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A NOTE ON THE FORM OF SHIP ROLL DAMPING.(U)

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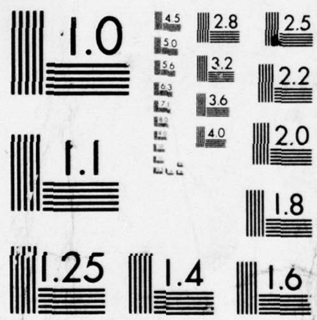
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STEVENS INSTITUTE OF TECHNOLOGY

CASTLE POINT STATION
HOBOKEN, NEW JERSEY 07030

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DAVIDSON LABORATORY

Report SIT-DL-76-1887

A NOTE ON THE FORM OF SHIP ROLL DAMPING

by

J.F. Dalzell

May 1976

Final Report - 1 October 1974 to 30 May 1976

Prepared for
David Taylor Naval Ship Research
and Development Center (1505)
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The objective of the present work was to develop an approximation to the conventional mixed linear-plus-quadratic ship roll damping model so that analytical obstacles could be overcome in the application of the functional series expansion to non-linear ship rolling. A mixed linear-plus-cubic approximation was found to be reasonable for this purpose. In the course of analyses there were indications that this model may be more generally valid than had initially been thought.

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Approved

Daniel Savitsky
Deputy Director

CONTENTS

INTRODUCTION

In most cases of practical application of seakeeping theory or experiment the problem comes ultimately to that of prediction of the statistics of maxima of ship response in realistic (random or irregular) seas. Methods of estimation of the magnitude of oscillatory ship motion statistics are reasonably well in hand so long as the motion being predicted can be assumed to be a linear function of regular wave height. The methods outlined by St. Denis and Pierson[1*] involve, in addition to the linearity assumption, the assumption that the wave process is Gaussian. These assumptions being fulfilled, the spectrum of response may be estimated, and from this a reasonable estimate of the statistics of maxima and minima can be formed.

In this context ship rolling continues to pose a problem. The often used single degree of freedom equation for roll (which is traceable to the work of W. Froude[2*]) is non-linear in damping moment. This same non-linearity, or the recognition of its possible occurrence, are to be found in many multi-degree-of-freedom analyses. In the case of both pitch and heave it has been possible to develop linear theoretical approaches to damping which conform reasonably well to observation. This has not been the case with rolling. There presently appears to be no theoretically based prediction method for roll which is completely free of empiricism with respect to roll damping.

Some direct tests have been made of the validity of the linear superposition technique for prediction of roll in irregular seas. Lalangas[3*]

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*1. St. Denis, M. and Pierson, W.J., Jr., "On the Motions of Ships in Confused Seas," SNAME, Vol. 61, 1953.

*2. "The Papers of William Froude," The Institution of Naval Architects, London, 1955.

*3. Lalangas, P., "Application of Linear Superposition Techniques to the Roll Response of a Ship Model in Beam Irregular Seas," D.L. Report 983, Davidson Laboratory, Stevens Institute of Technology, October 1963.

concluded that the method is reasonable for at-speed conditions. Other work tends to bear out this conclusion for at-speed conditions, but not for very low or zero ship speed.

In the case of the prediction of non-linear zero speed rolling in irregular seas, most work has involved the ordinary single degree of freedom roll equation with non-linear damping. The work of Kaplan[4*] and Vassilopoulos[5*] involves equivalent linearization techniques in the estimation of the variance of roll and the roll spectrum. Vassilopoulos includes a restoring moment non-linearity as well. In this technique nonlinear elements are replaced by linear elements chosen so as to minimize the resulting mean square errors for the case of random excitation. Yamanouchi[6*] approached the problem with a perturbation technique for the solution to the non-linear differential equation, obtaining solutions for the non-linear roll spectrum in terms of the sum of the linear spectrum and various convolutions.

In both the approaches just cited there is the implicit assumption that the statistics of rolling maxima are adequately described by the Rayleigh distribution. A partial vindication of this assumption has been made by numerical time domain simulations of a single degree of freedom roll equation.[7*] The results indicated that reasonably good predictions of up to the 1/10 highest averages of non-linear roll may be made from a knowledge of the roll spectrum (or equivalently the roll variance) and the assumption of a Rayleigh distribution of maxima -- despite the inclusion

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*4. Kaplan, P., "Lecture Notes on Non-Linear Theory of Ship Roll Motion in a Random Sea Way," ITTC Transactions, 1966.

*5. Vassilopoulos, L., "Ship Rolling at Zero Speed in Random Beam Seas with Non-Linear Damping and Restoration," Journal of Ship Research, Vol. 15, No. 4, December 1971.

*6. Yamanouchi, Y., "On the Effects of Non-linearity of Response on Calculation of the Spectrum," ITTC Transactions, 1966.

*7. Dalzell, J.F., "A Note on the Distribution of Maxima of Ship Rolling," Journal of Ship Research, Vol. 17, No. 4, December 1973.

of non-linearities within the normal range of magnitude observed in unstabilized ships and models.

In a quest for possible further improvements in techniques for prediction of non-linear rolling in random seas, two other general approaches for the solution of non-linear random vibration problems can be considered. The first is one of the variations of the Fokker-Planck equation method. Evaluation of roll response statistics according to this approach promise to be extremely difficult when the spectrum of excitation is not white (flat). Haddara[8*] uses a modified Fokker-Planck approach which results in estimates for roll variance for the case when the excitation spectrum is white.

The second alternative approach is the functional series model. In this approach there is a serious draw-back with respect to application to the usual non-linear rolling equation. This was pointed out some time ago by Vassilopoulos[9*]. If the functional series and a differential equation are to be related, it appears that the terms in the equation must be analytic for small values of the variables. This is not the case for the "quadratic" term ordinarily used to represent the damping non-linearity.

Because of reasonable success achieved[10*] with the application of the functional series approach to the added ship resistance problem, the possibilities of application to ship rolling were re-investigated. Among the attractions of this method is that as a conceptual framework it is suitable for any reasonably well behaved wave input (regular, transient, or random), and since it contains the completely linear system as a

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*8. Haddara, M.R., "A Modified Approach for the Application of Fokker-Planck Equation to the Nonlinear Ship Motions in Random Waves," International Shipbuilding Progress, Vol. 21, No. 242, October 1974.

*9. Vassilopoulos, L.A., "The Application of Statistical Theory to Non-Linear Systems to Ship Motion Performance in Random Seas," International Shipbuilding Progress, Vol. 14, No. 150, 1967.

*10. Dalzell, J.F., "Application of the Functional Polynomial Model to the Ship Added Resistance Problem," Eleventh Symposium on Naval Hydrodynamics, University College London, 1976.

special case it appears to have the potential of being a logical extention to present practice. As previously noted, the achievement of the more general advantages of the functional series model for ship rolling depend upon the approximation or replacement of the quadratic damping term with something more tractable. This report is the result of a step in this direction, which was to consider how an approximation to quadratic damping might be made and what the penalties might be.

AN APPROXIMATION TO THE QUADRATIC TERM

The single degree of freedom rolling equation may be considered in the form:

$$I\ddot{\varphi} + \underline{N}(\dot{\varphi}) + \underline{B}(\varphi) = F(t) \qquad (1)$$

where:

φ = roll angle

I = roll inertia

$\underline{N}(\dot{\varphi})$ = damping function

$\underline{B}(\varphi)$ = restoring function

$F(t)$ = excitation

t = time

In the literature there appear a number of forms of restoring function. None so far seen are fundamentally different than an odd series in φ:

$$\underline{B}(\varphi) = \sum_{j=1,3,\ldots} B_j \varphi^j \qquad (2)$$

In most applications only the first (linear) term is used. In any event the form of Eq. 2 presents no fundamental problem.

The classical damping function may be written in the form:

$$\underline{N}(\dot{\varphi}) = N_{21}\dot{\varphi} + N_{22}\,|\dot{\varphi}|\,\dot{\varphi} \qquad (3)$$

The absolute value in the second term of Eq. (3) is the issue.

The above form for roll damping has also been used both explicitly and implicitly in modern multi-degree-of-freedom analyses. There appear to have been extremely few deviations from the form of Eq. (3) in the last century. A few recent instances may be cited. Haddara[11*] replaced the second term with a cubic term in $\dot{\varphi}$ for illustrative purposes and possibly because the quadratic form was impossible to handle in his derivation. Lewison[12*] also replaced the quadratic term with a cubic term. The reason appears to have been largely to facilitate analog computer setup, though he remarked (without elaboration) that the cubic model better fitted the data. Takaki and Tasai[13*] performed forced oscillation experiments of a ship model with bilge keels, and found in their analysis of the roll damping that an improved fit of the data was obtained by adding a cubic term in $\dot{\varphi}$ to the standard model, Eq. (3).

The above was sufficient encouragement to consider the replacement of the quadratic term in Eq. (3) with an odd series in $\dot{\varphi}$. If it is assumed that the coefficients N_{21} and N_{22} in Eq. (3) are given, then the problem is to relate these to the coefficients of an odd expansion. One approach is to make a least square fit of an odd series to $|\dot{\varphi}|\dot{\varphi}$ over some assumed range of $\dot{\varphi}$, say $(-\dot{\varphi}_c < \dot{\varphi} < \dot{\varphi}_c)$. The fitting equation was therefore assumed in the following form:

$$|\dot{\varphi}|\dot{\varphi} = \sum_{k=1,3,\ldots} \alpha_k \dot{\varphi}^k/(\dot{\varphi}_c)^{k-2} \qquad (4)$$

It was further assumed that the odd series is truncated at the term with exponent K. In this case the (K+1)/2 normal equations in the unknown

*11. Haddara, M.R., "On Nonlinear Rolling of Ships in Random Seas," International Shipbuilding Progress, Vol. 20, No. 230, October 1973.

*12. Lewison, G.R.G., "Optimum Design of Passive Roll Stabilizer Tanks," The Naval Architect, RINA, January 1976.

*13. Takaki, M. and Tasai, F., "On the Hydrodynamic Derivative Coefficients for Lateral Motions of Ships." Transactions, West Japan Society of Naval Architects, No. 46, August 1973.

coefficients α_k become:

$$\sum_{k=1,3...}^{K} \frac{\alpha_k}{(k+j+1)} = \frac{1}{j+3}$$

$$\text{(with } j=1,3,...K) \qquad (5)$$

After solving for the α_k, the result for truncation of the series at the cubic term is:

$$|\dot{\varphi}|\ \dot{\varphi} \approx \frac{5}{16}\ \dot{\varphi}_c \dot{\varphi} + \frac{35}{48}\ \dot{\varphi}^3/\dot{\varphi}_c \qquad (6)$$

Figure 1 indicates the degree of approximation involved in Eq. (6). Only the positive half of the quadratic form is shown and the evaluation is made for normalized $|\dot{\varphi}|\ \dot{\varphi}$. The maximum deviations within the range of the fit ($\pm\ \dot{\varphi}_c$) amount to 3 or 4% of $\dot{\varphi}_c^2$. Because both the fitted coefficients are positive in Eq. (6), the general behavior of the approximation beyond the fitted range is the same as that of the quadratic term.

The solution of Eq. (5) for a truncation after the fifth power was also obtained. In this latter case approximation errors within the fitted range ($\pm\ \dot{\varphi}_c$) were about half those indicated in Fig. 1, but because one of the three coefficients (α_5) was negative, the behavior of the fitted function outside the range was most unlike that of $|\dot{\varphi}|\dot{\varphi}$, and accordingly the fifth power approximation was discarded in favor of an exploration of the possibilities of a cubic fit, Eq. (6).

If it is assumed that Eq. (6) is an adequate approximation, substitution in the damping function, Eq. (3), results in:

$$\underline{N}(\dot{\varphi}) \approx [N_{21} + \frac{5}{16}\ \dot{\varphi}_c\ N_{22}]\ \dot{\varphi} + \frac{35}{48}\frac{N_{22}}{\dot{\varphi}_c}\ \dot{\varphi}^3 = \underset{\sim}{N}\ (\dot{\varphi}) \qquad (7)$$

In approximating the non-linear damping elements of Eq. (1), it is the accuracy of $\underline{N}(\dot{\varphi})$ which matters. If the non-linear part of the damping is relatively weak within the range of absolute roll velocity for which the approximation is designed ($\dot{\varphi}_c$), the errors in $\underline{N}(\dot{\varphi})$ as approximated by Eq. (7) may be expected to be less than those implied by Fig. 1.

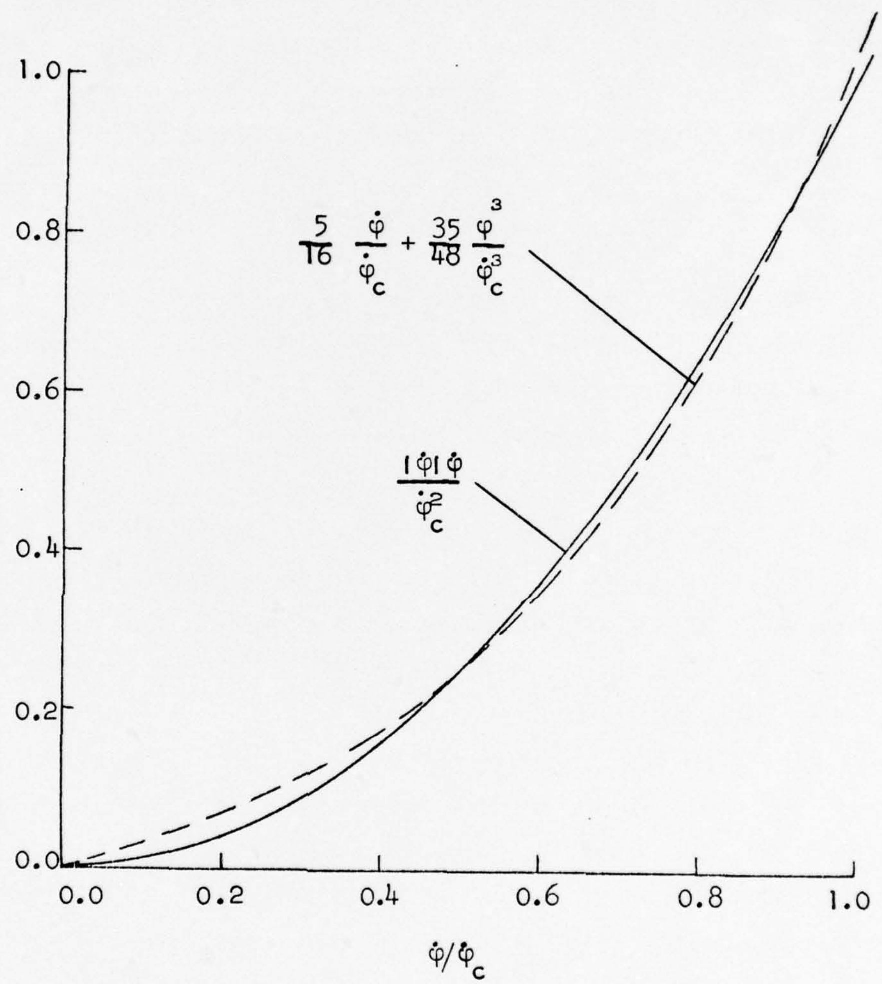

FIGURE 1. TWO TERM ODD SERIES FIT TO $|\dot{\varphi}|\dot{\varphi}$

Conceptually, Eq. (7) might be thought of as a sort of equivalent non-linearization. In the sense of making estimates of roll variance utilizing the functional series method, the form of Eq. (7) is suitable. However the presence of the parameter $\dot{\varphi}_c$ means in principle that estimates of both the roll and roll velocity variances would have to be made for an arbitrary choice of $\dot{\varphi}_c$, these used to estimate a reasonable solution domain of $\dot{\varphi}$, and the process iterated if required. This prospect was accepted pending some further studies of the cubic approximation.

THE THEORY FOR ANALYSIS OF ROLL EXTINCTION DATA

In order to assess further the approximation of Eq. (7) two things seemed in order. The first was to find some realistic magnitudes for N_{21} and N_{22}, and the second was to consider what violence to the left hand side of Eq. (1) was being done relative to observed behavior of ships.

As noted in the introduction, theoretical estimates of N_{21} or N_{22} appear for practical purposes to be totally lacking. Though theoretical estimates of N_{21} are made in the more sophisticated of modern ship motions algorithms, these seem invariably to come out too low, the typical procedure being to add something to account for "viscous" damping effects so as to bring computation into line with observation. In fact, what are considered realistic estimates of N_{21} and N_{22} are almost totally empirical. In the vast majority of studies where some distinction is made between "linear" and "quadratic" components of roll damping, the numerical results are obtained by analysis of ship or model sallying experiments. The data reduction approach has been basically the same since the time of W. Froude[2].

Minorsky[14*] (pp 186-188 and pp 192-196) derives the method according to the first approximation of Kryloff and Bogoliuboff. The basic result is the differential equation of the amplitude of response, assuming

*14. Minorsky, N., "Introduction to Non-Linear Mechanics," J.W. Edwards, Ann Arbor, 1947.

that damping is small, that excitation is zero and that angles are small. In particular if these assumptions are applied to Eq. (1) there results:

$$I\ddot{\varphi} + B_1\varphi + \underline{N}(\dot{\varphi}) = 0 \qquad (9)$$

Eq. (9) is assumed to be an adequate representation of the ship during the sallying experiment. The fundamental result from Minorsky[14] for the differential equation of roll amplitudes is:

$$\dot{Y} = -\frac{1}{2\pi I\omega}\int_0^{2\pi} \underline{N}(Y\omega\cos\epsilon)\cos\epsilon\, d\epsilon \qquad (10)$$

where:

ω = roll frequency = $\sqrt{B_1/I}$

Y = the roll amplitude in the sense that the rolling is represented in the form:

$$\varphi(t) = Y(t)\sin(\omega t + \text{constant})$$

Substituting the quadratic damping function, Eq. (3), in Eq. (10):

$$\dot{Y} = -\frac{1}{2\pi I\omega}\int_0^{2\pi} \left[N_{21}\; Y\omega\cos\epsilon + N_{22}Y^2\omega^2 \,|\cos\epsilon|\, \cos\epsilon\right]\cos\epsilon\, d\epsilon$$

$$= -\frac{N_{21}}{2I}\,Y - \frac{4N_{22}\omega}{3\pi I}\,Y^2 \qquad (11)$$

It is convenient for analysis of sallying data to make a change in the time variable in Eq. (11). That is, let

$$t = nT$$

where T = roll period = $2\pi/\omega$

making this change in Eq. (11):

$$\frac{dY}{dn} = -\frac{\pi N_{21}}{I\omega}\,Y - \frac{8N_{22}}{3I}\,Y^2 \qquad (12)$$

The typical analysis of the curve of declining angles obtained in a ship sallying experiment amounts to estimating dY/dn (the decrease in roll amplitude per cycle) as a function of mean amplitude and then choosing coefficients to make a best fit of the data to an equation of the form of Eq. (12). In particular,

$$-\frac{dY}{dn} = \delta_1 Y + \delta_2 Y^2 \tag{13}$$

is the form often used. Having estimates of δ_1 and δ_2 from the data, estimated values of N_{21} and N_{22} may be formed:

$$N_{21} = \frac{I\omega}{\pi} \delta_1 = \frac{B_1}{\pi\omega} \delta_1$$

$$N_{22} = \frac{3I}{8} \delta_2 = \frac{3B_1}{8\omega^2} \delta_2 \tag{14}$$

Froude found it convenient to estimate the decrease in roll per <u>half</u> cycle (dY/dm), and much of the available data is presented in this way. In this case the data is reduced according to:

$$-\frac{dY}{dm} = aY + bY^2 \tag{15}$$

so that in this system N_{21} and N_{22} may be estimated from Eq. (14) with $\delta_1 = 2a$ and $\delta_2 = 2b$.

Given equations of the form of (12), (13) or (15) the equation of the curve of declining angles from the sallying experiment may be found.[14] The result in terms of the notation of Eq. (15) may be written:

$$Y = \frac{Y_o \text{Exp}[-am]}{1 + \frac{b}{a} Y_o (1-\text{Exp}[-am])} \tag{16}$$

where the initial conditions: roll amplitude = Y_o at time m = 0 have been assumed.

Turning to the second point mentioned at the outset, once the numbers in the approximate damping function, Eq. (7), have been chosen the form of Eq. (1) changes. For example, for the zero excitation, small angle case as assumed for the ship sallying experiment, the original equation:

$$I\ddot{\varphi} + B_1\varphi + N_{21}\dot{\varphi} + N_{22}|\dot{\varphi}|\dot{\varphi} = 0$$

is changed to one of the form:

$$I\ddot{\varphi} + B_1 \varphi + N_{31} \dot{\varphi} + N_{33} \dot{\varphi}^3 = 0 \qquad (17)$$

The question is (apart from validity of empirical constants) will such a change imply responses which do not reconcile with observation. Accordingly, it is of interest to make-believe that Eq. (17) represents the ship in the sallying experiment. In this case the damping function would be:

$$\underline{\underline{N}}(\dot{\varphi}) = N_{31} \dot{\varphi} + N_{33} \dot{\varphi}^3 \qquad (18)$$

Substitution in Eq.(10) results in the differential equation of rolling amplitude:

$$\dot{Y} = -\frac{N_{31}}{2I} Y - \frac{3N_{33}\omega^2}{8I} Y^3 \qquad (19)$$

or, making the time variable change as was done to produce Eq. (12):

$$\frac{dY}{dn} = -\frac{\pi N_{31}}{I\omega} Y - \frac{3\pi N_{33}\omega}{4I} Y^3 \qquad (20)$$

Imagining that the results of a sallying experiment might be analyzed in a way similar to that for the quadratic model (Eq. 13), the coefficients γ_1 and γ_3 would be chosen so as to best fit the data to:

$$-\frac{dY}{dn} = \gamma_1 Y + \gamma_3 Y^3 \qquad (21)$$

Accordingly, the estimates of N_{31} and N_{33} from the experiment would become:

$$N_{31} = \frac{I\omega}{\pi} \gamma_1 = \frac{B_1}{\pi\omega} \gamma_1$$
$$N_{33} = \frac{4I}{3\pi\omega} \gamma_3 = \frac{4B_1}{3\pi\omega^3} \gamma_3 \qquad (22)$$

In Froude's method of fitting with half periods the fitting equation analogous to Eq. (15) might be written:

$$-\frac{dY}{dm} = cY + dY^3 \qquad (23)$$

and, as before, estimates of N_{31} and N_{33} could be formed from Eq.(22) by letting $\gamma_1 = 2c$ and $\gamma_3 = 2d$.

Finally, the equation of declining angles from the ship sallying experiment in which the damping is assumed to be the cubic model, may be derived from Eqs. (20), (21), or (23). Assuming, as with the quadratic model, that the initial roll amplitude is Y_o at time (m) = 0, the result in the notation of Eq. (23) comes out to be:

$$Y = \frac{Y_o \mathrm{Exp}[-cm]}{\left\{1 + \frac{d}{c} Y_o^2 (1 - \mathrm{Exp}[-2cm])\right\}^{1/2}} \qquad (24)$$

Accordingly, if an approximation of the nature of Eq. (7) is reasonable relative to available observations, the differences between the quadratic extinction model, Eq. (13) and the cubic extinction model, Eq. (21) should be small relative to the range and accuracy of data. Similarly, if the curves of declining roll angles are examined, Eq. (24) for the cubic model should not differ radically from Eq. (16) for the quadratic model within the range of observation.

SOME ANALYSES OF SHIP-SALLYING DATA

A review of recent texts (Lewis[15*], Korvin-Kroukovsky[16*], Vossers[17*], Blagoveshchensky[18*], for example) fairly convincingly indicates the originator of the mixed linear plus quadratic roll damping representation to be W. Froude.[2] It appears that there was no dispute at the time about the

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*15. Lewis, E.V., Chapter IX "Principles of Naval Architecture," SNAME, 1967.

*16. Korvin-Kroukovsky, B.V., "Theory of Seakeeping," SNAME, 1961.

*17. Vossers, G., "Behavior of Ships in Waves" - Volume IIC of Resistance, Propulsion and Steering of Ships, Technical Publishing Company, H. Stam. N.V., the Netherlands, 1962.

*18. Blagoveshchensky, S.N., "Theory of Ship Motions," Dover Publications, 1962.

representation of real fluid effects upon roll as being quadratic, only about the mixed model. Since some of the experimental data used by W. Froude in demonstrating his point are currently available[2] it was of interest to start with these. The data (which appeared in Naval Science, 1874, pp 220 of Ref. 2) involves the presumably faired points on the curves of declining angles for 5 sallying experiments on 4 ships, all of which apparently were built with bar keels.

The first point of the exercise was to make a comparison between the fits of Eqs. 16 and 24 to the original data. The experimental results had been plotted to an enlarged scale and it was possible to measure the roll amplitudes observed at each half cycle to within $\pm 0.05^{\circ}$ from these plots. Because it was not possible to quickly develop a direct least square fitting procedure for Eq. 16 or 24, an indirect approach was utilized. The first step was to make a least squares 5th order polynomial fit to the curves of declining angles (Y as function of m). From the resulting coefficients, an estimate of dY/dm was evaluated at each half cycle so that the result was a set of paired values of dY/dm and Y. These data were used in making least square fits of Eq. 15 and 23 so that the result was a first estimate of the coefficients a, b, c and d in Eq. 16 and 24. Given estimates of the coefficients and assuming that Y_o was the first roll amplitude shown on the data and that this amplitude corresponds to $m = 0$, Eq. 16 and 24 may be evaluated and the rms deviations from the original data computed. Trial and error perturbations of the initially estimated coefficients were made until an approximate minimum rms deviation was found or until rms deviations were 0.05° or lower, this being the probable error in reading the original data.

The results of the procedure are shown in Figures 2 through 5 for the data in Ref. 2. Figure 2 indicates curves of declining angles for two experiments on H.M.S. INCONSTANT. Each experiment was analyzed separately. (Froude had combined the two graphically in his analysis[2].) The logarithmic scale in this figure, as in those to follow, tends to accentuate reading errors for the lower ranges of roll amplitude. The fits of Eq. 16

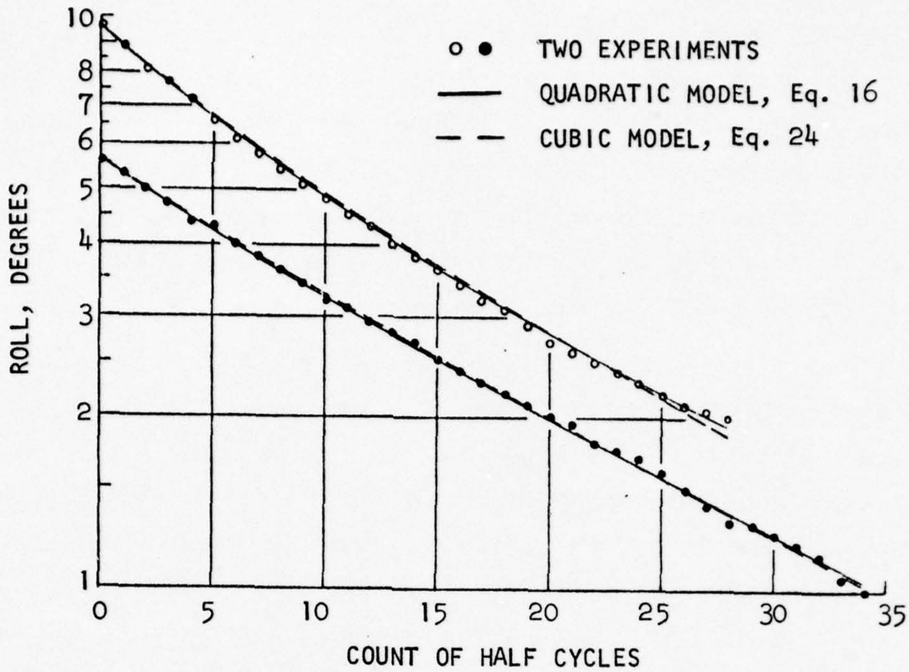

FIGURE 2. FIT OF EQUATIONS 16 AND 24 TO CURVE OF DECLINING ANGLES OBTAINED BY TWO EXPERIMENTS WITH H.M.S. INCONSTANT IN 1871

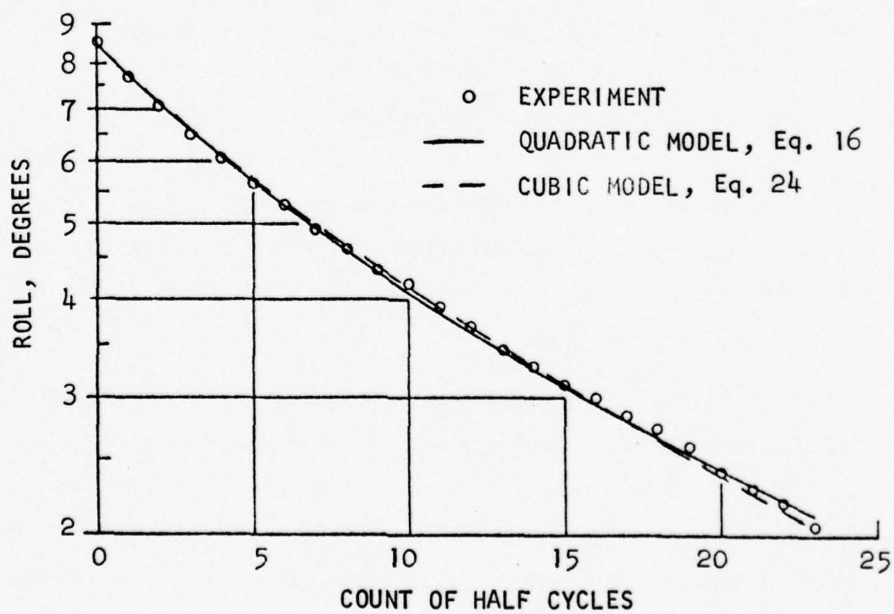

FIGURE 3. FIT OF EQUATIONS 16 AND 24 TO CURVE OF DECLINING ANGLES OBTAINED IN AN EXPERIMENT WITH H.M.S. VOLAGE IN 1871

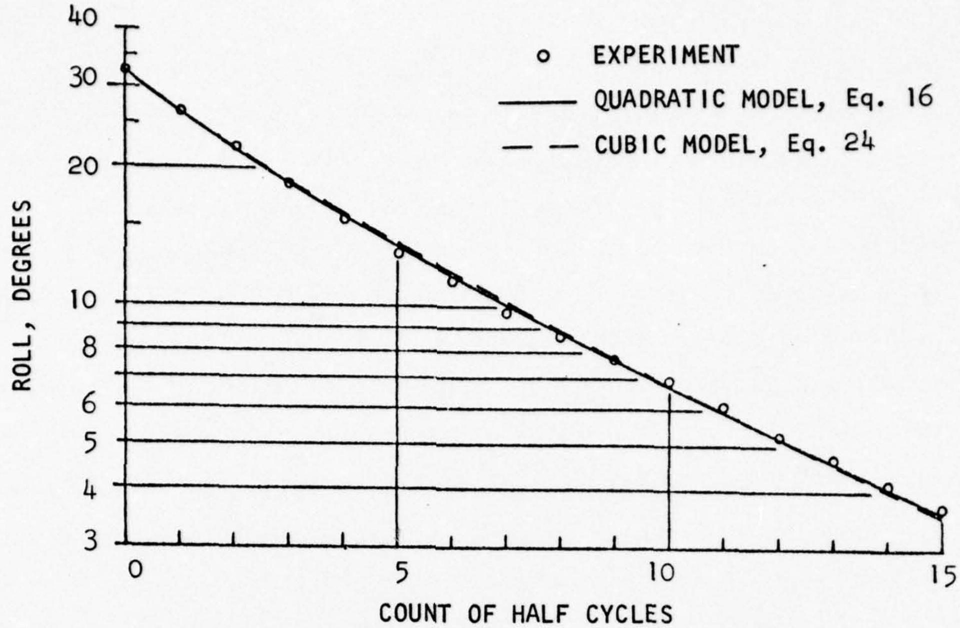

FIGURE 4. FIT OF EQUATIONS 16 AND 24 TO CURVE OF DECLINING ANGLES OBTAINED IN AN EXPERIMENT WITH ELORN IN 1872

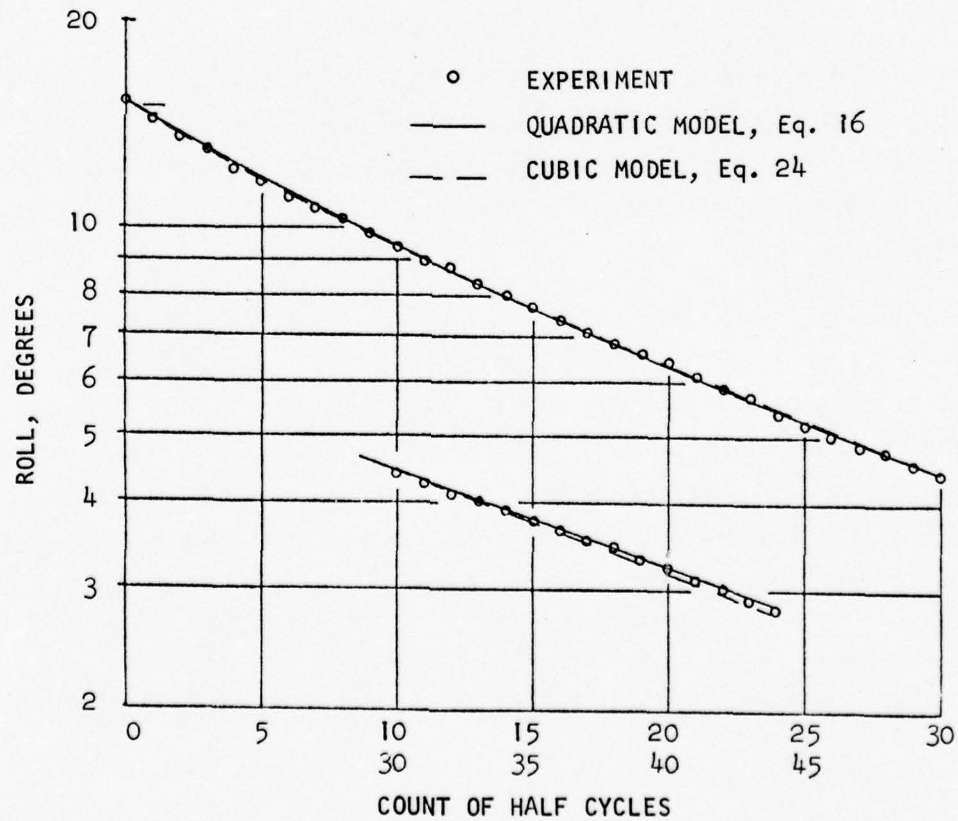

FIGURE 5. FIT OF EQUATIONS 16 AND 24 TO CURVE OF DECLINING ANGLES OBTAINED IN EXPERIMENT WITH H.M.S. SULTAN IN 1873

and 24 to data for H.M.S. VOLAGE is shown in Figure 3. Figure 4 involves fits to data obtained by French experimenters on a ship named ELORN. This experiment is notable for the large initial roll amplitude (32°), and it is regretted that the details of the ship or experiment could not be found. Figure 5 involves the declining angle curve for H.M.S. SULTAN, which, owing to the ship's light damping in roll, is unusually long, and accordingly the results were broken into two parts for plotting purposes in Figure 5.

Gawn[19*] presents results of several sallying experiments carried out by Froude on an ironclad warship, H.M.S. DEVASTATION. These were in the form of various declining angle curves plotted to relatively small scale. One of these results was picked arbitrarily, and the curve was read at each half cycle, excluding data for amplitudes less than 1° since the values could not be read much closer than $\pm 0.1^\circ$. The resulting data was subjected to the procedure outlined above, with results given in Figure 6.

Table 1 summarizes the numerical values of the coefficients used in evaluating Eq. 16 and 24 to produce the lines plotted in Figures 2 through 6. Values of rms deviations are also given. W. Froude's results for the coefficients "a" and "b" are shown in the table for the cases available.

On the whole, in Figures 2 through 6, there is not much to choose between the abilities of the quadratic model, Eq. 16, and that of the cubic model, Eq. 24, to represent the original data. In both cases, effort was made to produce fits having comparable rms deviations from the data. Within the range of data either would serve as a reasonable interpolator. In none of the cases shown would a purely linear damping function result in as good a fit, and it is strongly suspected that neither an

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*19. Gawn, R.W.L., "Rolling Experiments with Ships and Models in Still Water," The Institution of Naval Architects, Vol. 82, 1940.

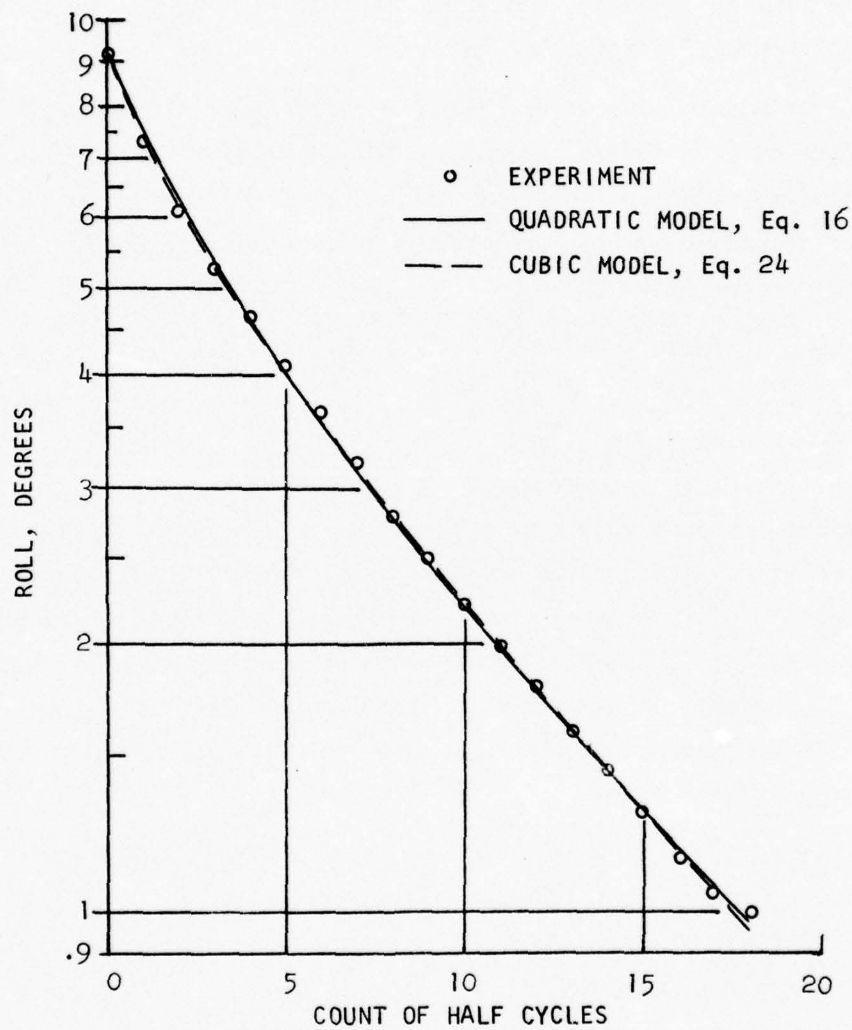

FIGURE 6. FIT OF EQUATIONS 16 AND 24 TO CURVE OF DECLINING ANGLES FOUND BY EXPERIMENT WITH H.M.S. DEVASTATION IN MAY 1873

assumed purely quadratic or an assumed purely cubic damping function would yield comparable fits. Relative to the present objectives, the results suggest that the replacement of the classical damping model, Eq. 3, with the cubic approximation, Eq. 7, can result in a quite reasonable approximation to the transient roll response so long as an appropriate choice of coefficients can be made.

Most modern results from sallying experiments are presented as roll extinction curves; that is, the numerical differentiation which provides the decrease in roll per cycle or half cycle is performed and the results analyzed or faired by the experimenter prior to presentation in the form of a plot of -dY/dm vs Y.

Gawn[19] presents three such sets of ship data. The first two sets to be discussed are from experiments carried out on a1912 vintage battleship (H.M.S. KING GEORGE V) and a 1918 destroyer (H.M.S. VIVIAN). In both cases Gawn[19] presents faired lines representing -dY/dm as a function of Y. For present purposes values of -dY/dm were taken off at equal intervals of Y and the results treated as data to which Eq. 15 and 23 were fitted by method of least squares. The resulting fitted lines are compared with points representing the original curves in Figures 7 and 8. There is again not much to choose between the fits, a somewhat surprising result in view of the likelihood that the original fairing of the two sets of data was done to the quadratic model.

The results of experiments on a third ship presented by Gawn[19] are of the same form but unfaired. The ship was a destroyer of a 1935 design (H.M.S. NUBIAN). There were six sallying experiments carried out. Values of dY/dm were originally derived from each and the results were pooled to form some 50 pairs of dY/dm and Y. These points were measured from the chart presented by Gawn and appear as circles in Figure 9. Least square fits of Eq. 15 and of Eq. 23 were made to the pooled data and the resulting fitted lines are also shown in Figure 9. The differences between the quadratic and cubic models are clearly less than the scatter of original data.

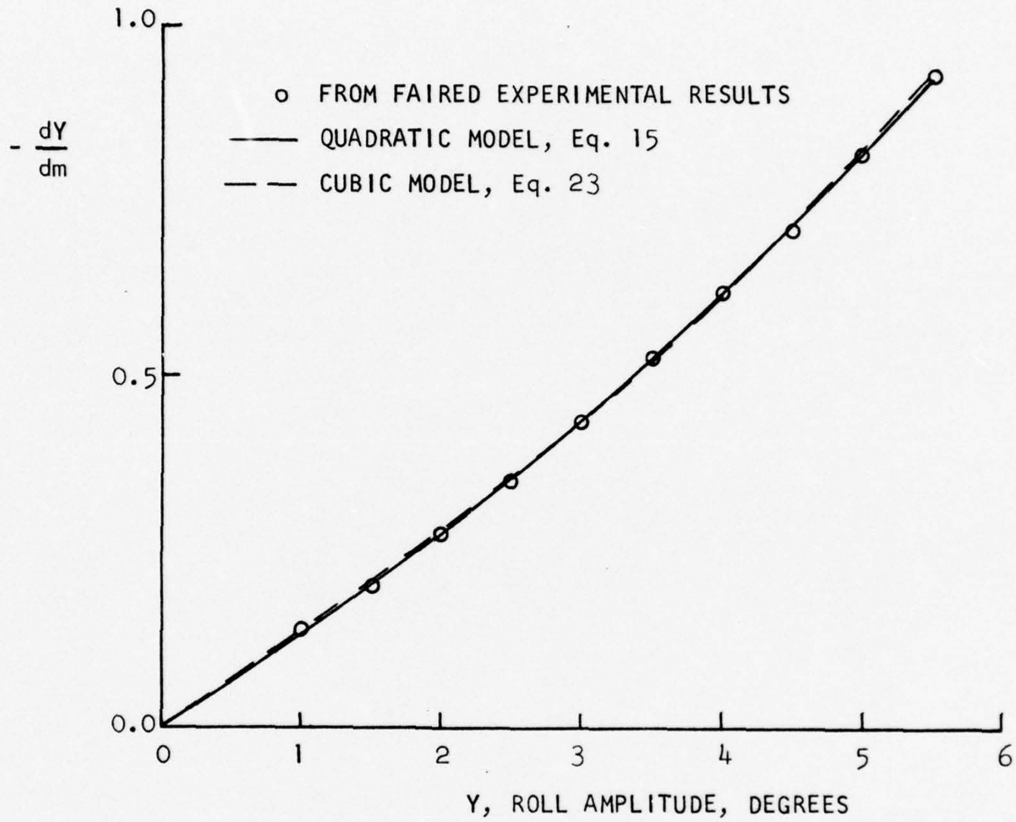

FIGURE 7. FIT OF EQUATIONS 15 AND 23 TO ROLL EXTINCTION CURVE OBTAINED FOR H.M.S. KING GEORGE V IN 1914

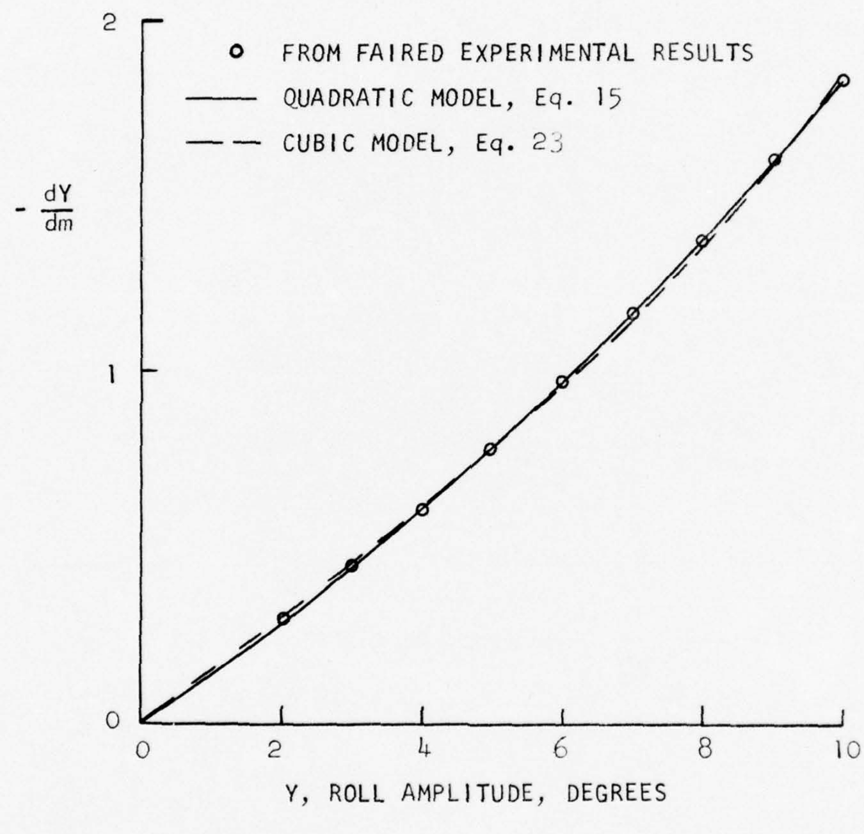

FIGURE 8. FIT OF EQUATIONS 15 AND 23 TO ROLL EXTINCTION CURVE OBTAINED FOR H.M.S. VIVIAN IN 1925

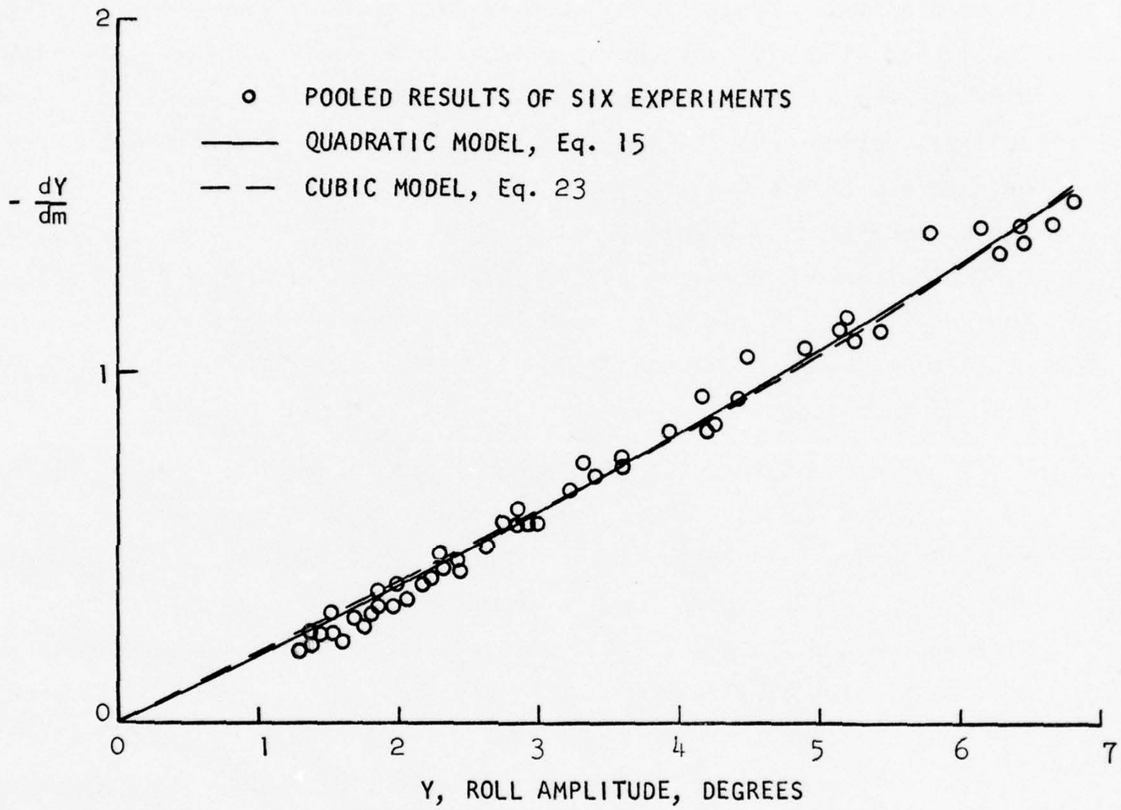

FIGURE 9. FIT OF EQUATIONS 15 AND 23 TO ROLL EXTINCTION DATA OBTAINED IN EXPERIMENTS WITH H.M.S. NUBIAN

Table 2 contains the numerical values of the coefficients obtained in the fits to roll extinction data shown in Figures 7 to 9

For present purposes, a selection of the available model experimental data was made and similar analyses carried out. Vossers[17] presents a set of unfaired model roll extinction data for one experiment in Figure 66 of his text. Figure 10 shows this data and the least-square fits of Eq. 13 and 21 (which are the equations corresponding to the roll extinction per whole cycle). Figure 11 indicates a similar result using model roll extinction data for one experiment presented by Blagoveshchensky[18] on page 640 of his text. Figure 12 is a third similar analysis for the pooled results of a number of experiments by Lalangas[3] on a Series 60, 0.60 block parent model at zero speed. Just as in Figure 9 for ship data, the results in Figures 10 through 12 for model data indicate that the differences between the quadratic and cubic models are less than or the same as data scatter.

Of the many model roll extinction experiments carried out by Martin, et al[20*], one was abstracted for analysis and this was the case where their model was bare (without bilge keels or artificial roughness) and was at zero speed. This extinction result was presented as a faired line. This line was read off at even increments of roll amplitude and the values appear as circles in Figure 13. The least square fits of both the quadratic and cubic models to the points representing the faired line are also shown. In this instance the cubic fit appears slightly better than the quadratic.

The last example picked was presented as a faired experimental result by Vossers[17] (p 116) and is the equivalent of a roll extinction curve. The result was obtained by Motora, et al[21*] by means of a forced

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*20. Martin, M., McLeod, C. and Landweber, L., "Effect of Roughness on Ship Rolling," Schiffstechnik, Bd. 7 - Heft 36, 1960.

*21. Motora, S., Shimizu, H. and Nishikido, T., "On the Measuring of the Damping Resistance of Roll Through a Large Angle by a Forced Oscillation Method," J. Zosen Kyokai, Vol. 100, 1957.

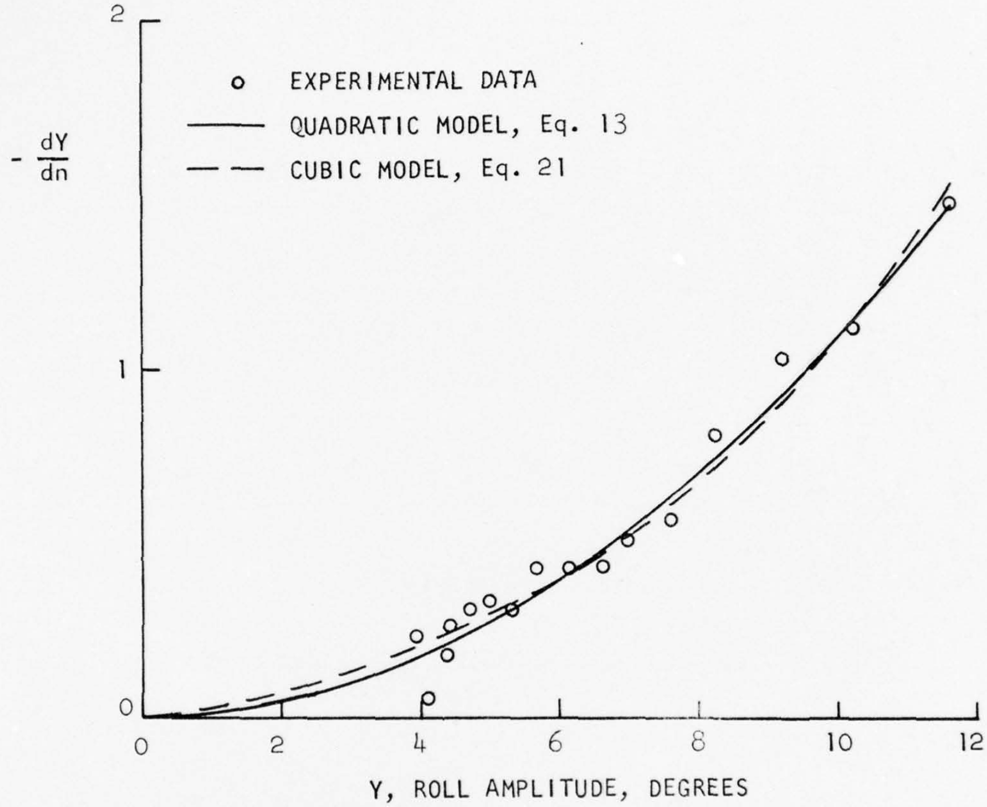

FIGURE 10. FIT OF EQUATIONS 13 AND 21 TO MODEL ROLL EXTINCTION DATA PRESENTED BY VOSSERS (REF. 17)

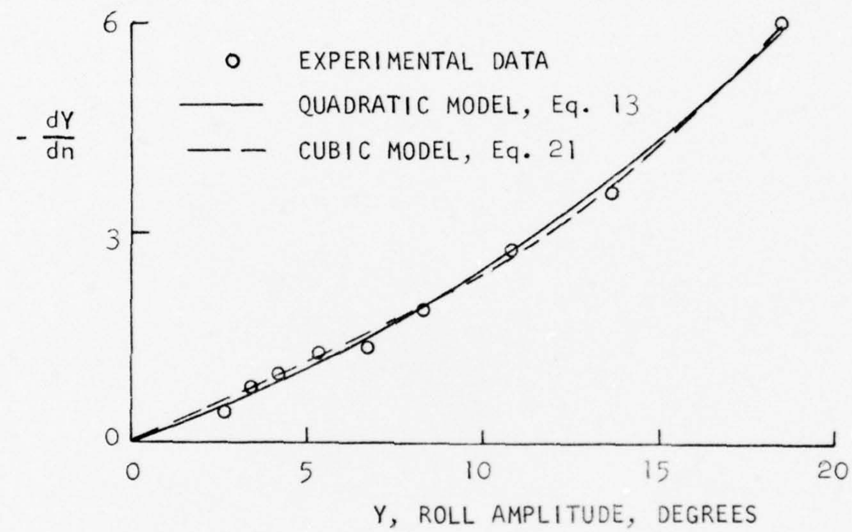

FIGURE 11. FIT OF EQUATIONS 13 AND 21 TO MODEL ROLL EXTINCTION DATA PRESENTED BY BLAGOVESHCHENSKY (REF. 18)

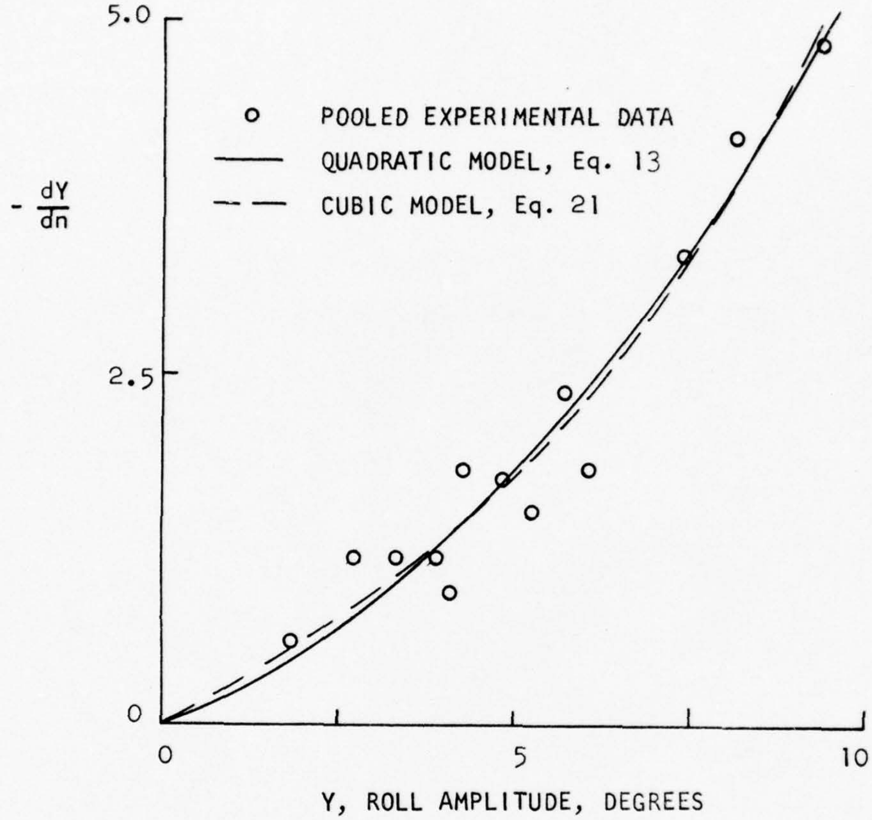

FIGURE 12. FIT OF EQUATIONS 13 AND 21 TO MODEL ROLL EXTINCTION DATA PRESENTED BY LALANGAS (REF. 3)

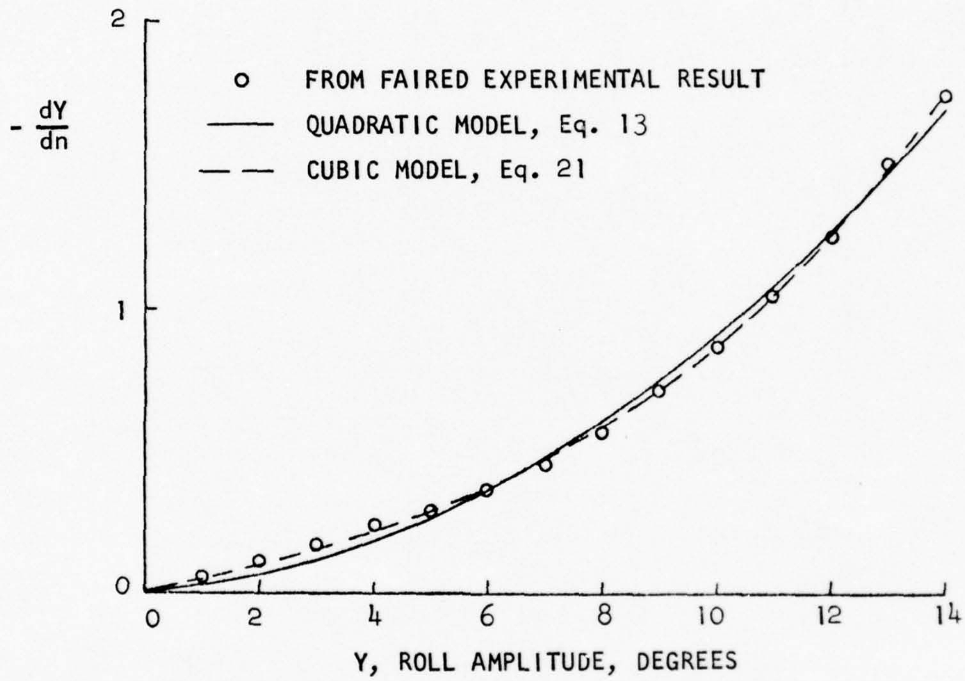

FIGURE 13. FIT OF EQUATIONS 13 AND 21 TO MODEL ROLL EXTINCTION DATA PRESENTED BY MARTIN (REF. 20)

oscillation technique, and is notable for the extremely large roll angles involved. As in the previous example, the faired data was read off at roughly uniform intervals of roll angle and the result treated as data in least square fits of Eq. 13 and 21. Figure 14 indicates the results. The cubic model yields a physically believable result, the quadratic model does not.

Table 3 includes all numerical values of the fitted coefficients for the model data of Figures 10 through 14.

Table 4 indicates some of the particulars of the ships and models involved in Figures 2 through 14. Particulars for two ships and two models could not be identified. As may be noted in Table 4, the data analyzed originates from a variety of ships and models. Taking ships and models together, the lengths ranged from 5 to 550 feet. The type of appendages range from nothing at all through bilge keels on relatively modern forms to bar keels in warships of a century ago. The initial angle in the sallying experiments (see Tables 1 through 3) ranged from 5.5 to 32^{o}. It appears from the results that the cubic approximation is qualitatively correct, and can be made to be quantitatively reasonable within the range of validity of the quadratic model.

The results in Tables 1 through 3 may be used to approximate the relative accuracy of the various damping function estimates. Given the scatter in even carefully conducted experiments (Figure 9, for example) the quadratic model with empirically determined coefficients itself appears not too firm an extrapolation beyond the range of roll velocity actually represented in the sallying experiment. Accordingly, it does not seem reasonable to compare accuracy of approximations outside this range.

Within the assumptions of the theory of the last section, the maximum instantaneous velocity involved in a particular sallying experiment may be approximated as the product of initial roll amplitude and the natural rolling frequency; that is, $(Y_o \omega)$. Thus for comparative

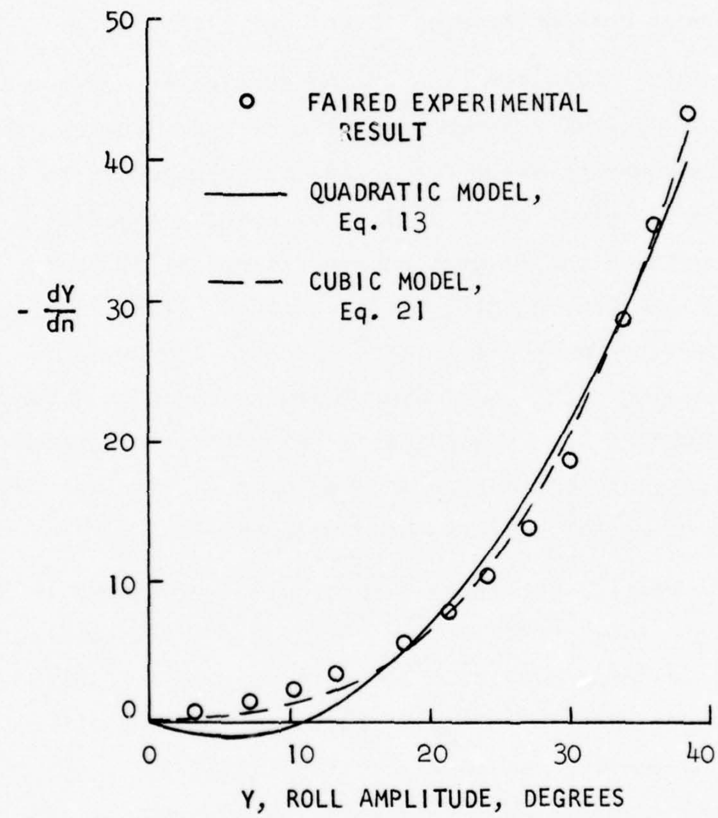

FIGURE 14. FIT OF EQUATIONS 13 AND 21 TO MODEL ROLL EXTINCTION DATA DERIVED FROM FORCED OSCILLATION EXPERIMENTS BY MOTORA, ET AL (REF. 21)

evaluation purposes, roll velocity of the experiments is assumed to lay within the range $(-Y_o\omega < \dot{\varphi} < Y_o\omega)$. Defining a normalized roll velocity as:

$$\Omega = \dot{\varphi}/Y_o\omega$$

The corresponding range for comparative evaluation becomes $(-1 < \Omega < 1)$. Defining the empirical quadratic damping function as $\underline{\hat{N}}(\Omega)$, (Eq. 3) and substituting the values of N_{21} and N_{22} from Eq. 14:

$$\underline{\hat{N}}(\Omega) = \frac{B_1 Y_o}{\pi}\left[\delta_1 + \frac{3\pi\delta_2 Y_o}{8}\,|\Omega|\right]\Omega \qquad (25)$$

The parameters δ_1, δ_2 and Y_o are empirically determined, and for the present case are contained in Tables 1 through 3. The parameter B_1 is conventionally estimated as displacement times transverse metacentric height, and these are given in Table 4 for some of the ships and models involved in the present study.

The cubic approximation to roll damping which was suggested is Eq. 7. This expression contains a parameter $\dot{\varphi}_c$ which is the assumed roll velocity range for which the approximation of Eq. 7 is designed. For present purposes, it is assumed that $\dot{\varphi}_c$ is some fraction of the maximum roll velocity range observed in experiment, say:

$$\dot{\varphi}_c = \epsilon Y_o \omega$$

With this definition the values of N_{21} and N_{22} given by Eq. 14 may be substituted into Eq. 7 to give the empirical cubic approximation, $\tilde{\hat{N}}(\Omega)$, analagous to Eq. 25:

$$\tilde{\hat{N}}(\Omega) = \frac{B_1 Y_o}{\pi}\,\Omega\left[\delta_1 + \frac{15\pi}{128}\,\delta_2 Y_o\,\epsilon + \frac{35\pi\,\delta_2 Y_o}{128\epsilon}\,\Omega^2\right] \qquad (26)$$

Considering the data for those cases where unfaired experimental results are given (Figures 9, 10, 11, 12) the basic experimental process appears typically one of fixed dynamic range. The magnitude of scatter appears independent of the magnitude of roll extinction and varies between approximately 4 and 10% of the highest value of roll extinction observed in the experiments. The implication is that the uncertainty in the fitted line for the extinction is closer to a constant, rather than to a percentage of extinction. It may be noted in Eq. 25 that the numerical coefficient of the second term is roughly unity, and thus the form of the quadratic damping function is very nearly that of the extinction, Figures 7 through 14. These charts could nearly be converted to those for empirical quadratic damping functions (Eq. 25) by simple scale changes. It thus appears reasonable to expect that the uncertainties in the estimation of the damping function, Eq. 25, are also constant rather than a percentage of the damping function for any roll velocity.

Following this, the difference between the approximation, Eq. 26 and the quadratic damping function may be formed and the result divided by $\hat{\underline{N}}(1)$ to form an estimate of the errors in the approximation relative to the value of damping function which corresponds to the highest roll velocity of the experiment. The result for positive Ω is:

$$\frac{\hat{\underset{\sim}{N}}(\Omega) - \hat{\underline{N}}(\Omega)}{\hat{\underline{N}}(1)} = \epsilon^2 \frac{\frac{5}{16}(\Omega/\epsilon) + \frac{35}{48}(\Omega/\epsilon)^3 - (\Omega/\epsilon)^2}{1 + \frac{8}{3\pi}\frac{\delta_1}{\delta_2 Y_o}} \tag{27}$$

If, as is implied by the approximation, Eq. 7, the errors are evaluated for the designed approximation range $(0 < \Omega < \epsilon)$ the maximum percentage error implied by Eq. 27 is about 4% for the case when $\epsilon = 1$ and there is no linear damping $(\delta_1 = 0)$. The error shrinks sharply for roll velocity approximation ranges less than the experimental range, and tends toward zero as the quadratic portion of damping (δ_2) gets small relative to the linear part. Evaluation of Eq. 27 with the coefficients of Tables 1 through 3 indicates that for typical mixes of linear and quadratic damping, and within the experimental range of roll velocity, the maximum error of the approximation might be expected to be 2 or 3% of the value of the function at the maximum experimental roll velocity. This magnitude of error appears comparable with the magnitude of experimental uncertainty, and it therefore appears that the approximation, Eq. 7 can be justified.

The degree of fit to observable data shown in Figures 2 through 14 for the cubic model suggests another approximation to roll damping, which is quite simply to assume that it follows the cubic model (Eq. 18) in the first place. In some of the results exhibited in Figures 2 through 13 the fit of the cubic model is slightly superior. In fact, for the large amplitude results of Figure 14 the fit is much more realistic.

If there are two analytical models which fit the observable data with roughly the same magnitude of error, the choice between the two must be made on bases other than the fit itself. In absence of such other considerations, the models may be considered equally good within the limitations of observable data -- outside the range of data both are extrapolations. Physically, it is expected that the damping function be odd in roll velocity and positive for positive roll velocity. Both the mixed quadratic and cubic models can be made to fit this criterion through choice of coefficients. The primary consideration in the long history of preference for the quadratic damping representation seems to be that the drag on a body in a real fluid is proportional to velocity squared if the velocity is high enough.

A REANALYSIS OF DATA ON PLATES AND CYLINDERS IN OSCILLATING FLOW

Since ships are ordinarily equipped with bilge keels or the equivalent to increase roll damping, it has appeared reasonable to consider the unsteady forces on oscillating plates as a fundamental aspect of the roll damping problem. The forces on fixed objects (plates, cylinders, etc.) in oscillating flow are the equivalent problem in many civil engineering problems, and a quadratic drag model is commonly used in this case. Indeed the Morison[22*] approach for forces on fixed structures is exactly this and is extensively used in ocean engineering.

*22. Morison, J.R., Johnson, J.W. and O'Brien, M.P., "Experimental Studies of Forces on Piles," Proceedings of the Fourth Coastal Engineering Conference, 1953.

Keulegan and Carpenter[23*] undertook a laboratory investigation of forces on cylinders and plates in an oscillating fluid with the objectives of clarifying the accuracy of the Morison predictor equation, and of attempting a correlation of mean experimentally determined drag coefficients. In this work, forces upon cylinder and plate specimens were actually measured, rather than inferred through considerations of the decay of free oscillations of a spring-mounted specimen, which is a more common laboratory technique.

The time domain predictor equation for the force (F) per unit length experienced by a cylindrical form, which was the starting point of the Keulegan and Carpenter[23] investigation, was:

$$F = C_m \rho V_m \dot{U} + \frac{1}{2} C_d \rho A |U| U \tag{28}$$

where C_m and C_d are inertia and drag coefficients, A is projected area of the cylinder per unit length, V_m is the volume per unit length, and U and $\dot{U}$ are the undisturbed velocity and acceleration of the fluid of mass density ρ.

Keulegan and Carpenter presented a brief two-dimensional theoretical momentum analysis for a circular cylinder from which they recovered the first (added mass) term of Eq. (28), and by assuming steady flow, the second. They were forced in effect to say that while the quadratic model for oscillatory drag and the existence of a drag coefficient for unsteady flow are plausible representations, they are not definite results of theory. As in the case of ship roll damping, the justification for the model lies entirely in how well it may be shown to work empirically.

The experiments of Keulegan and Carpenter involved the measurement of forces on cylinders of diameter D or plates of width, D, in a harmonically varying current. In setting up their data reduction procedures

*23. Keulegan, G.H. and Carpenter, L.H., "Forces on Cylinders and Plates in an Oscillating Fluid," NBS Report 4821, National Bureau of Standards, U.S. Department of Commerce, September 1956.

they assumed that the velocity, U:

$$U = -U_m \cos \omega t \tag{29}$$

where U_m is the velocity amplitude, ω is circular frequency and t is time. They chose a non-dimensional form of Eq. (28) in which the areas and volumes are replaced by the appropriate functions of D, and after substitution of Eq. (29) obtained Eq. (30) as a basic model:

$$\frac{F}{\rho U_m^2 D} = \frac{\pi}{4} C_m \cdot \frac{D\omega}{U_m} \sin \omega t - \frac{C_d}{2} |\cos \omega t| \cos \omega t \tag{30}$$

For practical reasons they chose to analyze their data in an odd Fourier Series so that their initial data reduction model was:

$$\frac{F}{\rho U_m^2 D} = \sum_{n=1,3...} A_n \sin n\omega t + \sum_{n=1,3...} B_n \cos n\omega t \tag{31}$$

Expanding $|\cos\omega t| \cos\omega t$ in a Fourier Series and substituting the results in the equivalent of Eq. (31) they obtained:

$$\frac{F}{\rho U_m^2 D} = A_1 \sin\omega t + B_1' |\cos\omega t| \cos\omega t + \Delta R \tag{32}$$

where:

$$\Delta R = \sum_{n=3,5...} A_n \sin n\omega t + \sum_{n=3,5...} B_n' \cos n\omega t \tag{33}$$

and the relationship between the primed and un-primed "B" coefficients in Eqs. (31), (32) and (33) are:

$$\begin{aligned} B_1' &= \frac{3\pi}{8} B_1 \\ B_3' &= B_3 - B_1/5 \\ B_5' &= B_5 + B_1/35 \\ &\cdots\cdot \end{aligned} \tag{34}$$

The function ΔR of Eq. (33) is what Keulegan and Carpenter called the "remainder function"; that is, it is effectively the error committed in assuming that the linear plus quadratic model (Eq. 30) fits the data. In their analysis of this error function they concluded that the fifth harmonic coefficients were within the order of error in reading the original records. Accordingly, all the basic results of their data reduction procedure are compressed into four coefficients (A_1, A_3, B'_1, and B'_3), and these are tabulated in[23] for each of 93 runs.

It was of interest to find out in what way the process had improved the original data reduction procedure, Eq. (31). The tabulations in[23] and Eq. (34) allow the original fit to be reconstructed, neglecting the fifth harmonics. (If B'_5 is assumed to be zero, $B_5 \approx 3\%$ of B_1, possibly also near data resolution.) This process was carried out for all the data and results in the availability of all the coefficients in Eq. (35) which will be called the "reconstructed model":

$$\frac{F}{\rho U_m^2 D} = A_1 \sin\omega t + A_3 \sin 3\omega t + B_1 \cos\omega t + B_3 \cos 3\omega t \tag{35}$$

In the report[23] the basic quadratic model was justified for the most part as an adequate approximation. This is written as Eq. (36) for illustration (and will be called the "K & C model"):

$$\frac{F}{\rho U_m^2 D} = A_1 \sin\omega t + B'_1 |\cos\omega t| \cos\omega t \tag{36}$$

Keulegan and Carpenter in justifying Eq. (36) had tabulated the peak non-dimensional force during the cycle for each run and an index of its position in the cycle. Equation (35) was evaluated and the magnitude and position of the peaks as shown by the "reconstructed model" were found. The results are shown in Tables 5, 6 and 7. Tables 5 and 6 include all of Keulegan and Carpenter's cylinder data, and Table 7 includes all their plate data. The second column of each table is the observed maximum force and the third is that computed from Eq. (35). The fourth and fifth columns are percent error in predicting the peak (the error in the "K & C model", Eq. (36) was copied from [23]). The reconstructed model reflects the observations as well or better than the K & C model in 50 out of 57 runs with cylinders (Tables 5 and 6) and in 33 out of 36 runs for plates (Table 7). The reconstruction for plates involves a 5% error at worst vice a 12% error for the K & C model.

The index of position of maximum used by Keulegan and Carpenter was a "phase" equal to $(\pi - \omega t_m)$ (where t_m is the time at which the peak occurred). These phases are given in degrees in the sixth column for the observed data. Columns 7 and 8 give the corresponding results for the reconstructed model and for Eq. (36). In many cases the reconstruction seems to yield a much better approximation to the observation than does Eq. (36). The reconstruction is only really badly out in two cases for cylinders (Runs 18 and 81). In both cases a detailed examination of the reconstructed data suggested that the original data must have had two peaks of nearly equal magnitude, or an ill-defined broad maximum.

Nothing in Tables 5 through 7 suggests that the basic odd Fourier expansion has been improved by the quadratic model. It is probable that to achieve comparable or better accuracy two terms from the remainder function (Eq. 33) must be added to Eq. (36).

Keulegan and Carpenter include in[23] four charts depicting original force data after non-dimensionalization. These charts appear herein as Figures 15 through 18. In each, the variation of force over one oscillatory cycle is shown, as is Keulegan and Carpenter's evaluation of the quadratic model, Eq. (36). (In these charts $\theta = \omega t$.) To each of these charts an evaluation of Eq. (35) has been added. It is shown as a series of circles corresponding to computed points during the cycle.

In Figure 15 there is not much to choose between the data, the reconstruction, and the quadratic model. This is because the A_1 term (added mass) dominates.

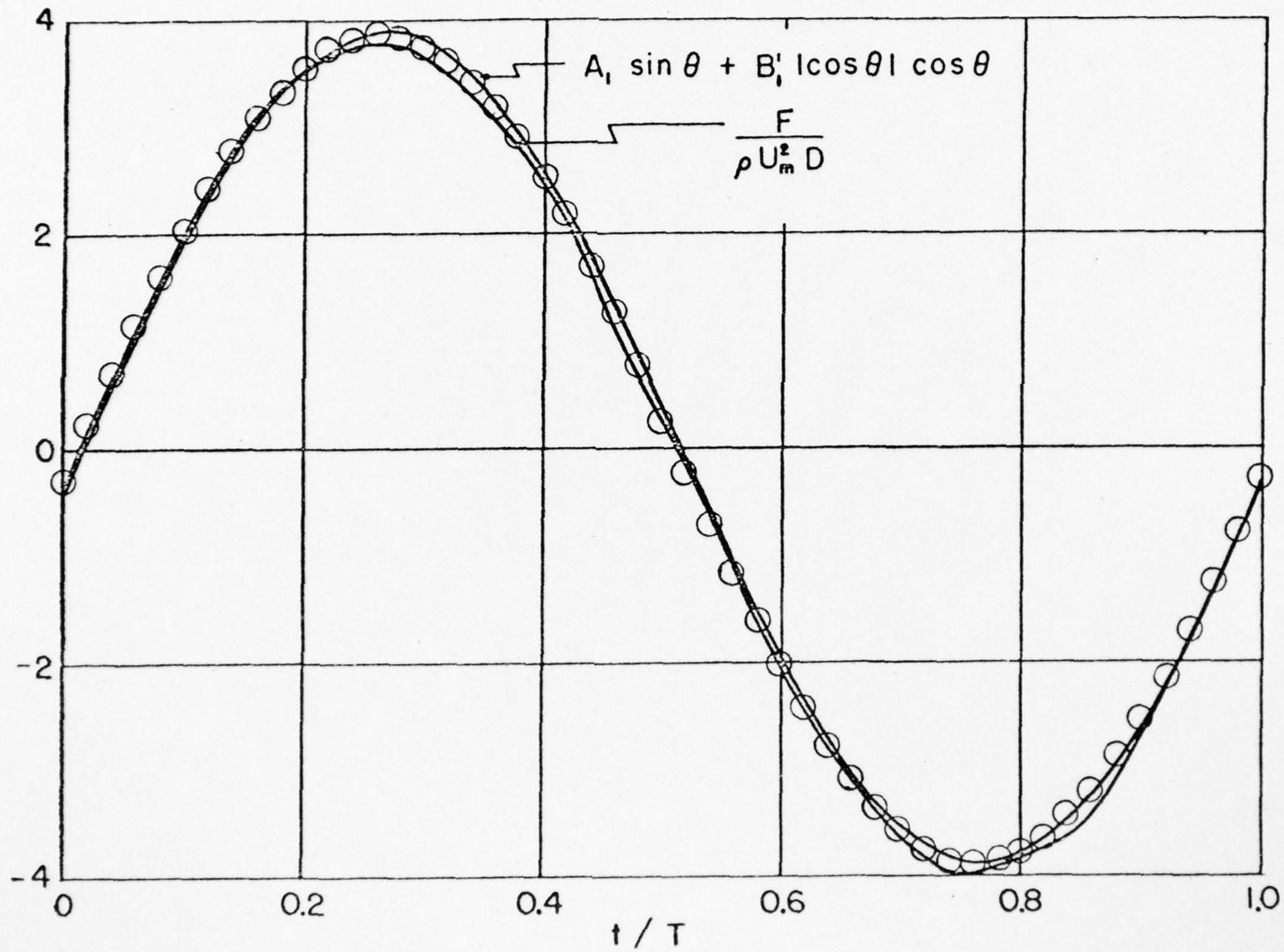

FIGURE 15. COMPARISON OF MEASURED AND COMPUTED FORCES ON A CYLINDER (FROM REF. 23, RUN 9)

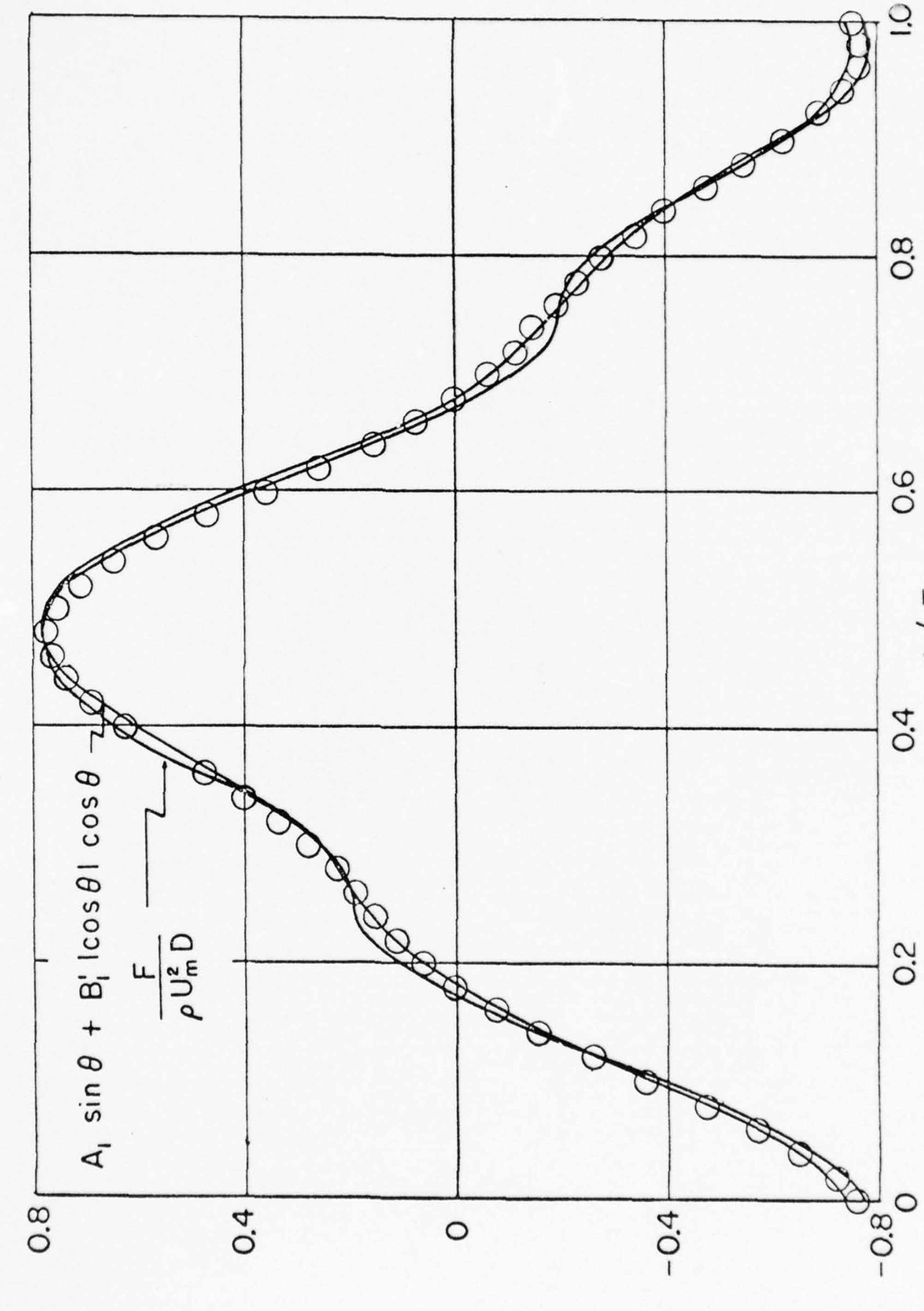

FIGURE 16. COMPARISON OF MEASURED AND COMPUTED FORCES ON A CYLINDER (FROM REF. 23, RUN 93)

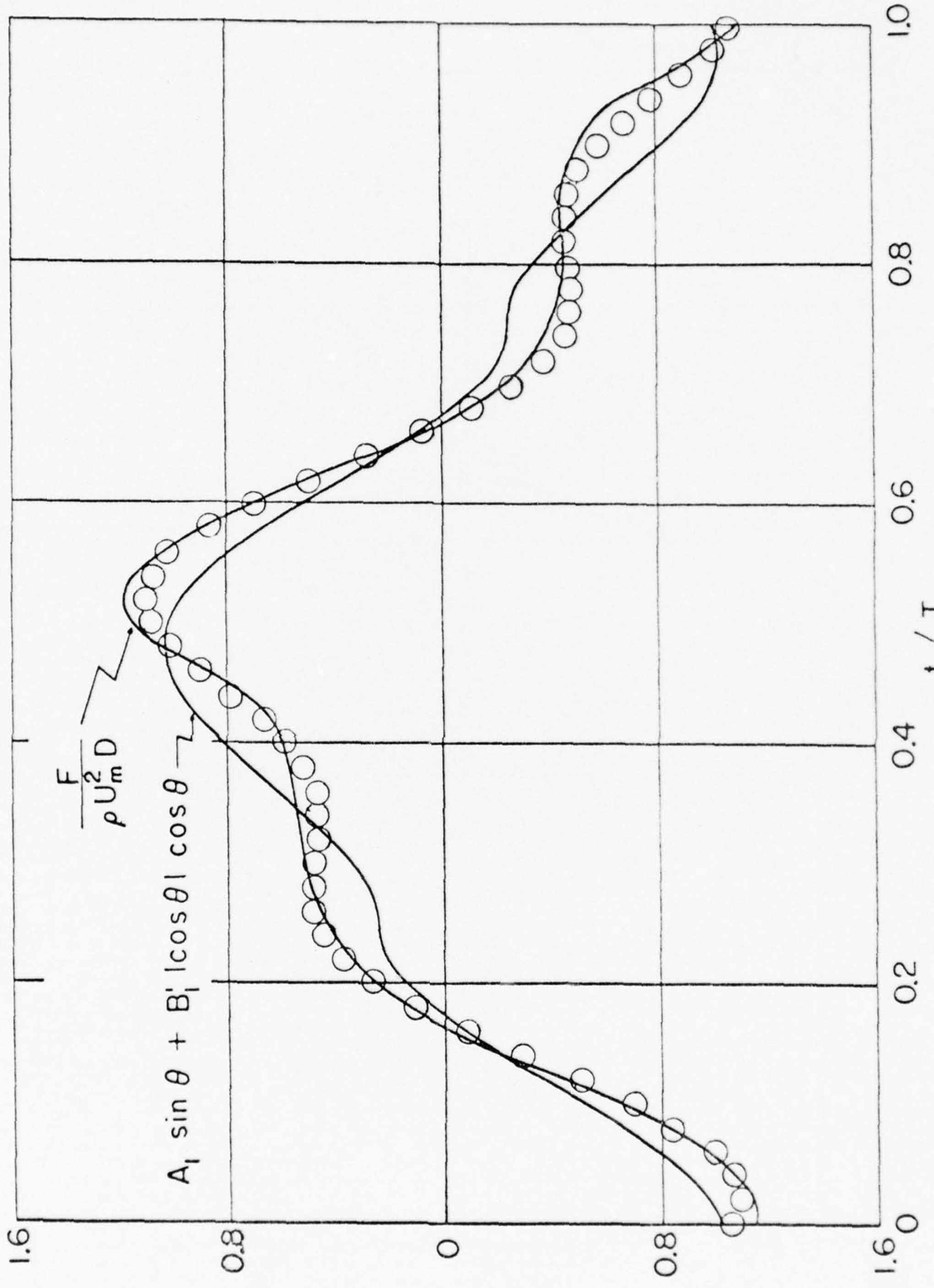

FIGURE 17. COMPARISON OF MEASURED AND COMPUTED FORCES ON A CYLINDER (FROM REF. 23, RUN 82)

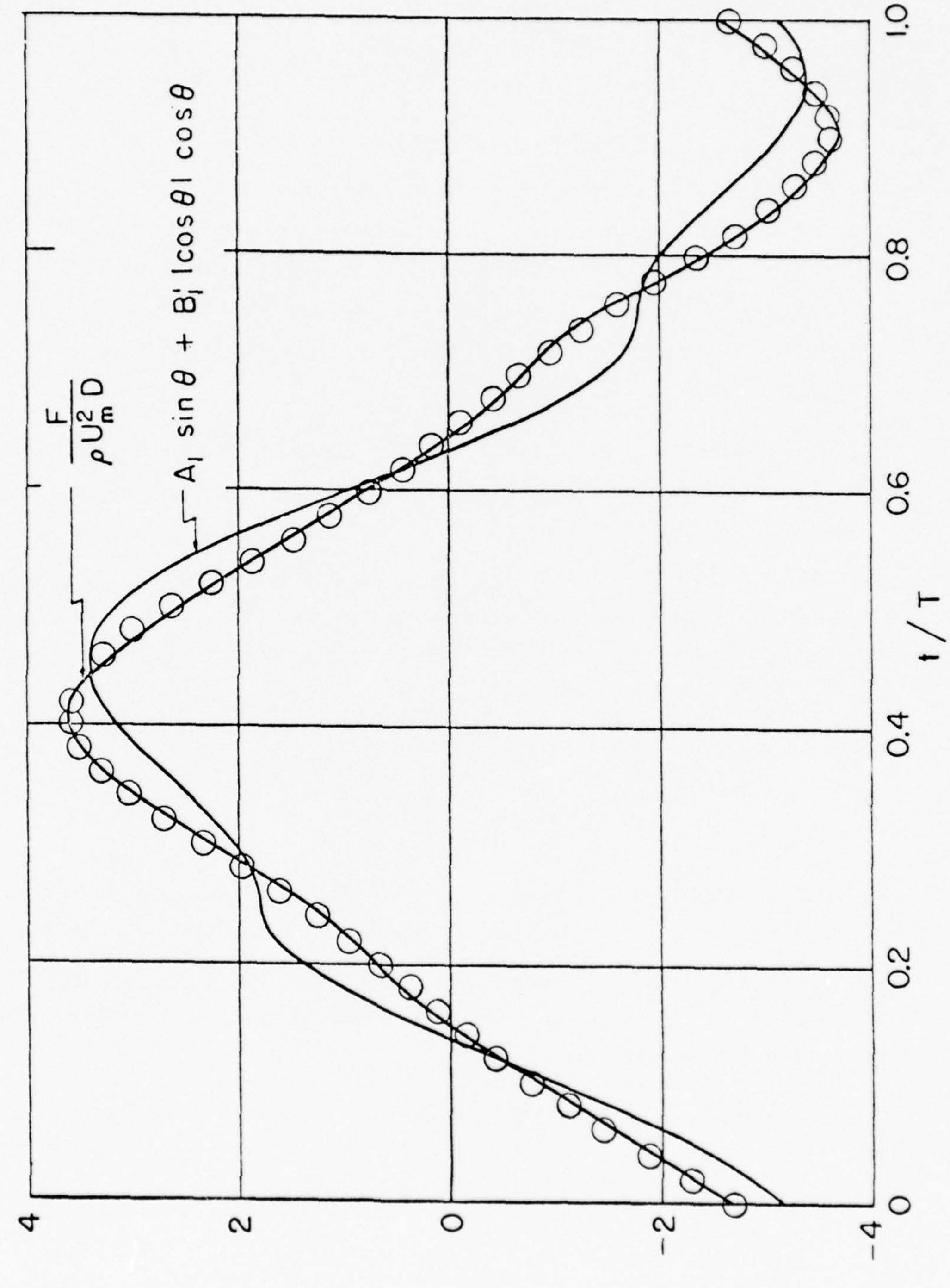

FIGURE 18. COMPARISON OF MEASURED AND COMPUTED FORCES ON A PLATE (FROM REF. 23, RUN 54)

In Figure 16 the reconstruction is a reasonable representation of the data, as is the quadratic model except for the regions near $t/T = .25$ and .75. There the velocity is zero, the sharp inflection to be expected from a term of the form $|U|U$ is apparent, and is not reflected by the original data.

In Figure 17 the reconstruction represents the data reasonably well, apart possibly from what might be a fifth harmonic contribution. The quadratic model on the other hand deviates strongly from the data. Again the characteristic inflections of the quadratic model at $(t/T) = .25$ and .75 are not reflected by the original data. Figure 18 indicates a similar situation. The details of the quadratic approximation do not agree with the data.

It appears that the quadratic model for the forces on an object in oscillatory flow has no particular magic. It does not appear that the single term $|\cos\omega t|\cos\omega t$ has the power to eliminate terms involving higher harmonics -- in fact the use of this form may at times create deviations from observation which would require the inclusion of more terms for a reasonably close representation of observation in the time domain.

Considering the form of the trigonometric identity for $\cos(3\omega t)$, Equation 35 implies a time domain prediction equation of the form:

$$F = P_1\dot{U} + P_3\dot{U}^3 + Q_1 U + Q_3 U^3 \qquad (39)$$

According to the results, this equation would be expected to be a better approximation in the time domain than Eq. 28 which contains the quadratic term. Preference in practice would be given to Eq. 28 if the drag coefficient (C_d) is really a constant. However, Keulegan and Carpenter show that this is not the case.

Going back to the ship rolling problem, a linear plus cubic representation for force on an oscillating plate translates into a linear plus cubic expansion in the roll damping produced by bilge keels, and it is possible to speculate that the similar model proposed as an approximation in the last section may be more generally valid than was thought.

CONCLUDING REMARKS

The reason for embarking on the present work was that the quadratic time domain representation for roll damping which has been in use for the past century is a serious analytical obstacle which, it appears, must be overcome if improvements in techniques for the prediction of non-linear rolling in random seas are sought. The basis for the acceptance of the mixed linear-plus-quadratic time domain roll damping model is almost entirely empirical, as are what are taken to be realistic coefficients in this model.

A mixed linear-plus-cubic model, in which the coefficients are functions of those of the linear-plus-quadratic model, was proposed as an approximation. The results of analyses indicate that this approximation is both quantitatively and qualitatively reasonable within the range and scatter of available experimental data.

The additional result that a linear-plus-cubic roll damping model fits experimental data about as well as (sometimes better than) the linear-plus-quadratic model, gives rise to the speculation that the cubic model might be closer to an "equivalent approach" than to an "approximation." Because the forces on plates in oscillating flow are fundamental to roll damping, a re-analysis of some experimental data on the forces on plates and cylinders was undertaken. The results of this analysis were that an odd series in velocity appeared to fit observable data as well as and sometimes significantly better than the quadratic model usually employed. This result is consistent with those for ship rolling data and reinforces the above speculation.

RECOMMENDATIONS

The analyses herein suffer from excessive empiricism, and the conclusions are accordingly weakened. However, there appears to be no alternative at present. It seems clear that fundamental advances in the understanding of the mechanism for and theory of the generation of forces on objects in oscillatory flow will have to be made before significant advances can be expected in the quantification of ship roll damping.

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Table 1

Coefficients Obtained in Fits to Declining Angle Data

Ship	Figure	Quadratic Model a	Quadratic Model b	RMS Dev.	Cubic Model c	Cubic Model d	RMS Dev.	Y_o	Froude's Analysis[2] a	Froude's Analysis[2] b
INCONSTANT	2	.0373	.00456	0.06°	.0507	.000319	0.1°	9.6°	.035	.0051
INCONSTANT	2	.0419	.00298	0.03°	.0466	.000410	0.03°	5.6°		
VOLAGE	3	.0296	.00729	0.04°	.0482	.000635	0.04°	8.5°	.028	.0073
ELORN	4	.110	.00304	0.22°	.130	.000090	0.30°	32.1°	.0947	.0044
SULTAN	5	.0250	.00192	0.07°	.0320	.000117	0.06°	15.2°	.02675	.001663
DEVASTATION	6	.0800	.0141	0.1°	.102	.00170	0.06°	9.2°	-	-

Table 2

Coefficients Obtained in Fits to Roll Extinction Data for Ships

Ship	Figure	Quadratic Model a	Quadratic Model b	Cubic Model c	Cubic Model d	Y_o	Gawn's Analysis[19] a	Gawn's Analysis[19] b
KING GEORGE V	7	.120	.00885	.135	.00114	5.5°	-	-
VIVIAN	8	.130	.00534	.147	.000379	10.0°	-	-
NUBIAN	9	.181	.00636	.196	.000630	6.8°	.175	.008

Table 3

Coefficients Obtained in Fits to Roll Extinction Data for Models

Source	Figure	δ_1	δ_2	γ_1	γ_3	Y_0
Vossers	10	.00219	.0109	.0428	.000663	11.6°
Blagoveshchensky	11	.164	.00834	.207	.000341	18.4°
Lalangas	12	.178	.0364	.282	.00285	9.4°
Martin	13	.0139	.00765	.0467	.000400	14.0°
Motora	14	-.414	.0380	.032	.000723	38.4°

Table 4

Particulars of Some of the Ships and Models for Which Comparisons are Made in Figures 2 through 14

Figure	Name/Source	Length (ft)	Beam (ft)	Displ. (tons)	$\overline{GM}_t$ (ft)	T (sec)
2	INCONSTANT	337.0	50.3	5200	2.8	15.97
3	VOLAGE	270.0	42.0	2990	2.9	11.59
5	SULTAN	325.0	59.0	9205	2.5	17.65
7	KING GEORGE V	555.0	89.0	25550	5.53	14.7
8	VIVIAN	309.0	29.5	1185	2.3	9.1
9	NUBIAN	364.0	36.5	2541	2.96	9.8
11	BLAGOVESHCHENSKY	11.2	2.22	0.214	0.51	1.34
12	LALANGAS	5.0	0.67	0.0148	0.018	1.35
13	MARTIN	6.67	0.94	0.0333	0.091	1.51

TABLE 5

Comparison of Maximum Forces on Cylinders with K & C Model and Reconstructed Model

Run	Maximum Force		% Error		"Phase" of Maximum		
	Observed	Recon-structed	Recon-structed	K&C Model	Observed	Recon-structed	K&C Model
1	0.78	0.69	-11	-4	-6	6	27
2	0.95	0.93	-1	-5	74	82	50
3	1.10	1.07	-3	-7	73	78	47
4	1.34	1.33	0	-6	74	73	90
5	1.68	1.67	0	-4	78	76	90
6	1.86	1.85	0	-1	79	76	90
7	2.41	2.41	0	-2	72	73	90
8	2.95	2.94	0	-1	79	75	90
9	3.90	3.88	0	-1	87	82	90
10	1.03	0.89	-13	-15	-10	-8	11
11	0.97	0.87	-9	-4	-7	2	23
12	1.35	1.33	-1	-7	78	82	67
13	2.38	2.34	-1	-2	76	67	90
14	3.06	3.08	0	-1	79	76	90
15	0.95	0.90	-5	-7	-13	-11	7
16	1.13	1.05	-6	-11	-10	-11	7
17	1.30	1.17	-10	-14	-8	-6	10
18	1.13	1.10	-2	4	-1	82	30
19	1.18	1.07	-9	-9	-3	0	17
20	0.71	0.66	-6	-7	-1	-2	8
21	0.78	0.75	-3	-3	-2	-2	8
22	1.03	0.95	-8	-15	-5	-8	8
23	1.24	1.17	-6	-12	-2	-9	7
24	1.34	1.20	-10	-15	-6	-6	11
25	0.58	0.59	1	2	7	4	7
26	0.64	0.63	-1	-2	5	4	8
27	0.74	0.74	0	1	2	4	9
28	0.81	0.78	-3	-4	3	2	10
29	0.99	0.93	-5	-10	-4	-6	8
30	0.56	0.57	1	0	8	6	7

TABLE 6

Comparison of Maximum Forces on Cylinders with K & C Model and Reconstructed Model

	Maximum Force		% Error		"Phase" of Maximum		
Run	Observed	Recon-structed	Recon-structed	K&C Model	Observed	Recon-structed	K&C Model
31	0.67	0.66	-1	-1	8	6	8
32	0.71	0.71	0	1	9	8	9
33	0.76	0.76	0	1	11	8	11
34	0.57	0.57	0	-2	9	6	6
35	0.60	0.61	1	0	8	2	6
36	0.72	0.71	-1	0	6	4	6
37	0.72	0.73	0	1	5	4	7
38	0.76	0.77	1	0	12	6	8
39	0.56	0.56	0	-4	7	6	6
40	0.66	0.66	0	-2	10	4	5
41	0.70	0.71	1	3	6	4	5
78	1.18	1.07	-9	-14	0	-6	10
79	1.21	1.09	-10	-12	-2	-6	10
80	1.31	1.17	-10	-8	3	4	24
81	1.30	1.37	5	5	17	87	48
82	1.16	1.10	-5	-10	-5	-9	7
83	1.29	1.20	-6	-10	-2	-9	7
84	1.43	1.27	-11	-15	-2	-8	10
85	1.30	1.19	-8	-2	4	8	45
86	0.61	0.61	0	-2	9	6	6
87	0.67	0.67	0	-1	8	4	6
88	0.72	0.73	0	1	6	4	6
89	0.76	0.76	0	3	11	6	8
90	0.66	0.66	0	-2	8	6	7
91	0.70	0.71	1	0	8	6	7
92	0.70	0.72	3	3	9	6	7
93	0.77	0.78	1	1	8	6	7

TABLE 7

Comparison of Maximum Forces on Plates
with K&C Model and Reconstructed Model

Run	Maximum Force: Observed	Maximum Force: Reconstructed	% Error: Reconstructed	% Error: K&C Model	"Phase" of Maximum: Observed	"Phase" of Maximum: Reconstructed	"Phase" of Maximum: K&C Model
42	5.22	5.05	-3	-9	27	26	17
43	5.16	5.08	-1	-8	25	24	16
44	5.47	5.53	1	-3	22	22	17
45	5.97	6.01	0	-5	24	24	20
46	6.57	6.61	0	-2	29	24	19
47	3.46	3.37	-2	-12	39	35	20
48	4.13	4.06	-1	-8	32	29	17
49	4.73	4.64	-1	-8	32	28	17
50	5.32	5.26	-1	-10	27	28	19
51	6.70	6.64	0	-6	25	24	19
52	2.96	2.89	-2	-7	34	26	14
53	3.17	3.11	-1	-6	35	29	17
54	3.67	3.61	-1	-7	34	29	17
55	4.19	4.12	-1	-7	31	28	16
56	5.03	4.93	-1	-12	31	29	19
57	2.06	1.99	-3	0	25	18	4
58	2.23	2.13	-4	-3	29	18	5
59	2.54	2.49	-2	-5	37	28	13
60	3.10	3.06	-1	-8	40	33	19
61	1.53	1.59	3	8	23	15	8
62	1.96	1.92	-2	1	24	11	4
63	2.11	2.07	-1	-2	29	22	7
64	2.51	2.46	-1	-7	35	29	14
65	1.48	1.53	3	7	11	9	7
66	1.62	1.71	5	10	15	9	6
67	1.82	1.82	0	3	25	13	7
68	2.26	2.20	-2	-2	29	20	7
69	1.22	1.21	0	1	8	6	6
70	1.41	1.41	0	2	8	6	6
71	1.48	1.51	2	5	10	9	7
72	1.63	1.68	3	5	14	11	8
73	0.93	0.94	0	-1	11	8	7
74	1.03	1.03	0	-1	11	6	5
75	1.15	1.16	0	2	9	6	6
76	1.20	1.22	1	3	12	6	6
77	1.31	1.31	0	0	9	8	7

PRINCIPAL NOTATION

A	Area
A_n, B_n	Coefficients in odd Fourier expansion
a, b	Coefficients in quadratic fitting equation for roll extinction per half cycle
$\underline{B}(\varphi)$	Restoring function
B_1	Linear coefficient of restoring function
B'_n	Modified Fourier expansion coefficients
C_m, C_d	Inertia and drag coefficients
c, d	Coefficients in cubic fitting equation for roll excitation per half cycle
D	Diameter of cylinder or width of plate
F	Force on oscillating cylinder or plate
$F(t)$	Excitation
I	Roll inertia
$\underline{N}(\dot{\varphi})$	Damping function
$\underline{\hat{N}}(\Omega)$	Empirical quadratic damping function
$\underset{\sim}{N}(\dot{\varphi})$	Cubic approximation to the quadratic roll damping model
$\underset{\sim}{\hat{N}}(\Omega)$	Empirical cubic approximation to quadratic roll damping
$\underline{\underline{N}}(\dot{\varphi})$	Cubic damping function model
N_{21}	Coefficient of linear part of quadratic roll damping model
N_{22}	Coefficient of quadratic part of roll damping model
N_{31}	Coefficient of linear part of cubic roll damping model
N_{33}	Coefficient of cubic part of cubic roll damping model
n, m	Non-dimensional time

Principal Notation (cont'd)

t	Time
T	Roll period, period
U	Current velocity
V_m	Volume of cylinder per unit length
U_m	Amplitude of velocity of current
Y	Roll amplitude
Y_o	Initial roll amplitude in ship sallying experiments
γ_1, γ_3	Coefficients in cubic fitting equation for roll extinction per cycle
δ_1, δ_2	Coefficients in quadratic fitting equation for roll extinction per cycle
ϵ	Normalized approximation range
ρ	Mass density
θ	ωt
φ	Roll angle
$\dot{\varphi}_c$	Assumed range of roll velocity
Ω	Normalized roll velocity
ω	Frequency, roll frequency

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