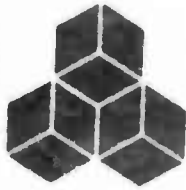


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CONSTITUTIVE EQUATIONS FOR FLUID-SATURATED
POROUS MEDIA

by

S. K. Garg

Topical Report

Sponsored by

Advanced Research Projects Agency

ARPA Order No. 2551

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by AFTAC/VSC, Patrick AFB FL 32929, under Contract No. F08606-75-C-0045.



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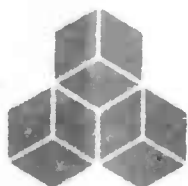
Principal Investigator and Phone No.

Dr. John M. Savino, (714) 453-0060, Ext. 455

Project Scientist and Phone No.

Dr. Ralph W. Alewine, III, (703) 325-8484

S³ Project No. 11014



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P. O. BOX 1420, LA JOLLA, CALIFORNIA 92038, TELEPHONE (714) 463-0060

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↘ conditions. These assumptions can, however, be relaxed at the expense of
some simplicity. ↗

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TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. THEORETICAL FORMULATION	2
III. ITERATIVE PROCEDURE FOR INCORPORATING THE THEORETICAL MODEL INTO COMPUTER CODES	7
IV. CONCLUDING REMARKS	11
REFERENCES	12

I. INTRODUCTION

In this report, we shall develop constitutive relations for fluid-saturated porous media, suitable for inclusion in standard hydrodynamic codes (e.g., CRAM or SKIPPER). The theoretical formulation is based on the models for fluid-saturated rock aggregates previously developed by Garg and Nur [1973] and Garg, et al. [1975]. In the present analysis, we shall assume that there is no relative motion between the fluid and the solid (no fluid diffusion); this assumption is equivalent to requiring that [Garg, et al., 1975]

$$X \gg \frac{C_0 k \rho_0}{\phi^2 \mu_f}$$

where

X - length scale of interest

μ_f - fluid viscosity

ρ_0 - density of porous media

C_0 - speed of sound in porous media

k - permeability of porous media

ϕ - porosity

It is straightforward to verify that the above inequality is satisfied for many field situations involving shock/seismic wave propagation in fluid-saturated porous media. The theoretical formulation will be outlined in Section II. A possible method for including the new constitutive model into standard hydrodynamic codes is described in Section III.

II. THEORETICAL FORMULATION

"No fluid diffusion" hypothesis implies that the fluid mass M in a volume V of the rock/fluid composite remains constant. Mathematically,

$$\frac{dM}{dt} = 0 . \quad (1)$$

Noting that

$$V_f = \text{volume occupied by the fluid} = \phi V , \quad (2a)$$

and defining

$$\rho_f = \text{fluid density} = M/V_f , \quad (2b)$$

we have from (1)

$$\rho_f = \rho_{0f} \frac{\phi}{\phi_0} \frac{V_0}{V} , \quad (3)$$

where subscript 0 denotes the value of the subscripted variable at $t = 0$. For a partially saturated media, fluid density as defined by (2b) is not equal to the real density of the fluid. (In other words, we replace the real fluid by an extended fluid occupied by the volume V_f .) This is particularly convenient for treating an initially partially saturated medium which upon shock loading (and resulting pore compaction) becomes fully saturated; in this case the fluid pressure p_f will remain zero as long as ρ_f is less than some reference density (say ρ_0^f).

We now introduce the following mixture (solid/fluid) quantities:

$$\rho \text{ (mixture density)} = (1-\phi)\rho_s + \phi\rho_f \quad (4a)$$

$$\sigma_{ij} \text{ (mixture stress)} = -p_c S_{ij} + S_{ij} \quad (4b)$$

$$p_c \text{ (mixture pressure)} = (1-\phi)p_s + \phi p_f \quad (4c)$$

$$E \text{ (mixture internal energy/unit mass)} = \frac{(1-\phi)\rho_s E_s + \phi_f \rho E_f}{\rho} \quad (4d)$$

where

ρ_s = rock grain density

S_{ij} = mixture deviatoric stress tensor

p_s (p_f) = solid (fluid) pressure, and

E_s (E_f) = solid (fluid) internal energy per unit mass.

The mixture particle velocity \underline{v} is identically equal to the solid (fluid) velocity \underline{v}_s (\underline{v}_f).

Mixture density ρ , velocity \underline{v} , and internal energy E are governed by the usual balance equations for a single continuum:

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \underline{v} = 0 \quad (5a)$$

$$\rho \frac{D\underline{v}}{Dt} = \operatorname{div} \underline{\sigma} + \rho \underline{g} \quad (5b)$$

$$\rho \frac{DE}{Dt} = \underline{\sigma} : \nabla \underline{v} \quad (5c)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{x}} \quad (5c)$$

We will define the mixture strain-rate tensor to be the symmetric part of the velocity gradient. Thus

$$\dot{\underline{\epsilon}} = \frac{1}{2} [\nabla \underline{v} + (\nabla \underline{v})^t] \quad (6)$$

The strain-rate tensor may be decomposed into isotropic ($\dot{\epsilon}$) and deviatoric (\dot{e}) parts as follows:

$$\dot{\epsilon} = \frac{\dot{\epsilon}}{3} \underline{I} + \dot{e} \quad (7)$$

where \underline{I} is the unit tensor.

Mixture density ρ is related to ϵ through the relation:

$$\rho = \rho_0 \exp(-\epsilon) . \quad (8)$$

Also, we have

$$V_0/V = \rho/\rho_0 = \exp(-\epsilon) \quad (9)$$

Given ϵ (Eq. 6) and ϕ (see below), ρ , ρ_f and ρ_s may then be determined from Eqs. (8), (3) and (4a) respectively.

Now, we shall consider the question of energy partitioning between the solid and fluid phases. Let t_i and t_r denote the time of interest and the thermal relaxation time respectively. If $t_i \ll t_r$, the adiabatic assumption (no thermal exchange between the solid and fluid phases) may be employed. Similarly, the isothermal assumption ($T_f = T_s$) is appropriate in the reverse case, i.e., when $t_i \gg t_r$. In the intermediate range ($t_r \sim 0(t_i)$), it is necessary to adopt a constitutive model for heat flow between the two phases (see, e.g., Riney, et al. [1971]). In laboratory situations, the adiabatic assumption is generally adequate. Adiabatic assumption may not, however, be always appropriate in field applications. For the sake of simplicity, we will adopt the adiabatic assumption in the present analysis. (It cannot be over-emphasized that this assumption may be invalid in specific applications.) In this case, the fluid internal energy can be written as:

$$E_f = - \int_0^t p_f d \left(\frac{1}{\rho_f} \right) \quad (10)$$

Given E_f (Eq. 10) and E (Eq. 5c), E_s may be determined from Eq. (4d).

We now wish to prescribe the constitutive relations for the pressures (p_s, p_f) and deviatoric stresses \underline{S} . In the rock-fluid mixture, only the rock can sustain shear stresses; and one can therefore directly postulate shear laws for the entire composite. We will assume the solid to be elastic-plastic; the shear stresses are assumed to be restricted by a yield criterion. The shear laws are, therefore, as follows:

- (i) Elastic response or for unloading from yield surface ($\underline{S} : \underline{S} \leq 2Y^2$):

$$\underline{S} = 2\mu \underline{e} \quad (11)$$

Here μ denotes the shear modulus of the porous rock and Y is the yield stress in shear.

- (ii) Plastic response:

$$\underline{S} : \underline{S} = 2Y^2 \quad (12)$$

where

$$Y \equiv Y(\bar{P}) \quad (13a)$$

$$\bar{P} = p_c - p_f + \frac{1}{2} \left(\frac{J'_3}{2} \right)^{1/3}, \text{ and} \quad (13b)$$

$$\begin{aligned} J'_3 &= \text{third invariant of deviatoric stresses} \\ &= \text{Det } (\underline{S}). \end{aligned} \quad (13c)$$

In writing down Eq. (13a), we have assumed that the yield surface Y of a porous rock is governed by the "effective stress law" (see, e.g., Garg and Nur [1973]). A mixture model for pressures (p_s, p_f) can be formulated by assuming that the pressure law (p versus ρ and E relationship) for each constituent as a single continuum applies in the mixture. Thus

$$p_f \equiv p_f(\rho_f, E_f) \quad (14a)$$

$$p_s \equiv p_s(\rho_s, E_s) \quad (14b)$$

The above constitutive formulation is complete only when porosity ϕ is prescribed. Porosity ϕ will, in general, depend upon p_c, p_f, S, E_s, E_f , and the past loading history. In shock wave propagation through dry rocks, it is often assumed (perhaps without much justification) that ϕ depends only upon pressure p_c . For wet rocks, a similar model is formulated by assuming that ϕ is a function of only p_c and p_f (see Garg, et al. [1975]).

$$\phi = \phi(p_c, p_f) \quad (15)$$

Empirical observations under isotropic loading reveal that ϕ is a particularly simple function of p_c and p_f ; ϕ depends to a close approximation on $p_c - p_f$.

$$\phi = \phi(p_c - p_f) \quad (16)$$

Function $\phi(p_c - p_f)$ may be determined from hydrostatic tests with p_f kept at zero. This completes the theoretical formulation. In the next section, we shall outline an iterative procedure for incorporating this model into conventional hydrodynamic codes.

III. ITERATIVE PROCEDURE FOR INCORPORATING THE THEORETICAL MODEL INTO COMPUTER CODES

In this section, we will describe a possible iterative procedure for incorporating the theoretical model of Section II into standard hydrodynamic codes. This procedure has not been tested, and may require minor adjustments to ensure convergence. For the sake of convenience, the iterative procedure is presented below in a flow chart form. (In the following, superscript n denotes the value of the superscripted variable at time $t = n\Delta t$). Given strain-rate tensor $\dot{\epsilon}$ at time $t = (n+1)\Delta t$ and p_f^n , ϕ^n , E_f^n , p_c^n , ρ^n , S^n , ξ^n and E^n , the iteration procedure solves for p_f^{n+1} , ϕ^{n+1} , E_f^{n+1} , p_c^{n+1} , ρ^{n+1} , S^{n+1} , and E^{n+1} .

Iteration Procedure

(Saved arrays are p_f , ϕ , E_f , p_c , S , E)

Step 1. Initialize variables

$$p_c^{n+1} = p_c^n$$

$$p_f^{n+1} = p_f^n$$

$$\phi^{n+1} = \phi^n$$

$$E_s^{n+1} = [\rho^n E^n - \rho_f^n \phi^n E_f^n] / [\rho_s^n (1 - \phi^n)]$$

Step 2. Determine new strains

$$\dot{\underline{\epsilon}} = \text{Trace } \dot{\underline{\underline{\epsilon}}}$$

$$\dot{\underline{\underline{\epsilon}}} = \dot{\underline{\underline{\epsilon}}} - \frac{\dot{\underline{\epsilon}}}{3} \underline{\underline{I}}$$

$$\underline{\epsilon}^{n+1} = \underline{\epsilon}^n + \Delta t \dot{\underline{\epsilon}}$$

$$\underline{\underline{\epsilon}}^{n+1} = \underline{\underline{\epsilon}}^n + \Delta t \dot{\underline{\underline{\epsilon}}}$$

Step 3. Start of Iteration Loop

Step 3A. Calculate ρ^{n+1} , ρ_f^{n+1} , ρ_s^{n+1}

$$\rho^{n+1} = \rho_0 \exp(-\epsilon^{n+1})$$

$$\rho_f^{n+1} = \rho_{0f} \frac{\phi}{\phi^{n+1}} \exp(-\epsilon^{n+1})$$

$$\rho_s^{n+1} = (\rho^{n+1} - \phi^{n+1} \rho_f^{n+1}) / (1 - \phi^{n+1})$$

Step 3B. Calculate new shear stresses

$$\mu = \text{Max}([\mu_0 (1 - E_s^{n+1} / E_m)], 0)$$

(μ_0 ~ shear modulus at room temperature)

(E_m ~ specific internal energy for the solid phase at incipient melting)

$$\Delta \underline{\underline{S}} = 2\mu \Delta t \dot{\underline{\underline{\epsilon}}}$$

$$\underline{\underline{S}}^{n+1} = \underline{\underline{S}}^n + \Delta \underline{\underline{S}}$$

$$J_2^{n+1} = [\underline{\underline{S}}^{n+1} : \underline{\underline{S}}^{n+1}] / 2$$

$$J_3^{n+1} = \text{Det}[\underline{\underline{S}}^{n+1}]$$

$$\bar{p}^{n+1} = (p_c^{n+1} - p_f^{n+1}) + \frac{1}{2} \left(\frac{J_2^{n+1}}{2} \right)^{1/3}$$

$$Y^{n+1} = Y_0(\bar{P}) \times (1 - E_s^{n+1}/E_m)$$

$$Y^{n+1} = \text{Max}(Y^{n+1}, 0)$$

($Y_0(\bar{P})$ ~ yield strength at room temperature)

$$\text{If } \sqrt{J_2^{n+1}} > Y^{n+1}, \text{ then set } \underline{s}^{n+1} = \underline{s}^{n+1} \frac{Y^{n+1}}{\sqrt{J_2^{n+1}}}$$

Step 3C. Calculate new internal energies

$$\dot{v} = -\dot{\rho}/\rho^2 = \dot{\epsilon}/\rho^{n+1}$$

$$\dot{E} = -(p + q)^{n+1} \dot{v} + \underline{s}^{n+1} : \dot{\epsilon}$$

(q ~ artificial viscosity)

v ~ specific volume = $1/\rho$)

$$E^{n+1} = E^n + \Delta t \dot{E}$$

$$\Delta v_f = \Delta \left(\frac{1}{\rho_f} \right) = \frac{1}{\rho_f^{n+1}} \left[\frac{\phi^{n+1} - \phi^n}{\phi^{n+1}} + \dot{\epsilon} \Delta t \right]$$

(v_f ~ specific volume of fluid)

$$\Delta E_f = -p_f^{n+1} \Delta v_f$$

$$E_f^{n+1} = E_f^n + \Delta E_f$$

$$E_s^{n+1} = [\rho^{n+1} E^{n+1} - \rho_f^{n+1} \phi^{n+1} E_f^{n+1}] / [\rho_s^{n+1} (1 - \phi^{n+1})]$$

Step 3D. Calculate new pressures

$$p_s^{n+1} = p_s(E_s^{n+1}, \rho_s^{n+1})$$

$$p_f^{n+1} = p_f(E_f^{n+1}, \rho_f^{n+1})$$

$$p_c^{n+1} = (1 - \phi^{n+1})p_s^{n+1} + \phi^{n+1}p_f^{n+1}$$

Step 3E. Test for convergence

If $\text{abs}(p_c^{n+1} - p_c^n) \leq A p_c^n$ exit iteration loop

A = preset constant

Otherwise, continue to Step 3F

Step 3F. Calculate new porosity

$$\phi^{n+1} = \phi(p_c^{n+1} - p_f^{n+1})$$

Go to start of Iteration Loop

IV. CONCLUDING REMARKS

The preceding sections describe a simple constitutive model for fluid-saturated porous rocks. In developing the constitutive model, it was assumed that (1) there is no relative motion between the fluid and the solid, (2) no significant heat exchange occurs between the solid and fluid phases, and (3) porosity ϕ is a function of only the mixture pressure p_c and fluid pressure p_f . The last two assumptions may not be valid under certain field conditions. These assumptions can, however, be relaxed at the expense of some simplicity [Riney, et al., 1971; Garg, 1975].

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