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Stereographic Projection in the Joint Surveillance System

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STEREOGRAPHIC PROJECTION IN THE JOINT SURVEILLANCE SYSTEM

SEPTEMBER 1976

Prepared for

DEPUTY FOR SURVEILLANCE AND NAVIGATION SYSTEMS
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The stereographic projection process consists of using slant range, azimuth and height information to obtain radar coordinates, and transformation of the radar coordinates to obtain coordinates on a common plane.

This report describes analysis that was performed on the stereographic projection process. The results indicate that the SAGE/BUIC equations for stereographic ground range produce unacceptably large registration errors when extended to a

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large region. The results further indicate that the error can be corrected by a simple modification of the SAGE/ BUIC equations.

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INTRODUCTION

In air defense and air traffic control systems, data from the system radars are stereographically projected onto a common coordinate plane for presentation to system operators. The stereographic projection of radar data involves two steps; stereographic projection using slant range, azimuth and height information to obtain polar coordinates in a plane of projection centered at the radar site, and transformation of the radar coordinates into cartesian coordinates on a common coordinate plane.

The stereographic projection and transformation process is mathematically complex. Because of the computational complexity, several assumptions and approximations have been made to expedite processing time without unduly sacrificing accuracy. The present SAGE/BUIC equations for computing radar coordinates, although satisfactory for their intended usage, introduce unacceptable registration errors when extended to large regions as will be encountered in the Joint Surveillance System (JSS).

This report describes analysis that was performed on the stereographic projection process. Equations for obtaining stereographic ground range are derived. The derivation indicates that the SAGE/BUIC ground range equation lacks a scale factor vital to proper registration in large regions. The scale factor is a function of the radius of the earth at a radar site and the radius of the conformal sphere.

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SECTION II

EFFECT OF THE CONFORMAL SPHERE ON STEREOGRAPHIC PROJECTION

STEREOGRAPHIC PROJECTION

Stereographic projection is a method for mapping points in space onto a plane tangent to a sphere. This sphere is termed the conformal sphere and its radius is arbitrary. Figure I depicts the projection geometry for a cross section of the sphere. The cross section is obtained by passing a plane through the center of the sphere, the aircraft, and the point of tangency. The intersection of this plane and the sphere is a great circle. Mapping of point A in space onto the tangent plane BB′ results in the stereographic ground range DT. DT is obtained by passing a line from point O, opposite the point of tangency D, through point P, the point of projection, and intersecting the tangent plane. Point P is the intersection of the line containing point A and the center of the earth, point C, and the great circle. If angle DCP is designated as ψ, then angle DOP equals $\psi/2$ since DC=CO=CP. The stereographic ground range DT is determined as follows.

$$DT = DO \tan\left(\frac{\psi}{2}\right) \tag{1}$$

If DT is defined as R and DC, CO, CP are defined as E_c, then equation (1) takes the form:

$$R = 2E_c \tan\left(\frac{\psi}{2}\right)$$

$$= 2E_c\left[\frac{1 - \cos\psi}{1 + \cos\psi}\right]^{1/2} \tag{2}$$

The stereographic ground range R is therefore directly proportional to the radius of the conformal sphere E_c.

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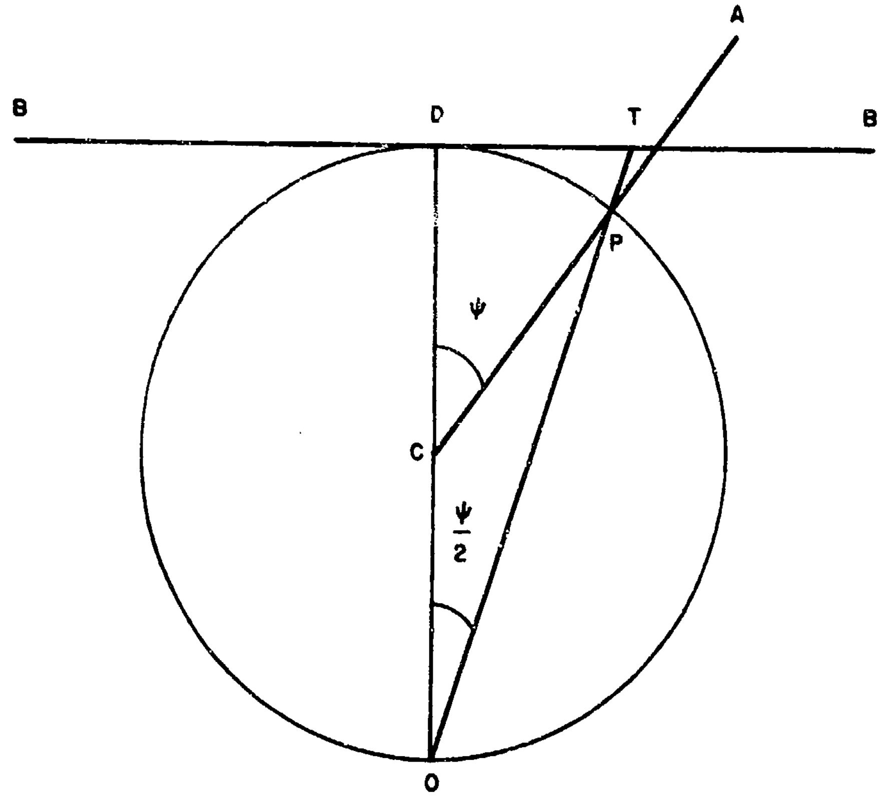

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Figure I. STEREOGRAPHIC PROJECTION

THE RADAR PLANE

Radar data are stereographically projected onto a plane centered at the reporting radar site and tangent to the conformal sphere. The plane will be termed the radar plane. The coordinate axes of the radar plane are oriented such that the positive Y axis is directed toward true north and the positive X axis towards east. Figure II shows the orientation of the coordinate axes on the radar plane. An aircraft's location on the plane is expressed in polar coordinates. The range R is the stereographic ground range, and the azimuth angle θ is the azimuth of the radar return. Azimuth angles are measured clockwise from the positive Y axis.

DETERMINATION OF STEREOGRAPHIC GROUND RANGE

Radar slant range, height of the aircraft above sea level and elevation of the site above sea level are used to determine the stereographic ground range of an aircraft on the radar plane. The angle ψ between the radar site, the center of the earth and the aircraft are used in equation (2) to calculate the stereographic ground range. The angle ψ can be calculated if the earth is assumed to be spherical. Figure III illustrates the radar geometry for a cross section of a spherical earth where E_s is the radius of the earth, h is the site elevation, S is the measured slant range and H is the aircraft height. From Figure III the angle ψ may be calculated from the law of cosines.

$$S^2 = (E_s + h)^2 + (E_s + H)^2 - 2(E_s + h)(E_s + H)\cos\psi$$

$$\cos\psi = 1 + \frac{(H - h)^2 - S^2}{2(E_s + h)(E_s + H)}$$

$$= 1 - \frac{F^2}{2(E_s + h)(E_s + H)} \qquad (3)$$

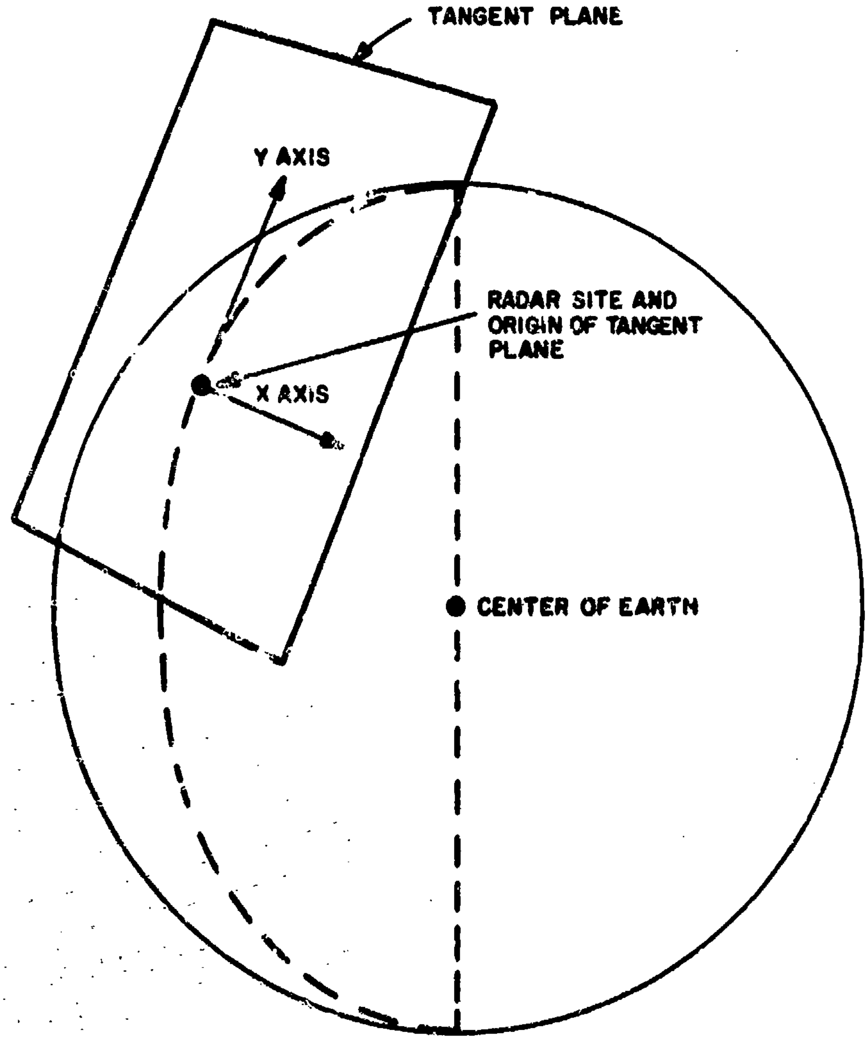

Figure II COORDINATE AXES ON RADAR PLANE

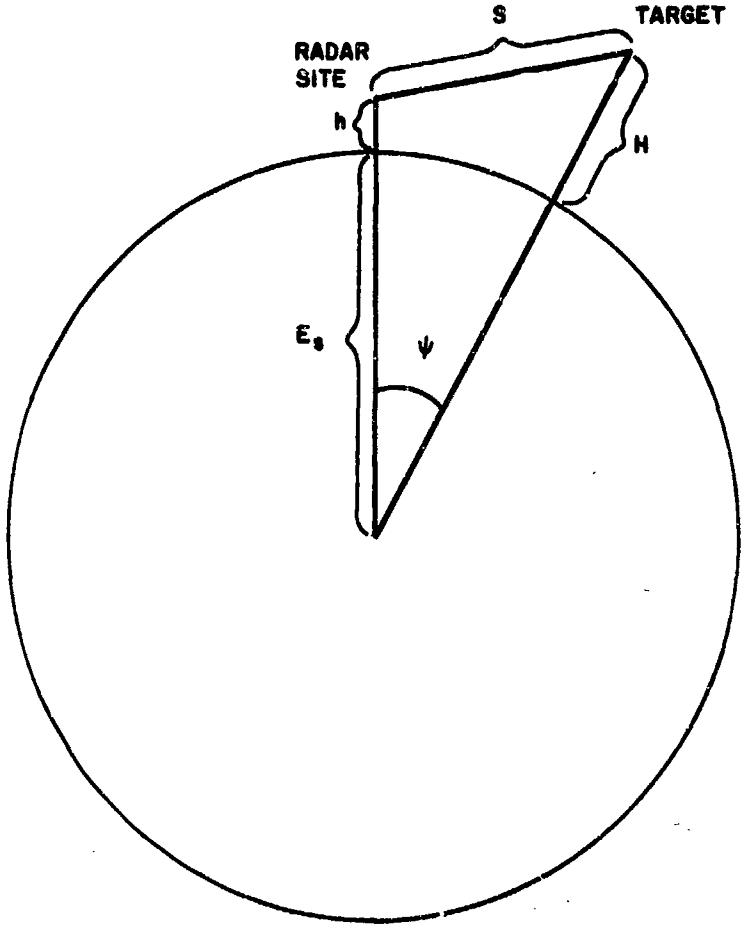

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Figure III RADAR OBSERVATION GEOMETRY

where:

$$F^2 = S^2 - (H - h)^2$$

Substituting the results of equation (3) into equation (2), the ground range R may be obtained as follows.

$$R = 2E_c \left[\frac{F^2}{4(E_s + h)(E_s + H) - F^2} \right]^{1/2}$$

$$= \frac{E_c F}{E_s} \left[1 + \frac{H + h}{E_s} + \frac{Hh}{E_s^2} - \frac{F^2}{4E_s^2} \right]^{-1/2} \qquad (4)$$

Reference 1 page 7 presents the SAGE/BUIC formulation of the ground range wherein it is assumed that E_c and E_s are equal and thus cancel. It will be shown that for the large regions that will be encountered in the JSS system, this assumption results in unacceptable registration errors.

APPROXIMATIONS TO GROUND RANGE

The accuracy of typical common digitizer search radar outputs is 0.25 nmi in range and 0.18° in azimuth. Registration errors are a combination of data errors, radar site location errors, and errors due to approximations in the stereographic projection process. Since equation (4) is computationally complex, an approximation which does not unduly sacrifice accuracy is used to expedite processing time. A maximum error, induced by approximation, of 0.18 nmi provides a reasonable compromise between processing requirements and registration accuracy. Four approximations are presented in this section; the series approximation, the first order approximation, the JSS approximation and the current SAGE/BUIC approximation.

The Series Approximation

Equation (4) may be written in the following form.

$$R \simeq \frac{E_c F}{E_s\left(1 + \frac{H + h}{E_s} + \frac{Hh}{E_s^2} - \frac{F^2}{4E_s^2}\right)^{1/2}} \tag{5}$$

The term within brackets in the denominator of equation (5) may then be expressed by the following series expansion.

$$(1 + x)^n = 1 + nx + \frac{n(n - 1)x^2}{2!} + \ldots$$

where:

$$x = \left(\frac{H + h}{E_s} + \frac{Hh}{E_s^2} - \frac{F^2}{4E_s^2}\right) \quad \text{and}$$

$$n = \frac{1}{2}$$

The maximum value of x encountered in a JSS region is 0.005. Since $x < < 1$, all but the first order term of the series may be ignored and equation (5) may be expressed as follows.

$$R \simeq \frac{E_c F}{E_s\left(1 + \frac{H + h}{2E_s} + \frac{Hh}{2E_s^2} - \frac{F^2}{8E_s^2}\right)} \tag{6}$$

The First Order Approximation

The bracketed term in the demoninator of equation (6) contains a first order term and two second order terms. The maximum value of $\frac{H + h}{2E_s}$ is 0.00248, the maximum value of $\frac{hH}{2E_s^2}$ is 0.000000721, and the maximum value of $\frac{F^2}{8E_s^2}$ is 0.000424. Since the second order terms are smaller than the first order term, they may be neglected and equation (6) may be expressed as follows.

$$R = \frac{E_c F}{E_s\left(1 + \frac{H + h}{2E_s}\right)} \tag{7}$$

The JSS Approximation

By replacing the aircraft height term (H) in the demoninator of equation (7) by a constant, equation (7) may be expressed as follows.

$$R = \frac{E_c F}{E_s\left(1 + \frac{(H_m/2) + h}{2E_s}\right)} \tag{8}$$

where:

H_m is the maximum expected aircraft altitude equal to 100,000 ft.

Equation (8) may be expressed in the following form.

$$R = CF$$

where:

$$C = \frac{E_c}{E_s\left(1 + \frac{(H_m/2) + h}{2E_s}\right)}$$

This is a particularly good approximation since the stereographic ground range is obtained from a simple scale multiplication of the quantity F. This greatly decreases the time required to process radar returns. The constant C is adaptation defined on a site-by-site basis.

The Current SAGE/BUIC Approximation

If E_c is assumed to be the same as E_s, equation (8) is expressed as follows.

$$R = \frac{F}{\left(1 + \frac{(H_m/2) + h}{2E_s}\right)} \tag{9}$$

EARTH MODEL

The earth is not a perfect sphere. Therefore, for precise calculations of stereographic ground range, a model for the geometric shape of the earth must be adopted. An appropriate first order representation is an ellipsoid. The ellipsoid is generated by revolving an ellipse about its semiminor axis. The earth model can therefore be specified by its semimajor axis or equatorial radius E_q and the eccentricity e. A cross section of the adopted earth model is shown in Figure IV. The eccentricity is defined as follows.

$$e^2 = 2f - f^2 \tag{10}$$

where:

$$f = \frac{E_q - E_p}{E_q}$$

E_p is the semiminor axis or polar radius

The International Ellipsoid of 1924 will be used for the purpose of this report; thus, E_q equals 3444.054 nmi and e^2 equals .00672267.

Mapping from Ellipsoid to Sphere

The stereographic projection equations have been derived for a sphere. It is therefore necessary to transform points on or above the ellipsoid to points on or above the sphere. This transformation must be conformal, i.e., angle preserving, if the final stereographic projection is to be conformal. Reference 2 page 86 derives the relationship between the ellipsoid and the conformal sphere. The mapping of points on or above a location on the ellipsoid onto the conformal sphere is performed as follows.

$$\lambda' = \lambda$$

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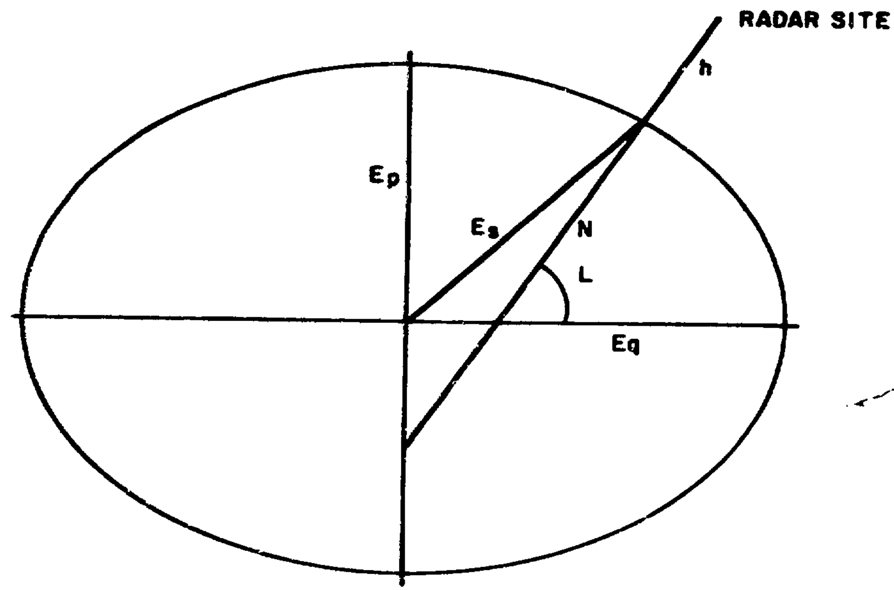

Figure IV A CROSS SECTION OF THE ADOPTED EARTH MODEL

$$\tan\left(\frac{\pi}{4} + \frac{\Phi}{2}\right) = \tan\left(\frac{\pi}{4} + \frac{L}{2}\right) \cdot \left[\frac{1 - e\sin L}{1 + e\sin L}\right]^{e/2} \qquad (11)$$

In these equations:

L, λ are the geographical latitude and longitude of the point on the ellipsoid

Φ, λ' are the latitude and longitude of the corresponding point on the conformal sphere. The latitude Φ is called the conformal latitude.

e is the eccentricity of the earth.

Since the mapping process involves only a transformation of latitudes, the stereographic projection equations are valid for the ellipsoid if the conformal latitude (Φ) is used in place of the geographic latitude (L).

Radius of the Earth to a Radar Site

In determination of the stereographic ground range, a spherical earth was used to compute the angle that subtends the radar site and the target. Since the earth is actually modeled by an ellipsoid, the use of a spherical earth in calculating stereographic ground range without introducing corrections to slant range and height, to allow for the conformal projection, will introduce a certain amount of error. To minimize this error, the radius of the spherical earth E_s is set equal to the distance from the center of the ellipsoid to the surface of the ellipsoid at the radar site as shown in Figure IV. The distance E_s is calculated as follows.

$$E_s = (X_2^2 + Y_s^2)^{1/2} \qquad (12)$$

where:

$$X_s = N \cos L$$

$$Y_s = (1 - e^2)\, N \sin L$$

$$N = \frac{E_q}{(1 - e^2 \sin^2 L)^{1/2}}$$

L is the geographic latitude.

Table I gives the radius of the Earth at latitudes between 0 and 90 degrees.

Table I

Earth Radius Versus Latitude

Geographic Latitude	Earth Radius (nmi)
0°	3444.054
10°	3443.707
20°	3442.708
30°	3441.173
40°	3439.286
50°	3437.273
60°	3435.375
70°	3433.824
80°	3432.810
90°	3432.458

Calculations show that the resulting error is greatest for a target north or south of the radar and does not exceed $\pm$ 0.03 nmi at a range of 250 nmi.

THE COMMON COORDINATE PLANE

In a large air surveillance region such as will be encountered in the JSS, several radars are linked together in order to display a composite air surveillance picture. It is therefore necessary to transform coordinates on the individual radar planes to coordinates on a common plane. The transformation process requires that the coordinates of the radars on the common plane and that angular rotations between the radar planes and the common plane be known.

The origin of the common plane or region center is defined as the center of the smallest circle that will circumscribe all radars tied into the air surveillance region. The common plane is tangent to the conformal sphere. The coordinates axes are oriented such that the positive Y axis is directed towards true north and the positive X axis is directed towards east. Coordinates on the common plane are expressed in cartesian coordinates.

Radar Coordinates on the Common Plane

A radar site or other known location can be stereographically projected onto the common coordinate plane. Reference 3 page 53 derives the equations necessary to project a point on the conformal sphere onto the common coordinate plane. The rectangular coordinates of a radar site or other known location X_r, Y_r on the common plane are obtained as follows.

$$X_r = 2E_c \frac{\sin\Delta\lambda\cos\phi}{1 + \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda} \quad (13)$$

$$Y_r = 2E_c \frac{\sin\phi\cos\phi_0 - \cos\phi\sin\phi_0\cos\Delta\lambda}{1 + \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda} \quad (14)$$

where:

ϕ, λ are the conformal latitude and longitude of the point to be projected

ϕ_0, λ_0 are the conformal latitude and longitude of the region center

$\Delta\lambda = \lambda_0 - \lambda$ if longitudes are measured positive west of the prime meridian

$= \lambda - \lambda_0$ if longitudes are measured positive east of the prime meridian

Angular Rotation

As indicated in reference 3 page 6, an angular rotation of the radar plane with respect to the common coordinate plane is necessary for the transformation process. The effect of the rotation is to make the axes of the radar plane more nearly parallel to the axes of the common plane. The angle of rotation is shown in reference 3 to be:

$$\beta = \tan^{-1}\left[\frac{-(\sin\phi + \sin\phi_0)\sin\Delta\lambda}{\cos\phi\cos\phi_0 + (1 + \sin\phi\sin\phi_0)\cos\Delta\lambda}\right] \tag{15}$$

Transformation of Radar Coordinates on the Common Plane

The equations for transformation of radar coordinates to coordinates in the common coordinate plane are derived in reference 3 pages 3 - 15. The exact transformation equations involve an infinite series. To lessen the processing requirement without unduly sacrificing accuracy a second order approximation is used. Rectangular coordinates X, Y are obtained as follows.

$$X = X_r + K(R\sin(\theta + \beta) + AR^2\sin[2(\theta + \beta) - \gamma]) \tag{16}$$

$$Y = Y_r + K(R\cos(\theta + \beta) + AR^2\cos[2(\theta + \beta) - \gamma]) \tag{17}$$

where:

$$K = 1 + \frac{\omega_r^2}{4E_c^2}$$

$$A = \frac{\omega_r}{4E_c^2}$$

$$\omega_r = (X_r^2 + Y_r^2)^{1/2}$$

$$\gamma = \tan^{-1}\left(\frac{X_r}{Y_r}\right)$$

R is the stereographic ground range

θ is the azimuth angle measured clockwise from north at the radar site

X_r, Y_r are the coordinates of the reporting radar on the common plane

Since the coordinates of an aircraft on the common plane are a function of the stereographic ground range R, any error in the stereographic ground range will appear as a misregistration on the common coordinate plane.

THE CONFORMAL SPHERE

The common coordinate plane and all radar planes are tangent to the conformal sphere. The radius of the conformal sphere is arbitrary, but is chosen to minimize the scale errors that will be encountered in the air surveillance region. Scale errors result in mapping from the ellipsoid to the conformal sphere and in mapping from the conformal sphere onto the tangent plane.

Scale Factor — Ellipsoid to Conformal Sphere

The scale factor associated with mapping from the ellipsoid to the conformal sphere is the ratio of a linear element on the conformal sphere to a corresponding linear element on the ellipsoid. From reference 2 page 86, the scale factor associated with a point of projection is:

$$k_1 = \frac{E_c \cos\phi}{N \cos L} \tag{18}$$

where:

ϕ is the conformal latitude of the point to be projected

L is the geographic latitude of the point to be projected

The scale factor is a function of the radius of the conformal sphere and the conformal and geographic latitude of the point to be projected. The scale factor can be expressed as follows.

$$k_1 = \frac{E_c}{E_q} \frac{(1 - e^2 \sin^2 L)^{1/2} \cos\phi}{\cos L}$$

$$= \frac{E_c}{E_q} k_1' \tag{19}$$

Table II gives values of k_1' for several different latitudes.

Table II

Scale Factor — Ellipsoid to Sphere versus Latitude

Geographic Latitude	Scale Factor k_1'
0°	1.00000000
10°	1.00010071
20°	1.00039099
30°	1.00083667
40°	1.00138493
50°	1.00197024
60°	1.00252203
70°	1.00297314
80°	1.00326821
90°	1.00337838

Scale Factor — Conformal Sphere to Plane

The scale factor associated with mapping from the conformal sphere to the plane of projection is the ratio of a linear element on the plane of projection to a corresponding linear element on the conformal sphere. The scale factor is therefore a function of the angular separation between the origin of the plane and the point of projection. The distance on the sphere between the origin of the plane and the point of projection is calculated as follows.

$$D = E_c \psi \tag{20}$$

The stereographic ground range R is given by equation (1) as follows.

$$R = 2E_c \tan\left(\frac{\psi}{2}\right) \tag{21}$$

The scale factor associated with mapping from the conformal sphere to the plane of projection is calculated as follows.

$$\begin{aligned} k_2 &= \frac{dR/_{d\psi}}{dD/_{d\psi}} \\ &= \sec^2\left(\frac{\psi}{2}\right) \\ &= \frac{2}{1 + \cos\psi} \end{aligned} \tag{22}$$

Table III presents the scale factor k_2 as a function of angular separation.

Table III

Scale Factor — Conformal Sphere to Plane versus Angular Separation

Angular Separation	Scale Factor k_2
0°	1.00000000
2°	1.00030468
4°	1.00121946
6°	1.00274658
8°	1.00488976
10°	1.00765427
12°	1.01104690
14°	1.01507605
16°	1.01975173
18°	1.02508563
20°	1.03109120

Aircraft and radar locations in the air surveillance region are presented on the common plane. The scale factor k_2, associated with the common plane is a function of the angular separation between the region center and the point of interest. From reference 3 page 23, the angular separation between the region center and a point of interest can be expressed in terms of their locations on the conformal sphere as follows.

$$\cos\psi = \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda \tag{23}$$

Combining equations (22) and (23) the scale factor k_2 is expressed as follows.

$$k_2 = \frac{2}{1 + \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda} \tag{24}$$

Calculation of the Radius of the Conformal Sphere

The total scale factor in mapping a point from the ellipsoid to the common coordinate plane is the product of k_1 and k_2. The total scale factor is expressed as follows.

$$k = \frac{2E_c\cos\phi}{N\cos L(1 + \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda)} \tag{25}$$

The scale factor expresses the ratio of a linear element on the common coordinate plane to the corresponding element on the ellipsoid. The scale factor therefore represents the ratio of the velocity on the common plane to the corresponding velocity on the ellipsoid. A unity scale factor is highly desirable since velocity on the common plane will represent actual ground speed. The scale error is defined as the difference between the scale factor at a point in the region and a unity scale factor. The scale error ε is expressed as follows.

$$\begin{aligned}\varepsilon &= \frac{2E_c\cos\phi}{N\cos L(1 + \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos\Delta\lambda)} - 1 \\ &= E_c A - 1 \end{aligned} \tag{26}$$

The extent of an air surveillance region is defined by the location of the radars that are tied into it. Since the scale error varies as a function of the location and separation of a point from the region center, it is highly desirable to minimize the maximum scale errors that will be encountered. From equation (26) the value of the scale error at a point can be varied by varying the radius of the conformal sphere E_c. It is therefore possible to obtain both positive and negative scale errors. To minimize the magnitude of the largest scale error, E_c is chosen such that the magnitude of the maximum negative scale error is equal to the maximum positive scale error. The radius of the conformal sphere is obtained as follows.

$$\varepsilon_{min} + \varepsilon_{max} = 0$$

$$E_c A_{min} - 1 + E_c A_{max} - 1 = 0$$

$$E_c (A_{min} + A_{max}) - 2 = 0$$

$$E_c = \frac{2}{A_{min} + A_{max}} \tag{27}$$

where A_{min} and A_{max} are the smallest and largest A calculated for the region center and all radars tied to the region. Substituting equations (19) and (24) into equation (2^) the value of A may be calculated as follows.

$$A = \frac{k_1' k_2}{E_q} \tag{28}$$

The values of A_{max} and A_{min} for a given region can therefore be determined from Tables II and III. The following general conclusions can be drawn from examination of the tables.

1. A_{min} usually corresponds to the region center since its value of k_2 is unity.
2. A_{max} usually corresponds to the most distant radar since it has the largest angular separation. If two radars have the same angular separation, A_{max} will correspond to the more northerly since it will have the larger k_1 value.

Table IV shows values of E_c calculated for a region center at a geographic latitude of 45° and radar sites directly north and south of the region center. Table IV shows that the radius of the conformal sphere decreases markedly as the region size increases.

Table IV

E_c Versus Region Size

Geographic Latitude of Most Distant Radar	Distance From Region Center	E_c
47°	120.012	3437.561
49°	240.[illegible]10	3435.787
51°	359.997	3435.963
53°	479.970	3429.088
57°	719.881	3418.170
61°	959.744	3403.004
65°	1199.563	3383.553
43°	120.026	3437.964
41°	240.064	3436.596
39°	360.116	3434.179
37°	480.179	3430.714
33°	720.342	3420.628
29°	960.553	3406.314
25°	1200.808	3387.749

Effect of E_c/E_s

The JSS stereographic ground range equation, equation (8), and the current SAGE/BUIC stereographic ground range equation, equation (9), differ by the scale factor E_c/E_s. The effect of E_c/E_s can be expressed as the difference between the two equations as follows.

$$\delta = F\Big/\left(1 + \frac{H_{m/2} + h}{2E_s}\right)\left(1 - \frac{E_c}{E_s}\right)$$

$$= R\left(1 - \frac{E_c}{E_s}\right) \qquad (29)$$

Since stereographic ground range is transformed into coordinates on the common plane, a difference in the ground ranges will result in a corresponding misregistration on the common plane. The quantity R in equation (29) represents an approximation. Since the design registration error budget in JSS is .18 nmi, R must differ from its actual value by no more than .18 nmi. (A detailed review of the errors induced by an error in R is beyond the scope of this report.) Therefore a difference calculated by equation (29) greater than .36 nmi will guarantee an unacceptable registration error if the SAGE/BUIC stereographic ground range is used. To show the effect of E_c/E_s as a function of region size, values of δ are shown in Table V for the radar locations, region center and values of E_c indicated in Table IV. The value of R was arbitrarily chosen to be 100 and 200 nmi. Region size is defined as the distance of the most distant radar from the region center.

Table V

Difference Between Ground Range Equations 8 and 9 versus Region Size

Geographic Latitude of Most Distant Radar	Approximate Region Size	E_c/E_s	δ (nmi) R=100nmi	δ (nmi) R=200nmi
47°	120	.9999	.009	.018
49°	240	.9995	.049	.098
51°	360	.9988	.120	.239
53°	480	.9978	.221	.442
57°	720	.9948	.517	1.033
61°	960	.9906	.937	1.875
65°	1200	.9852	1.485	2.969
43°	120	.9998	.021	.042
41°	240	.9993	.072	.145
39°	360	.9985	.154	.308
37°	480	.9973	.266	.533
33°	720	.9942	.581	1.163
29°	960	.9898	1.018	2.036
25°	1200	.9842	1.576	3.152

Examination of Table V reveals that for R equal to 200 nmi, values of δ will exceed .36 nmi for regions somewhere between 360 nmi and 480 nmi. This indicates that the SAGE/BUIC stereographic ground range equation will produce unacceptable results in large regions. Some of the current SAGE/BUIC regions exceed these limits, and all JSS regions will exceed these limits by a considerable margin. Section III of this report is devoted to depicting the registration errors produced by the SAGE/BUIC and JSS stereographic ground range equations in the seven JSS regions.

SECTION III

EFFECT OF THE SAGE/BUIC AND JSS STEREOGRAPHIC GROUND RANGE EQUATIONS ON REGISTRATION

INTRODUCTION

Accurate stereographic projection of radar data onto the common coordinate plane is vital to the operation of air defense and air traffic control systems. Large registration errors seriously downgrade the performance and stability of the active tracking algorithm. For each of the seven JSS regions, simulated radar data was used to demonstrate the effect of the SAGE/BUIC and JSS stereographic ground range equations on registration.

SIMULATED RADAR DATA

Radar slant range and azimuth data were generated for four aircraft locations at altitudes of 30,000, 45,000 and 60,000 ft in each JSS region. Appendix 1 indicates the algorithm used to produce the data. For a particular radar, a slant range-azimuth pair was generated only if the aircraft was within 250 nmi of the radar and above the radar horizon. Aircraft locations were chosen so that slant ranges for the reporting radars would be greater than 170 nmi.

Radar slant ranges were converted into stereographic ground ranges using the SAGE/BUIC and JSS stereographic ground range equations (equations (8) and (9)). Stereographic ground ranges were transformed into coordinates on the common plane using equations (16) and (17). Since the location of the aircraft was known, actual coordinates on the common plane were computed using equations (13) and (14). The registration error was obtained by taking the magnitude of the difference between the actual coordinates on the common plane and those obtained by the SAGE/BUIC and JSS stereographic ground range equations.

Appendix II presents the data for the seven JSS regions. For each region, there is a Site Data, a Simulated Radar Data, and a Registration Error table. In addition there is a figure showing the location of the region center, radar sites, and aircraft. The following information is given in the tables.

Site Data Table:

1. Approximate geographic latitude and longitude of the region center and radar sites.
2. Radius of the earth to each radar site.
3. Coordinates on the common plane for each radar site.
4. The radius of the conformal sphere.

Simulated Radar Data Table:

1. The geographic latitude and longitude and altitude of the aircraft in the region.
2. The designation of all reporting radars.
3. Slant range and azimuth data for all reporting radars.

Registration Error Table:

1. Stereographic ground range calculated using the SAGE/BUIC and JSS ground range equations.
2. Coordinates on the common plane obtained from the SAGE/BUIC and JSS ground range equations.
3. Coordinates on the common plane obtained from the actual aircraft locations.
4. The registration error induced by the SAGE/BUIC and JSS ground range equations.

Examination of the tables in Appendix II reveals that use of the SAGE/BUIC stereographic ground range equation resulted in registration errors that exceeded the .18 nmi JSS registration error budget in all cases tested. Use of the JSS stereographic ground range equation resulted in acceptable registration errors. Table VI summarizes the

TABLE VI

WORST-CASE REGISTRATION ERRORS IN EACH JSS REGION

JSS REGION	RADIUS OF THE CONFORMAL SPHERE	APPROXIMATE REGION SIZE (NM)	CASE #	SLANT RANGE (NM)	REGISTRATION ERRORS SAGE/BUIC	JSS
NORTHEAST	3428.842	507	12	179.161	0.584	0.016
NORTHWEST	3425.636	582	6	181.280	0.703	0.010
SOUTHEAST	3430.619	535	9	179.547	0.581	0.019
SOUTHWEST	3428.877	546	12	179.980	0.627	0.002
EASTERN CANADA	3411.221	844	12	179.475	1.433	0.015
WESTERN CANADA	3423.829	603	12a	181.173	0.684	0.031
ALASKAN	3427.488	439	9	182.171	0.439	0.030

worst case results for slant ranges of approximately 180 nmi in each of the seven JSS regions.

The omission of the factor E_c/E_s is responsible for the large registration errors produced by the SAGE/BUIC stereographic ground range equation. For a particular region, the value of E_c is constant. The most southerly radar will have the largest value of E_s. Therefore the value of δ as calculated by equation (29) will be largest for the most southerly radar. This indicates that the worst case errors will be produced by the most southerly radar, and the best case errors by the most northerly. Further examination of Appendix II supports this conclusion.

SECTION IV

CONCLUSIONS

The scale factor E_c/E_s should be included in the stereographic ground range equation to avoid large registration errors. This is especially important in large regions. The JSS stereographic ground range equation for processing returns with height data is:

$$R = \frac{E_c(S^2 - H^2)^{1/2}}{E_s\left(1 + \frac{H_{m/2} + h}{2E_s}\right)}$$

The present SAGE/BUIC stereographic ground range equations should be modified to reflect the scale factor E_c/E_s. The modification would require a change in the adaptation parameters on a site-by-site basis.

REFERENCES

1. J.J. Burke, An Improved Stereographic Coordinate Conversion Approximation for Radar Data Processing, MTR-2548, Contract F19628-73-C-0001, The MITRE Corporation, Bedford, MA, January 1973.

2. P. Thomas, Conformal Projections in Geodesy and Cartography, Special Publication 251, U.S. Coast and Geodetic Survey, 1952.

3. J.J. Burke, Stereographic Projection of Radar Data in a Netted Radar System, ESD-TR-73-210, AD 771544, November 1973.

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APPENDIX I

GENERATION OF SLANT RANGE AND AZIMUTH DATA

Radar slant range and azimuth data are calculated for aircraft locations using vector operations. The vector $\underline{V}$ from the center of the earth to a point on or above the ellipsoid is calculated as follows.

$$\underline{V} = \begin{cases} X_v = (N + H)\cos L\cos\lambda \\ Y_v = (N + H)\cos L\sin\lambda \\ Z_v = [N(1 - e^2) + H]\sin L \end{cases} \tag{A-1}$$

where:

H is the height of the point above the ellipsoid

λ is measured positive east of the prime meridian

The slant range S is computed as follows.

$$\underline{S} = \underline{T} - \underline{R}$$

$$S = |\underline{S}| \tag{A-2}$$

where:

$\underline{T}$, $\underline{R}$ are vectors from the center of the earth to the aircraft and radar respectively.

The aircraft must be above the radar horizon for a radar return to be possible. The aircraft is above the radar horizon if:

$$\underline{S} \cdot \underline{Z} > 0$$

where:

$\underline{Z}$ is a unit vector directed along the zenith of the reporting radar as follows:

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$$\underline{Z} = \begin{cases} X_z = \cos L_r \cos\lambda_r \\ Y_z = \cos L_r \sin\lambda_r \\ Z_z = \sin L_r \end{cases} \quad \text{(A-3)}$$

L_r, λ_r are the geographic latitude and longitude of the reporting radar

The azimuth angle θ is calculated as follows.

$$\theta = \tan^{-1}\left[\frac{\underline{S}\cdot\underline{E}}{\underline{S}\cdot\underline{N}}\right] \quad \text{(A-4)}$$

where:

$\underline{E}$ is a unit vector directed due east of the reporting radar as follows.

$$\underline{E} = \begin{cases} X_E = -\sin\lambda_r \\ Y_E = \cos\lambda_r \\ Z_E = 0 \end{cases} \quad \text{(A-5)}$$

$\underline{N}$ is a unit vector directed due north of the reporting radar as follows.

$$\underline{N} = \begin{cases} X_N = -\sin L_r \cos\lambda_r \\ Y_N = -\sin L_r \sin\lambda_r \\ Z_N = \cos L_r \end{cases} \quad \text{(A-6)}$$

APPENDIX II

SIMULATED DATA FOR THE SEVEN JSS REGIONS

Radar slant range and azimuth data were generated for four aircraft locations at altitudes of 30,000, 45,000 and 60,000 feet in each JSS region. For a particular radar, a slant range azimuth pair was generated if the aircraft was within 250 nmi of the radar and above the radar horizon. Radar slant ranges were converted into stereographic ground ranges using the SAGE/BUIC and JSS ground range equations. Stereographic ground ranges were transformed into coordinates on the common coordinate plane. The actual coordinates on the common plane were also obtained. The registration errors induced by the SAGE/BUIC and JSS ground range equations were calculated.

For each JSS region, there is a Site Data, Simulated Radar Data, and Registration Errors table. In addition, there is a figure showing radar site, aircraft, and region center locations.

Explanation of Tables

The Site Data Table shows the following.

a) Approximate latitude and longitude of the radar sites and region center.

b) The earth radius to each radar site calculated using equation (12).

c) Coordinates on the common plane for the radar sites calculated using equations (13) and (14).

d) The radius of the conformal sphere calculated using equation (27).

The Simulated Radar Data Table shows the following.

a) The latitude, longitude and altitude of the aircraft.

b) The slant range and azimuth for all reporting radars. A radar is a reporting radar if the target is within 250 nmi and above the radar horizon. Slant range and azimuth data are calculated using the algorithm in Appendix I.

The Registration Error Table shows the following.

a) The stereographic ground range calculated using the SAGE/BUIC and JSS ground range equations (equations (8) and (9)).

b) Coordinates on the common plane obtained from the SAGE/BUIC and JSS stereographic ground range equations using equations (16) and (17).

c) Coordinates on the common plane obtained from the actual location of the aircraft using equations (13) and (14).

d) The registration error induced by the SAGE/BUIC and JSS stereographic ground range equations.

TABLE BI

SITE DATA - NORTHEAST JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
	DESIGNATION	APPROXIMATE				
		LATITUDE	LONGITUDE	E_s	X	Y
1	BENSON, NC	35.4	78.6	3440.184	93.379	-495.951
2	BOSTON, MA	42.4	71.7	3438.806	416.372	-54.479
3	BUCKS HARBOR, ME	44.6	68.8	3438.361	499.454	89.369
4	CLEVELAND, OH	41.5	81.7	3438.987	-53.965	-131.216
5	DETROIT, MI	42.3	83.1	3438.826	-115.445	-81.965
6	DUBOIS, PA	41.1	78.8	3439.067	76.925	-154.757
7	EMPIRE, MI	44.8	86.1	3438.321	-238.529	73.930
8	FINLAND, MN	47.4	91.3	3437.795	-438.855	251.055
9	HARTFORD, CT	41.8	72.7	3438.926	348.913	-97.562
10	IVOR, VA	36.9	76.9	3439.896	173.299	-403.448
11	NEW YORK, NY	40.7	74.0	3439.147	295.843	-168.207
12	TREVOSE, PA	40.1	75.0	3439.267	252.658	-207.271
13	UTICA, NY	43.1	75.2	3438.665	232.255	-28.533
14	WASHINGTON, DC	38.9	77.0	3439.505	163.712	-283.874
15	REGION CENTER	43.7	80.5	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3428.842						

TABLE BII

SIMULATED RADAR DATA - NORTHEAST JSS REGION

CASE #	AIRCRAFT LOCATION LATITUDE	AIRCRAFT LOCATION LONGITUDE	ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
1	50.0	89.1	30	FINLAND, MN	179.102	28.432
2	50.0	89.1	45	FINLAND, MN	179.251	28.432
3	50.0	89.1	60	FINLAND, MN	179.434	28.432
4	47.0	83.2	30	EMPIRE, MI	179.600	41.591
5	47.0	83.2	45	EMPIRE, MI	179.749	41.591
6	47.0	83.2	60	EMPIRE, MI	179.931	41.591
7	40.0	85.7	30	DETROIT, MI	178.666	220.236
8a 8b	40.0	85.7	45	DETROIT, MI CLEVELAND, OH	178.815 199.594	220.236 244.455
9a 9b	40.0	85.7	60	DETROIT, MI CLEVELAND, OH	178.998 199.772	220.236 244.455
10	34.0	81.8	30	BENSON, NC	179.315	243.007
11	34.0	81.8	45	BENSON, NC	179.464	243.007
12	34.0	81.8	60	BENSON, NC	179.647	243.007

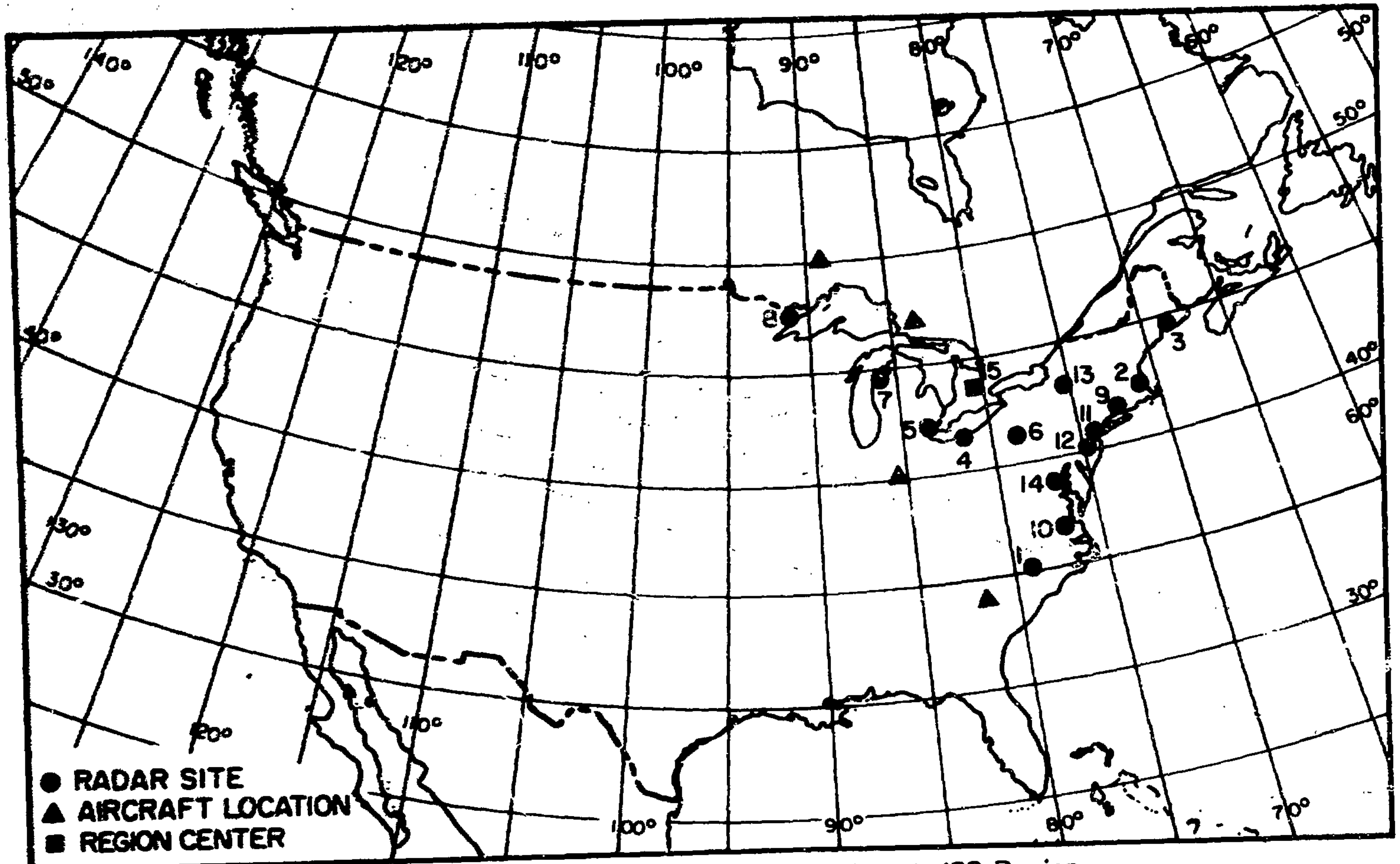

Fig. B1. Region Center, Radar Site, and Aircraft Locations – Northeast JSS Region

TABLE BIII

REGISTRATION ERRORS - NORTHEAST JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	178.820	178.355	-332.586	396.033	-332.864	395.656	-332.763	395.778	0.310	0.158
2	178.884	178.419	-332.548	396.085	-332.826	395.708	-332.763	395.778	0.374	0.094
3	178.949	178.483	-332.510	396.137	-332.788	395.760	-332.763	395.778	0.439	0.031
4	179.317	178.824	-110.443	199.647	-110.796	199.301	-110.676	199.402	0.339	0.156
5	179.382	178.888	-110.397	199.692	-110.750	199.346	-110.676	199.402	0.403	0.092
6	179.446	178.951	-110.352	199.737	-110.705	199.391	-110.676	199.402	0.467	0.030
7	178.335	177.867	-234.911	-214.665	-234.564	-214.280	-234.645	-214.383	0.387	0.131
8a 8b	178.448 199.219	177.931 198.632	-234.953 -235.096	-214.713 -214.574	-234.607 -234.562	-214.327 -214.328	-234.645	-214.383	0.451 0.490	0.068 0.099
9a 9b	178.512 199.290	177.994 198.703	-234.996 -235.161	-214.760 -214.604	-234.649 -234.627	-214.375 -214.358	-234.645	-214.383	0.515 0.561	0.010 0.031
10	179.033	178.443	-65.503	-580.774	-64.979	-580.495	-65.095	-580.572	0.455	0.132
11	179.097	178.507	-65.560	-580.805	-65.036	-580.526	-65.095	-580.572	0.520	0.075
12	179.161	178.571	-65.617	-580.835	-65.092	-580.556	-65.095	-580.572	0.584	0.016

TABLE BIV

SITE DATA - NORTHWEST JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
	DESIGNATION	APPROXIMATE LATITUDE	APPROXIMATE LONGITUDE	E_s	X	Y
1	BEACH, ND	46.9	104.0	3437.896	311.418	122.482
2	FINLEY, ND	47.5	98.0	3437.775	550.135	190.838
3	KALISPELL, MT	48.2	114.0	3437.634	-96.064	186.916
4	KENO, OR	42.1	121.9	3438.866	-458.105	-150.961
5	KLAMATH, CA	41.5	124.0	3438.987	-556.568	-173.879
6	MAKAH, WA	48.4	124.8	3437.594	-525.023	241.556
7	MALSTROM, MT	47.8	111.2	3437.714	16.132	161.551
8	MICA PEAK, WA	47.5	117.0	3437.775	-218.925	150.996
9	SALEM, OR	44.9	123.0	3438.301	-483.632	22.063
10	REGION CENTER	45.1	111.6	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3425.636						

TABLE BV

SIMULATED RADAR DATA - NORTHWEST JSS REGION

CASE #	AIRCRAFT LOCATION		ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE(NM)	AZIMUTH
	LATITUDE	LONGITUDE				
1	50.0	128.7	30	MAKAH, WA	181.212	303.515
2	50.0	128.7	45	MAKAH, WA	181.361	303.515
3	50.0	128.7	60	MAKAH, WA	181.543	303.515
4	39.0	126.2	30	KLAMATH, CA	180.949	214.707
5	39.0	126.2	45	KLAMATH, CA	181.097	214.707
6	39.0	126.2	60	KLAMATH, CA	181.280	214.707
7	50.0	108.1	30	MALMSTROM, MT	180.466	41.732
8	50.0	108.1	45	MALMSTROM, MT	180.615	41.732
9a 9b	50.0	108.1	60	MALMSTROM, MT BEACH, ND	180.797 248.353	41.732 320.177
10	50.0	95.5	30	FINLEY, ND	180.126	32.542
11	50.0	95.5	45	FINLEY, ND	180.275	32.542
12	50.0	95.5	60	FINLEY, ND	180.458	32.542

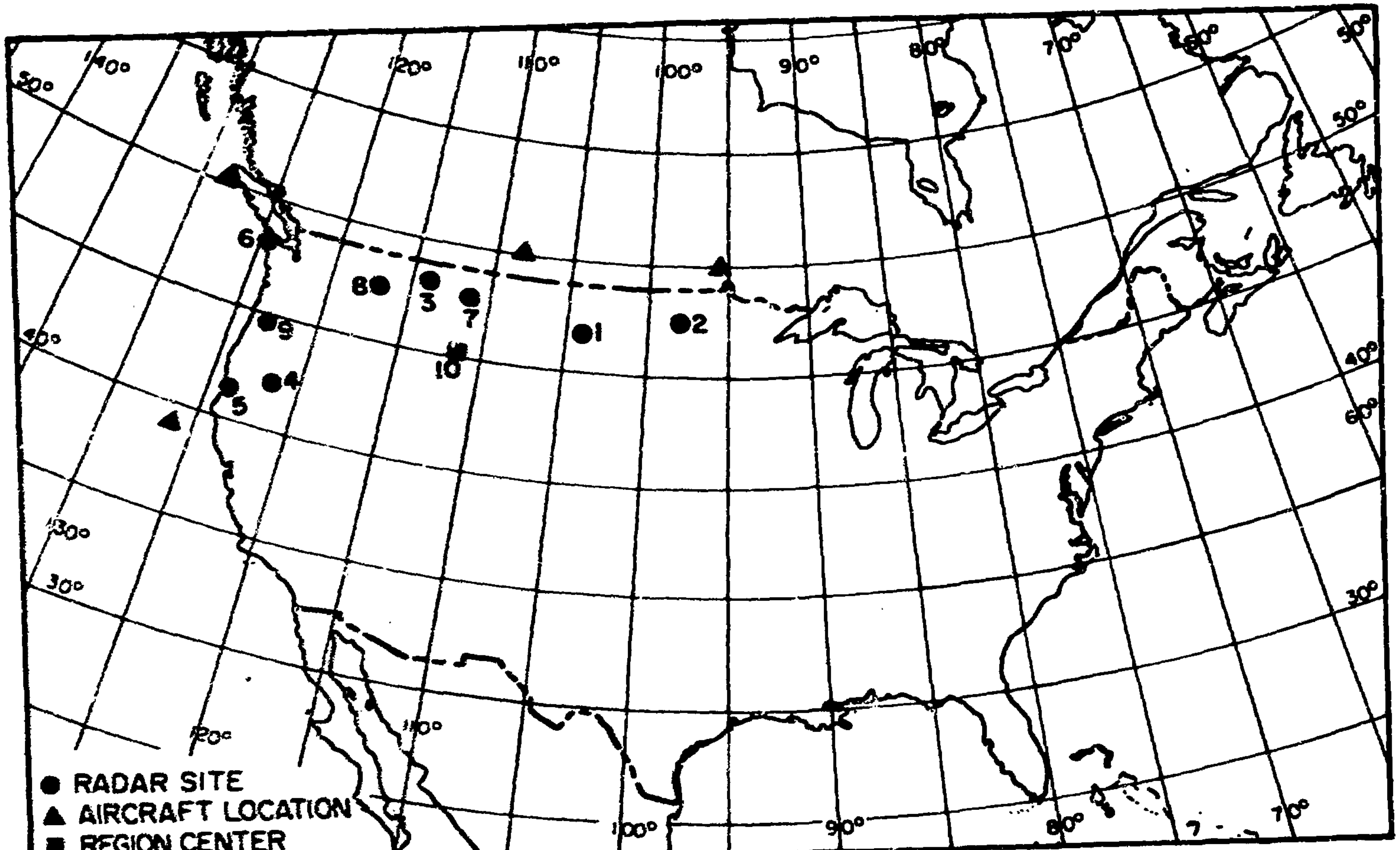

Fig. BII. Region Center, Radar Sites, and Aircraft Locations – Northwest JSS Region

TABLE BVI

REGISTRATION ERRORS - NORTHWEST JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
	BUIC/SAGE	JSS	BUIC/SAGE		JSS		ACTUAL		BUIC/SAGE	JSS
			X	Y	X	Y	X	Y		
1	180.928	180.299	-658.214	366.469	-657.751	366.033	-657.877	366.134	.475	161
2	180.993	180.364	-658.262	366.513	-657.798	366.078	-657.877	366.134	.541	.096
3	181.057	180.428	-658.309	366.559	-657.846	366.123	-657.877	366.134	.606	.033
4	180.635	179.964	-681.241	-306.922	-680.757	-306.922	-680.844	-306.510	.573	.136
5	180.729	180.029	-681.286	-306.970	-680.802	-306.452	-680.844	-306.510	.638	.071
6	180.794	180.093	-681.330	-307.018	-680.846	-306.500	-680.844	-306.510	.703	.010
7	180.182	179.550	135.567	296.718	135.148	296.244	135.262	296.[illegible]3	.476	.158
8	180.247	179.614	135.610	296.767	135.190	296.292	135.262	296.353	.541	.094
9a 9b	180.312 247.859	179.679 246.977	135.652 134.734	296.816 296.903	135.233 135.364	296.341 296.283	135.262	296.353	.606 .762	.031 .124
10	179.843	179.209	619.914	358.294	619.669	357.702	619.739	357.845	.482	.160
11	179.907	179.273	619.939	358.354	619.693	357.762	619.739	357.845	.547	.093
12	179.972	179.337	619.963	358.414	619.718	357.822	619.739	357.845	.612	.031

TABLE BVII

SITE DATA - SOUTHEAST JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
		APPROXIMATE				
	DESIGNATION	LATITUDE	LONGITUDE	E_s	X	Y
1	CROSS CITY, FL	26.7	81.7	3441.729	402.130	-142.802
2	DAUPHIN ISLAND, AL	30.3	88.1	3441.121	56.970	59.942
3	ELLINGTON, TX	29.6	95.2	3441.243	-312.965	25.924
4	GRAND BAY, AL	30.5	86.4	3441.086	144.730	73.363
5	JEDBERG, SC	33.4	80.0	3440.561	461.596	264.072
6	KEY WEST, FL	24.6	82.0	3442.058	393.289	-269.288
7	LACKLAND, TX	29.4	98.7	3441.277	-496.689	26.093
8	LAKE CHARLES, LA	30.2	93.2	3441.138	-207.382	57.280
9	MACDILL, FL	27.8	82.6	3441.548	350.265	-79.885
10	NEW ORLEANS, LA	30.5	89.9	3441.086	-36.178	71.718
11	OILTON, TX	27.8	99.2	3441.548	-530.944	-67.401
12	PATRICK, FL	28.6	80.7	3441.414	447.796	-25.737
13	RICHMOND, FL	25.7	80.4	3441.888	476.194	-197.948
14	TYNDALL, FL	30.1	85.6	3441.156	186.829	50.631
15	WHITE HOUSE, FL	29.9	81.8	3441.191	384.905	48.034
16	REGION CENTER	29.3	89.2	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3430.619						

TABLE BVIII

SIMULATED RADAR DATA - SOUTHEAST JSS REGION

CASE #	AIRCRAFT LOCATION		ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
	LATITUDE	LONGITUDE				
1	33.0	76.5	30	JEDBERG, SC	178.009	96.780
2	33.0	76.5	45	JEDBERG, SC	178.158	96.780
3	33.0	76.5	60	JEDBERG, SC	178.341	96.780
4	28.0	77.4	30	PATRICK, FL	178.609	100.822
5a 5b	28.0	77.4	45	PATRICK, FL RICHMOND, FL	178.758 212.001	100.822 48.810
6a 6b 6c	28.0	77.4	60	PATRICK, FL RICHMOND, FL CROSS CITY, FL	178.941 212.276 243.036	100.822 48.810 70.316
7	24.0	85.2	30	KEY WEST, FL	179.216	259.100
8	24.0	85.2	45	KEY WEST, FL	179.365	259.100
9	24.0	85.2	60	KEY WEST, FL	179.547	259.100
10	27.5	90.0	30	NEW ORLEANS, LA	179.798	181.704
11a 11b	27.5	90.0	45	NEW ORLEANS, LA DAUPHIN IS., AL	179.947 195.491	181.704 211.308
12a 12b 12c	27.5	90.0	60	NEW ORLEANS, LA DAUPHIN IS., AL LAKE CHARLES, LA	180.130 195.670 234.010	181.704 211.308 132.999

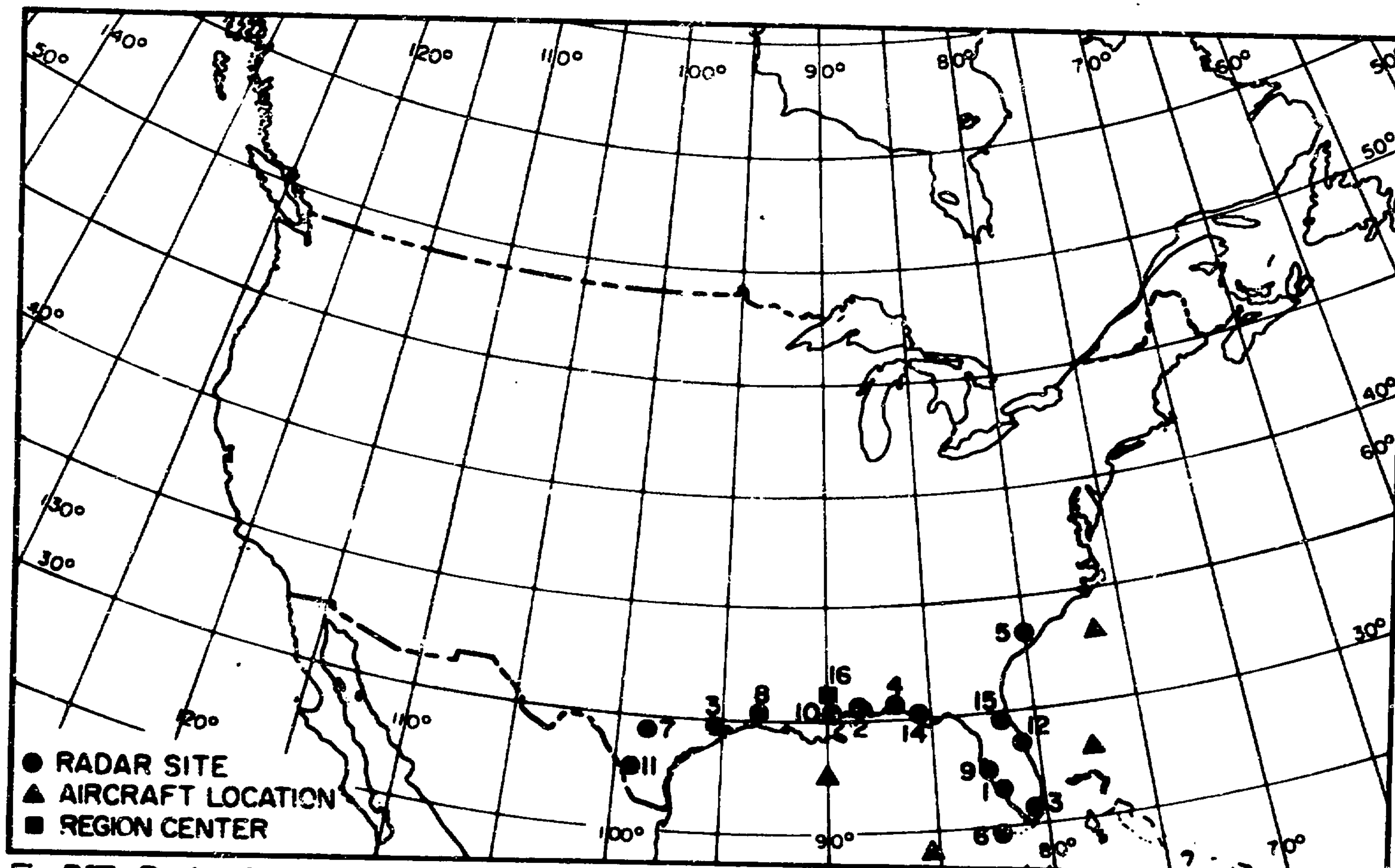

Fig. B III. Region Center, Radar Site and Aircraft Locations – Southeast JSS Region

TABLE BIX

REGISTRATION ERRORS - SOUTHEAST JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	177.728	177.215	640.580	257.611	640.063	257.631	640.205	257.610	.375	.144
2	177.791	177.278	640.644	257.609	640.126	257.628	640.205	257.610	.439	.081
3	177.855	177.341	640.708	257.607	640.190	257.626	640.205	257.610	.503	.022
4	178.327	177.769	625.953	-46.758	625.394	-46.692	625.535	-46.722	.420	.144
5a	178.391	177.832	626.017	-46.766	625.458	-46.700	625.535	-46.722	.484	.081
5b	211.718	211.025	625.920	-46.311	625.431	-46.808			.363	.135
6a	178.455	177.896	626.080	-46.773	625.521	-46.707	625.535	-46.722	.548	.020
6b	211.793	211.101	625.973	-46.256	625.484	-46.754			.639	.060
6c	242.545	241.763	626.158	-46.429	625.435	-46.741			.688	.102
7	178.934	178.340	219.097	-312.958	219.675	-312.814	219.539	-312.860	.453	.144
8	178.998	178.404	219.035	-312.973	219.613	-312.829	219.539	-312.860	.517	.080
9	179.062	178.467	218.973	-312.989	219.551	-312.845	219.539	-312.860	.581	.019
10	179.516	178.970	-42.575	-107.660	-42.556	-107.115	-42.560	-107.245	.416	.130
11a	179.580	179.035	-42.578	-107.724	-42.558	-107.179	-42.560	-107.245	.480	.065
11b	195.117	194.522	-42.826	-107.679	-42.522	-107.168			.510	.085
12a	179.645	179.099	-42.580	-107.789	-42.561	-107.244	-42.560	-107.245	.545	.001
12b	195.187	194.592	-42.862	-107.739	-42.558	-107.228			.580	.016
12c	233.523	232.810	-42.088	-107.695	-42.593	-107.192			.652	.062

TABLE BX

SITE DATA – SOUTHWEST JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
	DESIGNATION	APPROXIMATE				
		LATITUDE	LONGITUDE	E_s	X	Y
1	EL PASO, TX	31.8	105.9	3440.854	347.065	-257.259
2	LAKE HAVASU, AZ	34.5	115.0	3440.355	-113.710	-106.173
3	MT LAGUNA, CA	32.8	116.4	3440.671	-186.670	-205.609
4	OAKLAND, CA	38.0	122.8	3439.682	-477.514	126.934
5	ODESSA, TX	32.9	102.8	3440.653	498.919	-178.606
6	PASO ROBLES, CA	36.0	120.5	3440.070	-378.503	-2.762
7	PHOENIX, AZ	33.5	112.1	3440.542	30.020	-167.118
8	POINT ARENA, CA	38.9	123.7	3439.505	-513.764	185.416
9	SAN PEDRO, CA	33.8	118.3	3440.486	-279.209	-141.482
10	SILVER CITY, NM	32.9	108.8	3440.653	196.527	-199.258
11	REGION CENTER	36.3	112.7	–	0.000	0.000

RADIUS OF THE CONFORMAL SPHERE = 3428.877

TABLE BXI

SIMULATED RADAR DATA - SOUTHWEST JSS REGION

CASE #	AIRCRAFT LOCATION LATITUDE	AIRCRAFT LOCATION LONGITUDE	ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
1	39.0	127.5	30	POINT ARENA, CA	178.143	273.124
2	39.0	127.5	40	POINT ARENA, CA	178.292	273.124
3a 3b	39.0	127.5	60	POINT ARENA, CA OAKLAND, CA	178.475 229.828	273.124 286.604
4	34.0	123.2	30	PASO ROBLES, CA	179.237	228.788
5	34.0	123.2	45	PASO ROBLES, CA	179.386	228.787
6a 6b 6c	34.0	123.2	60	PASO ROBLES, CA OAKLAND, CA SAN PEDRO, CA	179.569 240.948 245.494	228.787 184.764 274.168
7	30.0	117.7	30	MT LAGUNA, CA	180.610	202.056
8	30.0	117.7	45	MT LAGUNA, CA	180.759	202.056
9a 9b	30.0	117.7	60	MT LAGUNA, CA SAN PEDRO, CA	180.942 230.076	202.056 172.168
10	28.8	106.0	30	EL PASO, TX	179.830	181.682
11	28.8	106.0	45	EL PASO, TX	179.980	181.682
12	28.8	106.0	60	EL PASO, TX	179.980	181.682

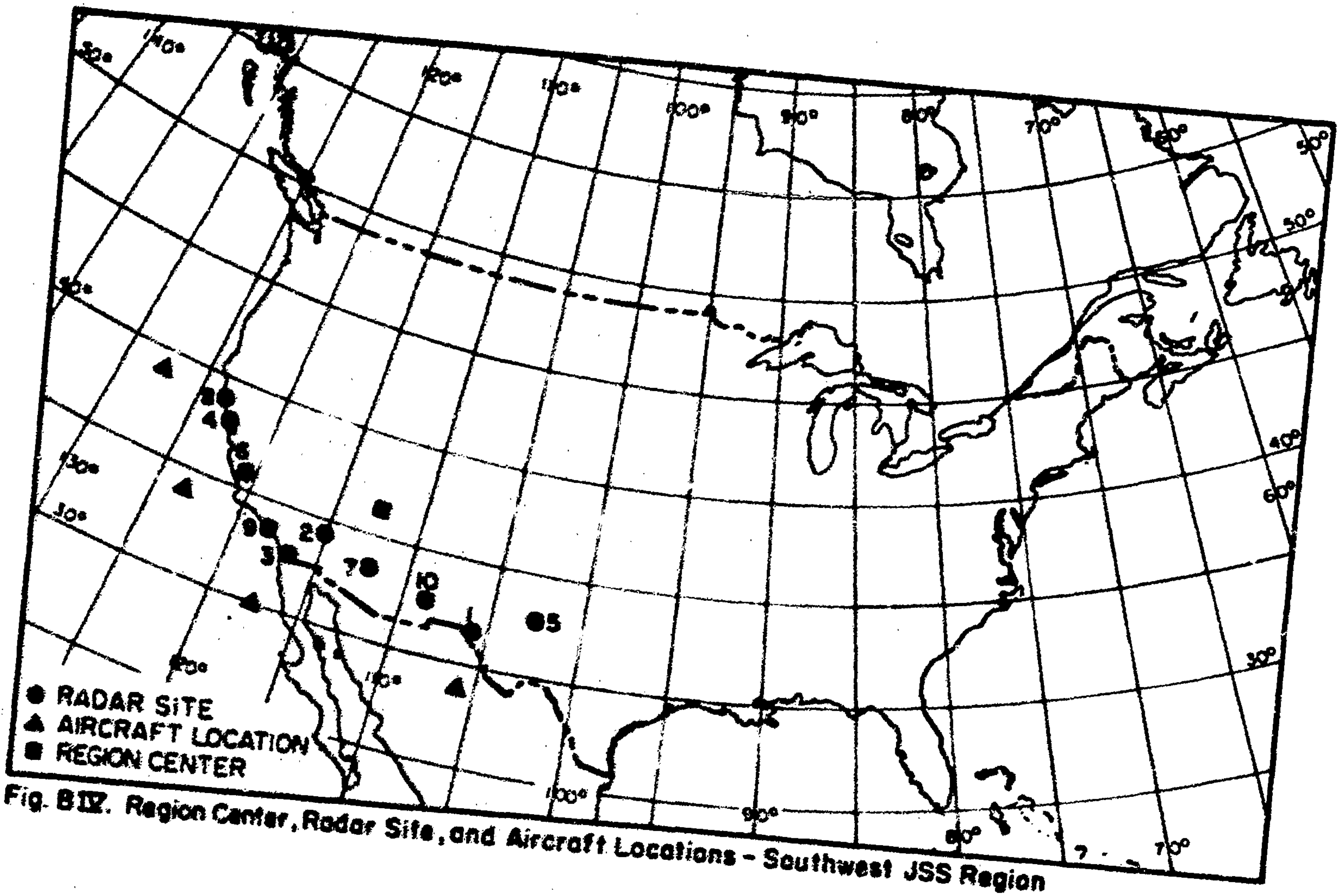

Fig. B IV. Region Center, Radar Site, and Aircraft Locations - Southwest JSS Region

TABLE XXII

REGISTRATION ERRORS — SOUTHWEST JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	177.861	177.313	-690.500	215.968	-689.953	215.854	-690.100	215.864	.408	.148
2	177.925	177.376	-690.563	215.959	-690.026	215.865	-690.100	215.864	.472	.084
3a 3b	177.989 229.341	177.439 228.622	-690.626 -690.685	215.970 216.136	-690.079 -690.015	215.875 215.856	-690.100	215.864	.536 .645	.026 .086
4	178.955	178.373	-522.635	-110.098	-522.167	-109.749	-522.270	-109.837	.449	.136
5	179.019	178.437	-522.686	-110.137	-522.218	-109.788	-522.270	-109.837	.514	.072
6a 6b 6c	179.083 240.458 245.002	178.501 239.763 244.176	-522.738 -522.399 -522.991	-110.176 -110.525 -109.715	-522.269 -522.260 -522.168	-109.826 -109.780 -109.823	-522.270	-109.837	.578 .700 .732	.011 .058 .102
7	180.327	179.710	-260.577	-370.620	-260.324	-370.055	-260.374	-370.179	.485	.134
8	180.392	179.774	-260.603	-370.680	-260.350	-370.114	-260.374	-370.179	.550	.070
9a 9b	180.652 229.589	179.839 228.815	-260.630 -260.311	-370.739 -370.905	-260.376 -260.376	-370.173 -370.132	-260.374	-370.179	.615 .729	.007 .047
10	179.568	178.924	353.489	-437.591	353.467	-436.963	353.471	-437.094	.497	.131
11	179.612	178.988	353.491	-437.655	353.469	-437.028	353.471	-437.094	.562	.066
12	179.677	179.052	353.493	-437.721	353.471	-437.092	353.471	-437.094	.627	.002

TABLE EXIII

SITE DATA – EASTERN CANADIAN JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
	DESIGNATION	APPROXIMATE LATITUDE	APPROXIMATE LONGITUDE	E_s	X	Y
C-1	BEAUSEJOUR, MAN	50.1	96.4	3437.253	-760.697	-9.238
C-2	GYPSUMVILLE, MAN	51.8	99.0	3436.916	-826.242	117.662
C-3	MONT APICA, QUE	48.0	71.8	3437.674	184.088	-232.865
C-4	LAC ST DENIS, QUE	46.0	75.0	3438.078	58.260	-357.336
C-5	ST MARGARETS, NB	47.0	65.5	3437.876	444.046	-266.025
C-6	SENNETERRE, QUE	48.3	77.8	3437.614	-55.714	-220.102
C-7	FALCONBRIDGE, ONT	46.2	81.0	3438.038	-190.642	-340.154
C-8	SIOUX LOOKOUT, ONT	50.2	91.9	3437.233	-590.264	-44.966
C-9	GOOSE BAY, LABR	53.6	60.2	3436.564	571.518	160.071
C-10	GANDER, NFLD	49.0	54.8	3437.473	839.591	-55.588
C-11	MOISIE, QUE	50.5	66.5	3437.173	375.718	-64.125
C-12	SYDNEY, NS	46.2	60.2	3438.038	668.436	-274.142
C-13	CHIBOUGAMAU, QUE	50.0	74.8	3437.273	61.488	-118.584
C-14	BARRINGTON, NS	43.5	65.8	3438.584	460.927	-475.666
C-15	LOWTHER, ONT	49.5	83.1	3437.373	-259.965	-137.306
C-16	REGION CENTER	52.0	76.4	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3411.221						

TABLE BXIV

SIMULATED RADAR DATA - EASTERN CANADIAN JSS REGION

CASE #	AIRCRAFT LOCATION		ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
	LATITUDE	LONGITUDE				
1	55.0	55.7	30	GOOSE BAY, LABR	179.313	60.176
2	55.0	55.7	45	GOOSE BAY, LABR	179.462	60.176
3	55.0	55.7	60	GOOSE BAY, LABR	179.644	60.176
4	52.4	72.0	30	CHIBOUGAMAU, QUE	178.906	35.157
5	52.4	72.0	45	CHIBOUGAMAU, QUE	179.055	35.157
6a 6b	52.4	72.0	60	CHIBOUGAMAU, QUE MOISIE, QUE	179.238 236.318	35.157 301.069
7	54.0	95.7	30	GYPSUMVILLE, MAN	178.618	40.902
8	54.0	95.7	45	GYPSUMVILLE, MAN	178.767	40.902
9a 9b	54.0	95.7	60	GYPSUMVILLE, MAN BEAUSEJOUR, MAN	178.950 236.246	40.902 6.039
10	42.0	62.3	30	BARRINGTON, NS	179.143	118.982
11	42.0	62.3	45	BARRINGTON, NS	179.292	118.982
12	42.0	62.3	60	BARRINGTON, NS	179.475	118.982

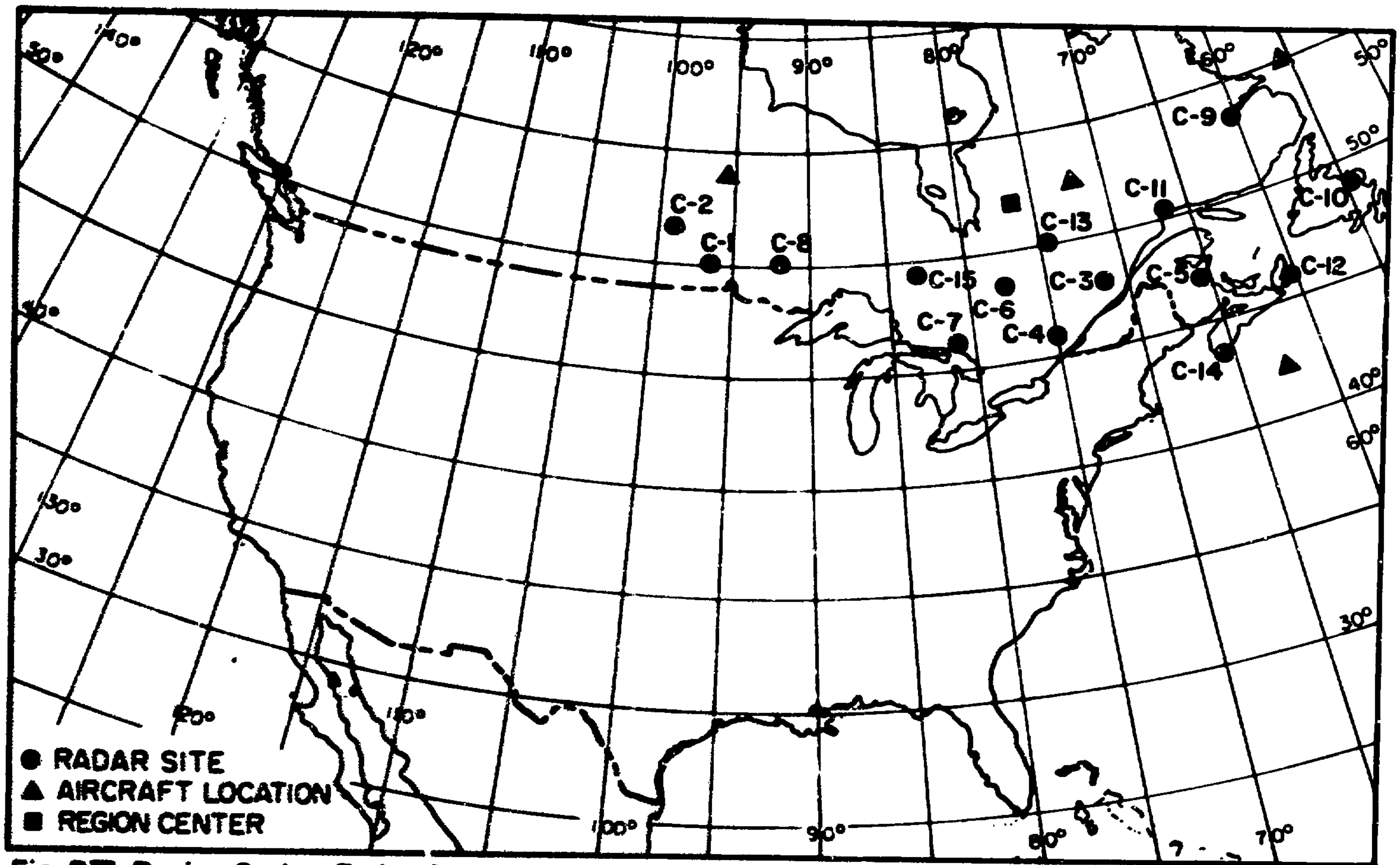

Fig. BV. Region Center, Radar Site, and Aircraft Locations – Eastern Canadian JSS Region

TABLE BXV

REGISTRATION ERRORS - EASTERN CANADIAN JSS REGION

CASE #	STEREOGRAHPIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	179.030	177.711	704.166	282.853	703.188	281.946	703.310	282.041	1.180	0.155
2	179.094	177.774	704.213	282.897	703.235	281.990	703.210	282.041	1.244	0.091
3	179.158	177.838	704.261	282.941	703.282	282.034	703.210	282.041	1.309	0.029
4	178.623	177.271	161.103	29.712	160.350	28.590	160.445	28.713	1.197	0.155
5	178.687	177.335	161.139	29.766	160.385	28.643	160.445	28.713	1.261	0.092
6a 6b	178.751 235.828	177.398 234.049	161.174 158.910	29.819 29.401	160.420 160.543	28.695 28.698	160.445	28.713	1.325 1.682	0.030 0.099
7	178.335	177.004	-671.725	211.072	-672.877	210.378	-672.738	210.450	1.189	0.157
8	178.399	177.067	-671.670	211.105	-672.823	210.412	-672.738	210.450	1.253	0.093
9a 9b	178.463 235.756	177.130 233.973	-671.615 -672.103	211.138 212.024	-672.768 -672.778	210.444 210.355	-672.738	210.450	1.318 1.697	0.030 0.103
10	178.860	177.438	629.795	-540.713	628.449	-540.196	628.575	-540.256	1.303	0.139
11	178.924	177.502	629.855	-540.737	628.509	-540.220	628.575	-540.256	1.368	0.075
12	178.988	177.565	629.916	-540.760	628.569	-540.243	628.575	-540.256	1.433	0.015

TABLE BXVI

SITE DATA — WESTERN CANADIAN JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
		APPROXIMATE				
	DESIGNATION	LATITUDE	LONGITUDE	E_s	X	Y
C-1	BEAUSEJOUR, MAN	50.1	96.4	3437.253	601.258	4.049
C-2	HOLBERG, BC	50.7	128.0	3437.133	-601.122	41.035
C-3	BALDY HUGHES, BC	53.8	122.8	3436.525	-378.533	189.690
C-4	BEAVERLODGE, ALTA	55.2	119.2	3436.256	-243.268	257.619
C-5	COLD LAKE, ALTA	54.2	110.5	3436.448	56.215	186.238
C-6	GYPSUMVILLE, MAN	51.8	99.0	3436.916	484.381	85.274
C-7	YORKTON, SASK	51.2	102.8	3437.034	349.035	28.033
C-8	DANA, SASK	52.0	105.9	3436.876	228.883	63.544
C-9	ALASKA, SASK	51.2	109.9	3437.034	82.707	7.216
C-10	PENHOLD, ALTA	51.9	114.0	3436.896	-70.348	48.789
C-11	KAMLOOPS, BC	50.8	120.1	3437.114	-302.978	-1.543
C-12	REGION CENTER	51.1	112.1	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3423.829						

TABLE XXVII

SIMULATED RADAR DATA - WESTERN CANADIAN JSS REGION

CASE #	AIRCRAFT LOCATION		ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
	LATITUDE	LONGITUDE				
1	51.0	132.7	30	HOLBERG, BC	179.783	277.577
2	51.0	132.7	45	HOLBERG, BC	179.931	277.577
3	51.0	132.7	60	HOLBERG, BC	180.114	277.577
4	57.0	123.5	30	BEAVERLODGE, ALTA	180.652	308.603
5a 5b	57.0	123.5	45	BEAVERLODGE, ALTA BALDY HUGHES, BC	180.801 194.178	308.603 353.191
6a 6b	57.0	123.5	60	BEAVERLODGE, ALTA BALDY, HUGHES, BC	180.983 194.357	308.603 353.191
7	57.0	108.6	30	COLD LAKE, ALTA	180.485	20.236
8	57.0	108.6	45	COLD LAKE, ALTA	180.634	20.236
9	57.0	108.6	60	COLD LAKE, ALTA	180.816	20.236
10	52.0	92.7	30	BEAUSEJOUR, MAN	180.843	49.402
11	52.0	92.7	45	BEAUSEJOUR, MAN	180.991	49.402
12a 12b	52.0	92.7	60	BEAUSEJOUR, MAN GYPSUMVILLE, MAN	181.173 234.886	49.402 84.586

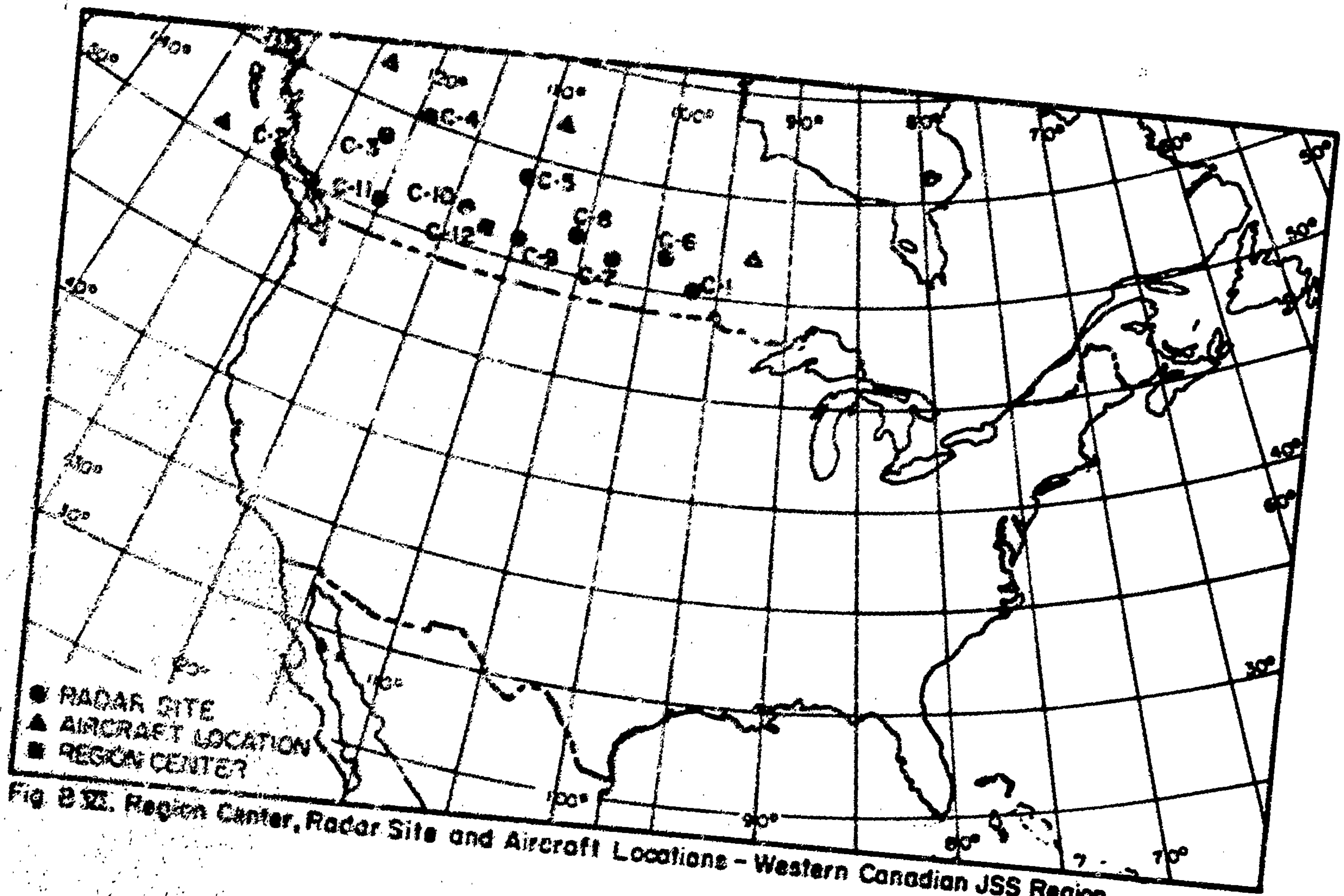

Fig. B XI. Region Center, Radar Site and Aircraft Locations - Western Canadian JSS Region

TABLE XXVIII

REGISTRATION ERRORS - WESTERN CANADIAN JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	179.350	178.856	-771.312	102.895	-770.872	102.655	-771.020	102.692	.551	.152
2	179.364	178.876	-771.393	102.917	-770.933	102.677	-771.020	102.692	.616	.088
3	179.628	178.934	-771.654	102.939	-770.994	102.699	-771.020	102.692	.681	.027
4	180.369	179.717	-372.917	384.060	-372.468	383.583	-372.591	383.682	.498	.158
5a 5b	180.431 191.975	179.761 191.049	-372.994 -372.450	384.085 384.288	-372.515 -372.593	383.628 383.569	-372.591	383.682	.563 .606	.094 .113
6a 6b	180.497 191.876	179.845 191.250	-373.010 -372.567	384.130 384.352	-372.562 -372.598	383.673 383.638	-372.591	383.682	.627 .675	.032 .044
7	140.202	179.541	114.441	356.919	114.665	356.293	114.724	356.442	.503	.160
8	140.266	179.603	114.903	356.990	114.686	356.354	114.724	356.442	.567	.096
9	140.318	179.669	114.922	[illegible]	114.707	356.414	114.724	356.442	.632	.032
10	140.559	179.925	711.118	149.246	710.909	148.678	711.015	148.796	.554	.158
11	140.624	179.919	711.172	149.299	710.948	148.730	711.015	148.796	.619	.094
12a 12b	140.624 216.397	179.991 211.506	711.416 711.791	149.350 149.042	710.987 710.926	148.782 148.799	711.015	148.796	.684	.031

TABLE BXIX

SITE DATA – ALASKAN JSS REGION

SITE #	SITE LOCATION			EARTH RADIUS TO SITE	COORDINATES ON THE COMMON PLANE	
		APPROXIMATE				
	DESIGNATION	LATITUDE	LONGITUDE	E_s	X	Y
1	CAPE LISBURNE	68.9	166.1	3433.972	-158.347	406.203
2	TIN CITY	65.6	168.0	3434.451	-228.125	214.751
3	KOTZEBUE	66.9	162.6	3434.256	-89.796	279.180
4	INDIAN MT	66.1	153.7	3434.375	124.323	233.333
5	CAMPION	64.7	156.7	3434.590	53.998	145.056
6	FORT YUKON	66.6	145.2	3434.300	323.211	293.186
7	MURPHY DOME	65.0	148.4	3434.543	263.517	183.669
8	TATILINA	62.9	156.0	3434.880	76.693	37.692
9	SPARREVON	61.1	155.6	3435.184	92.970	-69.769
10	FIRE ISLAND	61.2	150.2	3435.167	248.535	-49.613
11	KING SALMON	58.7	156.7	3435.607	65.641	-215.123
12	COLD BAY	55.3	162.9	3436.237	-140.719	-416.257
13	CAPE ROMANZOF	61.8	166.0	3435.064	-204.248	-18.689
14	CAPE NEWENHAM	58.6	162.1	3435.625	-103.432	-219.587
15	REGION CENTER	62.3	158.8	-	0.000	0.000
RADIUS OF THE CONFORMAL SPHERE = 3427.488						

TABLE BXX

SIMULATED RADAR DATA – ALASKAN JSS REGION

CASE #	AIRCRAFT LOCATION		ALTITUDE (K-FT)	REPORTING RADAR(S)	SLANT RANGE (NM)	AZIMUTH
	LATITUDE	LONGITUDE				
1	71.9	165.0	30	CAPE LISBURNE	182.265	6.500
2	71.9	165.0	45	CAPE LISBURNE	182.413	6.500
3	71.9	165.0	60	CAPE LISBURNE	182.595	6.500
4	64.0	174.0	30	TIN CITY	181.697	240.712
5	64.0	174.0	45	TIN CITY	181.845	240.712
6	64.0	174.0	60	TIN CITY	182.027	240.712
7	56.0	168.1	30	COLD BAY	181.840	285.529
8	56.0	168.1	45	COLD BAY	181.989	285.529
9	56.0	168.1	60	COLD BAY	182.171	285.529
10	58.2	150.0	30	FIRE ISLAND	180.750	177.983
11a 11b	58.2	150.0	45	FIRE ISLAND KING SALMON	180.899 213.608	177.983 95.237
12a 12b 12c	58.2	150.0	60	FIRE ISLAND KING SALMON SPARREVON	181.081 213.784 244.284	177.983 95.237 133.211

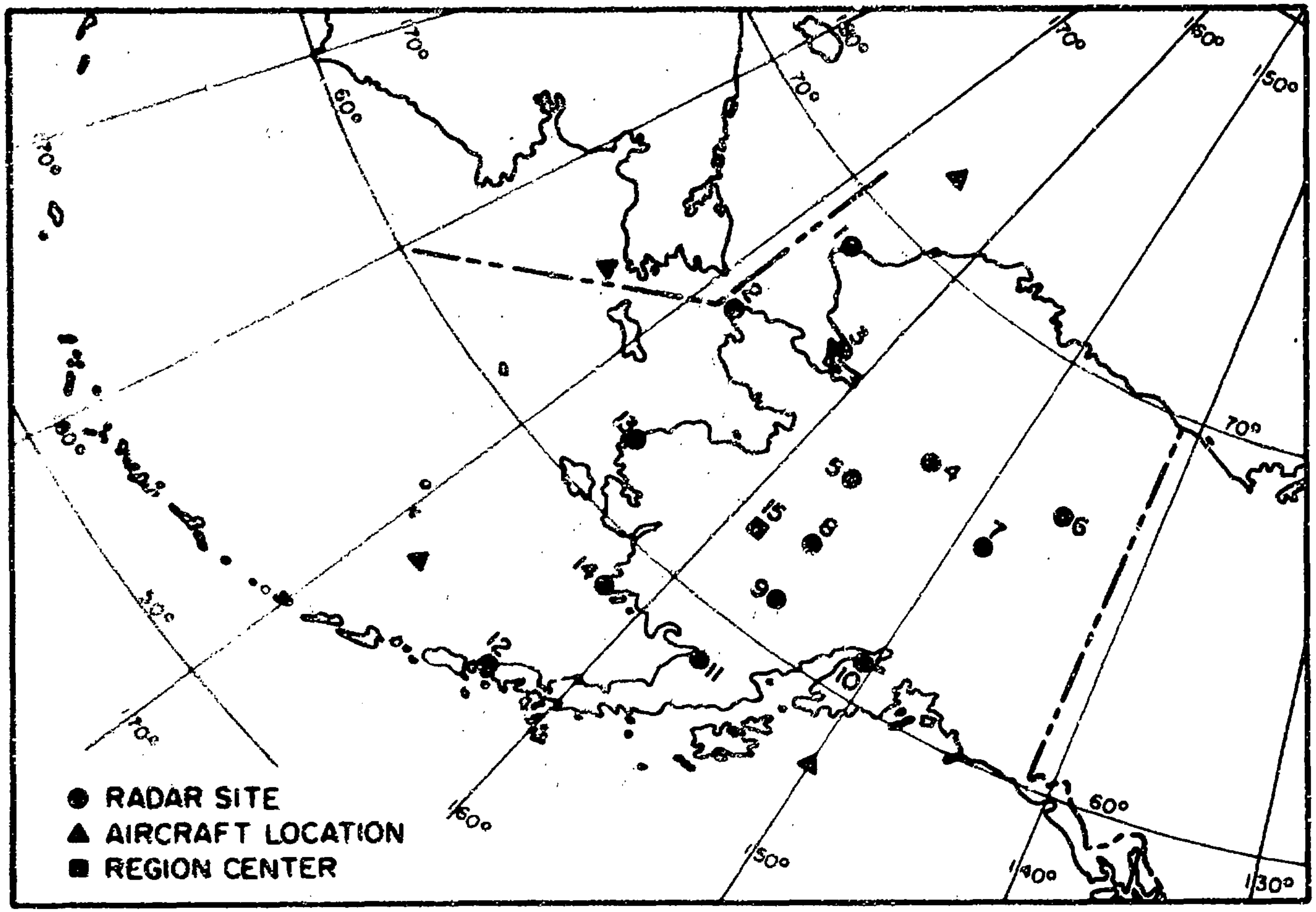

Fig. B VIII. Region Center, Radar Site, and Aircraft Locations - Alaskan JSS Region

TABLE XXXI

REGISTRATION ERRORS - ALASKAN JSS REGION

CASE #	STEREOGRAPHIC GROUND RANGE		COORDINATES ON THE COMMON PLANE						REGISTRATION ERROR (NM)	
			BUIC/SAGE		JSS		ACTUAL			
	BUIC/SAGE	JSS	X	Y	X	Y	X	Y	BUIC/SAGE	JSS
1	181.980	181.637	-116.542	584.333	-116.621	583.997	-116.582	584.149	.189	.157
2	182.045	181.702	-116.527	584.397	-116.606	584.061	-116.582	584.149	.254	.091
3	182.110	181.767	-116.512	584.461	-116.591	584.124	-116.582	584.149	.320	.026
4	181.413	181.045	-397.830	149.306	-397.487	149.439	-397.618	149.379	.224	.145
5	181.477	181.110	-397.891	149.282	-397.547	149.415	-397.618	149.379	.289	.080
6	181.542	181.174	-397.951	149.259	-397.607	149.392	-397.618	149.379	.354	.017
7	181.556	181.094	-312.948	-356.503	-312.511	-356.656	-312.662	-356.620	.309	.156
8	181.621	181.159	-313.010	-356.482	-312.572	-356.635	-312.662	-356.620	.374	.091
9	181.686	181.224	-313.071	-356.460	-312.633	-356.613	-312.662	-356.620	.439	.030
10	180.467	180.064	278.472	-227.893	278.405	-227.495	278.426	-227.623	.274	.129
11a 11b	180.532 213.225	180.129 212.722	278.483 278.805	-227.957 -227.626	278.416 278.301	-227.559 -227.597	278.426	-227.623	.339 .379	.065 .127
12a 12b 12c	180.596 213.301 243.793	180.193 212.797 243.247	278.493 278.881 278.794	-228.021 -227.630 -227.917	278.427 278.377 278.377	-227.622 -227.601 -227.563	278.426	-227.623	.404 .455 .471	.001 .053 .077