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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND CRITICAL
VELOCITIES FOR TARGET PENETRATIONS

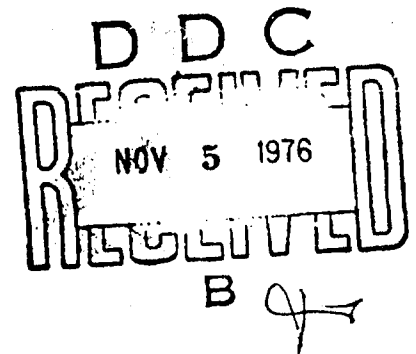
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JULY 1976



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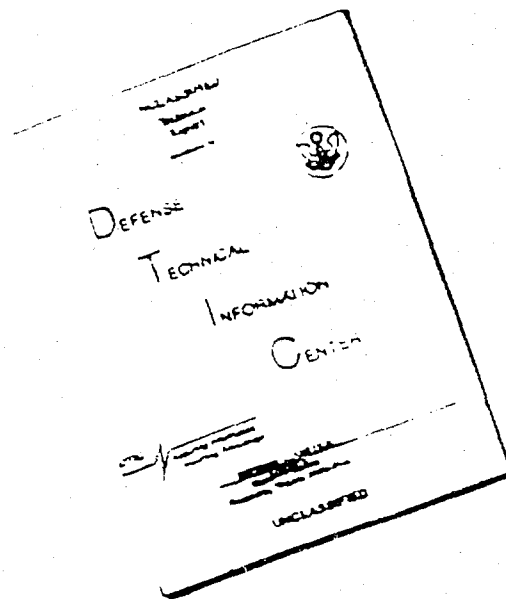
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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND
CRITICAL VELOCITIES FOR TARGET PENETRATIONS

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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND CRITICAL VELOCITIES FOR TARGET PENETRATIONS

D. CLARK
L. CROW
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1. INTRODUCTION

A number of procedures have been proposed for estimating from test data the functional relationship between V_r , the residual velocity of a projectile after penetration of a target and V_s , its striking velocity. Examples of these are models based on the hyperbolic [1] and exponential [4] relationships

$$V_r^2 = AV_s^2 + B \quad \text{and} \quad V_r = V_s - V_c e^{B(1 - V_s/V_c)},$$

respectively. In these models, V_c , the critical velocity is defined as the V_s intercept when $V_r = 0$.

Another penetration prediction model is the Johnson equation [2]

$$V_r = (V_s - V_c) [e^{K_4(V_s - V_c)^{K_5}} - 1]$$

where

$$V_c = K_1 (\sec \theta - 1) [e^{K_2(A/M \times 10^3)^{K_3}} - 1]$$

and θ is striking obliquity angle in degrees, A is fragment presented area in cm^2 , M is fragment weight in grams and K_1 , K_2 , K_3 , K_4 , and K_5 are constants to be estimated from the data. For this model the critical velocity is not directly estimated from test data. Instead, the critical velocity is determined by using the estimated V_c 's from an empirical model, such as the hyperbolic, fitted to several sets of data to estimate the constants K_1 , K_2 and K_3 .

For meaningful applications of these prediction models in ballistic studies, it is important that they realistically represent the relationship between striking and residual velocity and provide adequate estimates of the critical velocity V_c . Various applications of the penetration models mentioned above have demonstrated, however, that in many cases

they do not sufficiently describe this relationship.

In this report we propose a procedure based on the versatile three-parameter Weibull distribution function for estimating the relationship between the residual and striking velocities of a projectile from test data. This model has many shapes, as illustrated in the next section, which should make it useful for fitting ballistic data over a wide range of firing conditions for various types of projectiles. In Section 3 we discuss nonlinear estimation procedures for fitting this model to test data. These procedures utilize striking velocities with zero residuals to help estimate the three unknown parameters, including the critical velocity. In Appendix A we list a computer program for estimating these parameters and illustrate its use by fitting the Weibull model to several sets of penetration data. In Section 4 we compare the Weibull and hyperbolic models on eight sets of penetration data.

2. THE WEIBULL MODEL

Consider a continuous functional relationship $V_r = G(V_s)$, between striking velocity and residual velocity, with other impact conditions fixed. We assume that the function G describing this relationship satisfies the following conditions:

- (i) $G(V_s) = 0$ $0 \leq V_s \leq V_c$
- (ii) $G'(V_s) > 0$ $V_s > V_c$
- (iii) $V_s > G(V_s)$ $V_s > V_c$
- (iv) $V_s - G(V_s) \rightarrow 0$ $V_s \rightarrow \infty$

In practice, we are generally concerned with determining G over a finite range of V_s before any significant fragmentation of the projectile occurs. However, in the formulation of the problem, we assume the asymptotic property of condition (iv).

In the present paper we model the functional relationship as

$$G(V_s) = \begin{cases} 0 & V_s \leq V_c \\ V_s [1 - e^{-\lambda(V_s - V_c)^\beta}] & V_s > V_c \end{cases} \quad (2.1)$$

where $\lambda > 0$, $\beta > 0$ and $V_c > 0$. Observe that G satisfies conditions

(i) - (iv) and has (for $V_s \geq V_c$) the form $G(x) = xF(x)$ where

$F(x) = 1 - e^{-\lambda(x - \eta)^\beta}$, $x > \eta$, is the three-parameter Weibull distribution function with scale parameter λ , shape parameter β and location parameter η .

In Figure 1 we show some of the many shapes the function $G(x)$ can assume for various values of λ and β when $\eta = 100$. Condition (iv) implies, of course, that the line $y = x$ is an asymptote of $G(x)$. However, the second derivative $G''(x)$ is not necessarily less than zero for all x . Hence, as shown in Figure 1d, $G(x)$ may actually move away from the line $y = x$ over a finite range of x , for certain values of λ and β . However, the sign of $G''(x)$ will eventually change and $G(x)$ will approach the line $y = x$ asymptotically. This property of $G(x)$ is one of the characteristics which makes it a versatile model for fitting penetration data.

3. ESTIMATION PROCEDURES

The three unknown parameters λ , β and V_c in the model given by (2.1) can be estimated by the use of a nonlinear programming algorithm. A program, given in Appendix A, utilizing Marquardt's method [5] for nonlinear least squares has been developed by AMSAA for this application. Estimates of λ , β and V_c are determined as values $\hat{\lambda}$, $\hat{\beta}$ and $\hat{V}_c$, respectively, which minimize the root mean square error.

$$\text{ERMS} = \left(\frac{1}{N} \sum [V_r - G(V_s)]^2 \right)^{1/2},$$

where the summation is taken over all N pairs (V_s, V_r) such that $V_r > 0$.

The estimates $\hat{\lambda}$ and $\hat{\beta}$ are determined by this procedure with the constraint that they be greater than zero. Furthermore, to utilize the information associated with observed residual velocities that are zero, we perform this optimization with the additional constraint that $a \leq V_c < b$, where a and b are inputs to the program.

Marquardt's algorithm, which is an unconstrained optimization technique, has been modified to accommodate the constraints on λ , β and V_c . In most cases the algorithm converges to the solution within very few steps. If satisfactory results are not obtained by use of this method, one may elect to use another scheme such as those programmed by Wortman [6].

Regardless of the method employed to determine optimum values of the parameters, preliminary estimates λ^0 , β^0 , and V_c^0 must be established. Discretion should be exercised in order to determine the interval $[a, b)$ in which V_c is constrained to lie. It is often feasible to choose a to be

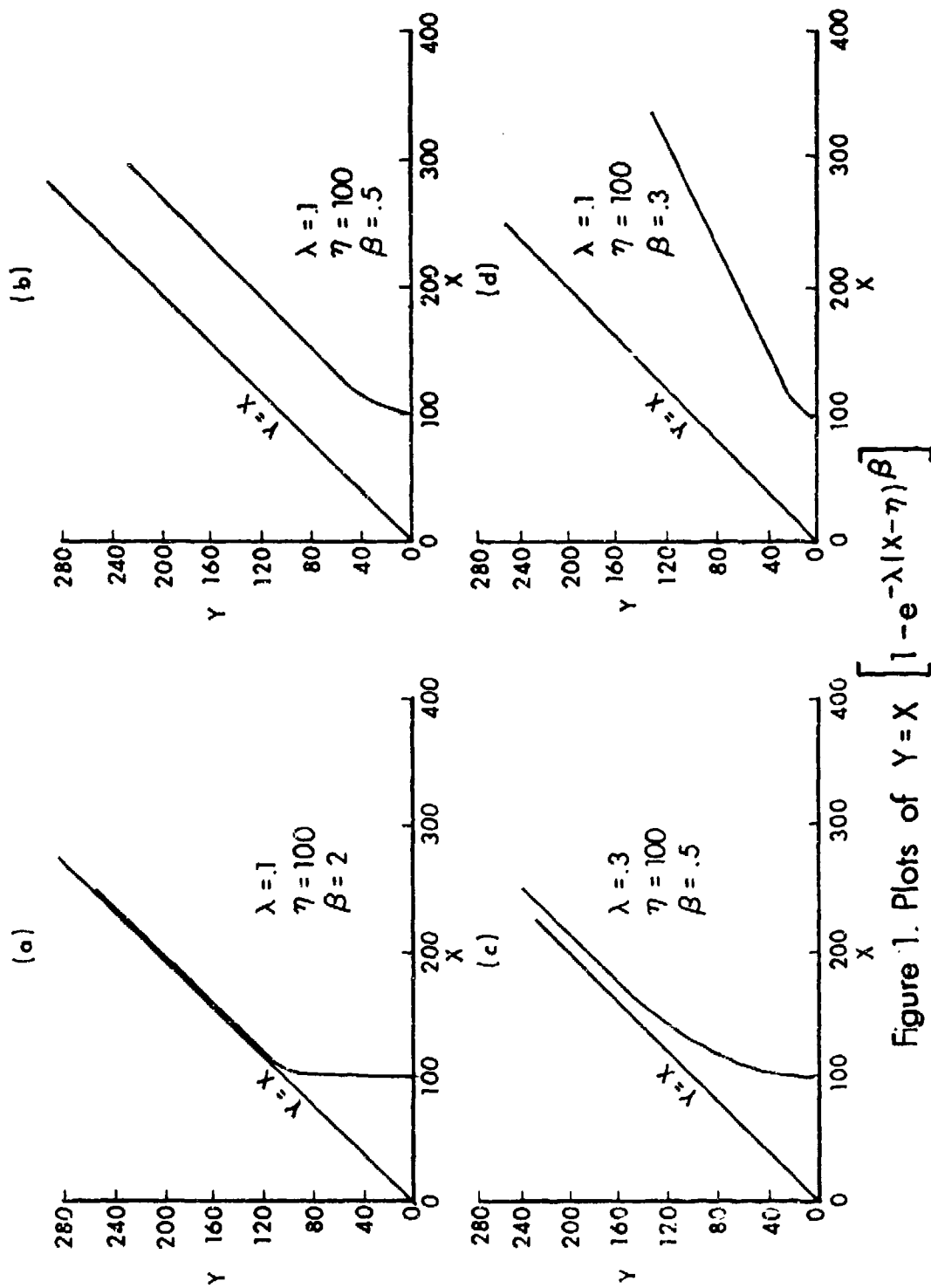


Figure 1. Plots of $Y = X \left[1 - e^{-\lambda(X-\eta)\beta} \right]$

the highest striking velocity for which the projectile did not perforate the target and b chosen to equal the lowest striking velocity for which perforation was achieved. Once this interval has been determined an initial estimate V_c^0 may be arbitrarily chosen within the interval.

With this choice of V_c^0 the equation (2.1) can be linearized to yield

$$\ln \ln \left(\frac{V_s}{V_s - V_r} \right) = \ln(\lambda) + \beta \ln(V_s - V_c^0).$$

In most cases initial estimates λ^0 and β^0 within the feasible region may be then obtained by using linear regression.

In Appendix A we illustrate the application of these procedures and the versatility of the model using several sets of penetration data.

4. COMPARISON OF WEIBULL AND HYPERBOLIC MODELS

In a recent study [3] published by the USA Ballistic Research Laboratories the hyperbolic model $V_r^2 = AV_s^2 + B$ was used to analyze the residual velocity of right circular cylinders after perforating doron body armor material. The cylinders considered in this study were made from 01 Tool Steel and heat treated to a hardness of $R_c 29 \pm 2$. The hyperbolic model was fitted to eight sets of penetration data from 2, 4, 16 and 64 grain cylinders at 0° and 45° obliquity.

We fitted the Weibull model to the eight sets of data considered in the above report and compared the results to those obtained from the hyperbolic model. These comparisons, with the fitted curves, are given in Tables 1-8 and Figures 2-9, respectively.

In four cases the ERMS's were slightly different for the two models (the hyperbolic ERMS's being lower in three of these cases), while in the other four cases the ERMS's for the Weibull model were significantly lower. Also, note that the hyperbolic model, in several cases estimated V_c lower than what one would expect based on the data. For example, consider the case of 2 grain steel at 45 degrees obliquity. The hyperbolic model estimated V_c at 530 m/s, but the experimental data had non-perforations for striking velocities as high as 629 m/s and perforations for striking velocities only as low as 605 m/s. The Weibull model estimate of V_c in this case is 607 m/s.

5. CONCLUSIONS

The Weibull model provides a versatile form which may be used to represent the relationship between the striking velocity and the residual velocity of a projectile fired into some target material. This model compares favorably with others proposed for this application.

In particular, it provides more satisfactory estimates of the critical velocity.

To date, this model has been used only to fit individual sets of data for which all conditions other than the striking velocity are held constant. No attempt has been made to physically interpret the values of the parameter estimates or to interpolate between test conditions. A model with those capabilities is needed for use in vulnerability and effectiveness models.

TABLE J

2 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR = -153388.9 + .5474 VS^2$

CRITICAL VELOCITY = 529.4 M/S

ERROR-RMS = 24.1 M/S

WEIBULL MODEL

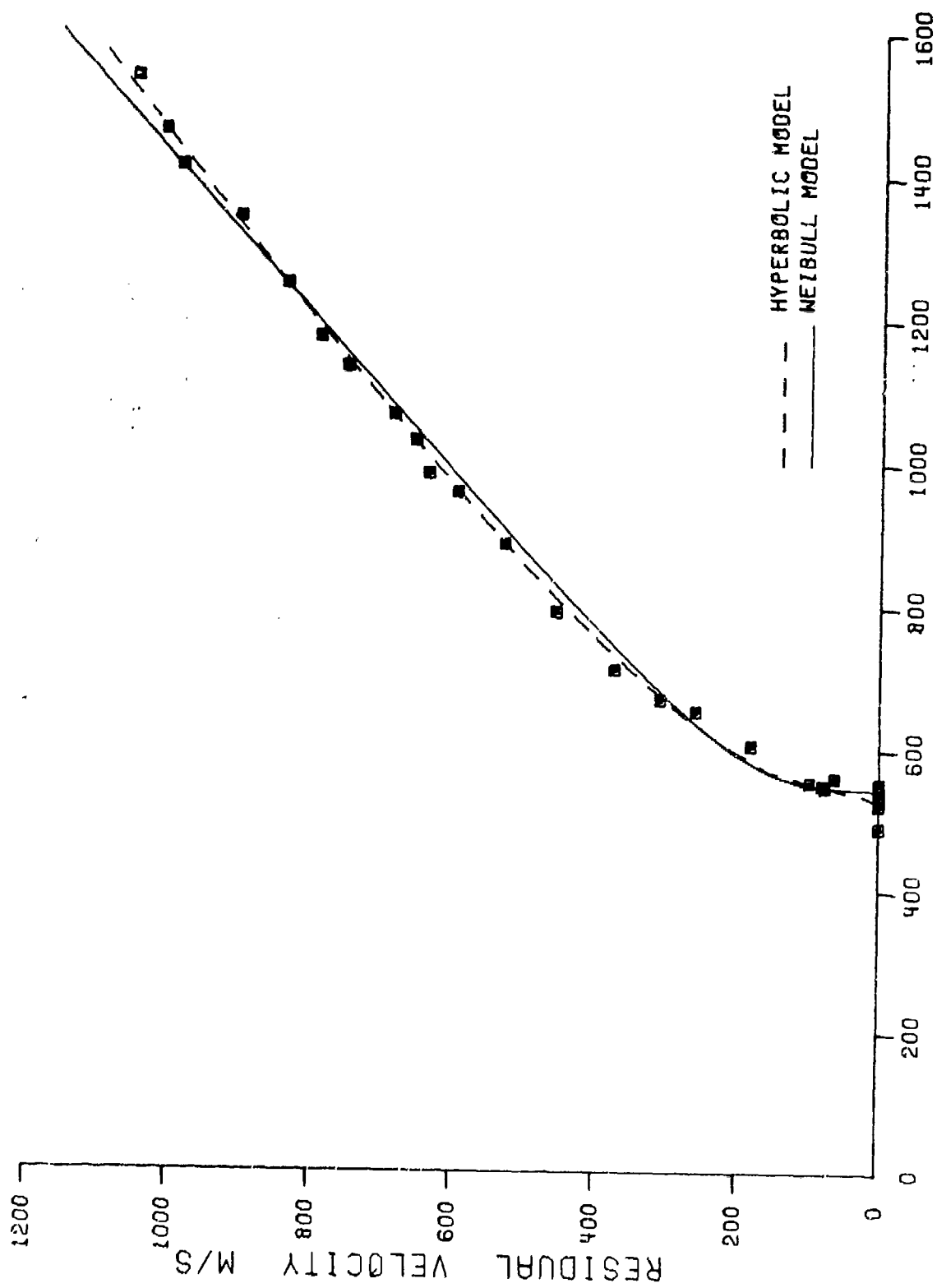
$VR/VS = 1 - \exp(-.0986(VS - 545.7)^{.36929})$

CRITICAL VELOCITY = 545.7 M/S

ERROR-RMS = 28.0 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
492.0	.0	524.0	.0
533.0	.0	537.0	.0
538.0	.0	550.0	.0
554.0	.0	547.0	76.0
551.0	80.0	555.0	98.0
561.0	63.0	607.0	181.0
655.0	259.0	671.0	309.0
713.0	374.0	794.0	455.0
888.0	528.0	960.0	594.0
986.0	636.0	1031.0	654.0
1068.0	685.0	1135.0	750.0
1176.0	789.0	1250.0	836.0
1342.0	902.0	1412.0	984.0
1461.0	1008.0	1534.0	1048.0



2 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 0 DEGREES OBLIQUITY
STRIKING VELOCITY M/S
FIGURE 2

TABLE 2

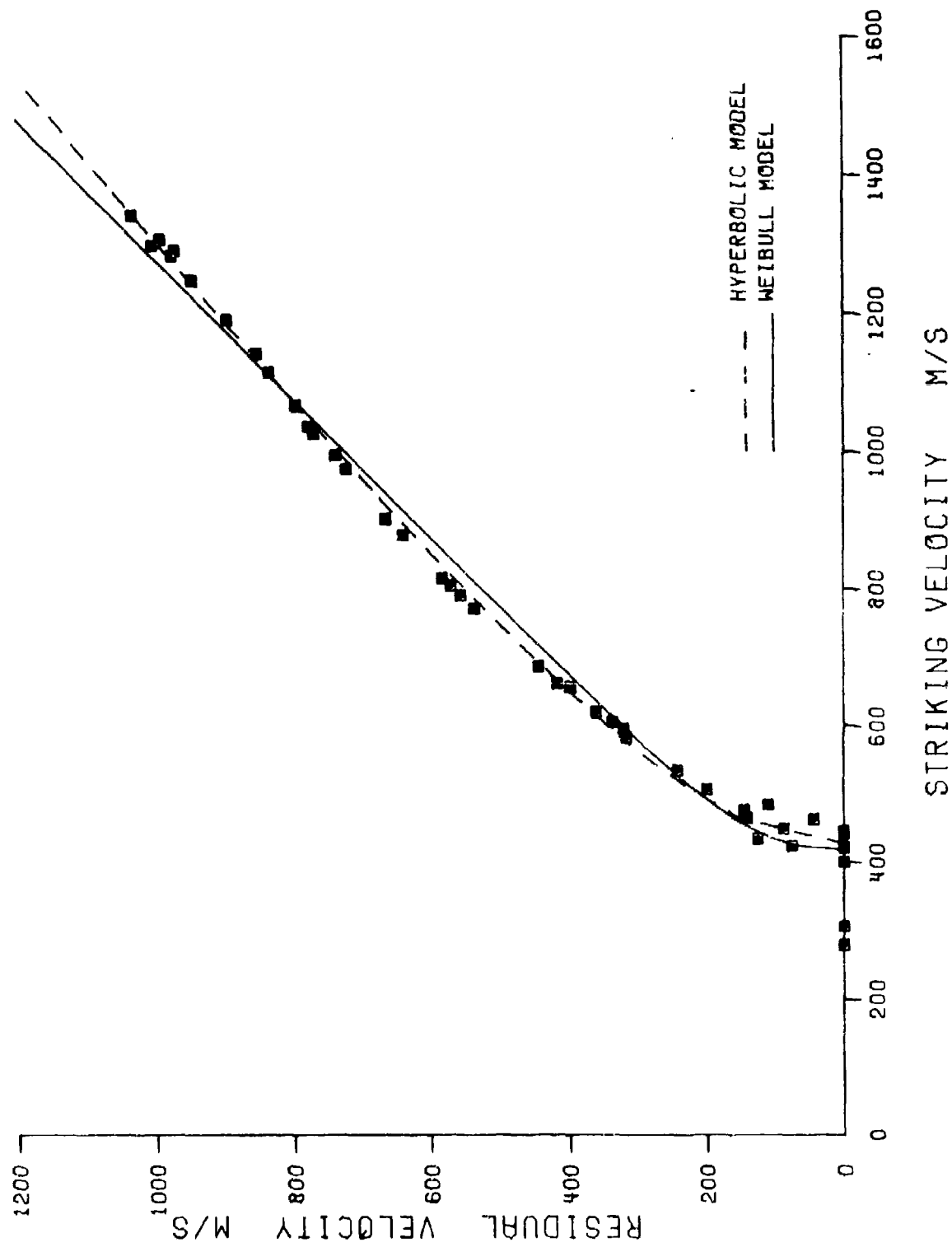
4 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -119248.1 + .6556 VS^2$
 CRITICAL VELOCITY = 426.5 M/S
 ERROR-RMS = 29.9 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.0777(VS - 421.2)^{.44043})$
 CRITICAL VELOCITY = 421.2 M/S
 ERROR-RMS = 34.7 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
281.0	.0	308.0	.0
402.0	.0	423.0	.0
431.0	.0	431.0	.0
441.0	.0	446.0	.0
447.0	.0	424.0	75.0
436.0	125.0	451.0	87.0
465.0	44.0	466.0	141.0
478.0	145.0	486.0	110.0
509.0	199.0	537.0	242.0
585.0	316.0	597.0	320.0
607.0	336.0	622.0	360.0
657.0	397.0	662.0	415.0
689.0	443.0	773.0	535.0
792.0	555.0	807.0	569.0
817.0	591.0	879.0	638.0
903.0	663.0	975.0	721.0
996.0	736.0	1027.0	768.0
1037.0	776.0	1067.0	794.0
1116.0	833.0	1143.0	851.0
1191.0	894.0	1247.0	945.0
1283.0	974.0	1290.0	969.0
1297.0	1002.0	1306.0	990.0
1341.0	1031.0		



4 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD BORON AT 0 DEGREES OBLIQUITY
FIGURE 3

TABLE 3

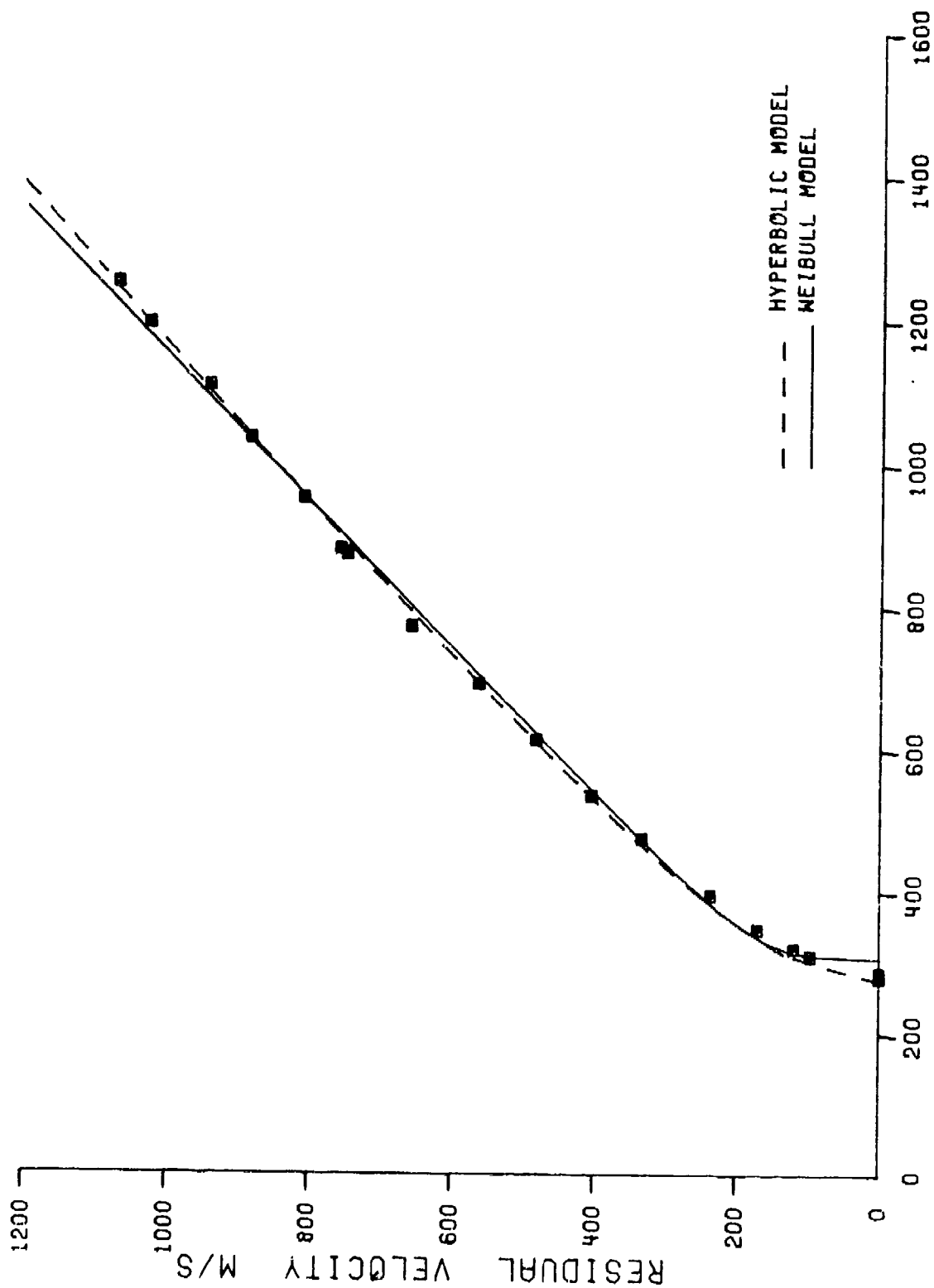
16 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -58815.3 + .7782 VS^2$
 CRITICAL VELOCITY = 274.9 M/S
 ERROR-RMS = 15.3 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.2440(VS - 309.1)^{.31345})$
 CRITICAL VELOCITY = 309.1 M/S
 ERROR-RMS = 16.1 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
282.0	.0	284.0	.0
287.0	.0	312.0	96.0
322.0	120.0	349.0	170.0
397.0	236.0	477.0	333.0
537.0	404.0	615.0	482.0
693.0	562.0	773.0	656.0
874.0	747.0	881.0	756.0
953.0	808.0	1037.0	882.0
1109.0	940.0	1195.0	1024.0
1251.0	1069.0		



16 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD CORON AT 0 DEGREES OBLIQUITY
FIGURE 4

TABLE 4

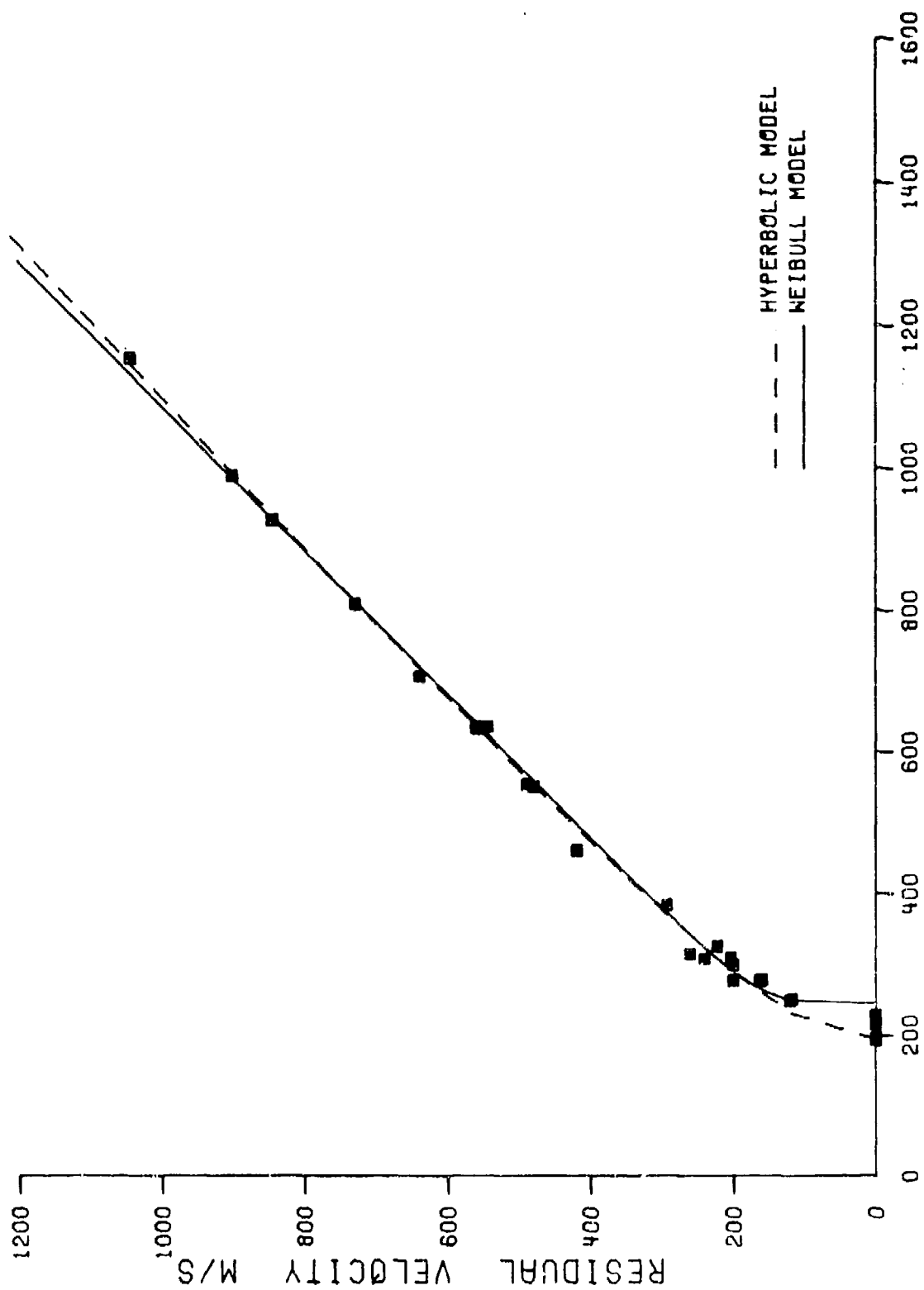
64 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -31554.4 + .8533 VS^2$
 CRITICAL VELOCITY = 192.3 M/S
 ERROR-RMS = 17.6 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.4322(VS - 245.0)^{.26232})$
 CRITICAL VELOCITY = 245.0 M/S
 ERROR-RMS = 16.6 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
193.0	.0	208.0	.0
223.0	.0	228.0	.0
249.0	121.0	250.0	118.0
277.0	163.0	277.0	200.0
278.0	160.0	300.0	200.0
308.0	240.0	309.0	203.0
314.0	260.0	326.0	222.0
384.0	293.0	461.0	418.0
551.0	479.0	554.0	488.0
633.0	558.0	635.0	543.0
635.0	560.0	707.0	638.0
809.0	728.0	927.0	843.0
989.0	899.0	1153.0	1043.0



64 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 0 DEGREES OBliquITY
FIGURE 5

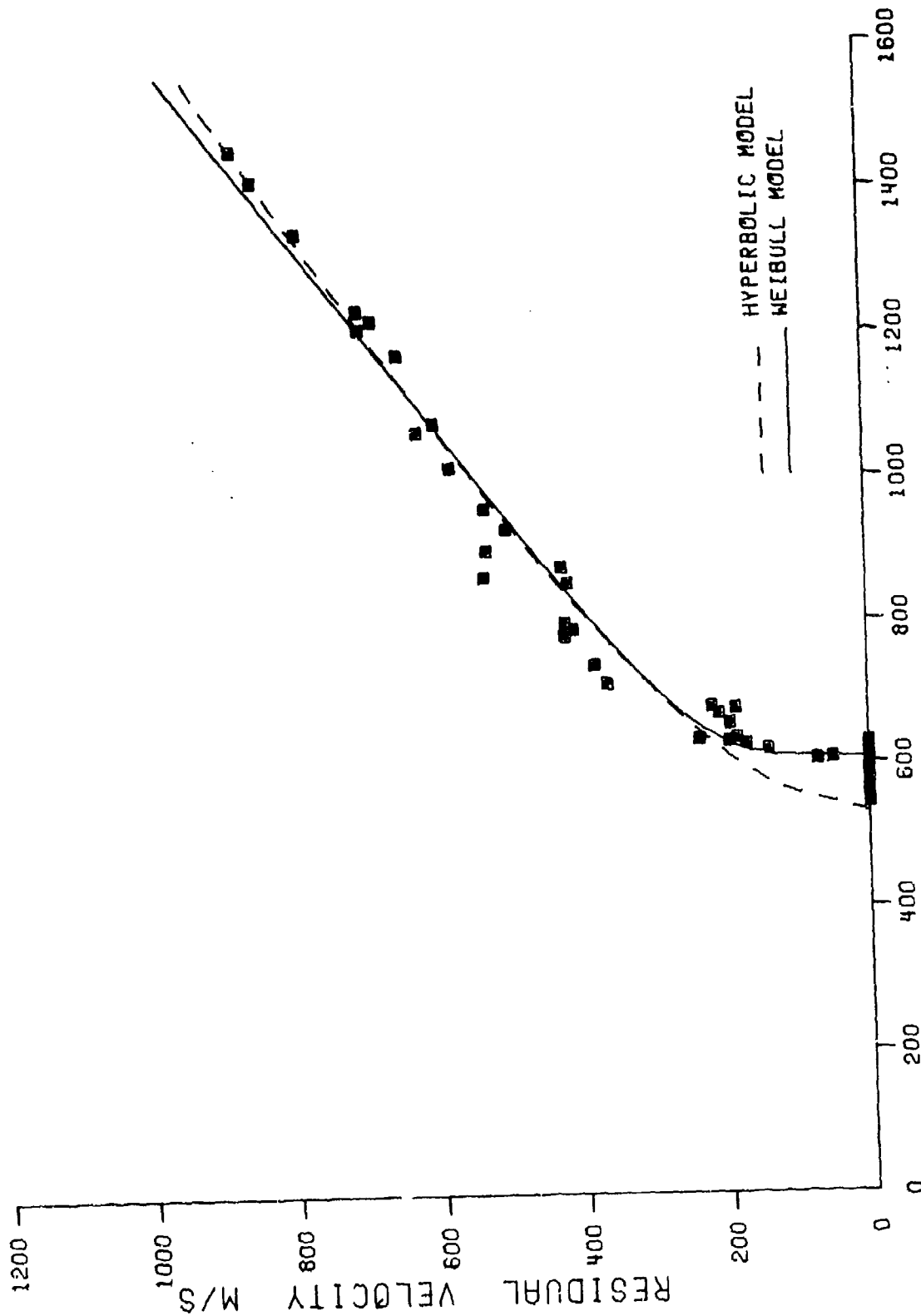
TABLE 5
2 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -114876.9 + .4091 VS^2$
CRITICAL VELOCITY = 529.9 M/S
ERROR-RMS = 51.3 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.1656(VS - 606.9)^{.25822})$
CRITICAL VELOCITY = 606.9 M/S
ERROR-RMS = 40.7 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
544.0	.0	561.0	.0
577.0	.0	587.0	.0
587.0	.0	600.0	.0
601.0	.0	609.0	.0
615.0	.0	617.0	.0
621.0	.0	629.0	.0
605.0	72.0	607.0	51.0
620.0	140.0	628.0	170.0
632.0	194.0	635.0	235.0
636.0	183.0	657.0	192.0
671.0	208.0	678.0	184.0
680.0	217.0	714.0	361.0
740.0	378.0	781.0	418.0
789.0	406.0	798.0	417.0
855.0	413.0	863.0	529.0
877.0	421.0	901.0	525.0
931.0	497.0	960.0	526.0
1018.0	574.0	1067.0	619.0
1078.0	595.0	1172.0	644.0
1209.0	696.0	1221.0	679.0
1235.0	698.0	1341.0	780.0
1414.0	839.0	1457.0	867.0



2 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
FIGURE 6

TABLE 6

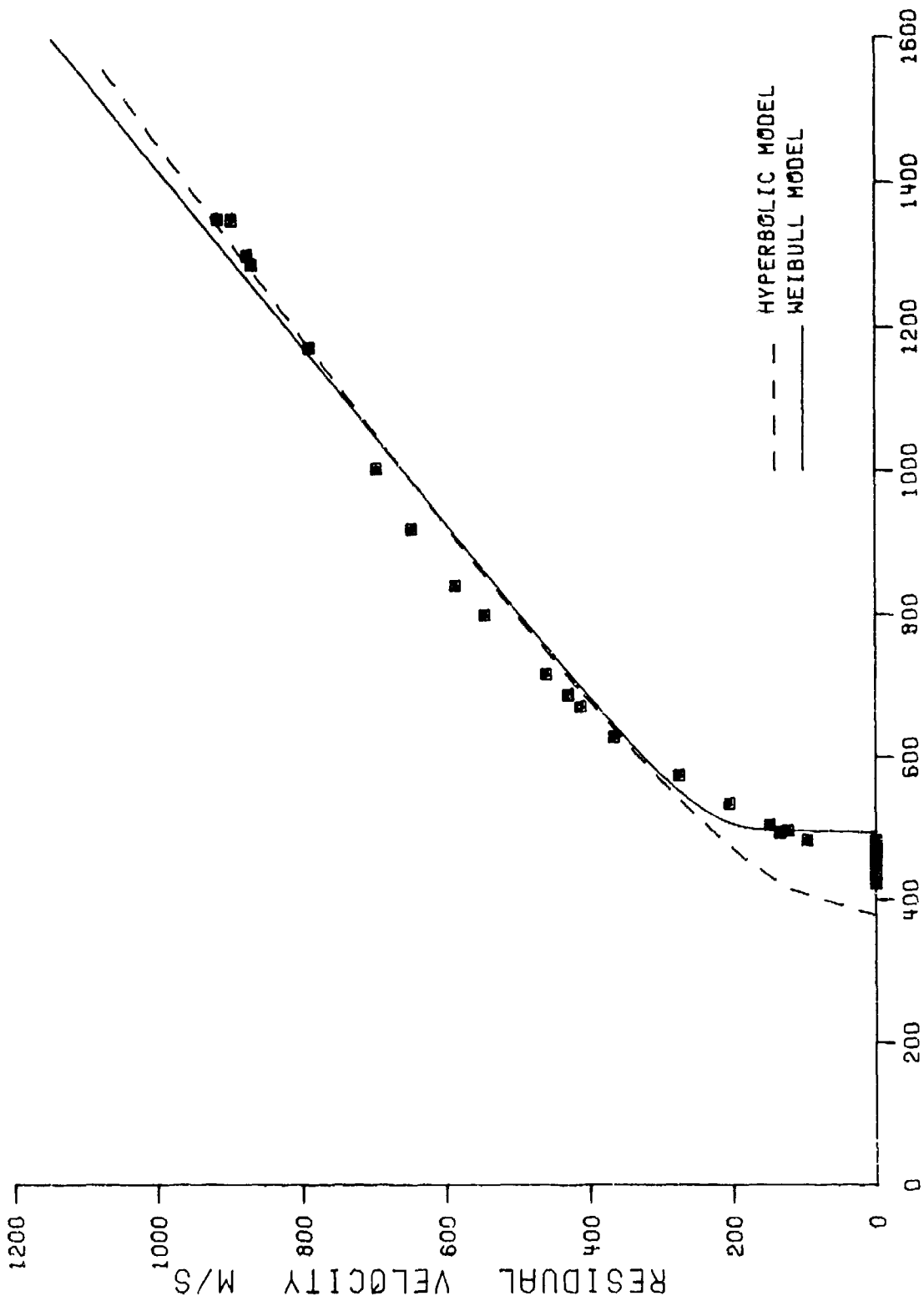
4 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -72019.2 + .5039 VS^2$
 CRITICAL VELOCITY = 378.1 M/S
 ERROR-RMS = 55.2 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.3015(VS - 494.3)^{.20376})$
 CRITICAL VELOCITY = 494.3 M/S
 ERROR-RMS = 40.9 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
423.0	.0	437.0	.0
454.0	.0	459.0	.0
467.0	.0	474.0	.0
475.0	.0	480.0	.0
481.0	.0	483.0	.0
483.0	96.0	495.0	134.0
497.0	123.0	505.0	148.0
536.0	204.0	575.0	275.0
629.0	366.0	671.0	411.0
688.0	428.0	717.0	459.0
799.0	544.0	840.0	585.0
919.0	646.0	1002.0	694.0
1171.0	787.0	1285.0	866.0
1299.0	872.0	1348.0	893.0
1348.0	912.0		



4 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
 STRIKING VELOCITY M/S
 FIGURE 7

TABLE 7

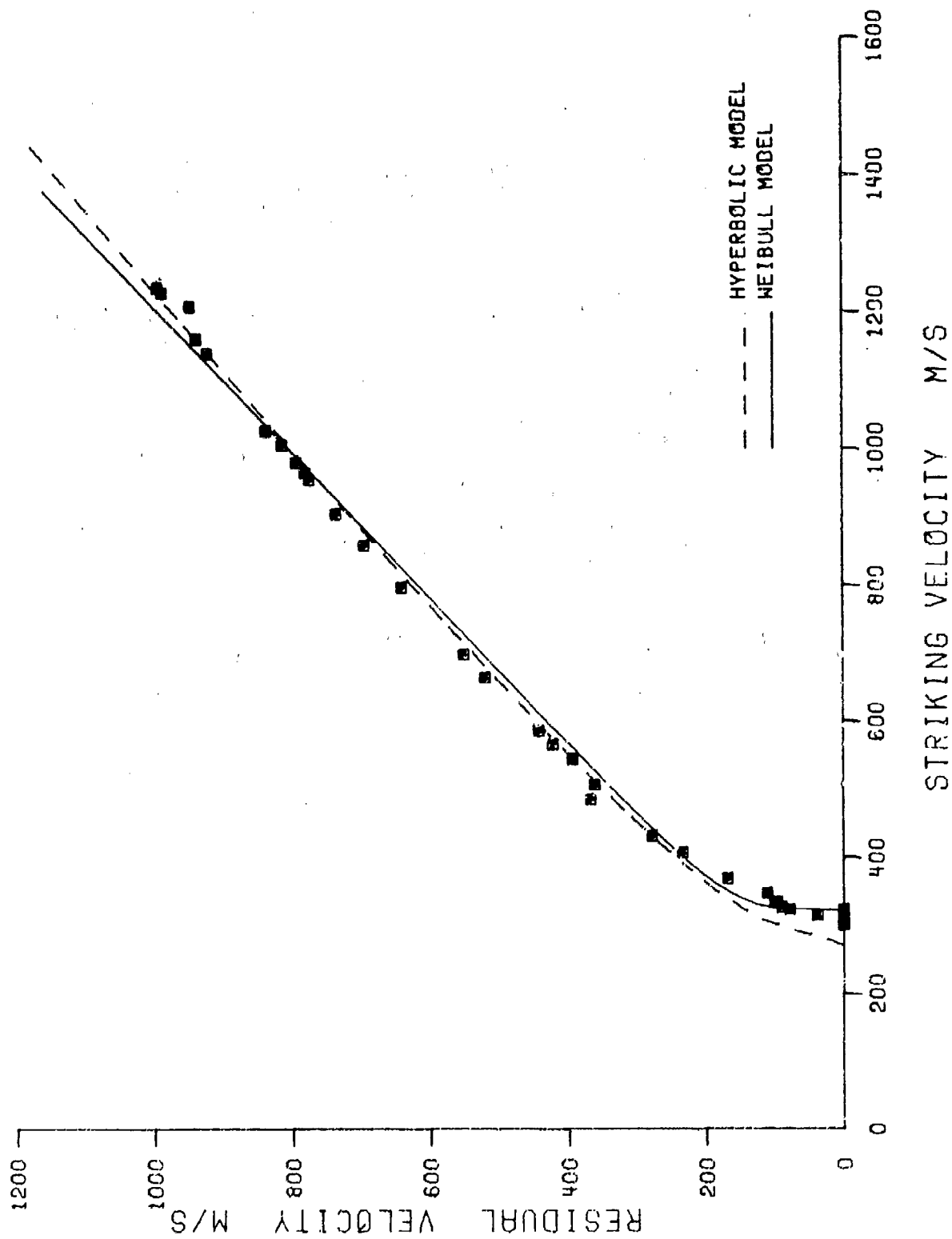
16 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -50639.3 + .6889 VS^2$
CRITICAL VELOCITY = 271.1 M/S
ERROR-RMS = 35.9 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.2515(VS - 323.3)^{.28592})$
CRITICAL VELOCITY = 323.3 M/S
ERROR-RMS = 26.6 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
302.0	.0	304.0	.0
304.0	.0	313.0	.0
324.0	.0	315.0	38.0
324.0	78.0	327.0	90.0
335.0	96.0	335.0	100.0
348.0	111.0	370.0	169.0
408.0	234.0	433.0	278.0
486.0	367.0	508.0	361.0
545.0	393.0	567.0	421.0
587.0	441.0	665.0	518.0
699.0	549.0	797.0	639.0
859.0	693.0	905.0	734.0
955.0	772.0	964.0	778.0
979.0	791.0	1005.0	811.0
1026.0	834.0	1138.0	919.0
1154.0	935.0	1207.0	944.0
1227.0	984.0	1234.0	990.0



16 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
FIGURE 8

TABLE 8

64 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -15795.6 + .7950 VS^2$

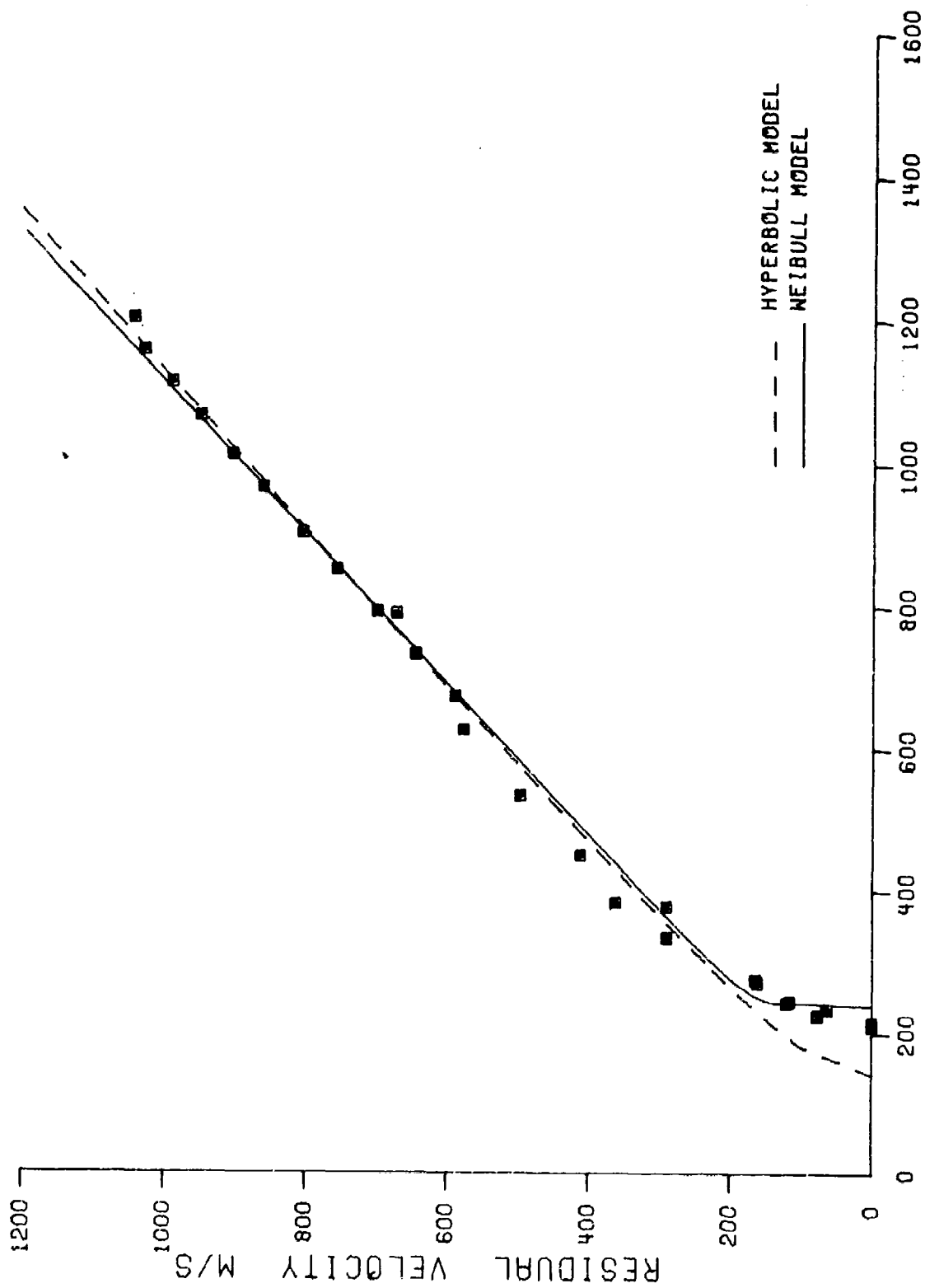
CRITICAL VELOCITY = 141.0 M/S
ERROR-RMS = 38.2 M/S

WEIBULL MODEL $VR/VS = 1 - \exp(-.6628(VS - 240.0)^{.18142})$

CRITICAL VELOCITY = 240.0 M/S
ERROR-RMS = 31.8 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
211.0	.0	212.0	.0
213.0	.0	215.0	.0
225.0	77.0	233.0	64.0
243.0	120.0	244.0	116.0
271.0	162.0	275.0	165.0
334.0	290.0	379.0	290.0
383.0	362.0	451.0	412.0
535.0	496.0	626.0	576.0
674.0	588.0	733.0	644.0
790.0	671.0	792.0	697.0
852.0	755.0	904.0	803.0
967.0	859.0	1013.0	902.0
1067.0	948.0	1114.0	987.0
1159.0	1028.0	1203.0	1042.0



64 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
FIGURE 9

REFERENCES

1. W. Bruchey, Jr., "A Comparison of the Residual Velocities of Various Fragment Simulating Projectiles and Actual Munitions Fragments After Penetrating Nylon Body Armor Material (U)," Ballistic Research Laboratories Interim Memorandum Report No. 22, Nov 1971 (CONFIDENTIAL) (no longer available).
2. W. Johnson, C. Collins and F. Kindred, "A Mathematical Model for Predicting Residual Velocities of Fragments After Perforating Helmets and Body Armor (U)," Ballistic Research Laboratories Technical Note No. 1705, October 1968. (CONFIDENTIAL) (AD# 394512)
3. W. Kokinakis, and F. H. Essig, "Penetration of Doron Body Armor Material By A Right Circular Cylinder Fragment Simulator," Ballistic Research Laboratories Memorandum Report No. 2445, March 1975.
4. P. G. Morfogenis, "A Learning Curve Type Equation Predicting Residual Velocity (U)," Ballistic Research Laboratories Memorandum Report No. 2477, April 1975.
5. D. W. Marquardt, "An Algorithm for Least Square Estimation of Nonlinear Parameter," J. SIAM, 2, 1963, pp. 431-441.
6. J. D. Wortman, "NLPROG (A Set of FORTRAN Programs to Find The Minimum of a Constrained Function)" Ballistic Research Laboratories Memorandum Report No. 1958, January 1969.

Appendix A

A FORTRAN computer program has been prepared to provide estimates of the parameters for the Weibull model described in this report. The program was written for use on the Ballistic Research Laboratories computers, BRLESC I and II. The program utilizes 16K memory locations. The CALCOMP plotter package is used to provide plots of the data and fitted curves.

The required input is as follows:

	Columns	
Card 1	1-79 80	Alphanumeric string used for title The symbol ">"
Card 2	1-10 11-20	Minimum value for critical velocity Maximum value for critical velocity
Card 3	1-10 11-20	Striking velocity Residual velocity

Repeat Card 3 as necessary to complete the data set. Follow the last Card 3 with a blank card.

After execution of the computation for a data set the program returns to read card 1 for another set.

A listing of the source program is provided on the following pages. To provide several samples of output and also to demonstrate the flexibility of the Weibull model the output from several data sets is also included. These data were obtained from firings of a 90 percent tungsten spheriod weighing approximately 7 grains. The firing conditions include two target materials, two thicknesses, and two angles of obliquity.

PROGRAM WBLRV	WBLRV 1
COMMON /HMX/ HMAX, HMIN	WBLRV 2
DIMENSION VS(100), VR(100), Z(100,2), A(2,3), C(3), R(100),	WBLRV 3
1 AF(100), SIG(3), T(3), ITL(8), F(100), TX(3), TY(3)	WBLRV 4
DIMENSION XZ(20), YZ(20)	WBLRV 5
EXTERNAL VREM	WBLRV 6
DATA TX(1), TX(2), TY(1), TY(2) /10WSTRIKING V, 8HELOCITY , 10HRES	WBLRV 7D
11DLAL , 9HVELOCITY / , TX(3), TY(3) /4HM/S>, 4HM/S>/	WBLRV 8D
INN=1	WBLRV 9
1 READ (5,11) (ITL(I),I=1,8)	WBLRV10R
WRITE (6,13) (ITL(I),I=1,8)	WBLRV11W
READ (5,16) HMIN,HMAX	WBLRV12R
WRITE (6,18) HMIN,HMAX	WBLRV13W
M=1	WBLRV14
NZ=C	WBLRV15
2 READ (5,12) VS(M),VR(M)	WBLRV16R
IF (VS(M).LE.0.) GO TO 4	WBLRV17
IF (VR(M).LE.0.) GO TO 3	WBLRV18
IF (VS(M).LE.HMIN) GO TO 3	WBLRV19
M=M+1	WBLRV20
GO TO 2	WBLRV21
3 NZ=NZ+1	WBLRV22
XZ(NZ)=VS(M)	WBLRV23
YZ(NZ)=VR(M)	WBLRV24
GO TO 2	WBLRV25
4 M=M-1	WBLRV26
IF (NZ.GT.0.) CALL SORTXY (XZ,YZ,NZ)	WBLRV27
CALL SORTXY (VS,VR,M)	WBLRV28
H=HMIN	WBLRV29
DO 5 K=1,M	WBLRV30
F(K)=ALOG(ALCG(VS(K)/(VS(M)-VR(K))))	WBLRV31
Z(K,1)=1.	WBLRV32
Z(K,2)=ALCG(VS(K)-H)	WBLRV33
5 CONTINUE	WBLRV34
IF (M.GT.2) GO TO 6	WBLRV35
WRITE (6,17) M	WBLRV36W
GO TO 1	WBLRV37
6 CALL GENLSQ (Z,100,F,M,A,2,2,C,R,AF,ERMS,SIG,T,DET,1)	WBLRV38
DO 7 I=1,M	WBLRV39
F(I)=VR(I)	WBLRV40
Z(I,1)=VS(I)	WBLRV41
7 Z(I,2)=VR(I)	WBLRV42
C(3)=C(2)	WBLRV43
C(2)=EXP(C(1))	WBLRV44
C(1)=H	WBLRV45
WRITE (6,19) (C(I),I=1,3)	WBLRV46W
CALL MRQLS (VREM,F,Z,C,2,M,100,3,ERMS,SIG,R,AF,.1E-5,.1E-3)	WBLRV47
B=C(3)	WBLRV48
XL=C(2)	WBLRV49
H=C(1)	WBLRV50
WRITE (6,20) H,XL,B,ERMS	WBLRV51W
WRITE (6,21)	WBLRV52W
IF (NZ.GT.0) WRITE (6,15) (XZ(I),YZ(I),I=1,NZ)	WBLRV53W
DO 8 I=1,M	WBLRV54
8 WRITE (6,14) VS(I),VR(I),AF(I),R(I)	WBLRV55W
YMAX=1500.	WBLRV56
YS=YMAX/6.	WBLRV57
XOR=0.	WBLRV58
XMAX=XOR+8.*YS	WBLRV59
CALL PLTVG (VS,VR,M,XOR,0.,VS,VS,TX,TY,2,5,INN,ITL)	WBLRV60
IF (NZ.GT.0) CALL PLTCCD (2,5,XZ(1),YZ(1),NZ)	WBLRV61
VS(1)=H	WBLRV62
VR(1)=0.	WBLRV63

DX=.C25*YS	WBLRV64
I=1	WBLRV65
9 VSAV=VR(1)+DX	WBLRV66
VST=VS(1)+DX	WBLRV67
IF (VST.GT.XMAX) GO TO 10	WBLRV68
I=I+1	WBLRV69
VS(I)=VST	WBLRV70
VR(I)=VS(I)*(1.-EXP(-XL*(VS(I)-H)**B))	WBLRV71
VSAV=VR(I)	WBLRV72
IF (I.GT.40) DX=.25*YS	WBLRV73
IF (VSAV.LT.YMAX) GO TO 9	WBLRV74
10 CALL PLTCCD (1,,VS(1),VR(1),I,0,XOR,XMAX,0.,YMAX)	WBLRV75
IAN=C	WBLRV76
GO TO 1	WBLRV77
	WBLRV78
	WBLRV79
11 FORMAT (PA10)	WBLRV80
12 FORMAT (2F10.0)	WBLRV81
13 FORMAT (1H1,30X,4A10/30X,3A10,A9/)	WBLRV82
14 FORMAT (19X,4F14.1)	WBLRV83
15 FORMAT (19X,2F14.1)	WBLRV84
16 FORMAT (2F10.5)	WBLRV85
17 FORMAT (5HONLY,13,25H DATA POINTS--FIT OMITTED)	WBLRV86
18 FORMAT (2CX,53HCRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FRM	WBLRV87
1CM,F7.1/2CX,3HTC ,F7.1,5H M/S./)	BETA WBLRV88
19 FORMAT (2CX,13HWEIBULL MODEL,8X,6HVC M/S,6X,24HLAMBCA	WBLRV89
1 ERMS//2CX,17HINITIAL ESTIMATES,F10.1,3X,2F10.6/)	WBLRV90
20 FORMAT (2CX,15HFINAL ESTIMATES,F12.1,3X,2F10.6,F7.1///)	WBLRV91
21 FORMAT (26X,49HSTRIKING RESIDUAL APPROXIMATION ERROR/2W	WBLRV92
16X,22HVELOCITY VELOCITY/18X,4(11X,3H M/S)/)	WBLRV93-
END	VREM 1
SUBROUTINE VREM (P,X,F,P,IC,M,K)	VREM 2
COMMON /HMX/ HMAX,HMIN	VREM 3
DIMENSION B(K), X(M), P(K)	VREM 4
IF (B(1).GT.HMAX) B(1)=HMAX	VREM 5
IF (B(1).LT.HMIN) B(1)=HMIN	VREM 6
XP=X(1)-P(1)	VREM 7
IF (XP.GT.0.) GO TO 1	VREM 8
F=C.	VREM 9
GO TO 2	VREM 10
1 XPL=ALOG(XP)	VREM 11
XPB=EXP(B(3)*XPL)	VREM 12
EX=EXP(-B(2)*XPB)	VREM 13
F=X(1)*(1.-EX)	VREM 14
2 IF (IC.NE.C) RETURN	VREM 15
IF (XP.GT.C) GO TO 3	VREM 16
P(1)=0.	VREM 17
P(2)=C.	VREM 18
P(3)=C.	VREM 19
RETURN	VREM 20
3 P(1)=-EX*X(1)*B(2)*B(3)*XPB/XP	VREM 21
P(2)=X(1)*XPB*EX	VREM 22
P(3)=P(2)*B(2)*XPL	VREM 23
RETURN	VREM 24
END	MRQLS 1
SUBROUTINE MRQLS (FORM,Y,X,B,M,N,NMAX,K,ERMS,SE,R,F,TAU,EPS)	MRQLS 2
DIMENSION Y(N), X(NMAX,M), B(K), SE(K), R(N), F(N)	MRQLS 3
DIMENSION A(20,20), SA(20,20), P(20), V(10), G(20), SG(20)	MRQLS 4
GNL=10.	MRQLS 5
ICT=0	MRQLS 6
XL=.01	

```

1 IC=C
  STEP=1.
  ICT=ICT+1
  PHI=0.
  DO 2 I=1,20
    G(I)=0.
  DO 2 J=1,20
2 A(I,J)=0.
  DO 5 I=1,N
  DO 3 J=1,M
3 V(J)=X(I,J)
  CALL FORM (P,V,F(I),P,IC,M,K)
  R(I)=Y(I)-F(I)
  PHI=PHI+R(I)**2
  DO 4 J=1,K
    G(J)=G(J)+R(I)*P(J)
  DO 4 L=J,K
4 A(J,L)=A(J,L)+P(J)*P(L)
5 CONTINUE
  ERMS=SQRT(PHI/FLOAT(N))
  DO 6 I=1,K
    SE(I)=SQRT(A(I,I))
6 G(I)=G(I)/SE(I)
  DO 7 I=1,K
  DO 7 J=1,K
    A(I,J)=A(I,J)/(SE(I)*SE(J))
    IF (J.GT.I) A(J,I)=A(I,J)
7 CONTINUE
  DO 8 I=1,K
    SG(I)=G(I)
  DO 8 J=1,K
8 SA(I,J)=A(I,J)
  IC=1
  XLM=XL/GNU
9 DO 10 I=1,K
10 A(I,I)=A(I,I)+XLM
  CALL MATINV (A,K,G,20,1,DET)
  IF (DET.EQ.0.) GO TO 17
11 DO 12 I=1,K
12 G(I)=B(I)+STEP*(G(I)/SE(I))
  PHIL=0.
  DO 14 I=1,N
  DO 13 J=1,M
13 V(J)=X(I,J)
  CALL FORM (G,V,F(I),P,IC,M,K)
  R(I)=Y(I)-F(I)
14 PHIL=PHIL+R(I)**2
  GO TO (15,19), IC
15 IF (PHIL-PHI) 16,16,17
16 XL=XLM
  GO TO 25
17 DO 18 I=1,K
    G(I)=SG(I)
  DO 18 J=1,K
18 A(I,J)=SA(I,J)
  IC=2
  XLM=XL
  GO TO 9
19 IF (PHIL-PHI) 25,25,20
20 IF (STEP.LT.1.) GO TO 24

```

```

MRQLS 7
MRQLS 8
MRQLS 9
MRQLS10
MRQLS11
MRQLS12
MRQLS13
MRQLS14
MRQLS15
MRQLS16
MRQLS17
MRQLS18
MRQLS19
MRQLS20
MRQLS21
MRQLS22
MRQLS23
MRQLS24
MRQLS25
MRQLS26
MRQLS27
MRQLS28
MRQLS29
MRQLS30
MRQLS31
MRQLS32
MRQLS33
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MRQLS38
MRQLS39
MRQLS40
MRQLS41
MRQLS42
MRQLS43
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MRQLS45
MRQLS46
MRQLS47
MRQLS48
MRQLS49
MRQLS50
MRQLS51
MRQLS52
MRQLS53
MRQLS54
MRQLS55
MRQLS56
MRQLS57
MRQLS58
MRQLS59
MRQLS60
MRQLS61
MRQLS62
MRQLS63
MRQLS64
MRQLS65
MRQLS66

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      IF (XL.GE.GNL*GNU) GO TO 22
21  XL=XL*GNU
    GO TO 17
22  GNCRM2=G.
    GNCRM2=C.
    DOT=C.
    DO 23 I=1,K
      GI=SG(I)*SE(J)
      DI=G(I)-B(I)
      DOT=DOT+GI*DI
      GNCRM2=GNCRM2+GI*GI
23  GNCRM2=GNCRM2+DI*DI
    DOT=DOT/SCRT(GNCRM2*GNCRM2)
24  IF (DOT.LE..7071) GO TO 21
    STEP=.5*STEP
    GO TO 11
25  DO 26 I=1,K
    IF (ABS(G(I)-B(I))/(TAU+ABS(B(I))).GT.EPS) GO TO 29
26  CONTINUE
27  ERMS=SQRT(PHIL/FLOAT(N))
    DO 28 I=1,K
28  B(I)=G(I)
    RETURN
29  DO 30 I=1,K
30  R(I)=G(I)
    IF (ICT.LT.50) GO TO 1
    GO TO 27

```

C

```

31  FORMAT (1H0,18HFAILED TO CONVERGE)
    END
    SUBROUTINE PLTVG (X,Y,N,XOR,YOR,XS,YS,TX,TY,M,NS,INN,ITL)
    DIMENSION X(N), Y(N), TX(2), TY(2), B(5000), XC(2), YC(2), ITL(8)
    IF (INN.NE.1) GO TO 1
    CALL PLTCCR (12.,1,B(1),R(5000))
    ICT=2
1   I=ICT+1
    ICT=MOD(I,3)
    IF (ICT.EQ.0) CALL PLTCCP
    XB=1.
    YB=1.+9.5*FLCAT(ICT)
    CALL PLTCCS (XB,YB,0.,0.,1.,1.)
    DO 3 I=1,2
      XP=10.5*FLOAT(I-1)
    DO 2 J=1,2
      YP=8.*FLCAT(J-1)
      XC(1)=XP
      XC(2)=XP
      YC(1)=YP-.25
      YC(2)=YP+.25
      CALL PLTCCD (1,0,XC(1),YC(1),2)
      XC(1)=XP-.25
      XC(2)=XP+.25
      YC(1)=YP
      YC(2)=YP
2   CALL PLTCCD (1,0,XC(1),YC(1),2)
3   CONTINUE
    CALL PLTCCCT (.15,TX(1),0.,1.,4.,.75)
    CALL PLTCCCT (.15,TY(1),1.,0.,9,3.)
    CALL PLTCCCT (.10,ITL(1),0.,1.,1.5,.5)
    XB=XB+1.5

```

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MRQLS67
MRQLS68
MRQLS69
MRQLS70
MRQLS71
MRQLS72
MRQLS73
MRQLS74
MRQLS75
MRQLS76
MRQLS77
MRQLS78
MRQLS79
MRQLS80
MRQLS81
MRQLS82
MRQLS83
MRQLS84
MRQLS85
MRQLS86
MRQLS87
MRQLS88
MRQLS89
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MRQLS92
MRQLS94
MRQLS95
MRQLS96
MRQLS97-
PLTVG 1
PLTVG 2
PLTVG 3
PLTVG 4
PLTVG 5
PLTVG 6
PLTVG 7
PLTVG 8
PLTVG 9
PLTVG10
PLTVG11
PLTVG12
PLTVG13
PLTVG14
PLTVG15
PLTVG16
PLTVG17
PLTVG18
PLTVG19
PLTVG20
PLTVG21
PLTVG22
PLTVG23
PLTVG24
PLTVG25
PLTVG26
PLTVG27
PLTVG28
PLTVG29
PLTVG30

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YB=YB+1.5
CALL PLTCCS (XB,YB,XOR,YOR,XS,YS)
XMAX=XOR+8.*XS
YMAX=YOR+6.*YS
CALL PLTCCA (XS,YS,XOR,XMAX,YOR,YMAX,0)
CALL PLTCCD (M,NS,X(1),Y(1),N)
CALL LABELA (XS,YS,XOR,XMAX,YOR,YMAX,1.,1.)
RETURN
END

```

```

*
*
*
C

```

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PLTVG31
PLTVG32
PLTVG33
PLTVG34
PLTVG35
PLTVG36
PLTVG37
PLTVG38
PLTVG39-

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END

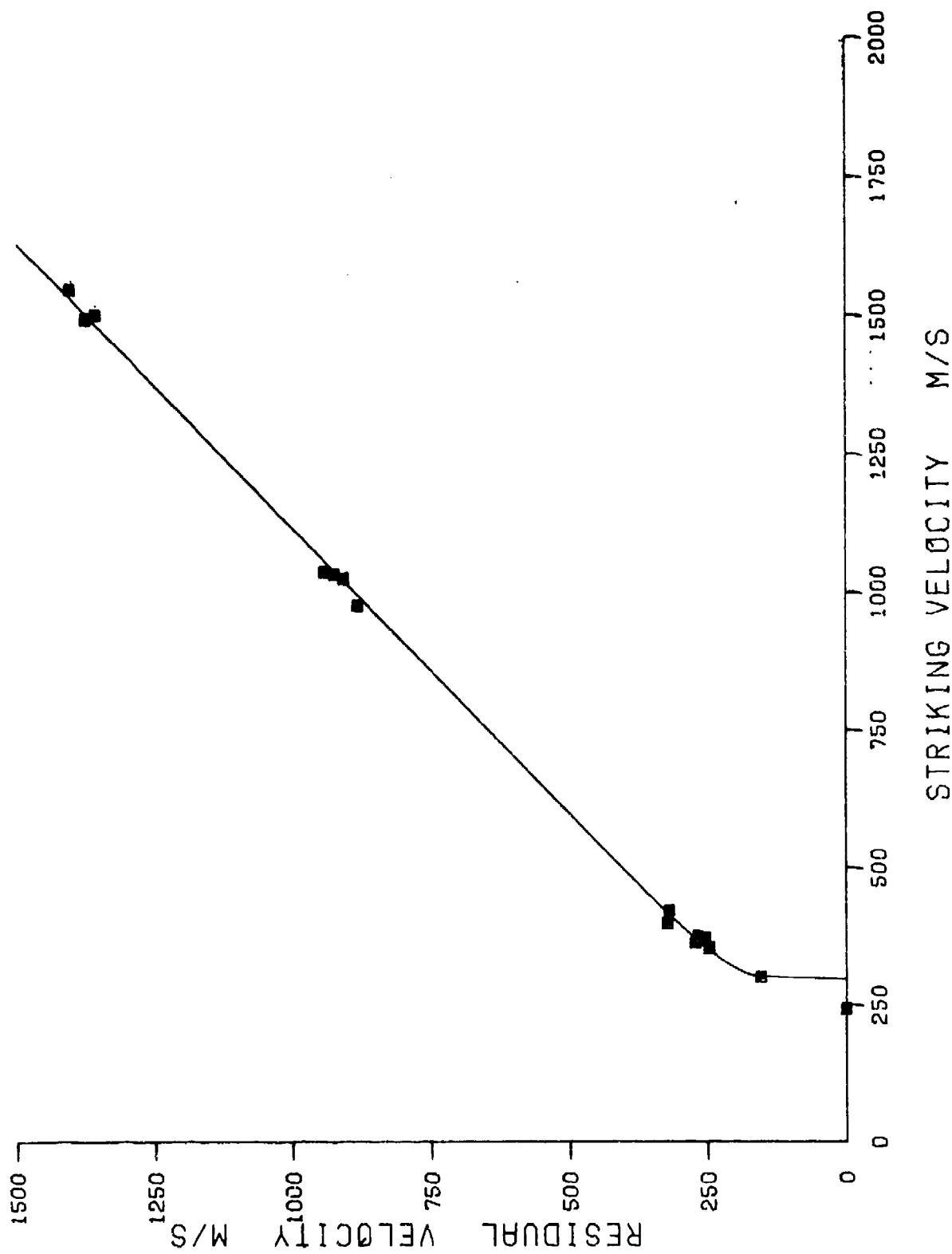
TABLE A-1

.45 GM TUNGSTEN SPHEROID INTO .15 CM
ALUMINUM AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 243.8
TO 310.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	243.8	.246400	.328262	
FINAL ESTIMATES	298.0	.504504	.222161	13.0

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
243.8	.0		
302.4	153.0	152.1	1.0
355.7	246.3	253.0	-6.7
363.3	271.0	262.0	9.0
373.4	253.3	273.4	-20.1
374.9	266.4	275.1	-8.7
399.9	321.3	302.2	19.1
422.8	318.2	325.9	-7.7
976.9	880.9	862.8	18.0
1026.3	905.9	910.4	-4.6
1033.3	921.7	917.2	4.5
1036.6	940.3	920.4	19.9
1494.1	1371.6	1363.3	8.3
1501.4	1353.0	1370.4	-17.4
1547.2	1400.6	1414.9	-14.3



.45 GM TUNGSTEN SPHEROID INTO .15 CM ALUMINUM AT 0 DEGREES OBLIQUITY
FIGURE A-1

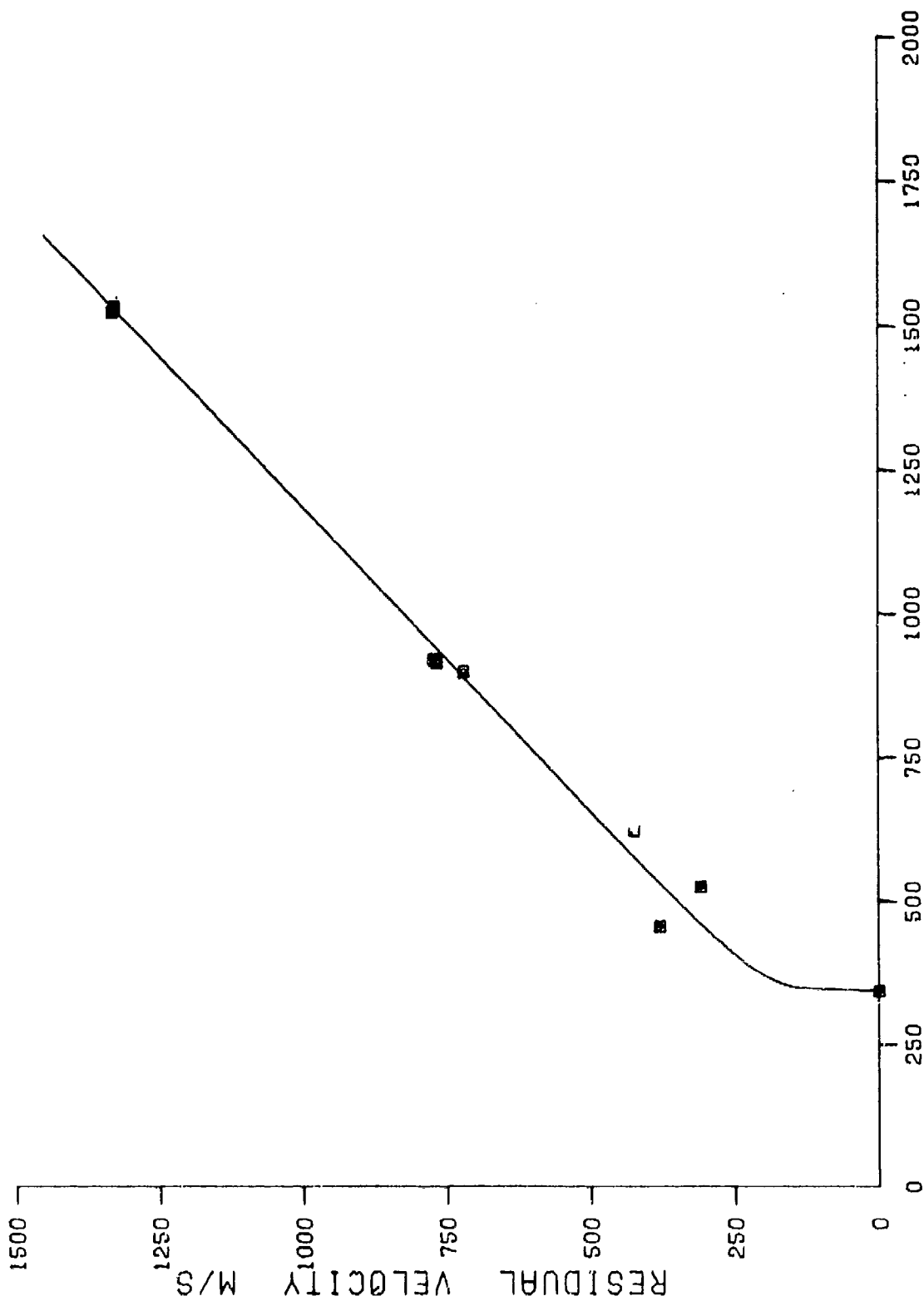
TABLE A-2

.45 GM TUNGSTEN SPHEROID INTO .32 CM
ALUMINUM AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 343.5
TO 460.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.426603	.214394	
FINAL ESTIMATES	343.5	.333039	.253525	40.7

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
343.5	.0		
456.9	379.5	305.6	73.9
526.4	307.8	375.2	-67.3
623.3	423.1	468.0	-44.9
899.2	718.7	727.1	-8.3
915.3	766.0	742.2	23.7
920.8	770.5	747.4	23.2
1524.0	1326.2	1318.0	8.1
1531.6	1322.2	1325.3	-3.1



.45 GM TUNGSTEN SPHEROID INTO .32 CM ALUMINUM AT 0 DEGREES OBLIQUITY
FIGURE A-2

TABLE A-2

.45 GM TUNGSTEN SPHEROID INTO .32 CM
ALUMINUM AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 343.5
TO 460.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.426603	.214394	
FINAL ESTIMATES	343.5	.333039	.253525	40.7

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
343.5	.0		
456.9	379.5	305.6	73.9
526.4	407.8	375.2	-67.3
623.3	423.1	468.0	-44.9
899.2	718.7	727.1	-8.3
915.3	766.0	742.2	23.7
920.8	770.5	747.4	23.2
1524.0	1326.2	1318.0	8.1
1531.6	1322.2	1325.3	-3.1

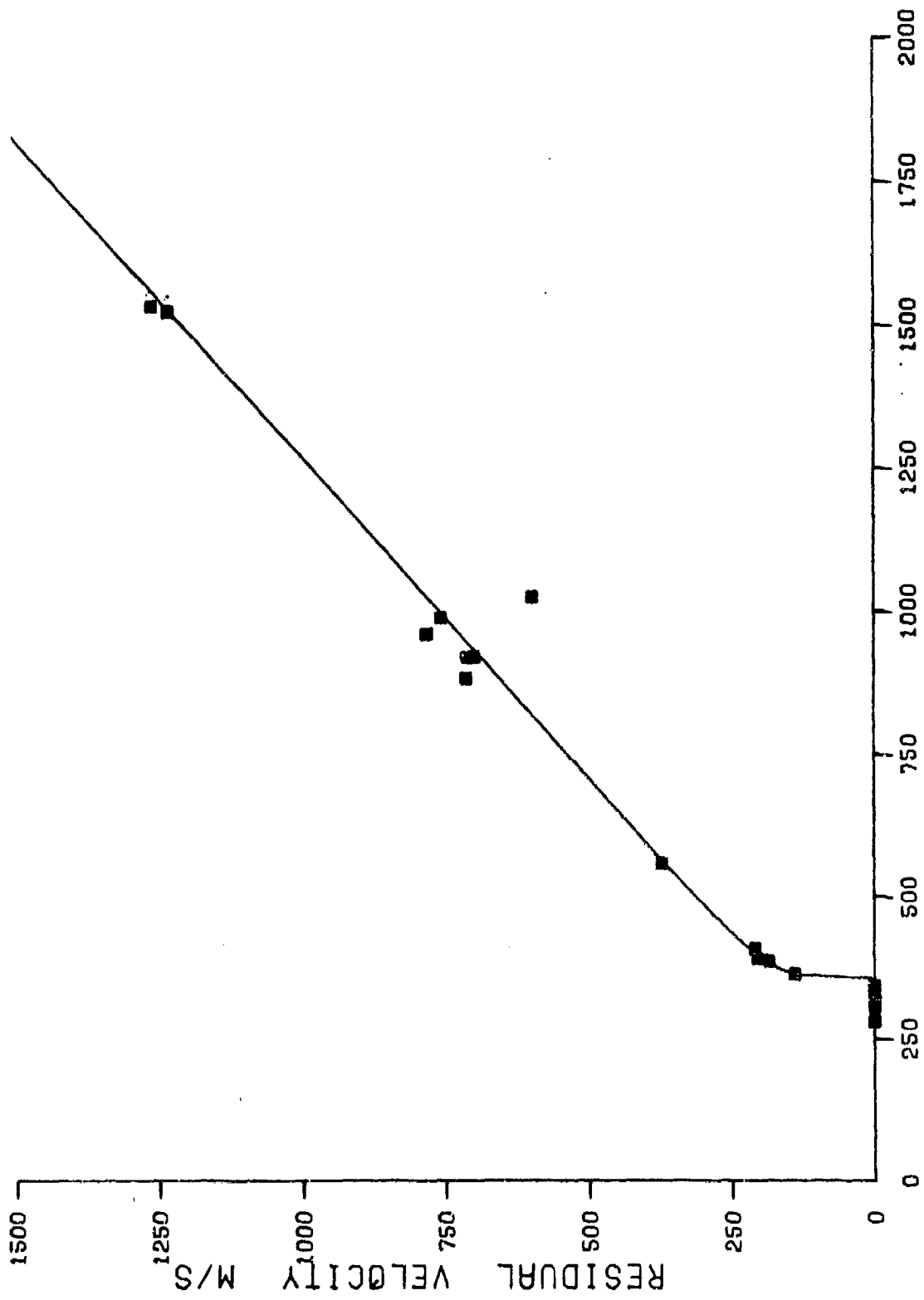
TABLE A-3

.45 GM TUNGSTEN SPHEROID INTO .15 CM
MILD STEEL AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 343.5
TO 380.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.213101	.292740	
FINAL ESTIMATES	358.1	.291155	.244159	56.8

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
281.6	.0		
305.4	.0		
318.5	.0		
334.1	.0		
343.5	.0		
365.5	138.4	138.0	.3
388.3	183.8	189.5	-5.7
391.4	204.2	194.1	10.1
410.3	208.2	219.3	-11.1
559.3	370.9	366.1	4.8
883.6	711.4	653.2	58.2
920.8	710.2	686.0	24.2
920.8	696.8	686.0	10.8
961.6	780.0	722.2	57.8
989.7	755.3	747.0	8.3
1026.3	596.5	779.5	-183.0
1524.0	1228.0	1226.2	1.9
1531.6	1257.9	1233.1	24.8



.45 GM TUNGSTEN SPHEROID INTO .15 CM MILD STEEL AT 0 DEGREES OBLIQUITY
 STRIKING VELOCITY M/S
 FIGURE A-3

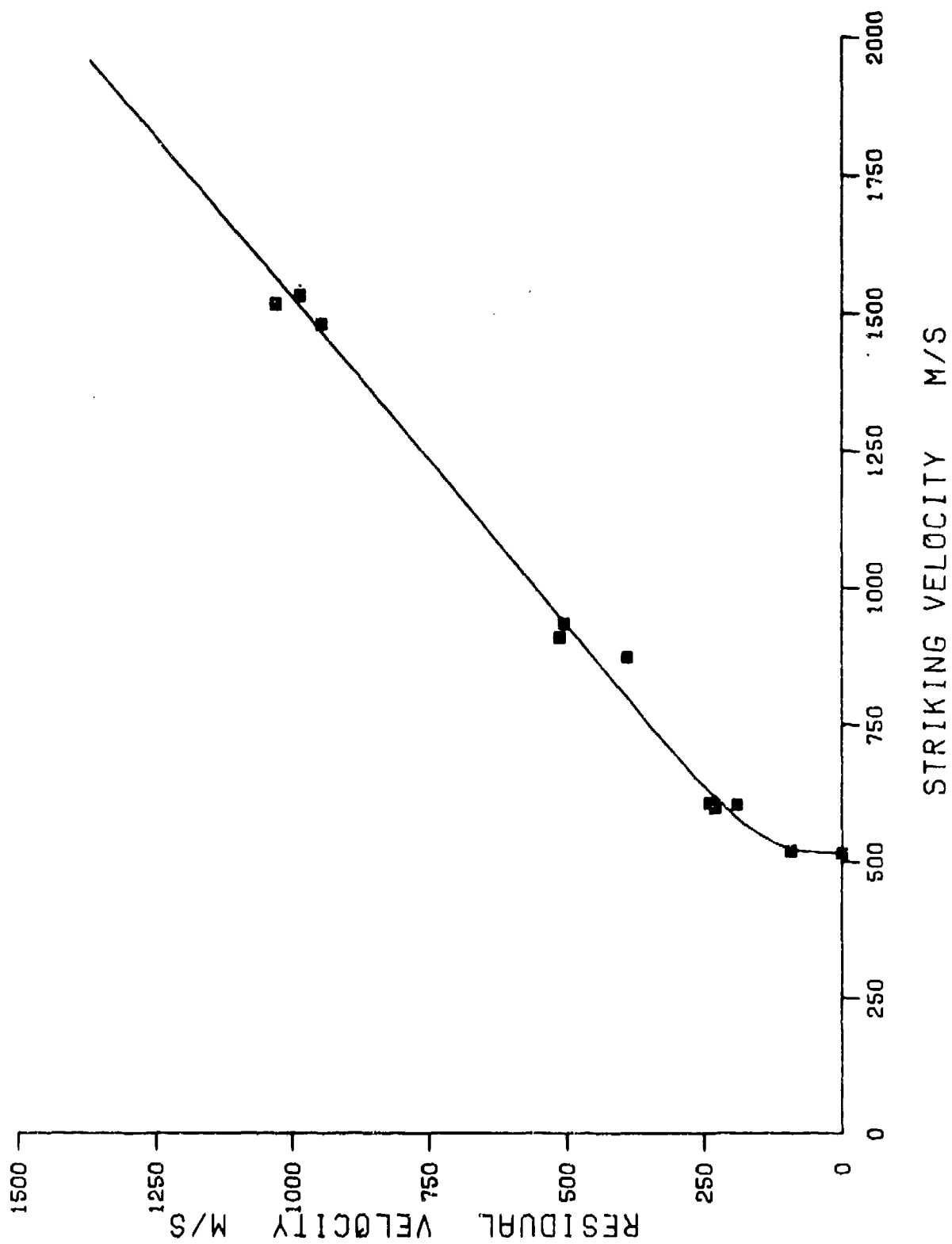
TABLE A-4

.45 GM TUNGSTEN SPHEROID INTO .32 CM
MILD STEEL AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 515.7
TO 530.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	515.7	.108057	.324210	
FINAL ESTIMATES	515.7	.089492	.356297	29.0

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
515.7	.0		
520.9	92.4	77.5	14.9
598.9	229.2	210.3	18.9
603.5	189.0	215.1	-26.2
607.2	239.0	219.0	20.0
873.3	389.5	451.2	-61.6
909.8	511.1	481.2	29.9
934.8	502.9	501.7	1.2
1479.5	942.1	954.0	-11.8
1516.4	1025.7	985.2	40.4
1531.6	982.1	998.2	-16.1



.45 GM TUNGSTEN SPHEROID INTO .32 CM MILD STEEL AT 0 DEGREES OBLIQUITY

FIGURE A-4

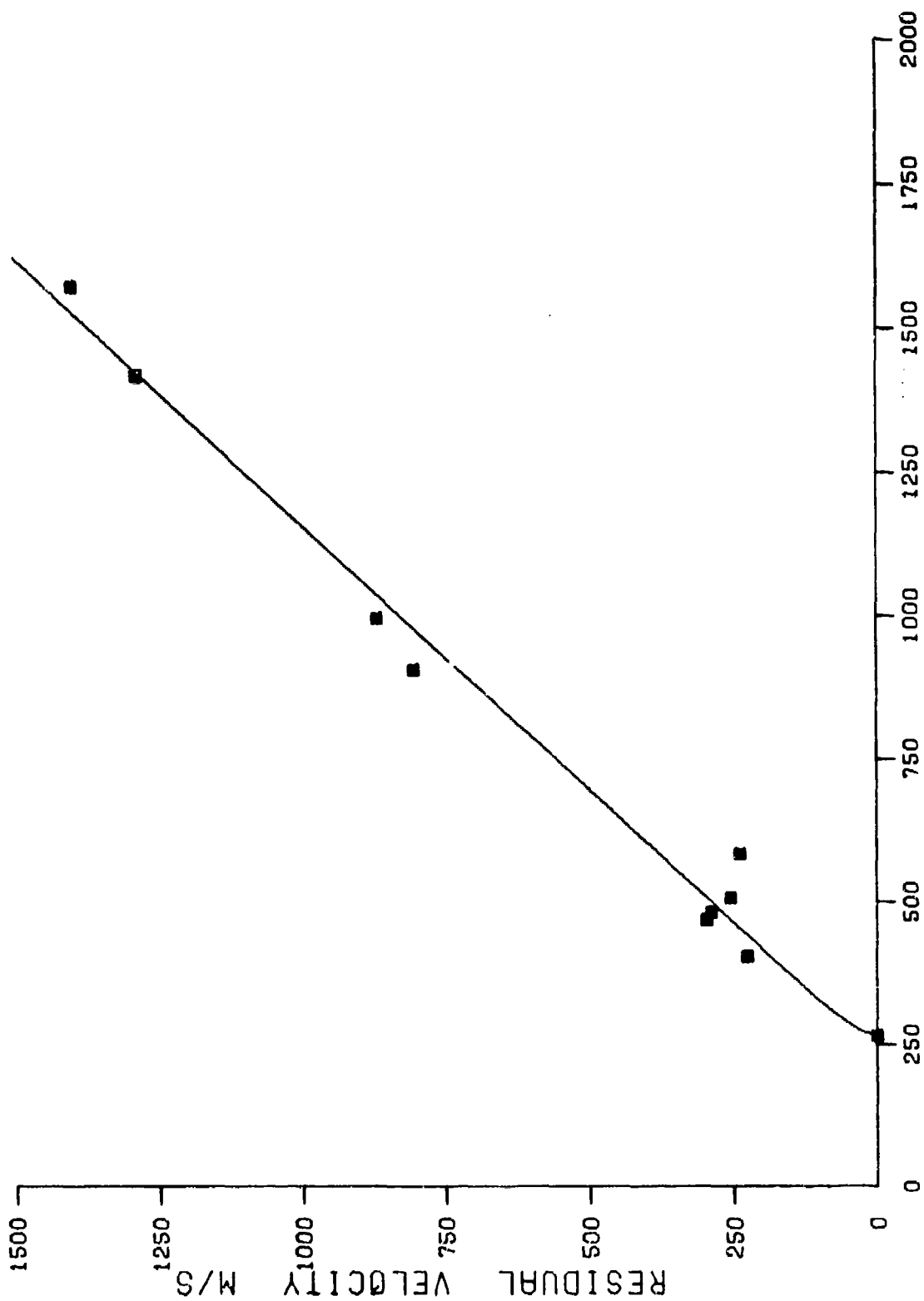
TABLE A-5

.45 GM TUNGSTEN SPHEROID INTO .15 CM
ALUMINUM AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 266.4
TO 410.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	266.4	.034665	.593401	
FINAL ESTIMATES	266.4	.028704	.623741	62.9

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
266.4	.0		
404.8	226.5	187.3	39.1
468.8	297.2	255.6	41.6
480.7	289.0	268.2	20.8
507.2	254.2	296.4	-42.2
584.9	238.4	379.3	-140.9
907.1	804.7	726.9	77.7
996.1	868.4	823.5	44.9
1417.6	1285.0	1279.7	5.4
1571.2	1397.8	1444.7	-46.9



.45 GM TUNGSTEN SPHEROID INTO .15 CM ALUMINUM AT 45 DEGREES OBLIQUITY
 STRIKING VELOCITY M/S
 FIGURE A-5

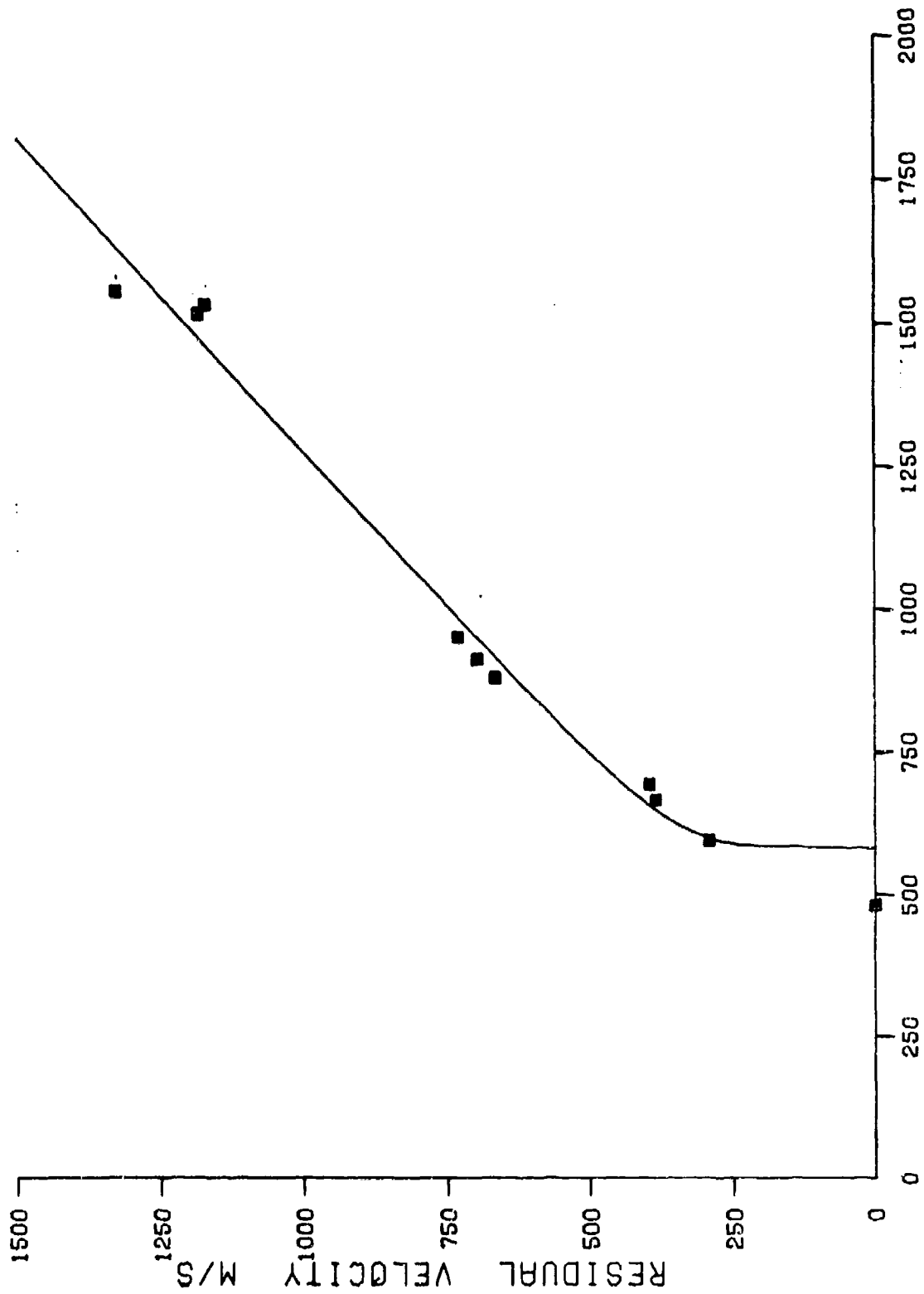
TABLE A-6

.45 GP TUNGSTEN SPHEROID INTO .32 CM
ALUMINUM AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 482.2
TO 600.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	RETA	ERMS
INITIAL ESTIMATES	482.2	.114710	.390246	
FINAL ESTIMATES	581.7	.344483	.226746	43.2

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
482.2	.0		
597.7	291.7	284.4	7.3
666.9	383.4	407.4	-23.9
695.9	395.6	442.0	-46.4
880.9	663.9	629.7	34.2
912.6	695.9	659.8	36.1
952.5	727.9	697.4	30.5
1516.4	1179.0	1217.7	-38.7
1531.6	1166.2	1231.7	-65.5
1555.1	1322.8	1253.3	69.5



.45 GM TUNGSTEN SPHEROID INTO .32 CM ALUMINUM AT 45 DEGREES OBLIQUITY
FIGURE A-6

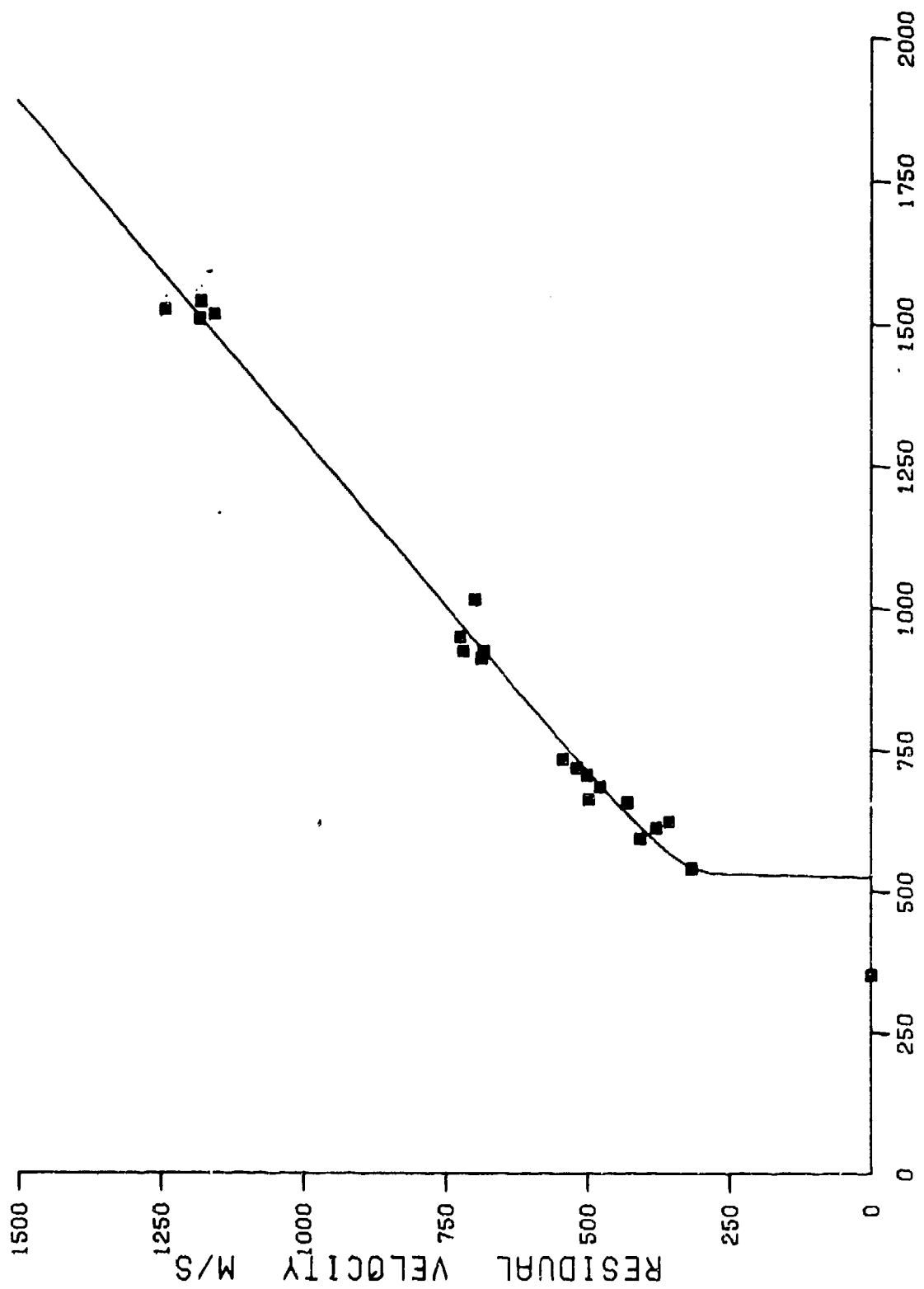
TABLE A-7

.45 GM TUNGSTEN SPHEROID INTO .15 CM
MILD STEEL AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 352.3
TO 550.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	352.3	.273901	.247218	
FINAL ESTIMATES	526.7	.604598	.132986	30.5

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
352.3	.0		
541.3	316.7	313.1	3.6
594.1	408.1	387.9	20.2
613.3	379.8	407.9	-28.2
623.3	356.6	418.0	-61.3
658.4	430.4	451.4	-21.0
662.6	498.0	455.3	42.7
686.4	477.9	477.0	1.0
705.6	501.4	494.2	7.2
718.7	518.5	505.8	12.7
734.6	542.2	519.7	22.5
912.6	685.8	672.4	13.4
923.5	716.6	681.7	34.9
923.5	681.8	681.7	.2
949.5	723.3	703.6	19.7
1015.9	697.1	759.7	-62.6
1508.8	1178.1	1175.9	2.1
1516.4	1153.1	1182.4	-29.3
1524.0	1239.0	1188.8	50.2
1539.5	1176.2	1202.0	-25.8



.45 GM TUNGSTEN SPHEROID INTO .15 CM MILD STEEL AT 45 DEGREES OBLIQUITY
 STRIKING VELOCITY M/S
 FIGURE A-7

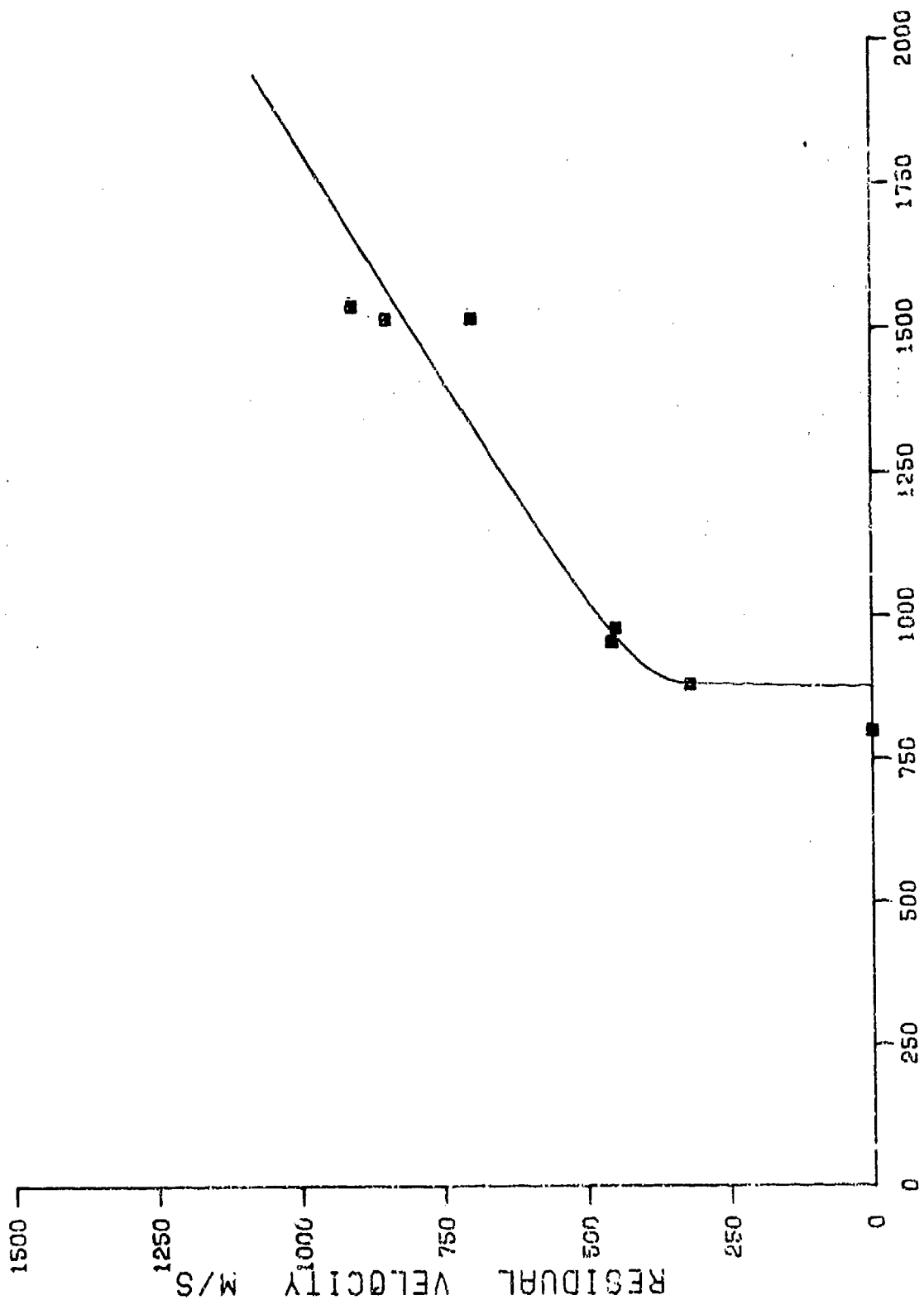
TABLE A-8

.45 GM TUNGSTEN SPHEROID INTO .32 CM
MILD STEEL AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 800.1
TO 890.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	800.1	.207160	.200348	
FINAL ESTIMATES	877.6	.396242	.101630	58.9

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
800.1	.0		
880.9	317.6	317.8	-.2
955.5	454.2	439.9	14.2
979.9	447.4	460.2	-12.8
1516.4	843.7	810.0	33.7
1516.4	695.6	810.0	-114.5
1539.5	903.4	824.4	79.0



STRIKING VELOCITY M/S
 .45 GM TUNGSTEN SPHEROID INTO .32 CM MILD STEEL AT 45 DEGREES OBLIQUITY
 FIGURE A-8

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