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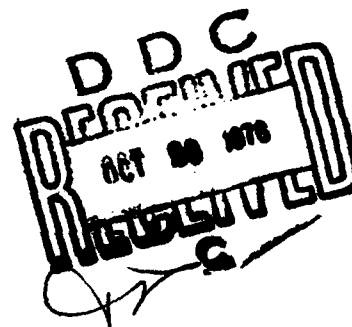
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A Circularly Polarized Yagi Antenna System for NTS-1 and NTS-2 Ground Stations

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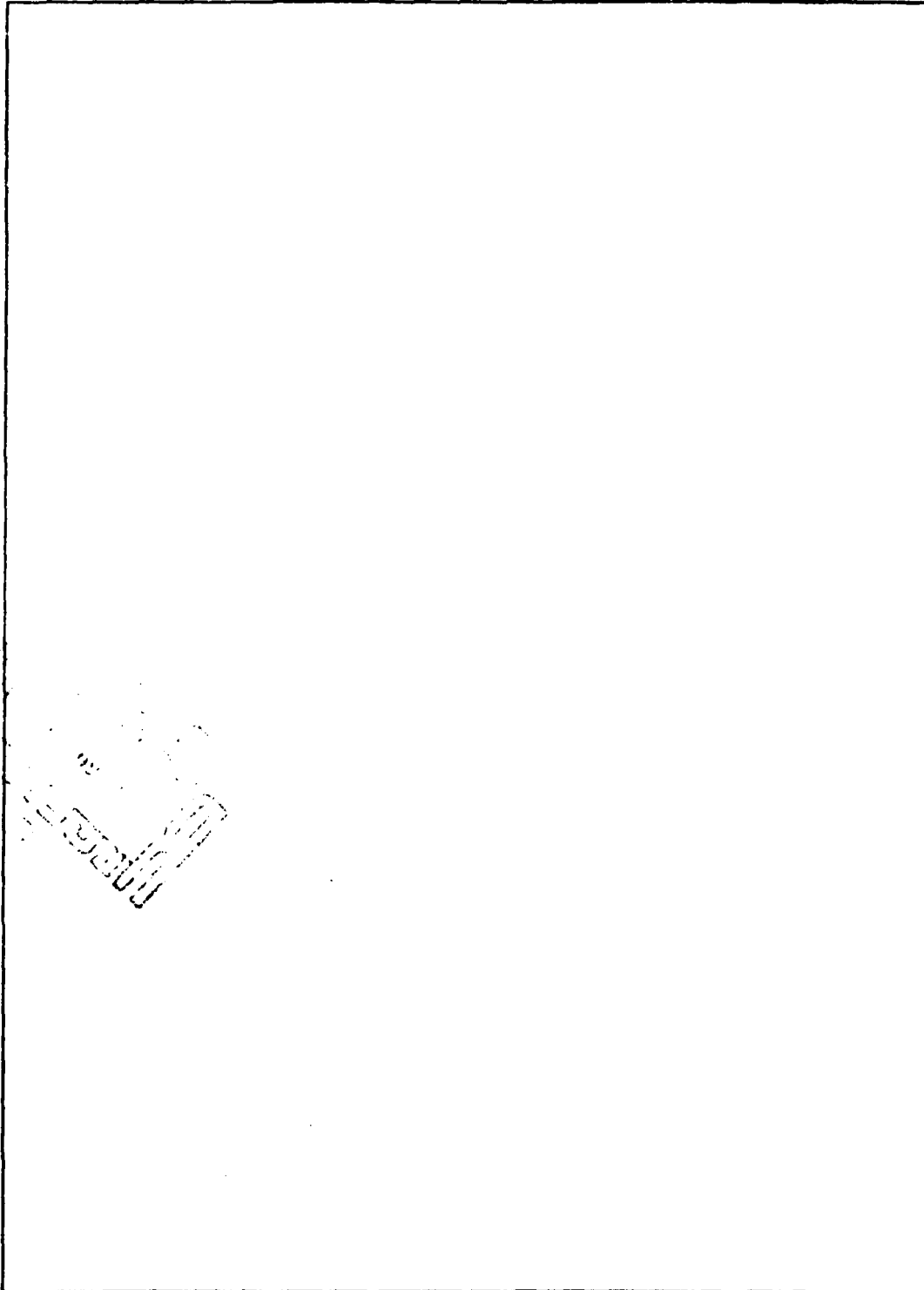
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A Circularly Polarized Yagi Antenna System
for
NTS-1 and NTS-2 Ground Stations

A. Introduction

While the NTS-1 and NTS-2 satellites are designed for users having hemispherical antenna patterns, there are many performance improvements that can be gained most easily by the use of a small antenna gain (10 dB) at the ground stations. The signal variation caused by Faraday rotation can be minimized by circular polarization. Accordingly two 10.5 foot yagi antennas were crossed to form a circular polarized device having a gain of at least 10 dB at 335 MHz.

B. Background

Previous experience with the original single yagi showed that the fed-dipole impedance was reduced to approximately 20-30 ohms due to other-element proximity and spacing. The coupling between the crossed yagis is minimal, and therefore the fed dipole impedance of both yagis remains around 20-30 ohms. However, now, both feeds are in parallel but not before one is connected through a 90 degree ($\lambda/4$ length) phase-delay line. This line, if not matched or not an exact $\lambda/4$, can give various degrees of connecting impedances versus frequency within the band of operation. The system feed was chosen to be the standard 50 ohms. Each fed-dipole was designed to present about 100 ohms so that when both are combined, the system impedance would match a 50 ohm line. The $\lambda/4$ phase delay line to one yagi was made of 93 ohm line, the closest commercially available impedance line. Thus, some slight mismatch was inherent at center design frequency.

The original single yagi system employed folded dipole impedance matching techniques. The crossed-yagi system employs a gamma match transformation in the fed-dipole with provision for adjusting rod-length and series capacity tune out of rod inductance. This facilitates fine tuning of the impedance in final assembly, measurements, and adjustments.

Note: Manuscript submitted August 25, 1976.

C. Description

The two identical yagis are described best by the sketches of Figure 1, 2 and 3. The boom length in Figure 1 is 10 ft - 5 inches and 1.25" (od) diameter. The element stock is 1/4 inch rod except for the driven elements (2, 2') which are 3/4 inch diameter. The prime numbered elements are crossed at 90° with the unprimed ones and are spaced 1.125 inches forward of their corresponding unprimed elements. Elements 1 and 1' = 17.82" long.

2 and 2' = 16.50" long. plus end discs

(Fig. 3)

3-13 and 3'-13' = 15.84" long.

Element spacing from end of boom is:

1	1 inch
2	9.25
3	12.25
4	15.5
5	18.75
6	25.75
7	39.57
8	53.43
9	67.3
10	81.15
11	95.00
12	108.87
13	122.75

The primed elements are spaced 1.125" forward.

D. Circularity

By definition, right hand circularity is that which rotates clockwise while looking in the direction of an outgoing signal or transmitted wave. Since the satellites transmit right hand circularly polarized waves (r.h.c.) the ground-based antennas must be r.h.c. To obtain r.h.c. the 90 degree phase lag must be applied to the dipole whose corresponding voltage end is to the right as viewed from the rear of the yagi. Thus the quarter wavelength delay line is applied to that dipole.

E. Impedance Characteristics

The impedance of a physical $\lambda/2$ dipole of finite thickness in free space will run $65-70\Omega + j30 - 40\Omega$ and may vary over a band as shown in Figures 4 and 5. These graphs are approximate at best and are extrapolated from the reference curves so marked. The reference curves are for a free-space dipole, but when associated with surrounding parasitic elements of various spacings, the dipole impedances may not be so reliably "straight-lined" vs.

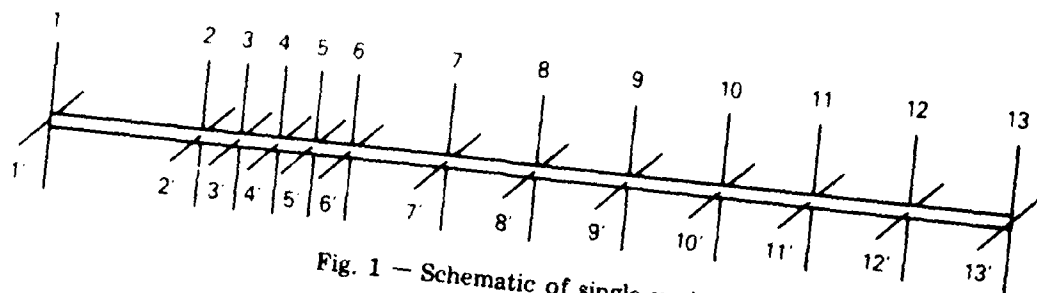


Fig. 1 — Schematic of single yagi

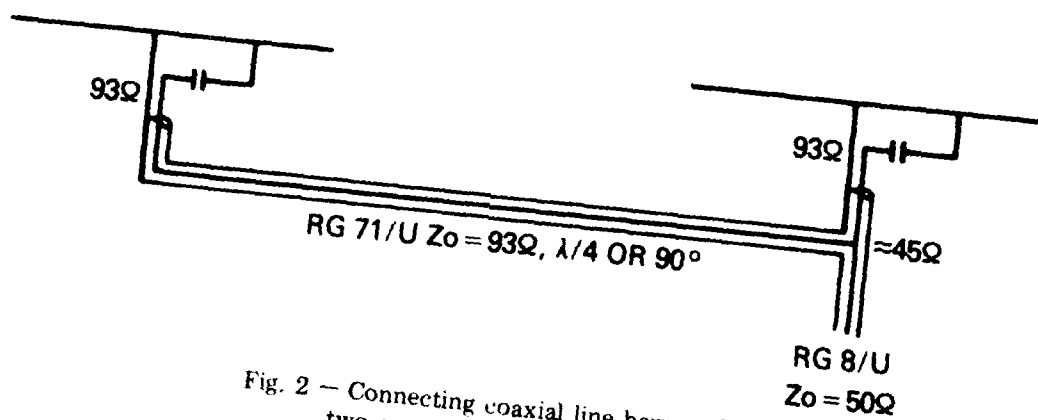


Fig. 2 — Connecting coaxial line harness for two crossed yagi fed dipoles

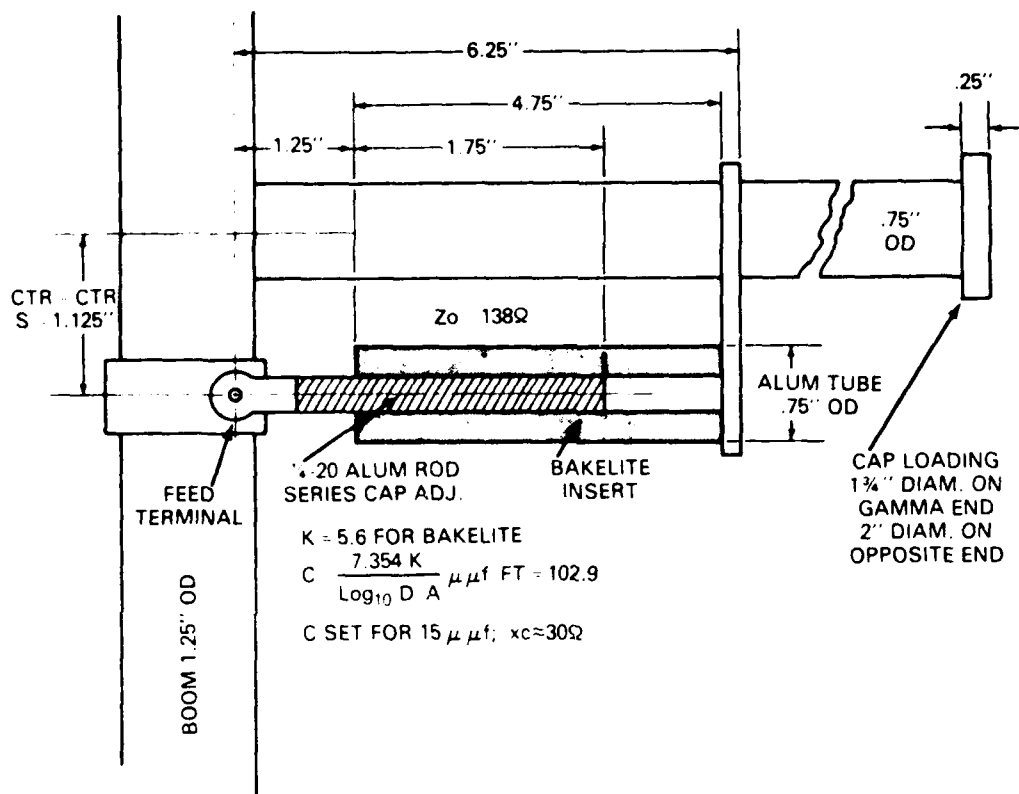


Fig. 3 — Dimensional schematic of gamma match elements

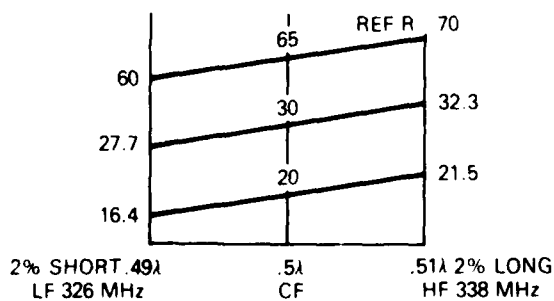


Fig. 4 — Dipole resistance characteristics vs bandwidth and other element effects

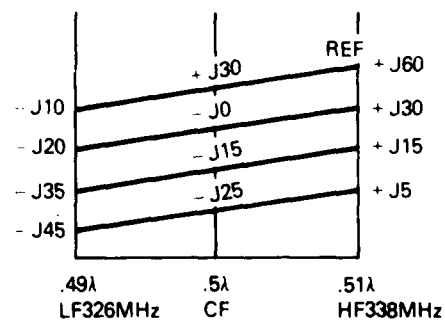


Fig. 5 — Dipole reactance characteristics under same conditions as in Fig. 4

frequency. Such complications may alter the agreement between calculated and measured impedance characteristics. In the impedance design calculations, the "straight-lines" dipole impedances were used. Therefore, the designed values may differ over the band from the measured values. The design values should be fairly close to the actual at or near center frequency. Since the dipole rods were manufactured to be 16.50" long and the half-wave resonant free-space length is 17.785 inches at 332 MHz., they were obviously too short by a considerable amount. This was confirmed in measurements. The reactive component of the feed impedance would have been in the vicinity of $-j70$ at center frequency instead of a desired $-j25$. Therefore the resonance was lowered by adding the discs at the ends of the two dipole driven elements.

The length of the dipole, due to length-to-diameter ratio, where $l/d = 17.785 \text{ inches}/.75 = 24$, should be reduced to the value $K(1/2) = .95 (17.785") = 16.89 \text{ inches}$. Thus the original dipole lengths of 16.50" were short by almost .4 inch and probably an additional amount shorter because of the paralleling of the gamma rod on a portion of each dipole stub.

Every indication was that a lowering of the dipole resonance was necessary by additions to the dipole lengths.

The use of Figures 5 and 6 demonstrate the impedance variation over a + 2% band and how the matching situation changes with frequency.

The capacity plates on the dipole ends are for optimizing the reactance term of the feed impedance of each dipole for full band coverage. The diameters of the loading discs differ on opposite ends of the dipoles to compensate for unequal loading due to the gamma rod hardware.

F. Impedance Calculations

The calculations will not be exact because, as previously mentioned, the "straight-line" adjusted values of $R \pm jx$ of Figures 4 and 5 were assumed. Almost certainly, a complicated set of curves would show if the numerous mutuals of all the parasitic elements were taken into account. The calculated value at center frequency (CF), most likely, will be a close "ball park" value with this assumption. In practice, the actual reactance can be tuned out to a desirable value at CF to give the desired results.

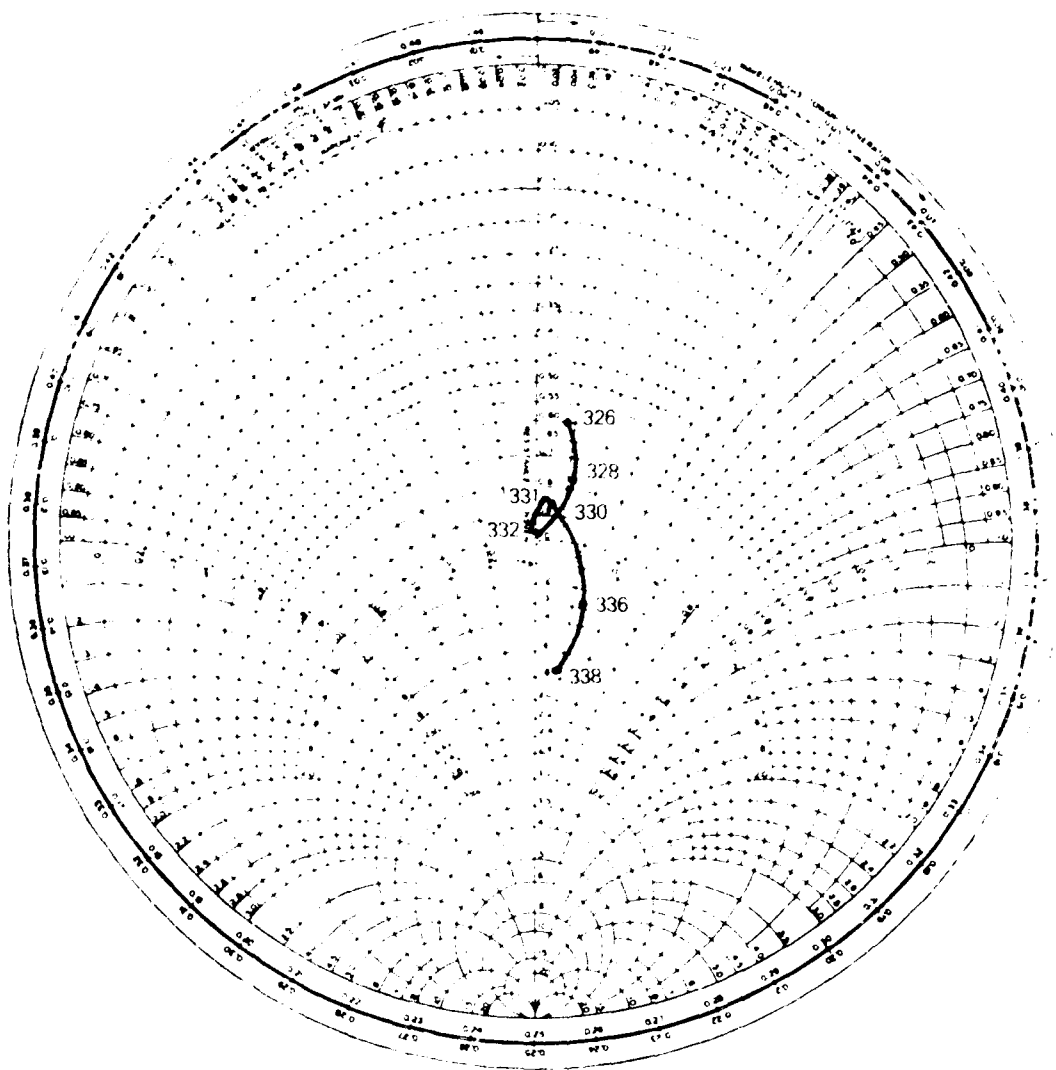


Fig. 6 — Calculated impedance for crossed yagi system

Assuming the fed-dipole impedance was adjusted to be capacitive or $-j25\Omega$, $Z_{\text{Feed}} = 20 - j25\Omega$ after all effects of other elements and dipole loading are combined at the feed point. Now the problem is to design a gamma-rod feed to transform this impedance to $93\Omega + j0\Omega$. The procedure which was chosen is described below. The appendix gives actual design figures. The center frequency of operating design is $f_0 = 332 \text{ MHz}$.

Step 1) As in a folded dipole, assume a diameter and rod-to-dipole spacing of, say, 4:1 impedance transformation ratio.

Step 2) The characteristic impedance of the gamma line is:

$$Z_0 = 276 \log \frac{2S}{d_1 d_2} = 132\Omega \text{ where: } S = \text{Ctr-Ctr line spacing} \\ d_1 d_2 = \text{line diameters}$$

Step 3) Choose a gamma rod length of 3.6 inches or $\emptyset = .101\lambda_0 = 36.4^\circ$ at $f_0 = 332 \text{ MHz}$; $\lambda_0 = 90.36 \text{ cm} = 35.57 \text{ inch}$.

Step 4) The impedance of the dipole where the gamma-rod attaches is:

$$Z_2 = Z_1 / \cos^2 \emptyset \text{ where } Z_1 \text{ is the impedance of the dipole center due to other element proximity, assumed here to be } R - jX = 20\Omega - j25\Omega.$$

Step 5) Find the load impedance at the antenna end of the gamma line or Z_2 times the factor of Step (1). Then normalize to Z_0 of the Rod line which in this case is 132Ω .

Step 6) Using a Smith Chart, transform the impedance of Step (5) to the feed terminal or $.101\lambda$ toward the generator.

Step 7) The inductive reactance of the rod itself, at f_0 is $X_p = j \tan \emptyset$.

Step 8) Using a Smith Chart, invert the load impedance of Step (6) to obtain admittance or $Y = G + jB$.

Step 9) Also invert the inductive reactance, X_p , of Step (7) to get $Y = G + jB$ and add together the Y's of Step (8) and Step (9) to get Y_T .

Step 10a) Using a Smith Chart, invert Y_T of Step (9) to get total impedance, Z_{T132} .

Step 10b) Convert the Z_{T132} to actual ohms by multiplying $(Z_{T132})(Z_0=132) = Z_{\text{actual}}$.

Now, one has the actual feed terminal impedance of each "identical" yagi without any tune-out of gamma-rod inductive reactance. This can be tuned out by the series capacity made up of the concentric threaded rod capacitor shown in Fig. 3, where, for the dimensions shown, is about 15 pico-farads presenting $X_C \approx 31\Omega$. This negative reactance, combined with the Z_{actual} of Step (10b) gives a new or adjusted actual feed terminal impedance of each identical yagi.

G. Combining the Two Yagis

Step 1) Normalize the Z adjusted of each yagi to $Z_0=93$ ohm which is the Z_0 of the 90 degree phase-shifter line.

Step 2) Combine both Yagis at the grand junction by, first, calculating the transformed impedance of one yagi through the $\lambda/4$ phase shifter section. Change this to $Y=G+jB$. The untransformed yagi impedance is also changed to admittance so that both can be added in parallel at the grand junction or system terminal.

It should be realized that $\lambda/4$ transformation of the one yagi impedance is essentially the equivalent of obtaining $Y=G+jB$ and the impedance of one can be considered the equivalent of the admittance. Therefore only one must be transformed and then both added (for identical yagis).

Step 3) Add both admittances at the grand terminal. Using the Smith Chart, invert this total admittance to a total feed impedance normalized to $Z_0=93\Omega$.

Step 4) The actual system terminal impedance is that of Step (3) times $Z_0=93\Omega$.

Step 5) The actual combined feed impedance of Step (4) may be divided by $Z_0=50\Omega$ of the main feed line to normalize it to RG8/U - 50 ohm line to give a Smith Chart impedance plot and to determine VSWR.

H. Calculated Results

A sample calculation of feed impedance is found in the appendix for certain assumed dipole feed characteristics as explained earlier. Again, due to these assumptions, the impedance agreement with the actual measured values may be close at f_o but are somewhat unpredictable away from f_o .

Table 1 and Figure 6 give calculated VSWR and impedance values vs frequency. Comparison of impedance in Figure 6 at $CF=f_o=332$ MHz shows a close agreement with the measured value in Figure 9 but a wide disagreement elsewhere. However the measured and calculated show acceptable values over the band.

Figure (7a,b) shows additional transmission line loss for 250 feet of RG 9/U cable. Thus a VSWR of 2:1 would add about 1 dB loss to line loss of 10 dB for 250 feet.

I. Measured Antenna Characteristics

The test equipment set-up consisted of a PRD 3302 VSWR and reflection angle indicator, an HP608 signal generator, Boonton 320A power amplifier and a Narda 441C VSWR amplifier. An HP5245L counter with 5293B plug-in was used to set frequency accurately. Figure 8 shows the block diagram of the impedance measuring set-up with the measured lengths of line which must be known to derive the impedance values.

The angle of reflection coefficient is (-) when the reflected voltage lags the incident. The angle of $\emptyset$ of the load (antenna termination) is

$$\emptyset = \emptyset_m + \Delta\emptyset + 720l/\lambda$$

where

$\emptyset_m$ = angle read from PRD

$\Delta\emptyset$ = correction angle from connection plane of PRD adapter to the "tee" center of the PRD = 70° at 332 MHz.

Table 2, shows the true values of the impedance at the antenna derived from $\emptyset_m$ degrees on the instrument for the particular lengths of connecting lines used in the measuring set-up. This crossed yagi impedance is plotted on the Smith Chart of Figure 9. It can be seen from comparison with Figure 6, that the impedances are close at center frequency and differ out beyond. This difference is due to the unknown factors and the simple assumptions used in the

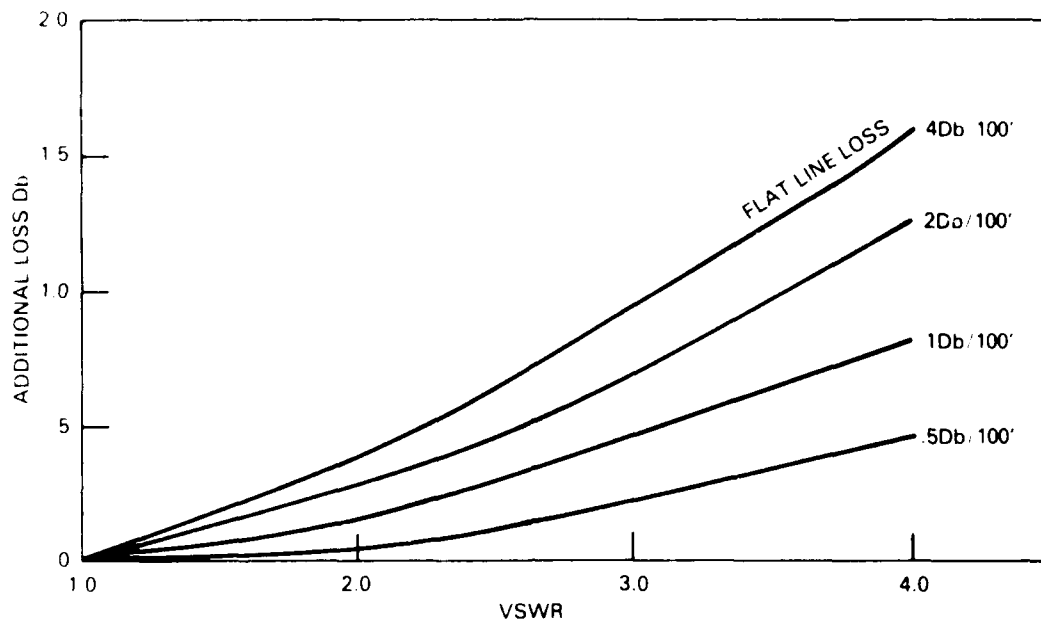


Fig. 7(a) — Loss added to flat line loss (approx.)

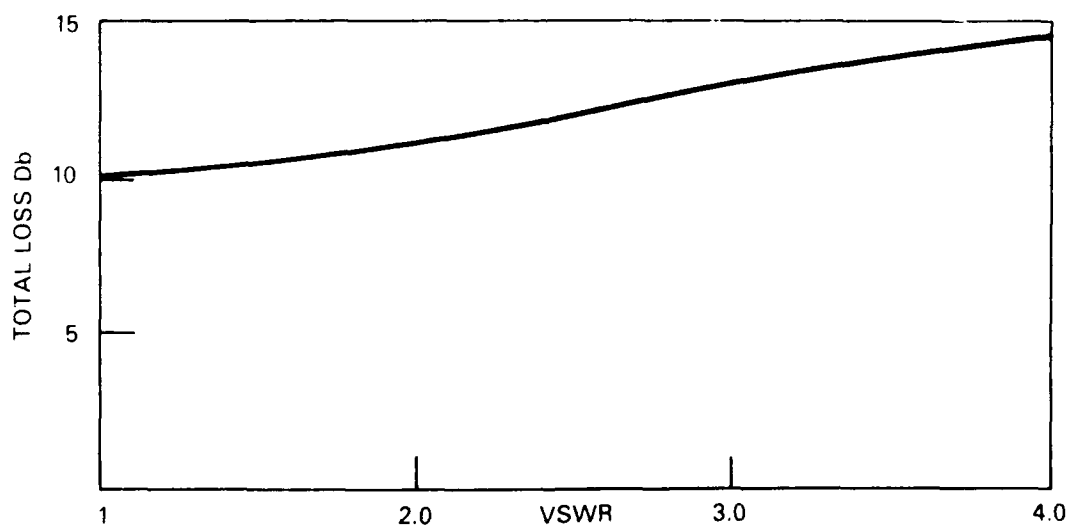


Fig. 7(b) — Typical loss vs VSWR for 250-ft length RG9/U cable at 4 dB/100 ft at 335 MHz

Table I

f, MHz	VSWR	Z
325	1.63	$.60 + j.09$
328	1.30	$.78 + j.11$
330	1.13	$.89 + j.07$
332	1.00	$.93 + j.0$
334	1.12	$.89 + j.02$
336	1.38	$1.26 + j.26$
338	1.78	$1.77 + j.13$

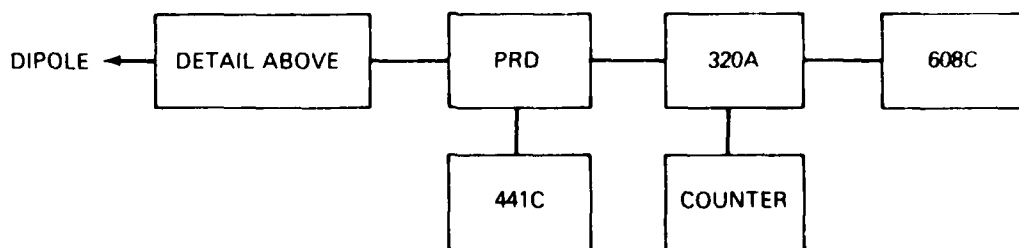
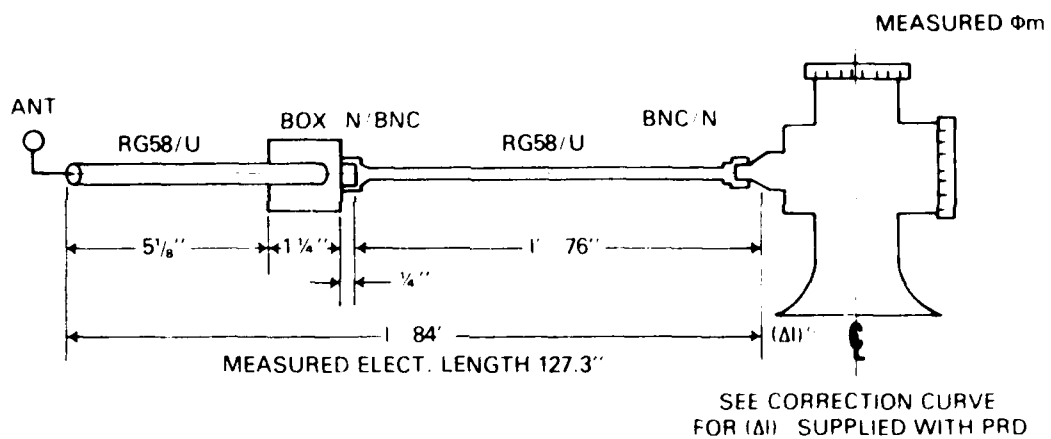


Fig. 8 — Test setup for antenna impedance measurements

TABLE 2
Value of Reflection Coeff. Angle

F(MHz)	λ	$\mathcal{L}\lambda$	$720(\mathcal{L}\lambda)^2 \text{Eq } \theta^0 + \Delta \theta^0 + \beta \theta^0 = \theta^0$	=Equiv θ (Ant)
326	92.03	3.51	2527 7.0 + 70 - 70 = +7	= +7
28	91.46	3.54	2540 28.88 + 70 - 55 = 43.8	= 43.8
30	90.90	3.56	2563 43.20 + 70 -115 = -1.8	= -1.8
332	90.36	3.58	2573 57.60 + 70 -175 = -47.40	= -47.4
34	89.82	3.60	2592 72.00 + 70 + 77 =218.0	=141.0
36	89.28	3.62	2605 86.40 + 70 - 55 =101.4	= 101.4
338	88.76	3.64	2620 100.80 + 70 -110 = 60.8	= 60.8

Table 3 shows the VSWR, the reflection angle and the derived antenna terminal impedance for the crossed yagi system.

TABLE 3
Crossed Yagi Impedance

<u>F(MHz)</u>	<u>VSWR</u>	<u>Angle</u>	<u>Z ant.</u>
326	1.27	+7	1.26 + j.0 2
28	1.75	+43.8	1.35 + j.56
30	1.9	- 1.8	1.9 - j.01
332	1.07	-47.40	1.07 - j.05
34	1.6	-140.0	.67 + j.21
36	1.9	+101.4	.75 + j.5
338	1.97	60.8	1.05 + j.7

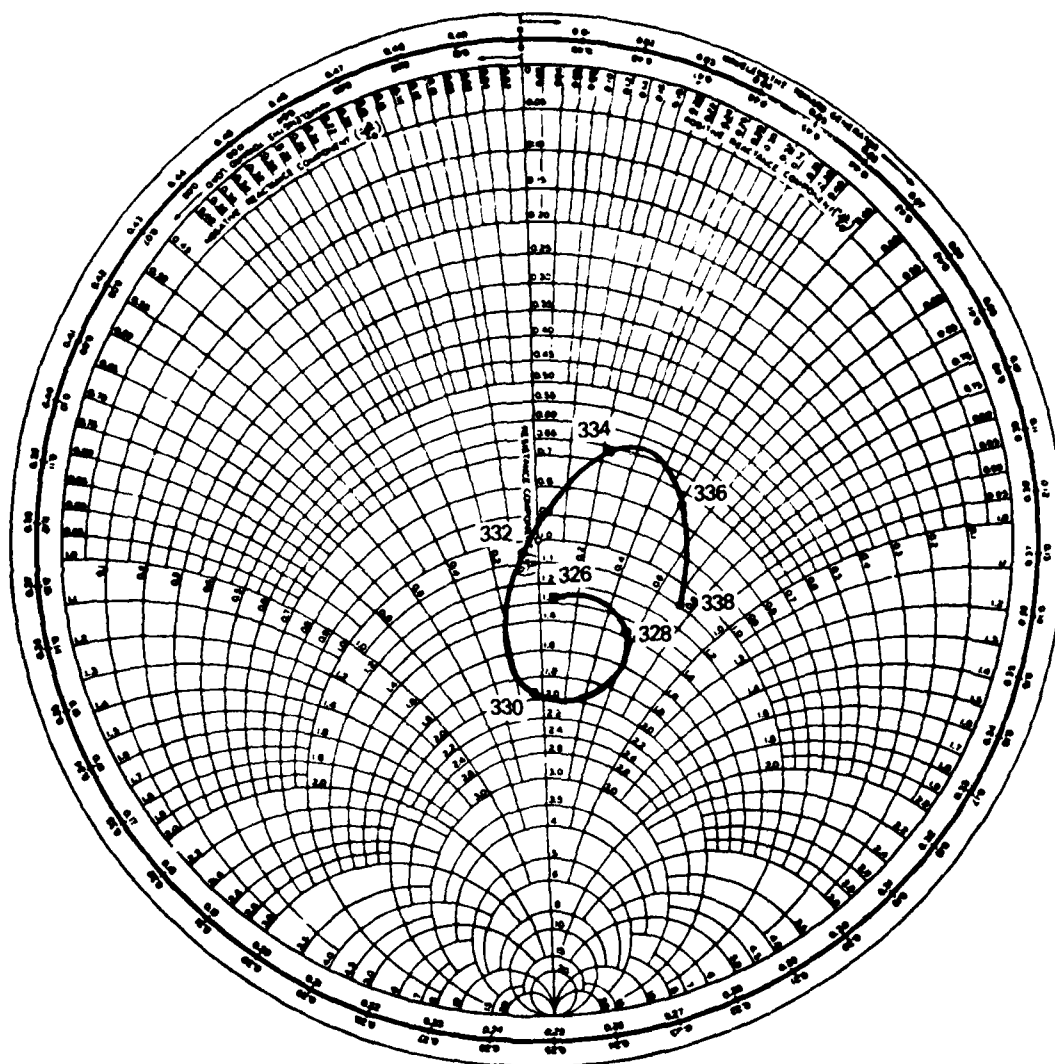


Fig. 9 — Measured impedance crossed yagi system

calculations. The actual VSWR and impedance values, however, are quite useable over the band.

Figure 10 shows VSWR for the Measured and Calculated Crossed Yagis. Included is a VSWR vs Freq. curve of an earlier single yagi of the folded dipole type driven element presently in use. This was included to show the similarity in VSWR's for the single and crossed system in that VSWR drops considerably on the low frequency side of the pass band where as the calculated one, due to simple assumptions described earlier, does not behave in this manner.

J. Patterns

Calculated axial ratio for the case of $Z=20-j25$ vs frequency looking "bore-sight" into the array is shown in Table 4 below.

TABLE 4

Axial Ratio vs. Frequency as Viewed from Array Axis

Freq (MHz)	Approx. Axial Ratio and Angle of Major Axis
328	3.7 $\angle -22^\circ$
330	1.9 $\angle -27^\circ$
332	1.06 $\angle +3.4^\circ$
334	1.97 $\angle +36^\circ$
336	1.41 $\angle 90^\circ$

Calculated axial ratio shows good circularity from 330 to 336 and acceptable circularity at 328 MHz.

The antenna field pattern at 332 MHz should be about 28° at the 3 dB level with the circularity obtained. The estimated gain should be about 10-12 dB.

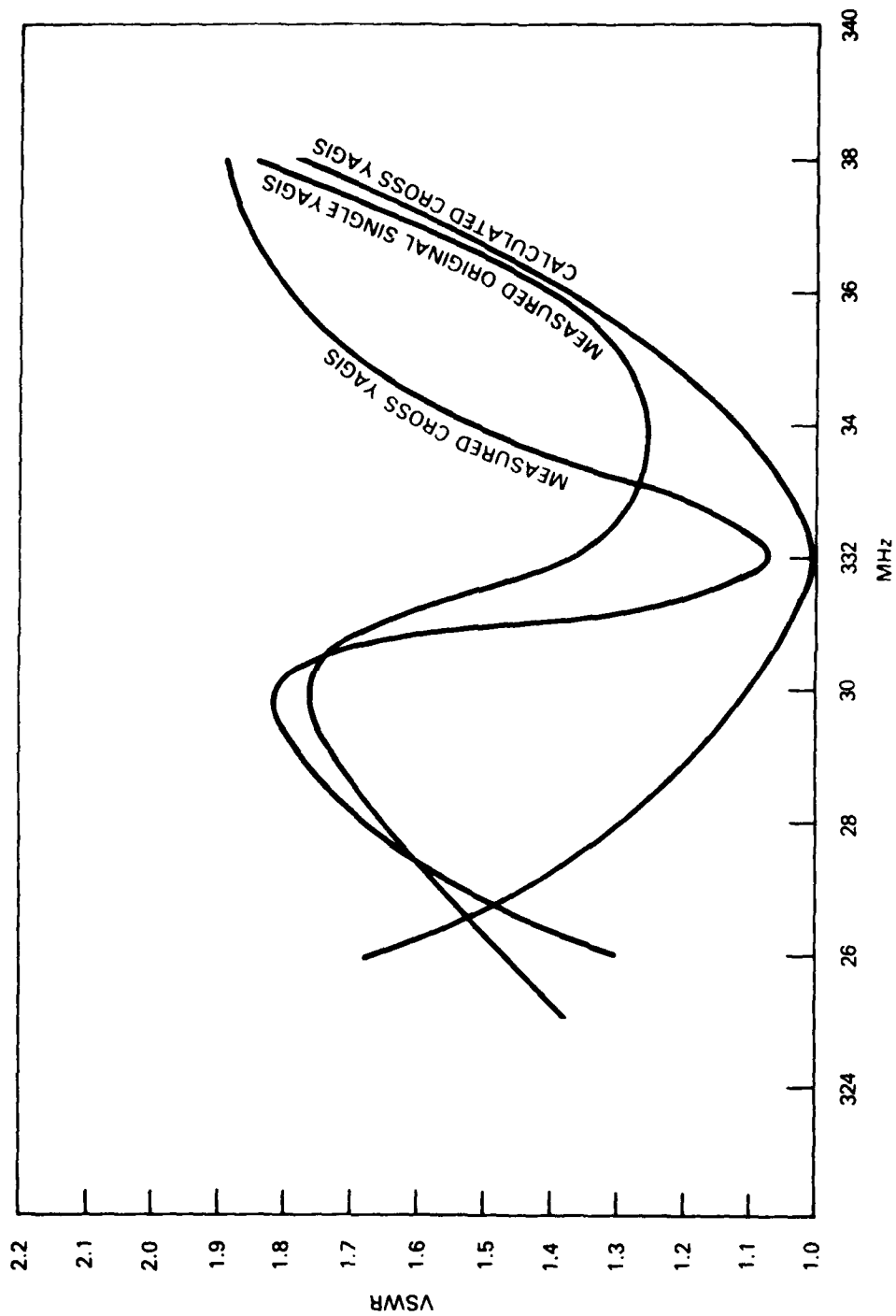


Fig. 10 — VSWR vs frequency for crossed yagi system and original single yagi with FD transformation

General References

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2. Krauss, John D., Antennas, McGraw-Hill 1950
3. Reference Data for Radio Engineers, 5th Edition, Howard W. Sons & Co., Inc. (ITT), March, 1969.
4. Reed, H. R. and Russell, Carl M., Ultra High Frequency Propagation; John Wiley and Sons, Inc., New York 1953 (Chapter 8).

APPENDIX

Sample Calculation of Dipole Feed Impedance

Case I. Assume $Z_1 = 20 - j25$ at $f_o = 332$

FD Step-up ratio of Gamma Rod = 4:1

Rod length = 3.6", Z_o of Rod line = 132Ω

MHz	λ_{cm}	λ_{in}	$3.6"/\lambda$	θ°	f/f_o	$R \pm jX = Z_1$
326	92.03	36.23	.0993	35.77	.982	18.55 - j45
28	91.46	36.01	.0999	35.99	.988	19.00 - j38.34
30	90.90	35.79	.1007	36.25	.994	19.52 - j31.67
332	90.36	35.57	.1012	36.44	1.000	20.00 - j25
34	89.82	35.36	.1018	36.65	1.006	20.48 - j15
36	89.28	35.15	.1024	36.87	1.012	21.00 - j 5
338	88.76	34.94	.1030	37.09	1.018	21.45 + j 5

Step (4) Impedance at Dipole where Gamma Rod attaches

Freq MHz	$Z_1 \div \cos^2 \theta = Z^2$
326	$(18.5 - j45) \div \cos^2 35.77 = 28.03 - j68.18$
28	$(19.0 - j38) \div \cos^2 35.9 = 28.79 - j57.6$
30	$(19.5 - j32) \div \cos^2 36.25 = 30.00 - j49.23$
332	$(20.0 - j25) \div \cos^2 36.44 = 30.77 - j38.46$
34	$(20.5 - j15) \div \cos^2 36.65 = 32.03 - j23.44$
36	$(21.0 - j 5) \div \cos^2 36.87 = 32.81 - j 7.8$
338	$(21.5 + j 5) \div \cos^2 37.09 = 33.59 + j 7.8$

Step (5) $(Z_2)(\text{Ratio in Step (1)}) = Z_2 \times 4$
 and normalize to $Z_0 = 132$

326	28.03 - j 68.18 (4/132 = .03) = .85 - j 2.05
28	28.79 - j 57.6 (.03) = .86 - j 1.73
30	30.00 - j 49.23 (.03) = .90 - j 1.48
332	30.77 - j 38.46 (.03) = .92 - j 1.15
34	32.03 - j 23.44 (.03) = .96 - j 0.70
36	32.81 - j 7.8 (.03) = .98 - j 0.23
338	33.59 + j 7.8 (.03) = 1.01 + j 0.23

Step (6) With the Smith Chart transform impedance of
 Step (5) to the feed terminal or go toward
 the generator " ℓ/λ " of the gamma rod length

326	.85 - j 2.05 $\rightarrow$.0993 λ = .21 - j .62
28	.86 - j 1.73 $\rightarrow$.0999 λ = .24 - j .52
30	.90 - j 1.48 $\rightarrow$.1007 λ = .29 - j .44
332	.92 - j 1.15 $\rightarrow$.1012 λ = .37 - j .35
34	.96 - j 0.70 $\rightarrow$.1018 λ = .54 - j .20
36	.98 - j 0.23 $\rightarrow$.1024 λ = .78 - j .08
338	1.01 + j 0.23 $\rightarrow$.1030 λ = 1.25 + j .02

Step(7) The inductive reactance of the rod, itself, is
 $X_p = \tan \theta$

Freq. MHz		Xp
326	=	+ j.720
28		+ j.726
30		+ j.733
332		+ j.738
34		+ j.744
36		+ j.750
338		+ j.756

Step(8) Invert the load impedance of Step (6) to get

$$Y = G + jB$$

Freq. MHz	Y
326	.5 + j 1.45
28	.71 + j 1.59
30	1.05 + j 1.60
332	1.44 + j 1.38
34	1.7 + j .61
36	1.27 + j .12
338	.80 - j .03

Step(9) Invert the Inductive Reactance, X_p , to get $Y = G + jB$
and add to Y of Step (8).

					Y Total
326	(.5 + j 1.45)	+	(-j 1.388)	=	.5 + j.06
28	(.71 + j 1.59)	+	(-j 1.377)	=	.71 + j.21
30	(1.05 + j 1.60)	+	(-j 1.364)	=	1.05 + j.24
332	(1.44 + j 1.38)	+	(-j 1.355)	=	1.44 + j.02
34	(1.7 + j .61)	+	(-j 1.344)	=	1.7 - j .73
36	(1.27 + j.12)	+	(-j 1.333)	=	1.27 - j 1.21
338	(.80 - j.03)	+	(-j 1.323)	=	.80 - j 1.29

Step 10a Invert Y total of Step (9) on Smith Chart to get
 Z total; 10b) $= (Z_T)(Z_0 = 132) = Z$ actual

	<u>Z Total</u>	<u>Z Actual</u>
326	1.96 - j.2	258.7 - j25.4
28	1.3 - j.36	171.6 - j47.5
30	.94 - j.23	124.1 - j29.0
332	.68 - j.02	89.8 - j2.64
34	..49 + j.23	64.7 - j30.4
36	.41 + j.41	54.1 - j54.1
338	.36 + j.57	47.5 - j75.2

This is for one yagi.

Now combine the two yagis after one goes through a $\lambda/4$
phase delay line of $Z_0 = 93\Omega$.

G. Combining the Two Yagis

At this point the procedure normally calls for a tune-out with capacitive reactance at the gamma-rod feed. However, due to a judicious selection of assumed antenna values, gamma line characteristic impedance and other adjustments, not easily reached in practice, the reactance value was slightly capacitive but close enough to zero to not require any compensation. The Z actual of step (10a) can be used in its unadulterated form. Normalize this Z actual to $Z_0 = 93\Omega$ and also find its value at the other end of the $\lambda/4$ phase line (Steps G1 and G2).

MHz	Z_{93} One Yagi	Z_{93} One Yagi inverted .25 λ
326	2.78 - j.27	.36 + j.06
28	1.85 - j.51	.50 + j.15
30	1.33 - j.31	.71 + j.18
332	.97 - j.03	1.03 + j.03
34	.70 - j.33	1.38 + j.26
36	.58 + j.58	.82 - j.82
338	.51 + j.85	.55 - j.90

3. Change both Z_{93} values to Y_{93} and add them to get a total Y_{93T} . Then change this to Z_{93T} by going directly opposite around the Smith Chart. Z to Y of the single Yagi is the same as the other single yagi transformed through $\lambda/4$ phase shift line and vice-versa. Therefore, merely add both columns of step (2) to get Y_{93} total and invert this to Z_{93} total at the "Grand Feed" terminal.

MHz	Y_{93} Total	Z_{93} Total
326	3.14 - j.21	.32 + j.05
28	2.35 - j.36	.42 + j.06
30	2.04 - j.13	.48 + j.04
332	2.00 + j.0	.50 + j.0
34	2.08 - j.07	.48 + j.01
36	1.40 - j.24	.68 + j.14
338	1.06 - j.05	.96 + j.07

The actual impedance is the normalized Z_{93} times 93Ω or

Actual Impedance at the "Grand-Feed" point is

MHz	Z actual-GF
326	29.76 + j 4.65
28	39.06 + j 5.58
30	44.64 + j 3.72
332	46.5 + j 0.0
34	44.64 + j .93
36	63.24 + j13.02
338	88.35 + j 6.51

Normalize Z actual-GF to 50 ohm to obtain a Smith Chart impedance plot and SWR graph figures

MHz	Z_{50}	VSW4
326	.60 + j .09	1.68
28	.78 + j .11	1.30
30	.89 + j .07	1.13
332	.93 + j0.0	1.00
34	.89 + j .02	1.12
36	1.26 + j .26	1.38
338	1.77 + j .13	1.78