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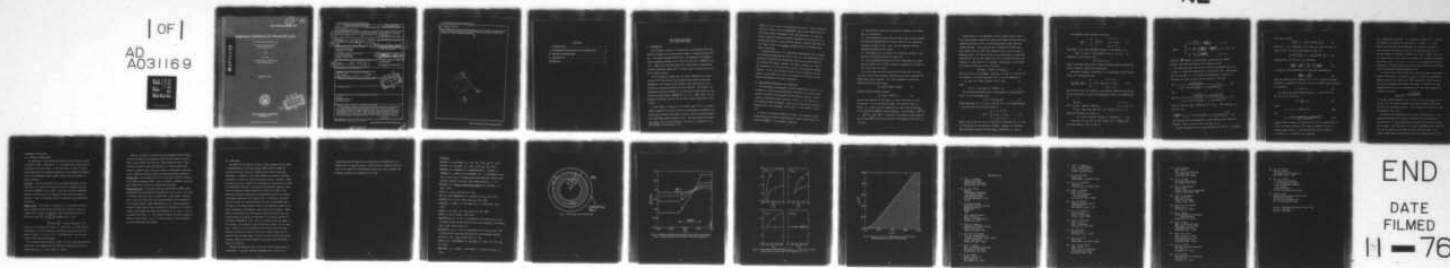
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Rigid Rotor Equilibrium of a Strong Ion Layer

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September 1976



NAVAL RESEARCH LABORATORY
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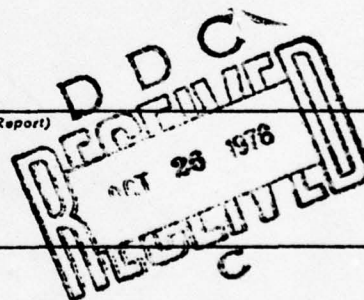
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20. Abstract (Continued)

layer equilibria exist only for small rotational velocities, and for a small range of the conducting wall radius, the frequency of rotation can be considerably greater than the cyclotron frequency corresponding to the externally applied magnetic field.

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RIGID ROTOR EQUILIBRIUM OF A STRONG ION LAYER

I. INTRODUCTION

The conceptual simplicity of rigid rotor equilibria make them very attractive models for the theoretical studies of charged particles layers. Such rigid rotorequilibria within the framework of the Maxwell-Vlasov equations (DAVIDSON et al., 1969, 1970 and 1974 ; ANDERSON et al., 1971 ; MARX, 1968) have been applied in recent years to magnetically confined nonneutral plasmas and to electron layers associated with the Astron reactor concept.

In the nonneutral plasmas case, the model (DAVIDSON et al., 1969, 1970 and 1974) was based on the nonrelativistic limit and the axial diamagnetic field as well as the effect of the conducting wall surrounding the gyrating electrons has been neglected. By contrast, the diamagnetic field was included in the theoretical model studied in connection with the E-layer (ANDERSON et al., 1971 ; MARX, 1968), but the assumed distribution function did not allow radial confinement of the E-layer.

In this paper, we make use of the steady state ($\partial/\partial t = 0$), Vlasov-Maxwell equations to study the rigid rotor equilibrium properties of a space charge neutral proton layer (p-layer), that is confined radially by an externally applied magnetic field. The main difference between the present work and that previously carried out in connection to E-

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layers is in the choice of the distribution function. The selected distribution function (CHU and KAPETANAKOS, 1975) in the present work not only allows radial confinement of the gyrating protons, but also is simple enough that explicit, analytic expressions can be obtained for the particle density, current density, magnetic field and the radii of the p-layer, without any a priori assumptions about the radial dimensions of the layer.

A brief description of the equilibrium configuration and an outline of the general assumptions are given in Sec. II. The equilibrium properties are calculated for the specific choice of the proton distribution function $f_1^0(H - \omega P_\theta) = (m\bar{n}/2\pi) \delta(H - \omega P_\theta - k_1)$, where m , $\bar{n}$, ω and k_1 are constants, H is the total energy and P_θ is the canonical angular momentum. In Sec. III, we review briefly results related to the stability of the p-layer and in Sec. IV we discuss the relevance of the present equilibrium to pertinent experiments.

II. EQUILIBRIUM CONFIGURATION AND ASSUMPTIONS

The equilibrium configuration is shown schematically in Fig. 1. It consists of a space charge neutralized, infinitely long, proton layer confined radially by an externally applied magnetic field B_0 . We introduce a cylindrical polar coordinate system (r, θ, z) , with the z axis along the axis of symmetry. The p-layer rotates about the axis of symmetry with a mean azimuthal velocity $V_\theta^0(r)$ in the negative θ -direction.

To make the theoretical analysis tractable, the following simplifying assumption are made in describing the p-layer equilibrium by the steady-state $\left(\frac{\partial}{\partial t}\right) = 0$ Vlasov-Maxwell equations:

- (a) Equilibrium properties are azimuthally symmetric and independent of z-coordinate.
- (b) The equilibrium electric field of the p-layer is neutralized by an electron background and the current carried by the background electrons is equal to zero. Thus, the self magnetic field is generated entirely by the p-layer.
- (c) The magnetic flux enclosed by the conducting cylinder is conserved for the entire life of the layer, and
- (d) The equilibrium distribution function describing the p-layer does not depend upon the axial velocity of protons, i.e., there is not motion along the z-axis.

It is well known, that any distribution function that is a function only of the single particle constants of the motion satisfies the steady-state Vlasov equation. For a time independent, azimuthally symmetric field the total energy

$$H = mv_r^2/2 + mv_\theta^2/2 + mv_z^2/2, \quad (1)$$

and the canonical angular momentum

$$P_\theta = mrv_\theta + q r A_\theta^0(r)/c, \quad (2)$$

are the two constants of the motion. In eqs. (1) and (2) m is the proton mass, $q = e$ is the proton charge, c the speed of light in vacuum, v_r , v_θ and v_z are the components of proton velocity and $A_\theta^0(r)$ is the equilibrium magnetic vector potential. Note that $A_\theta^0(r) = A_\theta^{\text{ext}}(r) + A_\theta^{\text{self}}(r)$, where $A_\theta^{\text{ext}}(r)$ describes the externally applied magnetic field B_0 and $A_\theta^{\text{self}}(r)$ describes the axial self magnetic field $B_z^s(r)$ of the layer.

Experimentally, it is desirable to form p-layers without shear in their mean azimuthal velocity of rotation and without all the particles composing the layer having the same energy and the same canonical angular momentum. Such equilibria are in general more stable than those having shear in their mean azimuthal velocity and no energy or momentum spread. For these reasons, we are considering an equilibrium distribution function than depends on H and P_θ through the linear combination, $H - \omega P_\theta$, where ω is a constant. Specifically, the distribution function is assumed to be of the form

$$f_i^0(H - \omega P_\theta) = (m\bar{n}/2\pi) \delta(H - \omega P_\theta - k_1), \quad (3)$$

where m , $\bar{n}$ and k_1 are constants. Using Eq. (1) with $v_z = 0$ and Eq. (2), the argument of the delta function in Eq. (3) may be expressed as

$$H - \omega P_\theta - k_1 = (m/2) [(v_\theta - r\omega)^2 + v_r^2] + U(r), \quad (4)$$

where

$$U(r) = - (qr/c) A_\theta^0(r)\omega - mr^2\omega^2/2 - k_1. \quad (5)$$

The equilibrium density profile corresponding to the distribution of Eq. (3) can be computed from

$$n^0(r) = \int f_i^0(H - \omega P_\theta) d^2v. \quad (6)$$

Substituting Eqs. (3), (4) and (5) into Eq. (6), it is straightforward to show that the proton density profile is

$$n^0(r) = \begin{cases} 0, & 0 \leq r < a_1, \\ \bar{n}, & a_1 \leq r \leq a_2, \\ 0, & a_2 < r < b, \end{cases} \quad (7)$$

where a_1 and a_2 are the roots of function $U(r)$ defined in Eq. (5), and b is the radius of the conducting cylinder. Equation (7) indicates that the density profile has sharp radial boundaries at a_1 and a_2 .

The azimuthal current density is given by

$$J_{\theta}^0(r) = \begin{cases} 0, & 0 \leq r < a_1, \\ \bar{q}\bar{n} V_{\theta}^0(r) & a_1 \leq r \leq a_2, \\ 0 & a_2 < r \leq b, \end{cases} \quad (8)$$

where $V_{\theta}^0(r)$ is the mean azimuthal velocity of the p-layer, i.e.,

$$V_{\theta}^0(r) = \langle v_{\theta}(r) \rangle = (1/\bar{n}) \int v_{\theta}^0 d^2v = \omega r, \quad a_1 \leq r \leq a_2. \quad (9)$$

Similarly, it can be shown that

$$V_r^0 = \langle v_r(r) \rangle = 0.$$

Thus, the p-layer rotates with a constant angular velocity ω around its axis of symmetry (rigid-rotor equilibrium).

The vector potential corresponding to the azimuthal current density of Eq. (8) is determined from

$$\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} [r A_{\theta}^0(r)] = \begin{cases} 0 & , 0 \leq r < a_1, \\ -(4\pi/c) \bar{q}\bar{n}r\omega, & a_1 \leq r \leq a_2, \\ 0 & , a_2 < r < b. \end{cases} \quad (10)$$

The solution of Eq. (10) subject to the boundary conditions that A_{θ}^0 and B_z are continuous at a_1 and a_2 and $\int_0^b 2\pi B_z r dr = \pi b^2 B_0$ (flux conservation) is

$$A_{\theta}^0(r) = \begin{cases} k_2 r, & 0 \leq r \leq a_1 \\ k_2 r - \pi \bar{n} q \omega (r^2 - a_1^2)^2 / 2cr, & a_1 \leq r \leq a_2, \\ k_2 r - \pi \bar{n} q \omega [(a_2^2 - a_1^2) (2r^2 - a_1^2 - a_2^2)] / 2cr, & a_2 < r \leq b, \end{cases} \quad (11)$$

where the constant k_2 is given by

$$k_2 = B_0/2 + (\pi \bar{n} q \omega / 2c) (a_2^2 - a_1^2) [2 - (a_1^2 + a_2^2)/b^2]. \quad (12)$$

Using Eqs. (11) and (12) and the relation $B^0(r) = \frac{1}{r} \frac{\partial}{\partial r} [r A_{\theta}^0(r)]$,

the axial magnetic field is given by

$$B_z^0(r) = \begin{cases} B_0 + \frac{4\pi I_\ell}{c} \left[1 - \frac{(a_1^2 + a_2^2)}{2b^2} \right] & , 0 \leq r < a_1 \\ B_0 + \frac{4\pi}{c} I_\ell \left[\frac{(a_2^2 - r^2)}{(a_2^2 - a_1^2)} - \frac{(a_1^2 + a_2^2)}{2b^2} \right] & , a_1 \leq r \leq a_2, (13) \\ B_0 - \frac{4\pi}{c} I_\ell \left[\frac{(a_1^2 + a_2^2)}{2b^2} \right] & , a_2 < r \leq b, \end{cases}$$

where $I_\ell = \frac{q\bar{n}\omega}{2} (a_2^2 - a_1^2)$ is the azimuthal current per unit length.

Figure 2 shows the ratio $B_z^0(r)/B_0$ as a function of r/b .

The inner and outer radii of the p-layer are determined from $U(a_1) = U(a_2) = 0$, where the function $U(r)$ is given by Eq. (5). Substituting Eq. (11) into Eq. (5), it is rather straightforward but tedious to show that the radii a_1 and a_2 can be expressed in the form

$$a_1^2 = - \frac{16k_1 \lambda^{*2} / (m\omega^2 b^2)}{1 \pm \left\{ 1 + 16(\lambda^*/b)^2 \left[\left(\frac{\Omega_0}{\omega} + 1 \right) + 4k_1 / (m\omega^2 b^2) \right] \right\}^{1/2}}, \quad (14)$$

and

$$a_2^2 = b^2 + \frac{8\lambda^{*2} \left[\left(\frac{\Omega_0}{\omega} + 1 \right) + 2k_1 / (m\omega^2 b^2) \right]}{1 \pm \left\{ 1 + 16(\lambda^*/b)^2 \left[\left(\frac{\Omega_0}{\omega} + 1 \right) + 4k_1 / (m\omega^2 b^2) \right] \right\}^{1/2}}, \quad (15)$$

where $\Omega_0 = qB_0/mc$, $\omega_{pi}^2 = 4\pi\bar{n}q^2/m$ and $\lambda^* = c/\omega_{pi}$. Note from Eqs. (14)

and (15) that

$$\frac{2(a_2^2 - a_1^2)}{b^2} = 1 \pm \left\{ 1 + 16(\lambda^*/b)^2 \left[\left(\frac{\Omega_0}{\omega} + 1 \right) + 4k_1 / (m\omega^2 b^2) \right] \right\}^{1/2}, \quad (16)$$

and since $a_2^2 > a_1^2$, the denominator in these equations must always be positive. Then, in order for a_1 to be real, $k_1 \leq 0$.

Radial confinement of the p-layer requires that $a_2^2 \leq b^2$. It follows

then from (15) that

$$\left[\left(\frac{\Omega_0}{\omega} + 1 \right) - 2|k_1| / (m\omega^2 b^2) \right] \leq 0, \quad (17)$$

where $|k_1| = -k_1$. Furthermore, since $(a_2^2 - a_1^2)$ should be real, the radical in Eq. (16) should be positive or zero, i.e.,

$$16(\lambda^*/b)^2 \left[\left(\frac{\Omega_0}{\omega} + 1 \right) - 4|k_1| / (m\omega^2 b^2) \right] \geq -1. \quad (18)$$

Combining Eqs. (17) and (18), it is obtained

$$\frac{4|k_1|}{m\omega^2 b^2} - \frac{b^2}{16\lambda^{*2}} \leq \left(\frac{\Omega_0}{\omega} + 1 \right) \leq \frac{2|k_1|}{m\omega^2 b^2}. \quad (19)$$

In order for both inequalities to be satisfied simultaneously,

$$\frac{2|k_1|}{m\omega^2 b^2} \leq \frac{b^2}{16\lambda^{*2}}. \quad (20)$$

When the conducting wall is absent, i.e., $b \rightarrow \infty$, equilibrium exists, provided ω is negative for positive Ω_0 and its absolute value satisfied the inequality

$$0 < |\omega| \leq \Omega_0. \quad (21)$$

Introducing the dimensionless variable $\lambda_0 = (2|k_1|/m)^{1/2} \Omega_0^{-1}$, Eq. (19) can be written as

$$\omega_1 \leq \left(\frac{\omega}{\Omega_0} \right) \leq \omega_3, \quad (22)$$

where

$$\omega_1 = [-1 - (1 + 4\lambda_0^2/b^2)^{1/2}]/2, \quad (23a)$$

and

$$\omega_3 = \frac{-1 - \{1 + (8\lambda_0^2/b^2)[1 + (b/4\lambda^*)^2]\}^{1/2}}{2[1 + (b/4\lambda^*)^2]}. \quad (23b)$$

Figure 3 shows ω_1 and ω_3 as a function of b/λ_0 for several values of λ^*/λ_0 . For small values of λ^*/λ_0 (high ion density) there is a small range of b/λ_0 , in which equilibrium exists for values of $|\omega|/\Omega_0 \gg 1$. In general such equilibria

are of appreciable thickness. For intermediate values of λ^*/λ_0 (Fig 3c), the frequency of rotation is restricted to $0 < |\omega|/\Omega_0 < 1$. In the special case that the various parameters of the system are tuned in such a way that the equality of Eq. (20) is satisfied, then $\omega_1 \approx \omega_3$ and the frequency of rotation is given by Eq. (23a). Finally (Fig. 3d), for large values of λ^*/λ_0 (small ion density), the normalized frequency of rotation $(\omega/\Omega_0) \approx \omega_1 \approx \omega_3 \approx -1$, i.e., as it is expected all the ions rotate with a frequency that is near the cyclotron frequency corresponding to the externally applied magnetic field.

It is easy to be shown from Eq. (16), that in the thin layer approximation the frequency of rotation $(\omega_{\text{thin}}/\Omega_0)$ is given by an expression similar to that of Eq. (23b), with $(\lambda^*)^2$ replaced by $(\lambda^*)^2/\epsilon$, where ϵ is a positive quantity much smaller than unity. If $(b/4\lambda^*) < 1$, the inequalities of Eq. (22) are satisfied and the thin layer frequency of rotation is given by

$$(\omega_{\text{thin}}/\Omega_0) \approx \frac{-1 - \sqrt{1 + 8\lambda_0^2/b^2}}{2}.$$

By contrast, when $\epsilon(b/4\lambda^*)^2 \gg 1$ and $\lambda_0/\lambda^* \gg 1$, both ω_3 and $(\omega_{\text{thin}}/\Omega_0)$ go to zero, i.e. for high ion density, thin layer equilibria exist only with very low rotational frequency. This conclusion is similar to that of LOVELACE et al., (1974).

In order for the inequalities of Eq. (22) to be satisfied $\omega_3 \geq \omega_1$. Using the expressions for ω_1 and ω_3 from Eq. (23), this relation is plotted in Fig. 4. The inequality is satisfied only in the upper half of the figure. For fixed values of λ_0 and b , equilibria exist provided that the ion density in the layer exceeds a critical value. This clearly demonstrates the importance of the self magnetic field in the

confinement of the layer.

III. REVIEW OF INSTABILITIES

The stability of the equilibrium discussed in the previous section is presently under investigation. It is expected that results will be available in the near future. In the mean time, in order to obtain some insight about the stability properties of the rigid rotor equilibria, we are reviewing several unstable modes that are pertinent to these equilibria.

Diocotron: The diocotron mode in a space charge nonneutral E-layer consisting of electrons that encircle the axis of symmetry has been studied by NOCENTINI et al., (1968). They found that the sufficient condition for stability is the absence of shear in the velocity of rotation. Thus, the diocotron mode is not present in any rigid rotor equilibrium.

Negative mass: The stability condition for a space charge neutral E-layer composed of relativistic electrons that encircle the axis of symmetry and confined in a magnetic field configuration that has a weak field index $n \left[= - \frac{r}{B_z} \frac{\partial B_z}{\partial r} \right]$ is (LANDAU et al., 1966)

$$\left[\Delta / (2m_0 c r_0) \right]^2 > v g \gamma / \alpha, \text{ (stability), (24)}$$

where m_0 is the electron rest mass, $g = c^2 L / (2\pi r_0)$, r_0 is the equilibrium radius, L is the inductance of the beam in cgs units, Δ is the momentum spread $\alpha = 1/(1-n) - \gamma^{-2}$, $v = Ne^2 / (2\pi r_0 m_0 c^2)$, and N is the number of electrons in the system.

For a strong, nonrelativistic p-layer, both the field index and the coefficient α are negative and the stability condition [Eq. (24)] is always satisfied, i.e., the p-layer is stable.

However, it should be noticed that the theoretical work of Landau and Neil is based on the assumption that the field index n is small, which is not probably the case for a field reversing p-layer. Thus definite conclusions about the stability of a strong p-layer with respect to negative mass instability cannot be drawn before recalculating the stability condition without the weak field index assumption.

Tearing mode: This instability is not of any great concern, because it can be easily stabilized (MARX, 1968) by placing a perfectly conducting wall sufficiently close to the charged particle layer.

Precessional mode: It has been predicted theoretically (FURTH, 1965) and demonstrated experimentally (CHRISTOFILOS et al., 1969) in the case of E-layers, that this mode is present only in weak layers. Furthermore, it has been shown that the precessional mode can be stabilized by a toroidal magnetic field (CHRISTOFILOS et al., 1972) produced by currents flowing along an axial conductor or cantilever or by a sufficiently strong quadrupole Ioffe field (WOODALL et al., 1975). Thus, it is expected this mode is only present whenever the layer strength is below a critical level, i.e., it is only of small, if any, importance for field reversing proton layers.

IV. CONCLUSIONS

The number of ions required to form a field reversing ring or layer is approximately by the factor (m_i/m_e) higher than the number of electrons required to form an electron ring or layer of the same dimensions. In addition, the current density of presently available ion sources is lower than those of electron sources. Therefore, field reversing ion rings or layers can be realized by using hollow ion beams of considerable dimensions and passing them through a magnetic cusp. Since the canonical angular momentum is a function of radius, such methods of injection correspond to an appreciable spread in the canonical angular momentum of the injected ions. Furthermore, the force that an ion sees as it passes through the cusp is approximately proportional to its radial distance and thus an appreciable spread in the energy transverse to the magnetic field lines will be present. Therefore, only equilibria that are based on distributions functions that include spread in P_θ and $E_\perp$ are pertinent to the present methods of injections (KAPETANAKOS et al., 1976) of intense, pulsed ion beams. In this respect, our selection of the distribution function is very realistic. However, it is not presently clear which must be the exact injection conditions in order to form an ion layer in which all the particles have the same $H - \omega P_\theta$. But even if such conditions were known, transient effects during the buildup of the layer could drastically change the situation.

Although the assumption that all the ions have the same $H - \omega P_\theta$ is questionable, it has the important advantage that it results to a

simple distribution function that allows explicit expressions to be obtained for the particle density, current density, magnetic field and radii of the layer. Such a distribution function is very convenient for studying the stability properties of the layer.

REFERENCES

- DAVIDSON, R.C. and KRALL, N.A., Phys. Rev. Letts. 23, 833 (1969).
- DAVIDSON, R.C. and KRALL, N.A., Phys. Fluids 13, 1543 (1970).
- ANDERSON, D.V., BENFORD, G.A., BRETTSCHEIDER, M., KILLEEN, J.,
NEWCOMB, W.A., MIRIN, A.A. and RENSINK, M.E., in Proceedings of the
Fourth Inter. Conf. on Plasma Physics and Controlled Nuclear Fusion
Research, Madison, Wisconsin, 1971 (IAEA-CN-2B/A-10.) p. 137.
- DAVIDSON, R.C., Theory of Nonneutral Plasmas (W.A. Benjamin, Inc.,
Mass., 1974).
- MARX, K.D., Phys. Fluids 11, 357 (1968).
- CHU, K.R. and KAPETANAKOS, C.A., Nuclear Fusion 15, 947 (1975).
- LOVELACE, R.V., et al., Phys. Fluids 17, 2081 (1974).
- NOCENTINI, A., BERK, H.L. and SUDAN, R.N., J. Plasma Phys. 2, 311
(1968).
- LANDAU, R. and NEIL, V.K., Phys. Fluids 9, 2412 (1966).
- FURTH, H., Phys. Fluids 8, 2020 (1965).
- CHRISTOFILOS, N.C., et al., in Proceedings of the Third Inter. Conf.
on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk,
1968, (IAEA, Vienna, 1969) 967.
- CHRISTOFILOS, N.C., et al., in Proceedings of the Fourth Inter. Conf.
on Plasma Physics and Controlled Nuclear Fusion Research, Madison,
Wisconsin, 1971. (IAEA, Vienna, 1972), p. 119.
- WOODALL, D.M., FLEISCHMANN, H.H. and BERK, H.L., Phys. Rev. Lett. 34,
260 (1975).
- KAPETANAKOS, C.A., GOLDEN, J. and YOUNG, F., Nuclear Fusion 16, 151
(1976).

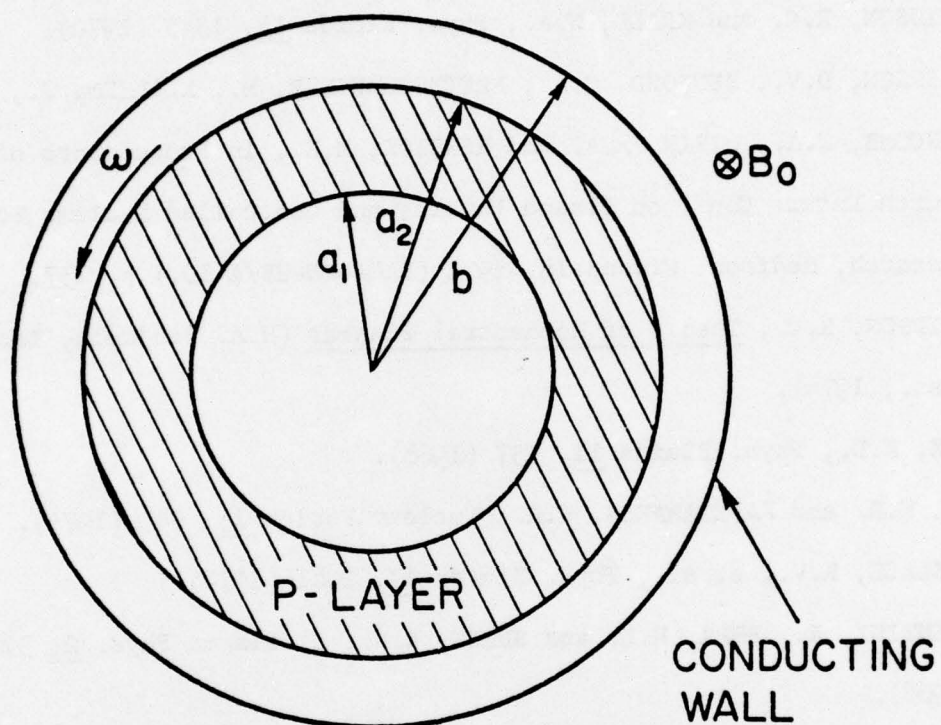


Fig. 1 — Cross section of the long ion layer

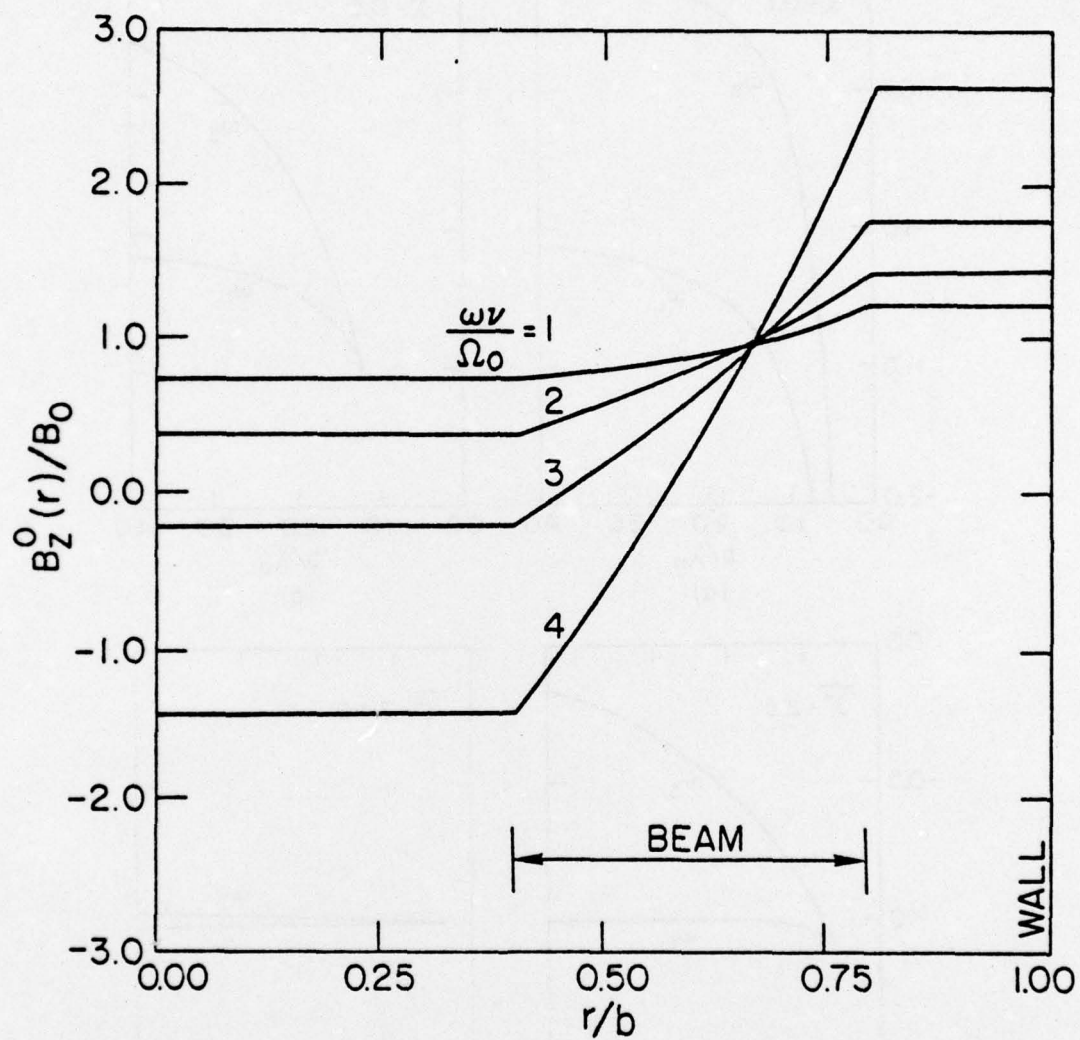


Fig. 2 — Equilibrium magnetic field profiles for various values of the parameter $\omega\nu/\Omega_0$. The ratios a_2/b and a_1/b are equal to 0.8 and 0.4 respectively.

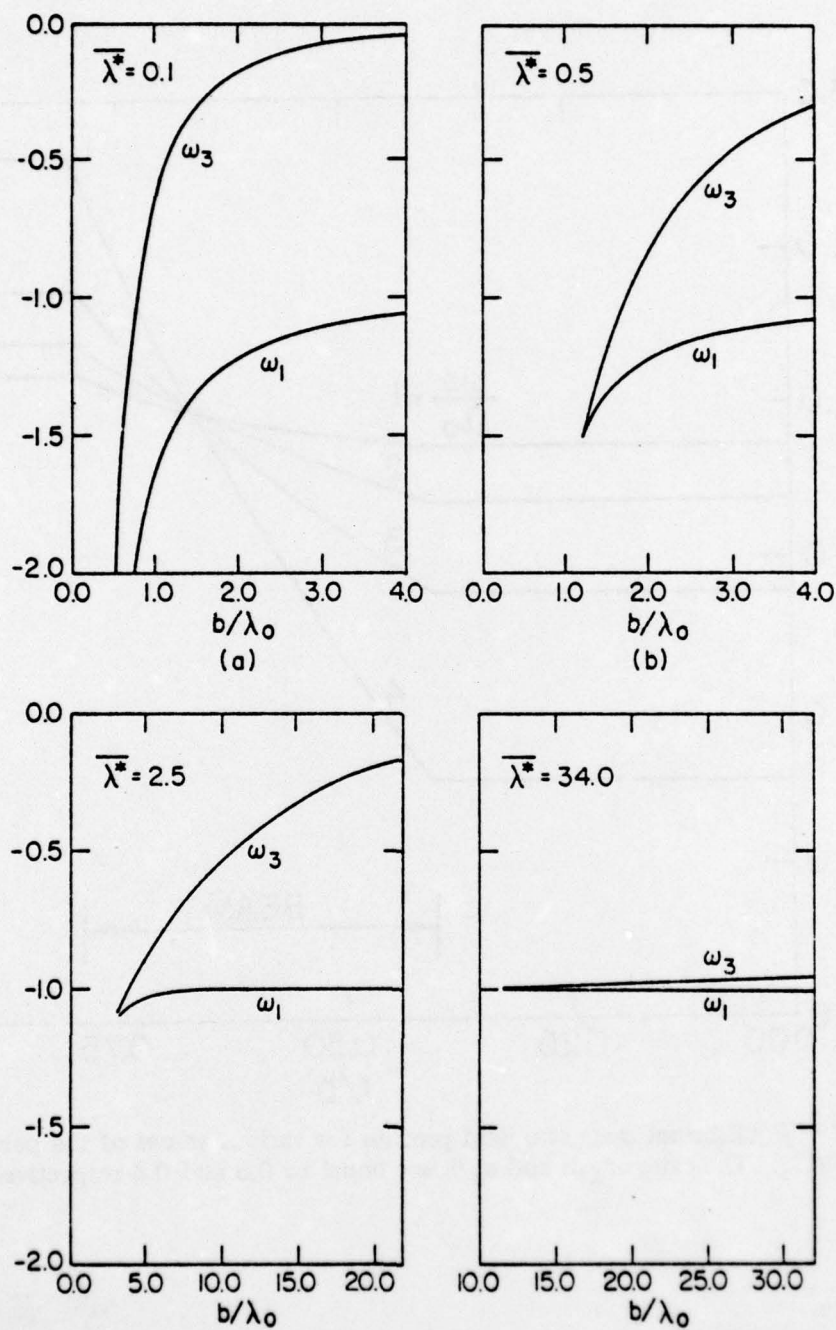


Fig. 3 — Allowed region of the rotational frequency $\omega_1 \leq (\omega/\Omega_0) \leq \omega_3$ as a function of λ_0/b , for various values of the parameter $\lambda^*/\lambda_0 = \bar{\lambda}^*$

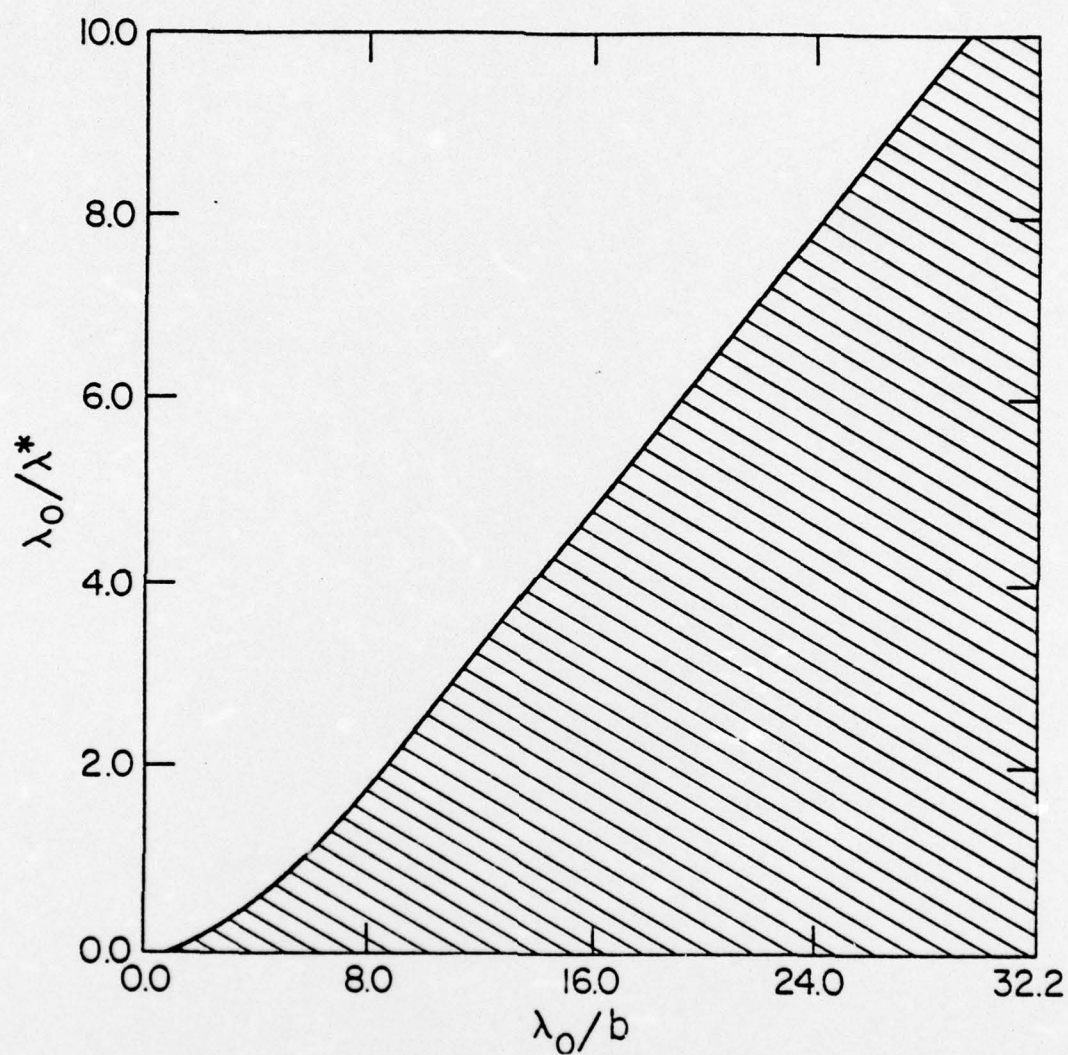


Fig. 4 — Plot of λ_0/λ^* vs. λ_0/b . Equilibria exist only for values of these parameters at the upper half of the figure.

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