{array}{lll}
p = C_{D_i} & \text{domain } (t_i, t_{i+1}] & \tilde{p} = C_D \\
p = A^i & \text{domain } (t_s^{A^i}, t_E^{A^i}] & \tilde{p} = A \\
p = e_i(t_s^{(1)}) & \text{domain } (t_s^{(1)}, t_E^{(1)}] & \tilde{p} = e_i(t_s) \\
p = kr & \text{domain } (t_s^{(1)}, t_E^{(1)}] & \tilde{p} = \tilde{kr}
\end{array}
\qquad (78)
$$

If p is a drag or thrust parameter

$$\psi_p(t) = 0 \qquad \text{if } t \leq t_s(p)$$

Since the difference of two solutions of (75) is a linear combination of the basic solutions of (74) we have

$$\psi_p(t) - \psi_{\tilde{p}}(t) = \psi(t)\alpha \qquad \text{if } t_s(p) < t \leq t_E(p) \qquad \text{and } \alpha \text{ in } R^6$$

Evaluating at $t = t_s(p)$ gives

$$\psi_p(t_s(p)) - \psi_{\tilde{p}}(t_s(p)) = \psi(t_s(p))\alpha$$

or

$$\alpha = -\psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p))$$

and

$$\psi_p(t) = \psi_{\tilde{p}}(t) - \psi(t)\psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p))$$

When $t > t_E(p)$ we wish a solution of (74) with $\psi_p(t_E(p))$ as the value at $t_E(p)$.

$$\psi_p(t) = \psi(t)\alpha \qquad \text{for } \alpha \text{ in } R^6$$

Evaluating at $t = t_E(p)$ gives

$$
\begin{aligned}
\psi(t_E(p))\alpha &= \psi_p(t_E(p)) \\
&= \psi_{\tilde{p}}(t_E(p)) - \psi(t_E(p))\psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p)) \\
\alpha &= \psi^{-1}(t_E(p))\psi_{\tilde{p}}(t_E(p)) - \psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p)) \\
\psi_p(t) &= \psi(t)[\psi^{-1}(t_E(p))\psi_{\tilde{p}}(t_E(p)) - \psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p))]
\end{aligned}
$$

We can summarize these results in

$$\psi_p(t) = \begin{cases} 0 & t \leq t_s(p) \\ \psi_{\tilde{p}}(t) - \psi(t)\psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p)) & t_s(p) < t \leq t_E(p) \\ \psi(t)[\psi^{-1}(t_E(p))\psi_{\tilde{p}}(t_E(p)) - \psi^{-1}(t_s(p))\psi_{\tilde{p}}(t_s(p)) & t_E(p) < t \end{cases} \tag{79}$$

If p is an orbit parameter $e_i(t_s(1))$ then ψ_p is a solution of (74). As such it is a linear combination of the six basic solutions $\psi = [\psi_{e_1}, \ldots \psi_{e_6}]$, at epoch t_s.

$$\psi_p(t) = \psi(t)\alpha \qquad \alpha \text{ in } R^6$$

Using ψ_{epoch} to represent the set of six solutions at some epoch gives

$$\psi_{t_s^{(1)}}(t) = \psi_{t_s}(t)\alpha = \psi(t)\alpha$$

where ψ is assumed to have epoch t_s and α is six by six

Evaluating at $t = t_s^{(1)}$ gives

$$\psi_{t_s(1)}(t_s(1)) = \frac{\partial x(t_s(1))}{\partial e(t_s(1))} = \psi(t_s(1))\alpha$$

$$\alpha = \psi^{-1}(t_s(1))T(t_s(1))$$

where T is given by

$$T(t) = \begin{cases} \dfrac{\partial x(t)}{\partial e(t)} & \text{if } e \text{ is the set of six osculating orbital elements} \\ I & \text{if } e \text{ is the state vector} \end{cases} \tag{80}$$

Summarizing gives

$$\psi_{t_s(1)}(t) = \psi(t)\psi^{-1}(t_s(1))T(t_s(1)) \tag{81}$$

Rather than use the expression for $\psi_{t_s^{(1)}}$ in this form we will restate it in a form similar to that of a model parameter p.

Recognizing that for an orbit parameter the domain will be $(t_s(1), t_E]$ we need only consider the case where t is in the domain. Again we have two solutions of (75) and they differ by a linear combination of basic solutions for (74).

With $p = e_i(t_s(1))$ and $\tilde{p} = e_i(t_s)$ we have

$$\psi_p(t) - \psi_{\tilde{p}}(t) = \psi(t)\alpha \qquad \alpha \text{ in } R^6$$

Evaluating at $t_s(1)$ gives

$$\psi_p(t_s(1)) - \psi_{\tilde{p}}(t_s(1)) = \psi(t_s(1))\alpha$$

$$\alpha = \psi^{-1}(t_s(1))[\psi_p(t_s(1)) - \psi_{\tilde{p}}(t_s(1))]$$

Unlike the model parameters

$$\psi_p(t_s(1)) = [T(t_s(1))]_{i\text{th. column}} \neq 0$$

Summarizing gives an alternate form of (81),

$$\psi_p(t) = \psi_{\tilde{p}}(t) - \psi(t)[\psi^{-1}(t_s(1))\psi_{\tilde{p}}(t_s(1)) - \psi^{-1}(t_s(1))\psi_p(t_s(1))]$$

$$= \psi_{\tilde{p}}(t) - \psi(t)[I_{\text{column } i} - \psi^{-1}(t_s(1))[T(t_s(1))]_{\text{column } i}] \tag{82}$$

For p any dynamic parameter define

$$\chi_p(t) = \begin{cases} I - \psi^{-1}(t)T(t) & p = \text{orbit set} \\ \psi^{-1}(t)\psi_{\tilde{p}}(t) & p = \text{model set} \end{cases} \tag{83}$$

Using χ_p rewrite (79) and (82) as

$$\psi_p(t) = \begin{cases} \begin{cases} T_p(t_s(p)) & p \text{ in orbit set} \\ 0 & p \text{ not in orbit set} \end{cases} & t \leq t_s(p) \\ \psi_{\tilde{p}}(t) - \psi(t)\chi_p(t_s(p)) & t_s(p) < t \leq t_E(p) \\ \psi(t)(\chi_p(t_E(p)) - \chi_p(t_s(p))) & t > t_E(p) \end{cases} \tag{84}$$

Optimization Problem

We are given a nonlinear dynamic process described by (71). We linearize this process about a reference trajectory obtained from (71) and obtain the linear process described by (74) and (75). Linearizing our observations about the reference trajectory gives observation residuals depending linearly on the linear process above.

Letting 0 denote observations and D the associated computed values on the reference trajectory gives

$$0 - D = \frac{\partial D}{\partial p}\Delta p + \nu \tag{85}$$

where ν is noise

Assuming

1. No correlation exists between the true state Δp of the process and the measurement noise ν.

2. The covariance of the true state C_p is known and nonsingular.

3. The covariance of the measurement noise C_ν is known and nonsingular.

The Gauss-Markoff theorem states that the linear minimum variance unbiased estimate of Δp is obtained by minimizing the quadratic form

$$H = \left(0 - D - \frac{\partial D}{\partial p} \Delta p\right)^* C_\nu^{-1} \left(0 - D - \frac{\partial D}{\partial p} \Delta p\right) + \Delta p^* C_p^{-1} \Delta p \tag{86}$$

If we further assume that the measurement noise is uncorrelated between passes then C_ν will be block diagonal and

$$H = \sum H_{\text{pass}}$$

The solution to this problem via Gauss Markoff is

$$\Delta p = \left(\frac{\partial D^*}{\partial p} C_\nu^{-1} \frac{\partial D}{\partial p} + C_p^{-1}\right)^{-1} \frac{\partial D^*}{\partial p} C_\nu^{-1}(0 - D) \tag{87}$$

Equation (87) is referred to as the normal equation, and the contribution from each pass is referred to as the pass normal equation. The ingrediants of the pass normal equation are

$$B_{\text{pass}} = \frac{\partial D^*}{\partial p} C_\nu^{-1} \frac{\partial D}{\partial p} \qquad E_{\text{pass}} = \frac{\partial D^*}{\partial p} C_\nu^{-1}(0 - D) \tag{88}$$

where C_ν is a diagonal matrix of variances determined in Filter

$$C_\nu = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{bmatrix} \tag{89}$$

At this point ($B_{\text{pass}}, E_{\text{pass}}$) must be computed, and for ease of computation we will let $C_\nu = I$. The terms $(0 - D)$ and $\partial D/\partial$ bias can be computed from (48) – (61). For p a dynamic parameter

$$\frac{\partial D}{\partial p} = \left.\frac{\partial D}{\partial x}\right|_{x(t,p0)} \left.\frac{\partial x}{\partial p}\right|_{(t,p0)}$$

The first term, $\partial D/\partial x$, can be computed by (48) – (61) and the second term, $\partial x/\partial p$, is ψ_p of (84).

The formulae in (84) express ψ_p as a function of $\psi_{\tilde{p}}$, the canonical perturbed trajectory. Thus, $\partial D/\partial p$ is a function of $\partial D/\partial \tilde{p}$. Using this result the pass normal equation will be a function of a canonical pass normal equation $(B_{\tilde{p},\tilde{q}}, E_{\tilde{p}})$. It was this canonical equation that was produced in Filter by (65). It remains then as a task of this section to carry out the expansion process which will present the needed normal equation as a function of the canonical one.

Expansion of Pass Matrices

Let p and q be dynamic parameters with associated canonical parameters $\tilde{p}$ and $\tilde{q}$.

$$B_{pq} = \sum_i \frac{\partial D_i}{\partial p} \frac{\partial D_i}{\partial q} = \sum_i B_{pq}(t_i) = \frac{\partial D^*}{\partial p} \frac{\partial D}{\partial q}$$

Using (84) we have five cases to consider in calculating $B_{pq}(t)$.

(1) t is in the domain of both p and q.

(2) t is in the domain of p and past the domain of q.

(3) t is past the domain of p and in the domain of q.

(4) t is past the domains of both p and q.

(5) t is before the domain of p or q.

Case (1)

$$B_{pq}(t) = \frac{\partial D_t}{\partial p} \frac{\partial D_t}{\partial q} = \frac{\partial D_t^*}{\partial p} \frac{\partial D_t}{\partial q}$$

$$\frac{\partial D_t}{\partial p} = \frac{\partial D_t}{\partial x} \frac{\partial x}{\partial p} \qquad x = x(t,p) = \begin{pmatrix} x_1(t,p) \\ \vdots \\ x_6(t,p) \end{pmatrix}$$

$$B_{pq}(t) = \frac{\partial x^*}{\partial p} \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \frac{\partial x}{\partial q}$$

Using (84) gives

$$B_{pq}(t) = [\psi_{\tilde{p}}(t) - \psi(t)\chi_p(t_s(p))]^* \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} [\psi_{\tilde{q}}(t) - \psi(t)\chi_q(t_s(q))]$$

$$= \psi_{\tilde{p}}^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \psi_{\tilde{q}}(t) - \psi_{\tilde{p}}^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \psi(t)\chi_q(t_s(q))$$

$$- \chi_p^*(t_s(p))\psi^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \psi_{\tilde{q}}(t)$$

$$+ \chi_p^*(t_s(p))\psi^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \psi(t)\chi_q(t_s(q))$$

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial \tilde{p}} \frac{\partial D_t}{\partial \tilde{q}} - \frac{\partial D_t^*}{\partial \tilde{p}} \frac{\partial D_t}{\partial \tilde{e}} \chi_q(t_s(q))$$

$$- \chi_p^*(t_s(p)) \frac{\partial D_t^*}{\partial \tilde{e}} \frac{\partial D_t}{\partial \tilde{q}}$$

$$+ \chi_p^*(t_s(p)) \frac{\partial D_t^*}{\partial \tilde{e}} \frac{\partial D_t}{\partial \tilde{e}} \chi_q(t_s(q))$$

where $e = e(t_s(1))$ is the set of six orbital elements with epoch $t_s(1)$ and $\tilde{e} = e(t_s)$, the six orbital elements at epoch t_s of the canonical perturbed trajectory. Recall from (82) that $\psi(t)$ is the matrix $(\psi_{\tilde{e}}, \ldots \psi_{\tilde{e}_6}) = (\partial x(t)/\partial \tilde{e}_1, \ldots \partial x(t)/\partial \tilde{e}_6)$.

We can express these results by

$$B_{pq}(t) = B_{\tilde{p}\tilde{q}}(t) - B_{\tilde{p}\tilde{e}}(t)\chi_q(t_s(q))$$

$$- \chi_p^*(t_s(p))B_{\tilde{e}\tilde{q}}(t)$$

$$+ \chi_p^*(t_s(p))B_{\tilde{e}\tilde{e}}(t)\chi_q(t_s(q))$$

If we assume that all of the data points within a pass fall in the same case, of cases (1) thru (5), then we can sum $B_{pq}(t)$ over the individual points giving

$$B_{pq} = B_{\tilde{p}\tilde{q}} - B_{\tilde{p}\tilde{e}}\chi_q(t_s(q)) - \chi_p^*(t_s(p))B_{\tilde{e}\tilde{q}} + \chi_p^*(t_s(p))B_{\tilde{e}\tilde{e}}\chi_q(t_s(q)) \tag{90}$$

where

$$B_{\tilde{p}\tilde{q}} = \frac{\partial D^*}{\partial \tilde{p}} \frac{\partial D}{\partial \tilde{q}}$$

Case (2)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p}\ \frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p}\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \frac{\partial x}{\partial q}$$

$$= [\psi_{\tilde{p}}(t) - \psi(t)\chi_p(t_s(p))]^*\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \psi(t)[\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

$$= \psi_{\tilde{p}}^*(t)\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \psi(t)[\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

$$- \chi_p^*(t_s(p))\psi^*(t)\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \psi(t)[\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

$$= \frac{\partial D_t^*}{\partial \tilde{p}}\ \frac{\partial D_t}{\partial \tilde{e}}\ [\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

$$- \chi_p^*(t_s(p))\ \frac{\partial D_t^*}{\partial \tilde{e}}\ \frac{\partial D_t}{\partial \tilde{e}}\ [\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

These results can be expressed by

$$B_{pq}(t) = [B_{\tilde{p}\tilde{e}}(t) - \chi_p^*(t_s(p))B_{\tilde{e}\tilde{e}}]\,[\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

Again summing over individual data points gives

$$B_{pq} = [B_{\tilde{p}\tilde{e}} - \chi_p^*(t_s(p))B_{\tilde{e}\tilde{e}}]\,[\chi_q(t_E(q)) - \chi_q(t_s(q))] \tag{91}$$

Case (3)

By symmetry we have $B_{pq} = B_{qp}$ and

$$B_{pq} = [B_{\tilde{q}\tilde{e}} - \chi_q^*(t_s(q))B_{\tilde{e}\tilde{e}}]\,[\chi_p(t_E(p)) - \chi_p(t_s(p))]$$

Using $B_{pq} = B_{pq}^*$ we rewrite the above as

$$B_{pq} = [\chi_p(t_E(p)) - \chi_p(t_s(p))]^*\,[B_{\tilde{e}\tilde{q}} - B_{\tilde{e}\tilde{e}}\chi_q(t_s(q))] \tag{92}$$

Case (4)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p}\ \frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p}\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \frac{\partial x}{\partial q}$$

$$= [\chi_p(t_E(p)) - \chi_p(t_s(p))]^*\,\psi^*(t)\ \frac{\partial D_t^*}{\partial x}\ \frac{\partial D_t}{\partial x}\ \psi(t)[\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

$$= [\chi_p(t_E(p)) - \chi_p(t_s(p))]^*\,B_{\tilde{e}\tilde{e}}^{(t)}\ [\chi_q(t_E(q)) - \chi_q(t_s(q))]$$

Summing over data points gives

$$B_{pq} = [\chi_p(t_E(p)) - \chi_p(t_s(p))]^* B_{\tilde{e}\tilde{e}} [\chi_q(t_E(q)) - \chi_q(t_s(q))] \tag{93}$$

Case (5)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p} \frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p} \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial x} \frac{\partial x}{\partial q}$$

In this case t comes before the domain of one of the parameters p or q. As the trajectory is unaffected at time t due to a change in the parameter with domain later than t, we have

$$B_{pq}(t) = 0$$

Summing over the data points gives

$$B_{pq} = 0 \tag{94}$$

Let p be a dynamic parameter and q be a bias parameter. Again we have several cases.

(1) t is in the domain of p.

(2) t is past the domain of p.

(3) t is before the domain of p.

Case (1)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p} \frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p} \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial q}$$

$$= [\psi_{\tilde{p}}(t) - \psi(t)\chi_p(t_s(p))]^* \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial q}$$

$$= \psi_{\tilde{p}}^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t^*}{\partial q} - \chi_p^*(t_s(p))\psi^*(t) \frac{\partial D_t^*}{\partial x} \frac{\partial D_t}{\partial q}$$

$$= B_{\tilde{p}\tilde{q}}(t) - \chi_p^*(t_s(p)) B_{\tilde{e}q}(t)$$

For a bias parameter q we will define its associated canonical parameter by $\tilde{q} = q$. Using this definition and summing over the data points gives

$$B_{pq} = B_{\tilde{p}\tilde{q}} - \chi_p^*(t_s(p))B_{\tilde{e}\tilde{q}} \tag{95}$$

Case (2)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p}\frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p}\frac{\partial D_t^*}{\partial x}\frac{\partial D_t}{\partial q}$$

$$= [\chi_p(t_E(p)) - \chi_p(t_s(p))]^* \psi^*(t)\frac{\partial D_t^*}{\partial x}\frac{\partial D_t}{\partial q}$$

$$= [\chi_p(t_E(p)) - \chi_p(t_s(p))]^* B_{\tilde{e}\tilde{q}}(t)$$

Summing over the data points gives

$$B_{pq} = [\chi_p(t_E(p)) - \chi_p(t_s(p))]^* B_{\tilde{e}\tilde{q}} \tag{96}$$

Case (3)

$$B_{pq}(t) = \frac{\partial D_t^*}{\partial p}\frac{\partial D_t}{\partial q} = \frac{\partial x^*}{\partial p}\frac{\partial D_t^*}{\partial x}\frac{\partial D_t}{\partial q}$$

In this case the trajectory is unaffected at time t due to a change in the value of p. $\partial x/\partial p$ will be zero and summing over the data points gives

$$B_{pq} = 0 \tag{97}$$

Let p and q be bias parameters. In this event p and q remain fixed giving

$$B_{pq} = B_{\tilde{p}\tilde{q}} \tag{98}$$

We turn now to the righthand side of the normal equation (87). Again we consider E_p where p is either a dynamic or bias parameter. For p a bias parameter E_p will remain fixed, and when p is a dynamic parameter we have three cases. Without going into the details—as they are quite similar to the previous work—we will give the resulting formulae.

For p a dynamic parameter we have

Case (1): Data contained in the domain of p

$$E_p = E_{\tilde{p}} - \chi_p^*(t_s(p))E_{\tilde{e}} \tag{99}$$

Case (2): Data past the domain of p

$$E_p = [\chi_p(t_E(p)) - \chi_p(t_s(p))]^* E_{\tilde{e}} \tag{100}$$

Case (3): Data before the domain of p

$$E_p = 0 \tag{101}$$

For p a bias parameter we have

$$E_p = E_{\tilde{p}} \tag{102}$$

Formulae (90) thru (102) define the expansion function, *Exp*, going from parameter space to canonical parameter space. Exp can be given by an n by $n + k$ matrix which operates on the canonical normal matrix to bring it to the desired normal matrix. The number $n + k$ is the dimension of the parameter set for a given problem, e.g.

Definition of Exp

$$Exp = \begin{bmatrix} Exp_{\tilde{e},e} & Exp_{\tilde{e},p} \\ Exp_{\tilde{q},e} & Exp_{\tilde{q},p} \end{bmatrix} \tag{103}$$

where e is the orbit set and p,q model or bias parameters.

$$Exp_{\tilde{e},e} = \psi^{-1}(t_s(1))T(t_s(1))$$

$$Exp_{\tilde{e},p} = \begin{cases} 0 & \text{if data before domain } p \\ -\chi_p(t_s(p)) & \text{if data in domain } p \\ \chi_p(t_E(p)) - \chi_p(t_s(p)) & \text{if data past domain } p \end{cases}$$

$$Exp_{\tilde{q}e} = 0$$

$$Exp_{\tilde{q},p} = \begin{cases} 1 & \tilde{q} = \tilde{p} \text{ and data in domain } p \\ 0 & \tilde{q} \neq \tilde{p} \text{ or data not in domain } p \end{cases}$$

Bias parameters are also covered by this formula if we define the domain of a bias parameter to be the duration of its pass and recall that the trajectory does not depend on bias parameters, i.e. $\psi_{\text{bias}} = \partial x / \partial_{\text{bias}} = 0$. Defining χ_{bias} as $\psi^{-1}\psi_{\text{bias}}$ implies $\chi_{\text{bias}} = 0$ as well.

Using *Exp* we can now state (90) thru (102) in

$$B = Exp^* \tilde{B} Exp \text{ and } E = Exp^* \tilde{E} \tag{104}$$

where $(\tilde{B}, \tilde{E})$ is the canonical normal matrix $(B_{\tilde{p},\tilde{q}}, E_{\tilde{p}})$.

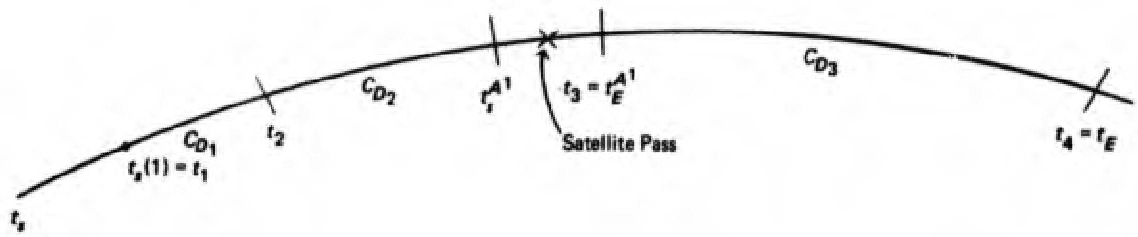
Parameter Name		Parameter Label	Number
	Orbit	$e_1, ..., e_6$	6
Model	Drag	$C_{D_1} ... C_{D_{NCD}}$	NCD
	Thrust	$A^1 ... A^{NT}$	$3NT$
	Radiation	kr	1
	Polar Motion	$\Delta q, \Delta p, \Delta t, \dot{\Delta t}$	4
Bias	Station	s_1, s_2, s_3	3
	Frequency	f_b	1
	Frequency Drift	$\dot{f}_b$	1
	Refraction	C_R	1

$$n + k = 6 + \text{NCD} + 3NT + 11$$

n = 6 orbit + 1 Drag (NCD $\neq$ 0) + 3 Thrust ($NT \neq 0$)
+ 1 Radiation + 3 Station + 1 Frequency +
1 Frequency Drift + 1 Refraction
= maximum of 17

Diagram 11

Diagrams (12) and (13) shows *Exp* and $\tilde{B}$ for the pass indicated below



Radiation Update

If the least square fit interval is $[t_s^{(1)}, t_E^{(1)}]$ and $t_s^{(1)} \neq t_s$ then it is necessary to update the normal matrix to reflect solving for kr rather than $\widetilde{kr}$. The parameter kr has epoch $t_s^{(1)}(\psi_{kr}(t_s^{(1)}) = 0)$ while $\widetilde{kr}$ has epoch $t_s(\psi_{\widetilde{kr}}(t_s) = 0)$.

Although (104) covers this situation the actual procedure employed is slightly different. The method used is equivalent to (104) but is employed after all other expansion has taken place.

$Exp =$

	$e_1 \cdots e_6$	C_{D_1}	C_{D_2}	C_{D_3}	A^1	kr	Bias
$\tilde{e}_1 \cdots \tilde{e}_6$	$\psi^{-1}(t_s(1))T(t_s\ 1))$	$\chi_{C_{D_1}}(t_2) - \chi_{C_{D_1}}(t_1)$	$-\chi_{C_{D_2}}(t_2)$	0	$-\chi_A(t_s^{A^1})$	$-\chi_{kr}(t_s(1))$	0
C_D	0	0	1	0	0	0	0
A	0	0	0	0	1	0	0
$\widetilde{kr}$	0	0	0	0	0	1	0
Bias	0	0	0	0	0	0	1

Diagram 12

Canonical Pass Matrix

$$\tilde{B}_{pass} = \begin{bmatrix} B_{\tilde{e},\tilde{e}} & B_{\tilde{e},C_D} & B_{\tilde{e},A} & B_{\tilde{e},\widetilde{kr}} & B_{\tilde{e},\widetilde{bias}} \\ \cdot & B_{C_D,C_D} & \cdot & \cdot & \cdot \\ \cdot & \cdot & B_{A,A} & \cdot & \cdot \\ \cdot & \cdot & \cdot & B_{\widetilde{kr},\widetilde{kr}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & B_{\widetilde{bias},\widetilde{bias}} \end{bmatrix} \quad E_{bias} = \begin{bmatrix} E_{\tilde{e}} \\ E_{C_D} \\ E_A \\ E_{\widetilde{kr}} \\ E_{\widetilde{bias}} \end{bmatrix}$$

Diagram 13

Updating the pass matrix for radiation means calculating all terms correlated to kr. Using (104) we can do this by calculating all terms in the kr column by

$$B_{kr} = Exp^{*}\widetilde{B}Exp_{kr} \tag{105}$$

Below is the expanded pass matrix of (104), for the example in Diagram (12), where $\widetilde{kr}$ has not yet been updated to kr.

$$B_{\text{pass}}^{\text{Exp.}} = \begin{bmatrix} B_{e,e} & B_{e,C_{D_1}} & B_{e,C_{D_2}} & B_{e,C_{D_3}} & B_{e,A^1} & B_{e,\widetilde{kr}} & B_{e,b} \\ \cdot & B_{C_{D_1},C_{D_1}} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & B_{C_{D_2},C_{D_2}} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & B_{C_{D_3},C_{D_3}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & B_{A^1,A^1} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & B_{\widetilde{kr},\widetilde{kr}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B_{b,b} \end{bmatrix}$$

where e is orbit, C_{D_i} drag, A^1 thrust, $\widetilde{kr}$ radiation and b represents bias parameters.

We desire to change the $\widetilde{kr}$ column of this matrix into B_{kr}, the kr column in (105).

Using Diagrams 12 and 13 we have

$$\widetilde{B}Exp_{kr} = \begin{bmatrix} -B_{\widetilde{e},\widetilde{e}}\,X_{kr}(t_s^{(1)}) + B_{\widetilde{e},\widetilde{kr}} \\ -B_{C_D,\widetilde{e}}\,X_{kr}(t_s^{(1)}) + B_{C_D,\widetilde{kr}} \\ -B_{A,\widetilde{e}}\,X_{kr}(t_s^{(1)}) + B_{A,\widetilde{kr}} \\ -B_{\widetilde{kr},\widetilde{e}}\,X_{kr}(t_s^{(1)}) + B_{\widetilde{kr},\widetilde{kr}} \\ -B_{\widetilde{\text{bias}},\widetilde{e}}\,X_{kr}(t_s^{(1)}) + B_{\widetilde{\text{bias}},\widetilde{kr}} \end{bmatrix}$$

We can now apply Exp^* giving

$$
Exp^*\widetilde{B}Exp_{kr} = \begin{bmatrix}
-T^*(t_s^{(1)})\psi^{-1*}(t_s^{(1)})B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) + T^*(t_s^{(1)})\psi^{-1*}(t_s^{(1)})B_{\widetilde{e},\widetilde{kr}} \\
-(\chi^*_{C_{D_1}}(t_2) - \chi^*_{C_{D_1}}(t_1))B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) + (\chi^*_{C_{D_1}}(t_2) - \chi^*_{C_{D_1}}(t_1))B_{\widetilde{e},\widetilde{kr}} \\
\chi^*_{C_{D_2}}(t_2)B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) - \chi^*_{C_{D_2}}(t_2)B_{\widetilde{e},\widetilde{kr}} - B_{C_D,\widetilde{e}}\chi_{kr}(t_s^{(1)}) + B_{C_D,\widetilde{kr}} \\
0 \\
\chi^*_A(t_s^{A1})B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) - \chi^*_A(t_s^{A1})B_{\widetilde{e},\widetilde{kr}} - B_{A,\widetilde{e}}\chi_{kr}(t_s^{(1)}) + B_{A,\widetilde{kr}} \\
\chi^*_{kr}(t_s^{(1)})B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) - \chi^*_{kr}(t_s^{(1)})B_{\widetilde{e},\widetilde{kr}} - B_{\widetilde{kr},\widetilde{e}}\chi_{kr}(t_s^{(1)}) + B_{\widetilde{kr},\widetilde{kr}} \\
-B_{\widetilde{\text{bias}},\widetilde{e}}\chi_{kr}(t_s^{(1)}) + B_{\widetilde{\text{bias}},\widetilde{kr}}
\end{bmatrix}
$$

Diagram 14

Using the above and the observation

$$
\begin{aligned}
B_{\widetilde{e},\widetilde{e}}\chi_{kr}(t_s^{(1)}) &= B_{\widetilde{e},\widetilde{e}}\psi^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) \\
&= B_{\widetilde{e},x(t_s^{(1)})}\psi_{\widetilde{kr}}(t_s^{(1)}) \\
&= B_{\widetilde{e},x(t_s^{(1)})}T(t_s^{(1)})T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) \\
&= B_{\widetilde{e},e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)})
\end{aligned}
$$

allows us to express the result as a function of the expanded terms.

$$
Exp^*\widetilde{B}Exp_{kr} = \begin{bmatrix}
-B_{e,e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{e,\widetilde{kr}} \\
-B_{C_{D_1},e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{C_{D_1},\widetilde{kr}} \\
-B_{C_{D_2},e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{C_{D_2},\widetilde{kr}} \\
-B_{A1,e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{A1,\widetilde{kr}} \\
\begin{bmatrix}
-B_{\widetilde{kr},e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{\widetilde{kr},\widetilde{kr}} \\
+\psi^*_{\widetilde{kr}}(t_s^{(1)})T^{-1*}(t_s^{(1)})B_{e,e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) \\
-\psi^*_{\widetilde{kr}}(t_s^{(1)})T^{-1*}(t_s^{(1)})B_{e,\widetilde{kr}}
\end{bmatrix} \\
-B_{\text{bias},e}T^{-1}(t_s^{(1)})\psi_{\widetilde{kr}}(t_s^{(1)}) + B_{\text{bias},\widetilde{kr}}
\end{bmatrix}
$$

An examination of this expression shows that the first two terms of the inner bracket are similar to other terms in the column. The third and fourth terms of the inner bracket are obtained by multiplying the orbit result on the left by $-[T^{-1}(t_s^{(1)})\psi_{k\tilde{r}}(t_s^{(1)})]^*$. This leads to the following algorithm for updating the radiation parameter:

After expanding the pass matrix for all except the radiation parameter, multiply the orbit columns on the right by $-T^{-1}(t_s^{(1)})\psi_{k\tilde{r}}(t_s^{(1)})$ and add the result to the radiation column. Next multiply the resulting orbit radiation terms by $-[T^{-1}(t_s^{(1)})\psi_{k\tilde{r}}(t_s^{(1)})]^*$ on the left and add the result to the radiation term. This of course corrects the radiation column and we must similarly correct the radiation row.

Definition

$$Up_{kr} = I + \left[0, \begin{matrix} -T^{-1}(t_s^{(1)})\psi_{k\tilde{r}}(t_s^{(1)}) \\ 0 \\ I_{kr,kr} \\ 0 \end{matrix}, 0\right] \tag{106}$$

Using Up_{kr} the algorithm for radiation update is

$$\begin{aligned} B &= Up_{kr}^* B_{\text{pass}}^{\text{Exp.}} Up_{kr} \\ E &= Up_{kr}^* E_{\text{pass}}^{\text{Exp.}} \end{aligned} \tag{107}$$

Bias and Station Elimination

After expanding the pass matrix, including polar motion expansion and radiation update, the bias parameters are formally eliminated.

$$\begin{bmatrix} B_{00} & B_{0b} \\ B_{b0} & B_{bb} \end{bmatrix} \begin{pmatrix} \Delta p_0 \\ \Delta p_b \end{pmatrix} = \begin{pmatrix} E_0 \\ E_b \end{pmatrix}$$

where b represents bias parameters and 0 represents all others.

$$B_{00}\Delta p_0 + B_{0b}\Delta p_b = E_0$$

$$B_{b0}\Delta p_0 + B_{bb}\Delta p_b = E_b$$

$$\Delta p_b = B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$B_{00}\Delta p_0 + B_{0b}B_{bb}^{-1}(E_b - B_{b0}\Delta p_0) = E_0$$

$$(B_{00} - B_{0b}B_{bb}^{-1}B_{b0})\Delta p_0 = E_0 - B_{0b}B_{bb}^{-1}E_b$$

Definition

$$B_{\text{pass}}^{\text{Exp.}}(\text{bias eliminated}) = B_{00} - B_{0b}B_{bb}^{-1}B_{b0}$$

$$B_{\text{pass}}^{\text{Exp.}}(\text{bias eliminated}) = E_0 - B_{0b}B_{bb}^{-1}E_b$$

After bias elimination, all pass matrices are added together to form an arc matrix. Care is taken to augment the arc matrix when summing in station coordinates from different stations. After the arc matrix is formed, a station elimination is performed for all station coordinates, leaving only dynamic parameters and polar motion parameters in the final arc matrix.

A Priori Data

A Priori data, C_p^{-1} of (87), is accommodated by the program in the form of an observation of each parameter equal to its initial value, with standard error of sigma parameter (σ_p).

This results in the value $1/\sigma_p^2$ being added to the diagonal term of the normal equation. In the case of bias parameters, the weight $1/\sigma_p^2$ is added prior to bias elimination. In the case of station parameters, $1/\sigma_p^2$ is added after the arc matrix has been formed but prior to station elimination. All other weights can be added after station elimination. This results in the arc normal matrix

$$\begin{aligned} B_{\text{ARC}} &= B_{\text{ARC}}(\text{Station Eliminated}) + S \\ E_{\text{ARC}} &= E_{\text{ARC}}(\text{Station Eliminated}) + S(P_0 - P) \end{aligned} \tag{108}$$

where P_0 represents initial values, P present values, and S is the weight matrix

$$S = \begin{bmatrix} 1/\sigma_{p_1}^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_{p_n}^2 \end{bmatrix} = C_p^{-1} \tag{109}$$

Navigation and Bias Solutions

There are three forms of navigation solution and two bias solutions. The first form of navigation solution took place in the Filter and consisted in navigating the orbit while holding the station coordinates fixed. This process was used to extract orbit error from the residuals and permit editing of the data. Form Two Navigation is used in the solution area of the program and consists of navigating the observing station while holding the orbit fixed. As the station coordinates are known to be correct, this process provides diagnostic information on the quality of the present orbit. After an orbit improvement is carried out, a linear estimate of the Form Two Navigation solution is determined for the improved orbit. This linear estimate is the Form Three Navigation solution.

The navigation diagnostics are used to edit passes from the final orbit solution. This process is described later under *Cross-Pass Filtering.*

Form One Navigation

This process was described in the Filter section.

Form Two Navigation

The bias section of the pass matrix is

$$B_{bb} = \begin{bmatrix} B_{ss} & B_{sf} \\ B_{fs} & B_{ff} \end{bmatrix} \qquad B_{ss} = \frac{\partial D^*}{\partial e_s} \frac{\partial D}{\partial e_s}$$

where s represents station, f represents frequency and refraction, and e_s represents earth fixed station coordinates.

Let ρ and $\dot{\rho}$ be the range and range-rate vectors, in earth fixed coordinates, evaluated at TCA. Define the rotation transformation by

$$R = [\hat{\rho}(\text{TCA}), \hat{\dot{\rho}}(\text{TCA}), \hat{\rho}(\text{TCA}) \times \hat{\dot{\rho}}(\text{TCA})] \tag{110}$$

If e is a vector with components e_i in the earth fixed system and r_i in the $\rho = \{\hat{\rho}, \hat{\dot{\rho}}, \hat{\rho} x \hat{\dot{\rho}}\}$ system then

$$e_i = \sum_j R_{ij} r_j$$

where R_{ij} is the rotation transformation above.

We wish to find $B_{ss}^R = B_{ss}$ in the ρ system.

$$B_{ss}^R = \frac{\partial D^*}{\partial r_s} \frac{\partial D}{\partial r_s}$$

where $r_s = (r_1 r_2 r_3)$ components given in the ρ system.

$$B_{ss}^R = \frac{\partial e_s^*}{\partial r_s} \frac{\partial D^*}{\partial e_s} \frac{\partial D}{\partial e_s} \frac{\partial e_s}{\partial r_s}$$

$$= R^* B_{ss} R$$

The bias section of the pass matrix can now be expressed in the ρ system by

$$B_{bb}^R = R^* B_{bb} R \qquad E_b^R = R^* E_b \tag{111}$$

where

$$R = \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix}$$

We now add additional a priori observations of each parameter where the observation is its initial value with standard error σ_{r_i}. This is done by adding the weight matrix W_b to B_{bb}^R.

$$W_b = \begin{bmatrix} \frac{1}{\sigma_{r_1}^2} & & & 0 \\ & \frac{1}{\sigma_{r_2}^2} & & \\ & & \frac{1}{\sigma_{r_3}^2} & \\ & 0 & & \frac{1}{\sigma_f^2} \end{bmatrix} \tag{112}$$

where $1/\sigma_f^2$ is a diagonal matrix with one term for each bias parameter.

Redefining B_{bb}^R by $B_{bb}^R + W_b$ the navigation equations become

$$B_{bb}^R \Delta r = E_b^R \tag{113}$$

In the event two parameter navigation solutions are desired (usual case), the sigma for the out of plane component r_3 is chosen as a small value. The parameter covariance matrix for this solution is given by $[B_{bb}^R]^{-1}$.

After each navigation solution the statistics, Δr_i, σ_{ri} and $(S/N)_i = \Delta r_i/[\sigma_{ri} + \sigma_{orbit}]$ are printed out. The value σ_{ri}^2 is the value from the diagonal of $[B_{bb}^R]^{-1}$, not the a priori, and σ_{orbit} is an input value.

Form Three Navigation

This form of navigation takes place after the orbit solution from (108).

$$\Delta p_0 = B_{ARC}^{-1} E_{ARC} \tag{114}$$

where $\Delta p_0 = \Delta p$ (orbit, drag, thrust, radiation, and polar motion).

We wish to apply Δp_0 and carry out the Form Two Navigation on the new orbit.

Setting Δp_{bias} to zero, so the corresponding bias solution will not be applied, we calculate the pass normal equation for the new orbit from $\Delta p_0 = \binom{\Delta p_0}{0}$.

$$\frac{\partial D^*}{\partial p}(p_0 + \bar{\Delta p}_0)\frac{\partial D}{\partial q}(p_0 + \bar{\Delta p}_0)\Delta q = \frac{\partial D^*}{\partial p}(p_0 + \bar{\Delta p}_0)(Obs. - D(p_0 + \bar{\Delta p}_0))$$

We obtain a linear estimate of this equation by expanding about p_0 and retaining only first partials.

$$\frac{\partial D^*}{\partial p}(p_0)\frac{\partial D}{\partial q}(p_0)\Delta q = \frac{\partial D^*}{\partial p}(p_0)(Obs. - D(p_0)) - \frac{\partial D^*}{\partial p}(p_0)\frac{\partial D}{\partial q}(p_0)\bar{\Delta p}_0$$

Since we are interested in the bias section of this equation for the purpose of navigation, we can write

$$B_{bb}(p_0)\Delta b = E_b(p_0) - B_{b,0}(p_0)\Delta p_0 \tag{115}$$

for the bias equation relative to the improved orbit.

Using (115) we can now carry out a Form Two Navigation obtaining diagnostic statistics on the new orbit. Included in these statistics are the values for frequency and refraction bias parameters from the navigation solution. When the program converges on a final orbit the last values for these parameters are printed under the title *Bias Solution (Navigated).*

A second form of bias solution, available as diagnostic statistics, is the one which was surpressed by setting Δp_{bias} to zero. This solution is the one corresponding to the orbit solution Δp_0. It is obtained by back-substituting Δp_0 into the bias elimination equations

$$\Delta p_b = B_{bb}^{-1}(E_b - B_{b0}\Delta p_0) \tag{116}$$

This bias solution can also be obtained at program convergence under the title *Bias Solution (From Orbit Solution).* For either type of bias solution straight line fits are carried out for the frequency bias solution yielding an average frequency bias and a standard derivation.

Define

$$\Delta f_{predicted} = A + B(\mathrm{TCA})$$

$$\Delta f = \text{frequency bias solution} \tag{117}$$

Let

$$\text{Sigma} = \left[\sum_{i=1}^{N} \frac{(\Delta f - \Delta f_{predicted})_i}{N}\right]^{1/2}$$

Reject those Δf_i for which

$$(\Delta f - \Delta f_{predicted})_i > 2.5 \text{ Sigma}$$

Take the M accepted passes and compute an average frequency bias

$$\Delta f_{\text{Average}} = \frac{1}{M} \sum_{i=1}^{M} \Delta f_i \qquad (118)$$

Compute A and B by

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} \sum_{i=1}^{N} (\text{TCA})_i^2 & -\sum_{i=1}^{N} (\text{TCA})_i \\ * & N \end{bmatrix} \frac{1}{F} \begin{pmatrix} \sum_{i=1}^{N} \Delta f_i \\ \sum_{i=1}^{N} (\text{TCA})_i \Delta f_i \end{pmatrix} \qquad (119)$$

$$F = N \sum_{i=1}^{N} (\text{TCA})_i^2 - \left[\sum_{i=1}^{N} (\text{TCA})_i \right]^2$$

N = Number of passes

Print A, B, Sigma, $\Delta f_{\text{Average}}$, N and M

Cross-Pass Filtering

1. Perform a navigation solution for each pass in the span of the orbit fit. Include all passes, even those not used in the orbit fit. Use a Form Two Navigation to initialize this process and continue with Form Three. Results are Δr_1 the radial solution, Δr_2 the tangential solution, and when desired Δr_3 the out of plane solution. Covariance is also obtained from $[B_{bb}^R]^{-1}$. The Δr_i values, their standard errors and RWS are printed.

$$\text{RWS } \Delta r_i = \left[\frac{\sum \left(\frac{\Delta r_i}{\sigma_{r_i}} \right)^2}{\sum \frac{1}{\sigma_{r_i}^2}} \right]^{1/2}$$

A test is made to determine if we should iterate again in the Cross-Pass Filter. Cross-Pass Filtering is stopped if the same passes are tagged on successive iterations or a maximum iteration number is reached.

2. Perform an absolute tolerance test on all navigation values.

$$|\Delta r_1| < T_1 \qquad |\Delta r_2| < T_2$$

3. Fit first order polynomials (p_1, p_2) to all value pairs $(\Delta r_1, \Delta r_2)$ which succeeded in passing the above absolute test, and were used in the previous solution.

4. Define sigma radial and sigma tangential by

$$\sigma_{r_i} = \left[\sum_{\text{pass}} \frac{(\Delta r_i(\text{pass}) - p_i(\text{pass}))^2}{N(\text{pass})} \right]^{1/2}$$

where N(pass) is the number of passes in the polynomial fits and p_i is evaluated at TCA of the pass.

5. Use σ_{r_i} and σ_{r_2} in a sigma test on all navigation solutions.

$$|\Delta r_i(\text{pass}) - p_i(\text{pass})| < F\sigma_{r_i}$$

Any pass successfully passing both sigma tests is labeled good.

6. Any pass now labeled good but labeled bad on the previous orbit solution is added to the solution. This is done by adding its bias eliminated normal equation to the previous arc equation, prior to station elimination. Passes now labeled bad but previously labeled good are similarly subtracted from the solution. A new solution is determined and we return to Number One.

Covariance Test (Short-Arc Mode)

The *Short Arc Selector* section discusses a selection process used in determining various data spans, each of which cover the same satellite revolution. These data spans take the form of a nominal span and various extensions of the nominal. Covariance Test is a procedure for carrying out an orbit determination over each span and selecting one of the results as the best orbit fit for the revolution of interest.

The Covariance Test procedure is described by referring to the example in Diagram 15. In this example the nominal short-arc span is from t_3 to t_6, the number of drag extensions, Q, on each side of the nominal is two, and the epoch for our work is t_1.

Procedure: Initially we process passes between t_3 and t_6 using t_1 as epoch. We include C_{D1} thru C_{D5} as drag parameters weighting C_{D1} and C_{D2} out of the solution using a priori sigma values close to zero. Denoting the sum of pass matrices in the nominal span by B_{Nom} and the sum in each drag segment by B_{CD_i} we form the following covariance matrices.

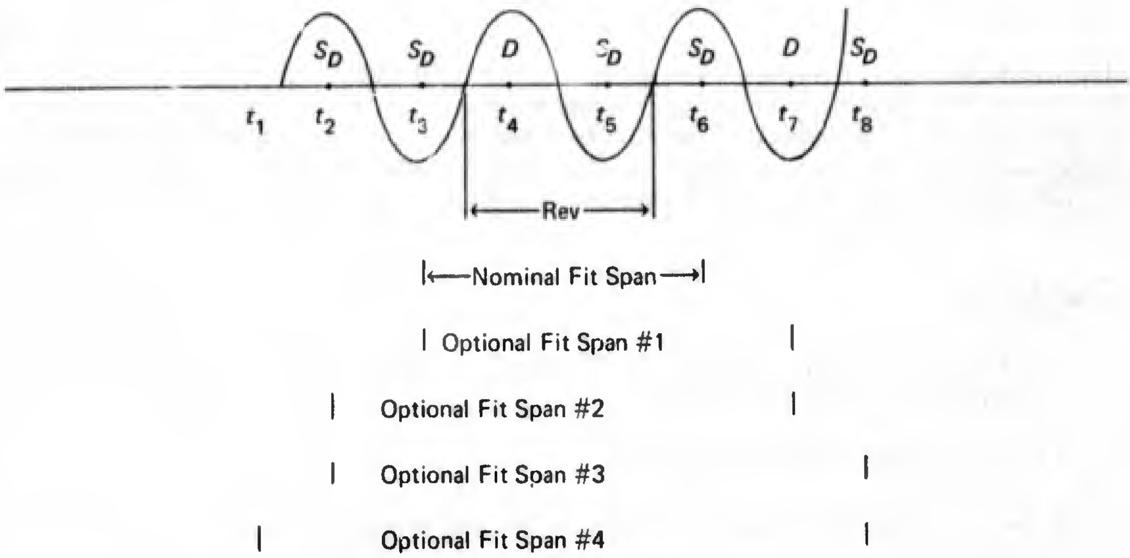
$$B_{\text{Nom}}^{-1}, (B_{\text{Nom}} + B_{CD6})^{-1}, (B_{\text{Nom}} + B_{CD6} + B_{CD2})^{-1}$$

$$(B_{\text{Nom}} + B_{CD6} + B_{CD2} + B_{CD7})^{-1} \quad \text{and}$$

$$(B_{\text{Nom}} + B_{CD6} + B_{CD2} + B_{CD7} + B_{CD1})^{-1}$$

S_D = Drag point end times determined by Short-Arc drag selection process.

D = Long-Arc reference trajectory drag end point times.



t_1 = epoch of fit

$t_2, t_3, t_4, t_5, t_6, t_7$ and t_8 are drag parameter end points.

Diagram 15

We propagate* each covariance matrix to the drag start and end times which occur in the revolution of interest. In the example this would be points t_4 and t_5.

*Propagation Procedure: Using (84) gives

$$B^{-1}(t) = \bar{\Psi}(t) B^{-1}(t_1) \bar{\Psi}^*(t) \tag{120}$$

$$\bar{\Psi}(t) = (\psi(t)\psi^{-1}(t_1)T(t_1), \ldots \psi_{C_{D_i}}(t) \ldots, \ldots \psi_{A^i}(t) \ldots)$$

$$\psi_{C_{D_i}}(t_j) = \begin{cases} \psi(t_j)[x_{C_{D_i}}(t_{i+1}) - x_{C_{D_i}}(t_i)] & t_j > t_i \\ 0 & t_j \leq t_i \end{cases}$$

$$\psi_{A^i}(t_j) = \begin{cases} \psi(t_j)[x_{A^i}(t_E^{A^i}) - x_{A^i}(t_s^{A^i})] & t_j > t_s^{A^i} \\ 0 & t_j \leq t_s^{A^i} \end{cases}$$

Letting $B^{-1}(t_i)$ be the propagated covariance we form the average positional covariance

$$\left[\sum_{i=4}^{5} \sum_{j=1}^{3} B_{ij}^{-1}(t_i)/2\right]^{1/2}$$

This value is compared with the input covariance tolerance. We go through this procedure for each of the covariance matrices listed, starting with B_{Nom}^{-1} and continuing along the list. If at some point the covariance tolerance is met then the process is terminated and that span, with associated covariance matrix, is used to procuce the orbit solution (short-arc solution) for the revolution. If the tolerance is not met then the short-arc solution is performed with the span of minimum average positional covariance.

Quality Point Index

A quality factor is computed for each short-arc and output as a potential diagnostic statistic. This factor is a sum of points computed as follows:

a. Count one point for each pass in the span.

b. Count one point for each pass with elevation at TCA greater than 35 degrees.

c. Count one point for each mid latitude pass. A pass is a mid latitude pass if the pass latitude λ satisfies

$$-m^\circ \leqslant \lambda \leqslant m^\circ$$

where m° is an input quantity.

d. Count -1/2 point for each pass that occurs in a given neighborhood of another pass. A pass with TCA(1) occurs in the neighborhood of a pass with TCA(2) if

$$|\text{TCA}(1) - \text{TCA}(2)| < \text{Input Neighborhood Tolerance}$$

Convergence Tests

Each pass has an error function H_{pass} associated with it.

$$H_{\text{pass}} = \sum_i (Obs - D)_i^2/\sigma_i^2$$

Using this error function we define the quantity signal to noise for a pass.

$$(\text{S/N})_i = \left[\sum_{j=1}^{N_i - \text{No. bias parameters}} (0 - D)_j^2/N_i\sigma_j\right]^{1/2} \tag{121}$$

Using signal to noise from each pass we define the arc signal to noise by

$$S/N = \left[\sum_{i=1}^{N} N_i (S/N)_i^2 \Big/ \sum_{i=1}^{N} N_i \right]^{1/2} \tag{122}$$

This value of signal to noise is output after an orbit improvement and labeled *Old Signal To Noise.*

$$(S/N)_i(\text{adjusted}) = \left[\frac{N_i (S/N)_i^2 - (E_b^* B_{bb}^{-1} E_b)_i}{N_i} \right]^{1/2} \tag{123}$$

$$S/N\ (\text{adjusted}) = \left[\frac{\sum_{i=1}^{N} [N_i (S/N)_i^2 - (E_b^* B_{bb}^{-1} E_b)_i]}{\sum_{i=1}^{N} N_i} \right]^{1/2} \tag{124}$$

$$= \left[\frac{\sum_{i=1}^{N} N_i (S/N)_i^2 (\text{adjusted})}{\sum_{i=1}^{N} N_i} \right]^{1/2}$$

Both of these quantities are output after an orbit solution and labeled *$(S/N)_i$ (adjusted)* and *Bias Reduced Signal-To-Noise* respectively. In addition if any station coordinates are solved for, the same procedure is used to compute *Station and Bias Reduced Signal-To-Noise.*

After an orbit solution the improvements, Δp_0, are used to compute a linear approximation to the station and bias reduced signal to noise of the new orbit. This approximation is labeled *Predicted Signal To Noise* and computed by

$$S/N\ (\text{Predicted}) = \left[\begin{pmatrix} \text{Station and Bias Reduced} \\ \text{Signal to Noise} \end{pmatrix}^2 - \frac{E_{\text{Arc}}^* \Delta p_0}{\sum_{i=1}^{N} N_i} \right]^{1/2} \tag{125}$$

When the difference between the station and bias reduced signal to noise of the present orbit and the predicted value for the next orbit differ by a sufficiently small amount, we do not recycle, i.e., do not integrate a new orbit. This is equivalent to having the change in variance (sum of squares of weighted residuals) be sufficiently small. Since variance is the quantity being minimized by our process, this will insure that re-integrating the trajectory, for the purpose of recycling, will not further reduce the variance. We make the recycling test by computing the percentage change in variance and testing against an input tolerance.

$$\text{Change in Variance} = E^*_{\text{Arc}} \Delta p_0$$

$$\text{Percent Change in Variance} = \frac{(100)\, E^*_{\text{Arc}} \Delta p_0}{\left(\sum_{i=1}^{N} N_i\right)\left(\begin{array}{c}\text{Bias and Station}\\ \text{Reduced Signal To Noise}\end{array}\right)^2} \tag{126}$$

If the recycling test fails we check against an input value for maximum cycle number. The cycle number starts out at an input value and is incremented each time the integration routine is entered. The program will continue its KFLOW after leaving the Combiner Solver area only if the percentage change in variance tolerance as well as the predicted signal to noise tolerance are satisfied. The recycling process is said to be converged if both tolerances are satisfied.

Derivation of Signal To Noise Computations

Using the definition for signal-to-noise of a pass and surpressing the weights for convenience gives

$$N_i (\mathrm{S/N})_i^2 = \text{Variance for Pass } (i) = H_{\text{pass}}(p)$$

for the variance of an orbit with parameter values p.

$$H_{\text{pass}}(p) = (0 - D(p))^* (0 - D(p))$$

We wish to compute $H_{\text{pass}}(p + \Delta p)$ which we will do using the approximation

$$D(p + \Delta p) \cong D(p) + \frac{\partial D}{\partial p} \Delta p$$

$$H_{\text{pass}}^{(p+\Delta p)} = (0 - D(p + \Delta p))^* (0 - D(p + \Delta p)) \cong \left(0 - D(p) - \frac{\partial D}{\partial p} \Delta p\right)^* \left(0 - D(p) - \frac{\partial D}{\partial p} \Delta p\right)$$

$$= (0 - D(p))^* (0 - D(p)) - (0 - D(p))^* \frac{\partial D}{\partial p} \Delta p - \Delta p^* \frac{\partial D^*}{\partial p} (0 - D(p))$$

$$+ \Delta p^* \frac{\partial D^*}{\partial p} \frac{\partial D}{\partial p} \Delta p$$

$$= H_{\text{pass}}(p) - 2(0 - D(p))^* \left(\frac{\partial D}{\partial p_0} \Delta p_0 + \frac{\partial D}{\partial p_b} \Delta p_b\right)$$

$$+ (\Delta p_0^*, \Delta p_b^*) \begin{bmatrix} \frac{\partial D^*}{\partial p_0} \frac{\partial D}{\partial p_0} & \frac{\partial D^*}{\partial p_0} \frac{\partial D}{\partial p_b} \\ \frac{\partial D^*}{\partial p_b} \frac{\partial D}{\partial p_0} & \frac{\partial D^*}{\partial p_b} \frac{\partial D}{\partial p_b} \end{bmatrix} \begin{pmatrix} \Delta p_0 \\ \Delta p_b \end{pmatrix}$$

$$\cong H_{pass}(p) - 2E_0^*\Delta p_0 - 2E_b^*\Delta p_b + \Delta p_0^* B_{00}\Delta p_0$$

$$+ \Delta p_b^* B_{b0}\Delta p_0 + \Delta p_0^* B_{0b}\Delta p_b + \Delta p_b^* B_{bb}\Delta p_b$$

From bias elimination we have

$$\Delta p_b = B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$H_{pass}^{(p+\Delta p)} \cong H_{pass}(p) - 2E_0^*\Delta p_0 - 2E_b^* B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$+ \Delta p_0^* B_{00}\Delta p_0 + (E_b^* - \Delta p_0^* B_{0b})B_{bb}^{-1}B_{b0}\Delta p_0$$

$$+ \Delta p_0^* B_{0b} B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$+ (E_b^* - \Delta p_0^* B_{0b})B_{bb}^{-1}B_{bb}B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$\cong H_{pass}(p) - 2E_0^*\Delta p_0 - 2E_b^* B_{bb}^{-1}E_b + 2E_b^* B_{bb}^{-1}B_{b0}\Delta p_0$$

$$+ \Delta p_0^* B_{00}\Delta p_0 + E_b^* B_{bb}^{-1}B_{b0}\Delta p_0 - \Delta p_0^* B_{0b}B_{bb}^{-1}B_{b0}\Delta p_0$$

$$+ \Delta p_0^* B_{0b}B_{bb}^{-1}E_b - \Delta p_0^* B_{0b}B_{bb}^{-1}B_{b0}\Delta p_0$$

$$+ E_b^* B_{bb}^{-1}E_b - E_b^* B_{bb}^{-1}B_{b0}\Delta p_0 - \Delta p_0^* B_{0b}B_{bb}^{-1}E_b$$

$$+ \Delta p_0^* B_{0b}B_{bb}^{-1}B_{b0}\Delta p_0$$

$$\cong H_{pass}(p) + \Delta p_0^*(B_{00} - B_{0b}B_{bb}^{-1}B_{b0})\Delta p_0$$

$$- 2(E_0^* - E_b^* B_{bb}^{-1}B_{b0})\Delta p_0 - E_b^* B_{bb}^{-1}E_b$$

$$\cong H_{pass}(p) + \Delta p_0^* B_{pass}^{Exp.}\text{(bias eliminated)}\,\Delta p_0$$

$$- 2\Delta p_0^* E_{pass}^{Exp.}\text{(bias eliminated)} - E_b^* B_{bb}^{-1}E_b$$

Summing over all passes yields

$$H(p + \Delta p) \cong H(p) + \Delta p_0^* B_{ARC}\Delta p_0 - 2\Delta p_0^* E_{ARC} - \sum_{\text{passes}} E_b^* B_{bb}^{-1}E_b$$

In this expression Δp_0 contains any possible station solutions. Using a similar argument on the 2nd and 3rd terms with respect to station elimination yields

$$H(p + \Delta p) \cong H(p) + \Delta p_0^* B_{ARC}\Delta p_0 - 2\Delta p_0^* E_{ARC} - \sum_{\text{stations}} E_s^* B_{ss}^{-1}E_s - \sum_{\text{passes}} E_b^* B_{bb}^{-1}E_b$$

however

$$B_{ARC}\Delta p_0 = E_{ARC}$$

giving

$$H(p + \Delta p) \cong H(p) - \sum_{\text{passes}} E_b^* B_{bb}^{-1} E_b - \sum_{\text{stations}} E_s^* B_{ss}^{-1} E_s - \Delta p_0^* E_{ARC}$$

This formula states that the variance for an orbit with parameter values $p + \Delta p$ can be estimated by the variance at p together with a bias, station and orbit reduction due to Δp.

New Variance $\cong$ Old Variance - Bias Reduction - Station Reduction - Orbit Reduction

The approximation for $H(p + \Delta p)$ given above leads in a natural way to the recycling and convergence criteria of the previous section.

Parameter Covariance Matrix

The arc normal matrix is obtained by formally eliminating the bias parameters from the expanded pass matrices and summing. An equivalent procedure would be to sum (direct sum) the pass matrices first and eliminate bias parameters last.

Let this later matrix be represented by

$$\begin{bmatrix} B_{00} & B_{0b} \\ B_{b0} & B_{bb} \end{bmatrix} \begin{pmatrix} \Delta p_0 \\ \Delta p_b \end{pmatrix} = \begin{pmatrix} E_0 \\ E_b \end{pmatrix}$$

where Δp_0 represents all non-bias parameters and Δp_b represents all bias parameters.

The parameter covariance matrix will now be given by $\epsilon(\Delta p \Delta p^*)$ where ϵ is the expectation function and

$$\Delta p = \begin{pmatrix} \Delta p_0 \\ \Delta p_b \end{pmatrix}$$

Formally eliminating Δp_b gives

$$\Delta p_b = B_{bb}^{-1}(E_b - B_{b0}\Delta p_0)$$

$$(B_{00} - B_{0b}B_{bb}^{-1}B_{b0})\Delta p_0 = (E_0 - B_{0b}B_{bb}^{-1}E_b)$$

or

$$B_{\text{Arc}}\Delta p_0 = E_{\text{Arc}}$$

$$\epsilon(\Delta p_0 \Delta p_0^*) = \epsilon(B_{Arc}^{-1} E_{Arc} E_{Arc}^* B_{Arc}^{-1})$$
$$= B_{Arc}^{-1} \epsilon(E_{Arc} E_{Arc}^*) B_{Arc}^{-1}$$

$$E_{Arc} E_{Arc}^* = (E_0 - B_{0b} B_{bb}^{-1} E_b)(E_0^* - E_b^* B_{bb}^{-1} B_{b0})$$
$$= E_0 E_0^* - E_0 E_b^* B_{bb}^{-1} B_{b0} - B_{0b} B_{bb}^{-1} E_b E_0^*$$
$$+ B_{0b} B_{bb}^{-1} E_b E_b^* B_{bb}^{-1} B_{b0}$$

Using our assumption that the data is uncorrelated and surpressing the weights for convenience yields $\epsilon((0 - D)(0 - D)^*) = I$.

Using $E = \partial D^* / \partial p \, (0 - D)$ and the bilinearity of ϵ gives

$$\epsilon(E_{Arc} E_{Arc}^*) = \frac{\partial D^*}{\partial p_0} \frac{\partial D}{\partial p_0} - \frac{\partial D^*}{\partial p_0} \frac{\partial D}{\partial p_b} B_{bb}^{-1} B_{b0}$$

$$- B_{0b} B_{bb}^{-1} \frac{\partial D^*}{\partial p_b} \frac{\partial D}{\partial p_0} + B_{0b} B_{bb}^{-1} \frac{\partial D^*}{\partial p_b} \frac{\partial D}{\partial p_b} B_{bb}^{-1} B_{b0}$$

$$= B_{00} - B_{0b} B_{bb}^{-1} B_{b0} - B_{0b} B_{bb}^{-1} B_{b0}$$
$$+ B_{0b} B_{bb}^{-1} B_{bb} B_{bb}^{-1} B_{b0}$$
$$= B_{00} - B_{0b} B_{bb}^{-1} B_{b0}$$
$$= B_{Arc}$$

Substituting this result in the above formula for $\epsilon(\Delta p_0 \Delta p_0^*)$ gives

$$\epsilon(\Delta p_0 \Delta p_0^*) = B_{Arc}^{-1} B_{Arc} B_{Arc}^{-1}$$
$$= B_{Arc}^{-1}$$

The above arguments still hold if $\epsilon((0 - D)(0 - D)^*)$ is a block diagonal covariance matrix, where each block represents a single pass. In addition, the inclusion of station parameters does not alter the result.

Pass Tags

Within Celest a system of pass tags is used in order to save information about bad passes and preserve this information for later use. The tags are stored in arrays on the Short-Arc-Selector File. LATAG is the long-arc tag array and ITAGS represents the short-arc arrays.

There are four basic paths involving tag processing.

Path 1. (Long-Arc Mode) In this path passes are read from the Pass Matrix File and checked against input to determine if any passes have been tagged using the pass or station delete option. Those tagged are so indicated and a prototype, NBB, of array LATAG is set up. After cross pass filtering is completed NBB is adjusted to include tage for all passes which were cross-pass filtered from the solution. At this time LATAG is written as a copy of NBB. This information can now be saved by cataloging the Short-Arc-Selector File.

Path 2. (Incomplete Short-Arc-Selector File attached) In this path LATAG has been set and Short-Arc-Selector writes all remaining information with the exception of the ITAGS array. The solution area of Celest initializes ITAGS for each short arc at the subset of LATAG contained in the short-arc span. If the drag extension option is on, then the short-arc is set at the largest span possible under that drag extension option. After Covariance Check selects a given span for the short-arc ITAGS is adjusted to that span. The final ITAGS array is then written on the Short-Arc-Selector File. This information may also be saved by cataloging the Short-Arc-Selector File.

Path 3. (Complete Short-Arc-Selector File attached) In this path LATAG and ITAGS are both set. In addition all other information on the Short-Arc-Selector File is present, and solutions are produced for each short arc as directed by this file. This situation can occur either by attaching a Short-Arc-Selector File made in a previous Celest run or one made in post analysis at the graphics console.

Path 4. (No Short-Arc-Selector File attached) In this path the LATAG array is written by checking input as in Path 1. At this point the procedure in Path 2 is followed.

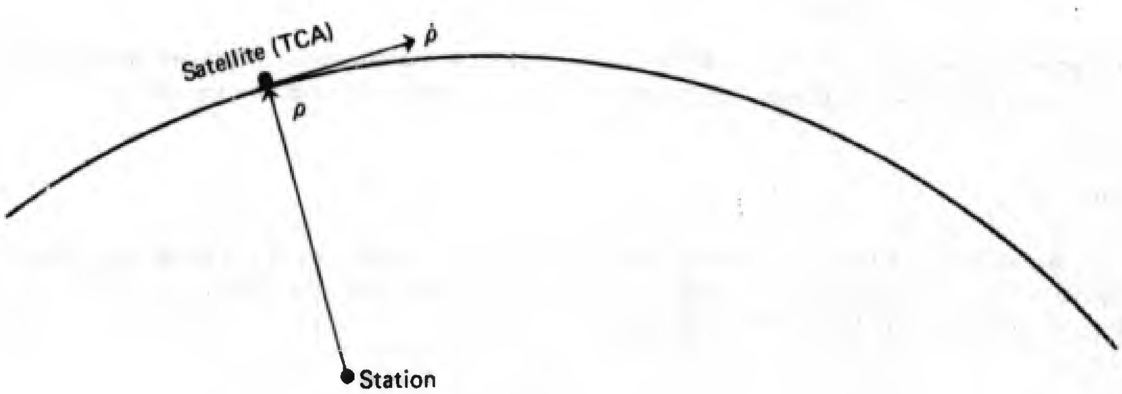
Station Analysis Report

Station analysis cards can be punched on option and contain diagnostic information useful to the satellite tracking stations. The card values are as follows:

1. STA – Station number extracted from the header message of the raw data.

2. Time (hr, min) – Time of the first data point.

3. TCA (sec) – Time of closest approach of the satellite obtained from the satellite orbit by searching for the point in time where the range (ρ) and range rate ($\dot{\rho}$) vectors are perpendicular to each other.



4. FREQ (MC) – The Q number from the raw data header message is used to determine a base frequency. The frequency given is this base frequency rounded to the nearest MHz.

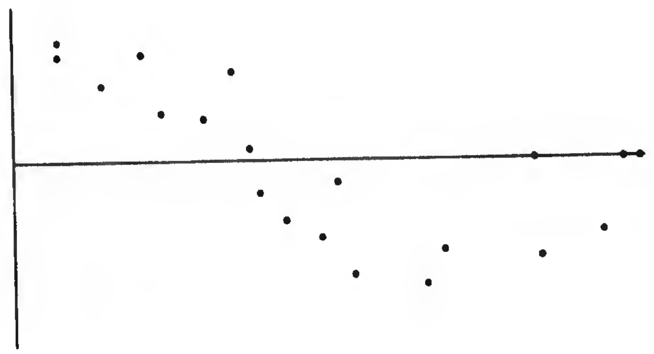
5. EL (Deg) – The satellite elevation at TCA computed from the satellite orbit.

6. PTS: Good – Total number of points left after passing through the Celest point filtering process.

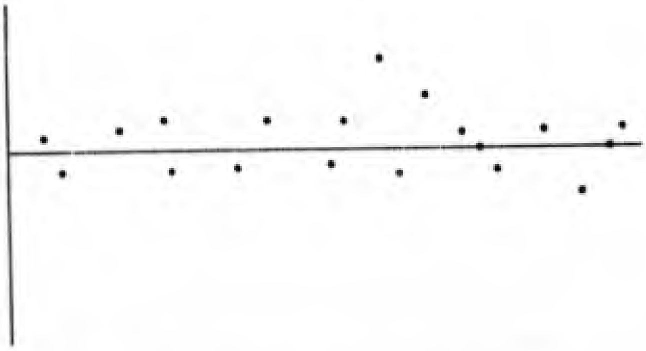
a. Points are filtered out in the Celest pre-processor if information is missing, values are too large, or the data fails a monotone test (see *Pre-Processor Reject Codes).*

b. Points are filtered out in the Celest filter by removing orbit error from the residuals and testing against 2.0 sigma.

7. Filt. Noise (cps/MHz or M) – Filtered noise is the standard deviation on the data after modeling error has been removed. In the case of Doppler this value is given in units of cps/MHz. In the cases of ITT, CCID or Geoceiver the value is given in units of meters.



Before modeling error removed



After modeling error removed

Filtered Noise = RMS of residuals after modeling error is removed. This value is scaled to 1 MH_z for Doppler data.

As a result of the point filtering process each data point is assigned a standard deviation (σ_i). The precise formula for calculating filtered noise is

$$\text{Filtered Noise} = \left[\sum_{i=1}^{N} \sigma_i^2 / N \right]^{1/2}$$

where N = number of good points + number of bias parameters used in filtering.

8. ELT. (M) – This is the along-track navigation error determined from the refined orbit. Holding the orbit fixed, the station is allowed to move in the along track ($\dot{\rho}$) and slant range (ρ) directions, in order to best fit the data from the pass. Since this movement is from a known position the result is tabulated as along track (ELT) and slant range (ELR) errors. The values represent a measurement of how well the final refined orbit fits the data of a given pass. (See diagram for #3)

9. ELR. (M) – Slant range error

10. DLT. F. (PPM)[(1)] – Delta frequency is the value of the frequency bias determined in the above navigation solution. Assuming that the satellite frequency has a constant bias during a given pass, then this number represents that bias in parts per million (ppm).

11. ACT – Action taken in the course of point filtering. Action label described below.

- A – No TCA
- B – Rejected in filter because to many points were filtered out.
- D – Rejected on TCA zenith angle test.
- E – Pass not balanced; i.e., the difference between the number of points on one side of TCA and the number of points on the other is greater than the balanced pass tolerance.

Passes are rejected for reasons other than the above, in the pre-processor. These reasons are listed in Pre-Processor under Reject Codes, but no indication is given on the Station Analysis Cards.

(1) A nominal value of oscillator frequency offset (in ppm) is associated with each satellite. On the basis of the Doppler data from a given pass, a correction, DLT. F. is calculated. The corrected absolute offset, in ppm, for the pass is the algebraic sum of the nominal offset and DLT. F.

$$\begin{aligned}\text{Absolute offset} &= \text{Nominal offset} + \text{DLT. F.} \\ &= \Delta V_s + \text{DLT. F.}\end{aligned}$$

The nominal offset is always negative. For example, if the nominal offset for a satellite is -80 ppm, a DLT. F. of + .04 would indicate an absolute offset of -80 + .04 = - 79.96 ppm.

Orbit Computation Report

The Orbit Computation Report is a restatement of all pertinent orbital information and confidence statistics in letter form. The only computation in this section is that giving mean orbital elements. This computation is carried out using the Brouwer method as described in Brouwer and Clemence (Ref. 2). The algorithm for this work is described in Appendix C.

Flow Chart

Diagram 16 is a flow chart summarizing this section.

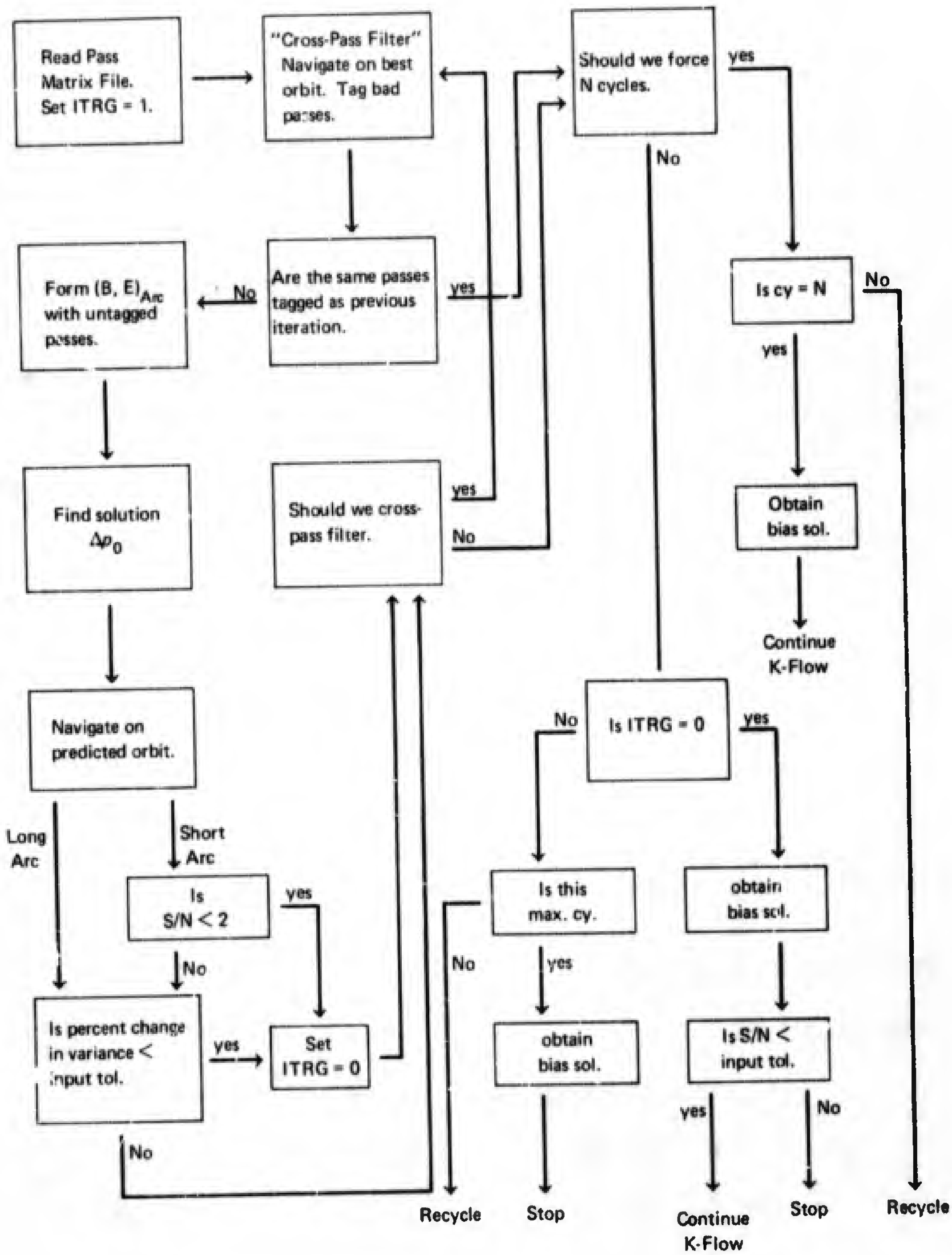


Diagram 16

PROPAGATOR

The propagator has two main functions:

1. Propagate the orbit improvements, Δp_0, over the reference trajectory adding the results to the reference values.

$$x_t(\text{new}) = x_t(\text{ref}) + \Delta x_t \tag{127}$$

where $x_t(\text{ref})$ is the reference trajectory value and Δx_t is the propagated orbit improvement at time t. The value $x_t(\text{new})$ is output in either inertial or earth fixed form.

2. Propagate the covariance for Δp_0, B_{Arc}^{-1}, so that it may be output at desired times in either inertial or earth fixed form. If C_t is the inertial position covariance at time t, then the earth fixed position covariance is given by

$$C_t(E.F.) = [\text{ABCD}]\, C_t\, [\text{ABCD}]^* \tag{128}$$

On option a local frame covariance can also be obtained by

$$C_t(R) = R C_t(E.F.) R^* \tag{129}$$

where

$$R = [\hat{r}, \dot{\hat{r}}, \hat{r x \dot{r}}]^{-1} \underset{\substack{\text{if}\\ \text{circular}\\ \text{orbit}}}{\cong} [\hat{r}, \dot{\hat{r}}, \hat{r x \dot{r}}]^* \tag{130}$$

r = earth fixed satellite position
$\dot{r}$ = earth fixed satellite velocity

COMPUTATION OF IMPROVED ORBIT

Recall from (84) that

$$\psi_p(t) = \begin{cases} 0 \;\; p \text{ not orbit, } T_p(t_s(p))\; p \text{ orbit} & t \leq t_s(p) \\ \psi_{\tilde{p}}(t) - \psi(t)\chi_p(t_s(p)) & t_s(p) < t \leq t_E(p) \\ \psi(t)[\chi_p(t_E(p)) - \chi_p(t_s(p))] & t > t_E(p) \end{cases}$$

Using χ define $\tilde{\chi}$ by

$$\tilde{\chi}_p(t) = \begin{cases} 0 & t \leq t_s(p) \\ -\chi_p(t_s(p)) & t_s(p) < t \leq t_E(p) \\ \chi_p(t_E(p)) - \chi_p(t_s(p)) & t > t_E(p) \end{cases} \tag{131}$$

when p is not an orbit parameter and

$$\tilde{\chi}_e(t) = \psi^{-1}(t_s(e))T_e(t_s(e))$$

when $p = e$ an orbit parameter. For each parameter p let β_p be its characteristic function which is zero outside its domain and one on its domain.

Define $\tilde{\psi}$ by

$$\tilde{\psi}_p(t) = \beta_p(t)\psi_{\tilde{p}}(t) \tag{132}$$

For a family of parameters $p = (p_1, \ldots p_n)$ we will use the convention that

$$\tilde{\psi}_p = (\tilde{\psi}_{p_1}, \ldots \tilde{\psi}_{p_n})$$

$$\tilde{\chi}_p = (\tilde{\chi}_{p_1}, \ldots \tilde{\chi}_{p_n})$$

Using these definitions it is clear that

$$\Delta x_t = \frac{\partial x_t}{\partial p} \Delta p_0 = [\psi(t)\tilde{\chi}_e(t), \psi(t)\tilde{\chi}_m(t) + \tilde{\psi}_m(t)]\begin{pmatrix}\Delta e_0 \\ \Delta m_0\end{pmatrix} \tag{133}$$

where the orbital elements in Δp_0 are Δe_0 and the model parameters are Δm_0. Expanding (133) gives

$$\Delta x_t = \psi(t)[\tilde{\chi}_e(t), \tilde{\chi}_m(t)]\begin{pmatrix}\Delta e_0 \\ \Delta m_0\end{pmatrix} + \tilde{\psi}_m(t)\Delta m_0$$

$$\Delta x_t = [\tilde{\psi}_e(t), \tilde{\psi}_m(t)]\begin{pmatrix}[\tilde{\chi}_e(t), \tilde{\chi}_m(t)]\begin{pmatrix}\Delta e_0 \\ \Delta m_0\end{pmatrix} \\ \Delta m_0\end{pmatrix}$$

$$\Delta x_t = \tilde{\psi}(t)\begin{pmatrix}\tilde{\chi}(t)\begin{pmatrix}\Delta e_0 \\ \Delta m_0\end{pmatrix} \\ \Delta m_0\end{pmatrix} \tag{134}$$

COVARIANCE COMPUTATION

The inertial covariance for x_t(new) is computed by $\epsilon(\Delta x_t \Delta x_t^*)$ where ϵ is the expectation function.

$$\epsilon(\Delta x_t \Delta x_t^*) = \epsilon\left(\tilde{\psi}(t)\begin{pmatrix}\tilde{\chi}(t)\Delta p_0 \\ \Delta m_0\end{pmatrix}(\Delta p_0^* \tilde{\chi}^*(t),\ \Delta m_0^*)\tilde{\psi}^*(t)\right)$$

$$= \tilde{\psi}(t)\begin{bmatrix}\tilde{\chi}(t)\epsilon(\Delta p_0 \Delta p_0^*)\tilde{\chi}^*(t) & \tilde{\chi}(t)\epsilon(\Delta p_0 \Delta m_0^*) \\ \epsilon(\Delta m_0 \Delta p_0^*)\tilde{\chi}^*(t) & \epsilon(\Delta m_0 \Delta m_0^*)\end{bmatrix}\tilde{\psi}^*(t)$$

or

$$\epsilon(\Delta x_t \Delta x_t^*) = \tilde{\psi}(t)\begin{bmatrix}\tilde{\chi}(t)B_{\text{Arc}}^{-1}\tilde{\chi}^*(t) & \tilde{\chi}(t)\begin{bmatrix}[B_{\text{Arc}}^{-1}]_{em} \\ [B_{\text{Arc}}^{-1}]_{mm}\end{bmatrix} \\ * & [B_{\text{Arc}}^{-1}]_{mm}\end{bmatrix}\tilde{\psi}^*(t) \tag{135}$$

ALGORITHM FOR COMPUTATIONS

$\tilde{\chi}(t)$ is a step function which changes value as the time parameter crosses a model parameter boundary. At each time point t, $\tilde{\chi}(t)$ is a $6 \times q$ matrix with q equal to the dimension of the dynamic parameter set p.

$$p = (e, m) = (e_1, \ldots, e_6, C_{D_1}, \ldots C_{D_{NCD}}, A', \ldots A^{NT}, kr)$$

where e represents orbital elements, m represents model parameters and

$$\text{dimension}(p) = 6 + NCD + 3\,NT + 1$$

For the purpose of describing this algorithm we will identify $\tilde{\psi}_{m_i}$ with its imbedding in the $6 \times q$ dimensional space of $\tilde{\chi}$, i.e.

$$\tilde{\psi}_{m_i} \rightarrow [0, 0, \ldots 0, \underset{\substack{\uparrow \\ m_i \text{ column}}}{\tilde{\psi}_{m_i}}, 0, \ldots, 0]$$

1. Initialize $(\tilde{\psi}, \tilde{\chi})$ at $(\tilde{\psi}(t_s(1)), \tilde{\chi}(t_s(1)))$.

2. $\tilde{\psi}$ changes at each time line, but $\tilde{\chi}$ is a step function which changes only at drag and thrust boundaries. When a drag or thrust domain is entered $\tilde{\chi}_m$ is adjusted by adding $-\chi_m(t_s(m))$ to its present value.

3. When a drag or thrust domain is exited, $\tilde{\chi}_m$ is adjusted by adding $\chi_m(t_E(m))$ to its present value.

If both a drag and thrust domain are entered or exited then adjustments must be made for both parameters.

In the event that the propagation step is greater than the drag or thrust segmentation intervals then adjustments for both entering and exiting a model domain must be made when such a domain is skipped over.

OUTPUT

The earth fixed position, velocity vector, and earth fixed position covariance are output at intervals ΔT. The inertial position, velocity vector, and covariance are output at intervals $m\Delta T$. Both m and ΔT are input values.

An additional output of positional standard errors in the radial, tangential, and out-of-plane directions can be obtained for diagnostic purposes.

SHORT-ARC-SELECTOR

GENERAL

The Short-Arc-Selector input is

A = Average drag length
TO = Start time of first revolution
TS(NSA) = Seconds before start of revolution to begin the nominal short-arc.
TE(NSA) = Seconds after end of revolution to end the nominal short-arc.
P = Period of the satellite
Q = Number of drag extensions
D(L.A.) = Long-arc drag decomposition times.

The revolution start times are given by

$$t_s(\text{Rev})(i) = To + (i - 1)P \tag{136}$$

The revolution end times are given by

$$t_E(\text{Rev})(i) = t_s(\text{Rev})(i + 1)$$

The nominal short-arc start and end times are given by

$$t_s(\text{NSA}) = t_s(\text{Rev}) - \text{TS(NSA)} \tag{137}$$

$$t_E(\text{NSA}) = t_E(\text{Rev}) + \text{TE(NSA)} \tag{138}$$

During the orbit integration process for a given long-arc span, $[t_s, t_E]$, a specific drag profile is used. The procedure for doing orbit fits over subintervals (short arcs) of $[t_s, t_E]$ requires the drag profile of a subinterval $[t_s^{(1)}, t_E^{(1)}]$ to include any long-arc drag end point which falls within its boundaries.

DRAG SELECTION

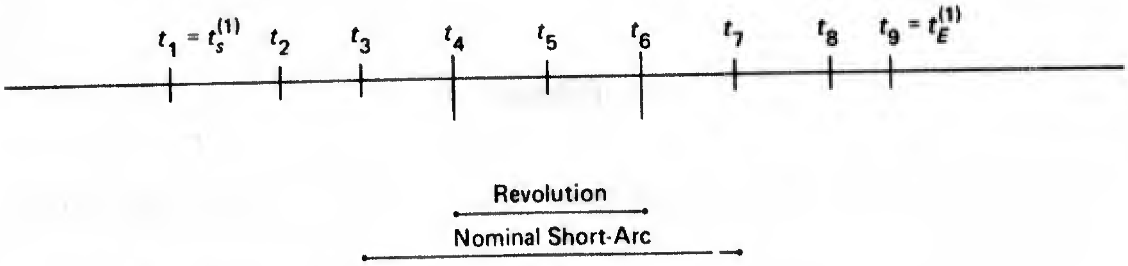
Define

$$\{T_1, T_2, T_3, T_4\} = \{t_s(\text{NSA}) - QA, t_s(\text{NSA}), t_E(\text{NSA}), t_E(\text{NSA}) + QA\}$$

where

$$T_1 = t_s(1) \text{ and } T_4 = t_E(1)$$

In Diagram 17, $T_1 = t_1$ $T_2 = t_3$ $T_3 = t_7$ and $T_4 = t_9$



A = 1/2 Revolution
Q = 2
TS(NSA) = TE(NSA) = 1/2 Revolution

Span 0 = all passes in the Nominal Short-Arc
Span 1 = Span 0 plus all passes in $[t_2, t_3]$
Span 2 = Span 1 plus all passes in $[t_7, t_8]$
Span 3 = Span 2 plus all passes in $[t_1, t_2]$
Span 4 = Span 3 plus all passes in $[t_8, t_9]$

Diagram 17

Initialize the short-arc drag profile points D(S.A.) by

$$D(S.A.) = \{T_1, T_2, T_3, T_4\} \cup \{D(L.A.) \cap [t_s^{(1)}, t_E^{(1)}]\}$$

Order D(S.A.) = $\{s_1, s_2, \ldots s_q\}$ so that $s_i < s_j$ for $i < j$ Partition $[S_i, S_{i+1}]$ as follows:

$$A_{i+1} = \frac{|S_{i+1} - S_i|}{A} = \frac{d_{i,i+1}}{A} \tag{139}$$

$$\alpha_{i+1} = [A_{i+1} + .5] \tag{140}$$

where [ ] is the greatest integer function.

If $\alpha_{i+1} = 0$ the resulting partition for $[s_i, s_{i+1}]$ is $\{s_i, s_{i+1}\}$.

If $\alpha_{i+1} \neq 0$ form the resulting partition for $[s_i, s_{i+1}]$ by

$$P_i = \left\{s_i,\ s_i + \left[\frac{d_{i,i+1}}{\alpha_{i+1}}\right],\ s_i + 2\left[\frac{d_{i,i+1}}{\alpha_{i+1}}\right],\ \ldots,\ s_i + (\alpha_{i+1} - 1)\left[\frac{d_{i,i+1}}{\alpha_{i+1}}\right],\ s_{i+1}\right\} \tag{141}$$

Set D(S.A.) = $\bigcup_{i=1}^{q-1} P_i$

The resulting drag profiles for all short arcs are stored on the Short-Arc-Selector File.

INTEGRATOR

Recalling (71), (74), and (75), the equations of motion and associated perturbation equations are given again by

$$\ddot{x} = G_1(x) + [C_D(t) + C_D]D(x, \dot{x}) + L(x, \dot{x})[A(t) + A] + krR(x) = G(x, \dot{x}, C_D, A, kr) \quad (71)$$

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h} \quad (74)$$

$$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{\partial G}{\partial \dot{x}} \dot{h} + \frac{\partial G}{\partial p} \qquad p = \{C_D, A, kr\} \quad (75)$$

The canonical perturbed trajectory is a solution to this set of equations with C_D and A set to zero.

At this point we will discuss the numerical integration technique used in solving (71), (74), and (75).

POLYNOMIAL APPROXIMATION

Let $y(x)$ be an $N + 1$ times continuously differentiable function. Let $x_1, \ldots x_{N+1}$ be $N + 1$ points with corresponding values $y(x_i)$. Let $p_N(x)$ be a polynomial of degree N such that $p_N(x_i) = y(x_i) i = 1, 2, \ldots N + 1$. Let $y(x) = p_N(x) + E(x)$ where E is the error function. We wish to determine E.

Definition

$$\pi(x) = (x - x_1)(x - x_2) \ldots (x - x_{N+1})$$

The function π vanishes at $x_1, x_2 \ldots x_{N+1}$ and the $N + 1$ derivative $\pi^{N+1}(x) = (N + 1)!$

Definition

$$F(x) = y(x) - p_N(x) - k\pi(x)$$

For any $\bar{x} \neq x_1, \ldots x_{N+1}$ choose k such that $F(\bar{x}) = 0$. This can be done since $\pi = 0$ at exactly $x_1, \ldots x_{N+1}$. Since $F = 0$ at $N + 2$ points, $F^{N+1} = 0$ at one point, i.e., $F^{N+1}(\epsilon) = 0$. Since $p_N^{N+1} = 0$ identically we have

$$F_{(\epsilon)}^{N+1} = y_{(\epsilon)}^{N+1} - k(N + 1)! = 0$$

$$k = y_{(\epsilon)}^{N+1} / (N + 1)!$$

$$F(x) = y(x) - p_N(x) - y_{(\epsilon)}^{N+1} \Big/ (N + 1)! \;\; \pi(x)$$

Since $F(\bar{x}) = 0$ we have

$$y(\bar{x}) = p_N(\bar{x}) + y^{N+1}(\epsilon) \Big/ (N+1)! \quad \pi(\bar{x})$$

ϵ is a function of $\bar{x}$ and $\bar{x}$ is arbitrary so

$$y(x) = p_N(x) + y^{N+1}(\epsilon) \Big/ (N+1)! \quad \pi(x) \tag{142}$$

Divided Differences

For the function $y(x)$ construct

$$y[x_0] = y(x_0)$$

$$y[x_0, x_1] = \frac{y[x_1] - y[x_0]}{x_1 - x_0} = \frac{y(x_1) - y(x_0)}{x_1 - x_0}$$

$$\vdots$$

$$y[x_0, x_1, \ldots x_k] = \frac{y[x_1, \ldots x_k] - y[x_0, \ldots x_{k-1}]}{x_k - x_0}$$

Backward Differences

For the function $y(x)$ construct

$$\nabla y(x_k) = y(x_k) - y(x_{k-1})$$

$$\nabla^r y(x_k) = \nabla [\nabla^{r-1} y(x_k)] \tag{143}$$

$$\nabla^{-1} \text{ defined by } y(x_k) = \nabla^{-1} y(x_k) - \nabla^{-1} y(x_{k-1})$$

Note that the backward differences and the divided differences are connected by

$$\nabla y(x_k) = y(x_k) - y(x_{k-1}) = y[x_k] - y[x_{k-1}] = y[x_{k-1}, x_k]\,(x_k - x_{k-1})$$

$$= y[x_{k-1}, x_k]\,h \quad \text{if } h = x_k - x_{k-1}$$

$$\nabla^r y(x_k) = r!\, h^r y[x_{k-r} \ldots x_{k-1} x_k]$$

$$\nabla^r y(x_{k+r}) = r!\, h^r y[x_k, \ldots x_{k+r-1} x_{k+r}]$$

From Divided Differences we have

$$y[x_0, x] = \frac{y(x) - y(x_0)}{x - x_0}$$

or

$$y(x) = y(x_0) + y[x_0, x](x - x_0)$$

also

$$y[x_1, x_0, x] = \frac{y[x_0, x] - y[x_1, x_0]}{x - x_1}$$

or

$$y(x) = y(x_0) + y[x_1 x_0](x - x_0) + y[x_1 x_0 x](x - x_0)(x - x_1)$$

Continuing gives

$$y(x) = y(x_0) + y[x_1 x_0](x - x_0) + y[x_2 x_1 x_0](x - x_0)(x - x_1) \qquad (144)$$
$$+ \ldots + y[x_N x_{N-1} \ldots x_0](x - x_0) \ldots (x - x_{N-1}) + E(x)$$

where

$$E(x) = y[x_N x_{N-1} \ldots x_0 x](x - x_0) \ldots (x - x_N)$$

Letting $p_N(x) = y(x) - E(x)$ defines p_N as a polynomial of degree N. It is clear that $p_N(x_i) = y(x_i)$ $i = 0, 1, \ldots N - 1, N$. It follows from the discussion on polynomial approximation that

$$E(x) = y^{N+1}(\epsilon) \Big/ (N + 1)! \quad \pi(x)$$

Let the $N + 1$ points $x_0, x_1, \ldots x_N$ be $x_{n+1}, x_n, \ldots x_{n+1-N}$. This gives, using (144)

$$y(x) = y(x_{n+1}) + y[x_n, x_{n+1}](x - x_{n+1}) + y[x_{n-1} x_n x_{n+1}](x - x_{n-1})(x - x_n) + \ldots$$
$$+ y[x_{n+1-N}, \ldots, x_{n+1}](x - x_{n+1}) \ldots (x - x_{n+2-N}) + E(x)$$

where

$$E(x) = y^{N+1}(\epsilon) \Big/ (N + 1)! \quad (x - x_{n+1}) \ldots (x - x_{n+1-N})$$

For equally spaced points $x_i - x_{i-1} = h$ and using the variable s defined by $x = x_n + sh$ we have

$$y(x_n + sh) = y(x_{n+1}) + \nabla y(x_{n+1})(s - 1) + \nabla^2 y(x_{n+1}) \frac{(s - 1)s}{2!}$$
$$+ \ldots + \nabla^N y(x_{n+1}) \frac{(s - 1)s(s + 1) \ldots (s + N - 2)}{N!} + E(s) \qquad (145)$$

INTEGRATION TECHNIQUE

Write the equations of motion (71) as

$$\dot{x} = y$$

$$\dot{y} = G(t, x, y) \qquad x_0 \text{ and } y_0 = \dot{x}_0 \text{ given} \tag{146}$$

Let $(x, y) = X$ and rewrite the above as

$$\dot{X} = \tilde{G}(t, x) \qquad X_0 \text{ given} \tag{147}$$

Applying the same technique to equations (74) and (75) permits the entire set to be expressed by

$$\dot{X} = \tilde{G}(t, X) \qquad X_0 \text{ given} \tag{148}$$

Solving (148) is equivalent to solving

$$X(t) = X_0 + \int_{t_0}^{t} \tilde{G}(T, X(T))dT = I(X)(t) \tag{149}$$

A solution to (149) is a function X with the property that $I(X) = X$. In particular we wish to find the solution values at time points $t_0, t_1, \ldots t_n, \ldots$ where $t_{i+1} = t_i + h$ and h is a constant integration step. All other values of X can then be determined by interpolation.

At this point we will assume that $\dot{X}(t)$ has been determined for $t = t_0, t_1, \ldots t_n$ and we wish to determine $X(t_{n+1})$.

Let $\tilde{G} = y$ in (145) and expand (145) to degree $N + 1$ assuming $n \geq N$. Substituting (145) into (149) gives

$$X(t_{n+1}) = X(t_n) + h \int_0^1 \Big[\tilde{G}(t_{n+1}, X(t_{n+1})) + (s-1)\nabla\tilde{G}_{n+1} + \frac{(s-1)s}{2!} \nabla^2\tilde{G}_{n+1} + \ldots$$

$$+ \frac{(s-1)s \ldots (s+N-2)(s+N-1)}{(N+1)!} \nabla^{N+1}\tilde{G}_{n+1} + E(s) \Big] ds$$

$$= I(X)(t_{n+1}) = \mathcal{J}(X(t_{n+1})) \tag{150}$$

Assuming that N can be chosen such that the integral of E is negligible, our problem reduces to finding a fixed point for $\mathcal{J}$. It is known that by making an appropriate estimate of $X(t_{n+1})$ and applying $\mathcal{J}$ successively we can converge to the correct value of $X(t_{n+1})$. The convergence will depend on h being sufficiently small.

An estimate of $X(t_{n+1})$ yields an estimate of $\dot{X}(t_{n+1}) = \tilde{G}(t_{n+1}, X(t_{n+1}))$. We will estimate $\dot{X}(t_{n+1})$ directly and use this estimate in (150). This gives an estimate of $X(t_{n+1})$ The velocity value can now be recomputed using $\tilde{G}$. This procedure can be iterated until convergence is reached, which is usually in one iteration. We express this technique in

$$X(t_{n+1}) = X(t_n) + h \int_0^1 \left[\dot{X}_{n+1} + (s-1)\nabla \dot{X}_{n+1} + \frac{(s-1)s}{2!} \nabla^2 \dot{X}_{n+1} \right.$$

$$\left. + \frac{(s-1)s \ldots (s+N-1)}{(N+1)!} \nabla^{N+1} \dot{X}_{n+1} \right] ds \tag{151}$$

Note that each backward difference will be an estimate as $\dot{X}(t_{n+1})$ is an estimate. When $\dot{X}(t_{n+1})$ is recomputed, using $\tilde{G}$, the backward differences must be corrected as a result of the recomputation.

At this point we rewrite (151) as

$$X_{n+1} = X_n + h \sum_{k=0}^{N+1} a_k \nabla^k \dot{X}_{n+1} \tag{152}$$

where

$$a_0 = 1$$

$$a_1 = -1/2$$

$$a_k = \int_0^1 \frac{(s-1)s \ldots (s+k-2)}{k!} \, ds \qquad k > 0$$

Equation (152) implies that

$$\nabla X_{n+1} = h \sum_{k=0}^{N+1} a_k \nabla^k \dot{X}_{n+1} \tag{153}$$

Recalling the definition of ∇^{-1}

$$X_{n+1} = h \sum_{k=0}^{N+1} a_k \nabla^{k-1} \dot{X}_{n+1} \tag{154}$$

$$X_{n+1} = h\nabla^{-1}\dot{X}_{n+1} + h \sum_{k=1}^{N+1} a_k \nabla^{k-1} \dot{X}_{n+1} = h\dot{X}_{n+1} + h\nabla^{-1}\dot{X}_n + h \sum_{k=1}^{N+1} a_k \nabla^{k-1} \dot{X}_{n+1}$$

$$= h\nabla^{-1}\dot{X}_n + h(1 + a_1)\dot{X}_{n+1} + h \sum_{k=2}^{N+1} a_k \nabla^{k-1} \dot{X}_{n+1}$$

$$= h\nabla^{-1}\dot{X}_n + ha_0'\dot{X}_{n+1} + h \sum_{k=1}^{N} a_{k+1} \nabla^k \dot{X}_{n+1}$$

$$X_{n+1} = h\nabla^{-1}\dot{X}_n + h\sum_{k=0}^{N} a'_k \nabla^k \dot{X}_{n+1} \tag{155}$$

$$a'_0 = 1 + a_1 = 1/2$$

$$a'_k = a_{k+1} \quad k > 0$$

If the backward differences $\nabla^k \dot{X}_{n+1}$ and the first sum $\nabla^{-1}\dot{X}_n$ are known, then an estimate of X_{n+1} can be computed from (155). As (155) was just used to compute X_n we do have values for $\nabla^{-1}\dot{X}_{n-1}$ and $\nabla^k \dot{X}_n$. Since we have discarded the error term E from (150) we are assuming that $\dot{X}$ can be represented by a polynomial. It is clear from the discussion on backward differences that for a polynomial p of degree $N + 1$, $\nabla^{N+1} p$ is a constant and higher differences are zero. Using this fact and the relationship

$$\nabla^i \dot{X}_{n+1} + \nabla^{i-1}\dot{X}_n = \nabla^{i-1}\dot{X}_{n+1} \tag{156}$$

We can work backwards from $\nabla^{N+1}\dot{X}_{n+1} = \nabla^{N+1}\dot{X}_n$, computing $\nabla^i \dot{X}_{n+1}$.

$\nabla^{-1}\dot{X}_{n-1}$ $\square$

$\dot{X}_n$ $\square$

$\nabla \dot{X}_n$ $\square$

$\nabla^2 \dot{X}_n$ $\square$

. .

. .

. .

$\nabla^N \dot{X}_n$ $\square$

$\nabla^{N+1}\dot{X}_n$ $\square$

.

.

.

Use (156) and $\nabla^i X = 0$ for $i > N + 1$ to fill the boxes. This procedure is referred to as extrapolating the difference table forward.

The integration procedure discussed earlier, just prior to formula (151), can now be restated as

1. Extrapolate the difference table forward from time line n to $n + 1$. This gives a first estimate for each of the backward differences $\nabla^k \dot{X}_{n+1}$ and the first sum $\nabla^{-1}\dot{X}_n$.

2. Use (155) to compute an estimate of X_{n+1}.

3. Use the present estimate of X_{n+1} in $\tilde{G}$ to compute a new estimate of $\dot{X}_{n+1}$. Compute the difference between the new $\dot{X}_{n+1}$ and the old $\dot{X}_{n+1}$. Difference = New $\dot{X}_{n+1}$ - Old $\dot{X}_{n+1}$. Add this difference to each term in the backward difference table at time line $n + 1$. Do not adjust any first sums.

4. Check to see if more iterations are desired. If no, go to Step Number One for the next time line. If yes, go to Step Number Two. Convergence would occur when X_{n+1} new differs from X_{n+1} old by less than a tolerance. In the program however, an input iteration number is used to decide how many iterations to use.

Although the previous discussion covers the numerical iteration technique it does not cover the explicit form of the equations used in Celest. Since $X = \binom{x}{y}$ where x is position and $y = \dot{x}$ velocity, we can decompose the equations into position and velocity components.

$$x_{n+1} = h\nabla^{-1}y_n + h\sum_{k=0}^{N} a'_k \nabla^k y_{n+1} \qquad (157)$$

$$y_{n+1} = h\nabla^{-1}\dot{y}_n + h\sum_{k=0}^{N} a'_k \nabla^k \dot{y}_{n+1}$$

Equation (157) gives the solution as a function of backward differences of velocity and acceleration. We desire formulae using accelerations only. We use the second equation of (157) in the first to obtain this result. Since N is arbitrary we will replace N by ∞ and truncate our final expressions. Equation (157) becomes

$$x_{n+1} = h\nabla^{-1}y_n + h\sum_{k=0}^{\infty} a'_k \nabla^k y_{n+1} \qquad (158)$$

$$y_{n+1} = h\nabla^{-1}\dot{y}_n + h\sum_{k=0}^{\infty} a'_k \nabla^k \dot{y}_{n+1}$$

Substituting the second equation in the first gives

$$x_{n+1} = h^2\nabla^{-2}\dot{y}_{n-1} + h^2\sum_{k=0}^{\infty} a'_k \nabla^{k-1}\dot{y}_n + h^2\sum_{k=0}^{\infty} a'_k \nabla^{k-1}\dot{y}_n + h^2\sum_{k=0}^{\infty}\sum_{\ell=0}^{\infty} a'_k a'_\ell \nabla^{k+\ell}\dot{y}_{n+1}$$

$$= h^2\nabla^{-2}\ddot{x}_{n-1} + 2h^2\sum_{k=0}^{\infty} a'_k \nabla^{k-1}\ddot{x}_n + h^2\sum_{k=0}^{\infty}\sum_{\ell=0}^{k} a'_\ell a'_{k-\ell} \nabla^k \ddot{x}_{n+1}$$

$$x_{n+1} = h^2[\nabla^{-2}\ddot{x}_{n-1} + \nabla^{-1}\ddot{x}_n] + 2h^2\sum_{k=1}^{\infty} a'_k \nabla^{k-1}\ddot{x}_n + h^2\sum_{k=0}^{\infty}\sum_{\ell=0}^{k} a'_\ell a'_{k-\ell} \nabla^k \ddot{x}_{n+1} \quad \text{since } a'_0 = 1/2$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + 2h^2\sum_{k=1}^{\infty} a'_k (\nabla^{k-1}\ddot{x}_{n+1} - \nabla^k\ddot{x}_{n+1}) + h^2\sum_{k=0}^{\infty}\sum_{\ell=0}^{k} a'_\ell a'_{k-\ell} \nabla^k \ddot{x}_{n+1}$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + h^2(2a_1' + a_0'a_0')\ddot{x}_{n+1} + 2h^2\sum_{k=2}^{\infty} a_k'\nabla^{k-1}\ddot{x}_{n+1} - 2h^2\sum_{k=1}^{\infty} a_k'\nabla^k\ddot{x}_{n+1}$$

$$+ h^2\sum_{k=1}^{\infty}\sum_{\ell=0}^{k} a_\ell' a_{k-\ell}'\nabla^k\ddot{x}_{n+1}$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + h^2(2a_1' + a_0'a_0')\ddot{x}_{n+1} + 2h^2\sum_{k=1}^{\infty} a_{k+1}'\nabla^{k}\ddot{x}_{n+1} - 2h^2\sum_{k=1}^{\infty} a_k'\nabla^k\ddot{x}_{n+1}$$

$$+ h^2\sum_{k=1}^{\infty}\sum_{\ell=0}^{k} a_\ell' a_{k-\ell}'\nabla^k\ddot{x}_{n+1}$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + h^2(2a_1' + a_0'a_0')\ddot{x}_{n+1} + 2h^2\sum_{k=1}^{\infty} (a_{k+1}' - a_k')\nabla^{k}\ddot{x}_{n+1}$$

$$+ h^2\sum_{k=1}^{\infty}\sum_{\ell=0}^{k} a_\ell' a_{k-\ell}'\nabla^k\ddot{x}_{n+1}$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + h^2(2a_1' + a_0'a_0')\ddot{x}_{n+1} + h^2\sum_{k=1}^{\infty}\left[2(a_{k+1}' - a_k') + \sum_{\ell=0}^{k} a_\ell' a_{k-\ell}'\right]\nabla^k\ddot{x}_{n+1}$$

$$2a_{k+1}' - 2a_k' + \sum_{\ell=0}^{k} a_\ell' a_{k-\ell}' = 2a_{k+2} - 2a_{k+1} + 2a_0'a_k' + \sum_{\ell=1}^{k-1} a_{\ell+1}a_{k-\ell+1} = 2a_{k+2} - 2a_{k+1} + a_{k+1}$$

$$+ \sum_{\ell=2}^{k} a_\ell a_{k-\ell+2} = 2a_{k+2} - a_{k+1} + \sum_{\ell=2}^{k} a_\ell a_{k-\ell+2}$$

$$= \sum_{\ell=0}^{k+2} a_\ell a_{k-\ell+2} \quad \text{as} \quad 2a_0 a_{k+2} = 2a_{k+2}$$

$$2a_1 a_{k+1} = -a_{k+1} \quad \text{and}$$

$$a_0' = 1 + a_1 = 1/2$$

$$x_{n+1} = h^2\nabla^{-2}\ddot{x}_n + h^2(2a_1' + a_0'a_0')\ddot{x}_{n+1} + h^2\sum_{k=1}^{\infty}\sum_{\ell=0}^{k+2} a_\ell a_{k+2-\ell}\nabla^k\ddot{x}_{n+1}$$

For $k = 0$ we have

$$\sum_{\ell=0}^{k+2} a_\ell a_{k+2-\ell} = \sum_{\ell=0}^{2} a_\ell a_{2-\ell} = a_0 a_2 + a_1 a_1 + a_2 a_0$$

$$= 2a_0 a_2 + a_1 a_1$$

$$= 2a_2 + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$= 2a_2 + \frac{1}{4}$$

Also $2a_1' + a_0' a_0' = 2a_2 + 1/4$. Therefore

$$x_{n+1} = h^2 \nabla^{-2} \ddot{x}_n + h^2 \sum_{k=0}^{\infty} \sum_{\ell=0}^{k+2} a_\ell a_{k+2-\ell} \nabla^k \ddot{x}_{n+1} \tag{159}$$

Since we are assuming that $\dot{x}$ is a polynomial of degree $N+1$, $\ddot{x}$ will be a polynomial of degree N. This implies that the kth backward difference of acceleration will be zero for k greater than N. Equation (159) becomes

$$x_{n+1} = h^2 \nabla^{-2} \ddot{x}_n + h^2 \sum_{k=0}^{N} \sum_{\ell=0}^{k+2} a_\ell a_{k+2-\ell} \nabla^k \ddot{x}_{n+1} \tag{160}$$

Equation (158) can now be expressed as

$$x_{n+1} = h^2 \nabla^{-2} \ddot{x}_n + h^2 \sum_{k=0}^{N} c_k \nabla^k \ddot{x}_{n+1}$$

(161)

$$\dot{x}_{n+1} = h \nabla^{-1} \ddot{x}_n + h \sum_{k=0}^{N} a_k' \nabla^k \ddot{x}_{n+1}$$

where

$a_k' = a_{k+1}$ for $k > 0$ $\quad a_0' = 1 + a_1 = 1/2$

$$c_k = \sum_{\ell=0}^{k+2} a_\ell a_{k+2-\ell}$$

$$a_k = \int_0^1 \frac{(s-1)s \cdots (s+k-2)}{k!}\, ds \qquad k > 0$$

$$a_0 = 1$$

$$a_1 = -\frac{1}{2}$$

A recurrence relation can be developed for a_k giving

$$k > 0 \qquad a_k = -\sum_{j=1}^{k} \frac{a_{k-j}}{1+j} \qquad a_0 = 1$$

Formula (161) is the form of the integration equations used in Celest.

Derivation of Recurrence Relations

$$f(x_0 + h) = f(x_0) + hDf(x_0) + \frac{h^2}{2!} D^2 f(x_0) + \cdots + \frac{h^n}{n!} D^n f(x_0) + \cdots$$

where D is the derivative. This formula can be expressed as

$$f(x_0 - h) - f(x_0) = -(hD)f(x_0) + \frac{(hD)^2}{2!} f(x_0) + \cdots + (-1)^n \frac{(hD)^n}{n!} f(x_0) + \cdots$$

or

$$-\nabla f(x_0) = \left[-(hD) + \frac{(hD)^2}{2!} + \cdots + (-1)^n \frac{(hD)^n}{n!} + \cdots \right]_{x_0} f$$

or

$$(1 - \nabla)_{x_0} f = \left[1 - (hD) + \frac{(hD)^2}{2!} + \cdots + (-1)^n \frac{(hD)^n}{n!} + \cdots \right]_{x_0} f$$

implying that

$$(1 - \nabla) = e^{-hD}$$

$$hD = \ln(1 - \nabla)$$

$$D = \frac{-\ln(1 - \nabla)}{h}$$

$$D = \frac{1}{h}\sum_{i=1}^{\infty} \frac{\nabla^i}{i}$$

From (161)

$$a_k = \int_0^1 \frac{(s-1)s\cdots(s+k-2)}{k!}\,ds \qquad k>0$$

$$a_0 = 1$$

If the integration had been for a partial step then the generalized formula would be

$$a_k(s_0) = \int_0^{s_0} \frac{(s-1)s\cdots(s+k-2)}{k!}\,ds \qquad k>0$$

$$a_0(s_0) = s_0$$

Define

$$\binom{s}{k} = \frac{s(s-1)\cdots(s-(k-1))}{k!}$$

Letting $h = 1$ we have

$$\nabla\binom{s}{k} = \binom{s}{k} - \binom{s-1}{k} = \binom{s-1}{k-1}$$

$$\int_0^{s_0} D\nabla^{-1}\left[\frac{(s-1)s\cdots(s+k-2)}{k!}\right]ds = \int_0^{s_0} D\nabla^{-1}\binom{s+k-2}{k}ds$$

$$= \int_0^{s_0} D\binom{s+k-1}{k+1}ds = \binom{s_0+k-1}{k+1}$$

also

$$\int_0^{s_0} D\nabla^{-1}\binom{s+k-2}{k}ds = \int_0^{s_0}\sum_{i=1}^{\infty} \frac{\nabla^{i-1}}{i}\binom{s+k-2}{k}ds$$

$$= \int_0^{s_0}\sum_{i=1}^{\infty} \frac{\binom{s+k-2-(i-1)}{k-(i-1)}}{i}\,ds$$

$$= \int_0^{s_0} \sum_{i=1}^{\infty} \frac{\binom{s+(k-i+1)-2}{(k-i+1)}}{i} \, ds$$

$$= \sum_{i=1}^{\infty} \frac{1}{i} \int_0^{s_0} \binom{s+(k-i+1)-2}{(k-i+1)} \, ds$$

$$= \sum_{i=1}^{\infty} \frac{1}{i} \, a^{(s_0)}_{k-i+1}$$

This implies that

$$\sum_{i=1}^{\infty} \frac{1}{i} a^{(s_0)}_{k-i+1} = \binom{s_0+k-1}{k+1}$$

or

$$a^{(s_0)}_k = \binom{s_0+k-1}{k+1} - \sum_{i=2}^{\infty} \frac{1}{i} \, a^{(s_0)}_{k-i+1}$$

or

$$a^{(s_0)}_k = \binom{s_0+k-1}{k+1} - \sum_{i=1}^{\infty} \frac{1}{i+1} \, a^{(s_0)}_{k-i}$$

or

$$a^{(s_0)}_k = \binom{s_0+k-1}{k+1} - \sum_{i=1}^{k} \frac{1}{i+1} \, a^{(s_0)}_{k-i}$$

when $s_0 = 1$ we have

$$a^{(1)}_k = a_k = -\sum_{i=1}^{k} \frac{a_{k-i}}{i+1}$$

INTEGRATION PROCEDURE

Given a difference table at time line n

$$\nabla^{-2}\ddot{X}(n-1),\ \nabla^{-1}\ddot{X}(n-1),\ \nabla^{0}\ddot{X}(n),\ \nabla^{1}\ddot{X}(n), \ldots,\ \nabla^{N}\ddot{X}(n)$$

proceed as follows:

1. Extrapolate the difference table from time line n to $n + 1$.

$$\nabla^{-1}\ddot{X}(n) = \nabla^{-1}\ddot{X}(n-1) + \nabla^{0}\ddot{X}(n)$$

$$\nabla^{-2}\ddot{X}(n) = \nabla^{-2}\ddot{X}(n-1) + \nabla^{-1}\ddot{X}(n)$$

$$\nabla^{N}\ddot{X}(n+1) = \nabla^{N}\ddot{X}(n)$$

$$\nabla^{k}\ddot{X}(n+1) = \nabla^{k}\ddot{X}(n) + \nabla^{k+1}\ddot{X}(n+1) \qquad k = N\text{-}1, \ldots 0$$

2. Use (161) to compute $X(n + 1)$ and $\dot{X}(n + 1)$.

3. Use (148) to compute $\ddot{X}(n + 1)$ and determine the difference between the computed and extrapolated values,

$$\text{Difference} = \ddot{X}(n+1)(\text{computed}) - \ddot{X}(n+1)(\text{extrapolated})$$

4. Add the difference computed in step 3 to each of the backward differences at time line $n + 1$. Do not alter the first and second sums.

5. Check to see if the number of iterations is equal to the input control. If no, go to Step 2; if yes, go to Step 6.

6. Enter smoothing process if indicated.

7. Begin processing next time line.

To begin the integration procedure a complete difference table is required at time line zero. This table is obtained by the starting procedure.

STARTING PROCEDURE

1. Compute $\ddot{X}(0)$ using (148) and set $\nabla^{k}\ddot{X}(0)$ to zero. Compute the first and second sums using (161).

$$\nabla^{-1}\ddot{X}(-1) = \frac{\dot{X}(0)}{h} - a_0'\ddot{X}(0)$$

$$\nabla^{-2}\ddot{X}(-1) = \frac{X(0)}{h^2} - c_0\ddot{X}(0)$$

2. Extrapolate the difference table forward to line one.

3. Use (161) to compute $X(1)$ and $\dot{X}(1)$.

4. Use (148) to compute $\ddot{X}(1)$ and determine the difference between the computed and extrapolated acceleration.

5. Correct the difference table by adding the difference computed in Step Four to the first backward difference of time line one.

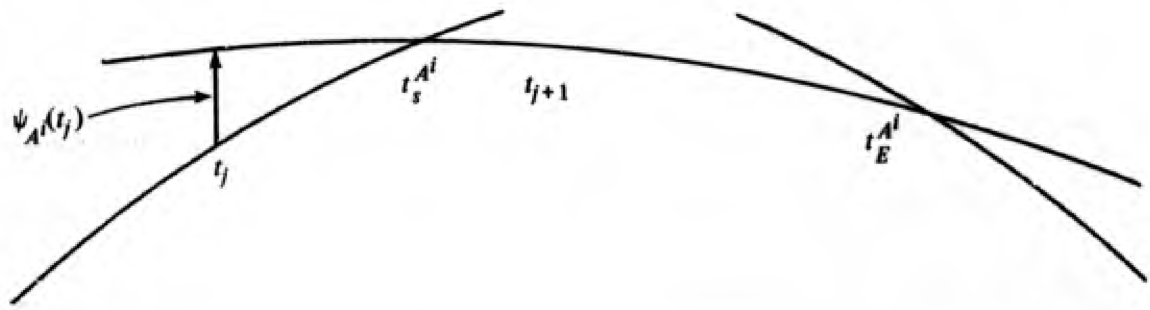
6. Begin processing the next time line. When Step Five is being executed on the pth ($p < N$) time line, correct the first p backward differences.

7. After processing time line N, the backward-difference table at line N is extrapolated backward to line zero holding the Nth difference constant.

8. Once the difference table at line zero is corrected, the first and second sums at line −1 are recomputed using (161). This completes one cycle of the starting procedure.

The program exits the starting procedure when the maximum absolute difference between successive position and velocity values of the N time lines satisfies separate input tolerances.

SMOOTHING PROCEDURE



The above diagram shows the effect on the trajectory due to a thrust $A^i = (A^i_{x_1}, A^i_{x_2}, A^i_{x_3})$ which starts at $t_s^{A^i}$ and ends at $t_E^{A^i}$.

When the thrust begins, the trajectory undergoes a perturbation from the path it would have followed had the thrust not occurred. The first order effect of this perturbation is given by ψ_{A^i}. ψ_{A^i} can be computed from the canonical thrust ψ_A and (84).

$$\psi_{A^i}(t) = \psi_A(t) - \psi(t)\chi_{A^i}(t_s^{A^i})$$

where

$$\psi_{A^i} = \begin{pmatrix} \psi_{A^i} \\ \dot{\psi}_{A^i} \end{pmatrix}$$

The perturbed position and velocity can now be computed to first order by

$$\bar{X}(t) = X(t) + \psi_{A^i}(t)A^i$$

$$\dot{\bar{X}}(t) = \dot{X}(t) + \dot{\psi}_{A^i}(t)A^i$$

Using (71) gives

$$\ddot{\bar{X}}(t) = G(X + \psi_{A^i}A^i, \dot{X} + \dot{\psi}_{A^i}A^i, A^i)$$

$$\ddot{\bar{X}} = G(X,\dot{X},0) + \frac{\partial G}{\partial X}\psi_{A^i}A^i + \frac{\partial G}{\partial \dot{X}}\dot{\psi}_{A^i}A^i + LA^i$$

$$= \ddot{X} + \left(\frac{\partial G}{\partial X}\psi_{A^i} + \frac{\partial G}{\partial \dot{X}}\dot{\psi}_{A^i} + L\right)A^i$$

$$= \ddot{X} + \ddot{\psi}_{A^i}A^i$$

Using ∇^k gives

$$\nabla^k \ddot{\bar{X}} = \nabla^k \ddot{X} + \nabla^k \ddot{\psi}_{A^i}A^i \tag{162}$$

where $\nabla^k \ddot{X}$ are the values obtained in Step Four of the integration procedure. $\nabla^k \ddot{\psi}_{A^i}$ are obtained from

$$\nabla^k \ddot{\psi}_{A^i}(t) = \nabla^k \ddot{\psi}_A(t) - \nabla^k \ddot{\psi}(t)\chi_{A^i}(t_s^{A^i}) \tag{163}$$

where $\nabla^k \ddot{\psi}_A$ and $\nabla^k \ddot{\psi}$ are also obtained from Step Four of the integration procedure. Using (161) compute new first and second sums

$$\nabla^{-1}\ddot{\bar{X}}_{j-1} = \frac{\dot{X}_j}{h} - \sum_{k=0}^{N} a_k' \nabla^k \ddot{\bar{X}}_j$$

$$\nabla^{-2}\ddot{\bar{X}}_{j-1} = \frac{X_j}{h^2} - \sum_{k=0}^{N} c_k \nabla^k \ddot{\bar{X}}_j \tag{164}$$

where it is assumed that $t_j = t_s^{A^i}$ so that $\bar{X}_j = X_j$ and $\dot{\bar{X}}_j = \dot{X}_j$.

Formulae (162) and (164) give a new difference table at time line j. It is this table that enters Step One of the integration procedure following smoothing. When $t_E^{A^i}$ is reached in the integration procedure, the smoothing process is entered with $-A^i$ replacing A^i.

The same smoothing procedure is used for drag end points. When leaving a drag segment with value C_{D_i} and entering one with value $C_{D_{i+1}}$, there is a parameter value change of $(C_{D_{i+1}} - C_{D_i})$. Replacing A^i by $(C_{D_{i+1}} - C_{D_i})$ gives the smoothing procedure for drag discontinuities.

BACKWARD INTEGRATION

The primary purpose of integrating in the backward direction is to provide values to be used in the eight-point interpolation routine.

The backward integration process is initialized by changing the initial conditions $(X, \dot{X})$ to $(X, -\dot{X})$ and changing the drag value C_D to $-C_D$. In addition $\tilde{\omega}$, the angular velocity of the earth, must be set to $-\tilde{\omega}$, and time increments must be subtracted from the initial time when reading the inertial-earth fixed rotation values and sun-moon ephemeris from the sun-moon file. Although these changes are sufficient to carry out the backward integration of position, we desire to have the associated partial derivatives referenced to the proper initial element or state vector values not values associated with $(X, -\dot{X})$.

Using the new variables $(X, -\dot{X})$ we desire partial derivatives

$$\frac{\partial(X, -\dot{X})}{\partial(X(0), \dot{X}(0))} \quad \text{or} \quad \frac{\partial(X, -\dot{X})}{\partial e_0}$$

where $e = e(X, \dot{X})$ is the coordinate to element transformation and

$$e_1 = a \qquad e_4 = i$$

$$e_2 = e \sin(w) \qquad e_5 = l + w$$

$$e_3 = e \cos(w) \qquad e_6 = \Omega$$

For each $(X, \dot{X})$ there is an associated $(X, -\dot{X})$. Associated to $(X, \dot{X})$ we have $e(X, \dot{X}) = (e_1, \ldots e_6)$. Associated to $(X, -\dot{X})$ we have $e(X, -\dot{X}) = (e_1', \ldots e_6')$. We wish to find $e = e(e')$, the functional relationship between e and e'. Since the orbit associated to $(X, -\dot{X})$ is geometrically the same but moving in the opposite direction to that of $(X, \dot{X})$ the ascending node will be 180° from that of $(X, \dot{X})$. This implies that

$$a' = a$$

$$e' = e$$

$$w' = 180 - w$$

$$i' = 180 - i$$

$$l' = 360 - l$$

$$\Omega' = 180 + \Omega$$

and that the orbital elements e' are related to e by

$$e_1' = a' = a = e_1$$

$$e_2' = e' \sin(w') = e \sin(180 - w) = e \sin(w) = e_2$$

$$e_3' = e' \cos(w') = e \cos(180 - w) = -e \cos(w) = -e_3$$

$$e_4' = i' = 180 - i = 180 - e_4$$

$$e_5' = l' + w' = 360 - l + 180 - w = 180 - (l + w) = 180 - e_5$$

$$e_6' = \Omega' = 180 + \Omega = 180 + e_6$$

Thus we can compute

$$\frac{\partial e'}{\partial e} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & -1 & & & \\ & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & 1 \end{bmatrix} \tag{165}$$

We can now compute

$$\left.\frac{\partial(X,-\dot{X})}{\partial e_0}\right|_{t_0} = \left.\frac{\partial(X,-\dot{X})}{\partial e_0'}\right|_{t_0} \frac{\partial e_0'}{\partial e_0}$$

$$= \left.\frac{\partial(X,-\dot{X})}{\partial e_0'}\right|_{t_0} \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & -1 & & & \\ & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & 1 \end{bmatrix} \tag{166}$$

and

$$\left.\frac{\partial(X,-\dot{X})}{\partial(X(0),\dot{X}(0))}\right|_{t_0} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & -1 \end{bmatrix} \tag{167}$$

giving proper initial values for the partial derivatives in both the element and coordinate mode.

We can now outline the integration procedure as follows:

1. Retain the initial conditions $(X, \dot{X})$
2. Determine dual conditions $(X, -\dot{X})$
3. Determine associated dual elements e'
4. Enter $(X, -\dot{X})$ and e' into the partial derivative routine and determine

$$\left.\frac{\partial(X,-\dot{X})}{\partial e'}\right|_{t_0}$$

In the coordinate mode this is

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & -1 \end{bmatrix}$$

5. Compute

$$\left.\frac{\partial(X,-\dot{X})}{\partial e'}\right|_{t_0} \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & -1 & & & \\ & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & 1 \end{bmatrix} = \left.\frac{\partial(X,-\dot{X})}{\partial e}\right|_{t_0}$$

6. Use

$$\left(X_0, -\dot{X}_0, \left.\frac{\partial(X,-\dot{X})}{\partial e}\right|_{t_0}\right)$$

as initial conditions for backward integration.

7. Integrate backwards N time lines and save $(X, \partial X/\partial e)(t_i)$ for each line.

8. Write the Perturbed Trajectory File for the backward lines by writing the Nth line first and proceeding to the first line.

9. Enter the forward integration procedure by using

$$\left(X_0, \dot{X}_0, \left.\frac{\partial(X,\dot{X})}{\partial e}\right|_{t_0}\right)$$

as initial conditions.

Remark

During the backward integration care must be taken in the perturbation equation for drag that the non-homogeneous term D is set to $-D$.

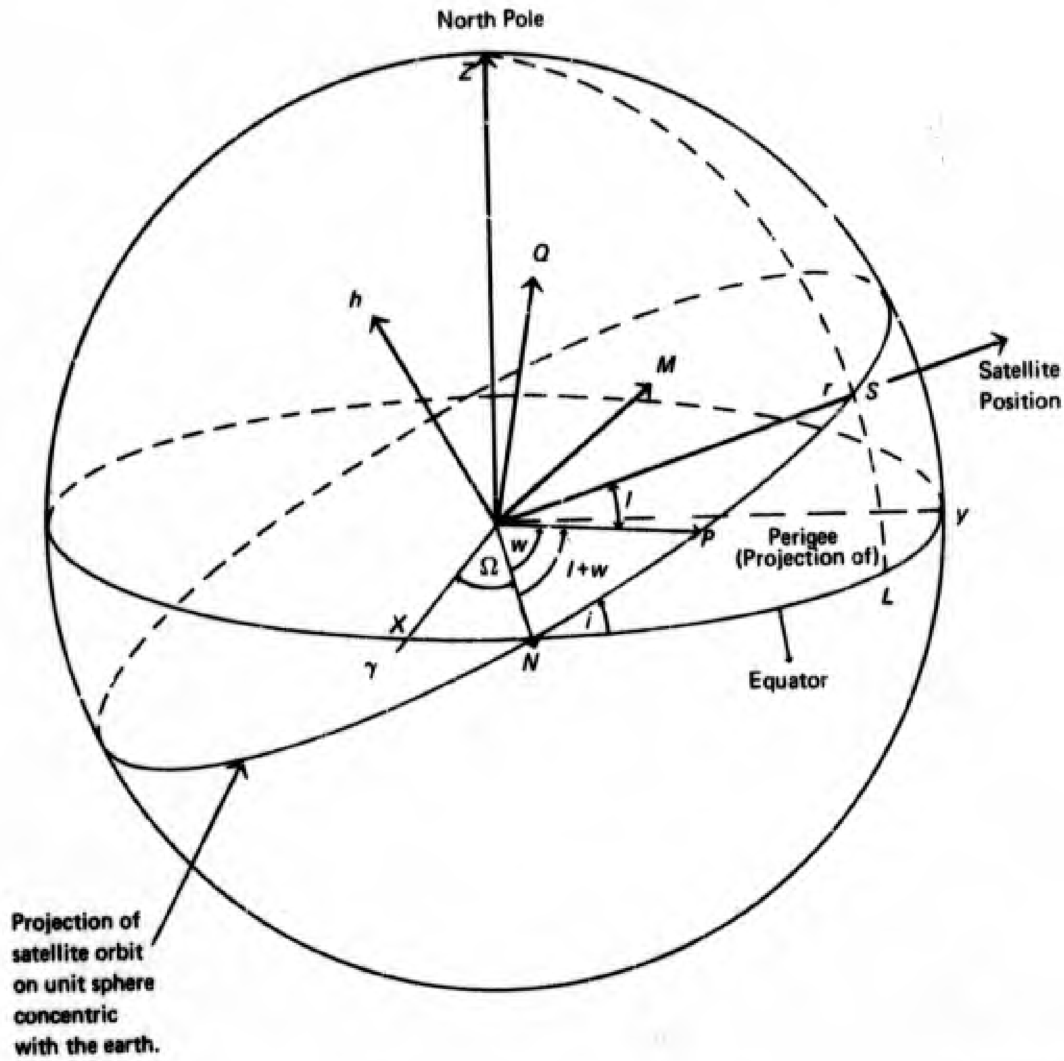


Diagram 18

Appendix A
POLAR MOTION

POLAR MOTION SOLUTION

Letting r_{s_0} denote the earth fixed station vector and $ABCD$ the inertial to earth fixed coordinate transformation of Appendix B gives

$$r_s = (ABCD)^* r_{s_0} \qquad r_{s_0} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

$$= (BCD)^* A^* r_{s_0}$$

$$A = \begin{bmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{bmatrix} \qquad \begin{array}{ll} \omega_1 = \Delta q & \omega_2 = \Delta p \\ \omega_3 = \tilde{\omega}(\Delta t + t\dot{\Delta} t) & \end{array} \qquad \text{(A-1)}$$

t = time in sec from the beginning of the year.

$$\frac{\partial r_s}{\partial \omega_1} = (BCD)^* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} r_{s_0}$$

$$= (BCD)^* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

$$= (BCD)^* \begin{pmatrix} 0 \\ s_3 \\ -s_2 \end{pmatrix}$$

$$\frac{\partial r_s}{\partial \omega_2} = (BCD)^* \begin{pmatrix} -s_3 \\ 0 \\ s_1 \end{pmatrix}$$

$$\frac{\partial r_s}{\partial \omega_3} = (BCD)^* \begin{pmatrix} s_2 \\ -s_1 \\ 0 \end{pmatrix}$$

$$\frac{\partial r_s}{\partial \dot{\Delta} t} = \frac{\partial r_s}{\partial \omega_3} \frac{\partial \omega_3}{\partial \dot{\Delta} t} = \frac{\partial r_s}{\partial \omega_3} \tilde{\omega} t$$

$$\frac{\partial r_s}{\partial \Delta t} = \frac{\partial r_s}{\partial \omega_3}\tilde{\omega}$$

Letting $p = (\Delta q, \Delta p, \Delta t, \dot{\Delta t})$ be the polar motion parameter set gives

$$\frac{\partial D_t}{\partial p} = \frac{\partial D_t}{\partial r_s}\frac{\partial r_s}{\partial p} = \frac{\partial D_t}{\partial r_s}(BCD)^* \begin{bmatrix} 0 & -s_3 & \tilde{\omega}s_2 & \tilde{\omega}ts_2 \\ s_3 & 0 & -\tilde{\omega}s_1 & -\tilde{\omega}ts_1 \\ -s_2 & s_1 & 0 & 0 \end{bmatrix}$$

$$= \frac{\partial D_t}{\partial r_s}(BCD)^*A^*A \begin{bmatrix} 0 & -s_3 & \tilde{\omega}s_2 & \tilde{\omega}ts_2 \\ s_3 & 0 & -\tilde{\omega}s_1 & -\tilde{\omega}ts_1 \\ -s_2 & s_1 & 0 & 0 \end{bmatrix}$$

$$= \frac{\partial D_t}{\partial r_s}\frac{\partial r_s}{\partial r_{s_0}} A \begin{bmatrix} 0 & -s_3 & \tilde{\omega}s_2 & \tilde{\omega}ts_2 \\ s_3 & 0 & -\tilde{\omega}s_1 & -\tilde{\omega}ts_1 \\ -s_2 & s_1 & 0 & 0 \end{bmatrix}$$

$$= \frac{\partial D_t}{\partial r_s}\frac{\partial r_s}{\partial r_{s_0}} AQ$$

$$= \frac{\partial D_t}{\partial r_{s_0}} AQ$$

$$B_{pp}(t) = \frac{\partial D_t^*}{\partial p}\frac{\partial D_t}{\partial p} = Q^*(t)A^*(t)\frac{\partial D_t^*}{\partial r_{s_0}}\frac{\partial D_t}{\partial r_{s_0}} A(t)Q(t)$$

$$= Q^*(t)A^*(t)B_{ss}(t)A(t)Q(t)$$

Evaluating AQ at TCA of the pass and summing over all data gives

$$B_{pp} = Q^*(TCA)A(TCA)B_{ss}A(TCA)Q(TCA)$$

$$= \bar{Q}^*B_{ss}\bar{Q} \qquad \text{(A-2)}$$

Following this procedure the formula for adjusting an expanded pass matrix to include polar motion parameters is

$$\begin{aligned} B_{pp} &= \bar{Q}^*B_{ss}\bar{Q} & B_{op} &= B_{os}\bar{Q} \\ B_{bp} &= B_{bs}\bar{Q} & B_{AP} &= B_{AS}\bar{Q} \\ B_{c_Dp} &= B_{c_Ds}\bar{Q} & B_{kr,p} &= B_{kr,s}\bar{Q} \\ && E_p &= \bar{Q}^*E_s \end{aligned} \qquad \text{(A-3)}$$

where p is the polar motion parameter set, b is the bias set, c_D is the drag set, A is the thrust set, o is the orbit set and kr is radiation pressure.

Before polar motion expansion the pass matrix appears as

$$\begin{bmatrix} B_{oo} & B_{oc_D} & B_{oA} & B_{okr} & B_{ob} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

After polar motion expansion the matrix appears as

$$\begin{bmatrix} B_{oo} & B_{oc_D} & B_{oA} & B_{okr} & B_{op} & B_{ob} \\ \cdot & \cdot & \cdot & \cdot & B_{c_Dp} & \cdot \\ \cdot & \cdot & \cdot & \cdot & B_{Ap} & \cdot \\ \cdot & \cdot & \cdot & \cdot & B_{krp} & \cdot \\ & & & & B_{pp} & \\ & & & & B_{bp} & \end{bmatrix}$$

POLAR MOTION PREDICTION

This is a discussion of the procedure used to obtain initial values for the polar motion parameter set $p = (\Delta q, \Delta p, \Delta t, \dot{\Delta t})$.

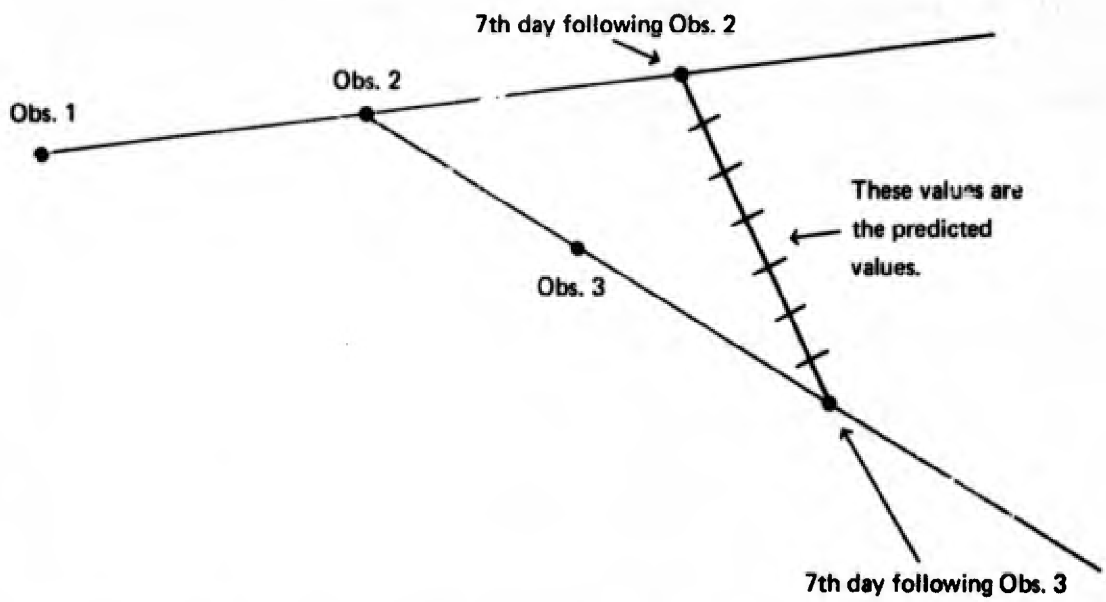
$(\Delta q, \Delta p)$: These values are routinely solved for in two-day navigation satellite orbit reductions. Using the most recent 200 days of $(\Delta q, \Delta p)$ values obtained from the above solutions, a least square fit is carried out fitting the following function to the values.

$$f(t) = A + B \cos(\omega t) + c \sin(\omega t) \qquad \text{(A-4)}$$

$$\omega = \frac{2\pi}{420} \quad \text{where } 420 = \text{Chandlerian period}$$

The results of carrying out separate fits for Δq and Δp are used to predict new values for the next seven days.

(Δt): UTC-UT1 values are obtained from the Naval Observatory on a weekly basis. When two values are in hand a straight line is determined.



Three observed values (one value per week from the Naval Observatory) are needed to initiate this process. The output predicted values (Δt_1) are those along the railroad track. One value per day is output for seven days. After seven days a new prediction is made incorporating a new observation from the Naval Observatory.

At the end of each year a straight line is fit to the Naval Observatory weekly values for that year. We denote this value by $\Delta t_2(t)$. The function Δt_2 is evaluated at the beginning of each day of the next year and these values are placed on the Sun-Moon File.

Definitions

Δt_1 = predicted values of UTC-UT1 using three observations from the Naval Observatory.

Δt_2 = predicted values obtained by fitting a straight line to the weekly UTC-UT1 Naval Observatory values of the preceding year.

$$\Delta t = \Delta t_1 - \Delta t_2$$

The value Δt is input to the Celest Program. As the program permits only one Δt input value, this value will adjust the straight line value Δt_2 to give Δt_1 for the epoch day. The resulting Δt_1 value will then drift by the straight line slope of Δt_2 on the Sun-Moon File. The value Δt is initialized to zero.

Appendix B
COORDINATE SYSTEMS

BASIC INERTIAL SYSTEM (X, Y, Z)

The reference date of the basic inertial system is taken to be either 1950.0 or zero sec of the day of the trajectory epoch.

X is the unit vector pointing toward the mean vernal equinox of the reference date.

Z is the unit vector pointing along the mean earth's spin axis of the reference date and positive in the northern hemisphere.

Y is the unit vector completing a right-handed, geocentric, orthogonal system with X and Z.

MEAN OF DATE SYSTEM (X', Y', Z')

This system differs from the basic inertial system only in that the reference date can be any time. The transformation transforming from the Basic Inertial System to the Mean of Date System is denoted by the symbol "D."

TRUE OF DATE SYSTEM (X'', Y'', Z'')

This is a geocentric system where

X'' is along the true vernal equinox at a given time.

Z'' is along the true earth spin axis at a given time.

Y'' completes a right-handed system.

The transformation going from the Mean of Date System to the True of Date System is denoted by the symbol "C."

EARTH FIXED SYSTEM (E', F', G')

This is a geocentric system where

E' is in the true equatorial plane in the direction of the Greenwich meridian.

G' is along the true earth spin axis positive in the northern hemisphere.

F' completes a right-handed orthonormal system.

The transformation going from the True of Date System to the Earth Fixed System is denoted by the symbol "B."

EARTH FIXED REFERENCE ELLIPSOID SYSTEM (E, F, G)

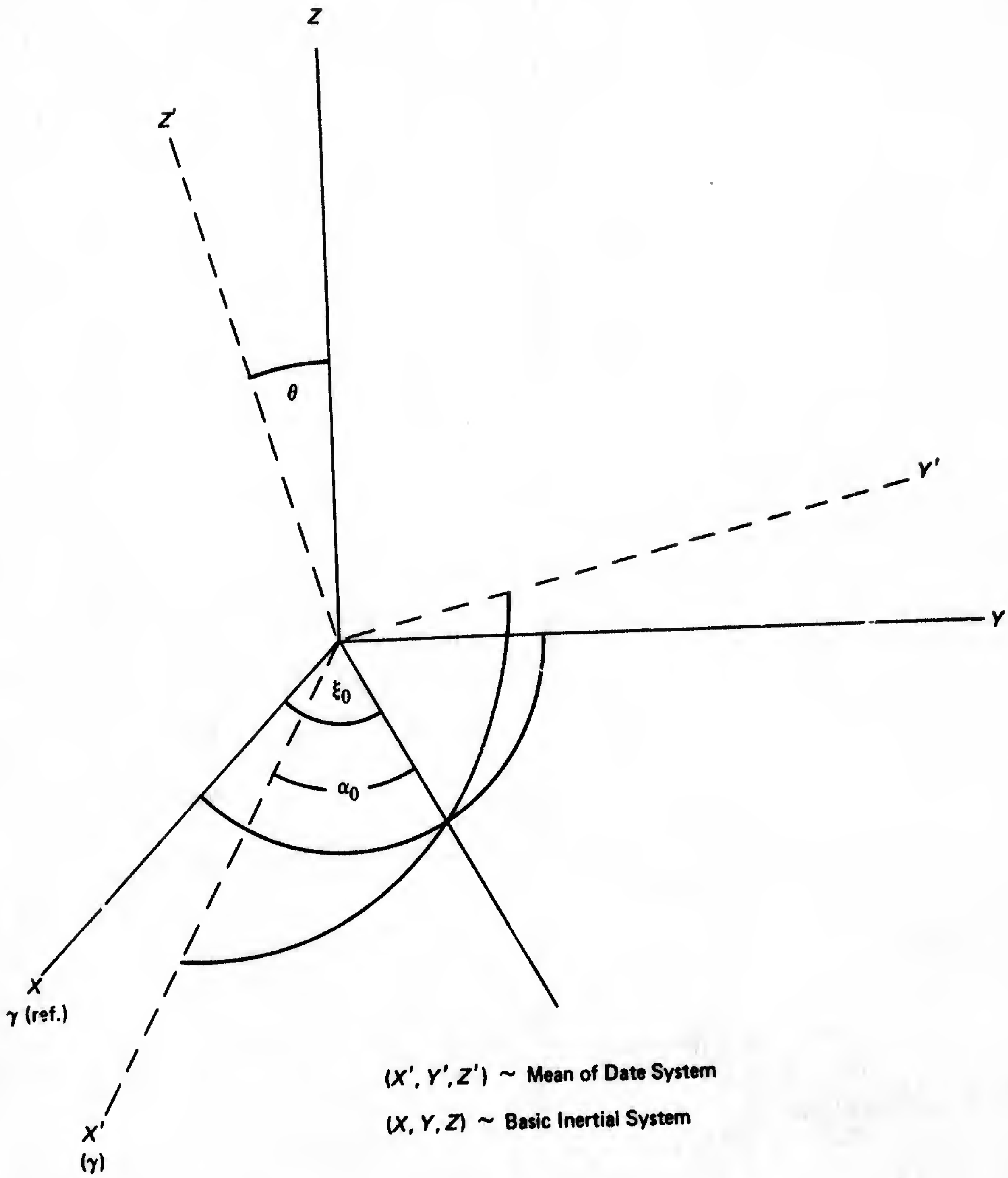
This is a geocentric system where

E points toward the Greenwich Meridian.

G points along the CIO pole and is positive in the northern hemisphere. The CIO pole is the average pole between 1900 and 1905.

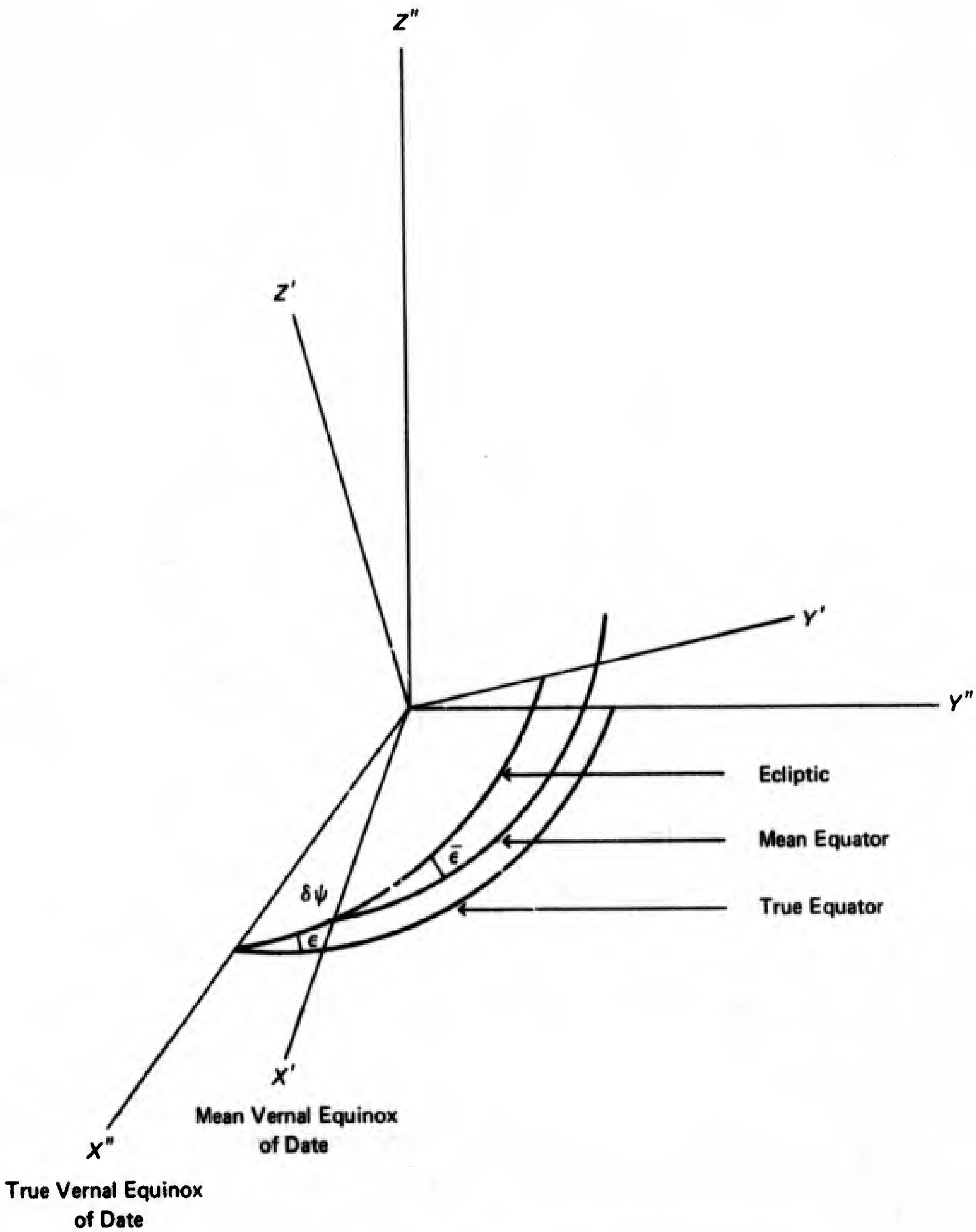
F completes a right-handed orthonormal system.

The transformation going from the Earth Fixed System to the Ellipsoid System is denoted by "A."



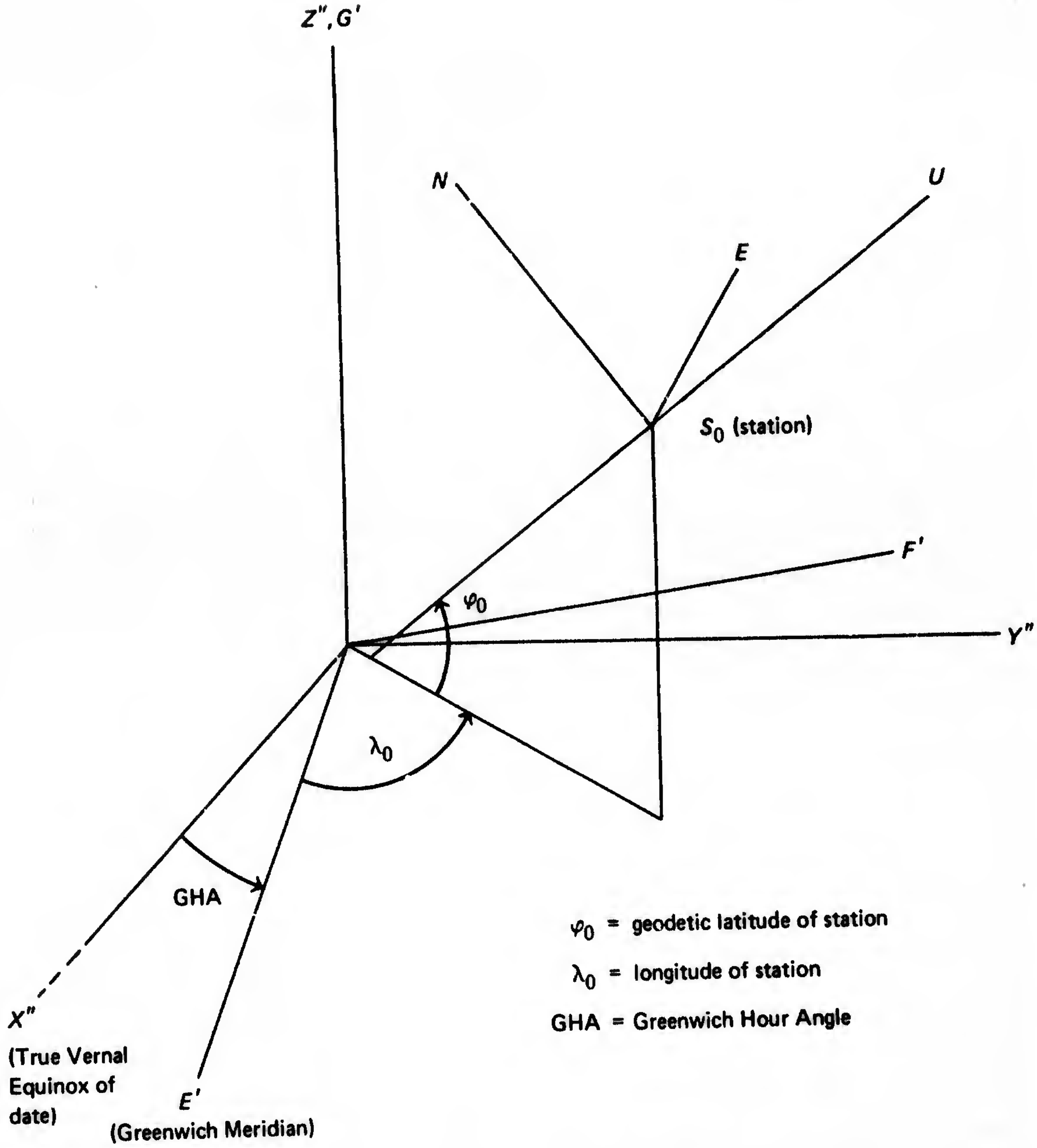
Relation between Basic Inertial System and Mean of Date System

Diagram B-1



Relation between Mean of Date System and True of Date System

Diagram B-2



Relation between True of Date System and Earth Fixed System

Diagram B-3

Appendix C
COORDINATE TRANSFORMATIONS

OSCULATING TO MEAN ORBITAL ELEMENTS

The Brouwer method is used to relate mean and osculating elements. This is expressed by

$$e_0 = B(e_m)$$

where e_0 is the osculating set and e_m is the mean set. Writing this expression as

$$e_m = e_m + e_0 - B(e_m) = \bar{B}(e_m) \quad \text{(C-1)}$$

we can observe that the problem of obtaining mean elements, given a set of osculating elements, is equivalent to finding a fixed point for $\bar{B}$. Due to the structure of $\bar{B}$ this can be done by making an initial estimate of e_m as e_0, defining subsequent estimates by $e_m(i+1) = \bar{B}(e_m(i))$ and iterating to convergence. The Celest Program uses one set of elements as osculating and reports another set as mean. Listing the osculating and mean sets below, we will let β stand for the transformation between the two sets.

Osculating		Mean
$e_1 = a$		a
$e_2 = e \sin(\omega)$		e
$e_3 = e \cos(\omega)$	$\xrightarrow{\beta}$	i
$e_4 = i$		l
$e_5 = l + \omega$		ω
$e_6 = \Omega$		Ω

Taking account of β we can rewrite C-1 as

$$e = e + e_0 - \beta^{-1} B \beta(e) \quad \text{(C-2)}$$

where B is understood to operate on the mean element set.

When convergence occurs the values $\beta(e)$ are reported as the Brouwer mean elements. Convergence occurs when

$$10^8(\Delta e_1)^2 + 10^{16}(\Delta e_2^2 + \Delta e_3^2) + 10^{14}(\Delta e_4^2 + \Delta e_5^2 + \Delta e_6^2) < 1$$

INERTIAL CARTESIAN TO OSCULATING ORBITAL ELEMENTS

The satellites' position and velocity are given by

$$r = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \dot{r} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix}$$

The energy integral gives

$$a = \frac{1}{\left(\frac{2}{|r|} - \frac{|\dot{r}|^2}{\mu}\right)} \tag{C-3}$$

where μ is the earth's gravitational constant. The angular momentum vector h is given by

$$h = r \times \dot{r} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

The magnitude of h is constant during Keplerian motion and gives the Kepler area constant.

Using the polar form of an ellipse gives

$$|r| = \frac{p}{(1 + e \cos(f))} = a(1 - e \cos(E)) \tag{C-4}$$

where p is the semi-latus rectum, $|h|/\mu$, f is the true anomaly, e the eccentricity and E the eccentric anomaly. From (C-4)

$$\dot{|r|} = ae\dot{E} \sin(E) \qquad e \cos(E) = 1 - \frac{|r|}{a} \tag{C-5}$$

implying

$$e \cos(E) = 1 - \frac{|r|}{a} \quad \text{and} \quad e \sin(E) = \frac{\dot{|r|}}{a\dot{E}} \tag{C-6}$$

From Keplers equation the mean anomaly l can be computed.

$$l = E - e \sin(E) \tag{C-7}$$

Since

$$\dot{l} = \text{mean motion} = \left(\frac{\mu}{a^3}\right)^{1/2} \tag{C-8}$$

we can use C-4, C-7 and C-8 to get

$$\dot{E} = \frac{c}{|r|}\left(\frac{\mu}{a^3}\right)^{1/2} \tag{C-9}$$

and we can use C-3, C-6 and C-9 to get

$$\mu_1 = e \sin(E) = \frac{|r|\,\dot{\overline{|r|}}}{(a\mu)^{1/2}}$$

$$\mu_2 = e \cos(E) = \frac{|r|\,|\dot{r}|^2}{\mu} - 1 \tag{C-10}$$

The eccentricity can be computed by

$$e = (\mu_1^2 + \mu_2^2)^{1/2} \tag{C-11}$$

The inclination comes from

$$i = \cos^{-1}\left(\frac{h_3}{|h|}\right) \tag{C-12}$$

Define

$$\hat{h} = \frac{h}{|h|} \qquad \hat{U} = \frac{r}{|r|} \qquad \hat{V} = \hat{h} \times \hat{U}$$

$$\begin{pmatrix} \hat{P} \\ \hat{Q} \end{pmatrix} = \begin{pmatrix} \cos(f) & -\sin(f) \\ \sin(f) & \cos(f) \end{pmatrix} \begin{pmatrix} \hat{U} \\ \hat{V} \end{pmatrix}$$

$$\hat{N} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \frac{\hat{h}}{\left| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \hat{h} \right|}$$

$$= \begin{pmatrix} -h_2 \\ h_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\Omega) \\ \sin(\Omega) \\ 0 \end{pmatrix}$$

It is clear, from Diagram 18, that $\hat{P}$ points to perigee and $\hat{N}$ points to the right ascension of the ascending node. We can calculate the right ascension Ω from

$$\Omega = \tan^{-1}\left(\frac{-h_1}{h_2}\right) \tag{C-13}$$

Define

$$\hat{M} = \hat{h} \times \hat{N}$$

This makes $\hat{M}$ and $\hat{N}$ an orthonormal system in the plane of the Kepler motion giving

$$\cos(\omega) = \hat{P} \cdot \hat{N} \qquad \sin(\omega) = \hat{P} \cdot \hat{M}$$

This permits the argument of perigee to be computed by

$$\omega = \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega)}\right) \tag{C-14}$$

The rate of change of ω and Ω is given by

$$\dot{\omega} = \frac{3\, C_{2,0}\left(\frac{\mu}{a^3}\right)^{1/2} a_e^2 (1 - 5\cos^2(i))}{4\, a^2(1 - e^2)^2} \tag{C-15}$$

$$\dot{\Omega} = \frac{3\, C_{2,0}\left(\frac{\mu}{a^3}\right)^{1/2} a_e^2 \cos(i)}{2\, a^2(1 - e^2)^2} \tag{C-16}$$

where a_e is the semi-major axis of the reference ellipsoid and $C_{2,0}$ is the coefficient of the second harmonic in the earth's oblateness expression.

The anomalistic period using Brouwer mean elements is given by

$$\overline{P_A} = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(1 - \frac{3\, C_{2,0} a_e^2 (3\sin^2(i) - 2)}{4\, a^2(1 - e^2)^{3/2}}\right) \tag{C-17}$$

The anomalistic period using osculating elements is given by

$$P_A = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(1 - \frac{3\, C_{2,0} a_e^2\, |h|^3 (3\sin^2(i)\, |\hat{r} \times \hat{N}|^2 - 1)}{2\, a^2\, [(\mu |r| (1 - e^2))]^3}\right) \tag{C-18}$$

The associated nodal periods can be obtained by adding the difference, $P_N - P_A$, to either of the above values, where mean or osculating elements respectively must be used to compute $P_N - P_A$.

$$P_N - P_A = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(\frac{3\, C_{2,0} a_e^2 (4 - 5\sin^2(i))}{4\, a^2(1 - e^2)^{1/2}(1 + e\cos(\omega))^2}\right) \tag{C-19}$$

OSCULATING ORBITAL ELEMENTS TO INERTIAL CARTESIAN

Kepler's equation can be written as

$$\ell + \omega = E + \omega - e \sin(E + \omega - \omega) \quad \text{(C-20)}$$

This is done because $\ell + \omega$ is one of our osculating elements. Let

$$m = \ell + \omega \qquad \text{and} \qquad \alpha = E + \omega$$

This gives

$$\alpha - e \sin(\alpha - \omega) = m$$

or

$$\alpha - e \sin(\alpha) \cos(\omega) + e \cos(\alpha) \sin(\omega) - m = 0 \quad \text{(C-21)}$$

The last expression is solved for α using Newton's method.

Having obtained α we have

$$e \sin(\alpha) = e \sin(E + \omega)$$

$$e \cos(\alpha) = e \cos(E + \omega)$$

giving

$$e \sin(E) = e \cos(\omega) \sin(\alpha) - e \sin(\omega) \cos(\alpha)$$

$$e \cos(E) = e \cos(\omega) \cos(\alpha) + e \sin(\omega) \sin(\alpha)$$

Recalling (C-4) we have

$$|r| = a(1 - e \cos(E))$$

$$= \frac{P}{1 + e \cos(f)} = \frac{a(1 - e^2)}{1 + e \cos(f)}$$

giving

$$\cos(f) = \frac{a}{|r|} (\cos(E) - e)$$

$$\sin(f) = \frac{a}{|r|} (1 - e^2)^{1/2} \sin(E) \quad \text{(C-22)}$$

The double angle formulae for sin and cos give

$$\sin(\omega + f) = \sin(\omega)\cos(f) + \cos(\omega)\sin(f)$$

$$\cos(\omega + f) = \cos(\omega)\cos(f) - \sin(\omega)\sin(f)$$

Using (C-22) gives

$$\sin(\omega + f) = \frac{a}{|r|}\left[\sin(\omega)(\cos(E) - e) + \cos(\omega)(1 - e^2)^{1/2}\sin(E)\right]$$

$$\cos(\omega + f) = \frac{a}{|r|}\left[\cos(\omega)(\cos(E) - e) - \sin(\omega)(1 - e^2)^{1/2}\sin(E)\right] \qquad \text{(C-23)}$$

The satellite position r can now be computed from

$$r = |r| K_1 K_2 K_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \text{(C-24)}$$

where $|r|$ is obtained from (C-4) and K_1, K_2, and K_3 are given below.

$$K_1 = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix}$$

$$K_3 = \begin{bmatrix} \cos(\omega + f) & -\sin(\omega + f) & 0 \\ \sin(\omega + f) & \cos(\omega + f) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The velocity $\dot{r}$ is given by

$$\dot{r} = \dot{|r|} K_1 K_2 K_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + |r| \dot{f} K_1 K_2 K_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{(C-25)}$$

where (using C-10)

$$\dot{|r|} = \frac{(a\mu)^{1/2}}{|r|} e \sin(E)$$

and differentiating (C-4) and using (C-22) give

$$\dot{j} = \frac{[\mu a(1 - e^2)]^{1/2}}{|r|^2}$$

EARTH FIXED CARTESIAN TO INERTIAL CARTESIAN

Let x, y, z represent components in the XYZ system of Diagram B-1. This is the Basic Inertial System. The earth fixed representation of (X, Y, Z) in the E, F, G system of time t is

$$\begin{pmatrix} e \\ f \\ g \end{pmatrix} = ABCD \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{C-26}$$

The D transformation accounts for the general precession of the earth's spin axis. The difference between the mean equator and equinox of time t and the mean equator and equinox of a reference date (for the Basic Inertial System) is due to this general precession.

Letting $D = D_t$ represent the transformation from the Basic Inertial System of time T_0 to the Basic Inertial System of time t, we decompose the computation as follows:

1. Compute D_{T_0+T} where $T_0 + T$ is the nearest day less than or equal to t.

2. Compute $D_{T_0+T+1\text{ day}}$.

3. Compute D_t by interpolation of the above two results. $T_0 + T \leq t \leq T_0 + T + 1$ day.

We now describe how to compute D_{T_0+T}. Let

γ(ref)	=	mean equinox of the reference date
γ	=	mean equinox of $T_0 + T$
ξ_0	=	right ascension of the ascending node of the mean equator of date ($T_0 + T$) on the mean equator of the reference date.
α_0	=	right ascension of the node measured in the mean equator of date from the mean equinox of date.
θ	=	inclination of the mean equator of date to the mean equator of reference.

be a description of the values in Diagram B-1.

$$D_{T_0+T}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}\cos\alpha_0 & -\sin\alpha_0 & 0\\ \sin\alpha_0 & \cos\alpha_0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\varphi_0 & \sin\varphi_0 & 0\\ -\sin\varphi_0 & \cos\varphi_0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$=\begin{bmatrix}d_{11} & d_{12} & d_{13}\\ d_{21} & d_{22} & d_{23}\\ d_{31} & d_{32} & d_{33}\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

From the *Explanatory Supplement to the Ephemeris*

$$\tilde{Z} = \alpha_0^0 - 90^\circ.0$$

$$\varphi_0 = 90^\circ.0 - \xi_0$$

$$\xi_0 = (2304.250'' + 396''\ T_0)T + 0.302''\ T^2 + 0.018''\ T^3$$

$$\tilde{Z} = \xi_0 + 0.791''\ T^2$$

$$\theta = (2004.682'' - 0.853''\ T_0)T - 0.426''\ T^2 - 0.042''\ T^3$$

where T_0 is the time, in tropical centuries, of the reference date measured from 1950.0 and T is the time, in tropical centuries, measured from T_0.

Expanding d_{ij} in a Taylor series in $\tilde{Z}, \theta, \varphi_0$ and replacing the arguments by their time series representation, we have

$$d_{11} = 1 - [(29696 + 26\ T_0)T^2 + 13\ T^3] \cdot 10^{-8}$$

$$d_{12} = -[(2234941 + 1355\ T_0)T + 676\ T^2 - 221\ T^3]\ 10^{-8}$$

$$d_{13} = -[(971690 - 414\ T_0)T - 207\ T^2 - 96\ T^3]\ 10^{-8}$$

$$d_{21} = -d_{12}$$

$$d_{22} = 1 - [(24975 + 30\ T_0)T^2 + 15\ T^3]\ 10^{-8}$$

$$d_{23} = -[(10858 + 2\ T_0)T^2]\ 10^{-8}$$

$$d_{31} = -d_{13}$$

$$d_{32} = d_{23}$$

$$d_{33} = 1 - [(4721 - 4\ T_0)T^2]\ 10^{-8}$$

$(TO(1), TO(2))$ = (yr, day) of the reference date of the Basic Inertial System. Δ_0 = greatest integer in $(TO(1) - 53/4 + 1)$ and T_0 is the Basic Inertial System epoch measured in tropical centuries from 1950.0.

$$T_0 = \frac{[(TO(1) - 50)\ 365 + \Delta_0 + TO(2) + .077 - 1]}{36524.2196}$$

Celest accepts two Basic Inertial System epoch times. One choice is 1950.0, the other is zero hours of the day of the trajectory epoch time. The input is in calendar days and the .077 references T_0 to 1950.0.

The value of T is computed from the same formula as T_0 and differenced with T_0.

The C transformation contains the nutation terms and transforms coordinates from the mean equator and equinox system of time t (Basic Inertial System of time t) to the true equator and equinox of time t. This transformation accounts for the nutation in longitude and obliquity. Let

$\delta\psi$ = nutation in longitude measured from the true vernal equinox to the mean vernal equinox of time t, measured in the ecliptic plane.

$\bar{\varepsilon}$ = mean obliquity

ε = true obliquity

$\delta\varepsilon$ = nutation in obliquity

be as described in Diagram B-2.

We can now compute

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = CD\begin{pmatrix} x \\ y \\ z \end{pmatrix} = CD_t\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \qquad \text{(C-27)}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix} \begin{bmatrix} \cos(\delta\psi) & -\sin(\delta\psi) & 0 \\ \sin(\delta\psi) & \cos(\delta\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{\varepsilon} & \sin\bar{\varepsilon} \\ 0 & -\sin\bar{\varepsilon} & \cos\bar{\varepsilon} \end{bmatrix}$$

$$= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Using $\varepsilon = \bar{\varepsilon} + \delta\varepsilon$, expanding C_{ij} in a power series about $(\delta\psi, \delta\varepsilon) = (0,0)$ and retaining second degree terms, we have

$$C_{11} = 1 - \frac{(\delta\psi)^2}{2}$$

$$C_{12} = -\delta\psi \cos\bar{\varepsilon}$$

$$C_{13} = -\delta\psi \sin\bar{\varepsilon}$$

$$C_{21} = \delta\psi(\cos\bar{\varepsilon} - \delta\varepsilon \sin\bar{\varepsilon})$$

$$C_{22} = 1 - \frac{(\delta\varepsilon)^2}{2} - \left(\frac{(\delta\psi)^2}{2}\right)\cos^2\bar{\varepsilon}$$

$$C_{23} = -\delta\varepsilon - \left(\frac{(\delta\psi)^2}{2}\right)\sin\bar{\varepsilon}\cos\bar{\varepsilon}$$

$$C_{31} = \delta\psi(\sin\bar{\varepsilon} + \delta\varepsilon\cos\bar{\varepsilon})$$

$$C_{32} = \delta\varepsilon - \left(\frac{(\delta\psi)^2}{2}\right)\sin\bar{\varepsilon}\cos\bar{\varepsilon}$$

$$C_{33} = 1 - \frac{(\delta\varepsilon)^2}{2} - \left(\frac{(\delta\psi)^2}{2}\right)\sin^2\bar{\varepsilon}$$

The B transformation goes from the inertial true equator and equinox system to the instantaneous earth fixed system (E', F', G'). Letting Λ be the longitude of the Greenwich Meridian from the true vernal equinox at time t gives

$$\begin{pmatrix} e' \\ f' \\ g' \end{pmatrix} = BCD \begin{pmatrix} x \\ y \\ z \end{pmatrix} = B \begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = \begin{bmatrix} \cos\Lambda & \sin\Lambda & 0 \\ -\sin\Lambda & \cos\Lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \qquad \text{(C-28)}$$

where $\Lambda = H_0 + \Delta H(t) + \tilde{\omega}(t - \Delta t_2)$

H_0 = Mean Greenwich hour angle at zero hours January one of the epoch year. Epoch in this case refers to the epoch of the trajectory integration. H_0 is either obtained from input or computed from the input Julian date d of midnight of the start of the epoch year.

$$H_0 = 1.74799048989 + (1.7202791487893\,E-02)D + (5.06424735759\,E-15)D^2 \pmod{2\pi}$$

$$D = d - 2436935.0$$

$\Delta H(t)$ = Equation of the equinox = difference between true and mean sidereal time. These daily values are stored on the Sun-Moon File at 0^h UTC.

$\tilde{\omega}$ = Earth's mean sidereal rotation rate determined by the time lapse between successive transits of the mean equinox. This value is usually set on input to .7292115855 E - 04 radians/sec.

t = time in sec from the beginning of the epoch year.

Δt_2 = Approximate (UTC-UT1) value. A more complete description is given under *Polar Motion* in Appendix A.

$$.7292115855\,E(-04) = \frac{15^\circ}{\text{hr}}\left(\frac{\text{Mean solar day}}{\text{Mean sidereal day}}\right)$$

$$= .7272205218\,E(-04)\left(\frac{1}{.997269566414}\right)$$

also

$$= .7272205218\,E-04 + \text{recession rate of equinox}$$

$$\text{recession rate} = .1991063831\,E-06$$

The A transformation corrects for polar motion by going from the instantaneous earth fixed system (E',F',G') to the reference ellipsoid system (E,F,G). The displacements from the CIO pole of the instantaneous axis of rotation are given as Δp and Δq in radians. The Δp value is measured from the pole in the direction of Greenwich while Δq is measured perpendicular to this direction, positively towards the west. In addition to the polar motion values the A transformation incorporates part of the seasonal correction (UTC-UT1). The (UTC-UT1) value Δt_1 was decomposed (Appendix A) as $\Delta t_1 = \Delta t_2 + \Delta t$. Rather than incorporate Δt_1 into the B transformation the program will use Δt_2 in B and place Δt in the A transformation. A drift value $\dot{\Delta t}$ is also placed in the A transformation and the program can consider $(\Delta p, \Delta q, \Delta t, \dot{\Delta t})$ as parameters of improvement.

We can now write the transformation as

$$\begin{pmatrix} e \\ f \\ g \end{pmatrix} = ABCD \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} e' \\ f' \\ g' \end{pmatrix} = \begin{bmatrix} 1 & -\tilde{\omega}(\Delta t + t\dot{\Delta t}) & \Delta p \\ \tilde{\omega}(\Delta t + t\dot{\Delta t}) & 1 & -\Delta q \\ -\Delta p & \Delta q & 1 \end{bmatrix} \begin{pmatrix} e' \\ f' \\ g' \end{pmatrix} \qquad \text{(C-29)}$$

where A is evaluated at the beginning and end of a day and linear interpolation is used for intermediate values. The value t in the transformation is measured from the beginning of the year.

DECOMPOSITION OF Δt_1

Recall the polar motion discussion in Appendix A where Δt_1 was decomposed as $\Delta t_2 + \Delta t$. The correct value to use in the angle Λ of the B transformation is Δt_1.

Define

$$\tilde{\Lambda} = H_0 + \Delta H(t) + \tilde{\omega}(t - \Delta t_1)$$

This gives

$$\tilde{\Lambda} = \Lambda - \tilde{\omega}\Delta t$$

Letting $\tilde{B}$ be the corresponding B transformation using $\tilde{\Lambda}$ gives

$$\tilde{B} = \begin{bmatrix} \cos\tilde{\Lambda} & \sin\tilde{\Lambda} & 0 \\ -\sin\tilde{\Lambda} & \cos\tilde{\Lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\Lambda - \tilde{\omega}\Delta t) & \sin(\Lambda - \tilde{\omega}\Delta t) & 0 \\ -\sin(\Lambda - \tilde{\omega}\Delta t) & \cos(\Lambda - \tilde{\omega}\Delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Expanding and using $\cos(\varepsilon) = 1$, $\sin(\varepsilon) = \varepsilon$ for ε small give

$$\tilde{B} = \begin{bmatrix} \cos\Lambda + \tilde{\omega}\Delta t \sin\Lambda & \sin\Lambda - \tilde{\omega}\Delta t\cos\Lambda & 0 \\ -\sin\Lambda + \tilde{\omega}\Delta t\cos\Lambda & \cos\Lambda + \tilde{\omega}\Delta t\sin\Lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} \cos\Lambda & \sin\Lambda & 0 \\ -\sin\Lambda & \cos\Lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} + \tilde{\omega}\Delta t \begin{bmatrix} \sin\Lambda & -\cos\Lambda & 0 \\ \cos\Lambda & \sin\Lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= B + \tilde{\omega}\Delta t \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\Lambda & \sin\Lambda & 0 \\ -\sin\Lambda & \cos\Lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= B + \begin{bmatrix} 0 & -\tilde{\omega}\Delta t & 0 \\ \tilde{\omega}\Delta t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} B$$

$$= \begin{bmatrix} 1 & -\tilde{\omega}\Delta t & 0 \\ \tilde{\omega}\Delta t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B$$

Thus the A transformation is defined as

$$A' = \begin{bmatrix} 1 & 0 & \Delta p \\ 0 & 1 & -\Delta q \\ -\Delta p & \Delta q & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tilde{\omega}\Delta t & 0 \\ \tilde{\omega}\Delta t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(C-30)}$$

retaining only first order differentials.

Appendix D
FORCE MODEL

Recalling (71), (74), and (75), the equations of motion can be expressed as

$$\ddot{x} = G(x, \dot{x}, p) = \Sigma A_i \qquad \text{(D-1)}$$

where each A_i is an acceleration term and p represents the set of model parameters.

The individual acceleration terms are

A_e = acceleration due to the gravitational field of the earth

A_s = acceleration due to the gravitational field of the sun

A_m = acceleration due to the gravitational field of the moon

A_d = acceleration due to atmospheric drag

A_r = acceleration due to radiation pressure from the sun

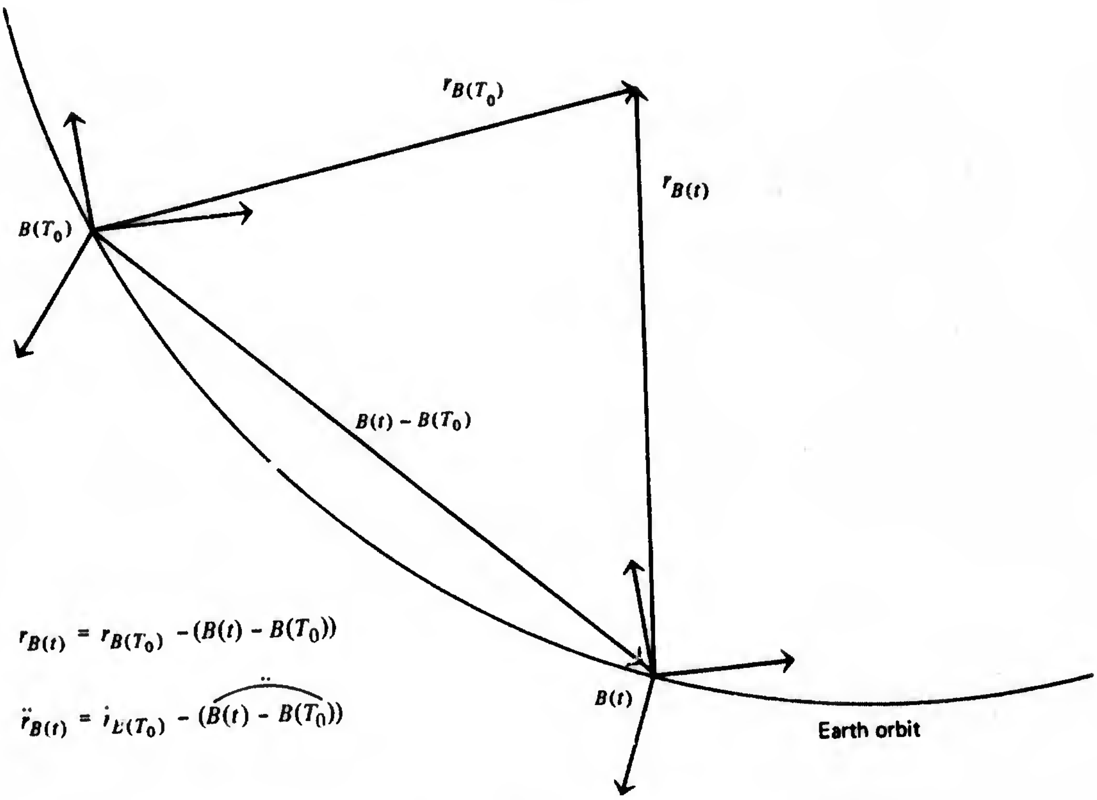
A_{ts} = acceleration due to the solar tidal distortion

A_{tm} = acceleration due to the lunar tidal distortion

A_t = acceleration due to vehicle thrust.

When we compute accelerations in the basic inertial system we must account for the fact that it is not an actual inertial frame. It moves with the earth while maintaining a fixed orientation to the stars. We can relate positions in our basic inertial system at any time to any actual inertial frame such as our basic system at some epoch time TO.

On the left side of the equation in Diagram D-1 we have the acceleration in the basic inertial system. On the right we have the acceleration in an actual inertial system contained in the first term. The second term, called indirect acceleration, is the acceleration due to the same sources acting on the basic system coordinate frame. Sources fixed in the basic system such as tides, earth mass, and atmosphere have no indirect contribution. Solar and lunar gravitational accelerations act on the earth as though it were a point, and are mass independent. Thus, their indirect contribution is just their acceleration evaluated at the origin of $B(t)$. Indirect radiation acceleration is proportional to the earth's surface to mass ratio and consequently is negligible.



$$r_{B(t)} = r_{B(T_0)} - (B(t) - B(T_0))$$

$$\ddot{r}_{B(t)} = \ddot{r}_{E(T_0)} - (\ddot{\overbrace{B(t) - B(T_0)}})$$

Diagram D-1

GRAVITATIONAL FIELD OF THE EARTH

The earth's potential in an earth fixed frame (E', F', G') is

$$V = \mu \sum_{n=0}^{N} \sum_{m=0}^{n} \left[a_e^n C_{nm} \frac{P_n^m \left(\frac{g'}{|r|}\right)}{|r|^{n+1}} \cos (m\lambda) + a_e^n S_{nm} \frac{P_n^m \left(\frac{g'}{|r|}\right)}{|r|^{n+1}} \sin (m\lambda) \right] \qquad \text{(D-2)}$$

where

$$r = \begin{pmatrix} e' \\ f' \\ g' \end{pmatrix} \qquad |r| = (e'^2 + f'^2 + g'^2)^{1/2}$$

a_e = Semi-major axis of the earth

P_n^m = Legendre polynomial

μ = Earth's gravity constant

λ = Longitude with respect to Greenwich

C_{nm}, S_{nm} = Gravity parameters

Since the coordinate system has its center at the earth's center of gravity we have $C_{10} = C_{11} = S_{11} = 0$.

The earth gravitational acceleration can now be given by

$$A_e = \nabla_R V = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix} \text{ in the } (XYZ) \text{ frame} \qquad \text{(D-3)}$$

For the purpose of calculation we rewrite (D-2) as

$$V = \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm} U_n^m + S_{nm} V_n^m] \quad n \neq 1 \qquad \text{(D-4)}$$

where

$$U_n^m = \mu a_e^n \frac{P_n^m \left(\frac{g'}{|r|}\right)}{|r|^{n+1}} \cos (m\lambda)$$

$$V_n^m = \mu a_e^n \left(\frac{P_n^m \frac{g'}{|r|}}{|r|^{n+1}} \right) \sin(m\lambda)$$

noting that $V_n^0 = C_{11} = C_{10} = S_{11} = 0$. Define the transformation E by

$$E = \frac{BCD}{a_e} \tag{D-5}$$

Introduce the longitude by

$$C(\lambda) = \sin(\theta)\cos(\lambda) = \frac{a_e}{|r|} \sum_{i=1}^{3} E_{1i} x_i$$

$$S(\lambda) = \sin(\theta)\sin(\lambda) = \frac{a_e}{|r|} \sum_{i=1}^{3} E_{2i} x_i \tag{D-6}$$

where

$$\theta = \cos^{-1}\left(\frac{g'}{|r|}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{inertial components of } r$$

Using the recurrence relations for Legendre polynomials we have recurrence relations for U and V given by

$$U_{n+1}^m = \frac{\rho}{(n-m+1)}\left[\frac{g'}{|r|}(2n+1)U_n^m - (n+m)\rho U_{n-1}^m\right]$$

$$V_{n+1}^m = \frac{\rho}{(n-m+1)}\left[\frac{g'}{|r|}(2n+1)V_n^m - (n+m)\rho V_{n-1}^m\right] \tag{D-7}$$

(D-7) is called horizontal stepping and (D-8) is called diagonal stepping

$$U_{n+1}^{n+1} = (2n+1)\rho\left[U_n^n C(\lambda) - V_n^n S(\lambda)\right]$$

$$V_{n+1}^{n+1} = (2n+1)\rho\left[V_n^n C(\lambda) + U_n^n S(\lambda)\right] \tag{D-8}$$

where

$$\rho = \frac{a_e}{|r|}$$

Using the values

$$U_0^0 = \frac{u}{|r|} \qquad U_1^0 = \frac{\mu a_e g'}{|r|^3}$$

$$V_0^0 = 0 \qquad V_1^0 = 0$$

we start at $n = 1, m = 0$ in the horizontal stepping equation and compute U_i^0, V_i^0 for $i = 2,3, \ldots N$. We then utilize diagonal stepping and calculate U_1^1, V_1^1. Returning to horizontal stepping enables the calculation of U_i^1, V_i^1 for $i = 2,3, \ldots N$. This process is repeated until $m = M$, where $M < N$. Note that

$$U_n^{-m} = \frac{(-1)^m (n-m)!}{(n+m)!} U_n^m$$

$$V_n^{-m} = \frac{(-1)^{m+1} (n-m)!}{(n+m)!} V_n^m$$

We can now compute D-3 by

$$\nabla_R V = E^* \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm} a_e \nabla U_n^m + S_{nm} a_e \nabla V_n^m] \tag{D-9}$$

where ∇_R is the gradient in the inertial system, ∇ is in the earth fixed, and a_e is the earth semi-major axis.

The recurrence relations for ∇U and ∇V can be given by

$$a_e \nabla U_n^m = \begin{bmatrix} \frac{1}{2} A_n^m U_{n+1}^{m-1} - \frac{1}{2} U_{n+1}^{m+1} \\ \\ -\frac{1}{2} A_n^m V_{n+1}^{m-1} - \frac{1}{2} V_{n+1}^{m+1} \\ \\ -(n-m+1) U_{n+1}^m \end{bmatrix} \tag{D-10}$$

$$a_e \nabla V_n^m = \begin{bmatrix} \frac{1}{2} A_n^m V_{n+1}^{m-1} - \frac{1}{2} V_{n+1}^{m+1} \\ \frac{1}{2} A_n^m U_{n+1}^{m-1} + \frac{1}{2} U_{n+1}^{m+1} \\ -(n-m+1) V_{n+1}^m \end{bmatrix} \quad \text{(D-10) (Continued)}$$

where

$$A_n^m = (n-m+1)(n-m+2)$$

GRAVITATIONAL FIELD OF THE SUN AND MOON

Coordinates of the sun and moon are stored on a file (sun-moon) at one-day and one-half-day intervals respectively. A sixth order Lagrangian interpolation procedure is used to obtain these values from the file.

The equation for acceleration due to the sun is

$$A_s = -\mu_s \left(\frac{r - r_s}{|r - r_s|^3} + \frac{r_s}{|r_s|^3} \right) \quad \text{(D-11)}$$

with a similar expression for the moon.

TIDAL DISTORTION

The gravitational (tidal) attraction of the sun and moon causes the earth to become elongated on an axis pointing toward the disturbing body. The redistribution of mass results in a perturbation of the earth's own gravitational field, which is represented by the potentials

$$U_s = -K_L \frac{\mu_s}{|r_s|^3} \frac{a_e^5}{|r|^3} P_2(\hat{r} \cdot \hat{r}_s)$$

$$U_m = -K_L \frac{\mu_m}{|r_m|^3} \frac{a_e^5}{|r|^3} P_2(\hat{r} \cdot \hat{r}_m) \quad \text{(D-12)}$$

where K_L is Love's constant and P_2 is the Legendre polynomial.

The associated acceleration is given by

$$A_{ts} = K_L \frac{\mu_s}{|r_s|^3} \frac{a_e^5}{|r|^5} \left[\left(-\frac{15}{2} (\hat{r} \cdot \hat{r}_s)^2 + \frac{3}{2} \right) r + 3(r \cdot \hat{r}_s) \hat{r}_s \right] \tag{D-13}$$

with a similar expression for A_{tm}.

RADIATION PRESSURE ACCELERATION

A shadow test is performed to determine if the satellite is in sunlight or shadow. If $r \cdot r_s \geqslant 0$ then the satellite is in sunlight. If $r \cdot r_s < 0$ then we compute $|r x \hat{r}_s|$. If $|r x \hat{r}_s|$ is less than a_e, the satellite is in shadow; and, if not, then it is in sunlight. When the satellite is in sunlight we compute the acceleration by

$$A_r = K_r \frac{s}{m} 10^{14} \frac{(r - r_s)}{|r - r_s|^3} \tag{D-14}$$

where s is the satellite cross-sectional area and m is satellite mass.

When the satellite is in shadow A_r is set to zero.

ATMOSPHERIC DRAG

The relative velocity of the satellite with respect to the atmosphere is

$$v_r = \dot{r} - \bar{\omega} x r \qquad \bar{\omega} = (CD)^* \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix}$$

where $(r, \dot{r})$ is the inertial satellite position and velocity. The acceleration of the satellite due to drag can now be expressed as

$$A_d = - C_D \rho \frac{s}{2m} |v_r| v_r \tag{D-15}$$

where m is the satellite mass, s the satellite cross-sectional area, and ρ the atmospheric density function defined by

$$\rho = \text{Exp}[Ah - B - (Ch^2 + Dh - E)^{1/2}]$$

where A, B, C, D and E are input values.

The height h is the satellite's geocentric altitude above its sub-point given by

$$h = |r| \left[1 - \frac{a_e}{\left(|r|^2 + \frac{e^2}{1 - e^2} z^2 \right)^{1/2}} \right]$$

$$= |r| - R$$

where R is the distance from the earth's center to the satellite sub-point.

$$R = \frac{a_e}{\left(1 + \frac{e^2}{1 - e^2} \sin^2 (\theta) \right)^{1/2}}$$

θ = geocentric latitude

THRUST

The satellite thrust provides acceleration

$$A_t = RA \tag{D-16}$$

where

$$R = [\hat{r}, \hat{\dot{r}}, \widehat{rx\dot{r}}] \quad \text{or} \quad R = I$$

$$A = \begin{pmatrix} A_r \\ A_{\dot{r}} \\ A_{rx\dot{r}} \end{pmatrix}$$

The values for A are specified in a thrust profile which gives A as a step function of time. The correct value for A at any time line is obtained by examining the profile. In the event that the $R = I$ option is selected the program will check the cycle number during integration and if it is the first cycle the input A values will be rotated to the inertial frame before they are used. Subsequent cycles will use the input values as given.

Appendix E
PERTURBATION EQUATIONS

Integrating 74 and 75 requires establishing initial conditions, which in the case of 7} are non-zero. If the element option is selected then $\partial x(t_s)/\partial e(t_s)$ is computed by

$$\frac{\partial r}{\partial a} = \frac{r}{a}$$

$$\frac{\partial r}{\partial e \sin(w)} = -\frac{1}{1-e^2}\left[\sin(w+E) + e\sin(w) - e\cos(w)\frac{e\sin E}{1+(1-e^2)^{1/2}}\right] r$$

$$+\left(\frac{a^3}{\mu}\right)^{1/2}\left[-\left(1+\frac{|r|}{a(1-e^2)}\right)\cos(w+E)\right.$$

$$\left.+\frac{|r|}{a(1-e^2)}\, e\cos(w)\, \frac{(e\cos E + (1-e^2)^{1/2})}{1+(1-e^2)^{1/2}}\right]\dot{r}$$

$$\frac{\partial r}{\partial e \cos(w)} = -\frac{1}{1-e^2}\left[\cos(w+E) + e\cos(w) + e\sin(w)\frac{e\sin(E)}{1+(1-e^2)^{1/2}}\right] r$$

$$+\left(\frac{a^3}{\mu}\right)^{1/2}\left[\left(1+\frac{|r|}{a(1-e^2)}\right)\sin(w+E)\right.$$

$$\left.-\frac{|r|}{a(1-e^2)}\, e\sin(w)\, \frac{(e\cos E + (1-e^2)^{1/2})}{1+(1-e^2)^{1/2}}\right]\dot{r}$$

$$\frac{\partial r}{\partial i} = \begin{pmatrix} z\sin\Omega \\ -z\cos\Omega \\ y\cos\Omega - x\sin\Omega \end{pmatrix}$$

$$\frac{\partial r}{\partial \ell} = \left(\frac{a^3}{\mu}\right)^{1/2}\dot{r}$$

$$\frac{\partial r}{\partial \Omega} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

$$\frac{\partial \dot{r}}{\partial a} = -\frac{1}{2}\,\frac{\dot{r}}{a}$$

$$\frac{\partial \dot{r}}{\partial e \sin(\omega)} = \frac{\left(\frac{\mu}{a^3}\right)^{1/2}}{1 - e^2} \left(\frac{a}{|r|}\right)^2 \left[\frac{a(1 - e^2)}{|r|} \cos(w + E) - e \sin E \sin(w + E) + \frac{e \cos(\omega)(1 - e^2 \cos^2 E)}{1 + (1 - e^2)^{1/2}}\right] r + \frac{1}{1 - e^2}\left[\sin(w + E) - \frac{e \cos(w) e \sin E}{1 + (1 - e^2)^{1/2}}\right] \dot{r}$$

$$\frac{\partial \dot{r}}{\partial e \cos(w)} = -\frac{\left(\frac{\mu}{a^3}\right)^{1/2}}{1 - e^2} \left(\frac{a}{|r|}\right)^2 \left[\frac{a(1 - e^2)}{|r|} \sin(w + E) + e \sin E \cos(w + E) + \frac{e \sin(w)(1 - e^2 \cos^2 E)}{1 + (1 - e^2)^{1/2}}\right] r + \frac{1}{1 - e^2}\left[\cos(w + E) + \frac{e \sin(w) e \sin E}{1 + (1 - e^2)^{1/2}}\right] \dot{r}$$

$$\frac{\partial \dot{r}}{\partial i} = \begin{pmatrix} \dot{z} \sin \Omega \\ -\dot{z} \cos \Omega \\ \dot{y} \cos \Omega - \dot{x} \sin \Omega \end{pmatrix}$$

$$\frac{\partial \dot{r}}{\partial \ell} = -\left(\frac{\mu}{a^3}\right)^{1/2} \left(\frac{a}{|r|}\right)^3 r$$

$$\frac{\partial \dot{r}}{\partial \Omega} = \begin{pmatrix} -\dot{y} \\ \dot{x} \\ 0 \end{pmatrix} \qquad \text{(E-1)}$$

where

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

and

$$\dot{r} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

PARTIALS FOR THE EARTH GRAVITATIONAL FIELD

Using (D-9) we can obtain $\nabla_R^2 V$ by

$$\nabla_R^2 V = E^* \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm} a_e^2 \nabla^2 U_n^m + S_{nm} a_e^2 \nabla^2 V_n^m] E \tag{E-2}$$

where

$$a_e^2 \nabla^2 U_n^m = \begin{bmatrix} \frac{1}{2} A_n^m a_e \nabla U_{n+1}^{m-1} - \frac{1}{2} a_e \nabla U_{n+1}^{m+1} \\ -\frac{1}{2} A_n^m a_e \nabla V_{n+1}^{m-1} - \frac{1}{2} a_e \nabla V_{n+1}^{m+1} \\ -(n-m+1) a_e \nabla U_{n+1}^m \end{bmatrix}$$

$$a_e^2 \nabla^2 V_n^m = \begin{bmatrix} \frac{1}{2} A_n^m a_e \nabla V_{n+1}^{m-1} - \frac{1}{2} a_e \nabla V_{n+1}^{m+1} \\ \frac{1}{2} A_n^m a_e \nabla U_{n+1}^{m-1} + \frac{1}{2} a_e \nabla U_{n+1}^{m+1} \\ -(n-m+1) a_e \nabla V_{n+1}^m \end{bmatrix} \tag{E-3}$$

The partials for C_{nm} and S_{nm} are given by

$$\frac{\partial \nabla_R V}{\partial C_{nm}} = E^* a_e \nabla U_n^m \qquad (n, m) \neq (0, 0)$$

$$\frac{\partial \nabla_R V}{\partial S_{nm}} = E^* a_e \nabla V_n^m \tag{E-4}$$

$$\frac{\partial \nabla_R V}{\partial \mu} = \frac{\nabla_R V}{\mu}$$

PARTIALS FOR THE SUN AND MOON FIELDS

Setting $p_s = |r - r_s|$ the derivatives are

$$\nabla A_{sx} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)^2 + p_s^2 \\ -3(x-x_s)(y-y_s) \\ -3(x-x_s)(z-z_s) \end{bmatrix}$$

$$\nabla A_{sy} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)(y-y_s) \\ -3(y-y_s)^2 + p_s^2 \\ -3(z-z_s)(y-y_s) \end{bmatrix} \qquad \text{(E-5)}$$

$$\nabla A_{sz} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)(z-z_s) \\ -3(y-y_s)(z-z_s) \\ -3(z-z_s)^2 + p_s^2 \end{bmatrix}$$

with similar results for the moon.

PARTIALS FOR TIDAL DISTORTION

$$\nabla A_{ts} = \frac{k_L}{2}\ \frac{\mu_s}{|r_s|^3}\ \frac{a_e^5}{|r|^5}\left\{\left[3 - 15\left(\frac{r^*r_s}{|r||r_s|}\right)^2\right] I + 6\,\frac{r_s r_s^*}{|r_s|^2} + \left[105\ \frac{r^*r_s^*}{|r||r_s|} - 15\right]\frac{rr^*}{|r|^2}\right.$$

$$\left. - 30\ \frac{r^*r_s}{|r||r_s|}\left[\frac{rr_s^* + r_s r^*}{|r||r_s|}\right]\right\} \qquad \text{(E-6)}$$

with similar results for the moon.

PARTIALS FOR RADIATION PRESSURE

Differentiating D-14 gives

$$\nabla A_{rx} = \frac{k_r s 10^{14}}{m p_s^5}\begin{pmatrix} -3(x-x_s)^2 + p_s^2 \\ -3(x-x_s)(y-y_s) \\ -3(x-x_s)(z-z_s) \end{pmatrix} \qquad \text{(E-7)}$$

$$\nabla A_{ry} = \frac{k_r s 10^{14}}{m p_s^5} \begin{pmatrix} -3(x-x_s)(y-y_s) \\ -3(y-y_s)^2 + p_s^2 \\ -3(y-y_s)(z-z_s) \end{pmatrix} \qquad \text{(E-7) (Cont.)}$$

$$\nabla A_{rz} = \frac{k_r s 10^{14}}{m p_s^5} \begin{pmatrix} -3(z-z_s)(x-x_s) \\ -3(z-z_s)(y-y_s) \\ -3(z-z_s)^2 + p_s^2 \end{pmatrix}$$

$$\frac{\partial A_r}{\partial k_r} = \frac{s}{m} 10^{14} \frac{(r-r_s)}{p_s^3}$$

PARTIALS FOR ATMOSPHERIC DRAG

From D-15 we have

$$\frac{\partial A_d}{\partial(r\dot{r})} = -\frac{C_D S}{2m}\left[\rho|v_r| \frac{\partial v_r}{\partial(r\dot{r})} + \rho v_r \frac{\partial|v_r|}{\partial(r\dot{r})} + |v_r|v_r \frac{\partial\rho}{\partial(r\dot{r})}\right] \qquad \text{(E-8)}$$

$$\frac{\partial A_d}{\partial C_D} = -\frac{\rho s}{2m}|v_r|v_r$$

Since $v_r = \dot{r} - \bar{\omega} x r = \dot{r} - \Omega r$ from (43)

$$\Omega = (CD)^* \begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD)$$

$$\frac{\partial v_r}{\partial r} = -\Omega = (CD)^* \begin{bmatrix} 0 & \tilde{\omega} & 0 \\ -\tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD)$$

$$\frac{\partial v_r}{\partial \dot{r}} = I$$

$$\frac{\partial v_r}{\partial(r\dot{r})} = \left[(CD)^* \begin{bmatrix} 0 & \tilde{\omega} & 0 \\ -\tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD), I\right]$$

where *CD* is set to *I* in the program.

$$\frac{\partial |v_r|}{\partial (r\dot{r})} = \frac{\partial (v_r^* v_r)^{1/2}}{\partial (r\dot{r})} = \frac{v_r^*}{|v_r|}\ \frac{\partial v_r}{\partial (r\dot{r})}$$

$$\frac{\partial \rho}{\partial (r\dot{r})} = \left[\frac{\partial \rho}{\partial r}\ ,\ \frac{\partial \rho}{\partial \dot{r}}\right] = \left[\frac{\partial \rho}{\partial r}\ ,\ 0\right]$$

From the expression for ρ

$$\frac{\partial \rho}{\partial r} = \frac{\partial \rho}{\partial h}\ \frac{\partial h}{\partial r}$$

$$\frac{\partial \rho}{\partial h} = \rho\left[A - \frac{(Ch + D/2)}{(Ch^2 + Dh - E)^{1/2}}\right].$$

$$\frac{\partial h}{\partial r} = \frac{1}{|r|^2}\left[h + \frac{(|r| - h)^3}{a_e^2}\right] r^* + \frac{(|r| - h)^3 e^2 (0, 0, z)}{|r|^2 a_e^2 (1 - e^2)}$$

PARTIALS FOR THRUST

Using (D-16) we have

$$\frac{\partial A_t}{\partial (r\dot{r})} = \frac{\partial R}{\partial (r\dot{r})}\ A \qquad \text{(E-9)}$$

$$\frac{\partial A_t}{\partial A} = R$$

where $\partial R / \partial (r\dot{r}) = 0$ if $R = I$ and $\partial R / \partial (r\dot{r})$ is given below if $R = [\hat{r}, \hat{\dot{r}}, r\hat{x}\dot{r}]$.

$$\text{Setting } r = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \dot{r} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \quad \text{gives}$$

$$\frac{\partial R}{\partial(r\dot{r})} = \left[\frac{\partial \hat{r}}{\partial(r\dot{r})} , \frac{\partial \hat{\dot{r}}}{\partial(r\dot{r})} , \frac{\partial \widehat{rx\dot{r}}}{\partial(r\dot{r})} \right]$$

$$\frac{\partial \hat{r}}{\partial(r\dot{r})} = \left[\frac{\partial \hat{r}}{\partial r} , \frac{\partial \hat{r}}{\partial \dot{r}} \right] = \left[\frac{\partial \hat{r}}{\partial r} , 0 \right]$$

$$\frac{\partial \hat{r}}{\partial r} = \frac{1}{|r|} \left[I - \frac{rr^{*}}{|r|^2} \right]$$

$$\frac{\partial \hat{\dot{r}}}{\partial(r\dot{r})} = \left[\frac{\partial \hat{\dot{r}}}{\partial r} , \frac{\partial \hat{\dot{r}}}{\partial \dot{r}} \right] = \left[0, \frac{\partial \hat{\dot{r}}}{\partial r} \right]$$

$$\frac{\partial \hat{\dot{r}}}{\partial \dot{r}} = \frac{1}{|\dot{r}|} \left[I - \frac{\dot{r}\dot{r}^{*}}{|\dot{r}|^2} \right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial(r\dot{r})} = \left[\frac{\partial \widehat{rx\dot{r}}}{\partial r} , \frac{\partial \widehat{rx\dot{r}}}{\partial \dot{r}} \right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial r} = - \frac{\Omega_{\dot{r}}}{|rx\dot{r}|} \left[I - rr^{*} \quad \frac{\Omega_{\dot{r}}^{*}\Omega_{\dot{r}}}{|rx\dot{r}|^2} \right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial \dot{r}} = \frac{\Omega_r}{|rx\dot{r}|} \left[I - \dot{r}\dot{r}^{*} \quad \frac{\Omega_r^{*}\Omega_r}{|rx\dot{r}|^2} \right]$$

where

$$rx\dot{r} = \Omega_r \dot{r} = - \dot{r}xr = - \Omega_{\dot{r}} r$$

$$\Omega_r = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \qquad \Omega_r^{*}\Omega_r = \begin{bmatrix} x_2^2 + x_3^2 & -x_1x_2 & -x_1x_3 \\ -x_1x_2 & x_1^2 + x_3^2 & -x_2x_3 \\ -x_1x_3 & -x_2x_3 & x_1^2 + x_2^2 \end{bmatrix}$$

Appendix F
GRAVITY PARAMETERS

On option a limited number of gravity parameters can be included in the orbit solution. The total number of gravity parameters can be as high as 32, but in no case can the total number of dynamic parameters exceed 39. A given gravity parameter $\tilde{g}$ will be c_{nm} or s_{nm}.

$$\frac{\partial G}{\partial \mu} = G/\mu$$

The other partials used in (75) are

$$\frac{\partial G}{\partial \tilde{g}} = (BCD)^* \begin{cases} \nabla U_n^m & \tilde{g} = c_{nm} \\ \nabla V_n^m & \tilde{g} = s_{nm} \end{cases}$$

The solutions $\partial x/\partial \tilde{g}$ are placed on the Perturbed Trajectory File and used in expanding the pass matrix to include gravity parameters.

GRAVITY INCLUSION

$$B\tilde{g}_1\tilde{g}_2 = \sum_i \frac{\partial D_i^*}{\partial \tilde{g}_1} \frac{\partial D_i}{\partial \tilde{g}_2} = \sum_i \frac{\partial x^*(t_i)}{\partial \tilde{g}_1} \frac{\partial D_i^*}{\partial x(t_i)} \frac{\partial D_i}{\partial x(t_i)} \frac{\partial x(t_i)}{\partial \tilde{g}_2}$$

If we approximate $\partial x(t_i)/\partial \tilde{g}$ by $\partial x(t_i)/\partial x(tcA)\ \partial x(tcA)/\partial \tilde{g}$ then

$$B\tilde{g}_1\tilde{g}_2 = \sum_i \frac{\partial x^*(tcA)}{\partial \tilde{g}_1} \frac{\partial x^*(t_i)}{\partial x(tcA)} \frac{\partial D_i^*}{\partial x(t_i)} \frac{\partial D_i}{\partial x(t_i)} \frac{\partial x(t_i)}{\partial x(tcA)} \frac{\partial x(tcA)}{\partial \tilde{g}_2}$$

$$= \frac{\partial x^*(tcA)}{\partial \tilde{g}_1} \left[\sum_i \frac{\partial D_i}{\partial x(tcA)} \frac{\partial D_i}{\partial x(tcA)} \right] \frac{\partial x(tcA)}{\partial \tilde{g}_2}$$

$$= \psi^*_{\tilde{g}1}(tcA)\psi^{-1*}(tcA)\left[\sum_i \psi^*(tcA)\frac{\partial D_i^*}{\partial x(tcA)}\frac{\partial D_i}{\partial x(tcA)}\psi(tcA)\right]\psi^{-1}(tcA)\psi_{\tilde{g}2}(tcA)$$

$$= \psi^*_{\tilde{g}1}(tcA)\psi^{-1*}(tcA)B_{\tilde{e}\tilde{e}}\psi^{-1}(tcA)\psi_{\tilde{g}2}(tcA)$$

$$= \psi^*_{\tilde{g}1}(tcA)\psi^{-1*}(tcA)\psi^*(t_s(1))T^{-1*}(t_s(1))B_{ee}T^{-1}(t_s(1))\psi(t_s(1))\psi^{-1}(tcA)\psi_{\tilde{g}2}(tcA)$$

where $B_{\tilde{e}\tilde{e}}$ is the orbital element section of the canonical pass matrix (Diagram 13), and B_{ee} is the same section at the epoch $t_s(1)$ of the expanded pass matrix. Continuing with the χ notation

$$\chi_g(t) = \psi^{-1}(t)\psi_{\tilde{g}}(t)$$

$$B_{\tilde{g}1\tilde{g}2} = \chi^*_{\tilde{g}1}(tcA)[T^{-1}(t_s(1))\psi(t_s(1))]^*B_{ee}[T^{-1}(t_s(1))\psi(t_s(1))]\chi_{\tilde{g}2}(tcA) \qquad \text{(F-1)}$$

Define the gravity inclusion transformation $ICL_{\tilde{g}}$ by

$$ICL_{\tilde{g}} = T^{-1}(t_s(1))\psi(t_s(1))\chi_{\tilde{g}}(tcA) \qquad \text{(F-2)}$$

During the expansion procedure an epoch change from t_s to $t_s(1)$ is carried out. If in $ICL_{\tilde{g}}$, $t_s(1)$ is taken as the present epoch of the pass matrix then the pass matrix can be further expanded to include gravity parameters by

$$B^{ICL}_{pass} = \overline{ICL}^*_{\tilde{g}} B^{Exp}_{pass} \overline{ICL}_{\tilde{g}}$$

$$E^{ICL}_{pass} = \overline{ICL}^*_{\tilde{g}} E^{Exp.}_{pass} \qquad \text{(F-3)}$$

Domain $(\tilde{g}) = (t_s, t_E]$

where

$$\overline{ICL}_{\tilde{g}} = \begin{bmatrix} I_{ee} & 0 & ICL_{\tilde{g}} & 0 \\ 0 & I_{mm} & 0 & 0 \\ 0 & 0 & 0 & I_{bb} \end{bmatrix} \qquad \text{(F-4)}$$

e = orbital parameters

m = model parameters

b = bias parameters

Expanding F-3 gives

$$B_{pass}^{ICL} = \begin{bmatrix} B_{ee} & B_{em} & B_{ee} ICL_{\tilde{g}} & B_{eb} \\ & B_{mm} & B_{me} ICL_{\tilde{g}} & B_{mb} \\ & & ICL_{\tilde{g}}^{*} B_{ee} ICL_{\tilde{g}} & ICL_{\tilde{g}}^{*} B_{eb} \\ * & & & B_{bb} \end{bmatrix}$$

$$E_{pass}^{ICL} = \begin{bmatrix} E_e \\ E_m \\ ICL_{\tilde{g}}^{*} E_e \\ E_b \end{bmatrix}$$

GRAVITY UPDATE

The Celest Program checks to see if the epoch of fit is different than the epoch of the perturbed trajectory. If it is, an epoch update for gravity is required to reference the gravity improvements to the new epoch time $t_s(1)$. We define the gravity update transformation by

$$Up_g = \begin{bmatrix} I_{\text{orbit, orbit}} & 0 & \bar{\chi}_g & 0 \\ 0 & I_{\text{model, model}} & 0 & 0 \\ 0 & 0 & I_{\text{gravity, gravity}} & 0 \\ 0 & 0 & 0 & I_{\text{bias, bias}} \end{bmatrix} \tag{F-5}$$

where $\bar{\chi}_g = - T^{-1}(t_s(1))(\psi_{\tilde{g}}(t_s(1)) - \psi_g(t_s(1)))$

$$\psi_g(t_s(1)) = \begin{Bmatrix} 0 & g \neq \mu \\ \psi_{\mu}(t_s(1)) & g = \mu \end{Bmatrix}$$

$$\psi_{\mu}(t_s(1)) = \begin{cases} 0 & \text{if the program is improving coordinates} \\ \begin{pmatrix} 0 \\ \dot{r}(t_s(1))/2\mu \end{pmatrix} & \text{if the program is improving elements} \end{cases}$$

Using the transformation Up_g, gravity update is accomplished by

$$B_{pass}^{updated} = Up_g^* B_{pass}^{ICL} Up_g$$

$$E_{pass}^{updated} = Up_g^* E_{pass}^{ICL} \tag{F-6}$$

The effect of applying Up_g to B_{pass}^{ICL}, as indicated in (F-6), can be obtained by multiplying the orbit columns on the right by $\overline{\chi}_g$ and adding the result to the gravity columns. Next multiply the orbit rows of the new B_{pass}^{ICL} by $\overline{\chi}_g^*$ on the left and add the result to the gravity rows. The final result is $B_{pass}^{updated}$. $E_{pass}^{updated}$ is obtained with the row operations only.

GRAVITY PROPAGATION

After having obtained a solution including gravity improvements Δg, the Δg values must be included in the propagation procedure. This can be accomplished using $\tilde{\psi}_g$ and $\tilde{\chi}_g$ defined below.

$$\tilde{\psi}_g(t) = \psi_{\tilde{g}}(t) \qquad t_s(1) \leq t \leq t_E(1)$$

$$\tilde{\chi}_g(t) = -\psi^{-1}(t_s(1))(\psi_{\tilde{g}}(t_s(1)) - \psi_g(t_s(1))) = -\chi_g(t_s(1)) \qquad t_s(1) \leq t \leq t_E(1) \tag{F-7}$$

where $\psi_g(t_s(1))$ is from (F-5).

In the algorithm for this work described in the *Propagator Section*, there are no changes. $\tilde{\chi}$ is initialized at $\tilde{\chi}_g(t_s(1))$ and never changed. $\tilde{\psi}_g$ is initialized at $\tilde{\psi}_g(t_s(1))$ and changes at every time line.

REFERENCES

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2. Brouwer, D. and Clemence, G. M., Methods of Celestial Mechanics, Academic Press, 1961.
3. ——————, Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, Her Majesty's Stationery Office.
4. Mark G. Tanenbaum. Alfred F. Buonaguro, Larry K. Beuglass and James W. O'Toole, Celest Mathematical Formulation (Unpublished notes).

Appendix G

B-INVERSE MATRIX FILE (TAPE24)

THE B-INVERSE FILE IS WRITTEN BY THE BSOLVR SECTION OF CELEST TO BE USED IN THE PROPAGATOR (COVAR) SECTION. IT CONSISTS OF THREE RECORD TYPES. RECORD TYPE 1 IS A HEADER RECORD FOR THE FILE AND APPEARS ONLY AS THE FIRST RECORD OF EACH FILE. RECORD TYPE 2 AND RECORD TYPE 3 ARE REPEATED FOR EACH SOLUTION. IF THE RUN PRODUCING THE FILE IS A LONG ARC RUN, THEN RECORD TYPES 2 AND 3 APPEAR ONLY ONCE. HOWEVER, IF THE RUN IS A SHORT ARC RUN, RECORD TYPES 2 AND 3 ARE REPEATED FOR EACH SHORT ARC.

FORMAT OF B-INVERSE FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	ID(1)	SATELLITE NO.
2	D	IDATE	TIME CLOCK VALUE OF RUN

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NREV	REV NUMBER OF SATELLITE
2	I	NPA	NUMBER OF PARAMETERS IN SOLUTION

RECORD TYPE 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	BINV	UPPER TRIANGULAR PART OF B-INVERSE MATRIX
2	F	.	
.	.	.	
* N	F	.	
N+1	F	DELP(1)	DELTA P VECTOR (PARAMETER IMPROVEMENTS)
N+2	F	.	
.	.	.	
** M	F	DELP(NPA)	

* WHERE N = NPA*(NPA+1)/2

** WHERE M = N+NPA

GRAVITY FILE (TAPE21)

THE GRAVITY FILE CONTAINS THE GRAVITY COEFFICIENTS USED TO COMPUTE THE GRAVITY CONTRIBUTATION TO THE FORCE EQUATIONS IN THE ORBIT GENERATION SECTION OF CELEST. THE FILE MAY BE CREATED IN THE INFTS SECTION BY READING THE COEFFICIENTS FROM CARDS.

RECORD NO. 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A	GNAME	GRAVITY SET NAME
2	I	NCSC	NUMBER OF INPUT CARDS CONTAINING C AND S VALUES
3	I	MGP	MAXIMUM ORDER-M
4	I	NGP	MAXIMUM DEGEE-N
5	F	CGP	ARRAY OF 703 VALUES CONTAINING THE
6	F	.	C GRAVITY COEFFICIENTS
.	.	.	.
.	.	.	.
* L	.	.	.
L+1	F	SGP	ARRAY OF 703 VALUES CONTAINING THE
L+2	.	.	S GRAVITY COEFFICIENTS
.	.	.	.
.	.	.	.
** K	.	.	.

* WHERE L=707
** WHERE K=1410

SHORT-ARC SELECTOR FILE (TAPE25)

RECORD NO. 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TO(1)	YEAR OF ARC EPOCH
2	F	TO(2)	DAY OF ARC EPOCH
3	F	TO(3)	SEC. OF ARC EPOCH
4	F	ID(1)	SATELLITE NUMBER (NSWC)
5	I	L	MAXIMUM NUMBER OF LONG ARC PASSES ALLOWED

RECORD NO. 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NLAP	NUMBER OF LONG ARC PASSES
2	I	NB(1)	ARRAY OF 300 WORDS CONTAINING LONG ARC PASS NUMBERS WITH TAGGED PASS NUMBERS SET NEGATIVE
.	.	.	
.	.	.	
301	.	NB(300)	
302	A	KSTA(1)	ARRAY OF 300 WORDS CONTAINING STATION NOS. OF THE LONG ARC PASSES
303	.	.	
.	.	.	
.	.	.	
601	.	KSTA(300)	
602	F	TCA(1)	ARRAY OF 300 WORDS CONTAINING THE TIME OF CLOSEST APPROACH OF THE SATELLITE TO THE STATION FOR EACH LONG ARC PASS
603	.	.	
.	.	.	
.	.	.	
901	.	TCA(300)	
902	F	PHD(1)	ARRAY OF 300 WORDS CONTAINING THE STATION LATITUDE (DEG) FOR EACH LONG ARC PASS
903			
.			
.			
1201	F	PHD(300)	

SHORT-ARC SELECTOR FILE (CONT.)

RECORD NO. 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NREV	NUMBER OF REVS CREATED BY SHORT-ARC-SELECTOR
2	I	NRVS	NUMBER OF REVS PROCESSED BY CELEST
3	I	NDPA	NUMBER OF LONG ARC DRAG PARAMETERS
4	I	NTPA	NUMBER OF LONG ARC THRUST PARAMETERS
5	F	P	PERIOD (SEC) OF SATELLITE
6	F	T0	START TIME OF 1ST REV (SEC. FROM EPOCH)
7	F	TRAT(7)	INTEGRATION INTERVAL (SEC)
8	F	ANGLEI	INCLINATION ANGLE (RADIANS) OF SATELLITE ORBIT
9	F	DELMID	DELTA MID LATITUDE (DEGREES) (USED FOR QUALITY FACTOR CALCULATION)
10	F	RNHOOD	NEIGHBORHOOD OF TCA (SEC) (USED FOR QUALITY FACTOR CALCULATION)

RECORD NO. 4

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	IRVS(1)	ARRAY OF 17 WORDS CONTAINING REV NOS.
2	I	IRVS(2)	PROCESSED BY CELEST
.	.	.	
.	.	.	
17	I	IRVS(17)	

SHORT-ARC SELECTOR FILE (CONT.)

RECORD NO. 5

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TREV(1)	START TIME OF FIRST REV (SEC. FROM EPOCH)
2	F	TREV(2)	END TIME OF FIRST REV (SEC. FROM EPOCH)
3	F	TREV(3)	START . . 2ND . . . .
4	F	TREV(4)	END . . 2ND . . . .
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
33	.	TREV(17)	START . . 17TH . . . .
34	.	TREV(17)	END . . 17TH . . . .

RECORD NO. 6

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	DTIMS(1)	ARRAY OF 20 WORDS CONTAINING THE LONG
2	F	DTIMS(2)	ARC DRAG TIMES (SEC. FROM EPOCH)
.	.	.	
.	.	.	
.	.	.	
20	F	DTIMS(20)	

SHORT-ARC SELECTOR FILE (CONT.)

RECORD NO. 7

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TTIMS(1,1)	START TIME OF 1ST THRUST(SEC FROM EPOCH)
2	F	TTIMS(2,1)	END TIME OF 1ST THRUST . . .
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
7	F	TTIMS(1,4)	START . . 4TH . . . .
8	F	TTIMS(2,4)	END . . 4TH . . . .

RECORD NO. 8

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NSDA(1,1)	REV NUMBER OF 1ST REV
2	I	NSDA(2,1)	NUMBER OF DRAG SEGMENTS FOR 1ST REV
3	I	NSDA(1,2)	REV NUMBER OF 2ND REV
4	I	NSDA(2,2)	NUMBER OF DRAG SEGMENTS FOR 2ND REV
.	.	.	. . . . .
.	.	.	. . . . .
.	.	.	. . . . .
33	I	NSDA(1,17)	
34	I	NSDA(2,17)	

RECORD NO. 9

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	DSA(1,1)	START TIME OF 1ST SHORT-ARC(SA) (SEC)
2	F	DSA(2,1)	END TIME OF 1ST DRAG SEGMENT OF 1ST SA
3	F	DSA(3,1)	. . . 2ND . . . . .
4	F	DSA(4,1)	. . . 3RD . . . . .
.	.	.	. . . .
.	.	.	. . . .
21	F	DSA(21,1)	. . . 20TH . . . . .
22	F	DSA(1,2)	START TIME OF 2ND SHORT-ARC (SEC)
23	F	DSA(2,2)	END TIME OF 1ST DRAG SEGMENT OF 2ND SA
24	F	DSA(3,2)	. . . 2ND . . . . .
.	.	.	. . . .
.	.	.	. . . .
357	F	DSA(21,17)	END TIME OF 20TH DRAG SEGMENT OF 17TH SA

SHORT-ARC SELECTOR FILE (CONT.)

RECORD NO. 10

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	CASA(1)	AVERAGE SA DRAG LENGTH FOR 1ST REV (SEC)
2	F	CASA(2)	. . . . . 2ND . .
3	F	CASA(3)	. . . . . 3RD . .
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
17	F	CASA(17)	. . . . . 17TH . .

RECORD NO. 11

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	KNDEXT(1)	NUMBER OF DRAG EXTENSIONS FOR 1ST REV
2	I	KNDEXT(2)	. . . . . 2ND .
3	I	KNDEXT(3)	. . . . . 3RD .
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
17	I	KNDEXT(17)	. . . . . 17TH .

RECORD NO. 12

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NSAP	NUMBER OF SHORT ARC PASSES
2-850	I	ITAGS(I,J) I=1,50 J=1,17	SHORT ARC PASS TAGS FOR JTH SHORT ARC(REV)

(O-C) FILE (TAPE7)

THE (O-C) FILE IS WRITTEN IN THE FILTER SECTION OF CELEST. USUALLY THE ADJUSTED (O-C) ARE WRITTEN, HOWEVER THERE IS AN INPUT OPTION TO THE FILTER SECTION TO WRITE THE UNADJUSTED VALUES VICE THE ADJUSTED. IF A PASS IS REJECTED IN THE FILTER THEN THE UNADJUSTED (O-C)S ARE WRITTEN. FOR ANY REJECTED POINTS WITHIN A PASS, THE TIME CORRESPONDING TO THE (O-C) FOR THAT POINT IS WRITTEN AS A NEGATIVE TIME.

RECORD NO. 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TCA	TIME OF CLOSEST APPROACH OF PASS
2	F	YEAR	YEAR OF PASS
3	F	DAY	DAY OF PASS
4	A	ISTA	STATION NUMBER
5	I	ICLAS	CLASS OF DATA
6	I	ITYPE	TYPE OF DATA
7	I	NO	NUMBER OF OBSERVATIONS IN PASS
8	F	TIMO(1)	TIME OF 1ST OBSERVATION
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
7+NO	F	TIMO(NO)	TIME OF LAST OBSERVATION
8+NO	F	OCADJ(1)	(O-C) OF 1ST OBSERVATION
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
7+2*NO	F	OCADJ(NO)	(O-C) OF LAST OBSERVATION
* M	I	SATNO1	SATELLITE NO. (NSWC)
M+1	I	SATNO2	SATELLITE NO. OF SECONDARY SAT.

* WHERE M = 7+2*NO+1

THE ABOVE RECORD IS REPEATED FOR EACH PASS

SUN-MOON FILE (TAPE19)

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	RECORD AND FILE ID (110XXX), WHERE 110 IS RECORD ID AND XXX IS FILE NO. IN ASCENDING ORDER BY YEAR
2	F	YEAR	YEAR OF SUN AND MOON DATA
3	F	ROD	NO. OF DAY RECORDS IN YEAR

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	RECORD AND FILE ID (111XXX)
2	F	DAY	DAY NUMBER OF DATA
3-5	F	XS(I)I=1,3	COORDINATES OF SUN AT ONE DAY INTERVALS
6-8	F	XM(I)I=1,3	COORDINATES OF MOON AT HALF DAY INTERVALS
9-11	F	XM(I)I=1,3	COORDINATES OF MOON AT HALF DAY INTERVALS
12	F	DLSI	NUTATION IN LONGITUDE OF THE SUN ZERO HOUR EPHEMERIS TIME
13	F	EP	OBLIQUITY OF ECLIPTIC ZERO HOUR EPHEMERIS TIME
14	F	DLEP	NUTATION IN OBLIQUITY (BESSELIAN DAY NUMBERS)(NEGATIVE OF COLUMN B FROM NAUTICAL ALMANAC)
15	F	DLT	ROTATIONAL CORRECTION, DETERMINED FROM YEARLY STRAIGHT LINE FIT TO UT1-UTC
16	F	DLH	EQUATION OF EQUINOX AT ZERO HOUR UT
17	F	P	POLAR MOTION VALUE (DISPLACEMENT TOWARD GREENWICH)
18	F	Q	POLAR MOTION VALUE (DISPLACEMENT WEST)
19	F	DUM	DUMMY WORD
20	F	DUM	DUMMY WORD

RECORD TYPE 2 IS REPEATED ROD TIMES.

SUN-MOON FILE (CONT.)

LAST RECORD OF TYPE 2 IS A DUMMY RECORD WITH NEGATIVE VALUE IN WORD 1

1	F	RTYPE	RECORD ID (-119) INDICATES END OF YEAR
2-20	F	DUM	DUMMY WORDS

END OF INFORMATION RECORD

1	F	RTYPE	RECORD ID (-199999) INDICATES END OF ALL DATA
2	F	FYEAR	FINAL YEAR ON FILE
3	F	TYEAR	NO. OF YEARS ON FILE

RECORD TYPES 1 AND 2 ARE REPEATED FOR ADDITIONAL YEARS OF DATA

DATA CLASS/PASS DIAGNOSTICS (TAPE27)

THE DATA CLASS FILE IS A DOUBLY INDEXED RANDOM ACCESS MS FILE WRITTEN IN THE BSOLVR SECTION OF CELEST. EACH RECORD IS 515 WORDS LONG. A RECORD CONTAINS INFORMATION FOR EACH PASS OF A GIVEN CLASS-TYPE IN THE ARC INTERVAL. THE MASTER INDEX IS THE INDEX OF THE SHORT-ARC (REV) AND THE SUBINDEX (K) IS THE POSITION OF THE CLASS-TYPE IN THE CLASS-TYPE ARRAY. THE FOLLOWING TABLE GIVES THE K INDEX AND THE CORRESPONDING CLASS-TYPE.

K	CLASS-TYPE		DESCRIPTION
1	7	5	SGLS FREQUENCY
2	7	7	DOPPLER FREQUENCY
3	9	4	RANGE DIFFERENCE (ITT)
4	9	5	RANGE DIFFERENCE (SGLS)
5	9	8	RANGE DIFFERENCE (GEOCEIVER)
6	9	9	RANGE DIFFERENCE (CCID)

RECORD FORMAT

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	SN	SIGNAL-TO-NOISE OF SHORT ARC
2	I	J	NUMBER OF PASSES IN THIS RECORD OF KTH CLASS-TYPE DATA
3	F	TO(2)	DAY OF SHORT ARC EPOCH
4	I	NBB(I)	PASS NUMBER OF ITH DATA PASS OF KTH TYPE
.	.		. . . .
3+J	.		. . . .
4+J	.	0	WORD OF ZERO
.	.	.	. . .
.	.	.	. . .
53	.	0	. . .
54	A	KSTA(I)	STATION NO. OF ITH DATA PASS OF KTH TYPE
.	.		. . . .
53+J	.		. . . .
.	.	ZEROES	
104	F	TTCA(I)	TCA FOR ITH DATA PASS OF KTH TYPE
	.		
.	.		. . . .
103+J	.	.	. . . .

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
154	F	ELV(I)	ELEVATION ANGLE ITH DATA PASS OF KTH TYPE
.			. . . .
.	.		. . . .
153+J	.		. . . .
204	F	PHO(I)	LATITUDE OF STA FOR ITH PASS OF KTH TYPE
.	.		. . . .
.	.		. . . .
203+J	.		. . . .
254	F	BSL(5,I)	FREQ. BIAS SOLUTION FOR ITH PASS OF K TYPE
.	.		. . . .
.	.	.	. . . .
253+J	.	.	. . . .
304	F	DRV(I)	RADIAL COMPONENT OF NAVIGATION SOLUTION
.	.	.	. . . . .
303+J	.	.	. . . . .
354	F	DRR(I)	TANGENTIAL COMP. OF NAVIGATION SOLUTION
.	.	.	. . . . .
353+J	.	.	. . . . .
404	F	SIGRAD(I)	RADIAL SIGMA OF NAVIGATION SOLUTION
.	.	.	. . . . .
403+J	F	.	. . . . .
454	F	SIGTAN(I)	TANGENTIAL SIGMA OF NAVIGATION SOLUTION
.	.	.	. . . . .
453+J	.	.	. . . . .
504	F	RWSDVT	RWS OF RADIAL COMPONENT OF NAVIGATION SOL.
505	F	RWSDRT	RWS OF TANGENTIAL COMP. OF NAVIGATION SOL.

THE ABOVE RECORD IS REPEATED FOR EACH CLASS-TYPE OF DATA IN EACH SHORT ARC FOR EACH SHORT ARC IN THE RUN GENERATING THE FILE.

STATION TABLE FILE (TAPE6)

THE STATION TABLE FILE IS A CDC FORTRAN EXTENDED RANDOM ACCESS (SYMBOLIC KEY) FILE WITH A MAXIMUM OF 100 RECORDS. ONE RECORD IS WRITTEN FOR EACH STATION. THE FILE IS CREATED BY A PROGRAM OUTSIDE OF THE CELEST PROGRAM USING WRITE-IN-PLACE. THE SYMBOLIC KEY IS THE ALPHANUMERIC STATION NUMBER.

RECORD FORMAT

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A	STA	STATION NUMBER
2	F	LAMBDA	GEODETIC LONGITUDE OF STATION (DEG)
3	F	PHI	GEODETIC LATITUDE OF STATION (DEG)
4	F	ALT	GEODETIC ALTITUDE OF STATION (KM)
6-15			NOT USED
16-385	F	TC	DAILY CLOCK ERROR FOR TYPE 7 OR 9 STATIONS WORD 16 = CLOCK ERROR FOR DAY 1, ETC.
16-385	I	NSEG	NUMBER OF SEGMENTS IN DAY FOR TYPE 4 OR 8 STATIONS
386-755	F	SEC1	EPOCH OF SEG 1 IN SEC OF DAY
756-1125	F	SEC2	EPOCH OF SEG 2 IN SEC OF DAY
1126-1495	F	SEC3	EPOCH OF SEG 3 IN SEC OF DAY
1496-1865	F	TC1	CLOCK ERROR AT EPOCH OF SEG 1
1866-2235	F	TC2	CLOCK ERROR AT EPOCH OF SEG 2
2236-2605	F	TC3	CLOCK ERROR AT EPOCH OF SEG 3
2606-2975	F	TC1D	DRIFT RATE OF CLOCK ERROR OF SEG 1
2976-3345	F	TC2D	DRIFT RATE OF CLOCK ERROR OF SEG 2
3346-3715	F	TC3D	DRIFT RATE OF CLOCK ERROR OF SEG 3

WORDS 386-3715 APPEAR ONLY FOR TYPE 4 OR TYPE 8 STATIONS, I.E., THE RECORD TERMINATES WITH WORD 385 FOR TYPE 7 OR 9 STATIONS.

RAW OBSERVATION FILE (TAPE10,11,12,13)

THE RAW OBSERVATION FILE IS A 7 TRACK, 556 BPI, BCD TAPE TRANSMITTED TO NSWC VIA A DATA LINK (MOHAWK, IBM 1401, ETC) FROM ANOTHER INSTALLATION (APL, ETC). THERE ARE THREE TYPES OF DATA RECEIVED (DOPPLER, GEOCEIVER, CCID). THE RECORD CONTENTS ARE DIFFERENT FOR THE DIFFERENT TYPES OF DATA. FOLLOWING ARE THE RECORD FORMATS FOR THE DIFFERENT TYPES OF DATA.

RECORD TYPE 1 (DOPPLER)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I3	IHDR(1)	DATA TYPE (007)
2	A4	IHDR(2)	APL STATION NUMBER
3	I6	HHB(3)	APL SATELLITE NUMBER
4	I4	IHDR(3)	Q-PAIR
5	I3	IHDR(4)	OBSERVATION YEAR
6	I4	IHDR(5)	OBSERVATION DAY
7	I5	IHDR(6)	RISE TIME
8	6X		FIRST TIME POINT IN PASS
9	6X		LAST TIME POINT IN PASS
10	I7	IHDR(7)	DTC, STATION CLOCK ERROR
11	I9	IHDR(8)	D/C, FREQUENCY STANDARD CORRECTION
12	I6	IHDR(9)	NC, NUMBER OF BEATS COUNTED
13	3X		FREQUENCY AND TIME STANDARD ORIGIN
14	A1	MESS	DATA OR NARRATIVE INDICATOR

RAW OBSERVATION FILE (CONT.)

RECORD TYPE 2 (DOPPLER)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I5	DATA(1)	TIME OF 1ST OBSERVATION
2	I7	DATA(2)	OBSERVATION VALUE OF 1ST POINT
.	.	.	
.	.	.	
.	.	.	
* N	I5	DATA(N)	TIME OF (N+1)/2 OBSERVATION
N+1	I7	DATA(N+1)	OBSERVATION VALUE OF (N+1)/2 POINT
* N+2	I5	DATA(N+2)	TIME OF ITH TIME CORRECTION POINT,I=1,40
N+3	I7	DATA(N+3)	VALUE OF ITH TIME CORRECTION POINT,I=1,40
N+4	I5	DATA(N+4)	TIME OF (N+2)/2 OBSERVATION
N+5	I7	DATA(N+5)	OBSERVATION VALUE OF (N+2)/2 POINT
.	.	.	
.	.	.	
.	.	.	
** M	.	.	

* THE TIME CORRECTION POINTS OCCUR RANDOMLY WITHIN THE RECORD. IF THE EVEN DATA WORD IS LESS THAN 1000000, THE DATA PAIR IS A TIME CORRECTION POINT.

** M LESS THAN OR EQUAL 600

RECORD TYPE 2 MAY REPEAT FOR A PASS IF MORE THAN 300 POINTS ARE IN THE PASS

RECORD TYPE 3 (DOPPLER)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A6	LOGRCD	SIGNIFIES END OF PASS.

RAW OBSERVATION FILE (CONT.)

RECORD TYPE 1 (GEOCEIVER)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I3	IHDR (1)	DATA TYPE (008)
2	I4	HHB (2)	NSWC SATELLITE NUMBER
3	A5	IHDR (2)	STATION NUMBER
4	I4	IHDR (3)	Q-PAIR
5	I3	IHDR (4)	OBSERVATION YEAR
6	I4	IHDR (5)	OBSERVATION DAY
7	I3	IHDR (12)	TEMPERATURE (DEG KELVIN)
8	I2	IHDR (10)	NUMBER OF POINTS
9	I3	IHDR (7)	PRESSURE
10	I3	IHDR (8)	RELATIVE HUMIDITY
11	A1		FORMAT NUMBER
12	I1	IHDR (9)	MODE NUMBER
13	A1	IHDR (14)	COMMENT NUMBER
14	A1	MESS	DATA OR NARRATIVE INDICATOR

RECORD TYPE 2 (GEOCEIVER)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I2	DATA (1)	HOUR, TIME OF OBSERVATION
2	I2	DATA (2)	MINUTE
3	I2	DATA (3)	SECOND
4	O6	DATA (4)	FRACTION OF SECOND
5	O8	DATA (5)	DOPPLER COUNT
6	O4	DATA (6)	REFRACTION

•
•
•

ITEMS 1-6 ARE REPEATED FOR EACH OBSERATION FOR UP TO 50

RECORD TYPE 3

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A6	LOGRCD	SIGNIFIES END OF PASS

RAW OBSERVATION FILE (CONT.)

RECORD TYPE 1 (CCID DATA)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I3	IHDR(1)	DATA TYPE (009)
2	A4	IHDR(2)	APL STATION NUMBER
3	I6	HHB(3)	APL SATELLITE NUMBER
4	I4	IHDR(3)	Q-PAIR
5	I3	IHDR(4)	OBSERVATION YEAR
6	I4	IHDR(5)	OBSERVATION DAY
7	I5	IHDR(6)	RISE TIME
8	6X		FIRST TIME POINT IN PASS
9	6X		LAST TIME POINT IN PASS
10	I7	IHDR(7)	DTC, STATION CLOCK ERROR
11	I9	IHDR(8)	DVC, FREQUENCY STANDARD CORRECTION
12	I3	IHDR(9)	J, SUCH THAT 2**J = NUMBER OF BEATS
13	A1	MESS	DATA OR NARRATIVE INDICATOR

RECORD TYPE 2 (CCID DATA)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I5	DATA(1)	TIME OF 1ST OBSERVATION
2	I7	DATA(2)	OBSERVATION VALUE OF 1ST POINT
.	.	.	
* N	I5	DATA(N)	TIME OF (N+1)/2 OBSERVATION
N+1	I7	DATA(N+1)	OBSERVATION VALUE OF (N+1)/2 POINT
* N+2	I5	DATA(N+2)	TIME OF ITH TIME CORRECTION POINT,I=1,40
N+4	I5	DATA(N+4)	TIME OF (N+2)/2 OBSERVATION
N+5	I7	DATA(N+5)	OBSERVATION VALUE OF (N+2)/2 POINT
N+3	I7	DATA(N+3)	VALUE OF ITH TIME CORRECTION POINT,I=1,40
.	.	.	
** M	.	.	

* THE TIME CORRECTION POINTS OCCUR RANDOMLY WITHIN THE RECORD. IF THE ODD DATA WORD IS GREATER THAN 90000, THE DATA PAIR IS A TIME CORRECTION POINT.

RECORD TYPE 3 (CCID DATA)

ITEM NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A6	LOGRCD	SIGNIFIES END OF PASS.

RECORD TYPES 1-3 ARE REPEATED FOR EACH PASS OF DATA. A TAPE MAY CONTAIN ALL THREE TYPES OF DATA IN ANY ORDER.

SATELLITE DATA FILE

THE SATELLITE DATA FILE IS CREATED BY AN OFFLINE PROGRAM. IT CONTAINS GENERAL FACTS ABOUT ALL SATELLITES THAT ARE RUN. THE INPUTS SECTION OF CELEST LOCATES THE RECORD FOR THE SATELLITE OF THE CURRENT RUN AND STORES THEM IN COMMON /CDSAT/.

RECORD FORMAT

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	ISATAB(1)	APL SATELLITE NUMBER
2	I	ISATAB(2)	NSWC SATELLITE NUMBER
3	I	ISATAB(3)	SPASUR SATELLITE NUMBER
4	I	ISATAB(4)	SATELLITE NAME
5	F	SATAB(5)	SATELLITE OFFSET FREQUENCY
6	F	SATAB(6)	ONE BIT DELAY
7	F	SATAB(7)	DOPPLER MAXIMUM
8	F	SATAB(8)	DOPPLER MINIMUM
9	F	SATAB(9)	MINIMUM TIME CORRECTION POINT GAP.
10	I	ISATAB(10)	MAXIMUM NUMBER OF TIME CORRECTION POINTS
11	I	ISATAB(11)	SATELLITE KIND 1=NAV, 2=GEOS, 3=NTS
12	I	ISATAB(12)	NOT USED

INITIAL CONDITIONS FILE (TAPE4)

THE INITIAL CONDITIONS FILE CONTAINS THE CONDITIONS FOR SPECIFIED TIMES. SEVERAL SETS OF CONDITIONS MAY BE ON THE FILE FOR A GIVEN TIME. WHEN THE INITIAL CONDITIONS ARE READ IN BY THE INPTS SECTION OF CELEST, THEY ARE WRITTEN ON THE FILE, THEN AT THE END OF EACH IMPROVEMENT CYCLE, THE IMPROVED CONDITIONS FOR THE GIVEN EPOCH ARE WRITTEN ON THE FILE. WHEN THE LAST CYCLE OF IMPROVEMENT HAS BEEN DONE, PROPAGATED CONDITIONS FOR THE TIME AT WHICH THE NEXT FIT IS TO BE DONE, ARE WRITTEN ON THE FILE. BY PROPER USE OF THE INPUT (TO) CARD, INITIAL CONDITIONS MAY BE READ OFF AND/OR ACCUMULATED TO THIS FILE.

FORMAT OF INITIAL CONDITIONS FILE

RECORD NO. 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NWD	NUMBER OF WORDS IN RECORD MINUS TWO
2	I	NB	RECORD IDENTIFING NUMBER =2 ORBGEN OR INPUTS =3 BSOLVR
3	I	ID(1)	SATELLITE NUMBER
4	F	TO(1)	YEAR OF EPOCH
5	F	TO(2)	DAY OF EPOCH
6	F	TO(3)	SEC. OF EPOCH
7	F	OI(1)	X)
8	F	OI(2)	Y } - INERTIAL COMPONENTS OF SATELLITE
9	F	OI(3)	Z) POSITION (KM)
10	F	OI(4)	XDOT)
11	F	OI(5)	YDOT } - INERTIAL COMPONENTS OF
12	F	OI(6)	ZDOT) SATELLITE VELOCITY (KM/SEC)
13	F	OI(7)	CD1 - COEFFICIENT OF DRAG FOR 1ST DRAG SEG
14	F	OI(8)	CD2 - . . . . 2ND . .
15	F	OI(9)	CD3 - . . . . 3RD . .
16	F	OI(10)	CD4 . . . . 4TH . .
17	F	OI(11)	CD5 . . . . 5TH . .
18	F	OI(12)	CD6 . . . . 6TH . .
19	F	OI(13)	CD7 . . . . 7TH . .
20	F	OI(14)	CD8 . . . . 8TH . .
21	F	OI(15)	CD9 . . . . 9TH . .
22	F	OI(16)	CD10 . . . . 10TH . .
23	F	OI(17)	CD11 . . . . 11TH . .
24	F	OI(18)	CD12 . . . . 12TH . .
25	F	OI(19)	CD13 . . . . 13TH . .
26	F	OI(20)	CD14 . . . . 14TH . .

RECORD NO. 1 (CONT.)

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
27	F	OI(21)	CD15 . . . . 15TH . .
28	F	OI(22)	CD16 . . . . 16TH . .
29	F	OI(23)	CD17 . . . . 17TH . .
30	F	OI(24)	CD18 . . . . 18TH . .
31	F	OI(25)	CD19 . . . . 19TH . .
32	F	OI(26)	CD20 . . . . 20TH . .
33	F	OI(27)	AX1)
34	F	OI(28)	AY1 }-COMPONENTS OF 1ST THRUST(KM/SEC**2)
35	F	OI(29)	AZ1)
36	F	OI(30)	AX2)
37	F	OI(31)	AY2 }-COMPONENTS OF 2ND THRUST(KM/SEC**2)
38	F	OI(32)	AZ2)
39	F	OI(33)	AX3)
40	F	OI(34)	AY3 }-COMPONENTS OF 3RD THRUST(KM/SEC**2)
41	F	OI(35)	AZ3)
42	F	OI(36)	AX4)
43	F	OI(37)	AY4 }-COMPONENTS OF 4TH THRUST(KM/SEC**2)
44	F	OI(38)	AZ4)
45	F	OI(39)	KR - RADIATION PRESSURE COEFFICIENT
		NGPA	NO. OF GRAVITY PARAMETERS
		IDGD	GRAVITY ID ARRAY (32)
		DLPGP(I), I=1,NGPA	DELTA P GRAVITY

RECORDS OF THE ABOVE FORMAT ARE REPEATED FOR EACH SET OF CONDITIONS. THE NUMBER OF SETS OF CONDITIONS ACCUMULATED TO THE FILE DEPENDS UPON THE USER.

PASS MATRIX FILE (TAPE5)

THE PASS MATRIX FILE CONTAINS THE B-MATRIX FOR EACH PASS OF DATA INCLUDED IN THE SPAN OF THE TRAJECTORY USED FOR FILTERING THE DATA. ONLY THE ACCEPTED PASSES ARE WRITTEN ON THE PASS MATRIX FILE. RECORD TYPE 1 IS A HEADER RECORD FOR THE FILE AND APPEARS ONLY AS THE FIRST RECORD OF EACH FILE. INFORMATION IN THIS RECORD IS COMPARED WITH THE SAME TYPE INFORMATION FROM THE TRAJECTORY IN THE BSOLVR SECTION OF CELEST. IF THERE IS A DIFFERENCE IN INFORMATION, AN ERROR STOP OCCURS. RECORD TYPES 2 THRU 5 ARE REPEATED FOR EACH DATA PASS.

FORMAT OF PASS MATRIX FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TRAT(1)	YEAR OF EPOCH OF TRAJECTORY
2	F	TRAT(2)	DAY " " "
3	A	ITIME	TIME CLOCK VALUE WHEN TRAJECTORY WAS MADE
4	I	ID(1)	SATELLITE NUMBER
5	A	SAT(1)	SATELLITE NAME
6	A	SAT(2)	SATELLITE NAME (CONT)

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	IPASS	PASS NO. - EACH DATA PASS READ FROM THE OBS FILE IS COUNTED. IF PASSES ARE REJECTED THE PASS NOS. WILL NOT BE CONSECUTIVE.
2	F	YEAR	YEAR OF OBSERVATION PASS
3	F	DAY	DAY OF OBSERVATION PASS
4	A	ISTA	STATION NO. OF OBSERVING STATION
5	I	ND	DATA CLASS
6	I	ITYPE	DATA TYPE
7	I	NO	NUMBER OF OBSERVATIONS IN PASS
8	I	ITOLP	TOTAL NUMBER OF PARAMETERS IN B-MATRIX, INCLUDING BIASES
9	I	NOF	NO. OF ACCEPTED POINTS IN PASS
10	I	IPS	PASS STATUS =0,GOOD,NOT =0,BAD(ALWAYS 0,NOW)
11	O	PL	PARAMETER LABEL

RECORD TYPE 2 (CONT.)

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
12	F	V	VARIENCE
13	F	ELEV	ELEVATION AT TCA OF PASS (DEG)
14	F	PHIO	STATION LONGITUDE (DEG)
15	F	FNS	FILTERED NOISE
16	F	FSS	SATELLITE FREQUENCY
17	F	XO	EARTH-FIXED X COMPONENT OF STATION POSITION
18	F	YO	EARTH-FIXED Y COMPONENT OF STATION POSITION
19	F	ZO	EARTH-FIXED Z COMPONENT OF STATION POSITION

RECORD TYPE 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	BMAT	LOWER TRIANGULAR PART OF B-MATRIX FOR PASS
2	F	.	
.	.	.	
.	.	.	
*N	F	.	
N+1	F	EVEC(1)	E-VECTOR FOR PASS
N+2	F	.	
.	.	.	
.	.	.	
.	.	.	
** M	F	EVEC (ITOLP)	

WHERE N =(ITOLP(ITOLP+1))/2
**WHERE M = N+ITOLP

RECORD TYPE 4

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NOBIAS	NO. OF BIASES
2	I	IHRMIN	RISE TIME OF PASS
.	.	.	
.	.	.	

RECORD TYPE 5

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TCA	TIME OF CLOSEST APPROACH OF PASS(SEC. FROM MIDNIGHT)
2	F	RMAT(1,1)	ROTATION MATRIX TO ROTATE FROM INERTIAL TO EARTH-FIXED AT TCA.
3	F	RMAT(1,2)	
.	.	.	
.	.	.	
.	.	.	
10	F	RMAT(3,3)	

RECORD TYPES 2 THRU 5 ARE REPEATED FOR EACH DATA PASS

INERTIAL PERTURBED TRAJECTORY FILE (TAPE9)

THE INERTIAL PERTURBED TRAJECTORY FILE CONSISTS OF TWO RECORD TYPES - RECORD TYPE 1 BEING A HEADER RECORD APPEARING ONLY ONCE, AND RECORD TYPE 2 WHICH CONTAINS THE POSITION OF THE SATELLITE AND PARTIALS OF POSITION WITH RESPECT TO EACH PARAMETER. ONLY THE CANONICAL PARTIALS WRT DRAG AND THRUST APPEAR ON THE TRAJ. RECORD TYPE 2 IS REPEATED FOR EACH TIME LINE OF THE TRAJECTORY.

FORMAT OF INERTIAL PERTURBED TRAJECTORY FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	RECORD NO. = 1
2	F	TRAT(1)	YEAR OF EPOCH OF TRAJECTORY
3	F	TRAT(2)	DAY " " " "
4	F	TRAT(3)	SEC " " " "
5	F	TRAT(4)	TVE-TO, TIME OF VERNAL EQUINOX MINUS EPOCH OF THE TRAJECTORY
6	F	TRAT(5)	INTERVAL AT WHICH TRAJ IS WRITTEN (SEC)
7	F	TRAT(6)	LAST TIME ON TRAJECTORY (SEC FROM EPOCH)
8	F	TRAT(7)	INTEGRATION INTERVAL USED WHEN CREATING THE TRAJECTORY (SEC)
9	I	IFLOW	KIND OF TRAJECTORY = 6,7,4,OR 5.
10	I	ID(1)	SATELLITE NUMBER
11	I	KCG(10)	INDICATES IF TRAJECTORY WAS MADE USING EPOCH OF DATE=1, OR EPOCH OF 1950= 0
12	I	ID(3)	IMPROVEMENT CYCLE NUMBER
13	A	IDATE	TIME CLOCK VALUE WHEN TRAJECTORY WAS MADE
14	I	NPAR	NUMBER OF PARAMETERS
15	O	LABELP	PARAMETER LABEL WORD
16	I	NOPA	NO. OF DRAGS USED WHEN MAKING TRAJECTORY
17	I	NTPA	NO. OF THRUST USED WHEN MAKING THE TRAJ
18	F	DTIM(1)	END TIME OF 1ST DRAG SEGMENT (SEC FROM EPOCH)
19	F	DTIM(2)	" " " 2ND " " " " "
.	F	.	" " " " " " " "
.	F	.	" " " " " " " "
.	F	.	
* N	F	DTIMS(NDPA)	END TIME OF LAST DRAG SEGMENT

* WHERE N=NOPA+17 OR N=18, IF NOPA=0

RECORD NO. 1 (CONT.)

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
N+1	F	TTIMS(1,1)	START TIME OF 1ST THRUST SEGMENT
N+2	F	TTIMS(2,1)	END " " " " "
.	F	.	
.	F	.	
.	F	.	
.	F	.	
**M-1	F	TTIMS(1,L)	START TIME OF LAST THRUST SEGMENT
M	F	TTIMS(2,L)	END " " " " "
M+1	F	OI(1)	X)
M+2	F	OI(2)	Y }- INERTIAL COMPONENTS OF SATELLITE
M+3	F	OI(3)	Z) POSITION AT EPOCH (KM)
M+4	F	OI(4)	XDOT)
M+5	F	OI(5)	YDOT }- INERTIAL COMPONENTS OF SATELITE
M+6	F	OI(6)	ZDOT) VELOCITY AT EPOCH (KM/SEC)
M+7	F	OI(7)	CD1 - COEFFICIENT OF DRAG FOR 1ST DRAG SEG
M+8	F	OI(8)	CD2 - . . . . 2ND . .
M+9	F	OI(9)	CD3 - . . . . 3RD . .
M+10	F	OI(10)	CD4 . . . . 4TH . .
M+11	F	OI(11)	CD5 . . . . 5TH . .
M+12	F	OI(12)	CD6 . . . . 6TH . .
M+13	F	OI(13)	CD7 . . . . 7TH . .
M+14	F	OI(14)	CD8 . . . . 8TH . .
M+15	F	OI(15)	CD9 . . . . 9TH . .
M+16	F	OI(16)	CD10 . . . . 10TH . .
M+17	F	OI(17)	CD11 . . . . 11TH . .
M+18	F	OI(18)	CD12 . . . . 12TH . .
M+19	F	OI(19)	CD13 . . . . 13TH . .
M+20	F	OI(20)	CD14 . . . . 14TH . .
M+21	F	OI(21)	CD15 . . . . 15TH . .
M+22	F	OI(22)	CD16 . . . . 16TH . .
M+23	F	OI(23)	CD17 . . . . 17TH . .
M+24	F	OI(24)	CD18 . . . . 18TH . .
M+25	F	OI(25)	CD19 . . . . 19TH . .
M+26	F	OI(26)	CD20 . . . . 20TH . .
M+27	F	OI(27)	AX1)
M+28	F	OI(28)	AY1 }-COMPONENTS OF 1ST THRUST(KM/SEC**2)
M+29	F	OI(29)	AZ1)

** WHERE M= N+2*(NTPA) OR M=N+2, IF NTPA=0

RECORD NO. 1 (CONT.)

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
M+30	F	OI(30)	AX2)
M+31	F	OI(31)	AY2 }-COMPONENTS OF 2ND THRUST(KM/SEC**2)
M+32	F	OI(32)	AZ2)
M+33	F	OI(33)	AX3)
M+34	F	OI(34)	AY3 }-COMPONENTS OF 3RD THRUST(KM/SEC**2)
M+35	F	OI(35)	AZ3)
M+36	F	OI(36)	AX4)
M+37	F	OI(37)	AY4 }-COMPONENTS OF 4TH THRUST(KM/SEC**2)
M+38	F	OI(38)	AZ4)
M+39	F	OI(39)	KR - RADIATION PRESSURE COEFFICIENT
		NGPA	NO. OF GRAVITY PARAMETERS
		IDGD	GRAVITY ID ARRAY (32)
		DLPGP(I),	I=1,NGPA DELTA P GRAVITY

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	CONSECUTIVE RECORD NO.=2,3,4...N
2	F	TI	TI-TO (SECONDS FROM EPOCH)
3	F	TRA(2)	X)
4	F	TRA(3)	Y }- INERTIAL COMPONENTS OF SATELLITE
5	F	TRA(4)	Z) POSITION AT TIME (TI) (KM)
6	F	TRA(5)	PARTIAL X AT (TI) WRT PARAMETER (1) AT (TO)
7	F	TRA(6)	. Y . . . . (1) . .
8	F	TRA(7)	. Z . . . . (1) . .
9	F	TRA(8)	. X . . . . (2) . .
10	F	TRA(9)	. Y . . . . (2) . .
11	F	TRA(10)	. Z . . . . (2) . .
.	.	.	. . . .
.	.	.	. . . .
.	.	.	. . . .
N-2	F	TRA(M-2)	. X . . . . NPAR . .
N-1	F	TRA(M-1)	. Y . . . . NPAR . .
N	F	TRA(M)	. Z . . . . NPAR . .

WHERE NPAR = NO. OF PARAMETERS
= 6 ORBIT +MINO(1,NDPA)+3*MINO(1,NTPA)+MINO(1,NRPA)

NDPA=NO. OF DRAG SEGMENTS
NTPA=NO. OF THRUST SEGMENTS
NRPA=NO. OF RADIATION PRESSURE PARAMETERS (0 OR 1)
N= 5+3*NPAR
M= 4+3*NPAR

EARTH-FIXED TRAJECTORY FILE (TAPE9)

THE EARTH-FIXED TRAJECTORY FILE CONSISTS OF TWO RECORD TYPES - RECORD TYPE 1 BEING A HEADER RECORD APPEARING ONLY ONCE, AND RECORD TYPE 2 WHICH CONTAINS THE EARTH-FIXED POSITION AND VELOCITY OF THE SATELLITE. RECORD TYPE 2 IS REPEATED FOR EACH TIME LINE OF THE TRAJECTORY.

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION

SAME AS RECORD TYPE 1 OF INERTIAL PERTURBED TRAJECTORY FILE.

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	CONSECUTIVE RECORD NO.=2,3,4...N
2	F	TI	TI-T0 (SECONDS FROM EPOCH)
3	F	XROTAT(1)	X)
4	F	XROTAT(2)	Y }- EARTH-FIXED COMPONENTS OF SATELLITE
5	F	XROTAT(3)	Z) POSITION AT TIME (TI) (KM)
6	F	VROTAT(1)	XDOT)
7	F	VROTAT(2)	YDOT }- EARTH-FIXED COMPONENTS OF SATELLITE
8	F	VROTAT(3)	ZDOT) VELOCITY AT TIME (TI) (KM/SEC)

INERTIAL STANDARD TRAJECTORY FILE (TAPE9)

THE INERTIAL STANDARD TRAJECTORY FILE CONSISTS OF TWO RECORD TYPES - RECORD TYPE 1 BEING A HEADER RECORD APPEARING ONLY ONCE, AND RECORD TYPE 2 WHICH CONTAINS THE POSITION AND VELOCITY OF THE SATELLITE. RECORD TYPE 2 IS REPEATED FOR EACH TIME LINE OF THE TRAJECTORY.

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION

SAME AS RECORD TYPE 1 OF INERTIAL PERTURBED TRAJECTORY FILE.

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	CONSECUTIVE RECORD NO. =2,3,4,...N
2	F	TI	TI-T0 (SECONDS FROM EPOCH)
3	F	X (1)	X)
4	F	X (2)	Y }- INERTIAL COMPONENTS OF SATELLITE
5	F	X (3)	Z) POSITION AT TIME (TI) (KM)
6	F	XD (1)	XDOT)
7	F	XD (2)	YDOT }- INERTIAL COMPONENTS OF SATELLITE
8	F	XD (3)	ZDOT) VELOCITY AT TIME (TI) (KM/SEC)

PROPAGATED TRAJECTORY FILE (EARTH-FIXED OR INERTIAL) (TAPE51)

THE PROPAGATED TRAJECTORY FILE MAY CONSIST OF ONE OR SEVERAL TRAJECTORIES DEPENDING UPON THE RUNNING OPTION SELECTED. IF RUNNING IN THE SHORT ARC MODE, THE TRAJECTORIES FOR EACH SHORT ARC ARE ACCUMULATED TO THE FILE WITH A RECORD CONTAINING A NEGATIVE RECORD NO. SEPARATING THE TRAJECTORIES. THE FILE CONTAINS TWO RECORD TYPES - TYPE 1 BEING A HEADER FOR EACH TRAJECTORY AND APPEARING ONLY ONCE FOR EACH TRAJECTORY, AND RECORD TYPE 2 CONTAINING THE POSITION AND VELOCITY OF THE SATELLITE FOR A GIVEN TIME. RECORD TYPE 2 IS REPEATED FOR EACH TIME LINE FOR EACH TRAJECTORY. THE LAST RECORD OF A TRAJECTORY IS A RECORD OF TYPE 2 CONTAINING A NEGATIVE VALUE FOR THE CONSECUTIVE RECORD NO.

FORMAT OF PROPAGATED TRAJECTORY FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	RECORD NO. = 1
2	F	TRAT(1)	YEAR OF EPOCH OF TRAJECTORY
3	F	TRAT(2)	DAY " " " "
4	F	TRAT(3)	SEC " " " "
5	F	TRAT(4)	TVE-TO, TIME OF VERNAL EQUINOX MINUS EPOCH OF THE TRAJECTORY
6	F	TRAT(5)	INTERVAL AT WHICH TRAJ IS WRITTEN (SEC)
7	F	TRAT(6)	LAST TIME ON TRAJECTORY(SEC FROM EPOCH)
8	F	TRAT(7)	INTEGRATION INTERVAL USED WHEN CREATING THE TRAJECTORY
9	I	IFLOW	KIND OF TRAJECTORY = 10, 11, 12,OR 13
10	I	ID(1)	SATELLITE NUMBER
11	I	KCG(10)	INDICATES IF TRAJECTORY WAS MADE USING EPOCH OF DATE=1, OR EPOCH OF 1950= 0
12	I	ID(3)	IMPROVEMENT CYCLE NUMBER
13	A	IDATE	TIME CLOCK VALUE WHEN TRAJECTORY WAS MADE
14	I	NREV	REVOLUTION NUMBER OF SATELLITE
15	I	NONE	DUMMY WORD

PROPGATED TRAJECTORY FILE (CONT)

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NB	CONSECUTIVE RECORD NO.=2,3,4...N
2	F	TI	TI-T0 (SECONDS FROM EPOCH)
3	F	TRA(2)	X) EARTH-FIXED OR INERTIAL COMPONENTS OF
4	F	TRA(3)	Y }- SATELLITE POSITION AT TIME (TI)
5	F	TRA(4)	Z)
6	F	TRA(5)	XDOT)
7	F	TRA(6)	YDOT }-EARTH-FIXED OR INERTIAL COMPONENTS
8	F	TRA(7)	ZDOT) OF SATELLITE VELOCITY AT TIME (TI)

TIME CORRECTED OBSERVATION FILE (TAPE14)

THE TIME CORRECTED OBSERVATION IS CREATED IN THE DATA PREP SECTION OF CELEST TO BE USED BY THE FILTER SECTION. THE FILE CONSISTS OF THREE RECORD TYPES. RECORD TYPE 1 IS A FILE HEADER AND APPEARS ONLY ONCE ON THE FILE. RECORD TYPE 2 IS A PASS HEADER CONTAINING INFORMATION PERTINENT TO THE PASS, AND RECORD TYPE 3 CONTAINS THE OBSERVATIONAL DATA FOR THE PASS. RECORD TYPE 2 AND 3 ARE REPEATED FOR A COLLECTION OF PASSES OVER SEVERAL OBSERVING STATIONS.

FORMAT OF TIME CORRECTED OBSERVATION FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	A	WORD	FILE HEADER WORD (OBS FILE)
2	I	ISAT	SATELLITE NUMBER (NSWC)

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	YEAR	YEAR OF OBSERVATIONS
2	F	DAY	DAY OF OBSERVATIONS
3	F	TTCA	PREDICTED TCA VALUE (SEC. FROM MIDNIGHT)
4	A	ISTA	OBSERVING STATION NUMBER
5	I	ICLAS	CLASS OF DATA
6	I	ITYPE	TYPE OF DATA
7	F	SPARE	NOT USED
8	I	NO	NUMBER OF OBSERVATIONS
9	I	NBI	UNUSED WORD (ORIGINALLY NO. OF BIASES)
10	F	TPHM(1)	TEMPERATURE
11	F	TPHM(2)	PRESSURE
12	F	TPHM(3)	HUMIDITY
13	I	ITI	UNUSED WORD INDICATES TIME IS (EMISSION OR RECEPTION)
14	I	IF	INDICATES IF PASS HAS BEEN FILTERED=0,NO
15	I	IP	PASS STATUS,IF=0,GOOD PASS

RECORD TYPE 2 (CONT.)

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
16	I	IQPR	Q-NUMBER
17	F	XLAM	STATION LONGITUDE (DEG)
18	F	PHID	STATION LATITUDE (DEG)
19	F	ALT	STATION ALTITUDE (KM)
*20	F	FS	SATELLITE FREQUENCY
21	F	DUM(1)	
22	F	DUM(2)	
23	F	DUM(3)	
24	F	DUM(4)	
THE FOLLOWING APPEAR ONLY ON THE UN-TIME CORRECTED FILE.			
25	I	NTC	NUMBER OF TIME CORRECTION WORDS PASSED FROM NAVIGATION SATELLITE (NTC .NE. 0)
26	F	TC(I)	I=1T.NTC (TIME CORRECTIONS)

RECORD TYPE 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	DA(I,1)	TIME OF 1ST OBSERVATION (SEC. FROM MIDNIGHT)
2	F	DA(I,2)	OBSERVATION VALUE
3	F	DA(I,3)	SIGMA FOR OBSERVATION
4	I	DA(I,4)	OBSERVATION TAG, IF=0, GOOD OBSERVATION
.	.	.	
.	.	.	

WORDS 1 THRU 4 ARE REPEATED FOR THE NO. OF OBSERVATIONS IN PASS

* FS = 4.E+08 FOR NAVIGATION SATELLITE
* FS = 3.24E+08 FOR GEODETIC SATELLITE

UN-TIME CORRECTED OBSERVATION FILE (TAPE8)

THE UN-TIME CORRECTED OBSERVATION FILE IS CREATED IN THE DATA PREP SECTION OF CELEST AND IS ONLY USED INTERNALLY WITHIN THE DATA PREP SECTION. IT IS INPUT TO THE SUBPROGRAM OPB WHICH PRODUCES THE TIME CORRECTED OBSERVATION FILE. THE FORMAT OF THE UN-TIME CORRECTED FILE IS IDENTICAL TO THE FORMAT OF THE TIME CORRECTED FILE ABOVE, WITH THE EXCEPTION OF WORDS 25 AND 26.

DIAGNOSTIC INFORMATION FILE (TAPE16)

THE DIAGNOSTIC INFORMATION FILE IS CREATED BY THE COVAR SECTION OF CELEST TO BE USED BY THE GRAPHICS OR POST ANALYSIS SECTION FOR CELEST. IT CONTAINS STATISTICAL DATA WHICH IS USED AS A MEASURE OF HOW GOOD THE FIT OVER A PARTICULAR SPAN IS. IT CONSISTS OF FOUR RECORD TYPES. RECORD TYPES 1,2, AND 4 APPEAR ONLY ONCE FOR A REV, BUT ARE REPEATED IF MORE THAN ONE REV IS PROCESSED. RECORD TYPE 3 IS REPEATED FOR EACH OUTPUT DELTA T DURING THE SPAN OF THE REV.

FORMAT OF DIAGNOSTIC INFORMATION FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TO(1)	YEAR OF EPOCH OF REV
2	F	TO(2)	DAY . . . .
3	F	TO(3)	SEC. . . . .
4	I	ID(1)	SATELLITE NUMBER
5	I	CS	SCALE FACTOR FOR COVARIANCE
6	I	NREV	REV NUMBER OF SATELLITE
7	F	TRAT(1)	YEAR OF EPOCH OF PERT TRAJ (LONG ARC)
8	F	TRAT(2)	DAY . . . .
9	F	TRAT(3)	SEC. . . . .
10	F	TOLT	OUTPUT DELTA T FOR EARTH-FIXED VALUES
11	I	MT	MULTIPLE OF TOLT TO OUTPUT INERTIAL VALUES

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TLT	LAST TIME FOR WHICH RECORD TYPE 3 IS WRITTEN (SEC. FROM REV EPOCH)

RECORD TYPE 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	TIM	TIME IN SEC. FROM REV EPOCH
2	F	BR(1,1))	CONFIDENCE IN THE TRAJECTORY IN THE
3	F	BR(2,2) }-	RADIAL, TANGENTIAL, AND OUT OF PLANE (RXV)
4	F	BR(3,3))	DIRECTION AT TIME (TIM)

RECORD TYPE 3 IS REPEATED FOR EACH OUTPUT TIME IN REV SPAN.

THE LAST TYPE 3 RECORD CONTAINS FOUR WORDS OF ZERO VALUE.

RECORD TYPE 4

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	SIG2R	AVERAGE SIGMA RADIAL FOR ENTIRE REV
2	F	SIG2V	AVERAGE SIGMA TANGENTIAL FOR ENTIRE REV
3	F	S2RXV	AVERAGE SIGMA (RXV) FOR ENTIRE REV

SHORT ARC EPHEMERES FILE (TAPE31)

THE SHORT ARC EPHEMERES FILE IS CREATED BY THE COVAR SECTION OF CELEST. IT IS THEN USED AS INPUT TO THE COVAR LIBRARY PROGRAM WHICH MAY ADD THE INFORMATION TO A MASTER SHORT ARC EPHEMERES FILE, SORT THE INFORMATION BY REV NUMBERS, AND/OR EXTRACT SELECTED REVS AND WRITE THE INFORMATION TO ANOTHER FILE IN BCD FORMAT. THE FILE CONSISTS OF TWELVE RECORD TYPES. RECORD TYPES 1 THRU 4 ARE HEADER RECORDS AND APPEAR ONLY ONCE FOR EACH REV. RECORD TYPES 5 THRU 11 CONTAIN THE EPHEMERIS AND COVARIANCE VALUES FOR EACH OUTPUT TIME IN THE REV SPAN.

FORMAT OF SHORT ARC EPHEMERES FILE

RECORD TYPE 1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NEG	NEGATIVE WORD INDICATING FIRST RECORD OF OUTPUT FOR ANOTHER REV
2	I	IOPC	NUMBER IDENTIFING SET OF OUTPUT RECORDS
3	F	TRAT(1)	YEAR OF EPOCH ON PERTURBED TRAJECTORY
4	F	TRAT(2)	DAY . . . . .
5	F	TRAT(3)	SEC . . . . .
6	F	TRAT(4)	TVE-TO, TIME OF VERNAL EQUINOX MINUS EPOCH OF THE TRAJECTORY
7	F	TRAT(5)	INTERVAL AT WHICH PERT TRAJ WAS WRITTEN
8	F	TRAT(6)	LAST TIME ON PERT TRAJ (SEC. FROM EPOCH)
9	F	TRAT(7)	INTEGRATION INTERVAL USED WHEN CREATING THE PERT TRAJ REVS PROCESSED

WORDS 3 THRU 9 ARE DUMMY WORDS TO MAKE RECORD TYPE 1 THE SAME LENGTH AS RECORD TYPE 2

RECORD TYPE 2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD	IDENTIFIES RECORD AS HEADER TYPE = 1
2	I	NCRD	SUB IDENTIFICATION OF HEADER = 1
3	I	ID2	IDENTIFING REV NO. FOR RUN (SATELLITE REV NO. MINUS INITIAL REV. NO FOR RUN PLUS 1)
4	F	TC(1)	YEAR OF EPOCH OF REV
5	F	TC(2)	DAY OF EPOCH OF REV
6	F	TC(3)	SEC OF EPOCH OF REV
7	F	TDLT	DELTA T AT WHICH EARTH-FIXED EPHEMERIS AND COVARIANCE IS WRITTEN
8	F	DM	MULTIPLE OF TDLT FOR WHICH INERTIAL EPHEMERIS AND COVARIANCE IS WRITTEN
9	F	DN	NO. OF EARTH-FIXED OUTPUT TIMES IN THE REV SPAN

RECORD TYPE 3

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD	IDENTIFIES RECORD AS HEADER TYPE = 1
2	I	NCRD	SUB IDENTIFICATION OF HEADER = 2
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	DI	SATELLITE NUMBER
5	F	TC(1)	YEAR OF EPOCH OF REV
6	F	TC(2)	DAY OF EPOCH OF REV
7	F	DUME	DUMMY WORD = 0
8	F	DUMD	DUMMY WORD = 0
9	F	AK	NUMBER OF RECORD TYPE 4"S TO FOLLOW = 1

RECORD TYPE 4

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD	IDENTIFIES RECORD AS HEADER TYPE = 1
2	I	NCRD	SUB IDENTIFICATION OF HEADER = 3
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	A	REVNO	HOLLERITH WORD (9H REV NO.)
5	I	NREV	REV NO. OF SATELLITE
6	A	ALPHA(1)	AN ALPHA MESSAGE TO IDENTIFY COVARIANCE OUTPUT. USER INPUTS MESSAGE ON INPUT CARD.
.	.	.	
.	.	.	
.	.	.	
11	A	ALPHA(6)	

RECORD TYPE 5

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=2	IDENTIFIES RECORD AS EARTH-FIXED EPHEMERIS
2	F	TA	TI-TJ, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	XE(1))	
5	F	XE(2) }-	PROPAGATED EARTH-FIXED COMPONENTS OF
6	F	XE(3))	SATELLITE POSITION AT TIME (TA)
7	F	XE(4))	
8	F	XE(5) }-	PROPAGATED EARTH-FIXED COMPONENTS OF
9	F	XE(6))	SATELLITE VELOCITY AT TIME (TA)

RECORD TYPE 6

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=3	IDENTIFIES RECORD AS EARTH-FIXED COVARIANCE
2	F	TA	TI-TO, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	C(1,1))	
5	F	C(2,1))	
6	F	C(2,2) }-	POSITION PORTION (LOWER TRIANGULAR PORTION
7	F	C(3,1))	ONLY) OF EARTH-FIXED PROPAGATED COVARIANCE
8	F	C(3,2))	MATRIX AT TIME (TA)
9	F	C(3,3))	

RECORD TYPES 7 THRU 11 ARE WRITTEN ONLY WHEN THE OUTPUT TIME IS A MULTIPLE OF THE OUTPUT DELTA T AS SPECIFIED BY INPUT (MT)

RECORD TYPE 7

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=4	IDENTIFIES RECORD AS INERTIAL EPHEMERIS
2	F	TA	TI-TO, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	X(1))	
5	F	X(2) }-	PROPAGATED INERTIAL COMPONENTS OF SATELLITE
6	F	X(3))	POSITION AT TIME (TA)
7	F	X(4))	
8	F	X(5) }-	PROPAGATED INERTIAL COMPONENTS OF SATELLITE
9	F	X(6))	VELOCITY AT TIME (TA)

RECORD TYPES 8 THRU 11 CONTAIN THE LOWER TRIANGULAR PORTION OF THE INERTIAL PROPAGATED COVARIANCE MATRIX AT TIME (TA)

RECORD TYPE 8

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=5	IDENTIFIES RECORD AS INERTIAL COVARIANCE
2	F	TA	TI-T0, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	B0(1,1)	
5	F	B0(2,1)	
6	F	B0(2,2)	
7	F	B0(3,1)	
8	F	B0(3,2)	
9	F	B0(3,3)	

RECORD TYPE 9

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=6	IDENTIFIES RECORD AS INERTIAL COVARIANCE
2	F	TA	TI-T0, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	B0(4,1)	
5	F	B0(4,2)	
6	F	B0(4,3)	
7	F	B0(4,4)	
8	F	B0(5,1)	
9	F	B0(5,2)	

RECORD TYPE 10

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=7	IDENTIFIES RECORD AS INERTIAL COVARIANCE
2	F	TA	TI-T0, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	B0(5,3)	
5	F	B0(5,4)	
6	F	B0(5,5)	
7	F	B0(6,1)	
8	F	B0(6,2)	
9	F	B0(6,3)	

RECORD TYPE 11

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	I	NCARD=8	IDENTIFIES RECORD AS INERTIAL COVARIANCE
2	F	TA	TI-T0, SECONDS FROM REV EPOCH
3	I	ID2	IDENTIFING REV NO. FOR RUN
4	F	B0(6,4)	
5	F	B0(6,5)	
6	F	B0(6,6)	

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