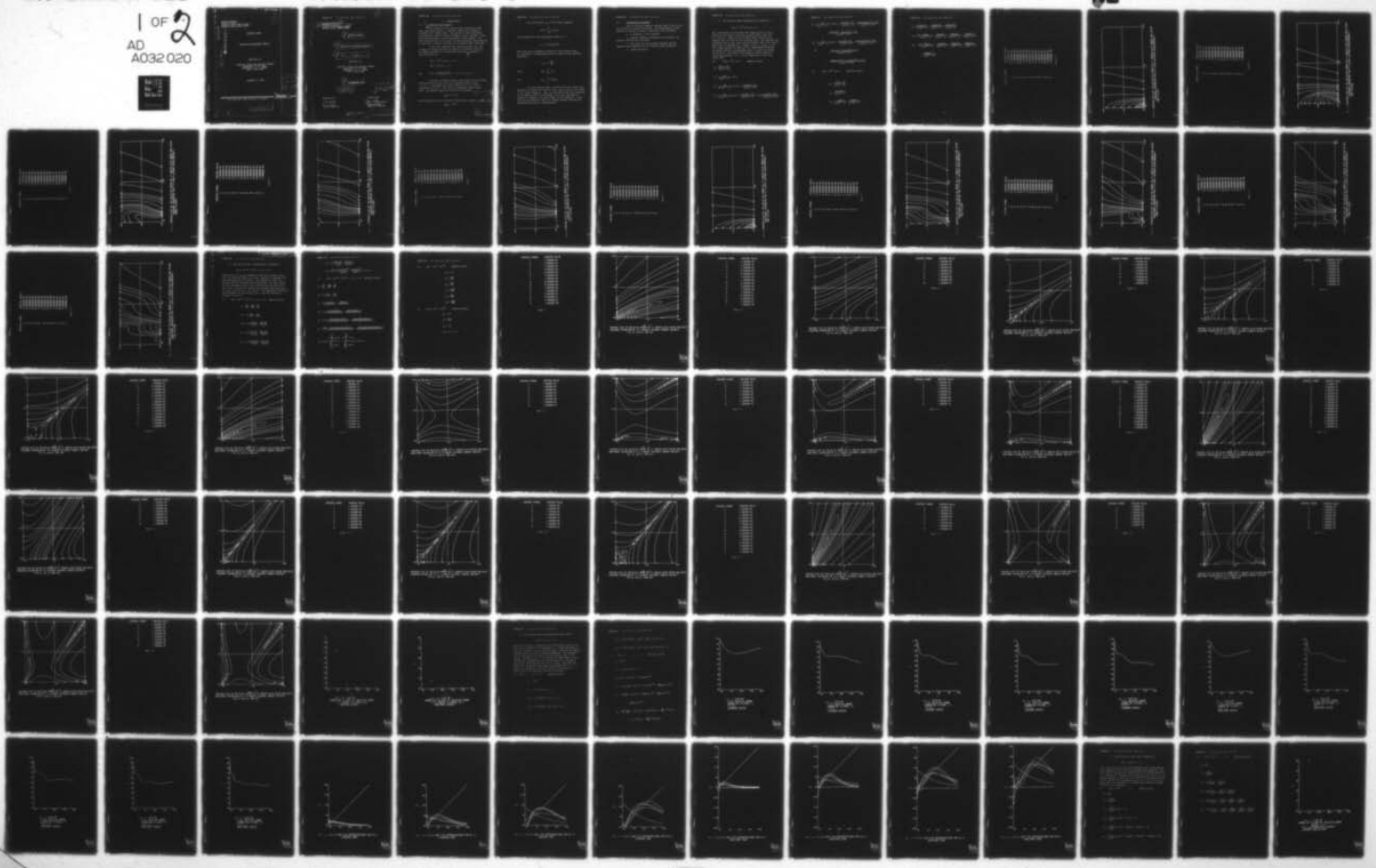
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ANALYSIS OF TRANSIENT SIGNALS.(U)
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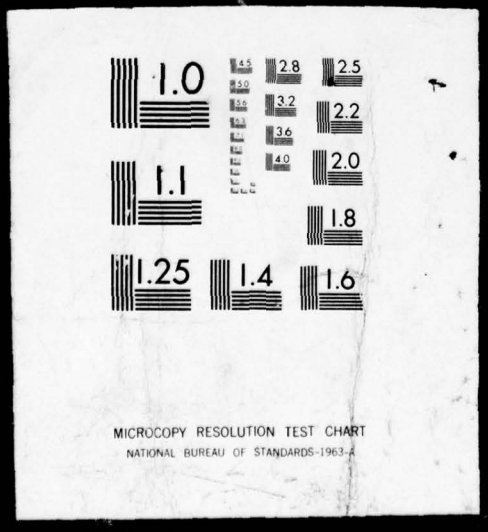
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TRACOR Project Number 20034

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TECHNICAL NOTE

ANALYSIS OF TRANSIENT SIGNALS

Submitted to:

Commander, Naval Ship Systems Command
Department of the Navy
Washington, D. C. 20360
Attn: Code 1622C

September 9, 1966

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James A. Downing

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181 p.

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Submitted by:

H. R. Courts

H. R. Courts
Project Director

Prepared by:

James Downing

James Downing
Engineer Scientist

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1. INTRODUCTION

1.1 Summary of the Problem

Past experience with passive sonar systems has shown that significant numbers of transient signals are produced by submarines and surface ships. The sources of these transients are mechanical, such as hatches slamming or tools dropping. Until now little systematic use has been made of these signals. The work presented here is the result of some preliminary investigations to determine a suitable technique for processing these transients.

It has been concluded that transient pulses fall into a number of classes and that all pulses in a specific class can be roughly described by an appropriate function. Examples of these functions are

$$f(t) = e^{-\alpha t} \cos \omega t, \ \alpha \geq 0 ,$$

$$f(t) = \alpha t e^{-\alpha t}, \ \alpha \geq 1 ,$$

and

$$f(t) = \frac{\alpha e^{-\alpha t} - \beta e^{-\beta t}}{\alpha - \beta}, \ \alpha \geq 0, \ \beta \geq 0, \ \alpha \neq \beta .$$

To design an optimum linear filter system for processing such transients one can select an orthonormal basis, $\Phi_i(t)$, $i = 1,2,\ldots$, and expand the signal function $f(t)$ in a linear combination of the Φ_i. Two sets of orthonormal functions were used: the Laguerre functions defined by the sequence

$$U_n(t) = t^n e^{-t}$$

and an arbitrary set of functions defined by the sequence

$$U_n(t) = e^{-nt} .$$

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The coefficients, c_i, of the linear expansion

$$f(t) \approx \sum_{i=1}^{N} c_i \Phi_i(t)$$

were determined in the least-squares sense, i.e.,

$$c_i = \int_{-\infty}^{\infty} f(t)\Phi_i(t)dt \ .$$

With this type of expansion a measure of the relative error, η_N, incurred in the approximation with N terms of the expansion is given by

$$\eta_N = 1 - \frac{E_N}{E_i} \ ,$$

where

$$E_N = \sum_{i=1}^{N} c_i^2 \ ,$$

and

$$E_i = \int_{-\infty}^{\infty} f^2(t)dt \ .$$

For each function under consideration error curves were developed to display the relative error, η_N, for one to five terms of the linear expansion. Where η_N also varies with some parameters of the function under consideration, plots were developed to display the changes in η_N with respect to the parameters. Other plots were developed to show the original function and the five approximation functions simultaneously.

1.2 Presentation of Results

Each of the seven numbered sections which follow covers the analysis of a specific function. The presentation of each analysis is similar in format, consisting of the following:

a. A definition of the function.

b. General comments pertaining to the function, its analysis, and the data presented.

c. The first five coefficients obtained from the Laguerre basis expansion and the arbitrary basis expansion.

d. Curves and tables.

2. The function under consideration is defined by

$$f(t) = e^{-\alpha t} \cos \omega t, \ \alpha \geq 0 \ .$$

The illustrations in this group show contour plots of $\eta(\alpha,\omega)$ which were developed to show how η_N varies with α and ω for $N = 1,2,3,4,5$. There are ten of these plots, five for the Laguerre case and five for the arbitrary case. For each plot the horizontal axis is the ω-axis and the vertical one is the α-axis. Each locus is identified by a symbol, 1,2,3,..., A,B,..., etc. On the page adjacent to each plot is a table which associates the symbol with the corresponding value of $\eta_N(\alpha,\omega)$ for points on the locus. The format of the numbers in these tables is scientific notation, i.e., E $\pm$ X is equivalent to the scale factor $10^{\pm X}$.

2.1 $f(t) = e^{-\alpha t} \cos \omega t$ Laguerre series

$$E_i = \frac{(2\alpha^2 + \omega^2)}{4\alpha(\alpha^2 + \omega^2)}$$

$$C_1 = \frac{\sqrt{2}}{(\alpha+1)^2 + \omega^2} \quad (\alpha+1)$$

$$C_2 = \frac{\sqrt{2}}{(\alpha+1)^2 + \omega^2} \left\{ - (\alpha+1) + 2\frac{[(\alpha+1)^2 - \omega^2]}{[(\alpha+1)^2 + \omega^2]} \right\}$$

$$C_3 = \frac{\sqrt{2}}{(\alpha+1)^2 + \omega^2} \left\{ (\alpha+1) - 4\frac{[(\alpha+1)^2 - \omega^2]}{[(\alpha+1)^2 + \omega^2]} + 4(\alpha+1)\frac{[(\alpha+1)^2 - 3\omega^2]}{[(\alpha+1)^2 + \omega^2]^2} \right\}$$

$$C_4 = \frac{\sqrt{2}}{(\alpha+1)^2 + \omega^2} \left\{ - (\alpha+1) + 6\frac{[(\alpha+1)^2 - \omega^2]}{[(\alpha+1)^2 + \omega^2]} - \frac{12(\alpha+1)[(a+1)^2 - 3\omega^2]}{[(\alpha+1)^2 + \omega^2]^2} \right.$$

$$\left. + \frac{8[(\alpha+1)^4 - 6(\alpha+1)^2\omega^2 + \omega^4]}{[(\alpha+1)^2 + \omega^2]^3} \right\}$$

$$C_5 = \frac{\sqrt{2}}{(\alpha+1)^2 + \omega^2} \left\{ (\alpha+1) - \frac{8[(\alpha+1)^2 - \omega^2]}{[(\alpha+1)^2 + \omega^2]} + \frac{24(\alpha+1)[(\alpha+1)^2 - 3\omega^2]}{[(\alpha+1)^2 + \omega^2]^2} \right.$$

$$- \frac{32[(\alpha+1)^4 - 6(\alpha+1)^2\omega^2 + \omega^4]}{[(\alpha+1)^2 + \omega^2]^3}$$

$$\left. + \frac{16(\alpha+1)[(\alpha+1)^4 - 10(\alpha+1)^2\omega^2 + 5\omega^4]}{[(\alpha+1)^2 + \omega^2]^4} \right\}$$

2.2 $f(t) = e^{-\alpha t} \cos \omega t$ Arbitrary series

$$E_i = \frac{(2\alpha^2 + \omega^2)}{4\alpha(\alpha^2 + \omega^2)}$$

$$C_1 = \frac{\sqrt{2}\ (\alpha+1)}{(\alpha+1)^2 + \omega^2}$$

$$C_2 = \frac{4(\alpha+1)}{(\alpha+1)^2 + \omega^2} - \frac{6(\alpha+2)}{(\alpha+2)^2 + \omega^2}$$

$$C_3 = \frac{3\sqrt{6}(\alpha+1)}{(\alpha+1)^2 + \omega^2} - \frac{12\sqrt{6}(\alpha+2)}{(\alpha+2)^2 + \omega^2} + \frac{10\sqrt{6}(\alpha+3)}{(\alpha+3)^2 + \omega^2}$$

$$C_4 = 2\sqrt{2}\left[\frac{4(\alpha+1)}{(\alpha+1)^2 + \omega^2} - \frac{30(\alpha+2)}{(\alpha+2)^2 + \omega^2} + \frac{60(\alpha+3)}{(\alpha+3)^2 + \omega^2} - \frac{35(\alpha+4)}{(\alpha+4)^2 + \omega^2}\right]$$

$$C_5 = \sqrt{10}\left[\frac{5(\alpha+1)}{(\alpha+1)^2 + \omega^2} - \frac{60(\alpha+2)}{(\alpha+2)^2 + \omega^2} + \frac{210(\alpha+3)}{(\alpha+3)^2 + \omega^2} - \frac{280(\alpha+4)}{(\alpha+4)^2 + \omega^2}\right.$$

$$\left. + \frac{126(\alpha+5)}{(\alpha+5)^2 + \omega^2}\right]$$

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.1

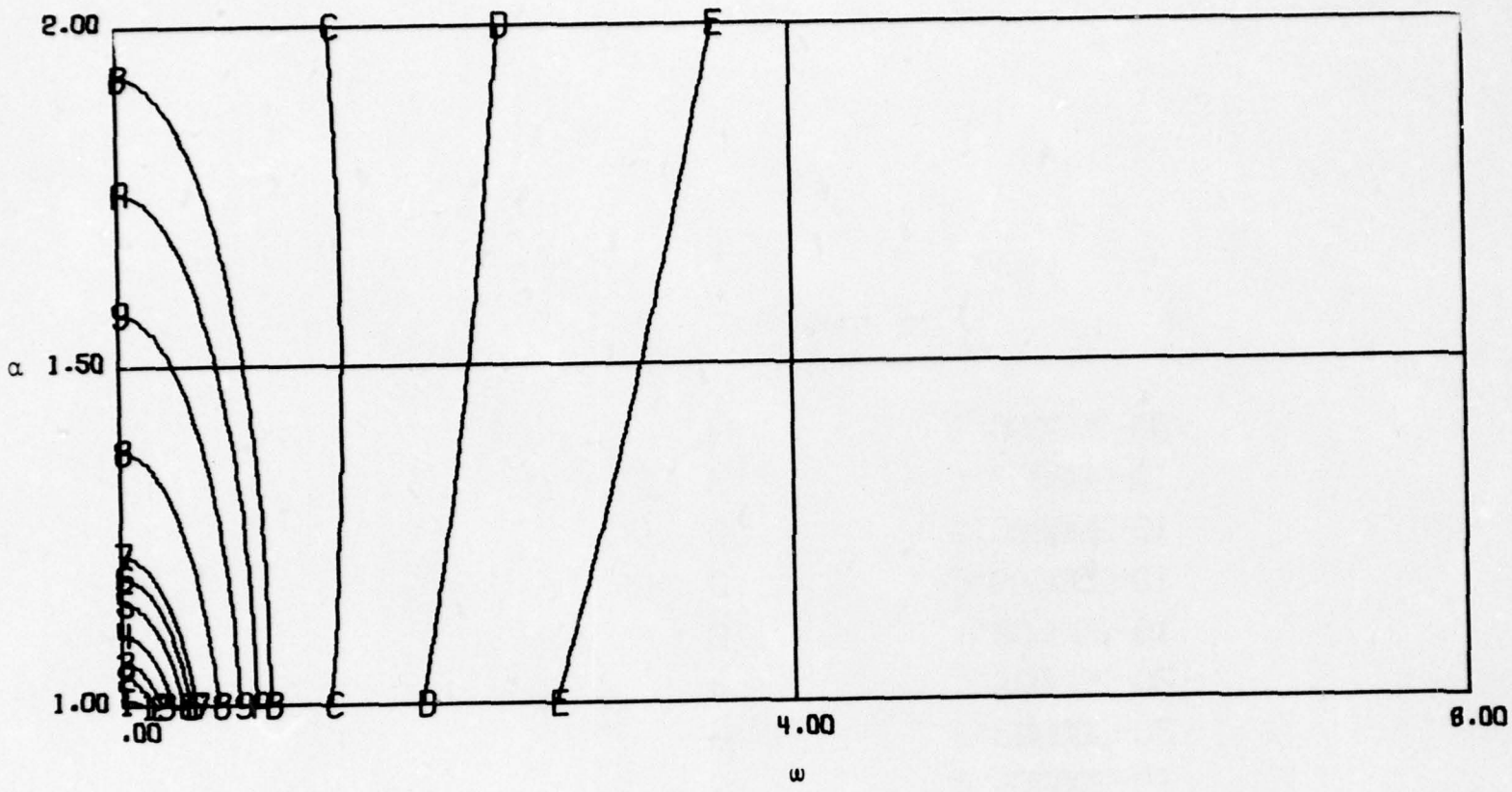


FIG. 2.1 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
LAGUERRE EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
ONE FILTER

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.2

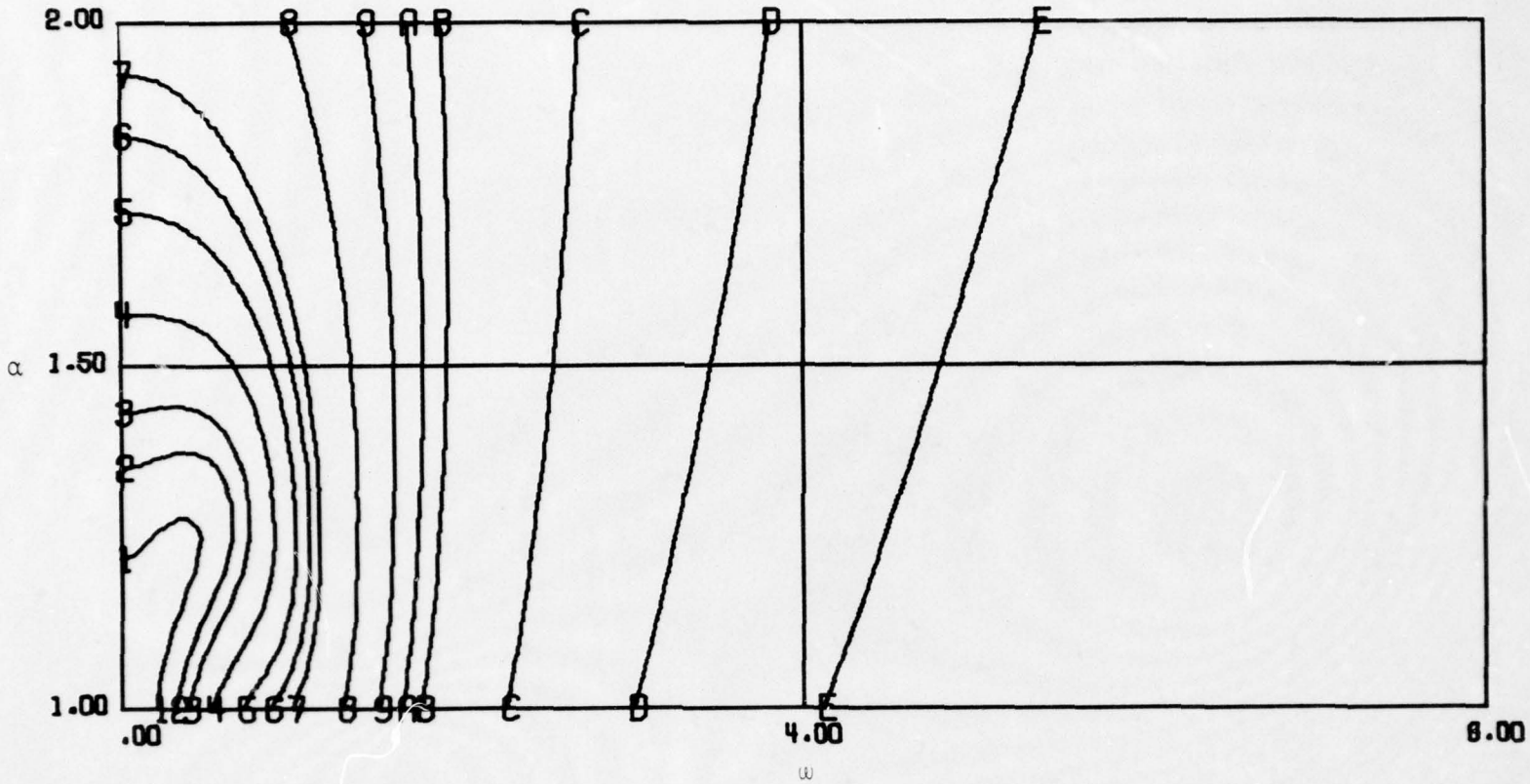


FIG 2.2 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
LAGUERRE EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
TWO FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.3

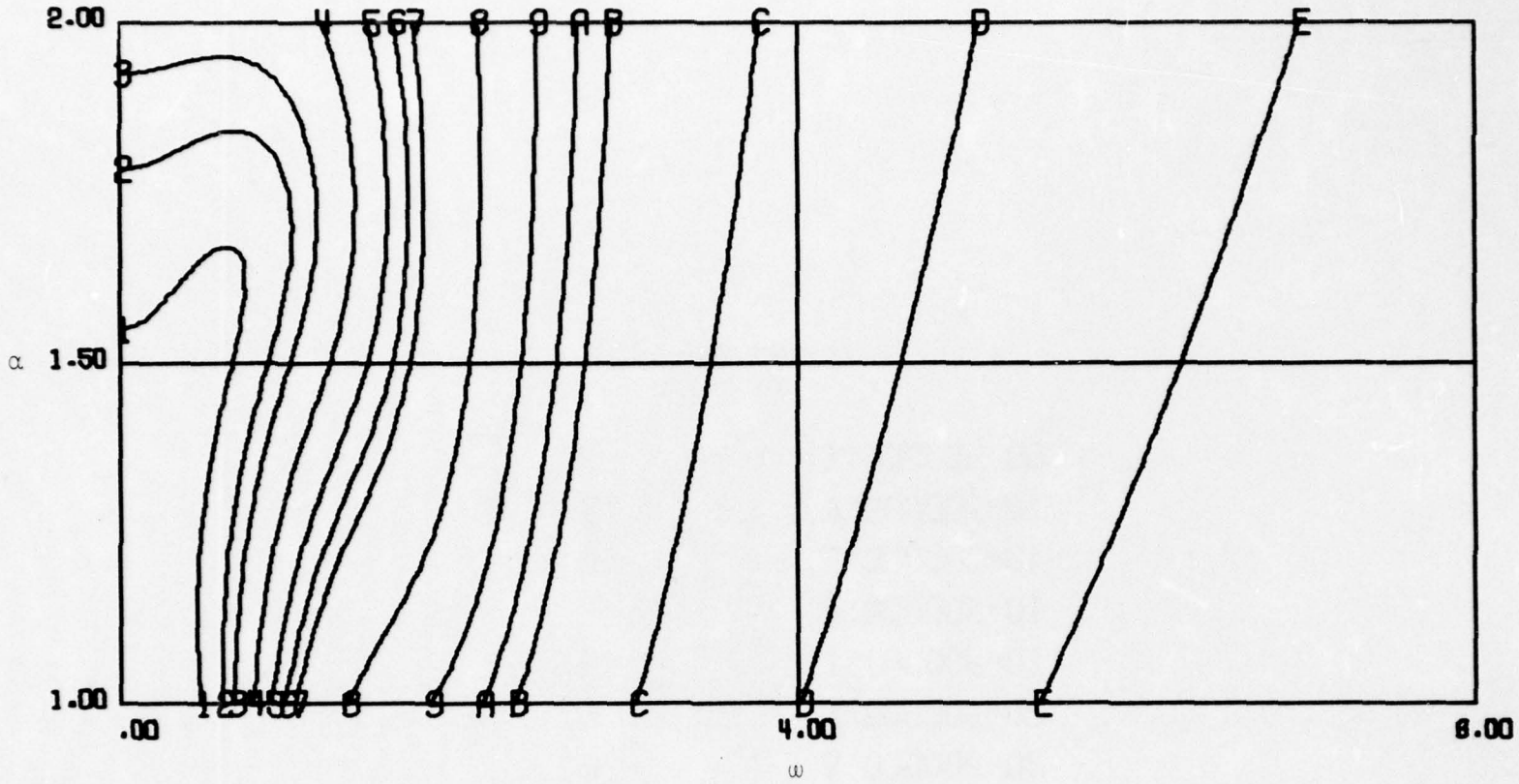


FIG 2.3 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
LAGUERRE EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
THREE FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.4

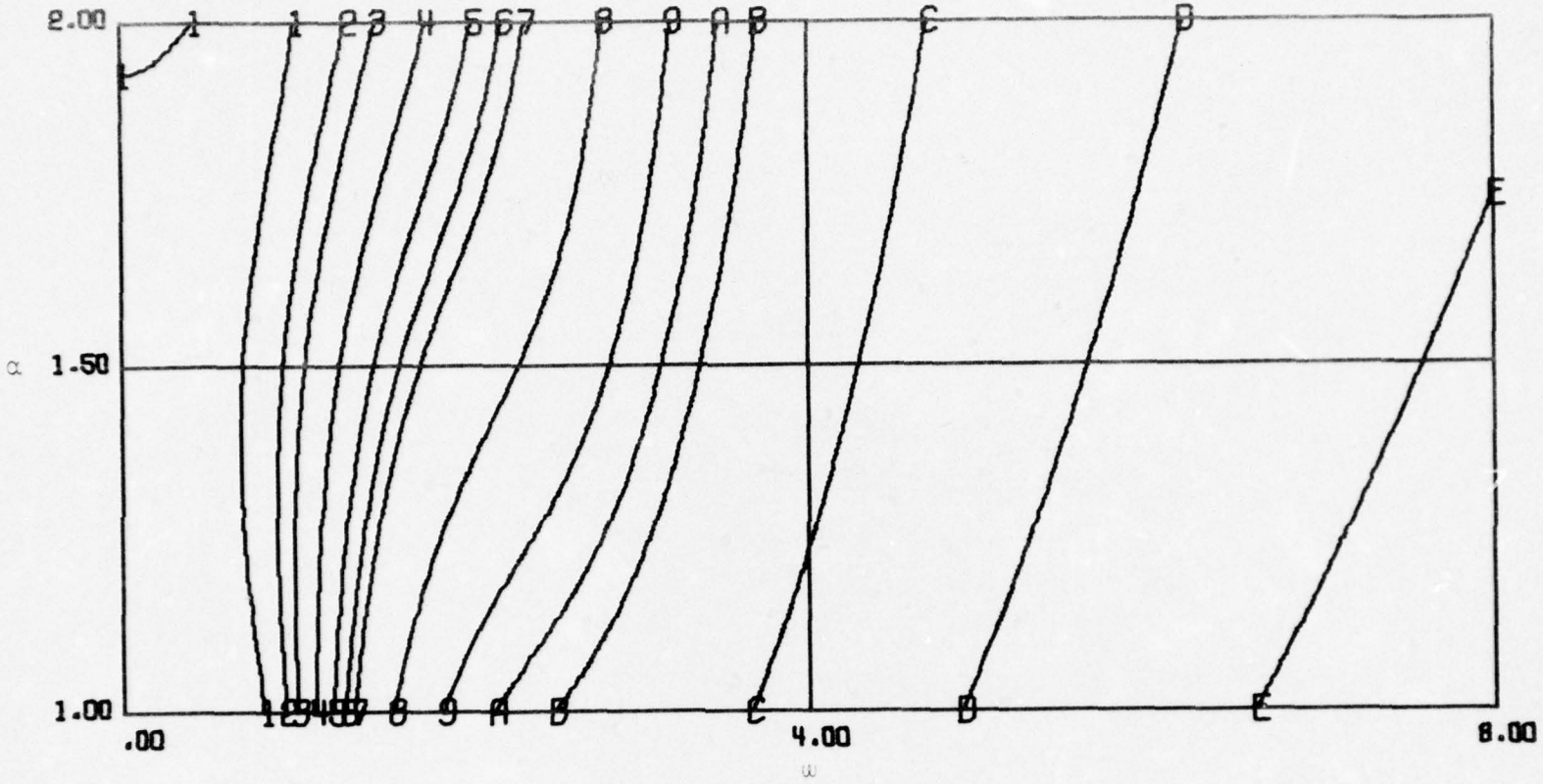


FIG. 2.4 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
LAGUERRE EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
FOUR FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.00000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.5

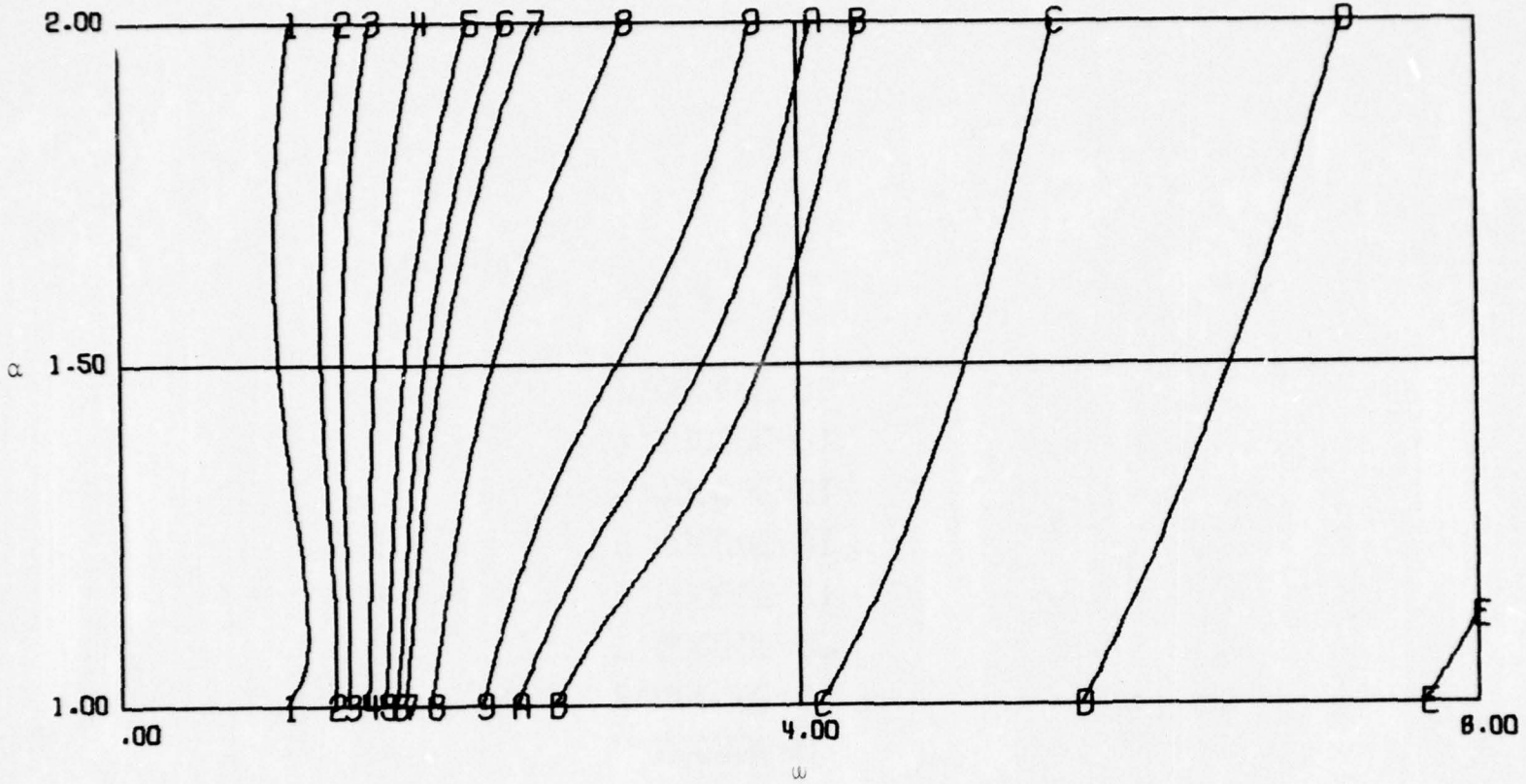


FIG. 2.5 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
LAGUERRE EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
FIVE FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.6

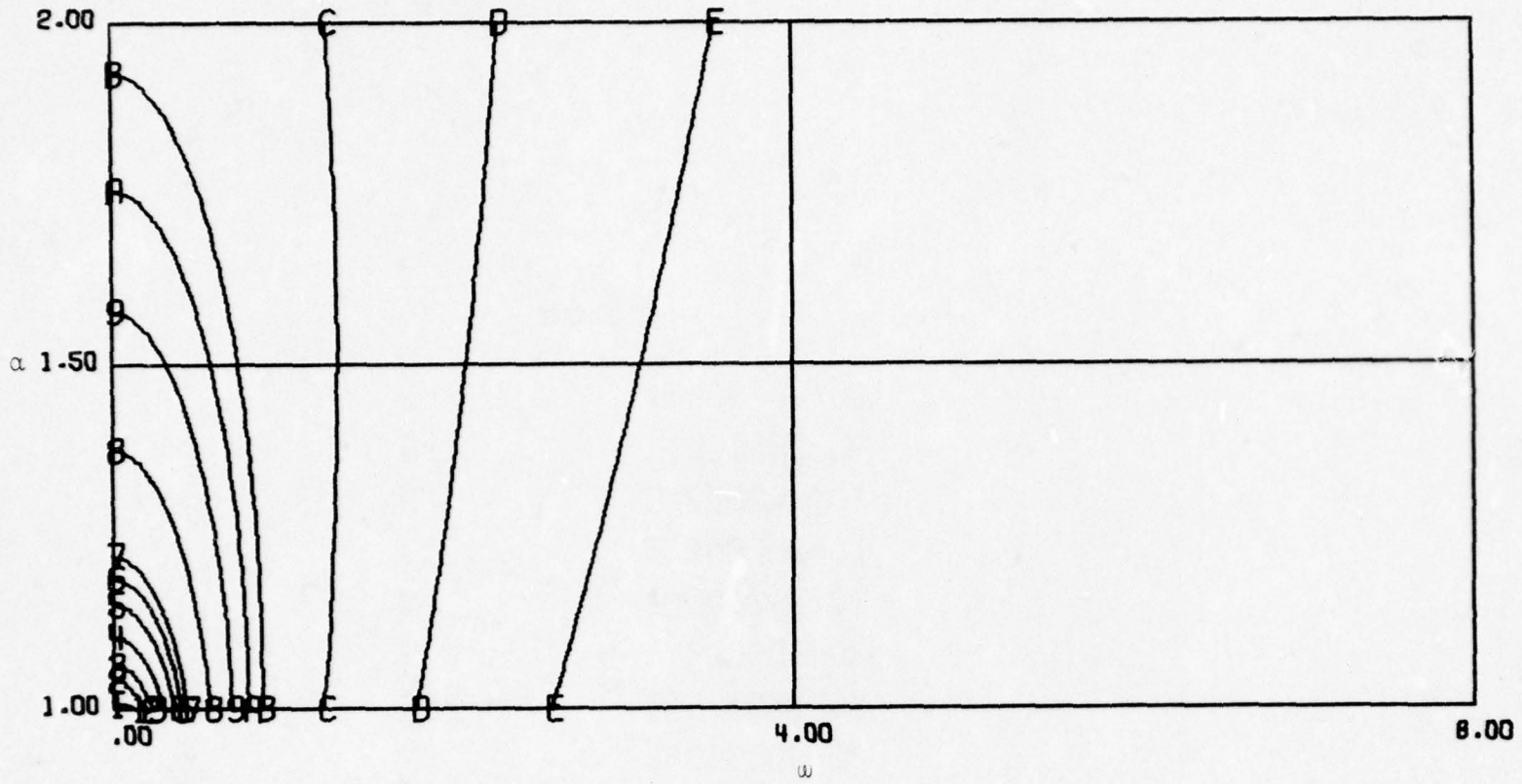


FIG. 2.6 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
ARBITRARY EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
ONE FILTER

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.7

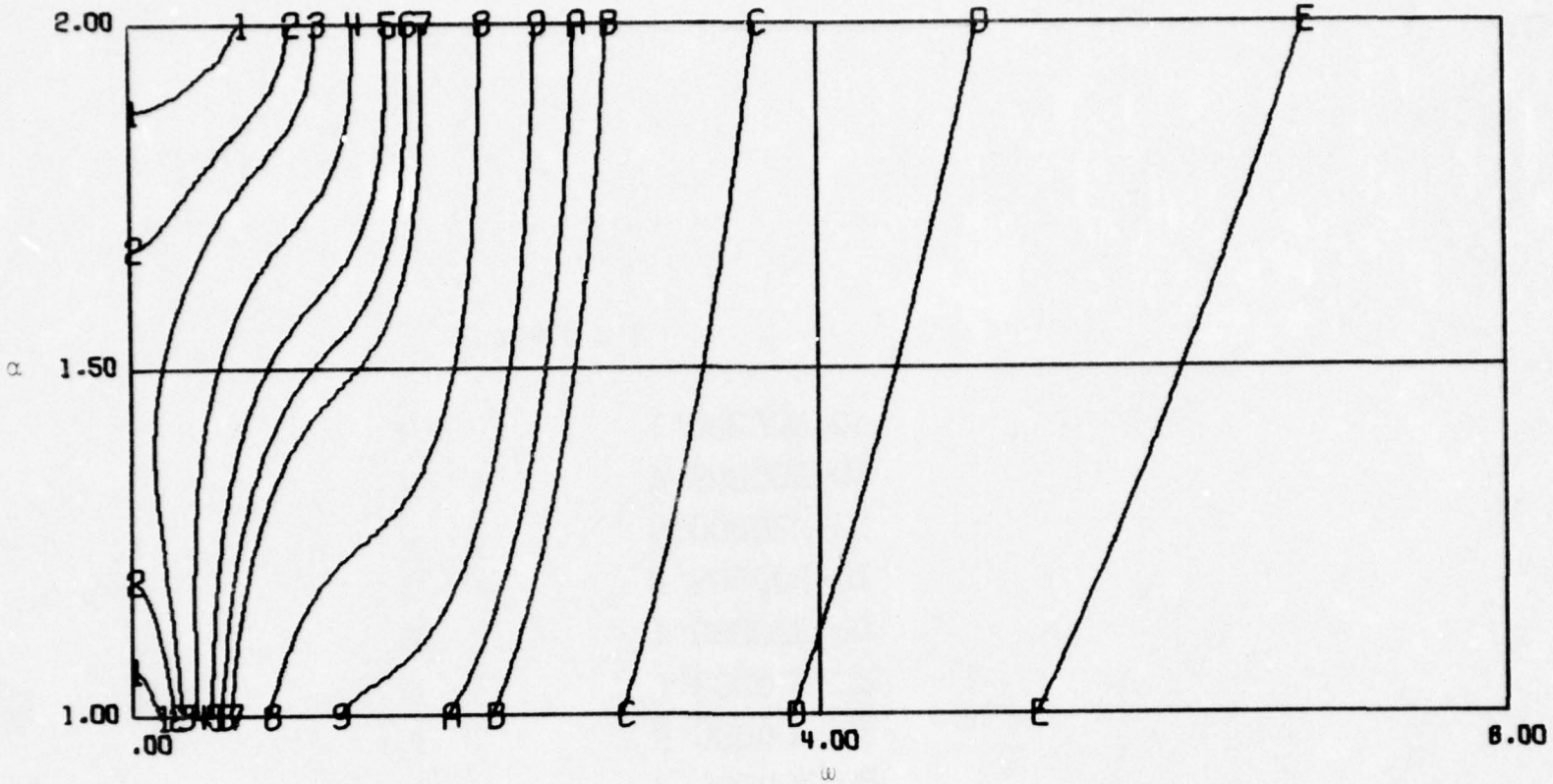


FIG. 2.7 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
ARBITRARY EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
TWO FILTERS

CONTOUR SYMBOL.	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.8

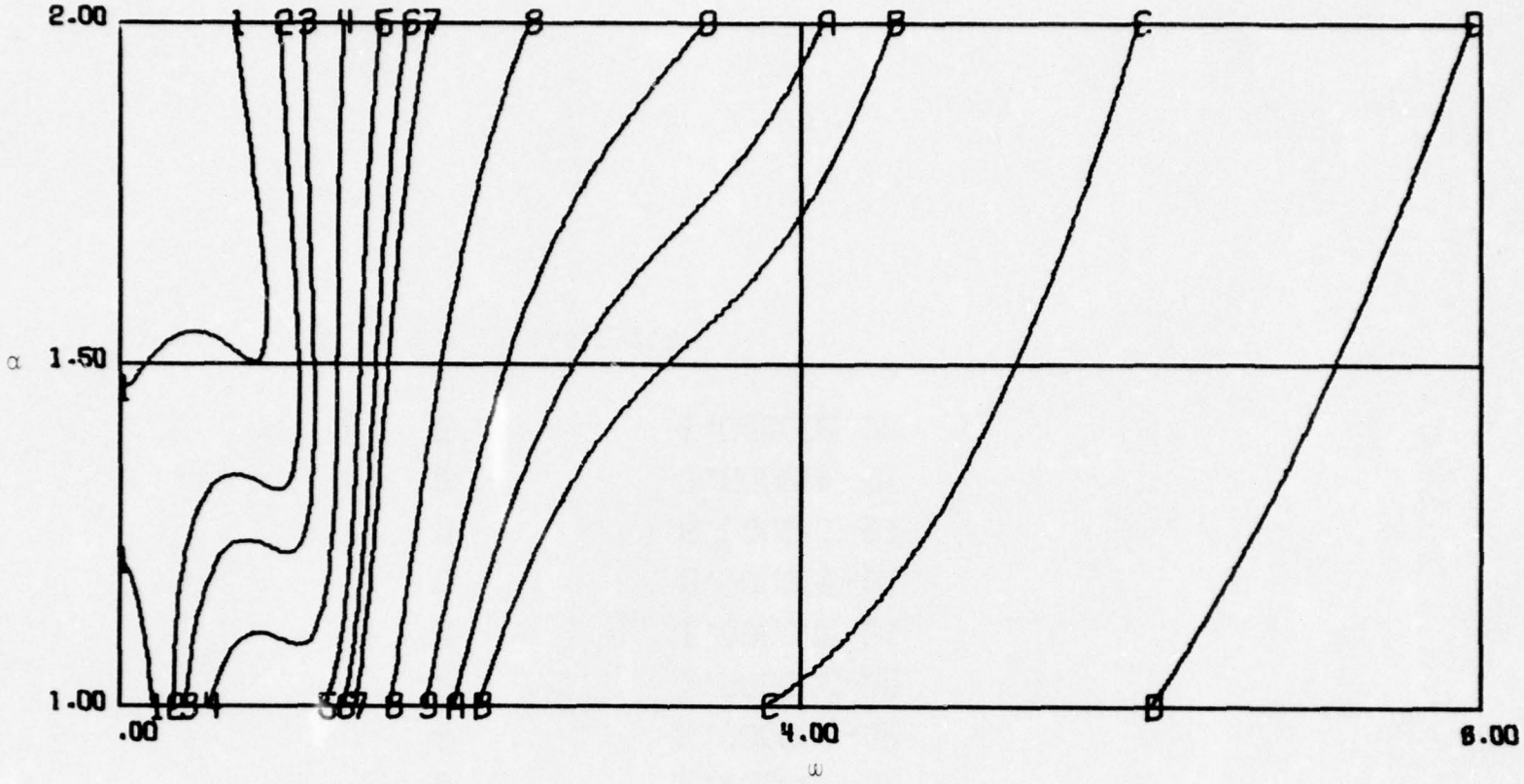


FIG. 2.8 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
ARBITRARY EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
THREE FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.9

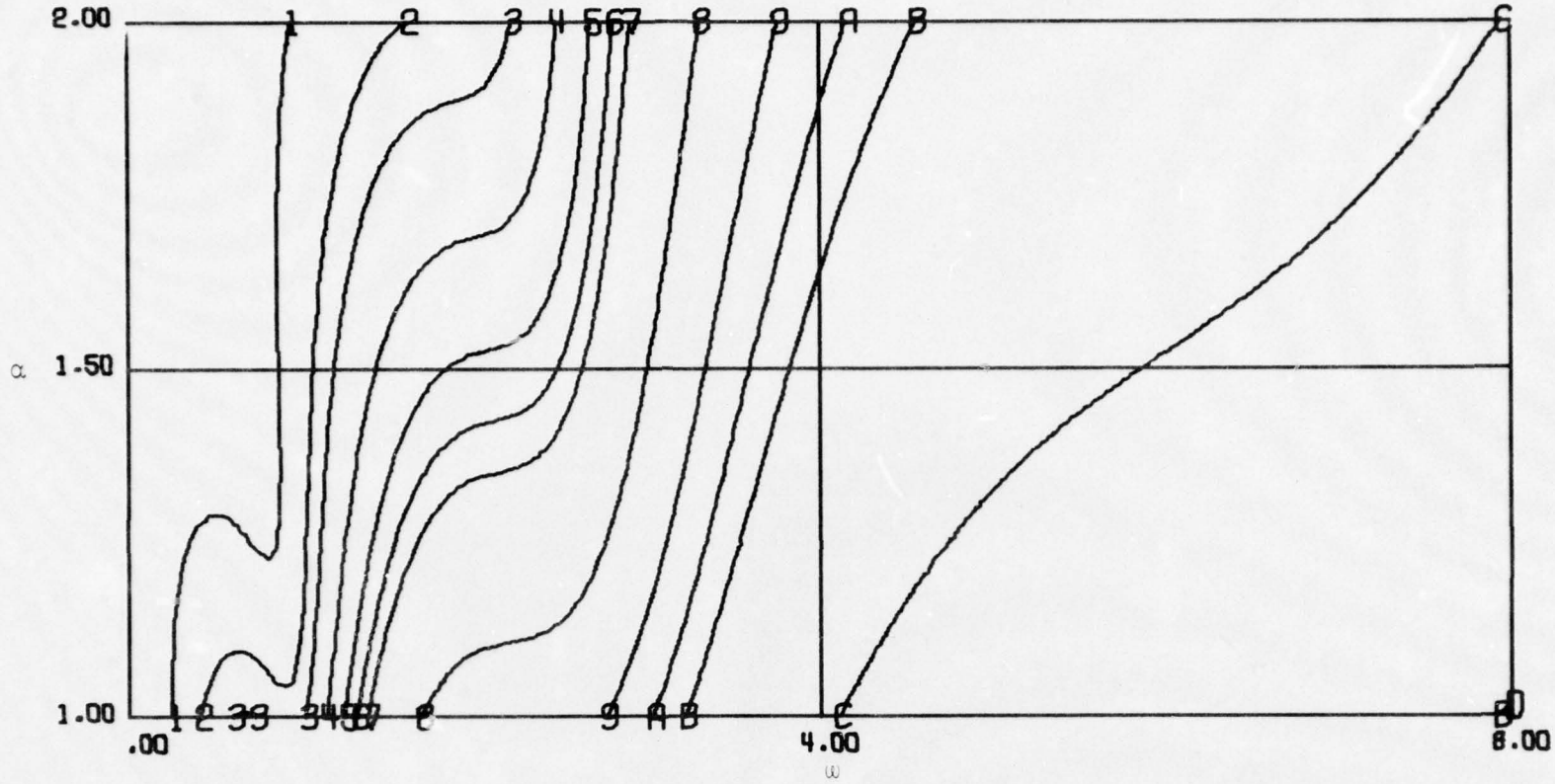


FIG. 2.9 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
ARBITRARY EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
FOUR FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.00000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 2.10

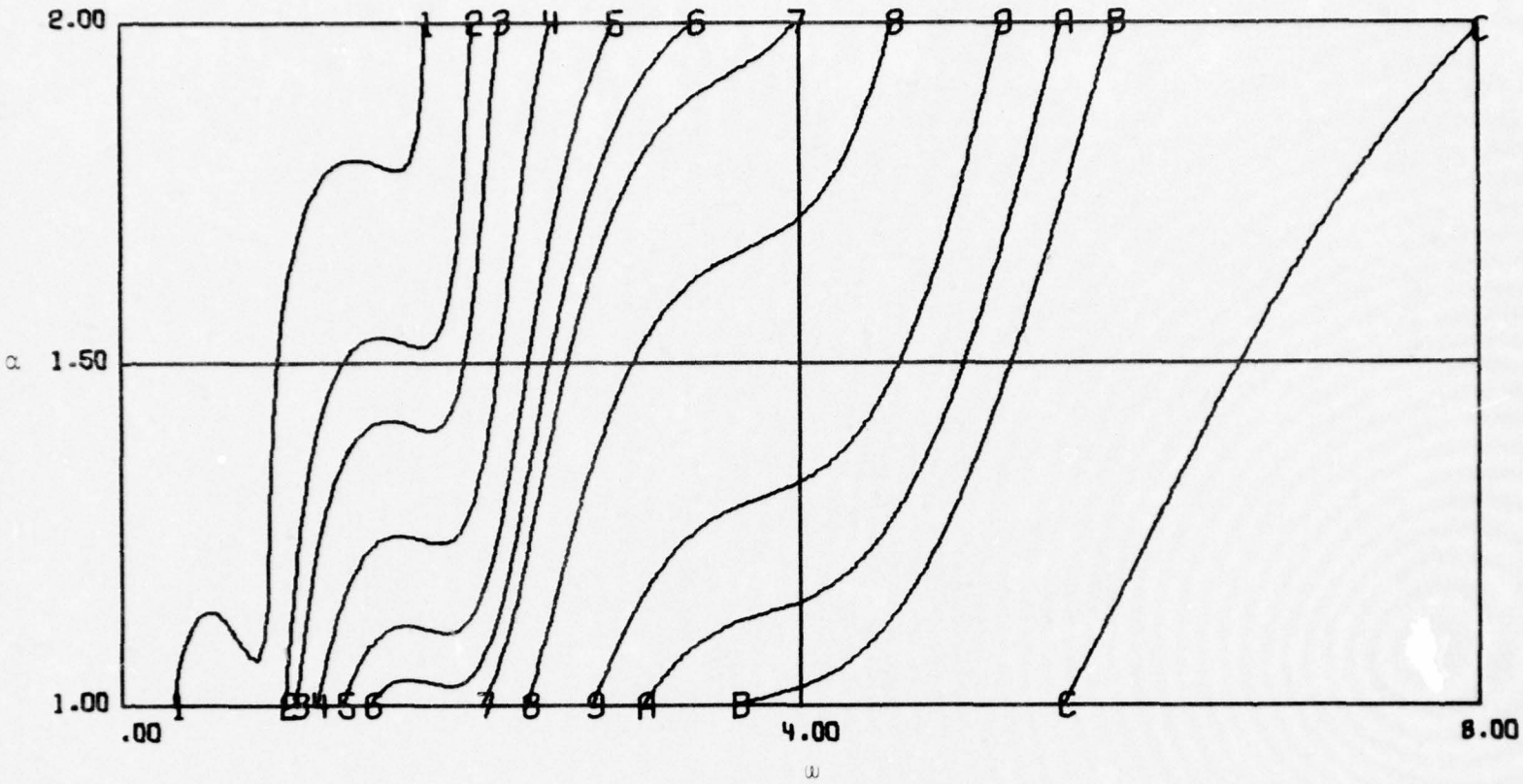


FIG. 2.10 CONTOUR PLOT OF THE RELATIVE ERROR AS IT VARIES WITH OMEGA AND ALPHA
ARBITRARY EXPANSION FOR F(T)=EXP(-ALPHA*T)*COS(OMEGA*T)
FIVE FILTERS

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3. The function under consideration is defined by

$$f(t) = Ae^{-\alpha t} - Be^{-\beta t}, \quad \alpha > 0, \quad \beta > 0 .$$

Contour plots of η_N were produced to show how η_N varies with α and β for selected values of A and B. Figures 3.1 through 3.10 were developed with A=1 and B=2. The second set, Figs. 3.11 through 3.20 were developed with A=3 and B=2. For each contour plot the horizontal axis is the α-axis and the vertical one is the β-axis. The last two figures of this group are point plots of η_N vs N for A=3, B=2, α=1, and β=2. The expansion coefficients were developed for this special case. They are included in Sections 3.3 and 3.4.

3.1 $f(t) = Ae^{-\alpha t} - Be^{-\beta t}$, $\alpha > 0$, $\beta > 0$. Laguerre series

$$E_i = \frac{A^2}{2\alpha} - \frac{2AB}{\alpha+\beta} + \frac{B^2}{2\beta}$$

$$C_1 = \sqrt{2}\left[\frac{A}{\alpha+1} - \frac{B}{\beta+1}\right]$$

$$C_2 = -\sqrt{2}\left[\frac{A(\alpha-1)}{(\alpha+1)^2} - \frac{B(\beta-1)}{(\beta+1)^2}\right]$$

$$C_3 = \sqrt{2}\left[\frac{A(\alpha-1)^2}{(\alpha+1)^3} - \frac{B(\beta-1)^2}{(\beta+1)^3}\right]$$

$$C_4 = -\sqrt{2}\left[\frac{A(\alpha-1)^3}{(\alpha+1)^4} - \frac{B(\beta-1)^3}{(\beta+1)^4}\right]$$

$$C_5 = \sqrt{2}\left[\frac{A(\alpha-1)^4}{(\alpha+1)^5} - \frac{B(\beta-1)^4}{(\beta+1)^5}\right]$$

$$C_n = (-1)^{n-1}\sqrt{2}\left[\frac{A(\alpha-1)^{n-1}}{(\alpha+1)^n} - \frac{B(\beta-1)^{n-1}}{(\beta+1)^n}\right], \quad n=1,2,\ldots$$

3.2 $f(t) = Ae^{-\alpha t} - Be^{-\beta t}$, $\alpha > 0$, $\beta > 0$. Arbitrary series

$$E_i = \frac{A^2}{2\alpha} - \frac{2AB}{\alpha+\beta} + \frac{B^2}{2\beta}$$

$$C_1 = \sqrt{2}\left[\frac{A}{\alpha+1} - \frac{B}{\beta+1}\right]$$

$$C_2 = 2\left[\frac{A(1-\alpha)}{(\alpha+1)(\alpha+2)} - \frac{B(1-\beta)}{(\beta+1)(\beta+2)}\right]$$

$$C_3 = \sqrt{6}\left[\frac{A(1-\alpha)(2-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)} - \frac{B(1-\beta)(2-\beta)}{(\beta+1)(\beta+2)(\beta+3)}\right]$$

$$C_4 = 2\sqrt{2}\left[\frac{A(1-\alpha)(2-\alpha)(3-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)} - \frac{B(1-\beta)(2-\beta)(3-\beta)}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}\right]$$

$$C_5 = \frac{\sqrt{10}}{2}\left[\frac{A(1-\alpha)(2-\alpha)(3-\alpha)(4-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)} - \frac{B(1-\beta)(2-\beta)(3-\beta)(4-\beta)}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)}\right]$$

$$C_n = K_n\left[\frac{A\prod_{i=1}^{n-1}(i-\alpha)}{\prod_{i=1}^{n}(\alpha+i)} - \frac{B\prod_{i=1}^{n-1}(i-\beta)}{\prod_{i=1}^{n}(\beta+i)}\right], \quad n=1,2,\ldots$$

3.3 $f(t) = 3e^{-t} - 2e^{-2t}$ Laguerre series

$$E_i = 1.5$$

$$C_1 = \frac{5\sqrt{2}}{6}$$

$$C_2 = \frac{2\sqrt{2}}{9}$$

$$C_3 = \frac{-2\sqrt{2}}{27}$$

$$C_4 = \frac{2\sqrt{2}}{81}$$

$$C_5 = \frac{-2\sqrt{2}}{243}$$

3.4 $f(t) = 3e^{-t} - 2e^{-2t}$ Arbitrary series

$$E_i = 1.5$$

$$C_1 = \frac{5\sqrt{2}}{6}$$

$$C_2 = 1/3$$

$$C_n = 0 \ , \ n \geq 3$$

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.1

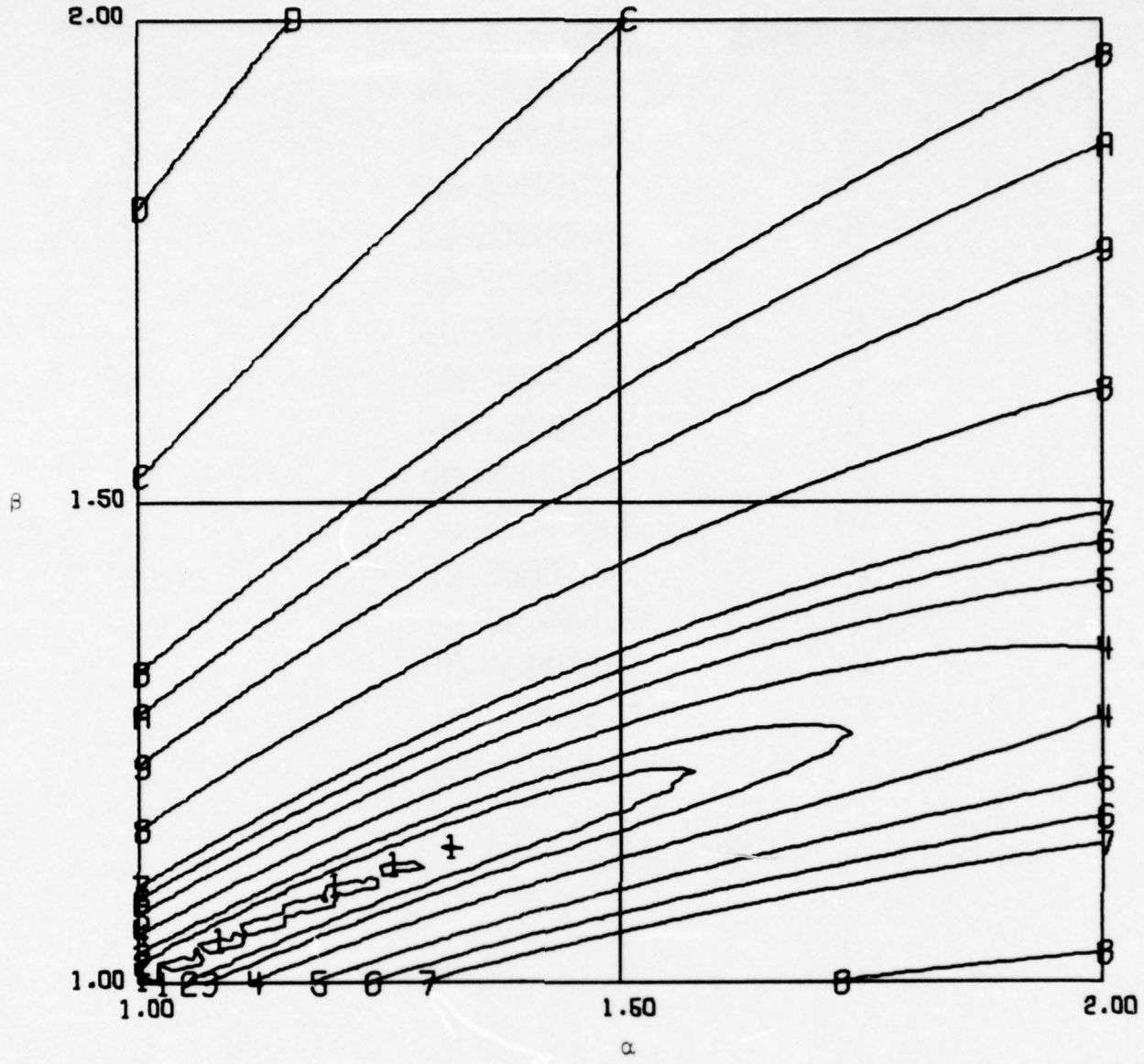


FIG. 3.1

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=1

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.2

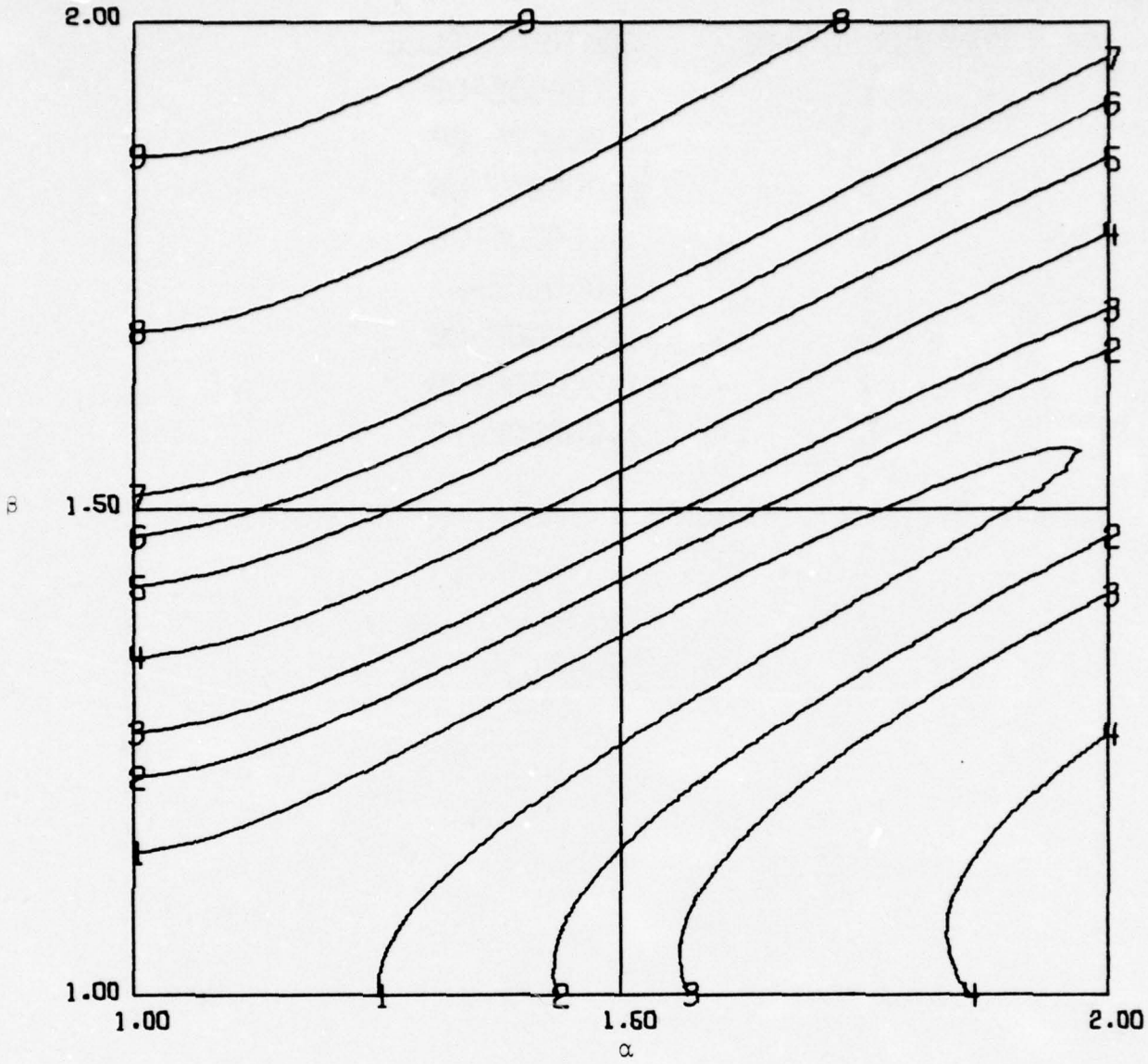


FIG. 3.2

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=2

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.3

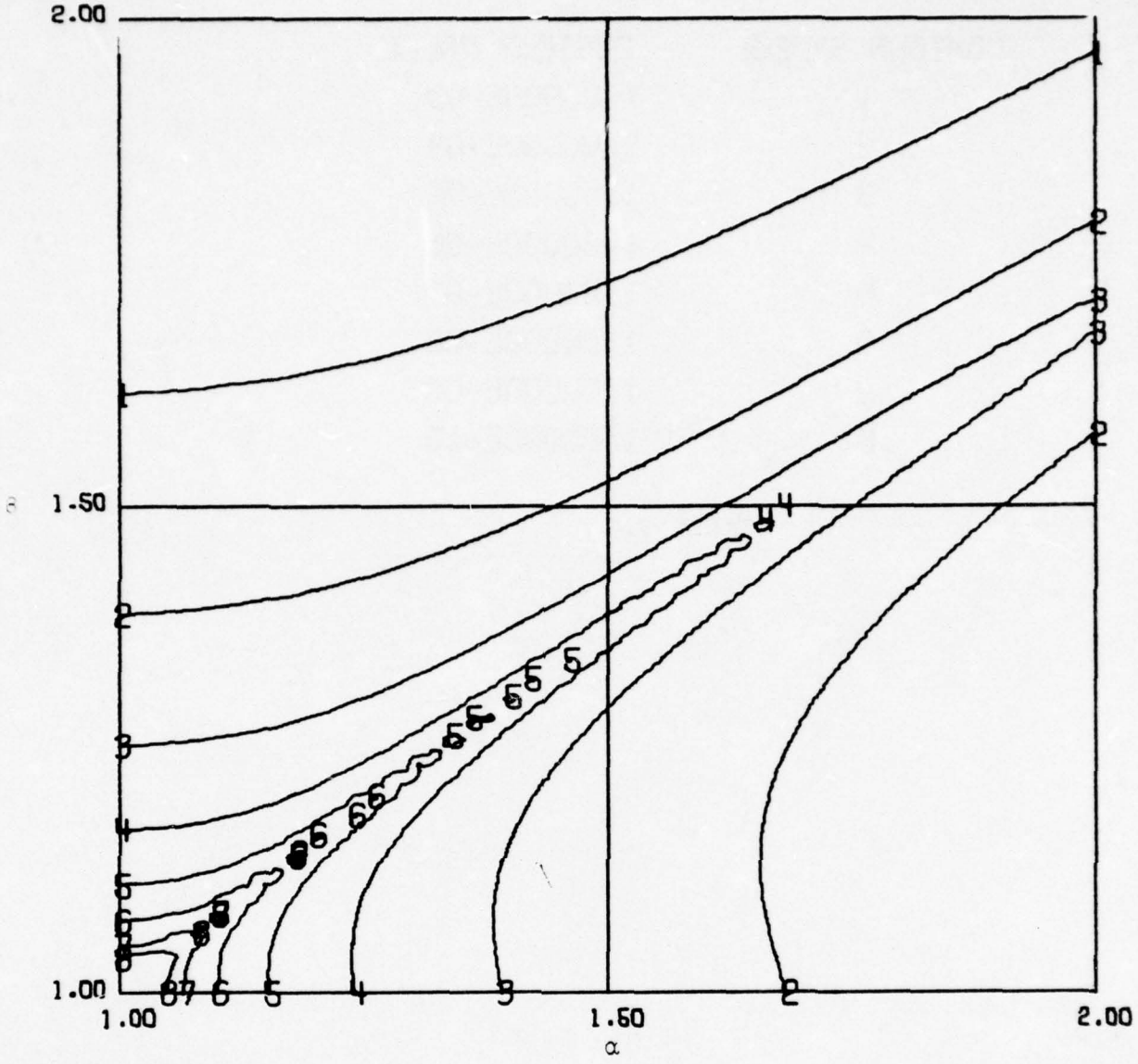


FIG. 3.3

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=3

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.4

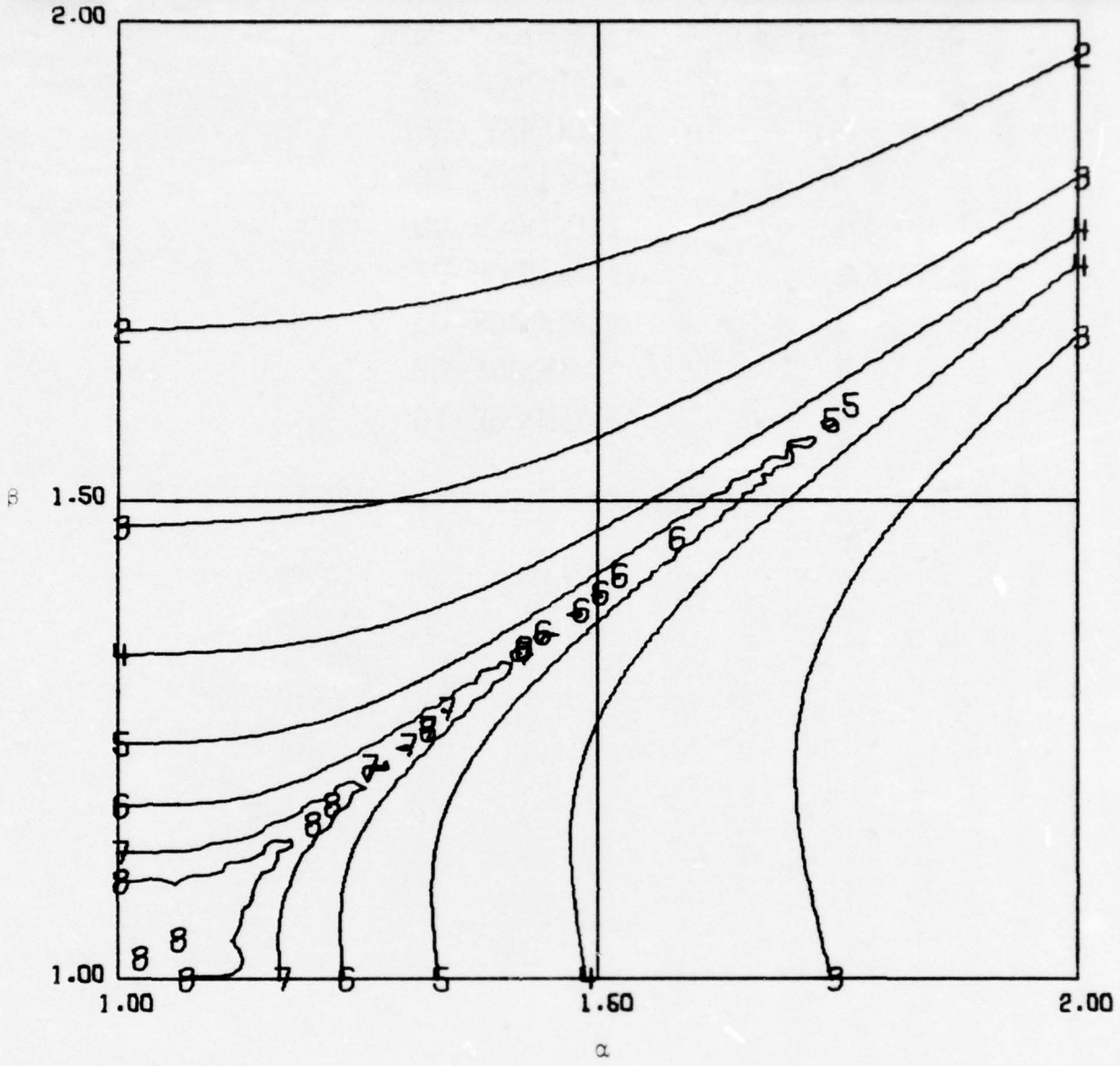


FIG. 3.4
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0. B=2.0. AND N=4

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.5

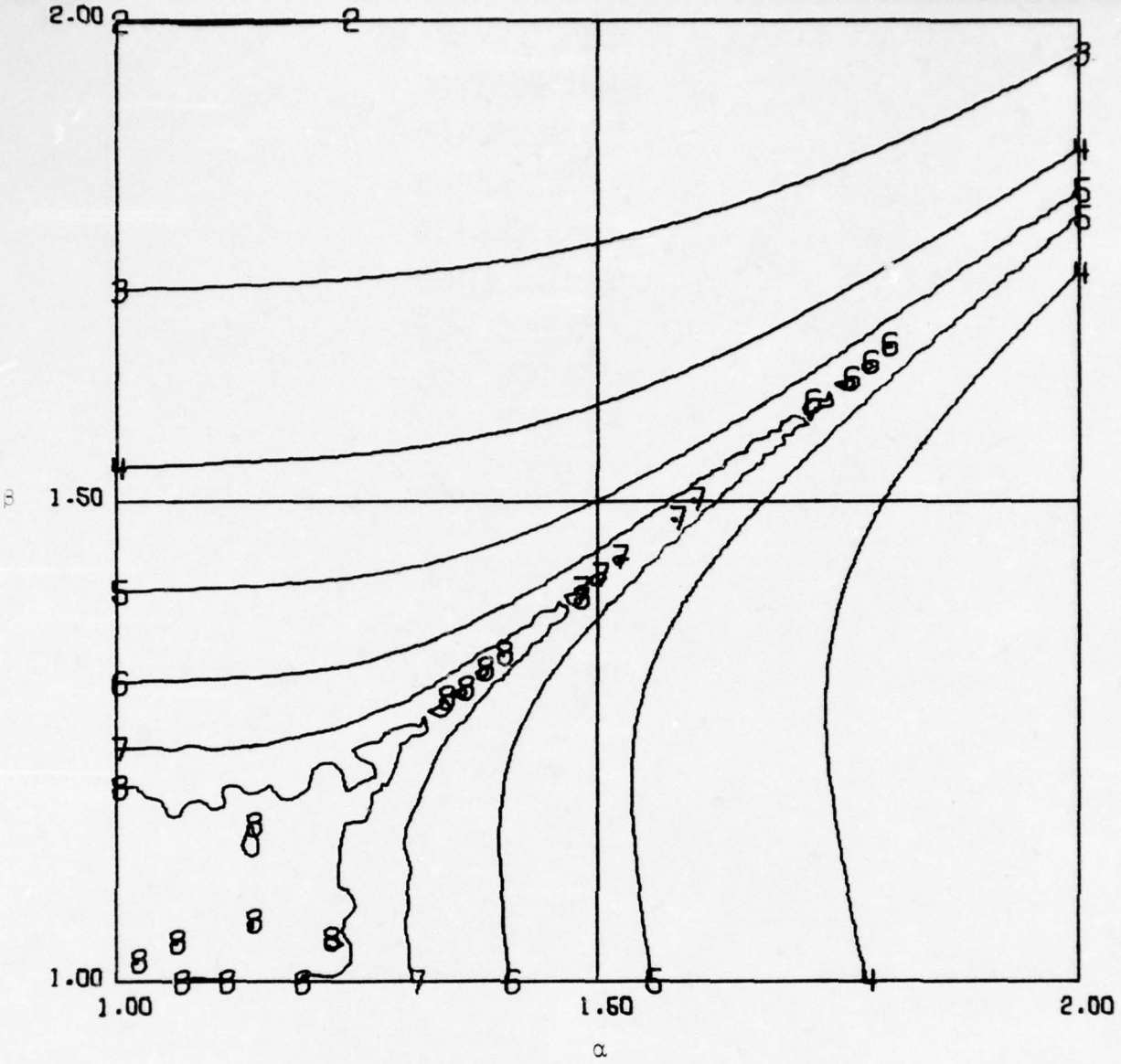


FIG. 3.5

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=5

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.6

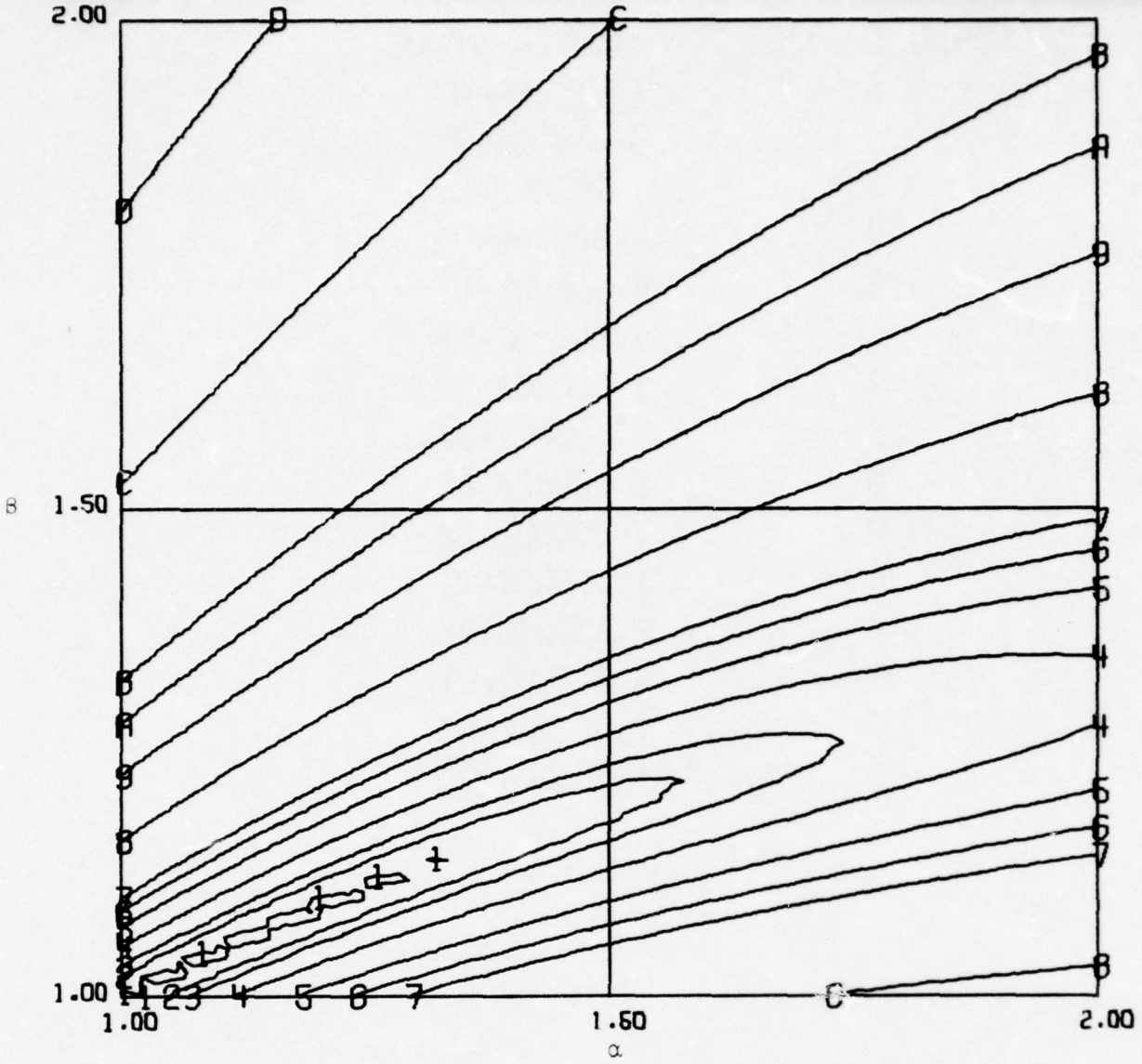


FIG. 3.6

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=1

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.7

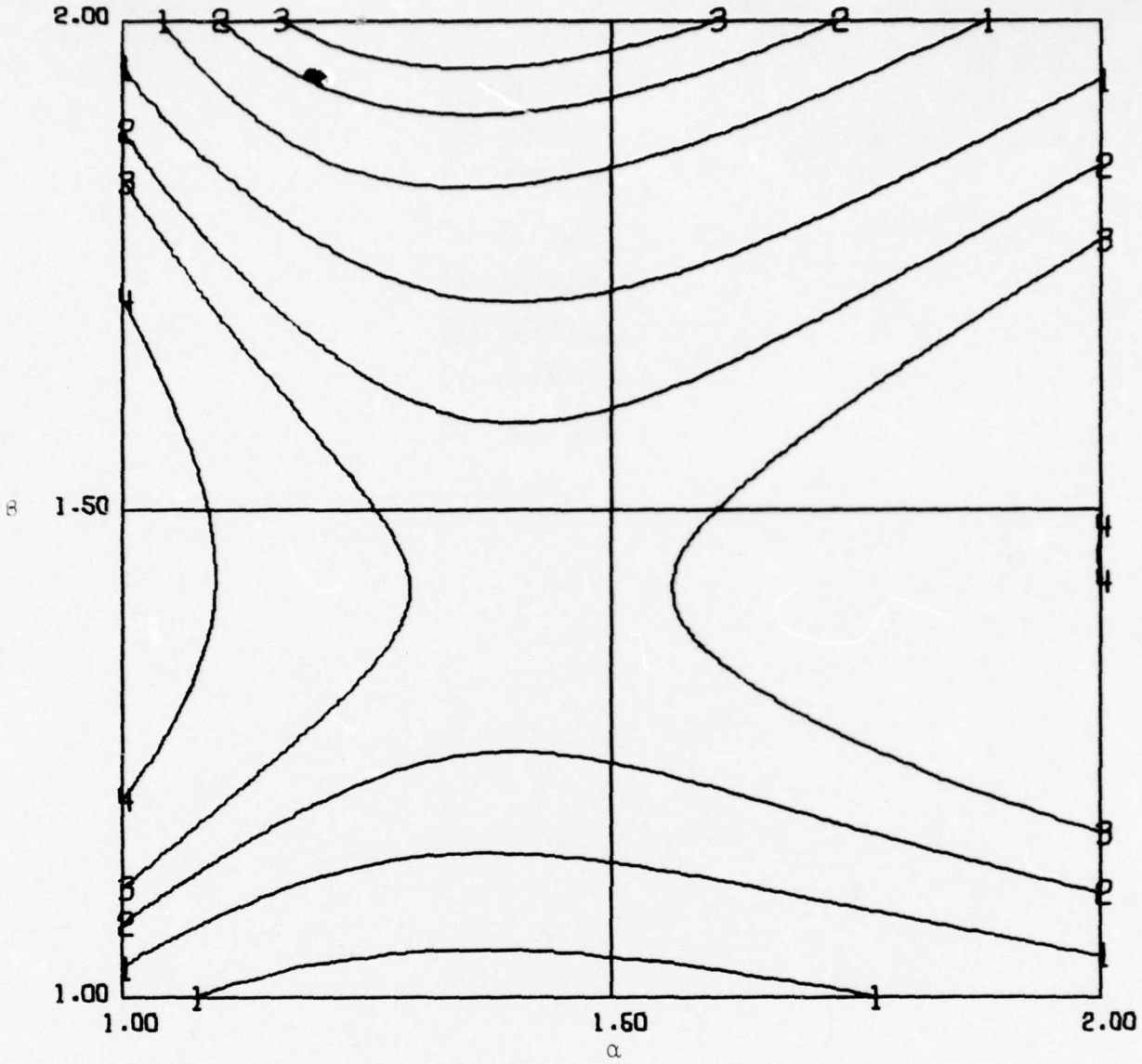


FIG. 3.7
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0. B=2.0. AND N=2

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.8

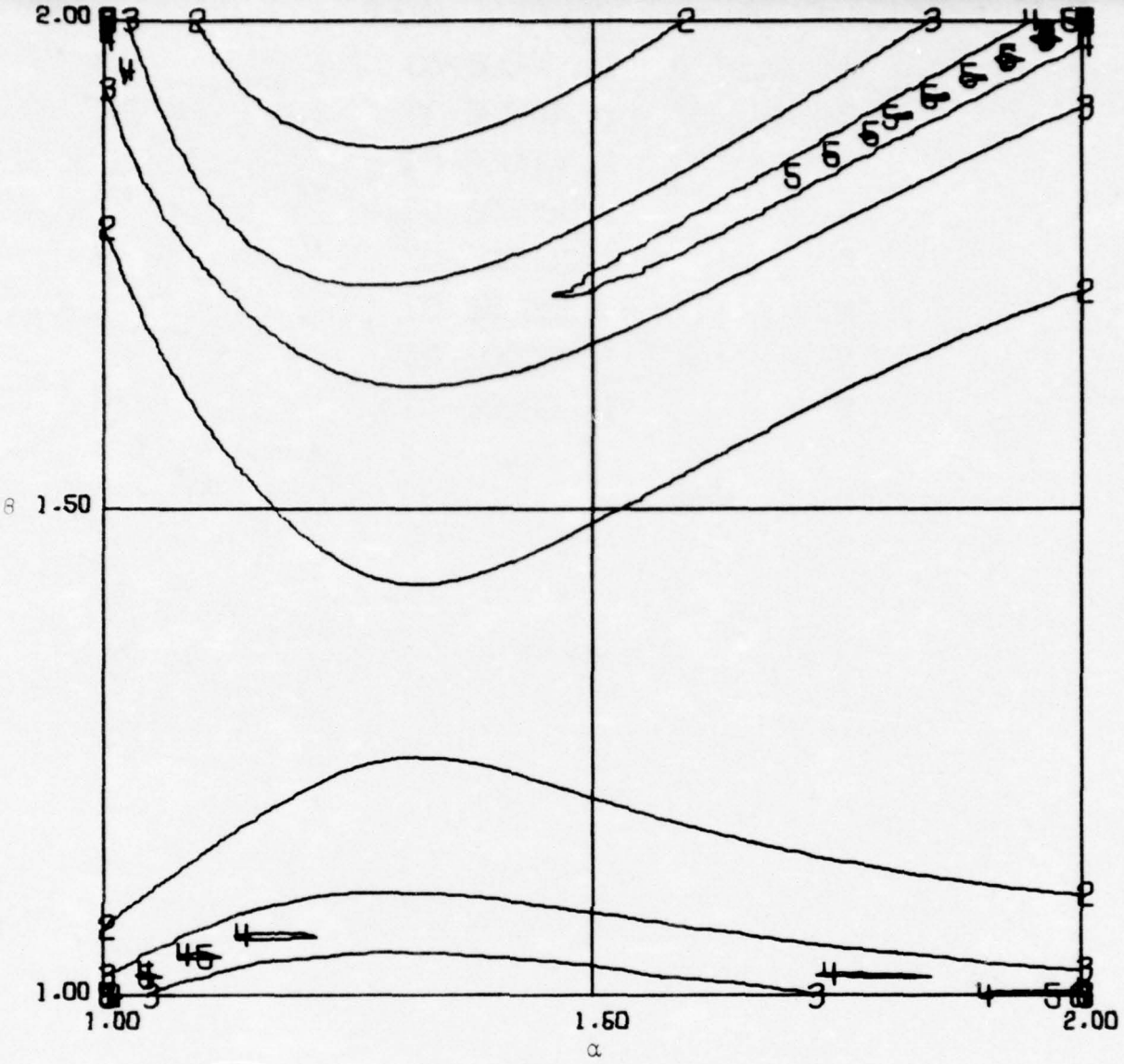


FIG. 3.8

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=3

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.9

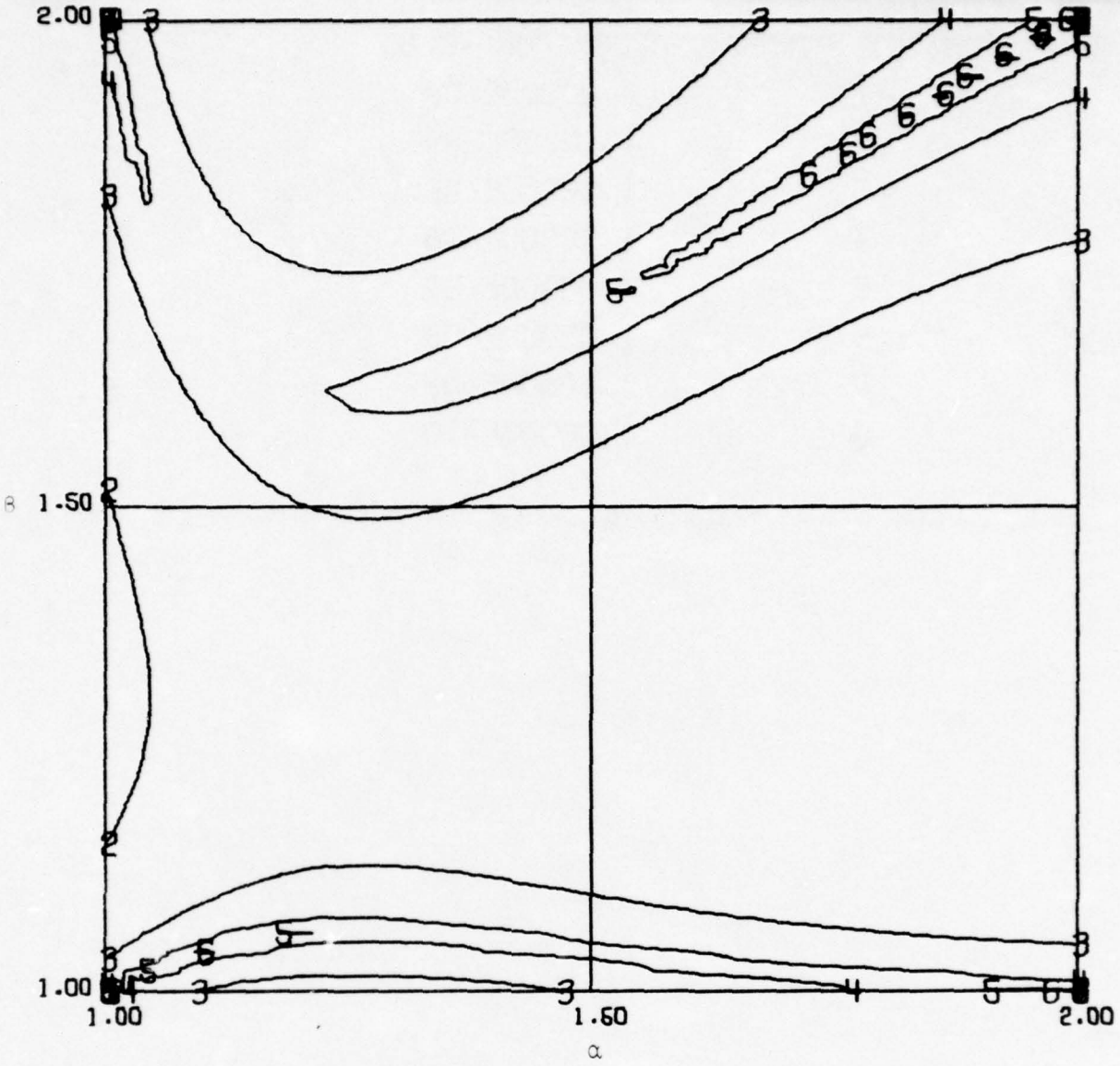


FIG. 3.9

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=4

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.10

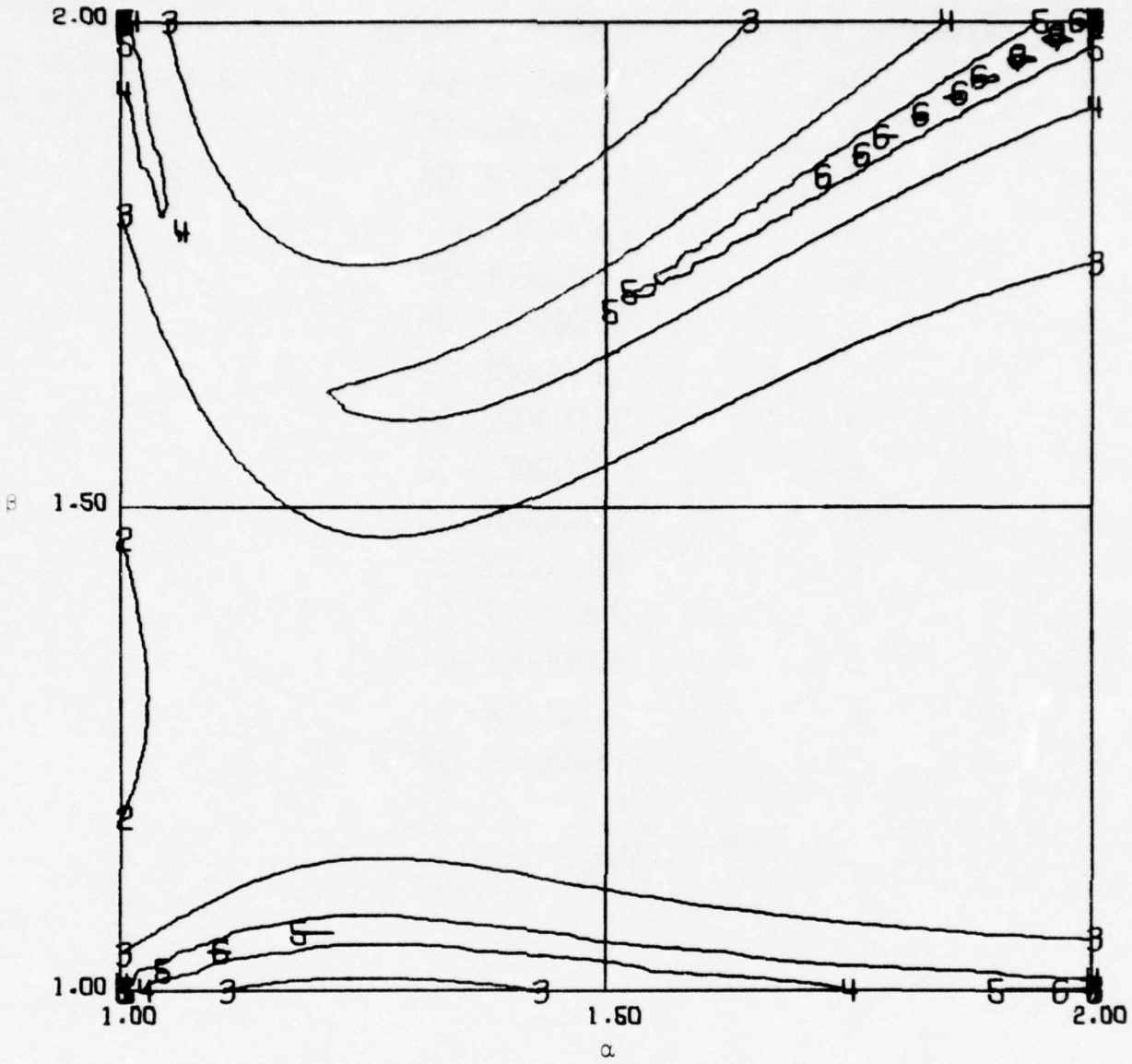


FIG. 3.10
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=1.0, B=2.0, AND N=5

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.11

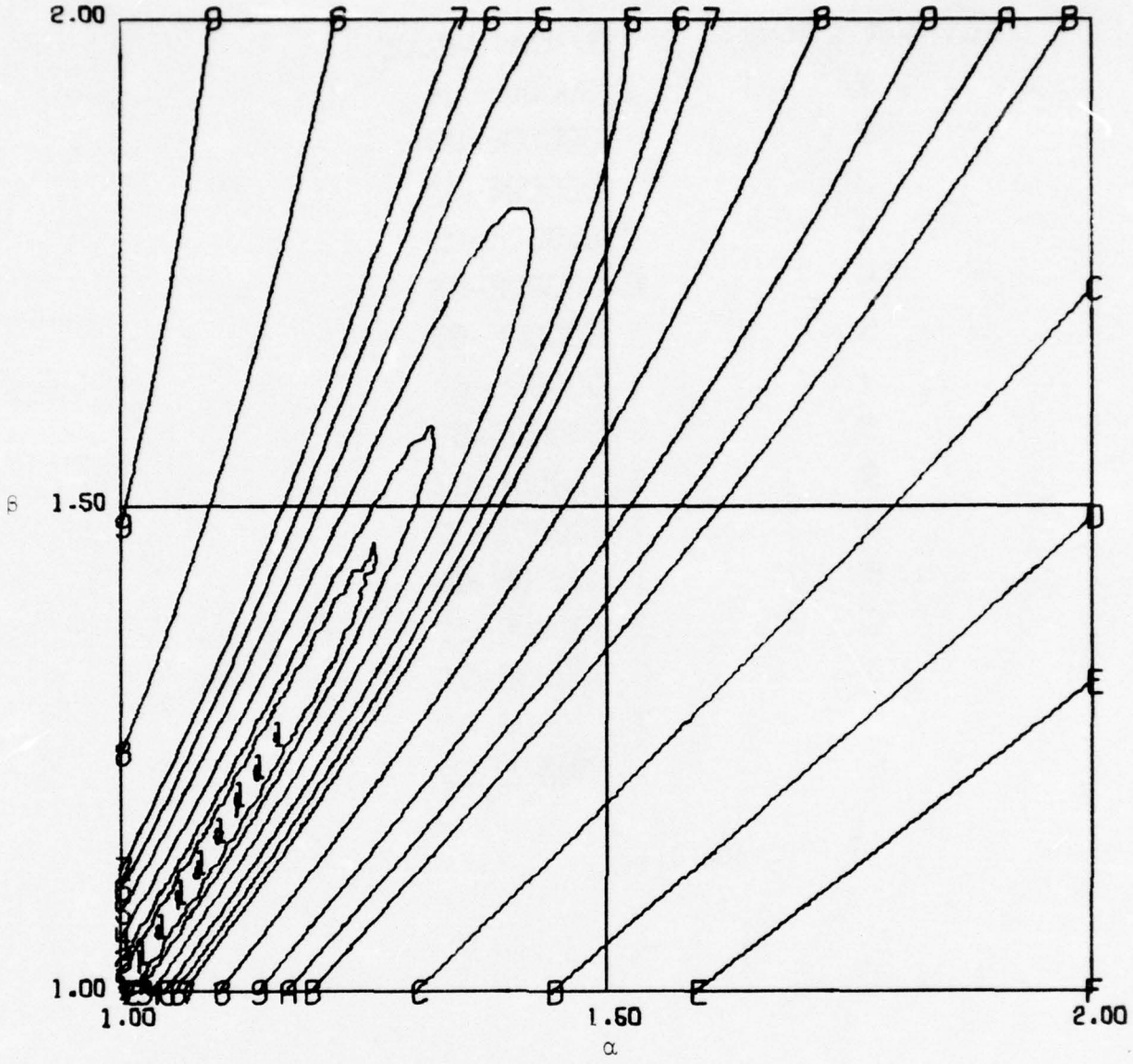


FIG. 3.11
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0. B=2.0. AND N=1

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.12

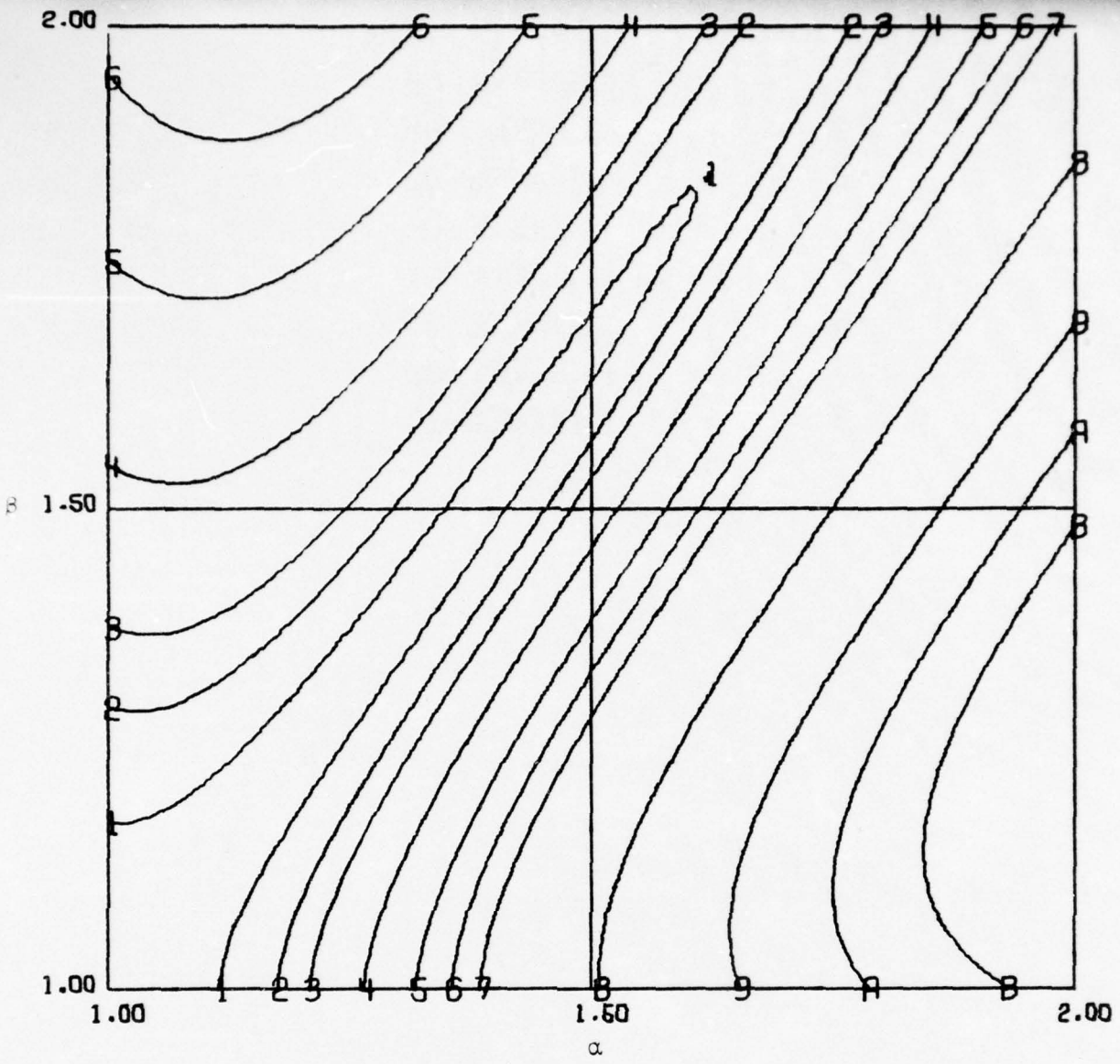


FIG. 3.12
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=2

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.13

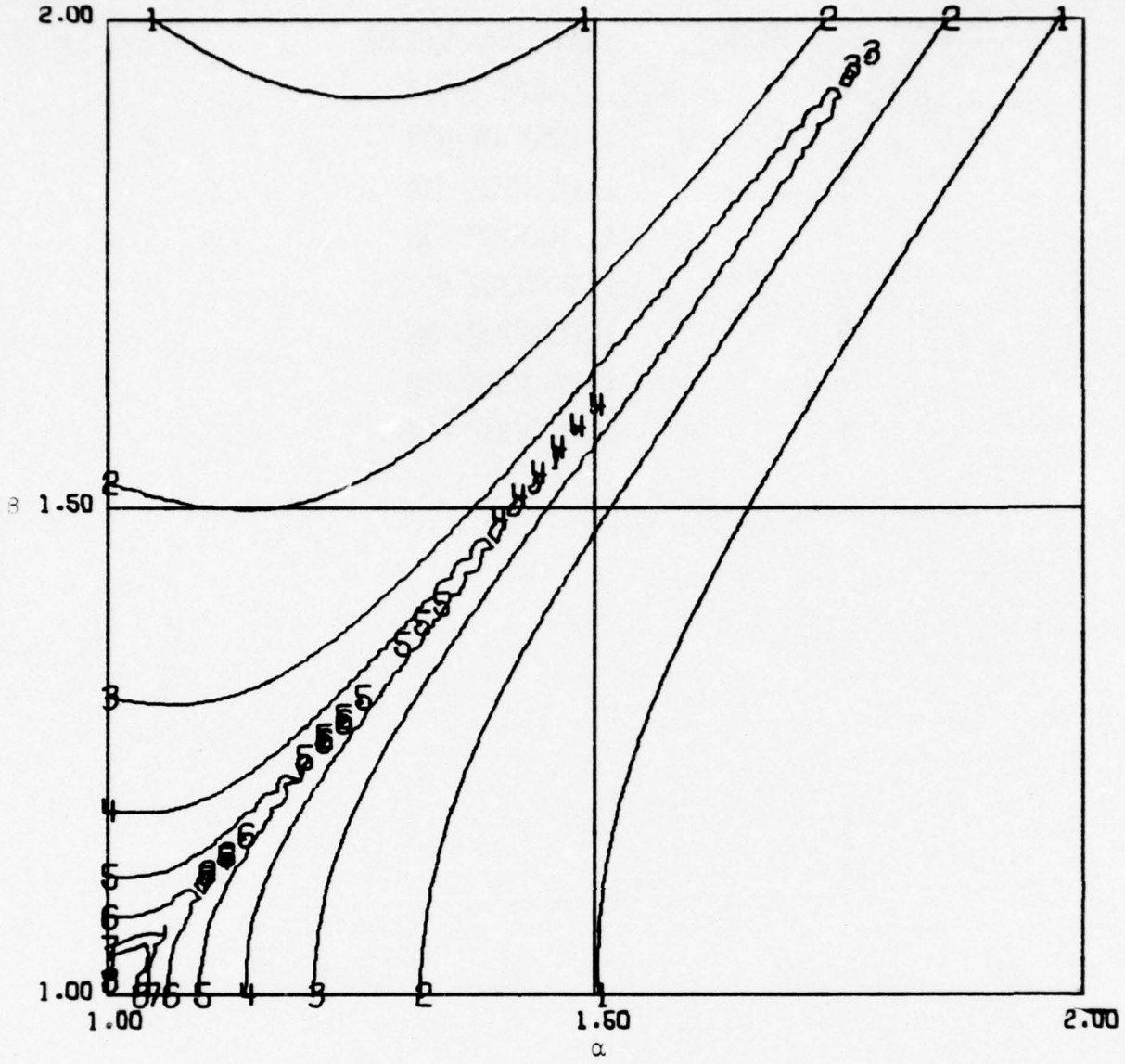


FIG. 3.13
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=3

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.14

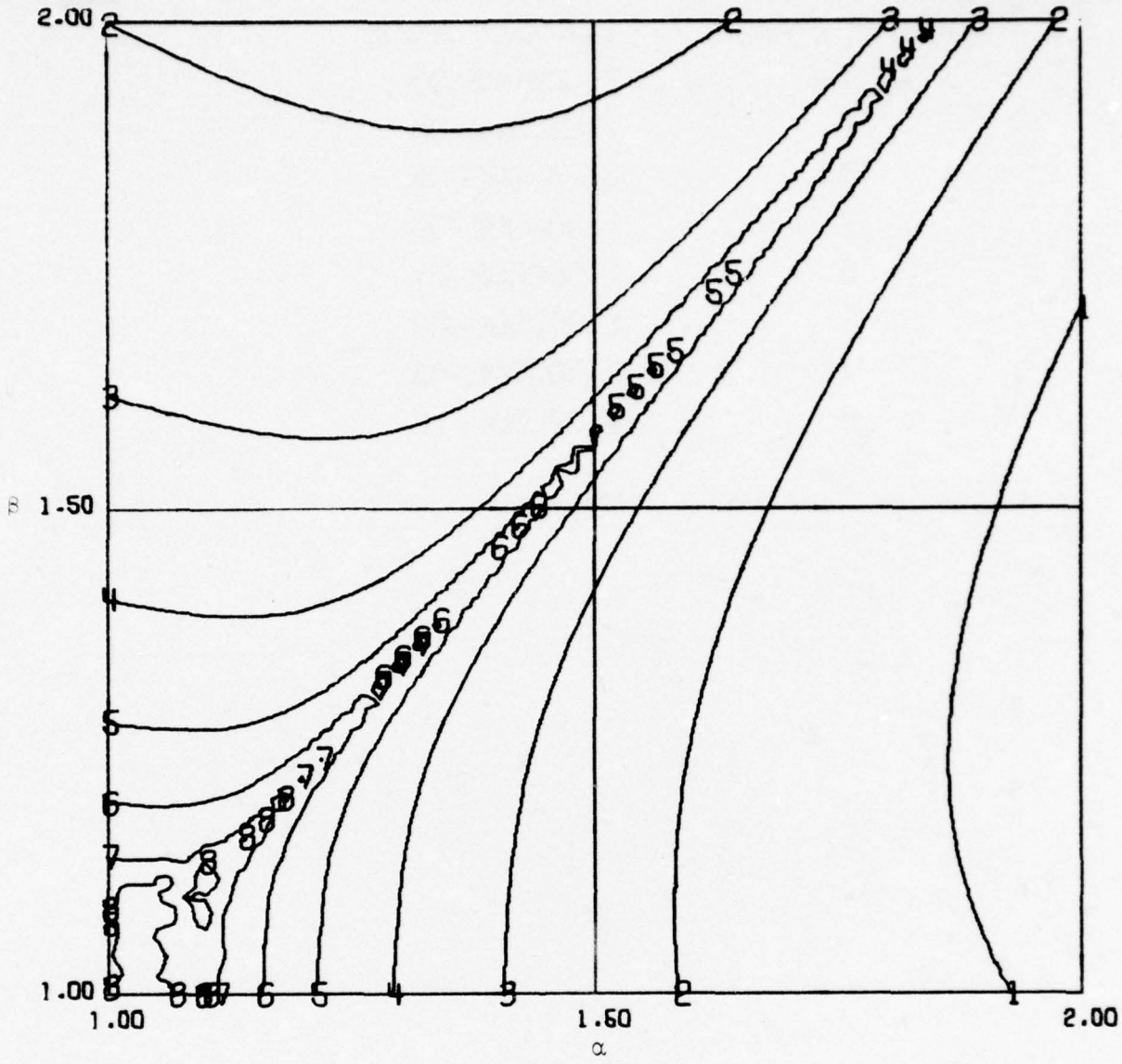


FIG. 3.14
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0. B=2.0. AND N=4

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.15

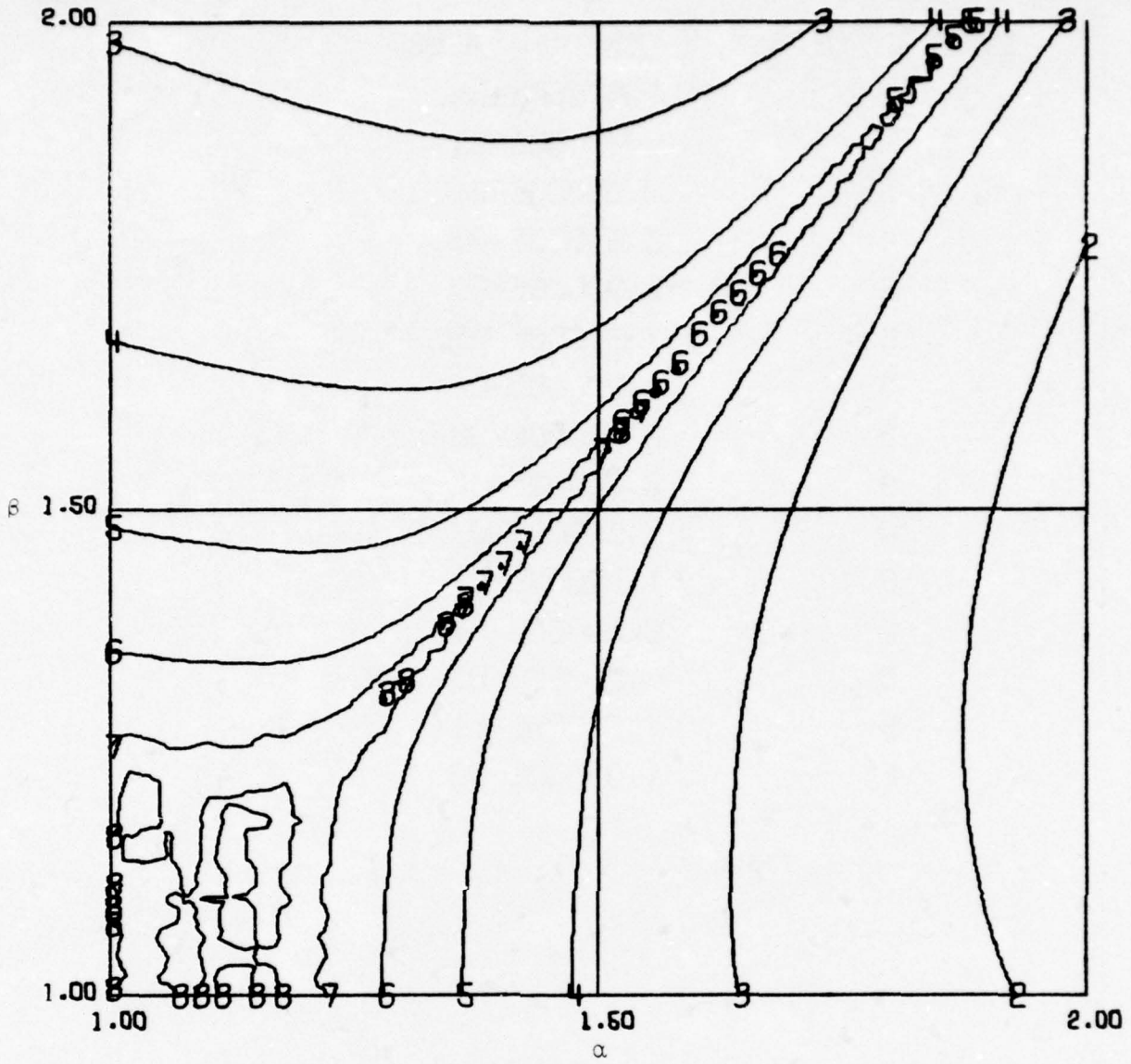


FIG. 3.15
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=5

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 3.16

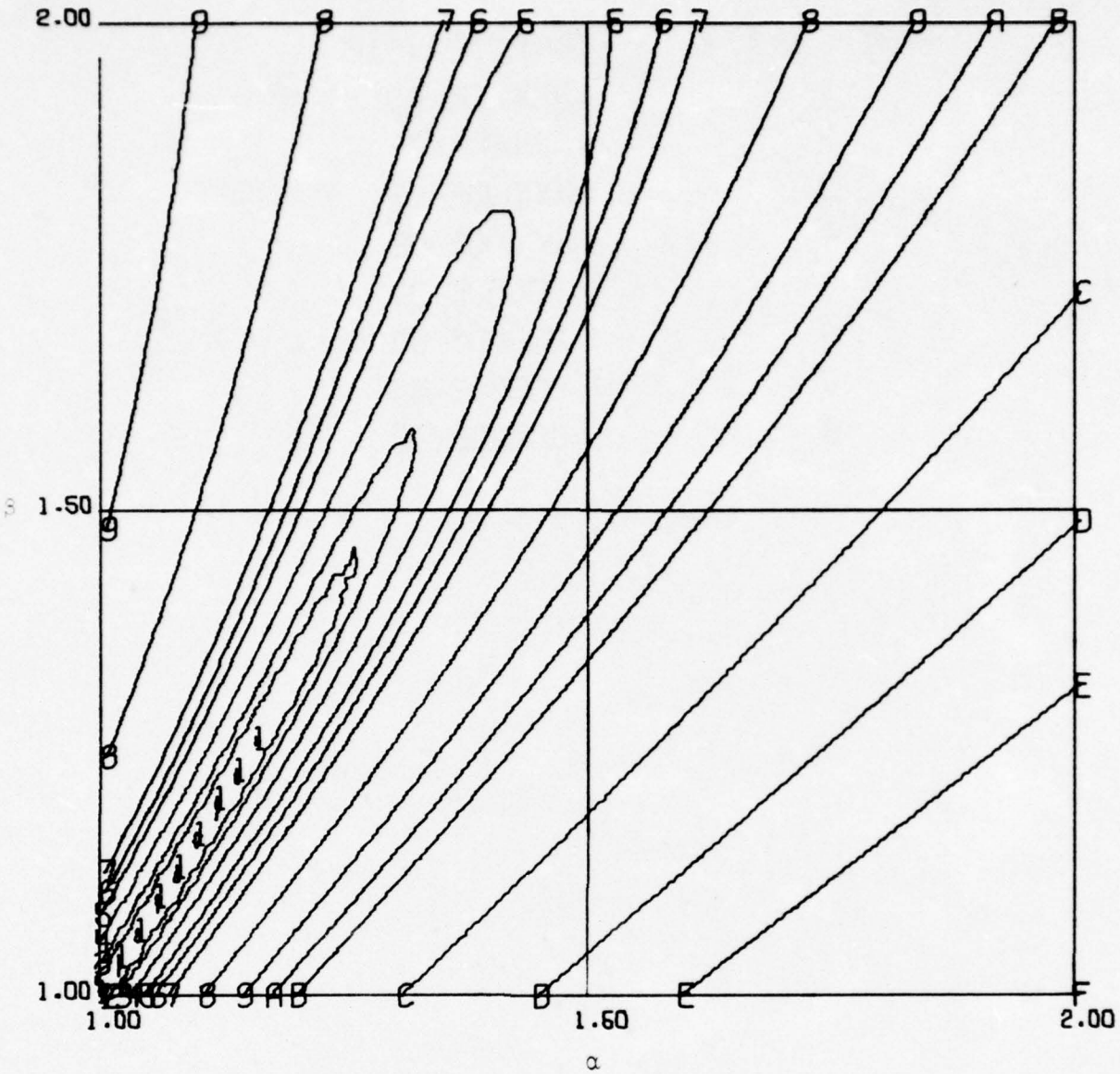


FIG. 3.16

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=1

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.17

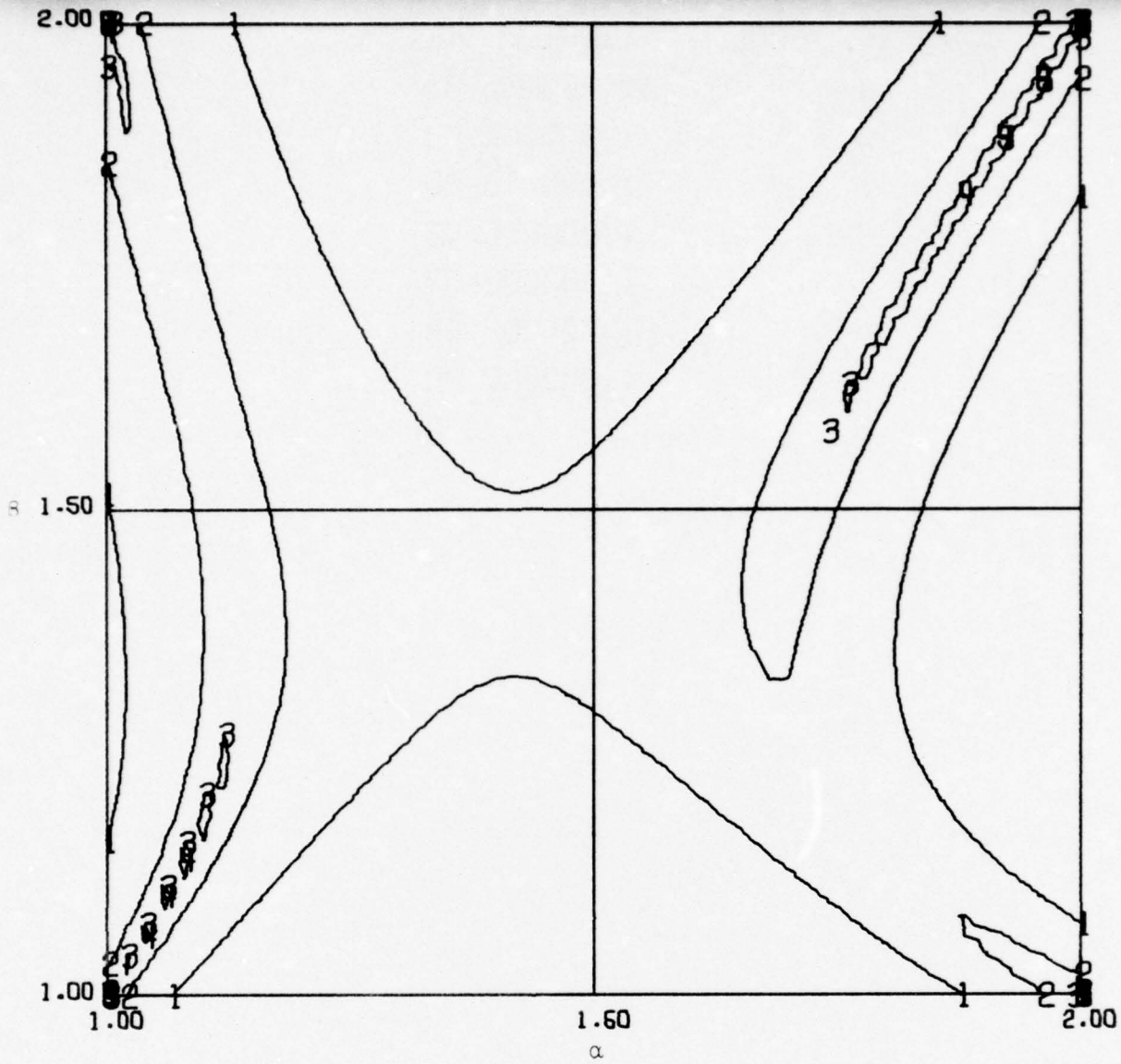


FIG. 3.17
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=2

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E 00
8	0

TABLE 3.18

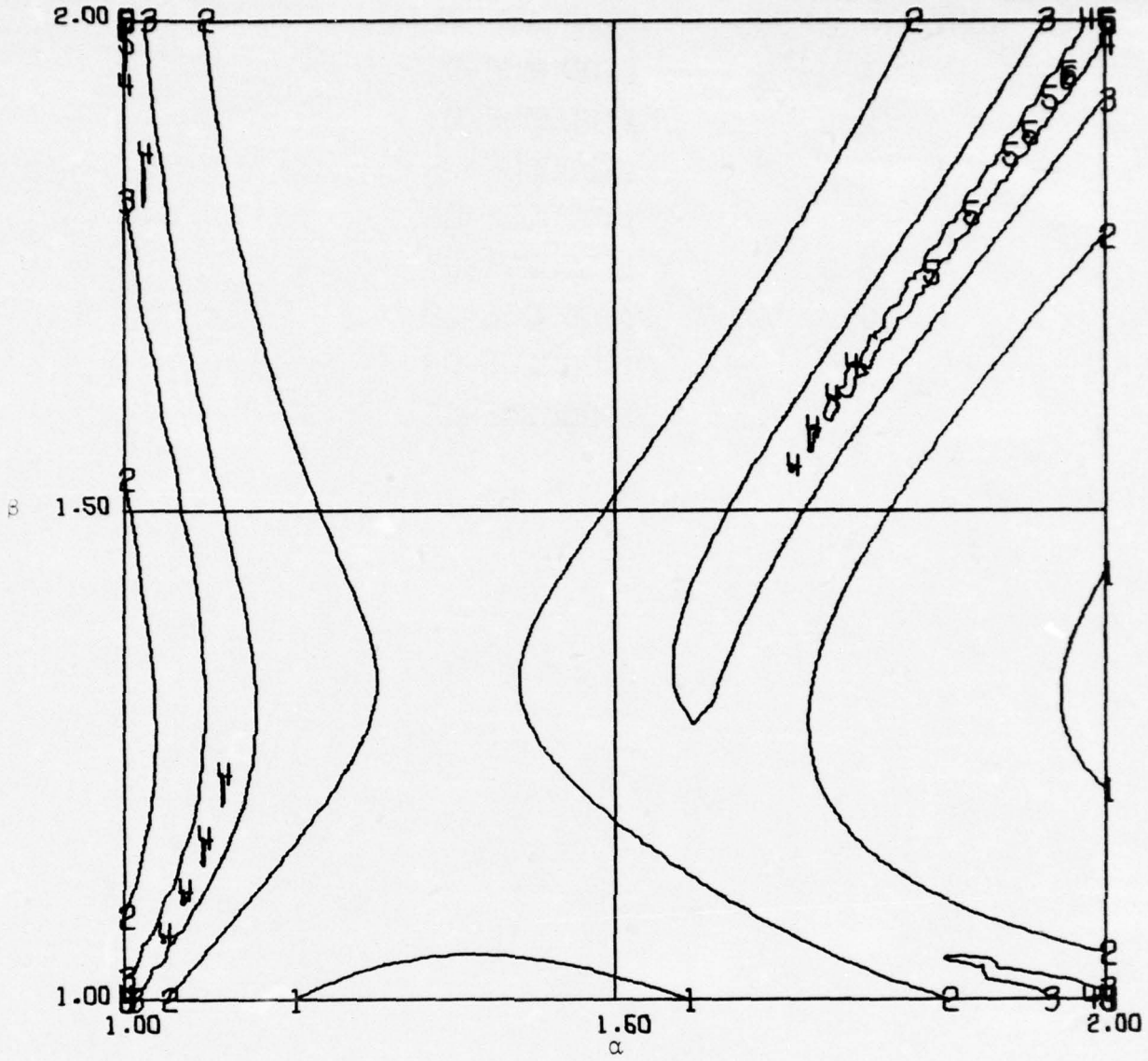


FIG. 3.18
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=3

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.19

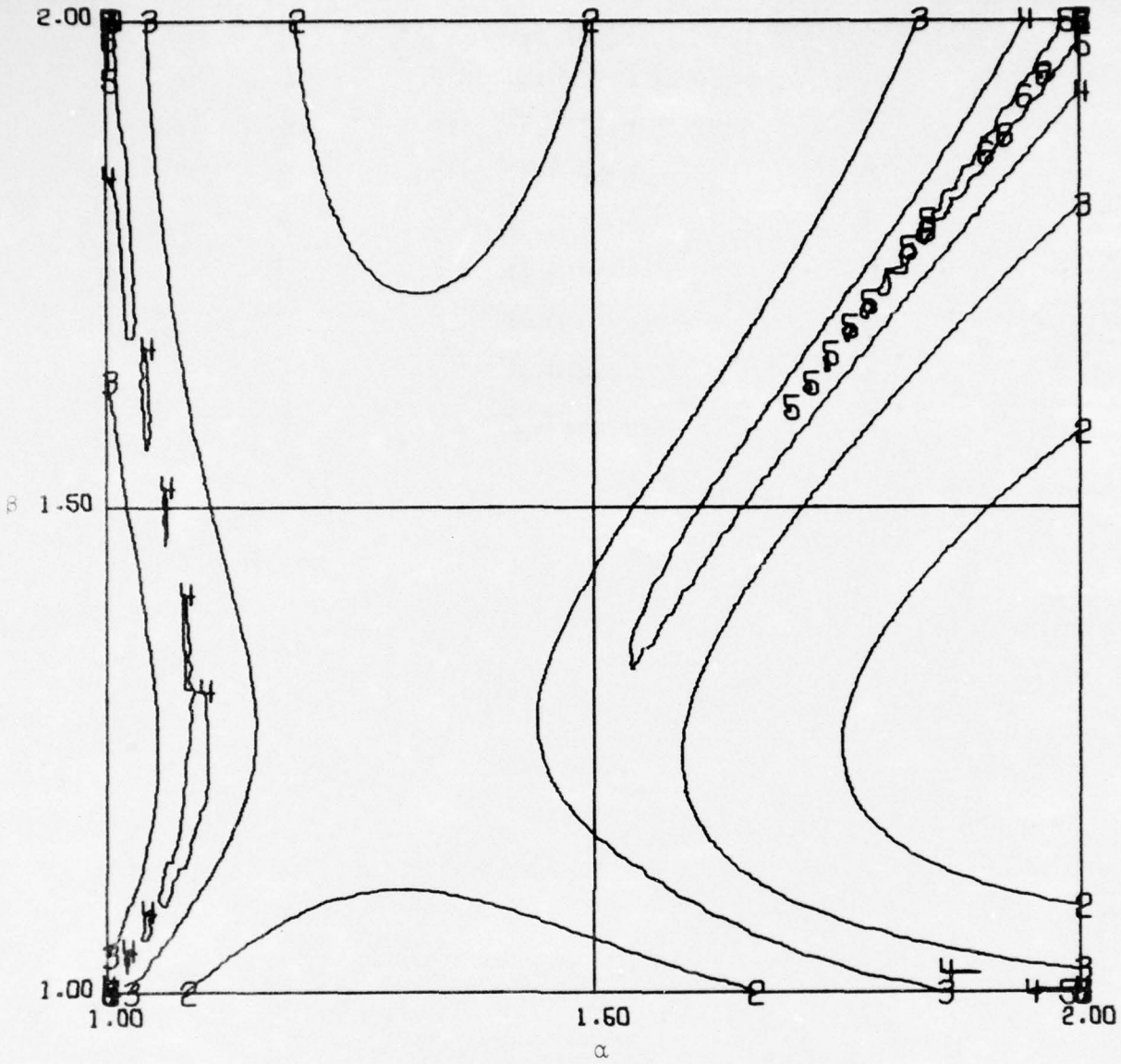


FIG. 3.19
CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0, B=2.0, AND N=4

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-03
2	1.00000E-04
3	1.00000E-05
4	1.00000E-06
5	1.00000E-07
6	1.00000E-08
7	1.00000E-09
8	1.00000E-10

TABLE 3.20

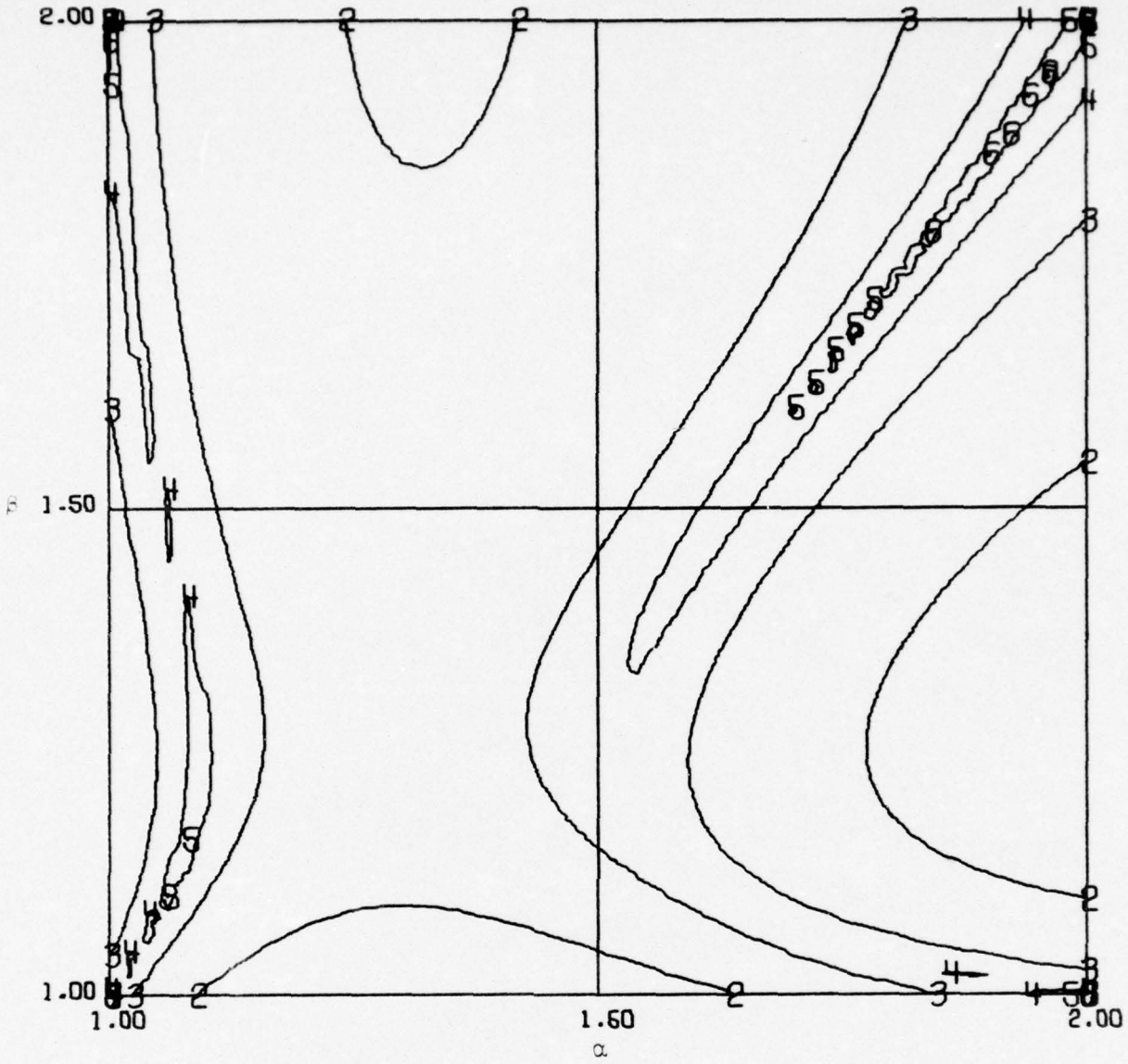


FIG. 3.20

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=A*EXP(-ALPHA*T)-B*EXP(-BETA*T)
A=3.0. B=2.0. AND N=5

TRACOR

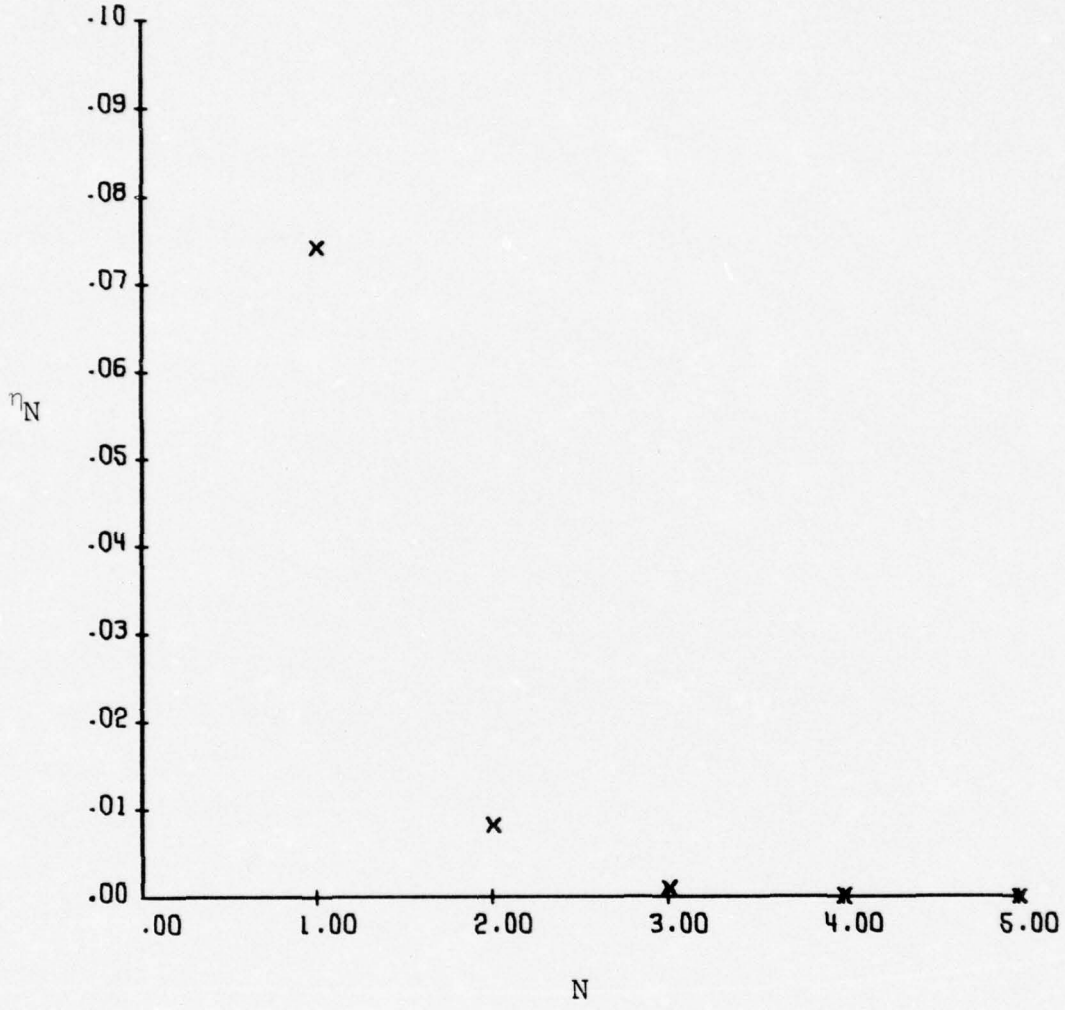


FIG. 3.21 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
F(T)=3*EXP(-T)-2*EXP(-2T)
LAGUERRE SERIES

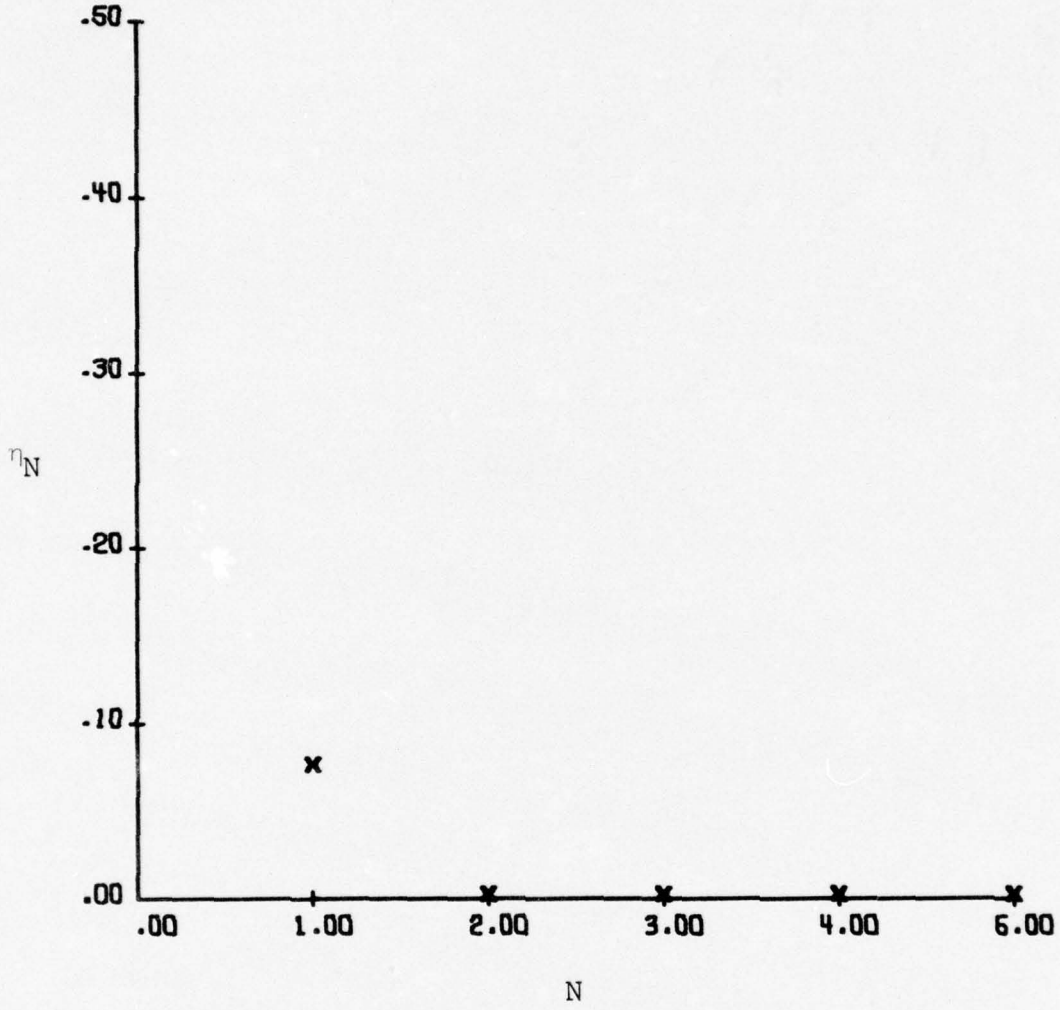


FIG. 3.22 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
F(T)=3*EXP(-T)-2*EXP(-2T)
ARBITRARY SERIES

4. The function under consideration in this case is

$$f(t) = t,\ 0 < t < A\ .$$

From the two pages of coefficients for this function one can see that η_N is a function of the parameter A. To show the affect of A on η ten plots of η_N vs A were developed: five for the Laguerre expansion and five for the arbitrary expansion. The horizontal axis of each plot is the A-axis and the vertical axis is the η_N-axis. Following the ten error plots are eight plots of f(t) and the five approximations for four values of A. In each of these plots the horizontal axis is the t-axis. Note that the curves for a given plot approximate the function for $0 < t \leq A$ where A is specified on the plot; therefore for values of t greater than A the approximations have no meaning.

4.1 $f(t) = t$ $\quad 0 < t < A$ Laguerre series

$$E_i = 1/2A^3$$

$$C_1 = -\sqrt{2}\left[e^{-A}\ (A+1) - 1\right]$$

$$C_2 = -\sqrt{2}\left[e^{-A}(2A^2 + 3A + 3) - 3\right]$$

$$C_3 = -\sqrt{2}\left[e^{-A}(2A^3 + 2A^2 + 5A + 5) - 5\right]$$

$$C_4 = -\sqrt{2}\left[e^{-A}(4/3A^4 - 2/3A^3 + 4A^2 + 7A + 7) - 7\right]$$

$$C_5 = -\sqrt{2}\left[e^{-A}(2/3A^5 - 2A^4 + 4A^3 + 4A^2 + 9A + 9) - 9\right]$$

4.2 $f(t) = t \qquad 0 < t < A$ Arbitrary series

$$E_i = 1/2A^3$$

$$C_1 = -\sqrt{2}\left[e^{-A}(A+1) - 1\right]$$

$$C_2 = 5/2 - 4(A+1)e^{-A} + 3/2(2A+1)e^{-2A}$$

$$C_3 = -\sqrt{6}\left[-\frac{10}{9} + 3(A+1)e^{-A} - 3(2A+1)e^{-2A} + \frac{10}{9}(3A+1)e^{-3A}\right]$$

$$C_4 = 2\sqrt{2}\left[\frac{47}{48} - 4(A+1)e^{-A} + \frac{15}{2}(2A+1)e^{-2A} - \frac{20}{3}(3A+1)e^{-3A} + \frac{35}{15}(4A+1)e^{-4A}\right]$$

$$C_5 = \frac{\sqrt{10}}{2}\left\{\frac{393}{225} - 10e^{-A}(A+1) + 30e^{-2A}(2A+1) - \frac{420}{9}e^{-3A}(3A+1) + 35e^{-4A}(4A+1) - \frac{252}{25}e^{-5A}(5A+1)\right\}$$

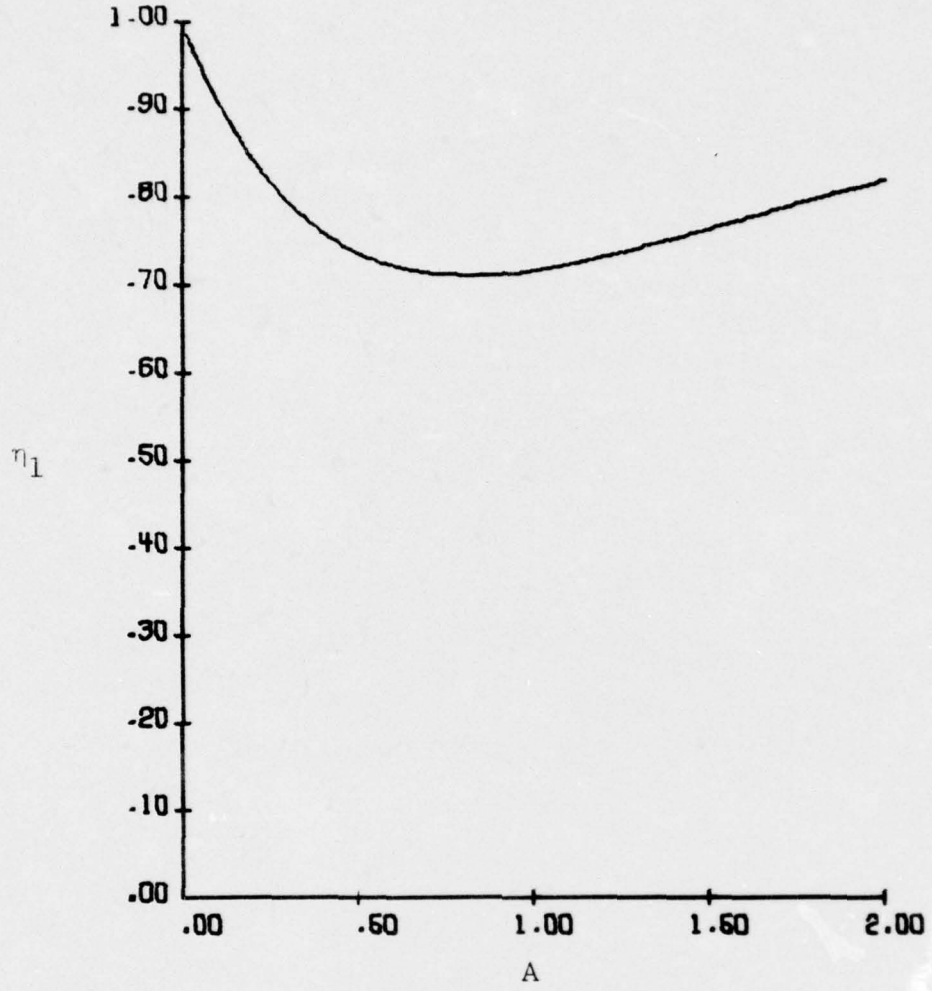


FIG. 4.1 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=T
LAGUERRE SERIES

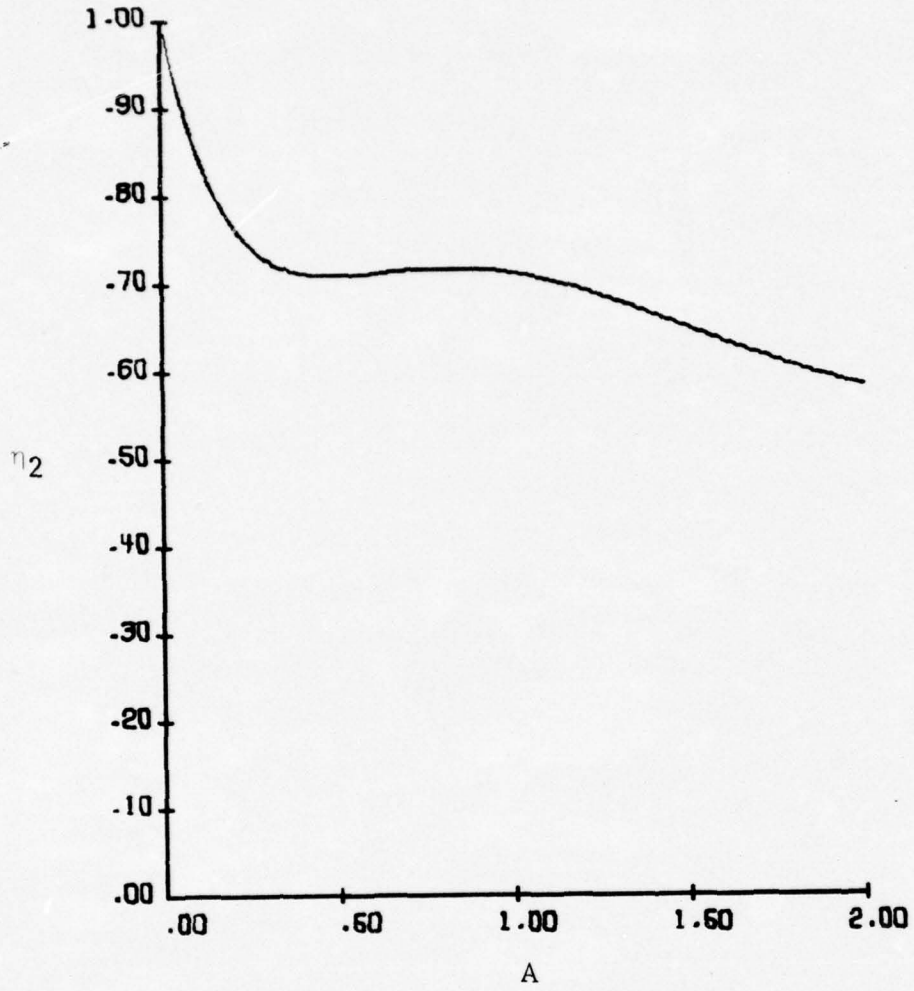


FIG. 4.2 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=T
LAGUERRE SERIES

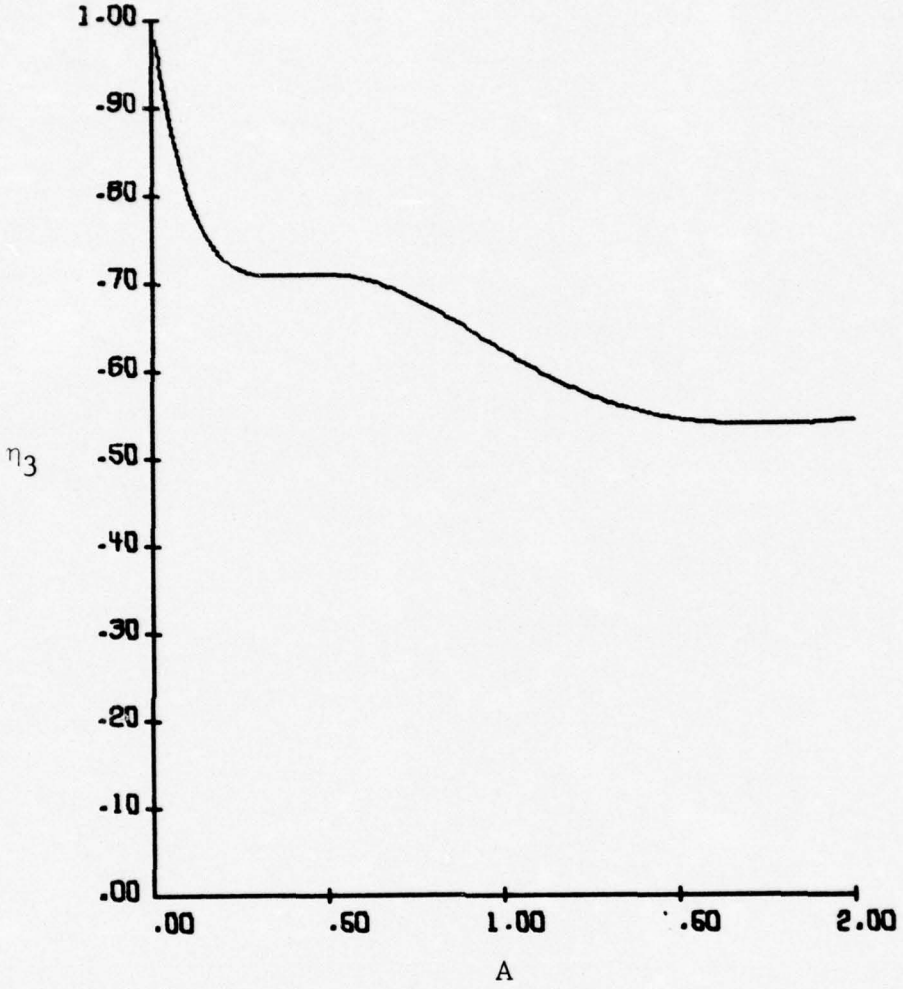


FIG. 4.3 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=T
LAGUERRE SERIES

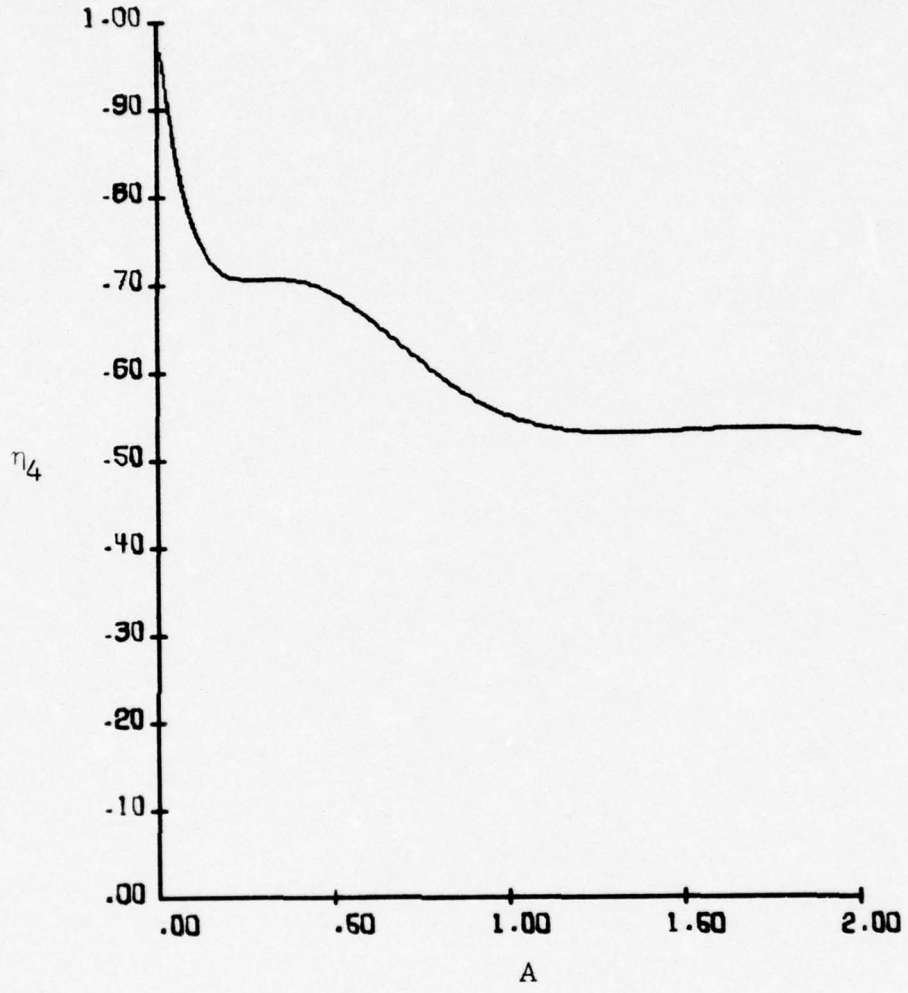


FIG. 4.4 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=T
LAGUERRE SERIES

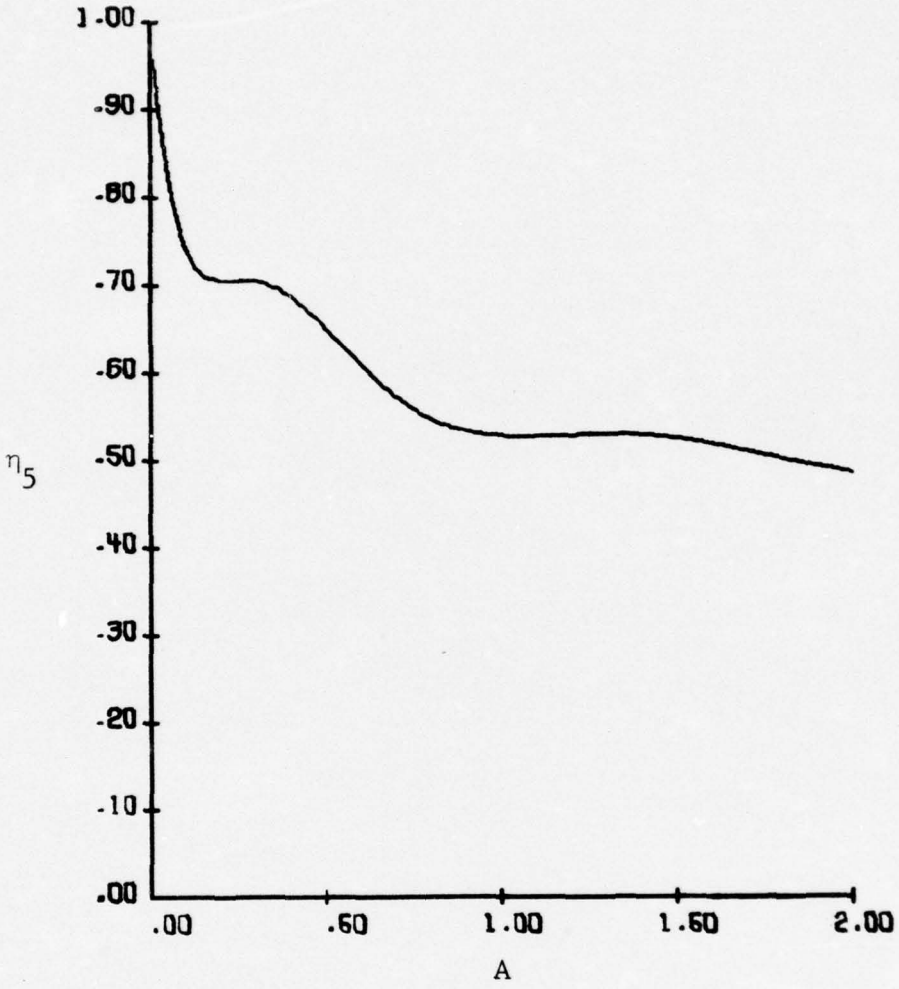


FIG. 4.5 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=T
LAGUERRE SERIES

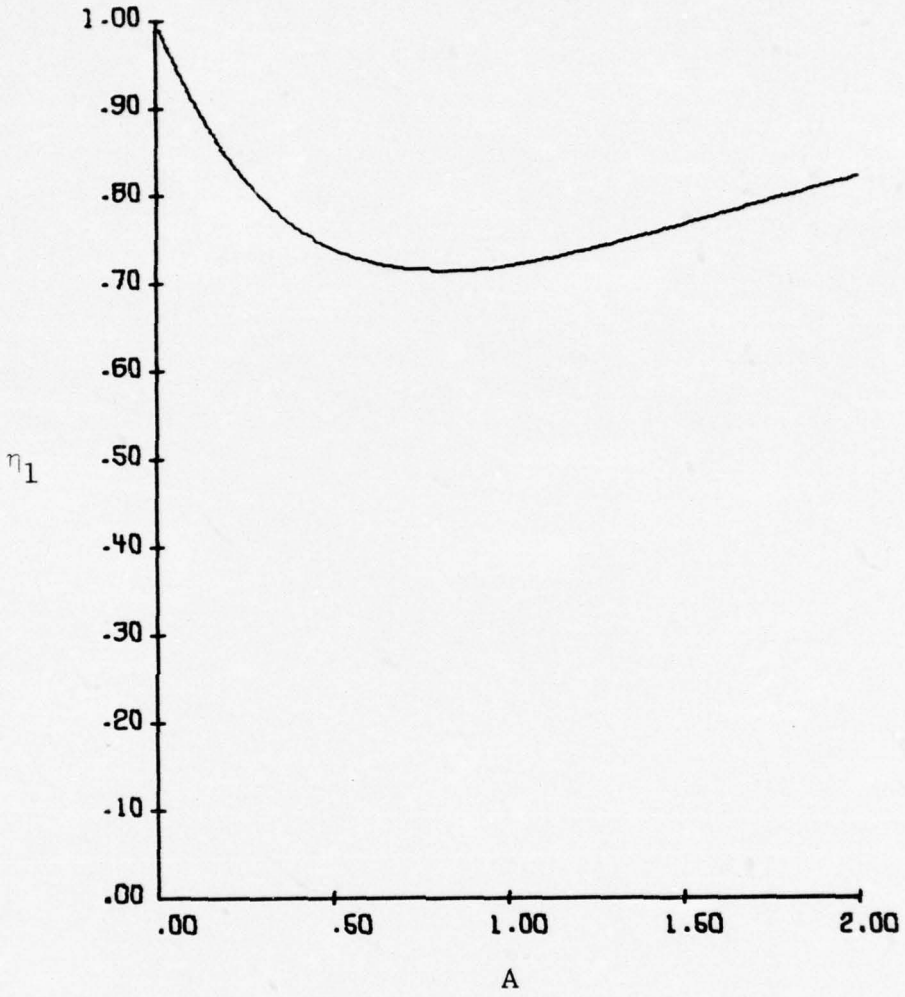


FIG. 4.6 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T) = T
ARBTIRARY SERIES

TRACOR

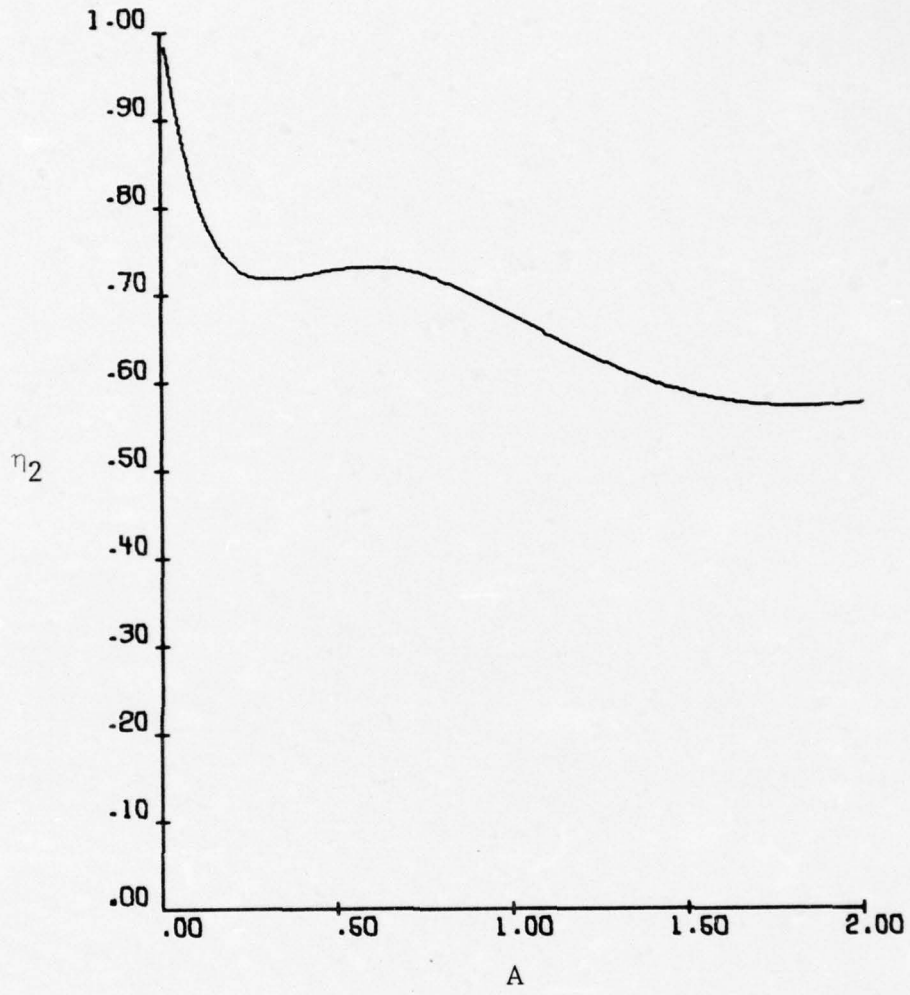


FIG. 4.7 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T) = T
ARBTIRARY SERIES

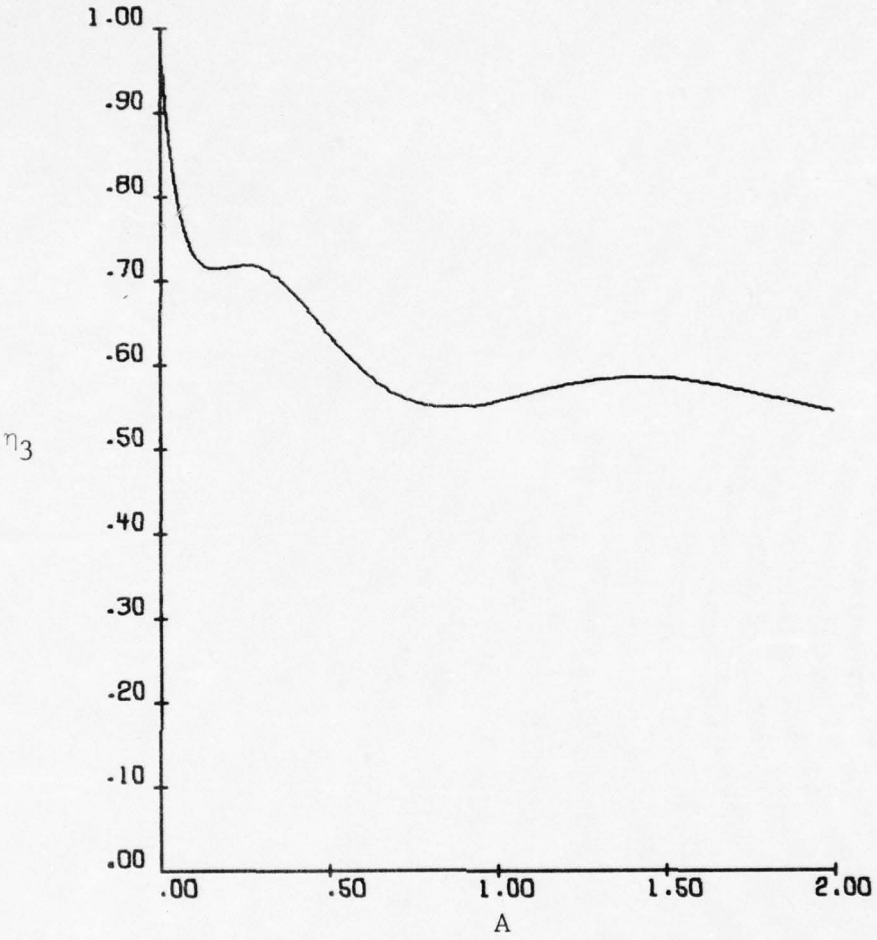


FIG. 4.8 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T) = T
ARBTIRARY SERIES

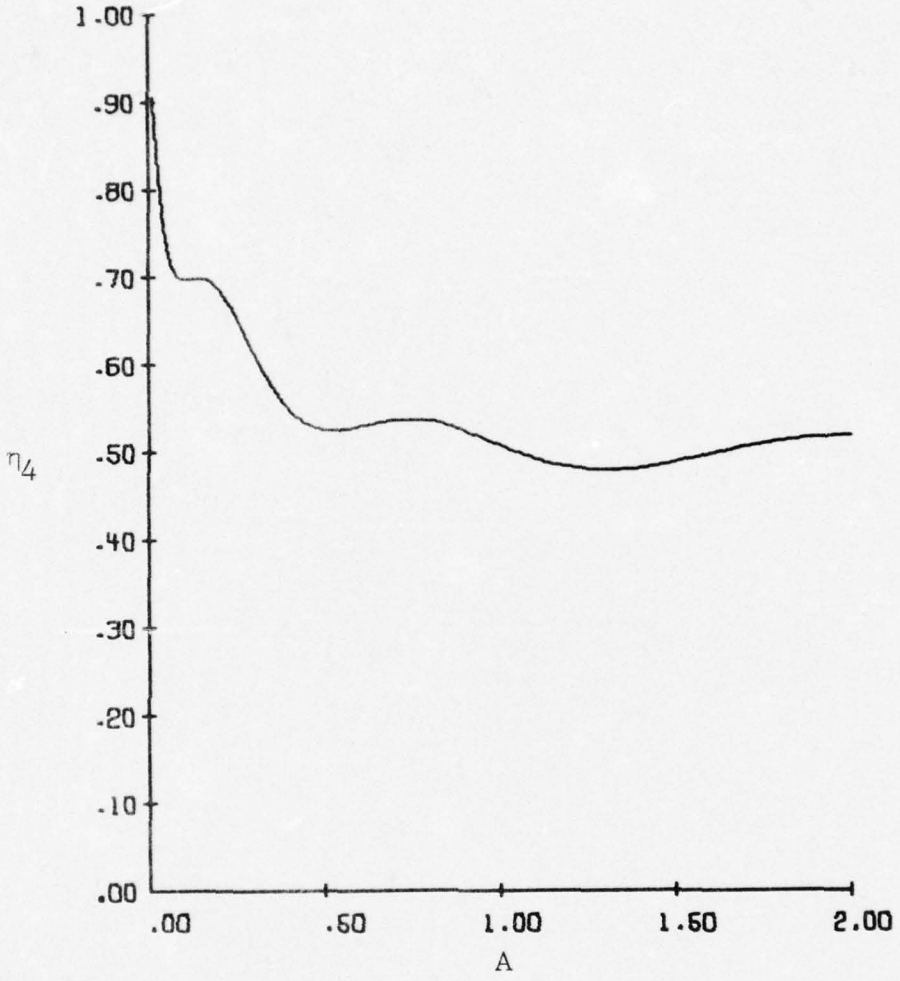


FIG. 4.9 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T) = T
ARBTIRARY SERIES

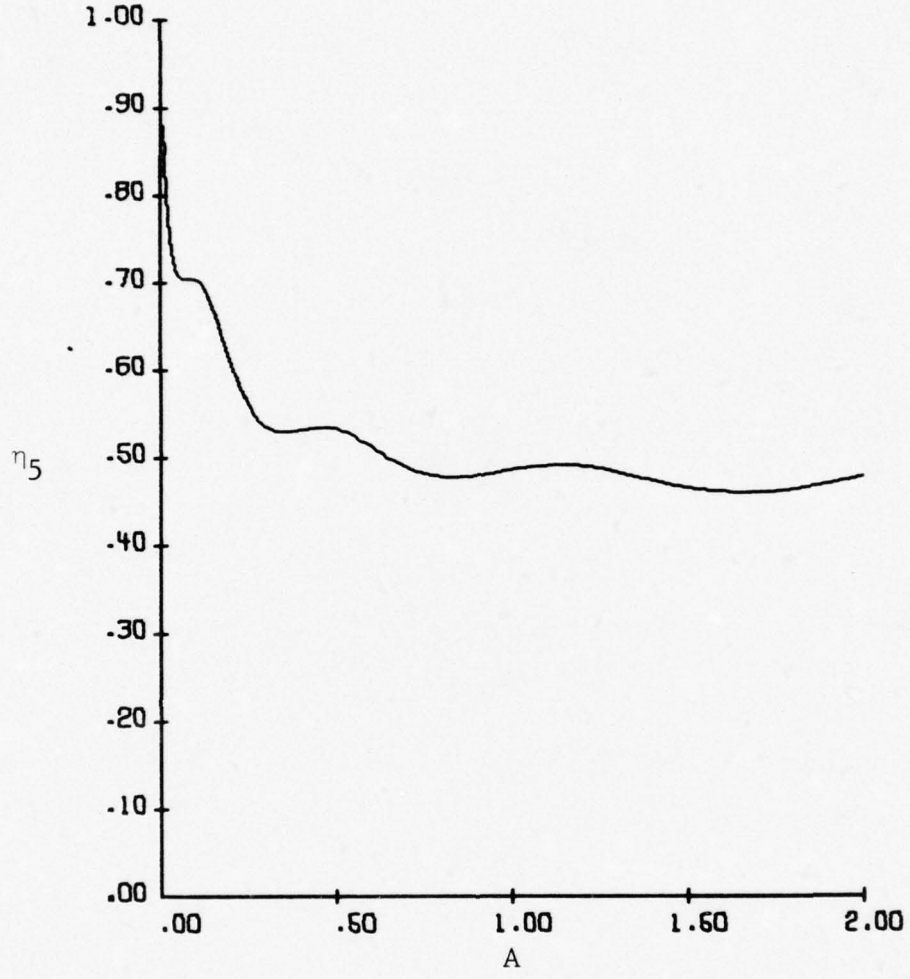


FIG. 4.10 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T) = T
ARBTIRARY SERIES

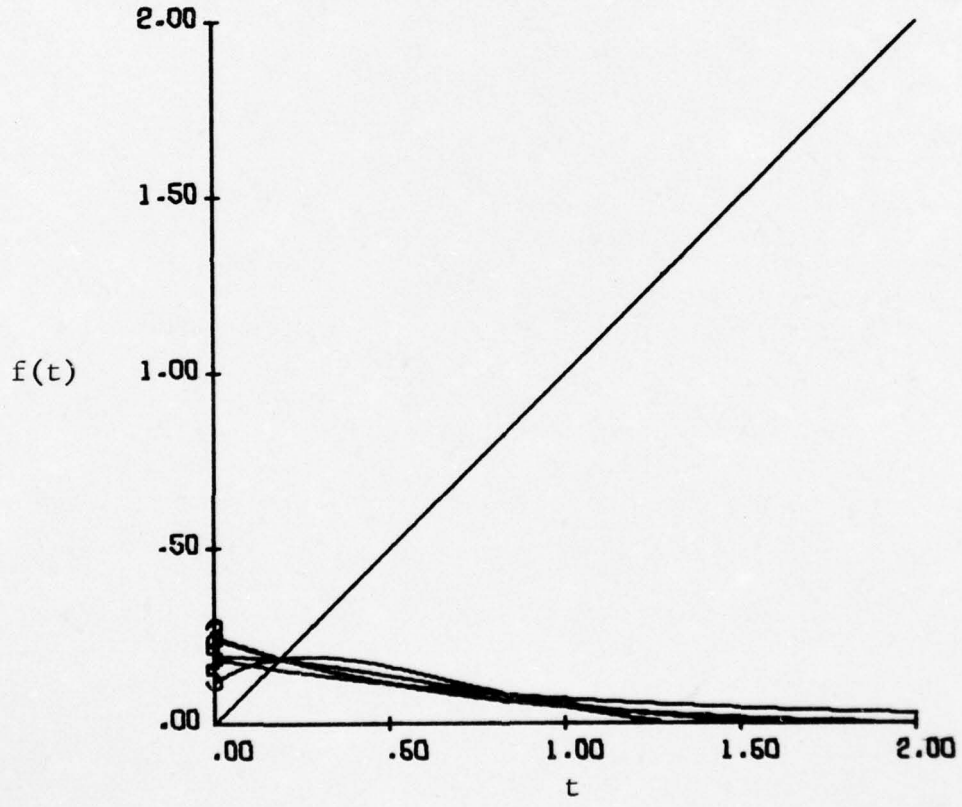


FIG. 4.11 F(T)=T AND FIVE APPROXIMATIONS FOR A=.5
LAGUERRE CASE

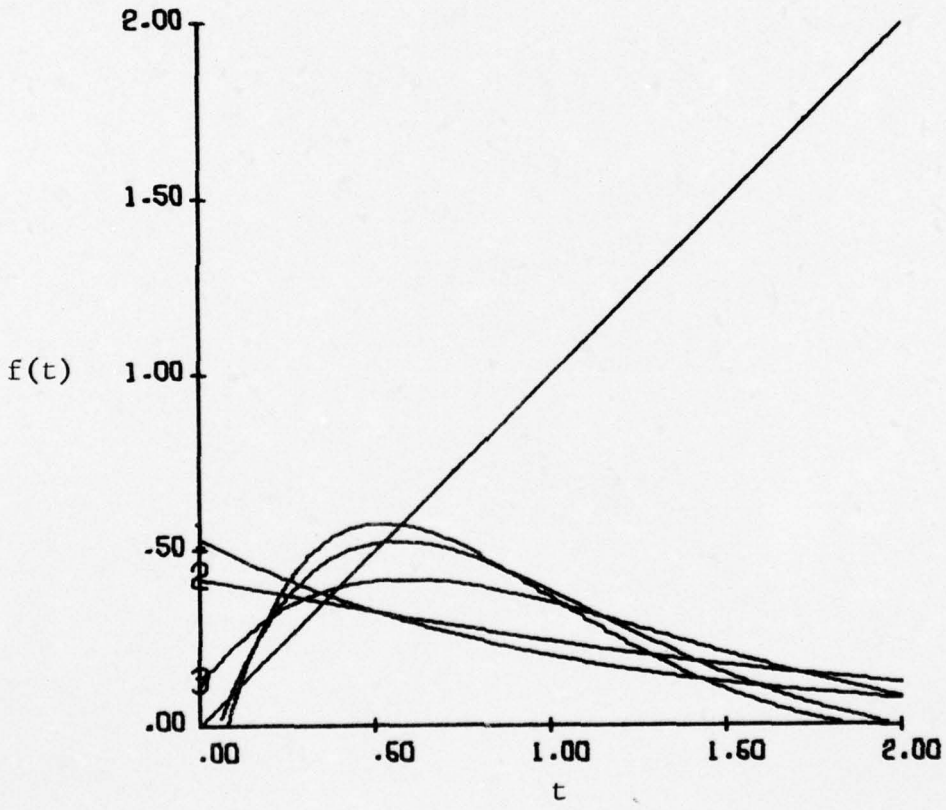


FIG. 4.12 F(T)=T AND FIVE APPROXIMATIONS FOR A=1.0 LAGUERRE CASE

TRACOR
6-70-44

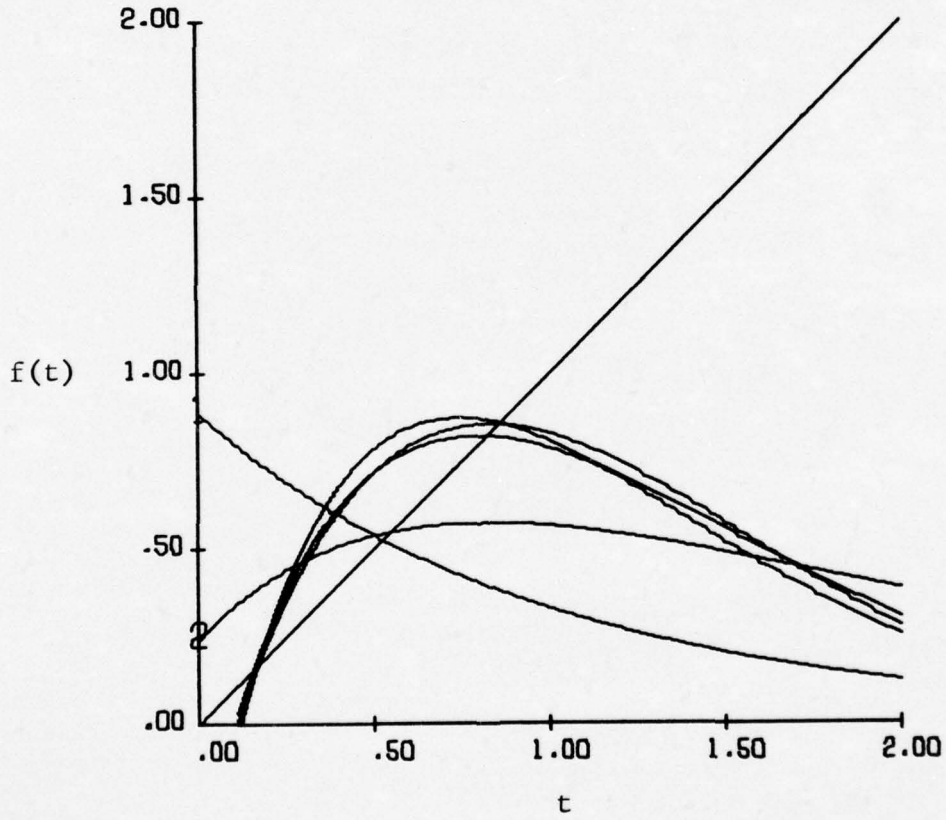


FIG. 4.13 F(T)=T AND FIVE APPROXIMATIONS FOR A=1.5
LAGUERRE CASE

TRACOR

6-70-45

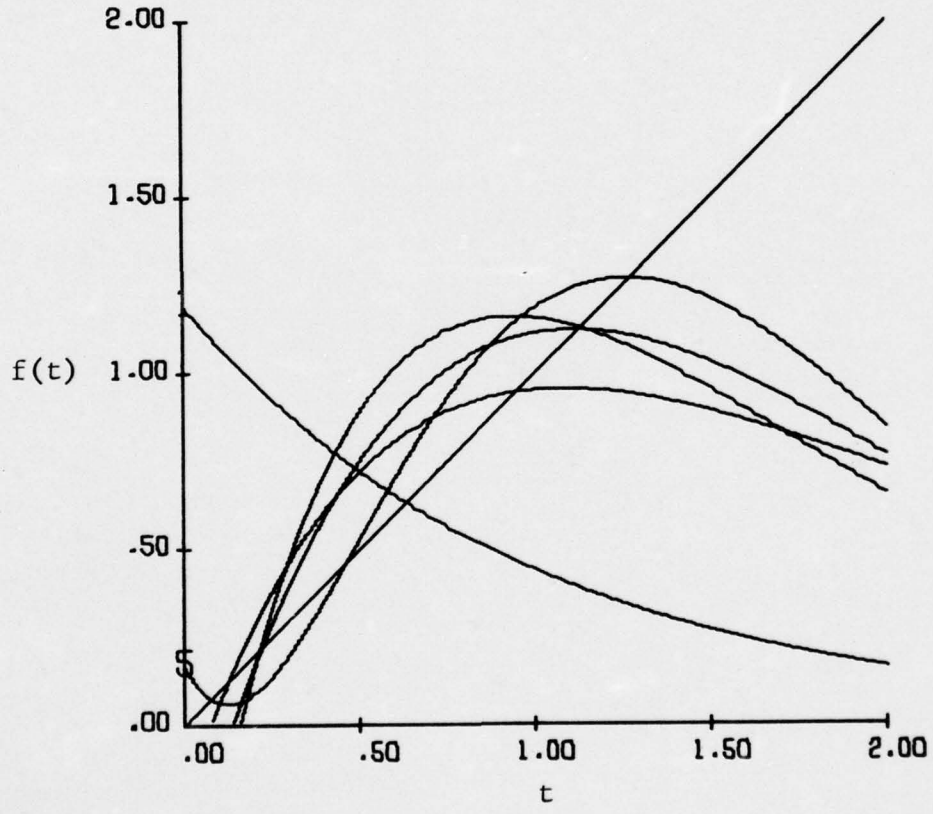


FIG. 4.14 F(T)=T AND FIVE APPROXIMATIONS FOR A=2.0 LAGUERRE CASE

TRACOR

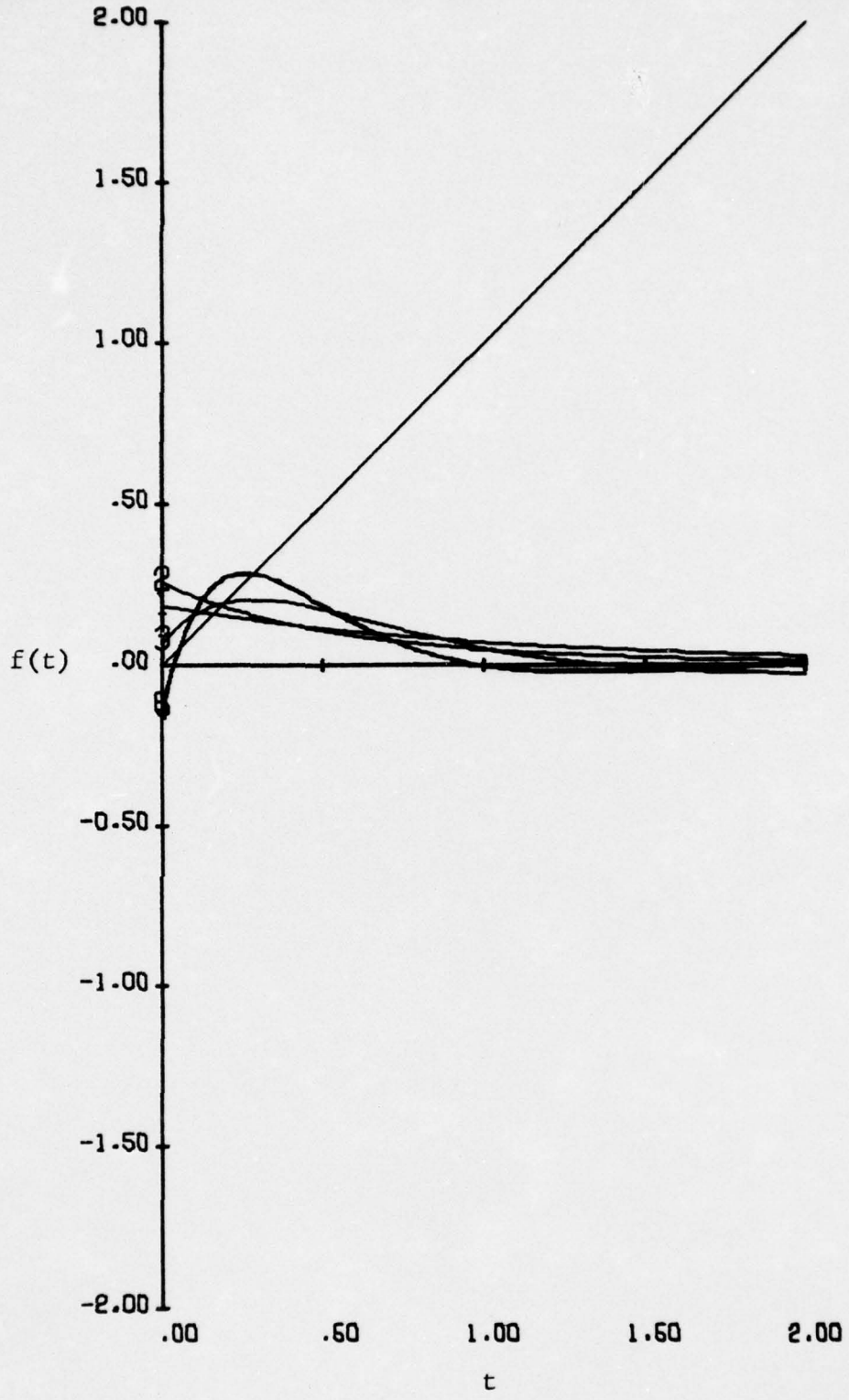


FIG. 4.15 F(T)=T AND FIVE APPROXIMATIONS FOR A=.5
ARBITRARY CASE

TRACOR

6-70-47

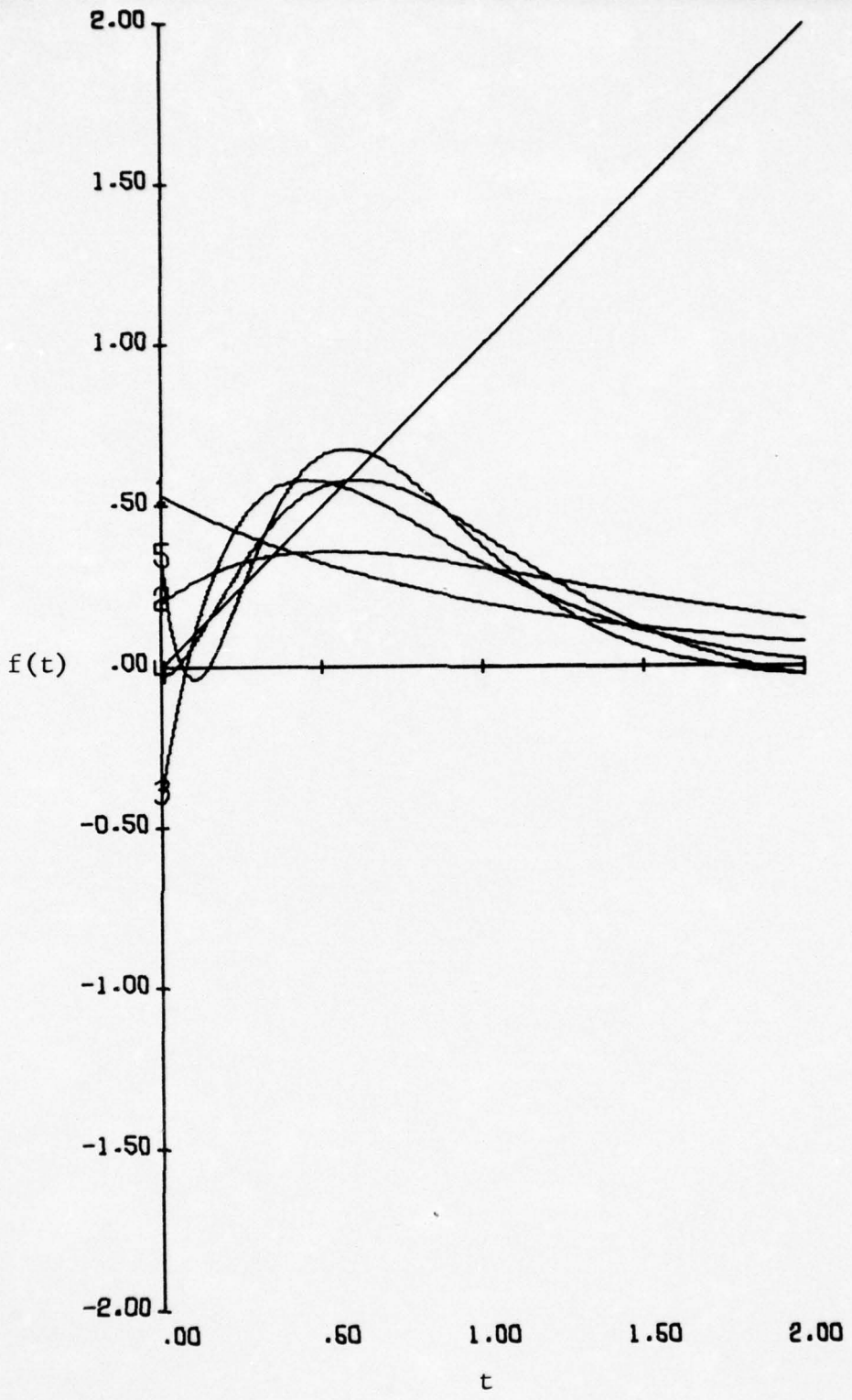


FIG. 4.16 F(T)=T AND FIVE APPROXIMATIONS FOR A=1.0 ARBITRARY CASE

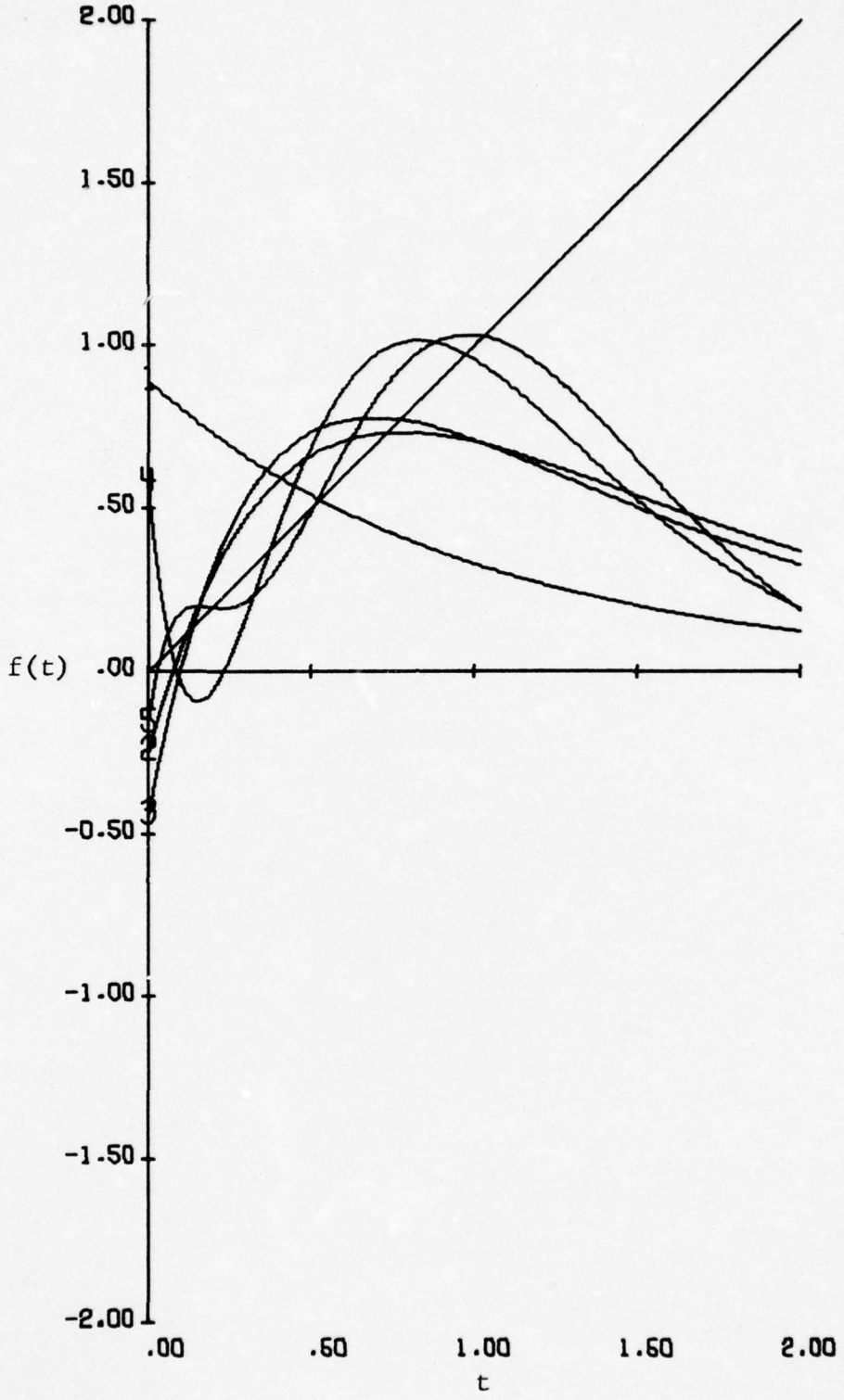


FIG. 4.17 F(T)=T AND FIVE APPROXIMATIONS FOR A=1.5 ARBITRARY CASE

TRACOR

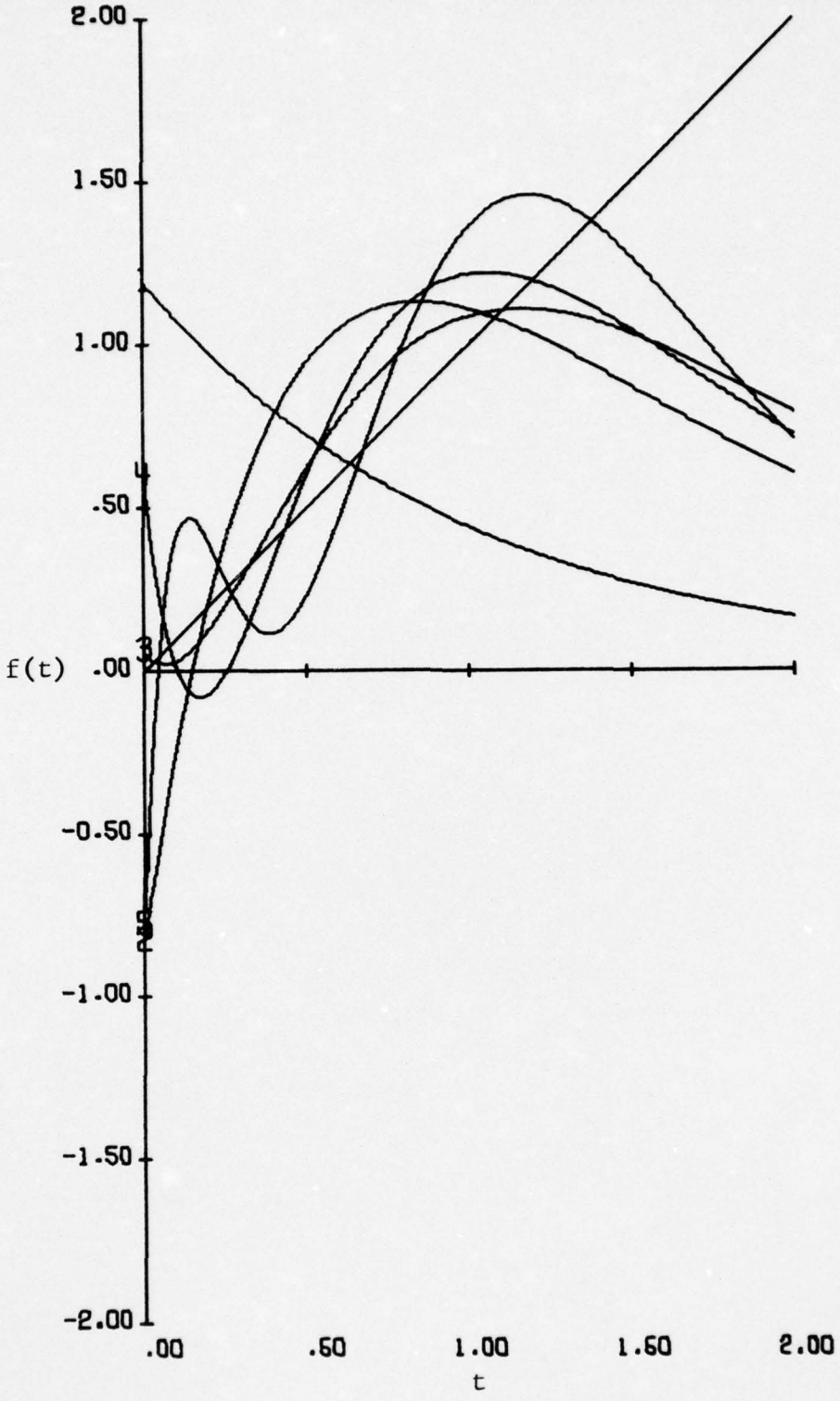


FIG. 4.18 F(T)=T AND FIVE APPROXIMATIONS FOR A=2.0 ARBITRARY CASE

5. The function in this case is defined by

$$f(t) = \alpha t e^{-\alpha t}, \quad \alpha \geq 1 .$$

Four error plots of η_N vs N were developed, two for the Laguerre basis expansion and two for the arbitrary basis expansion. The first plot of each case was produced with α=1 and the second with α=2. Figures 5.5 through 5.14 are plots of η_N vs A for $0 \leq A \leq 2$ and N=1 through 5. The final set of plots of this group are plots of f(t) and the five approximations for α=.5, 1.0, 1.5, and 2.0. These curves are identified by a number which denotes the number of terms in the expansion.

5.1 $f(t) = ate^{-\alpha t}$ $\alpha \geq 1$ Laguerre series

$$E_i = \frac{1}{4\alpha}$$

$$C_1 = \frac{\sqrt{2}\,\alpha}{(\alpha+1)^2}$$

$$C_2 = \frac{-\sqrt{2}\,\alpha}{(\alpha+1)^3}\left[(\alpha+1) - 4\right]$$

$$C_3 = \frac{\sqrt{2}\,\alpha}{(\alpha+1)^4}\left[(\alpha+1)^2 - 8(\alpha+1) + 12\right]$$

$$C_4 = \frac{-\sqrt{2}\,\alpha}{(\alpha+1)^5}\left[(\alpha+1)^3 - 12(\alpha+1)^2 + 36(\alpha+1) - 32\right]$$

$$C_5 = \frac{\sqrt{2}\,\alpha}{(\alpha+1)^6}\left[(\alpha+1)^4 - 16(\alpha+1)^3 + 72(\alpha+1)^2 - 128(\alpha+1) + 80\right]$$

5.2 $f(t) = \alpha t e^{-\alpha t}$ $\alpha \geq 1$ Arbitrary series

$$E_i = \frac{1}{4\alpha}$$

$$C_1 = \frac{\sqrt{2}\,\alpha}{(\alpha+1)^2}$$

$$C_2 = 2\alpha\left[\frac{2}{(\alpha+1)^2} - \frac{3}{(\alpha+2)^2}\right]$$

$$C_3 = \sqrt{6}\,\alpha\left[\frac{3}{(\alpha+1)^2} - \frac{12}{(\alpha+2)^2} + \frac{10}{(\alpha+3)^2}\right]$$

$$C_4 = 2\sqrt{2}\,\alpha\left[\frac{4}{(\alpha+1)^2} - \frac{30}{(\alpha+2)^2} + \frac{60}{(\alpha+3)^2} - \frac{35}{(\alpha+4)^2}\right]$$

$$C_5 = \sqrt{10}\,\alpha\left[\frac{5}{(\alpha+1)^2} - \frac{60}{(\alpha+2)^2} + \frac{210}{(\alpha+3)^2} - \frac{280}{(\alpha+4)^2} + \frac{126}{(\alpha+5)^2}\right]$$

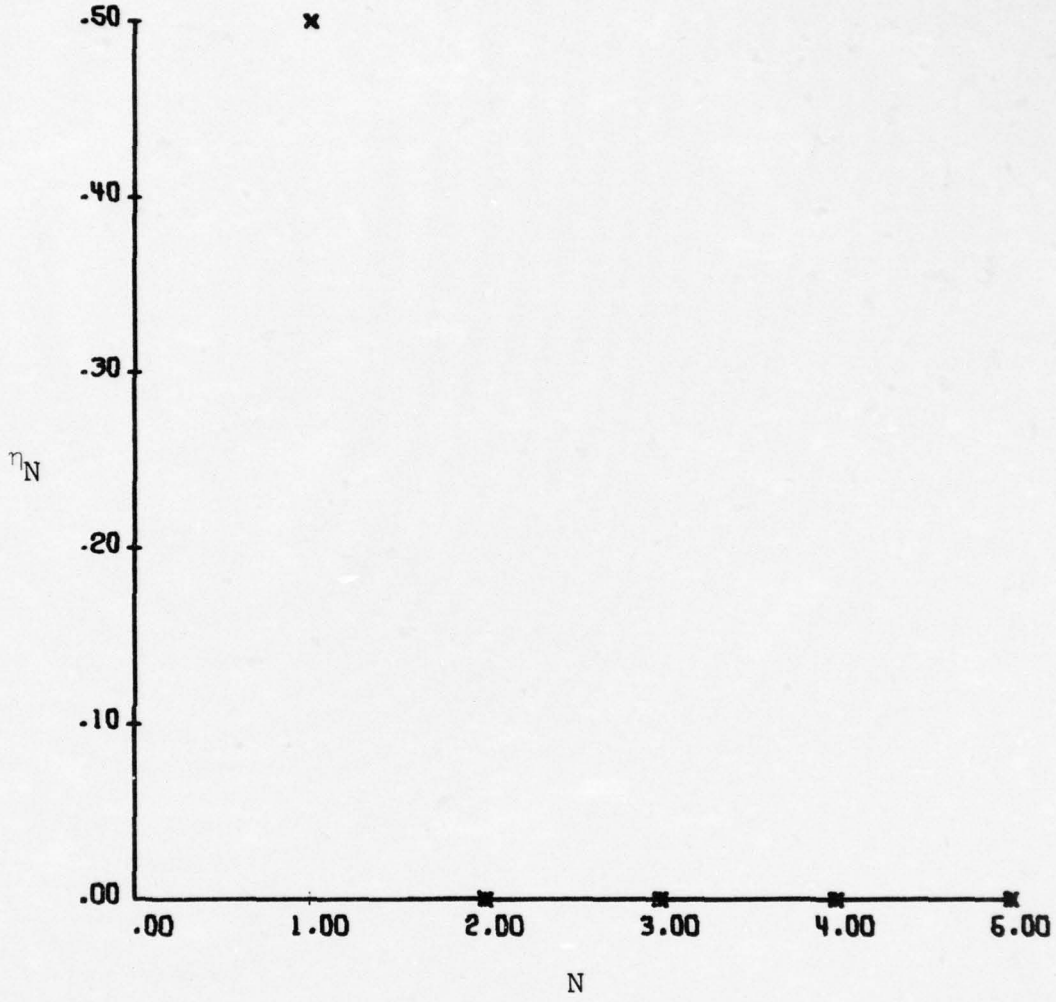
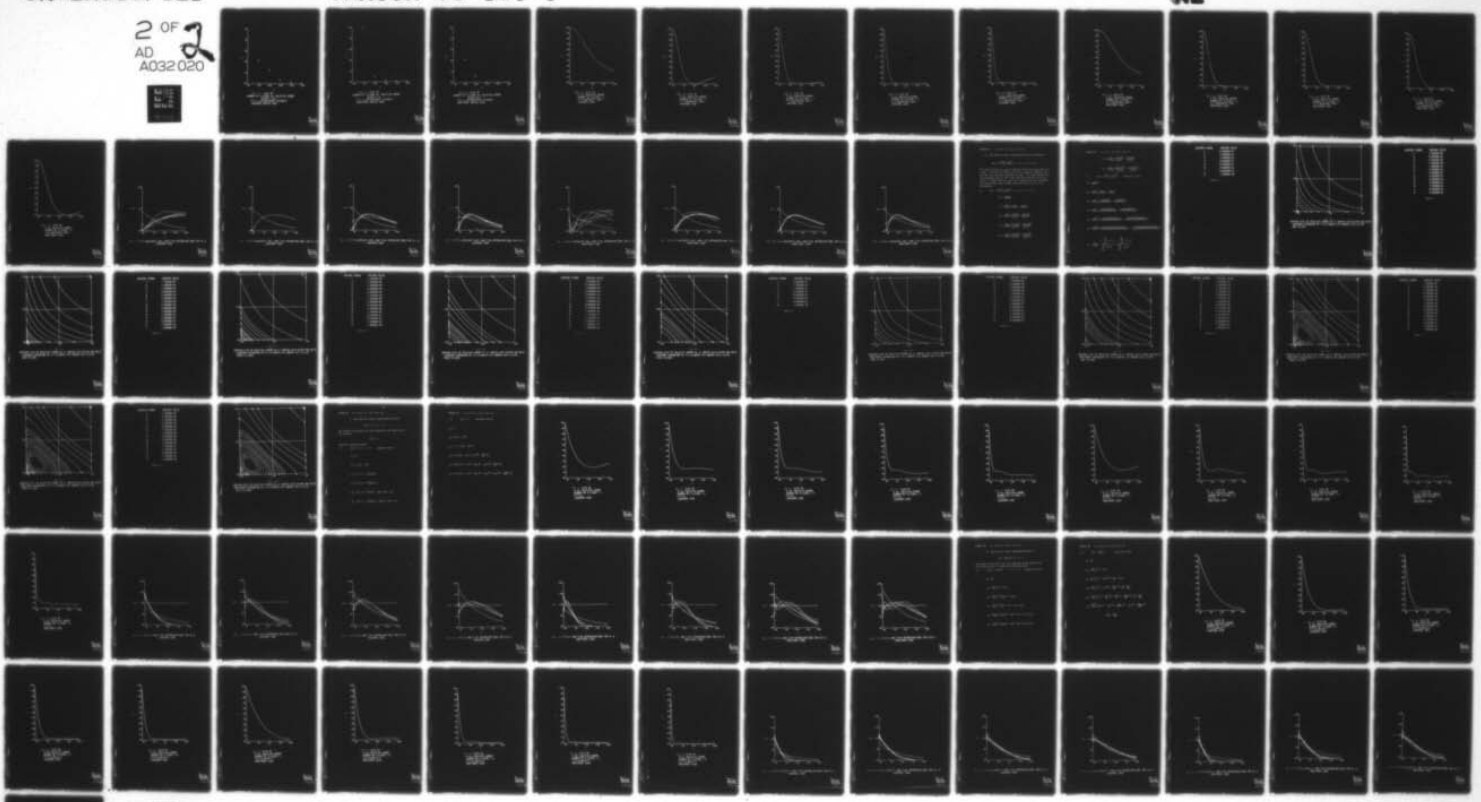


FIG. 5.1 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
ALPHA = 1.0
F(T) = ALPHA*T*EXP(-ALPHA*T)
LAGUERRE SERIES CASE

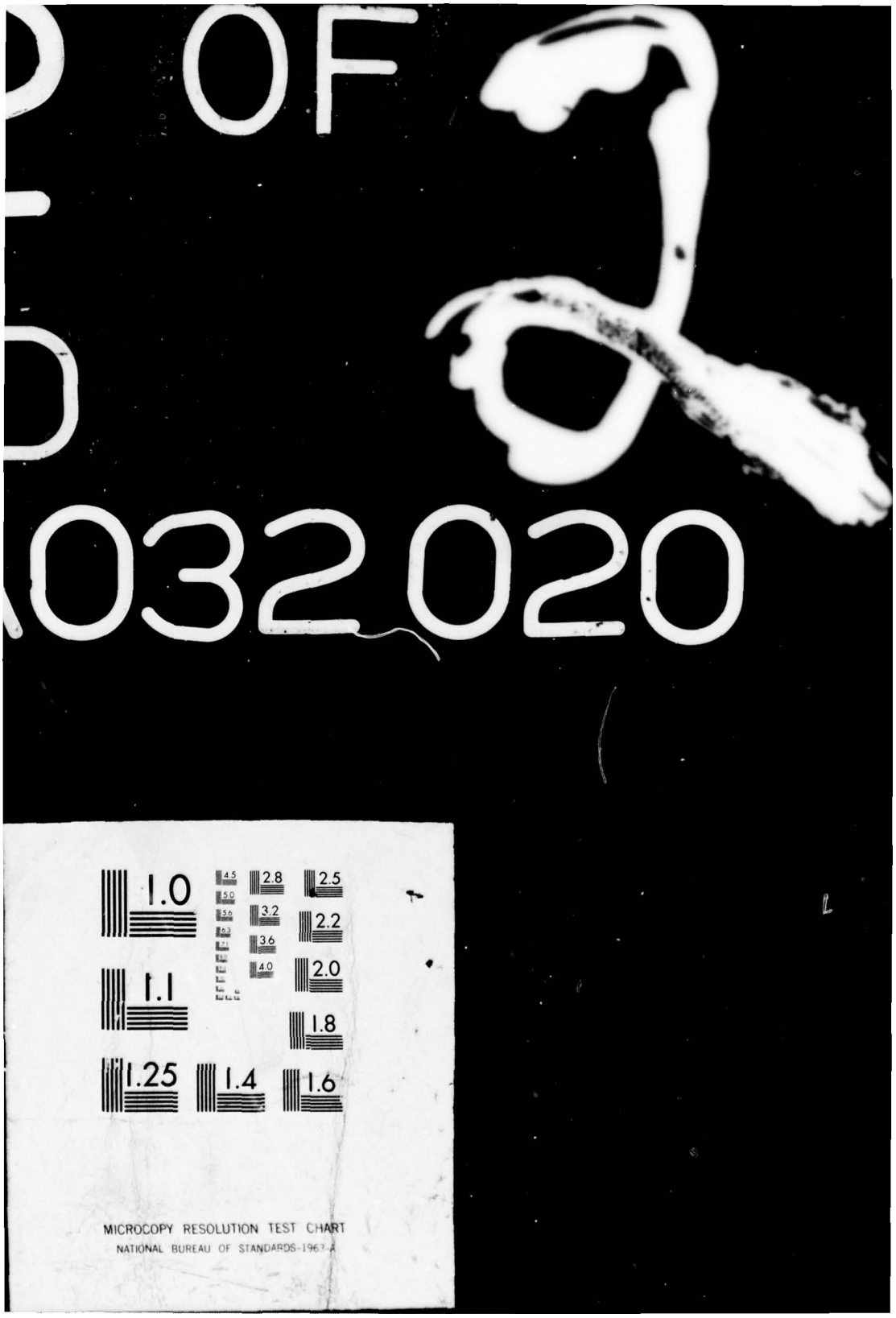
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ANALYSIS OF TRANSIENT SIGNALS.(U)
SEP 66 J A DOWNING NOBSR-93220
UNCLASSIFIED TRACOR-66-519-U NL

2 OF 2
AD A032 020



END

DATE FILMED
1-77


OF
032020
1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
7.1
8.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

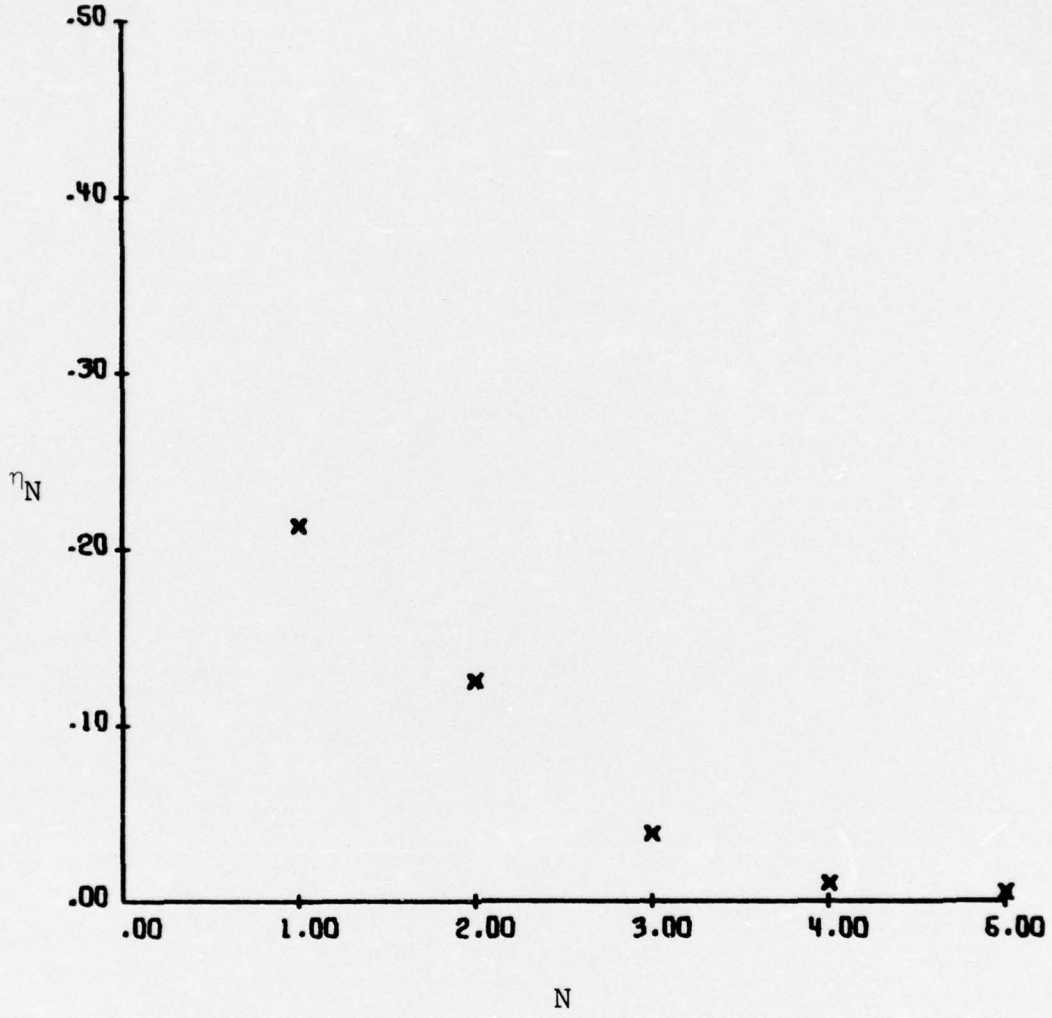


FIG. 5.2 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
ALPHA = 2.0
F(T) = ALPHA*T*EXP(-ALPHA*T)
LAGUERRE SERIES CASE

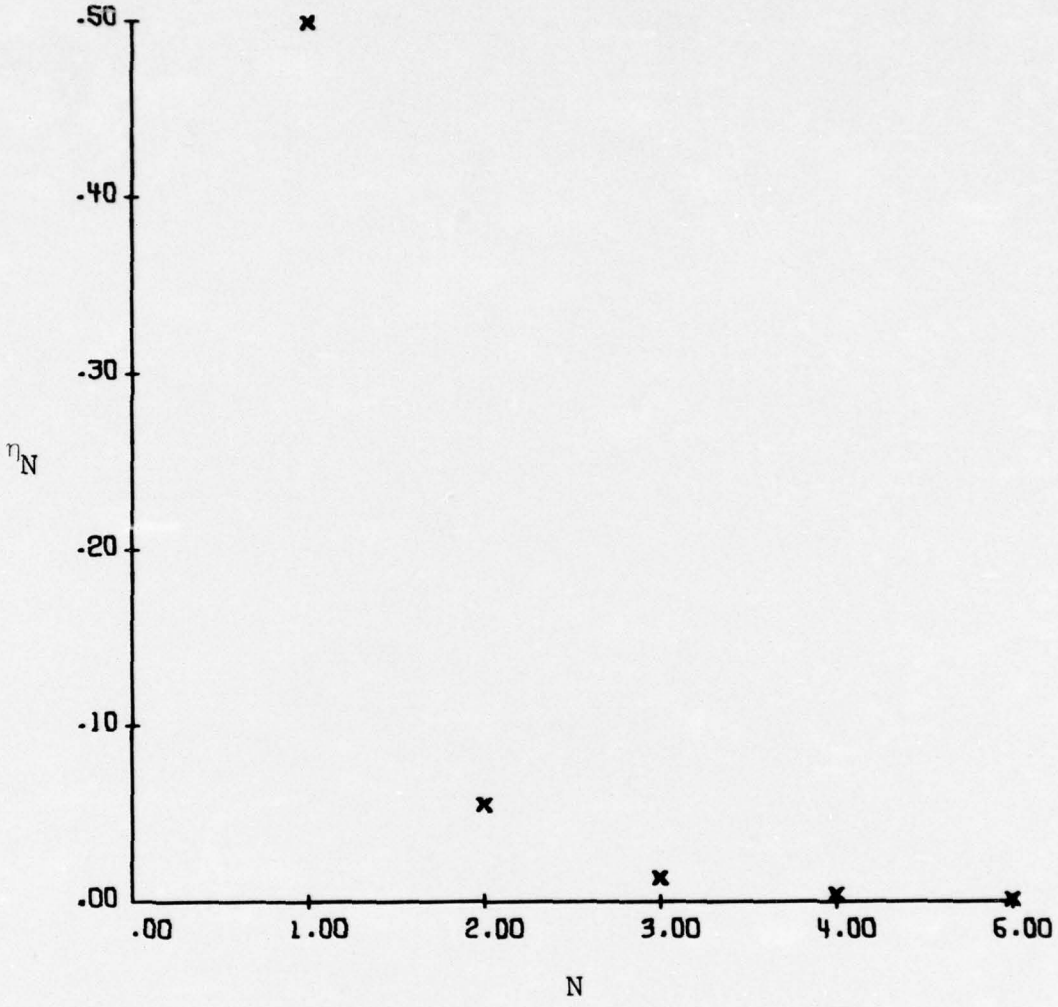


FIG. 5.3 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
ALPHA =1.0
F(T) = ALPHA*T*EXP(-ALPHA*T)
ARBITRARY SERIES CASE

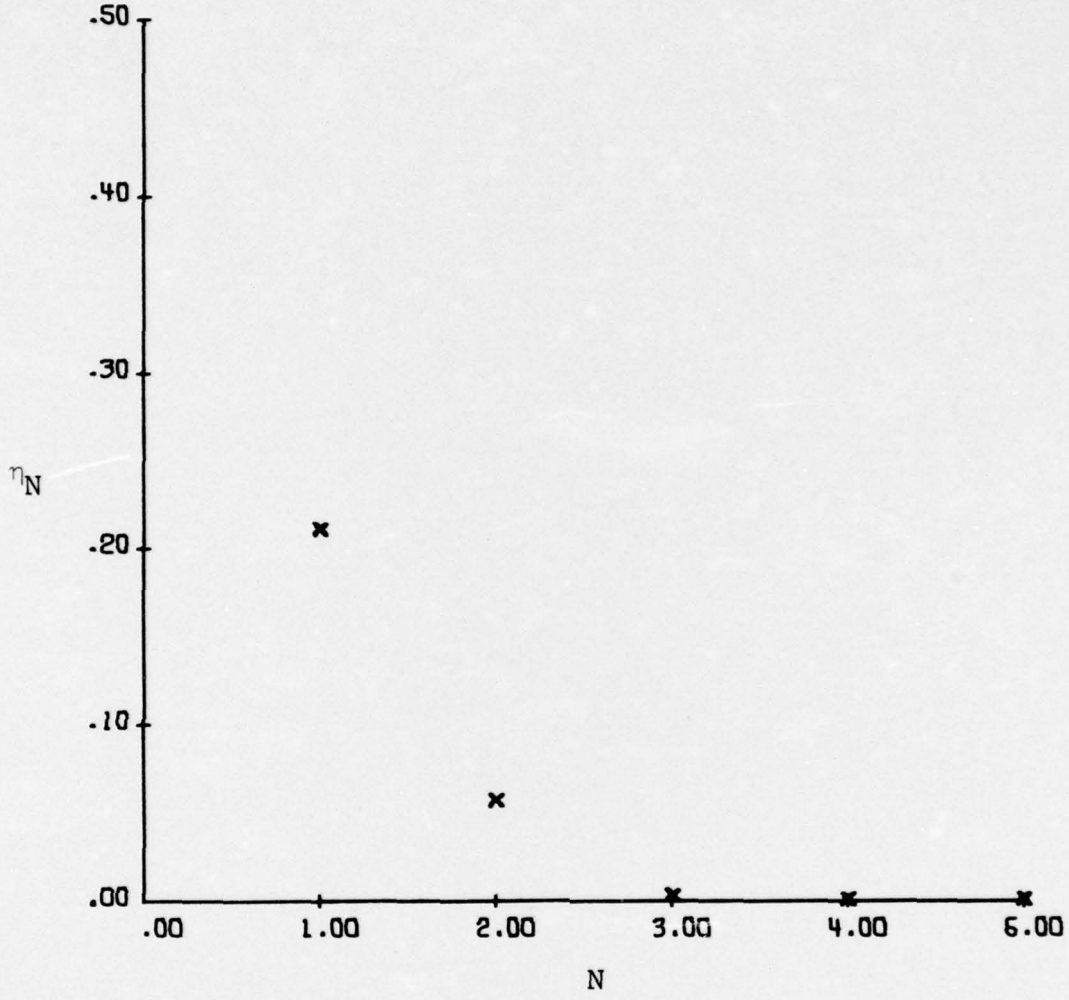


FIG. 5.4 PLOT OF
NUMBER OF FILTERS VS. RELATIVE ERROR
ALPHA =2.0
F(T) = ALPHA*T*EXP(-ALPHA*T)
ARBITRARY SERIES CASE

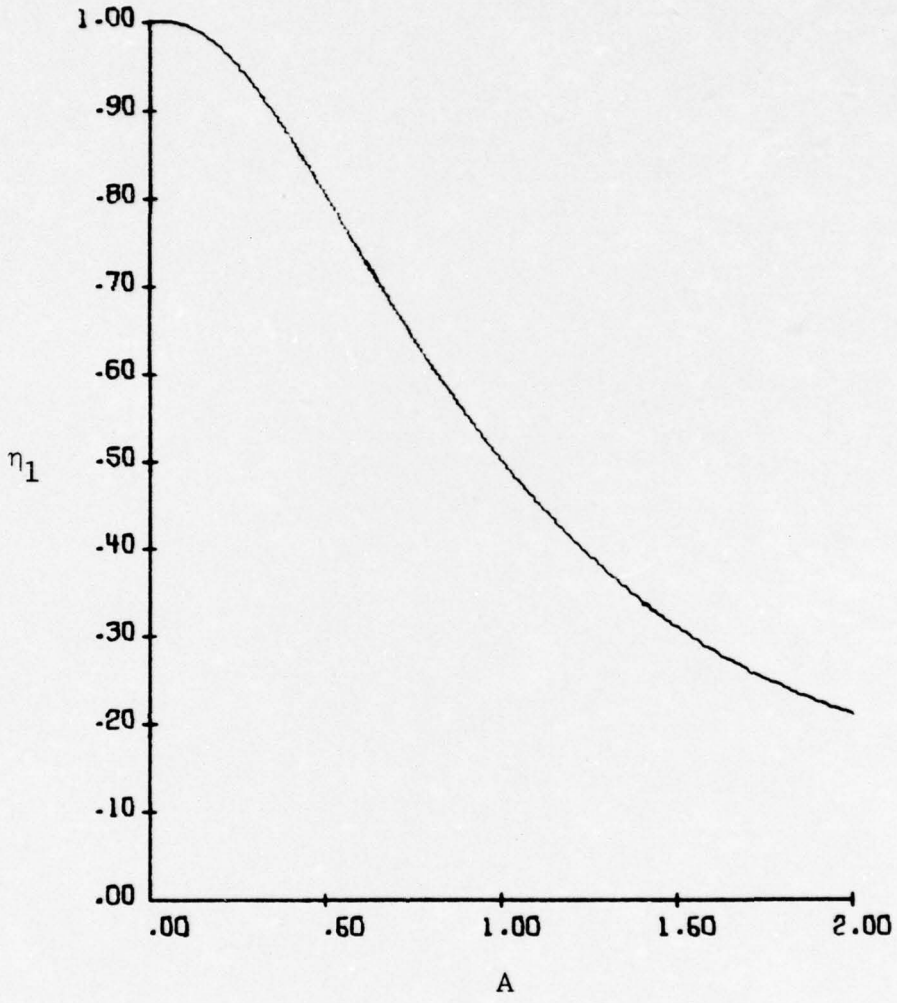


FIG. 5.5 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=A*T*EXPF(-A*T)
LAGUERRE CASE

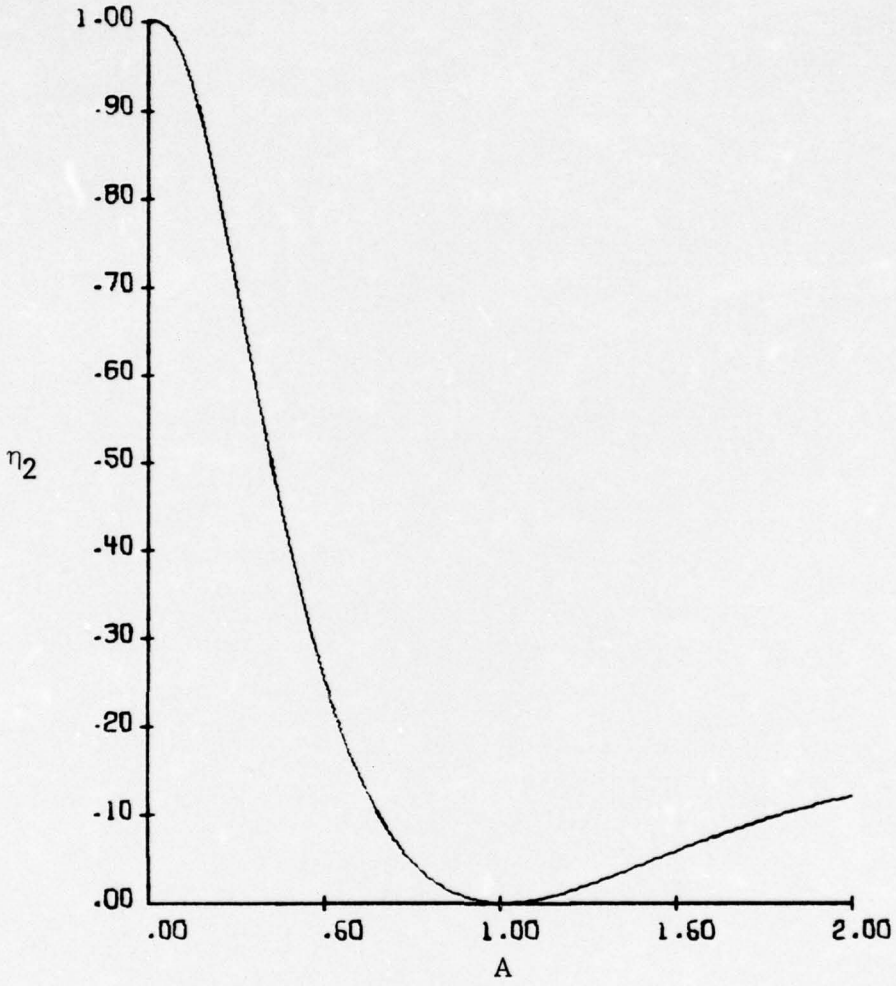


FIG. 5.6 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=A*T*EXPF(-A*T)
LAGUERRE CASE

TRACOR

6-70-56

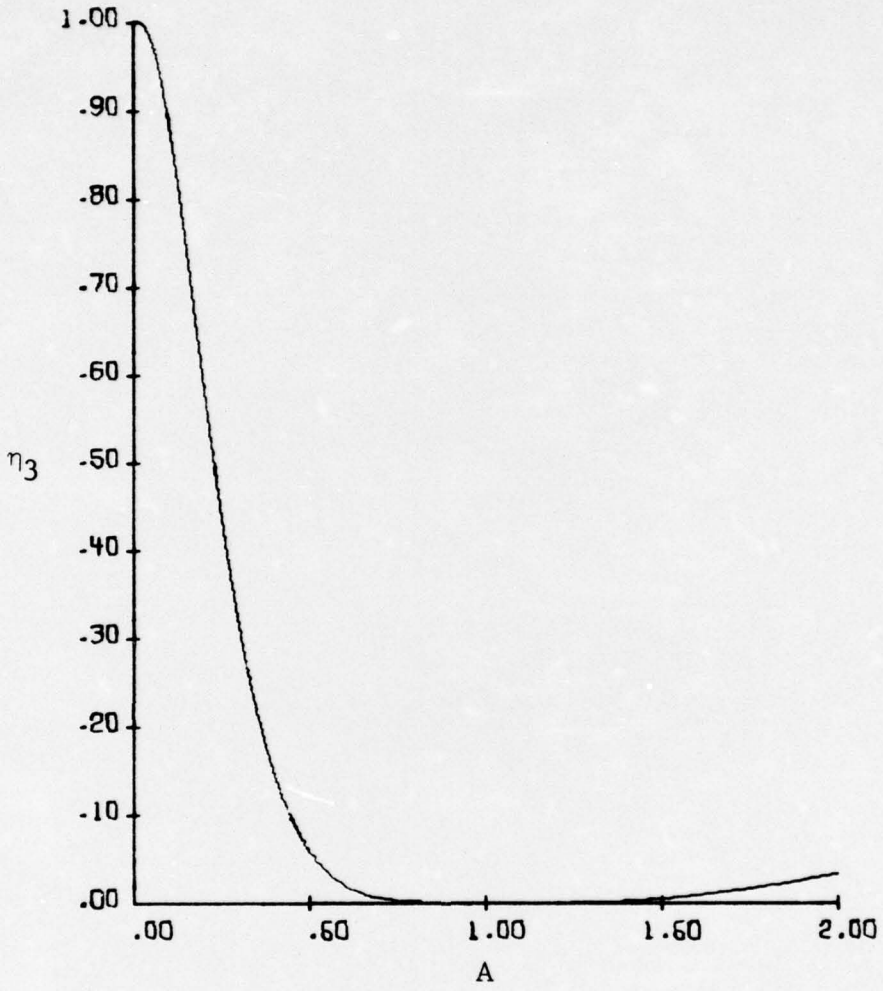


FIG. 5.7 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=A*T*EXPF(-A*T)
LAGUERRE CASE

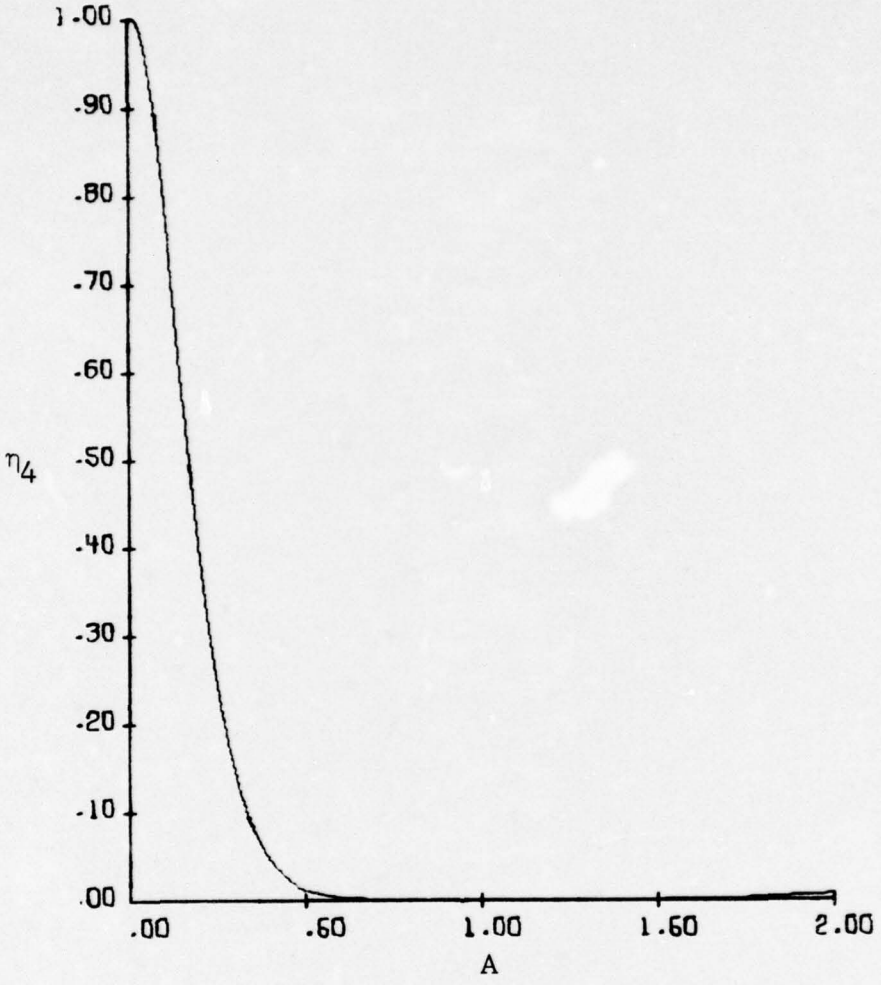


FIG. 5.8 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=A*T*EXPF(-A*T)
LAGUERRE CASE

TRACOR

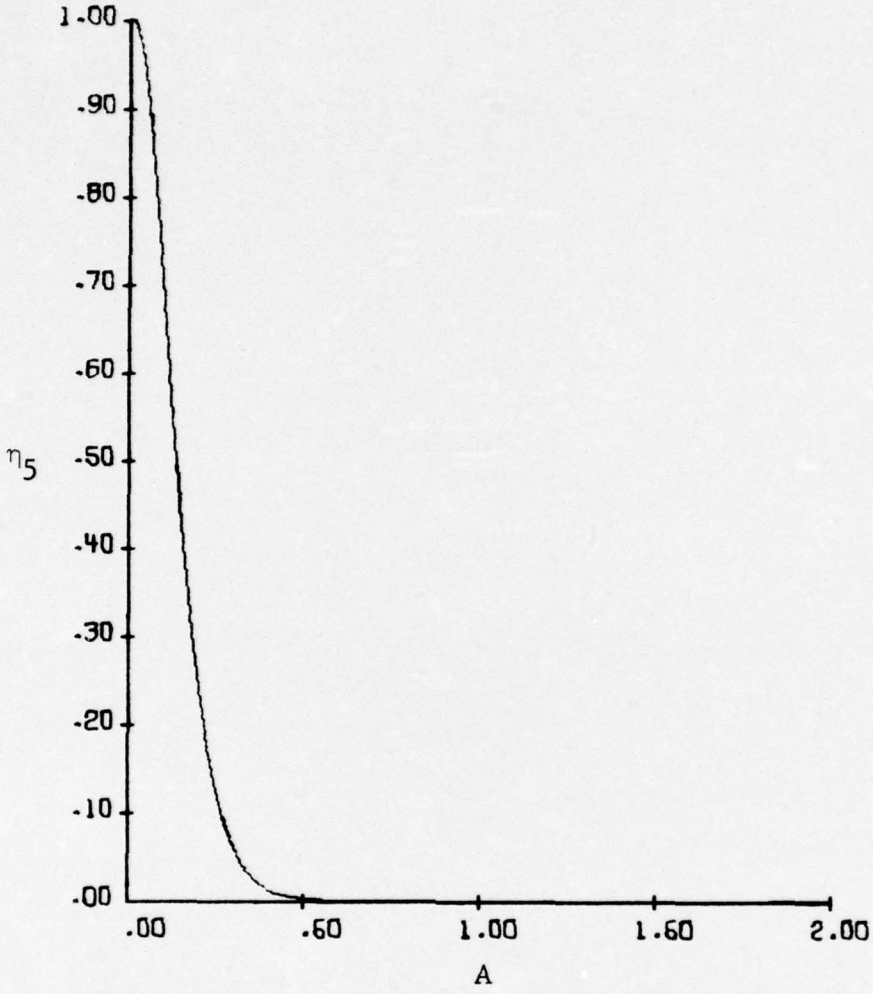


FIG. 5.9 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=A*T*EXPF(-A*T)
LAGUERRE CASE

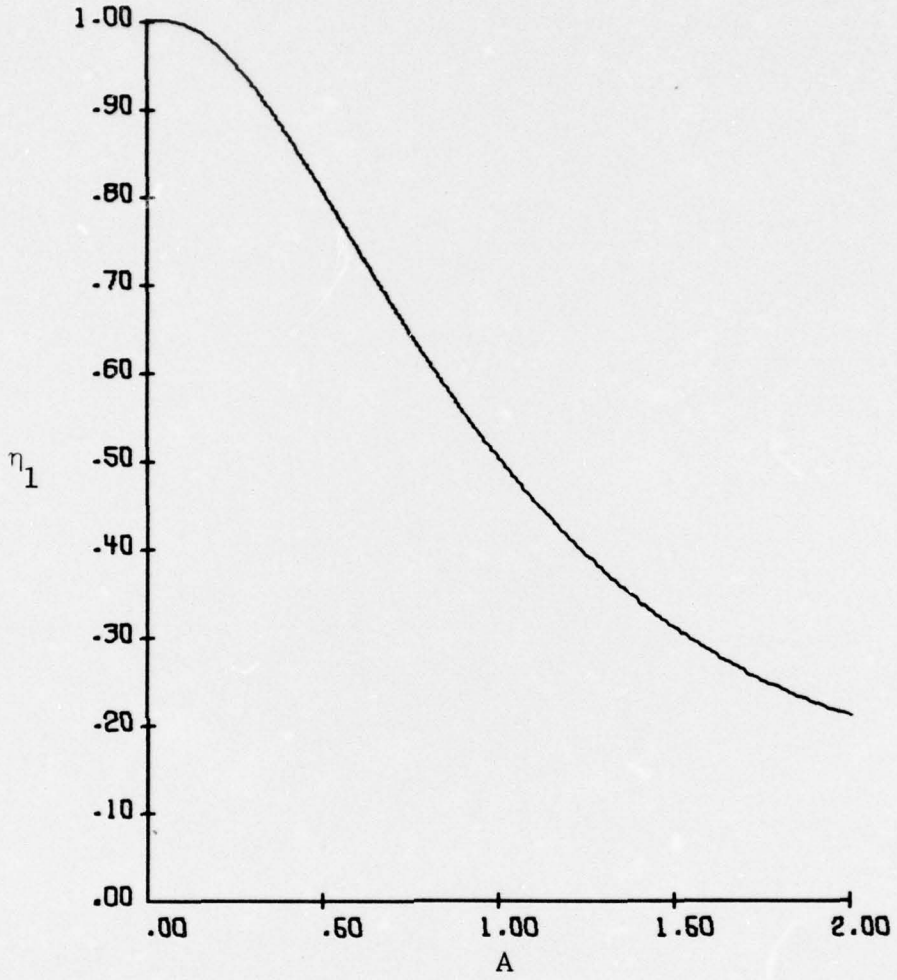


FIG. 5.10 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=A*T*EXPF(-A*T)
ARBITRARY CASE

TRACOR

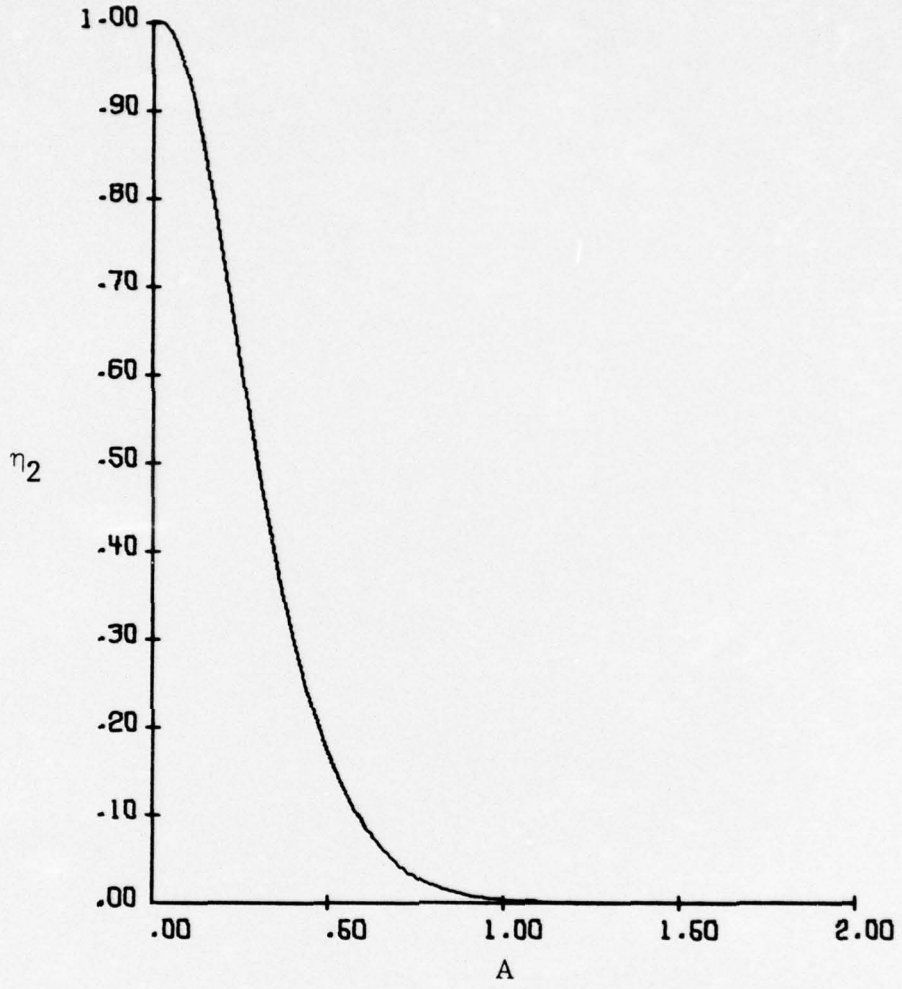


FIG. 5.11 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=A*T*EXPF(-A*T)
ARBITRARY CASE

TRACOR

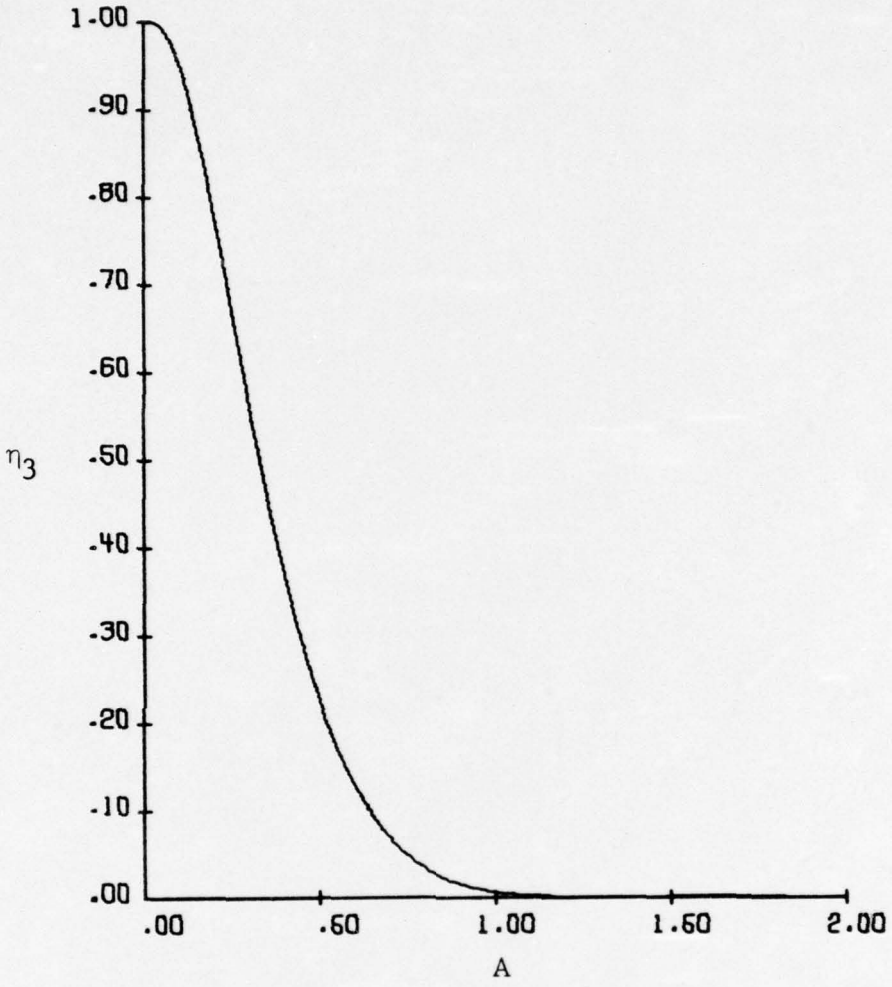


FIG. 5.12 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=A*T*EXPF(-A*T)
ARBITRARY CASE

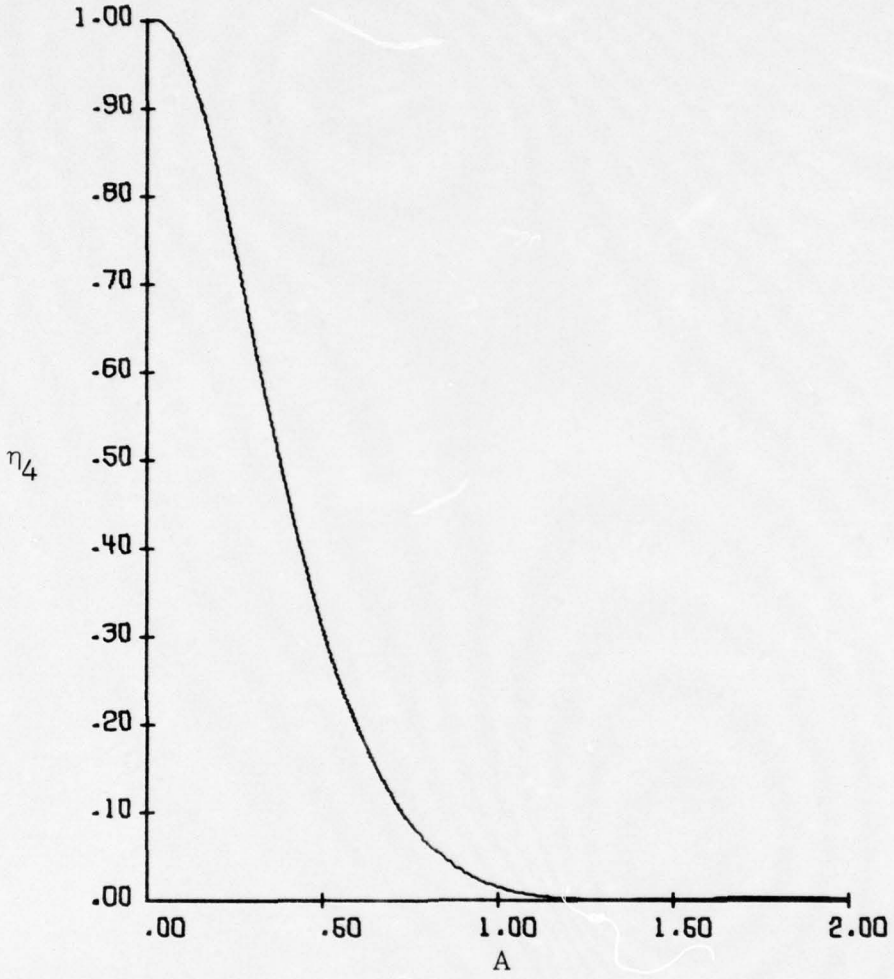


FIG. 5.13 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=A*T*EXPF(-A*T)
ARBITRARY CASE

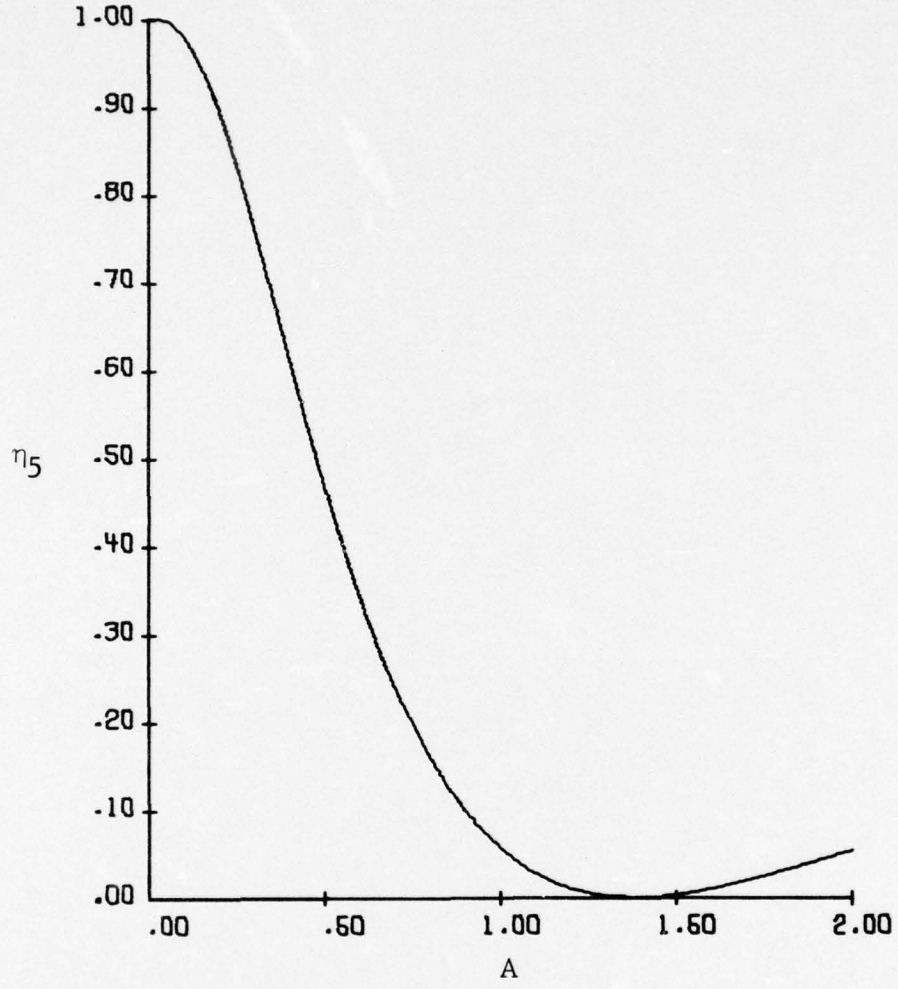


FIG. 5.14 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=A*T*EXPF(-A*T)
ARBITRARY CASE

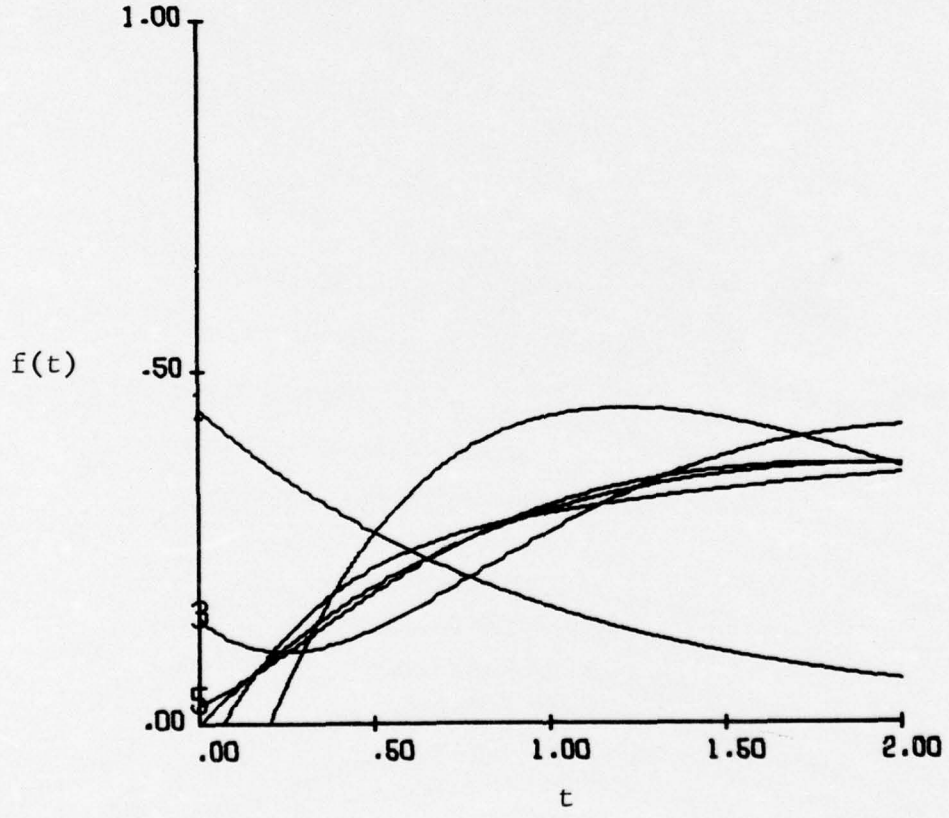


FIG. 5.15 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=.5
LAGUERRE CASE

TRACOR

6-70-65

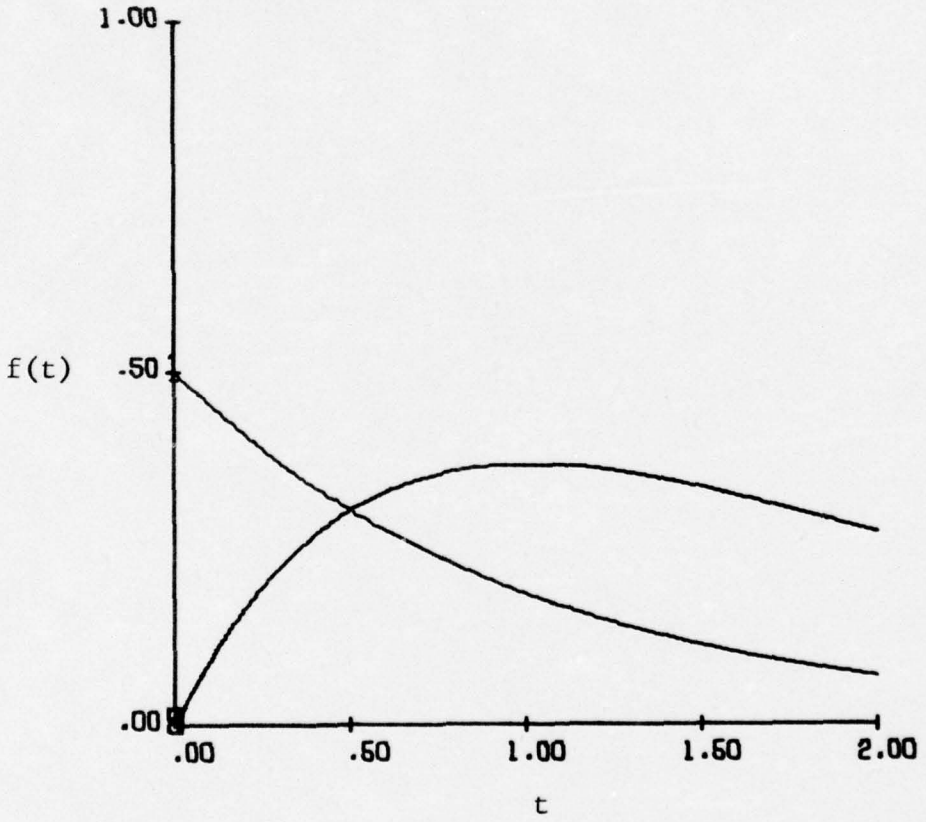


FIG. 5.16 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=1.0
LAGUERRE CASE

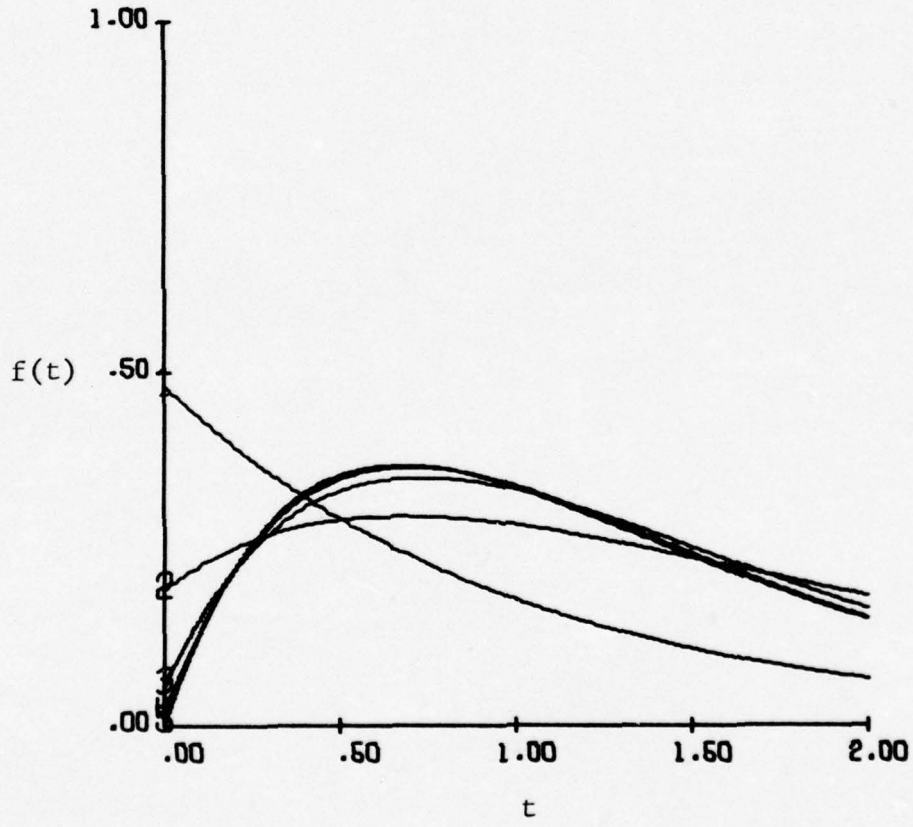


FIG. 5.17 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=1.5
LAGUERRE CASE

TRACOR

6-70-67

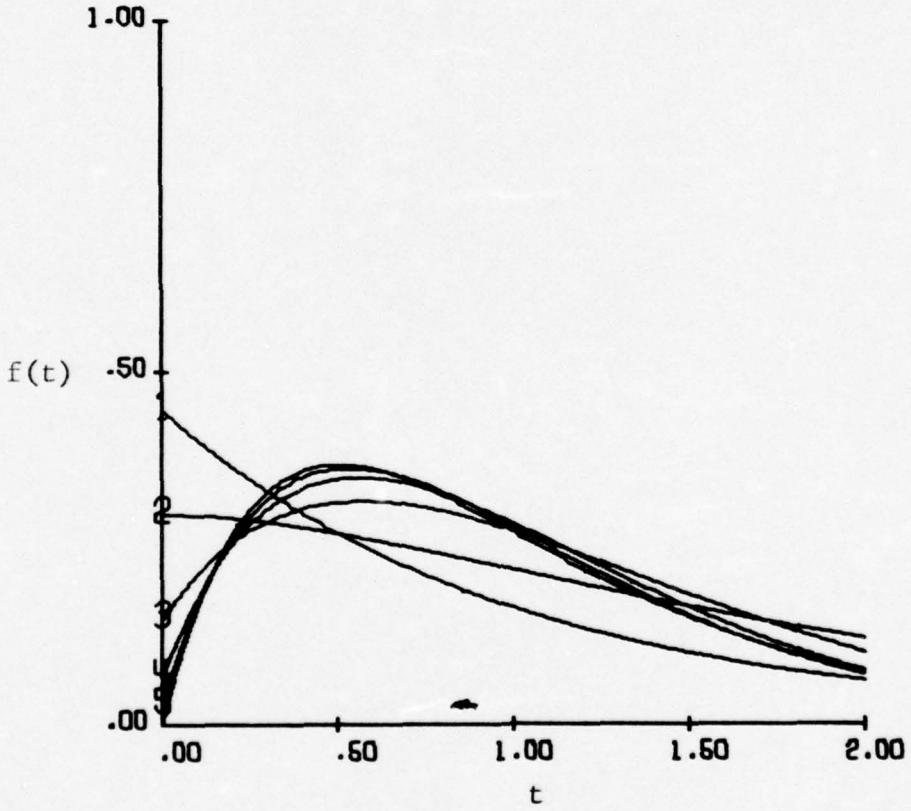


FIG. 5.18 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=2.0
LAGUERRE CASE

TRACOR

6-70-68

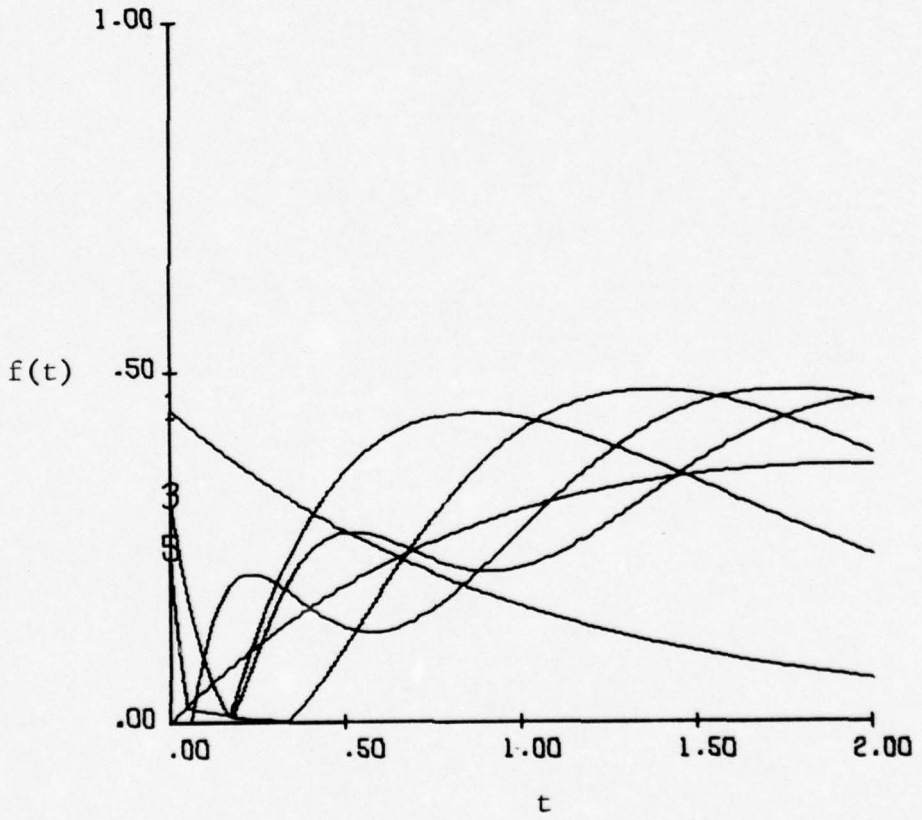


FIG. 5.19 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=.5
ARBITRARY CASE

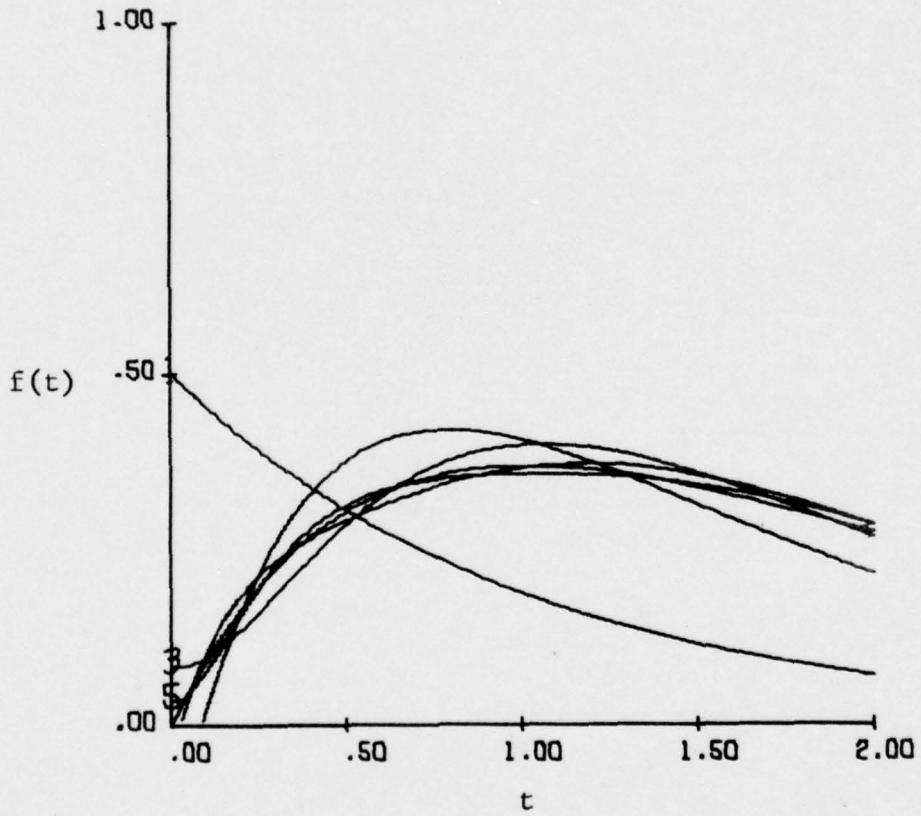


FIG. 5.20 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=1.0
ARBITRARY CASE

TRACOR

6-70-70

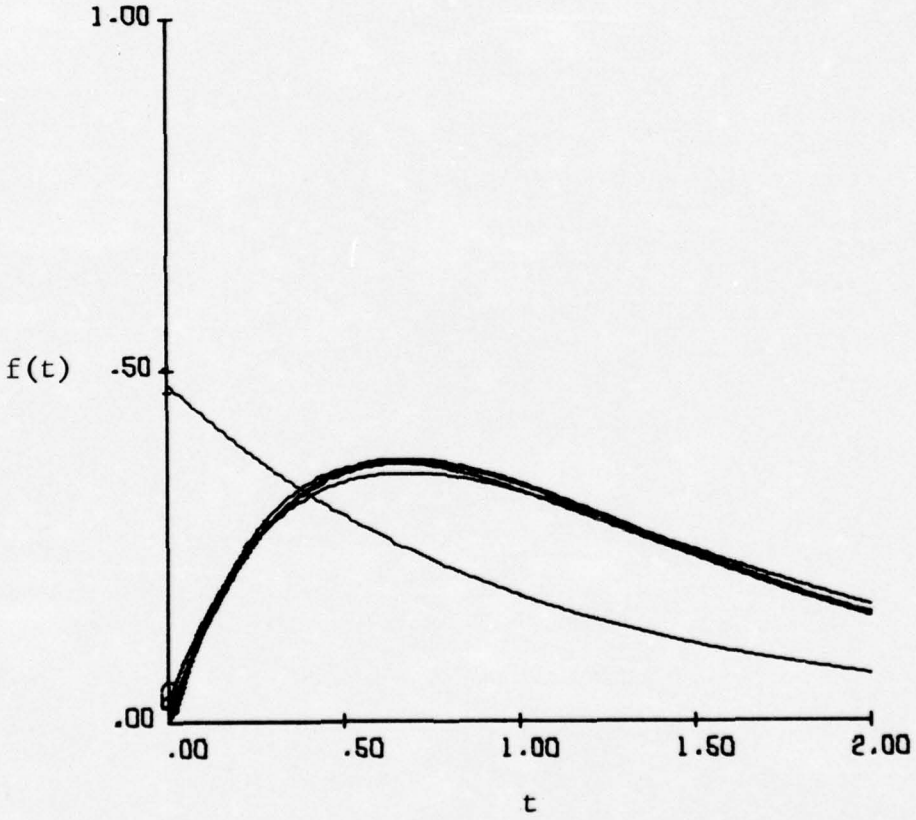


FIG. 5.21 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=1.5 ARBITRARY CASE

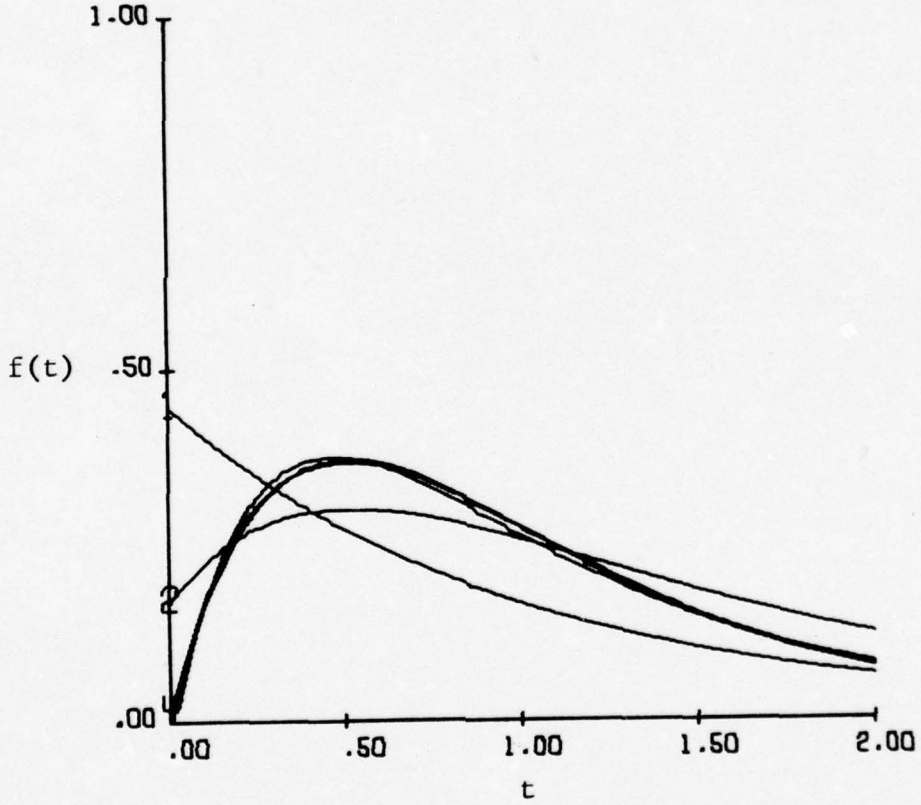


FIG. 5.22 F(T)=A*T*EXP(-A*T) AND FIVE APPROXIMATIONS FOR A=2.0
ARBITRARY CASE

6. The function under consideration here is defined by

$$f(t) = \frac{\alpha e^{-\alpha t} - \beta e^{-\beta t}}{\alpha - \beta}, \quad \alpha \geq 0, \quad \beta \geq 0, \quad \alpha \neq \beta .$$

Ten contour plots of η_N were produced to show how η_N varies with α and β. Five of the plots are for the Laguerre expansion and five for the arbitrary expansion of f(t). For each contour plot the horizontal axis is the α-axis and the vertical one is the β-axis. Neither f(t) nor $\eta_N(\alpha,\beta)$ is defined for $\alpha=\beta$, therefore when using these plots points along the line $\alpha=\beta$ are to be disregarded.

6.1 $$f(t) = \frac{\alpha e^{-\alpha t} - \beta e^{-\beta t}}{\alpha - \beta}, \quad \alpha \geq 0, \quad \beta \geq 0, \quad \alpha \neq \beta$$

$$E_i = \frac{1}{2(\alpha+\beta)}$$

$$C_1 = \frac{\sqrt{2}}{(\alpha-\beta)} \left[\frac{\alpha}{(\alpha+1)} - \frac{\beta}{(\beta+1)}\right]$$

$$C_2 = \frac{\sqrt{2}}{(\alpha-\beta)} \left[\frac{\alpha(1-\alpha)}{(\alpha+1)^2} - \frac{\beta(1-\beta)}{(\beta+1)^2}\right]$$

$$C_3 = \frac{\sqrt{2}}{(\alpha-\beta)} \left[\frac{\alpha(1-\alpha)^2}{(\alpha+1)^3} - \frac{\beta(1-\beta)^2}{(\beta+1)^3}\right]$$

$$C_4 = \frac{\sqrt{2}}{(\alpha-\beta)} \left[\frac{\alpha(1-\alpha)^3}{(\alpha+1)^4} - \frac{\beta(1-\beta)^3}{(\beta+1)^4}\right]$$

$$C_5 = \frac{\sqrt{2}}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)^4}{(\alpha+1)^5} - \frac{\beta(1-\beta)^4}{(\beta+1)^5}\right]$$

$$C_n = \frac{\sqrt{2}}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)^{n-1}}{(\alpha+1)^n} - \frac{\beta(1-\beta)^{n-1}}{(\beta+1)^n}\right]$$

6.2 $f(t) = \dfrac{\alpha e^{-\alpha t} - \beta e^{-\beta t}}{\alpha - \beta}$ Arbitrary series

$$E_i = \frac{1}{2(\alpha+\beta)}$$

$$C_1 = \frac{\sqrt{2}}{(\alpha-\beta)}\left[\frac{\alpha}{(\alpha+1)} - \frac{\beta}{(\beta+1)}\right]$$

$$C_2 = \frac{2}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)}{(\alpha+1)(\alpha+2)} - \frac{\beta(1-\beta)}{(\beta+1)(\beta+2)}\right]$$

$$C_3 = \frac{\sqrt{6}}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)(2-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)} - \frac{\beta(1-\beta)(2-\beta)}{(\beta+1)(\beta+2)(\beta+3)}\right]$$

$$C_4 = \frac{2\sqrt{2}}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)(2-\alpha)(3-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)} - \frac{\beta(1-\beta)(2-\beta)(3-\beta)}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}\right]$$

$$C_5 = \frac{1/2\sqrt{10}}{(\alpha-\beta)}\left[\frac{\alpha(1-\alpha)(2-\alpha)(3-\alpha)(4-\alpha)}{(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)} - \frac{\beta(1-\beta)(2-\beta)(3-\beta)(4-\beta)}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)}\right]$$

$$C_n = \frac{K_n}{(\alpha-\beta)}\left[\frac{\alpha \prod_{i=1}^{n-1}(i-\alpha)}{\prod_{i=1}^{n}(\alpha+i)} - \frac{\beta \prod_{i=1}^{n-1}(i-\beta)}{\prod_{i=1}^{n}(\beta+i)}\right]$$

CONTOUR SYMBOL	CONTOUR VALUE
1	6.50000E-01
2	7.00000E-01
3	7.50000E-01
4	8.00000E-01
5	8.50000E-01
6	9.00000E-01
7	9.50000E-01
8	1.00000E 00

TABLE 6.1

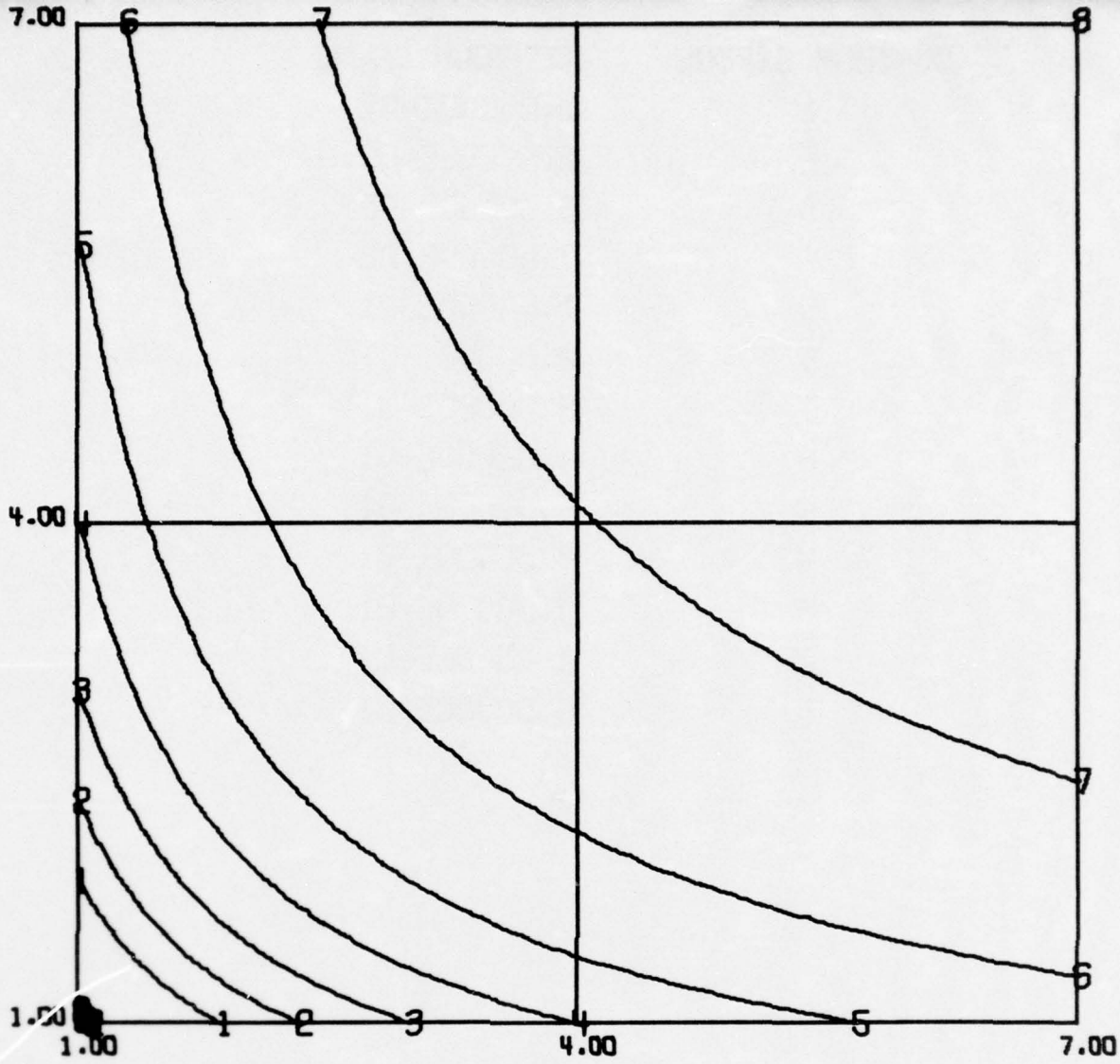


FIG. 6.1

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
ONE FILTER

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-02
2	2.50000E-02
3	5.00000E-02
4	7.50000E-02
5	1.00000E-01
6	2.00000E-01
7	3.00000E-01
8	4.00000E-01
9	5.00000E-01
A	6.00000E-01
B	7.00000E-01
C	8.00000E-01
D	9.00000E-01

TABLE 6.2

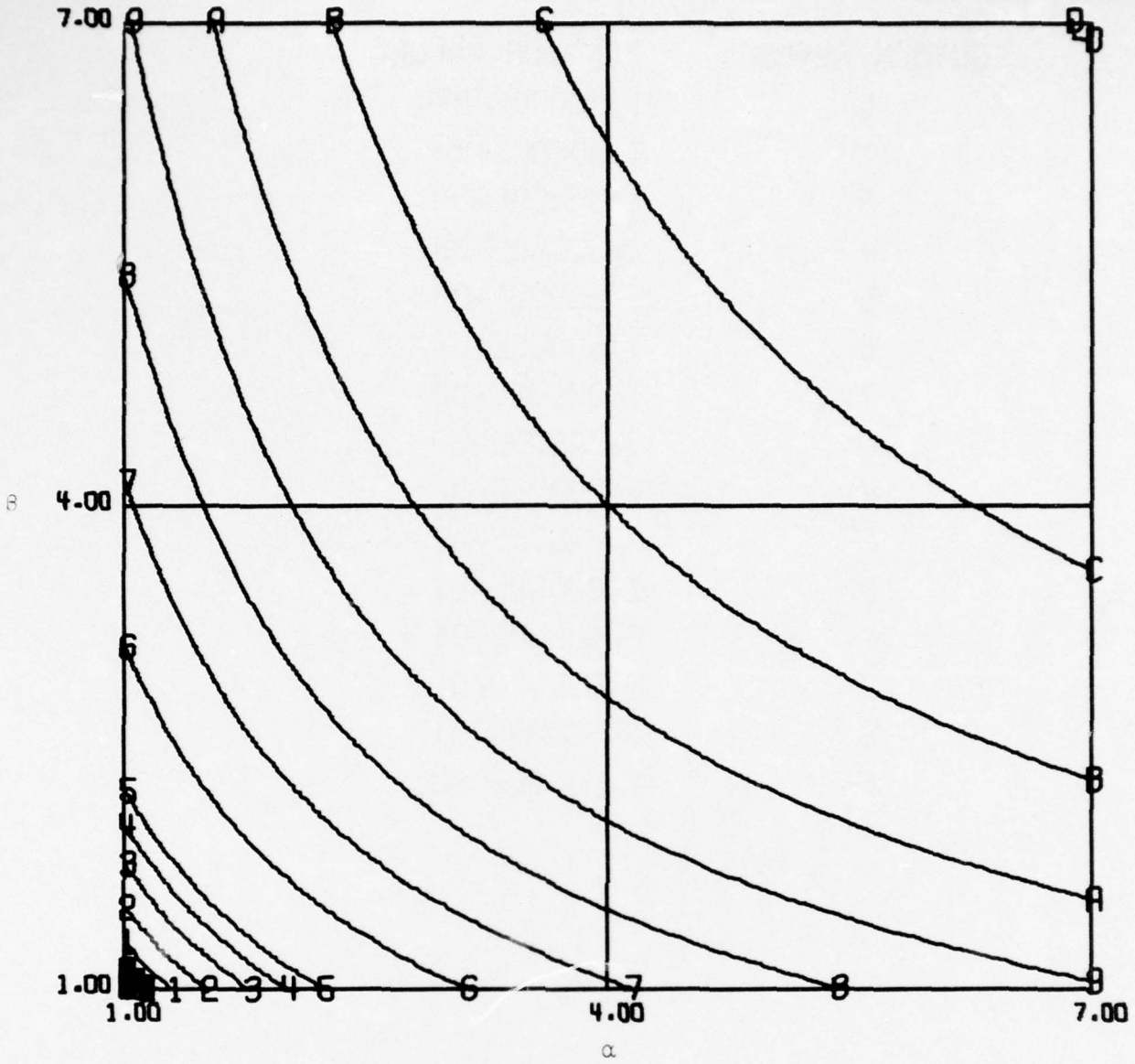


FIG. 6.2

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
TWO FILTERS

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.3

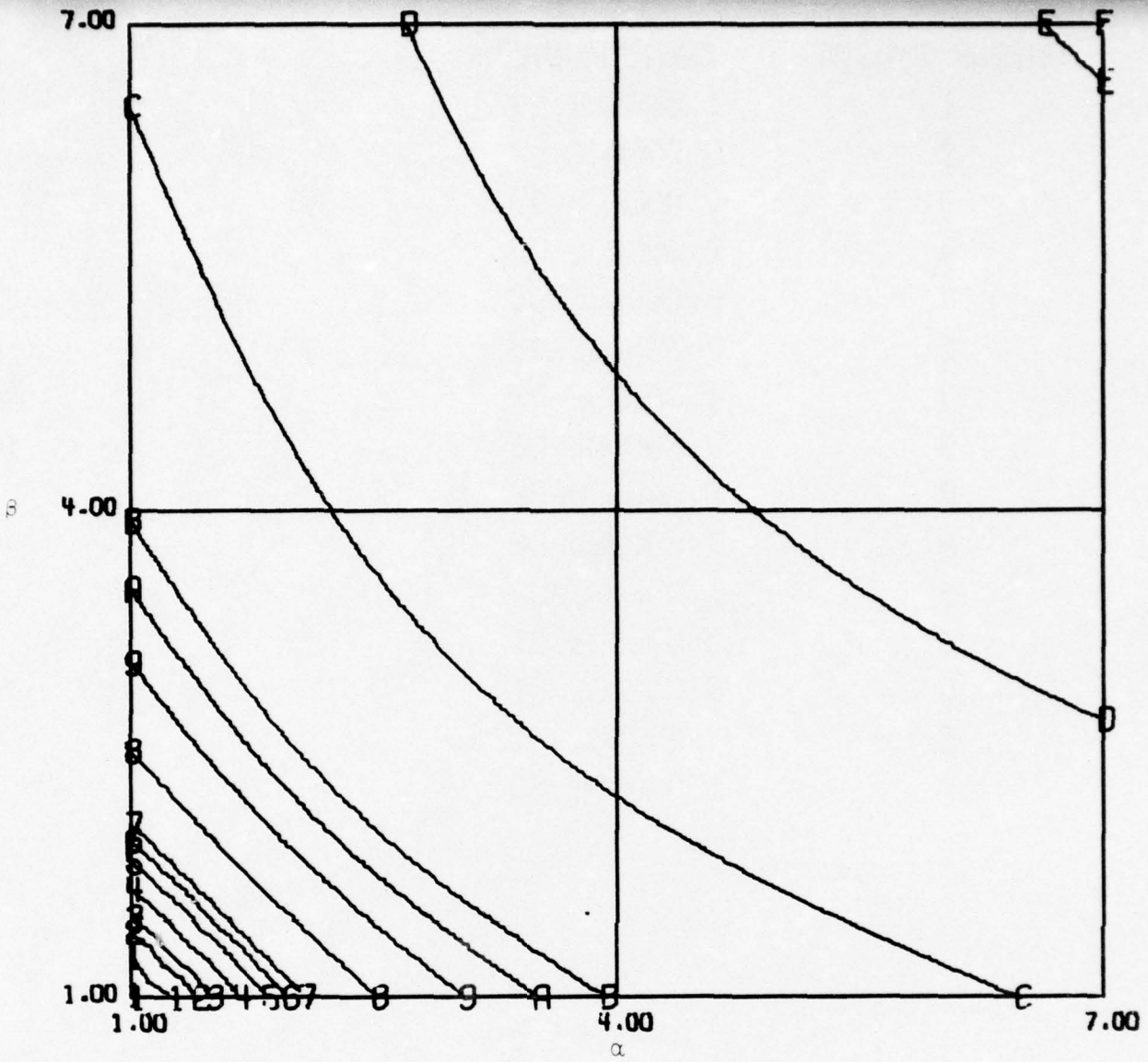


FIG. 6.3

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
THREE FILTERS

ONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.4

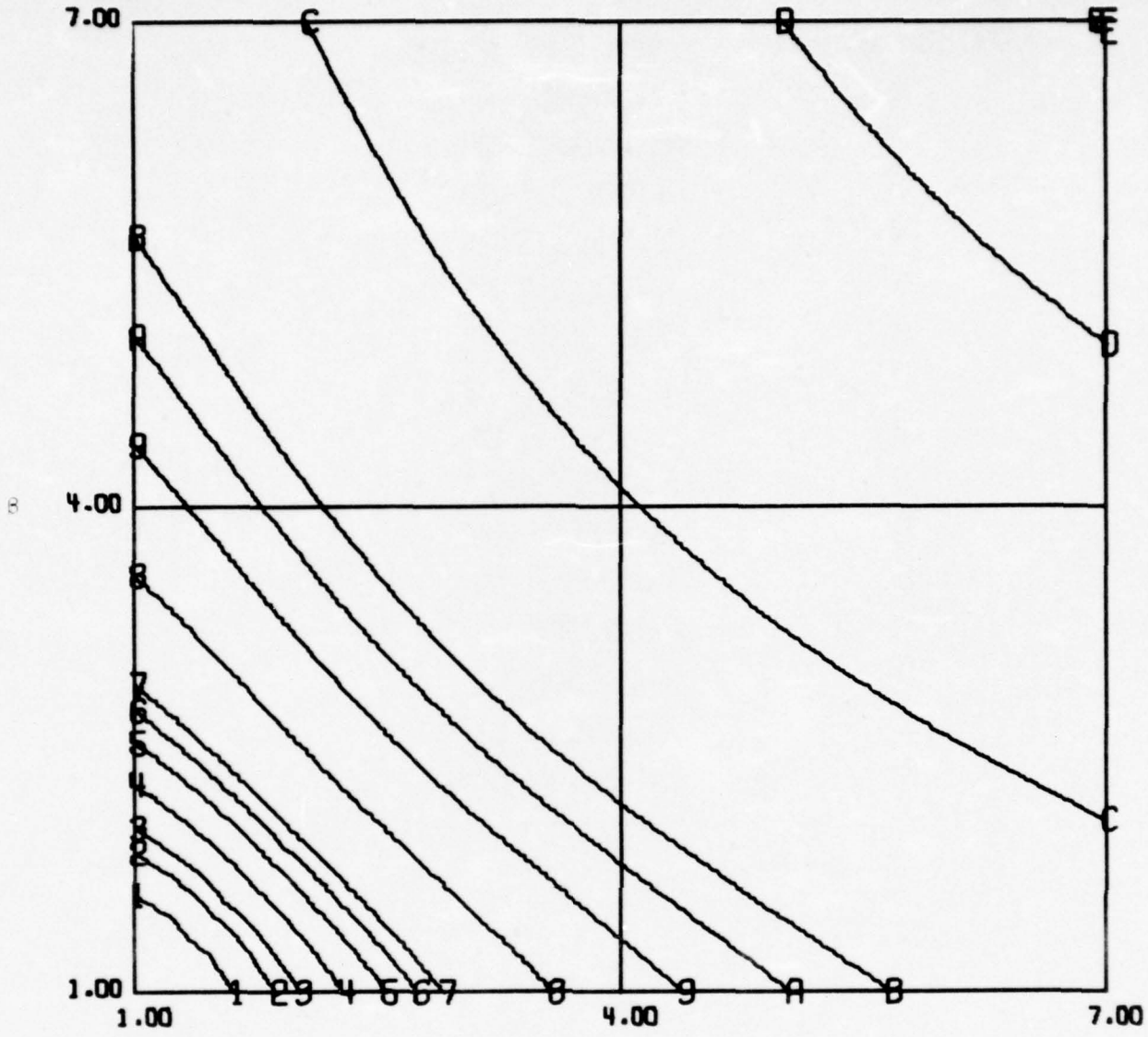


FIG. 6.4

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
FOUR FILTERS

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.5

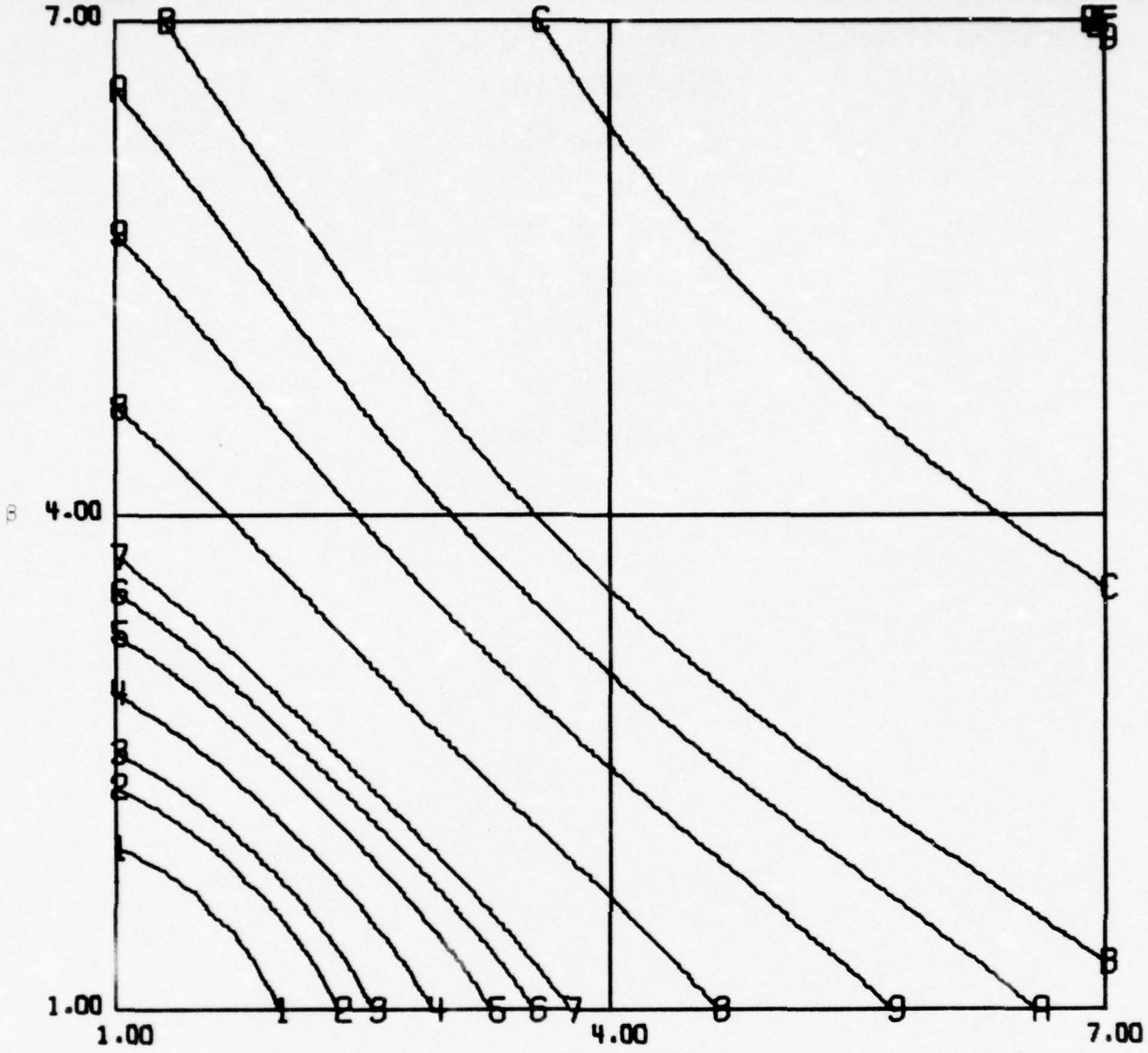


FIG. 6.5

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
LAGUERRE EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
FIVE FILTERS

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	6.50000E-01
2	7.00000E-01
3	7.50000E-01
4	8.00000E-01
5	8.50000E-01
6	9.00000E-01
7	9.50000E-01
8	1.00000E 00

TABLE 6.6

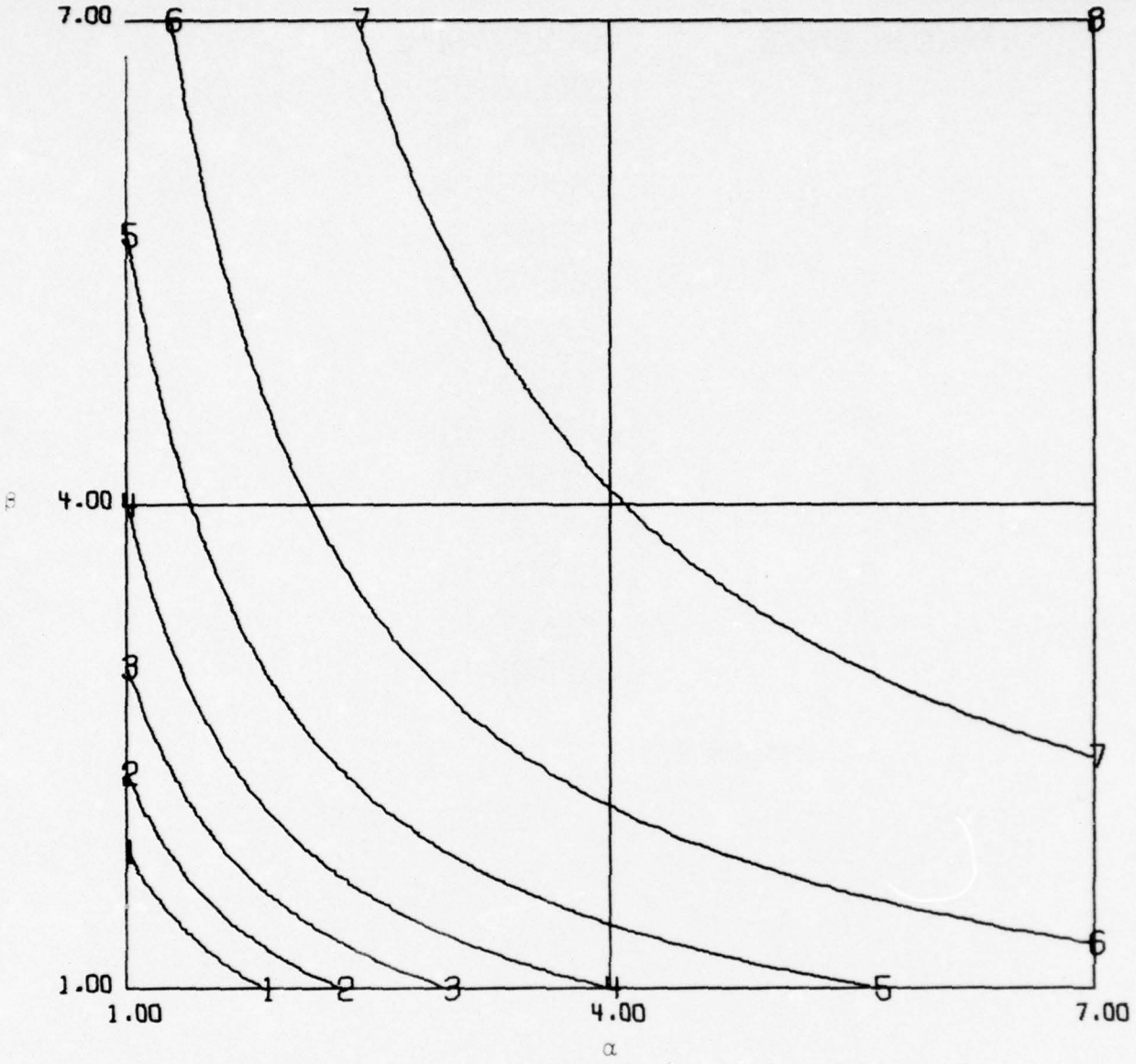


FIG. 6.6

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
ONE FILTER

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-02
2	2.50000E-02
3	5.00000E-02
4	7.50000E-02
5	1.00000E-01
6	2.00000E-01
7	3.00000E-01
8	4.00000E-01
9	5.00000E-01
A	6.00000E-01
B	7.00000E-01
C	8.00000E-01
D	9.00000E-01

TABLE 6.7

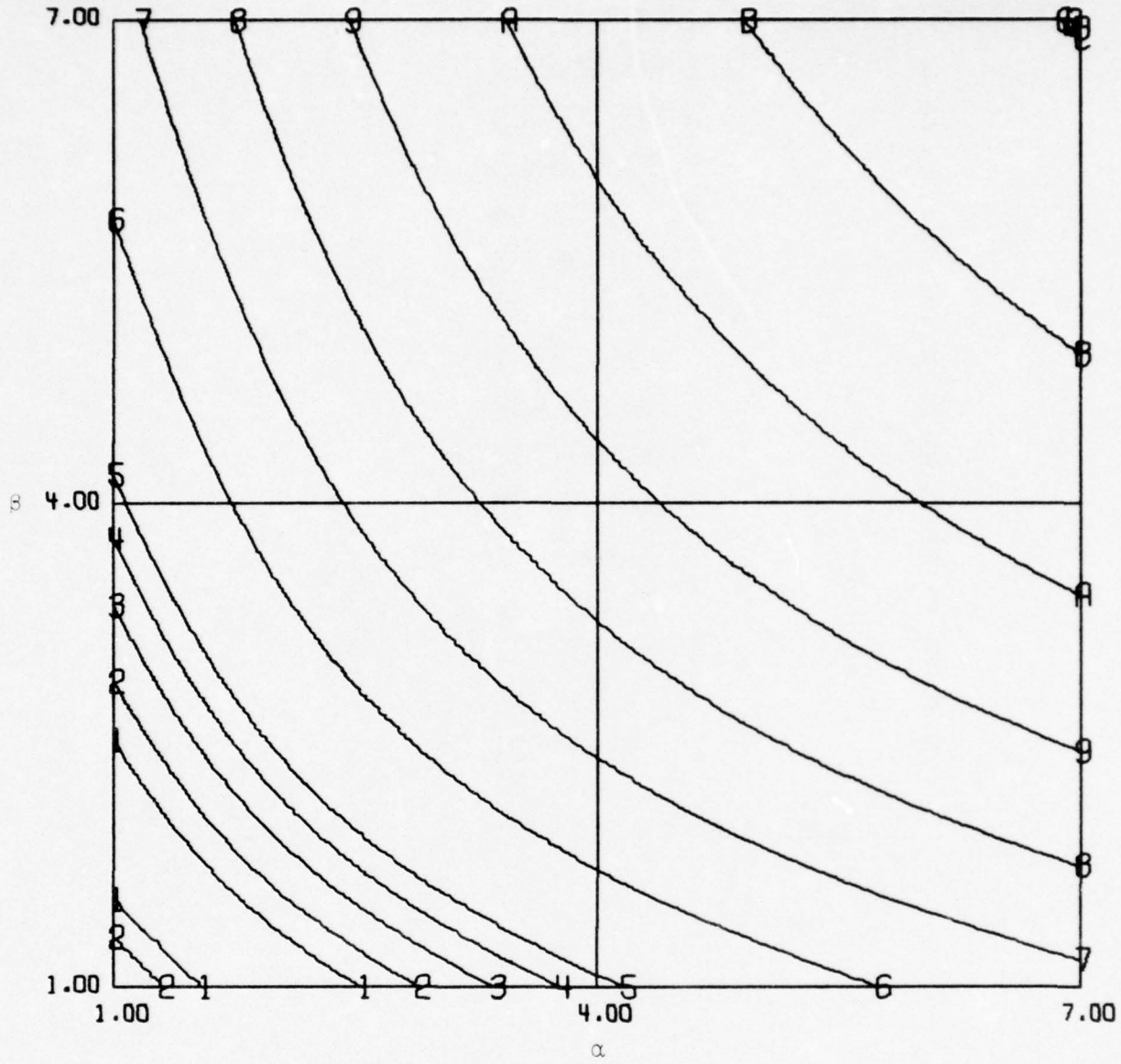


FIG. 6.7

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
TWO FILTERS

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.8

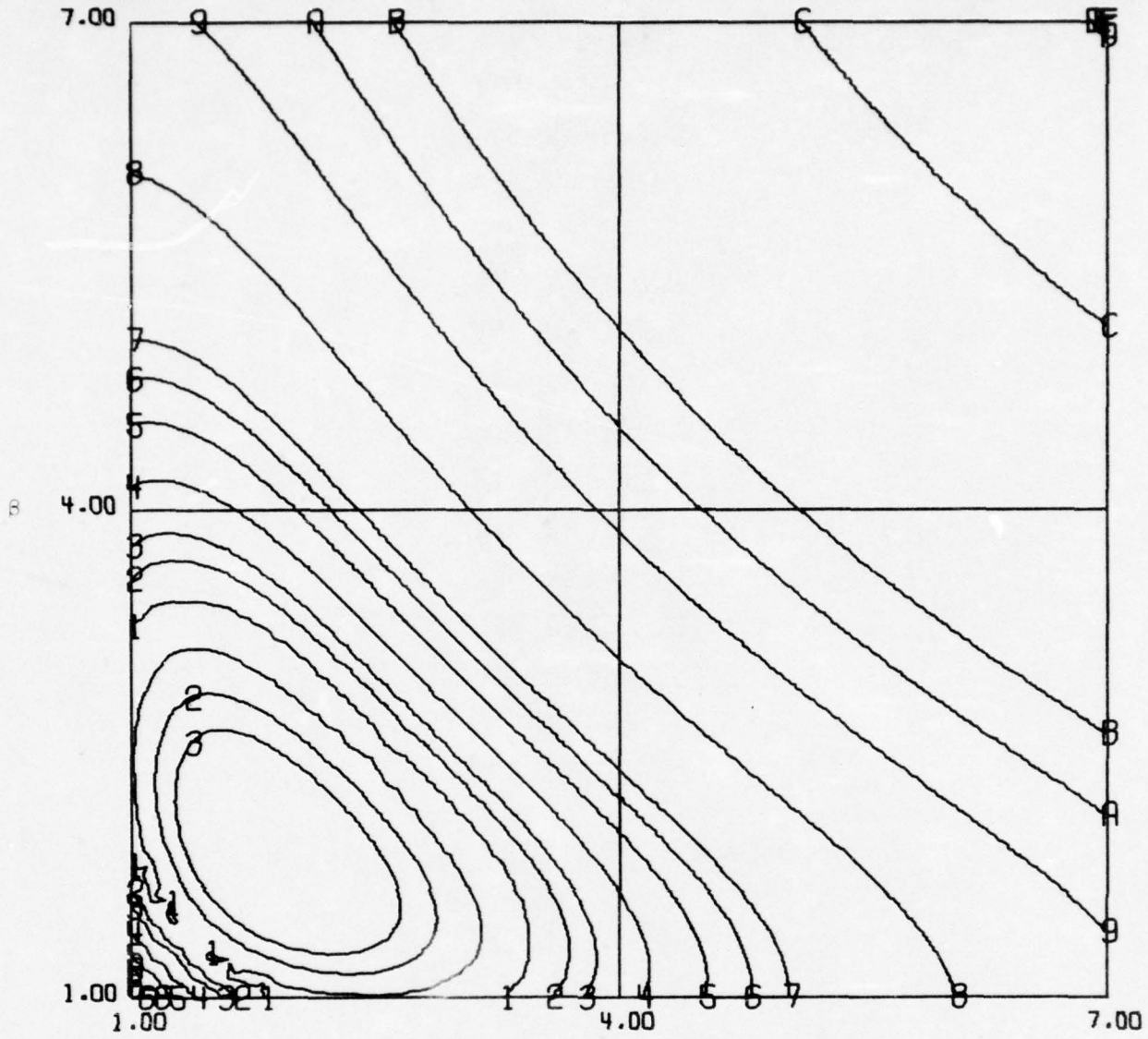


FIG. 6.8

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
THREE FILTERS

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.9

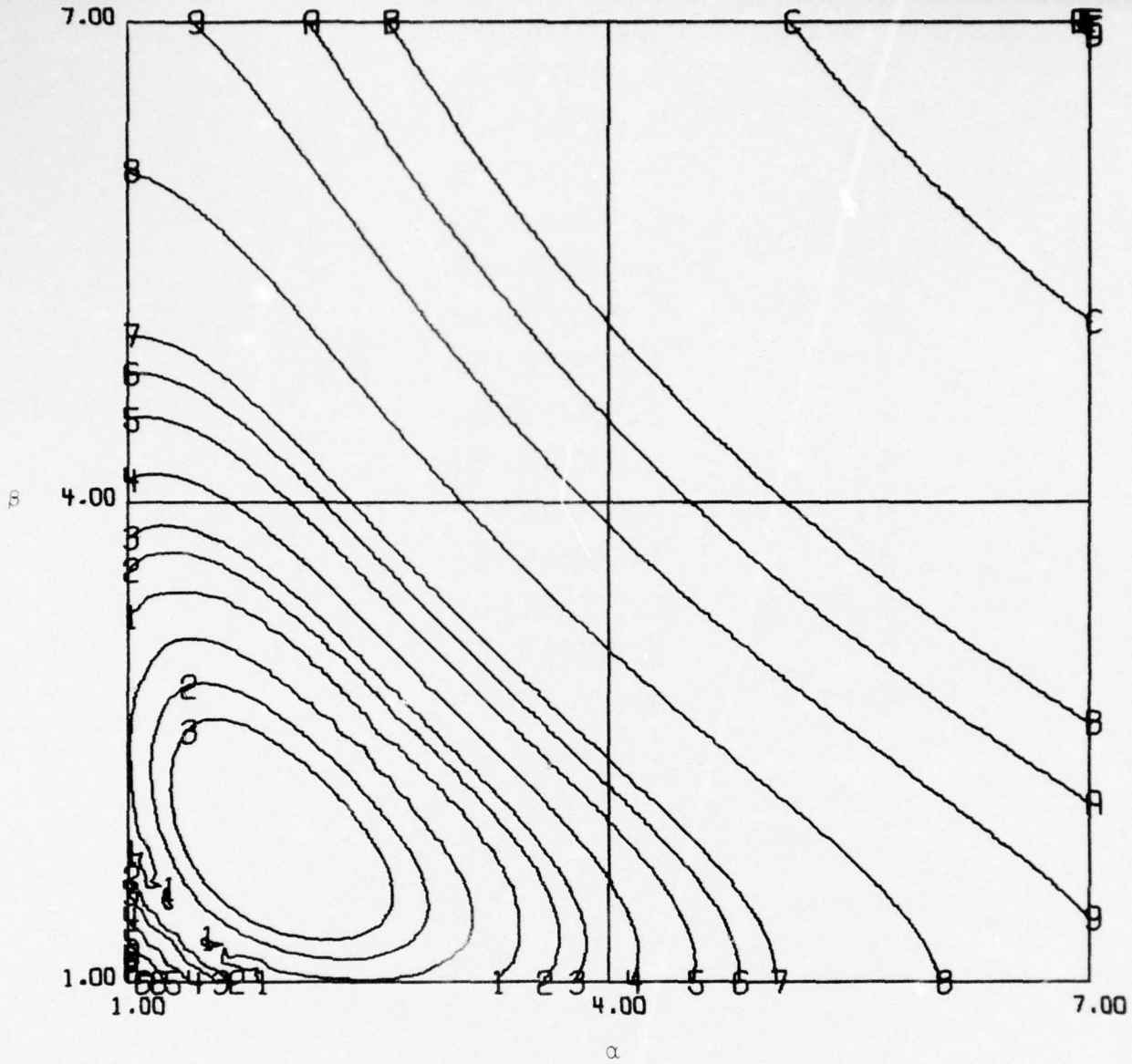


FIG. 6.9

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
FOUR FILTERS

TRACOR

CONTOUR SYMBOL	CONTOUR VALUE
1	1.00000E-04
2	5.00000E-04
3	1.00000E-03
4	2.50000E-03
5	5.00000E-03
6	7.50000E-03
7	1.00000E-02
8	2.50000E-02
9	5.00000E-02
A	7.50000E-02
B	1.00000E-01
C	2.50000E-01
D	5.00000E-01
E	7.50000E-01
F	1.00000E 00

TABLE 6.10

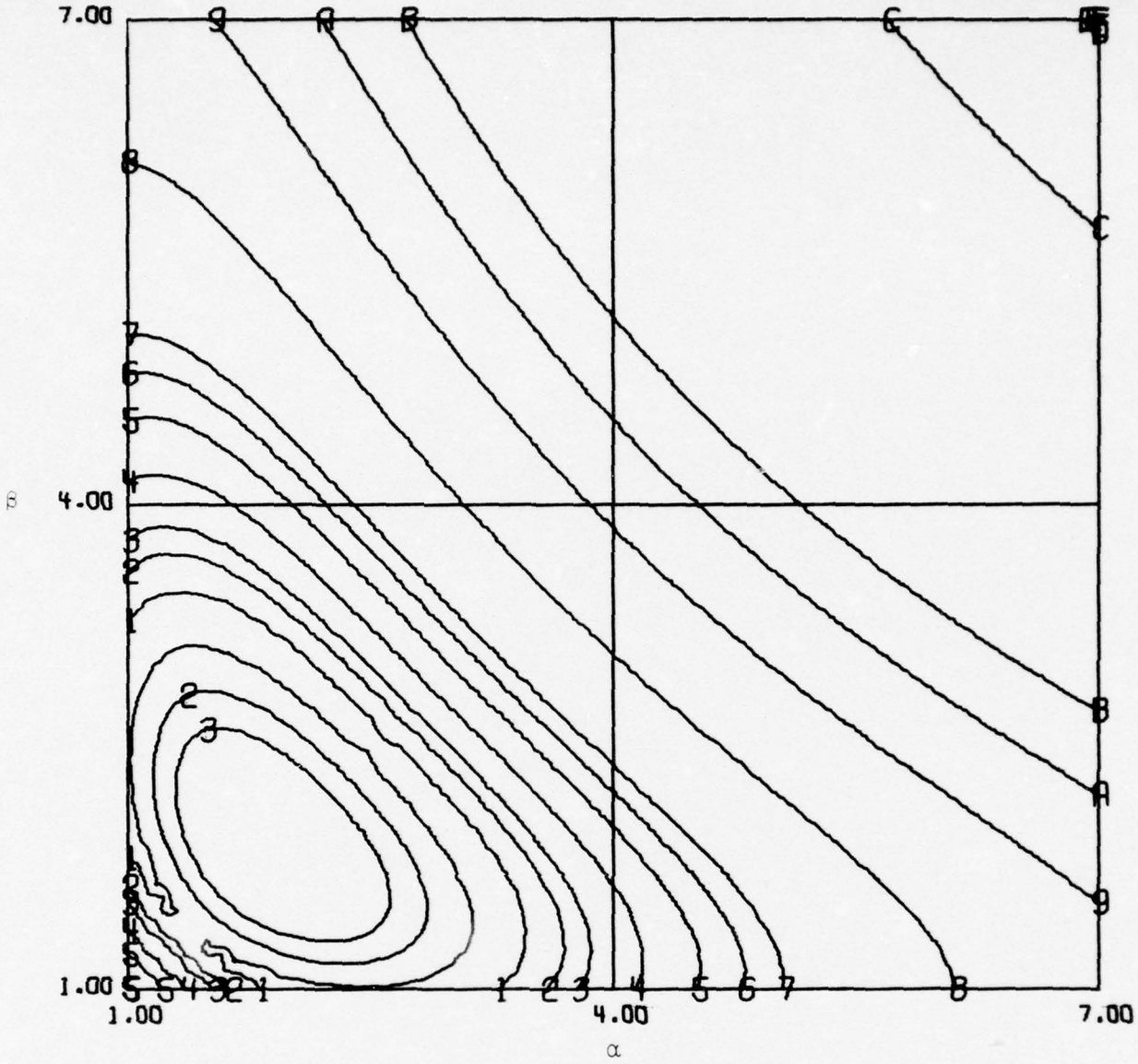


FIG. 6.10

CONTOUR PLOT OF RELATIVE ERROR AS IT VARIES WITH ALPHA AND BETA
ARBITRARY EXPANSION OF F(T)=(A*EXP(-AT)-B*EXP(-BT))/(A-B)
FIVE FILTERS

7. The functions under consideration here is

$$f(t) = 1,\ 0 < t < A\ .$$

The format of the plots for this function is the same as for the function

$$f(t) = t$$

which was described above.

7.1 $f(t) = 1,\ 0 < t < A$ Laguerre series

$$E_i = A$$

$$C_1 = \sqrt{2}(1 - e^{-A})$$

$$C_2 = \sqrt{2}\ [1 - e^{-A}(2A+1)]$$

$$C_3 = \sqrt{2}\ [1 - e^{-A}(2A^2+1)]$$

$$C_4 = \sqrt{2}\ [1 - e^{-A}(4/3A^3 - 2A^2 + 2A + 1)]$$

$$C_5 = \sqrt{2}\ [1 - e^{-A}(2/3A^4 - 8/3A^3 + 4A^2 + 1)]$$

7.2 $f(t) = 1$ Arbitrary series

$$E_i = A$$

$$C_1 = \sqrt{2}(1 - e^{-A})$$

$$C_2 = 1 - e^{-A}(4 - 3e^{-A})$$

$$C_3 = \sqrt{6}\left[1/3 - 3e^{-A} + 6e^{-2A} - \frac{10}{3}e^{-3A}\right]$$

$$C_4 = 2\sqrt{2}\left[1/4 - 4e^{-A} + 15e^{-2A} - 20e^{-3A} + \frac{35}{4}e^{-4A}\right]$$

$$C_5 = \sqrt{10}\left[1/5 - 5e^{-A} + 30e^{-2A} - 70e^{-3A} + 70e^{-4A} - \frac{126}{5}e^{-5A}\right]$$

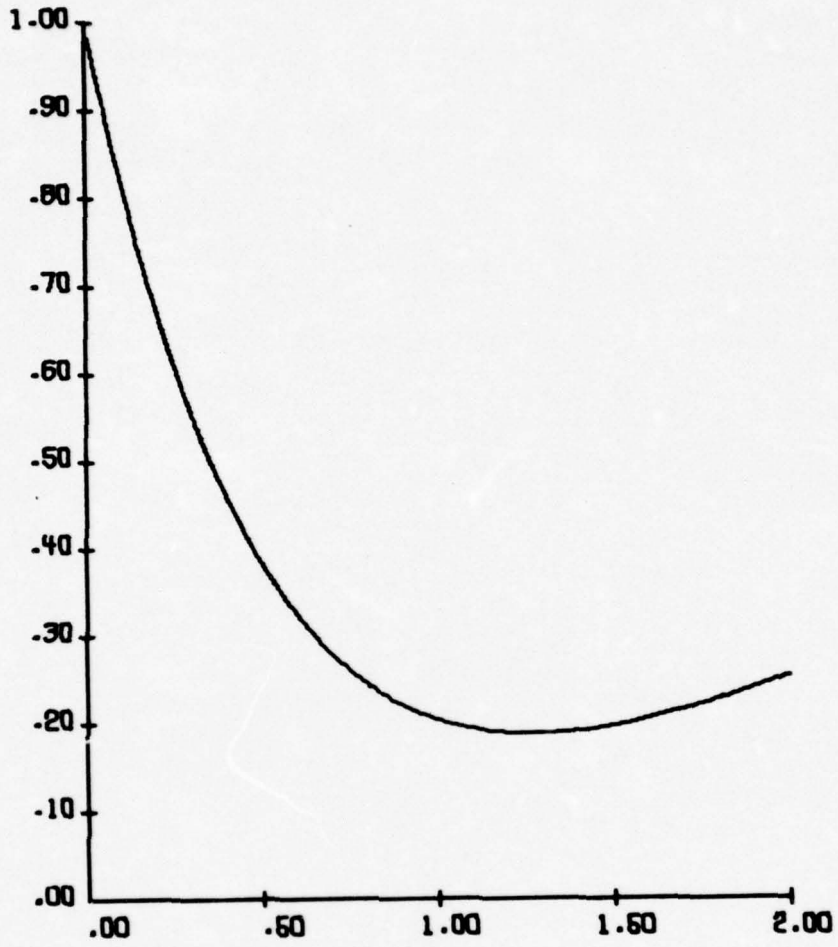


FIG. 7.1 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=1
LAGUERRE CASE

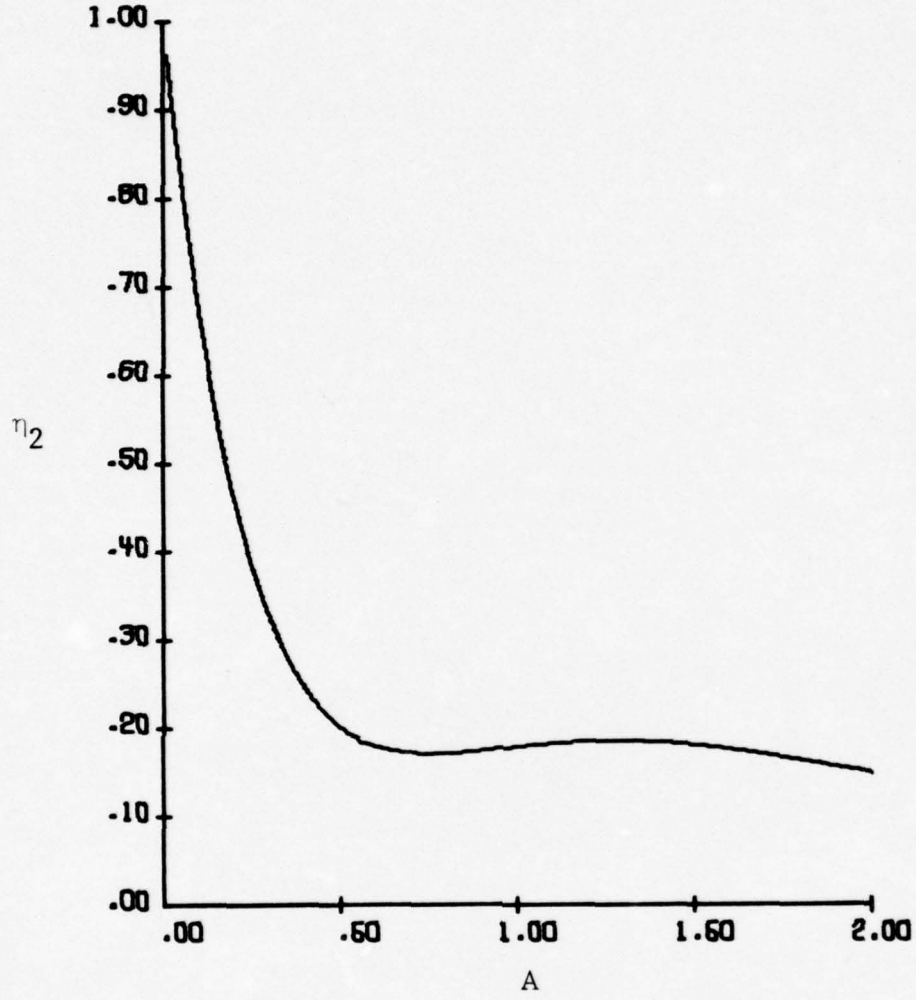


FIG. 7.2 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=1
LAGUERRE CASE

TRACOR

6-70-84

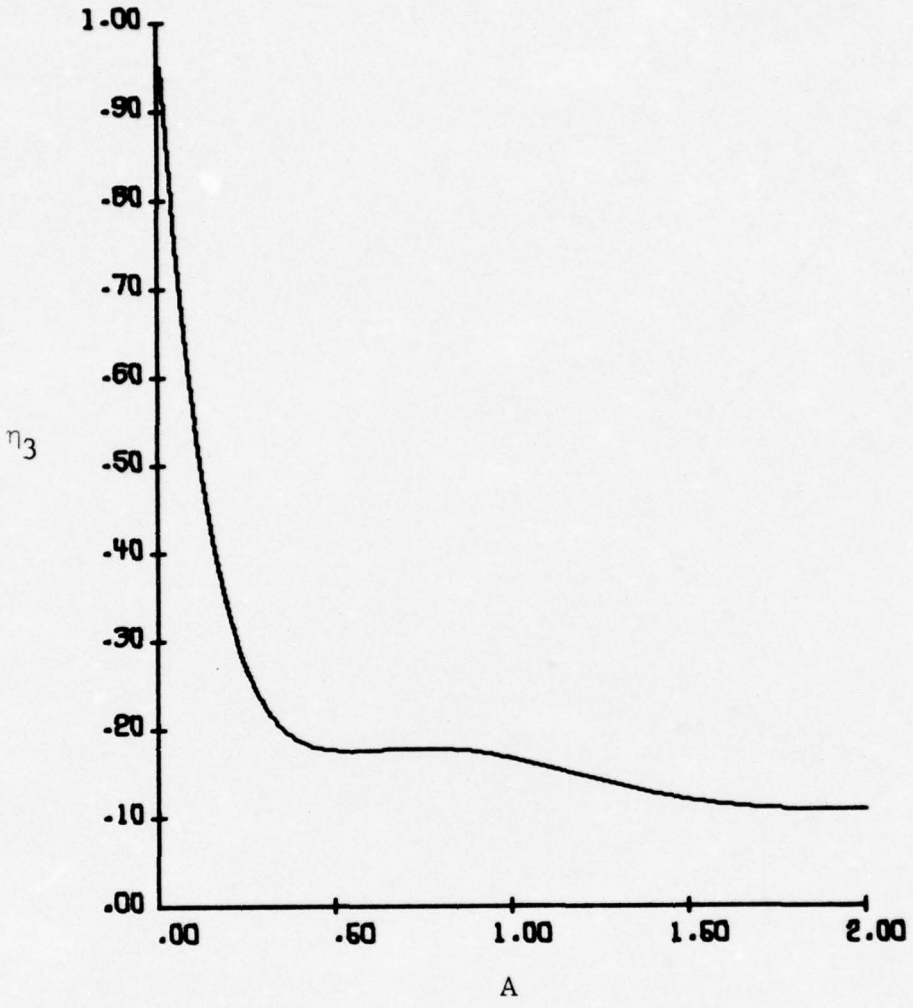


FIG. 7.3 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=1
LAGUERRE CASE

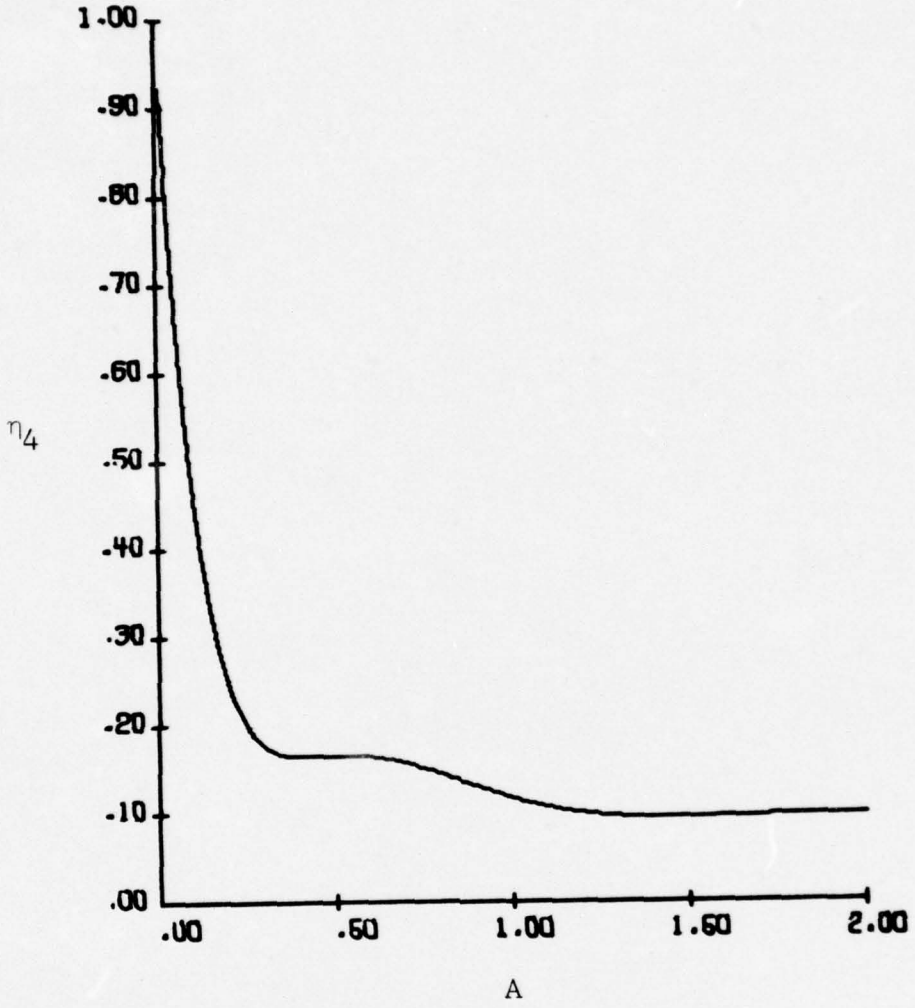


FIG. 7.4 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=1
LAGUERRE CASE

TRACOR

5-70-86

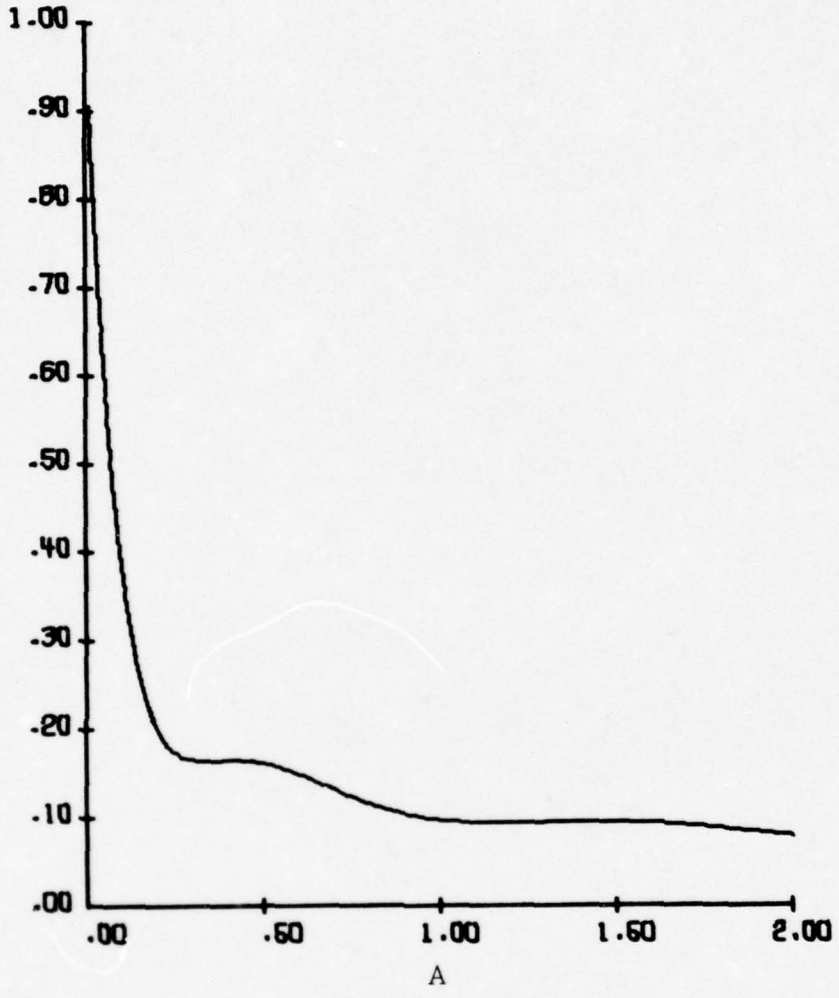


FIG. 7.5 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=1
LAGUERRE CASE

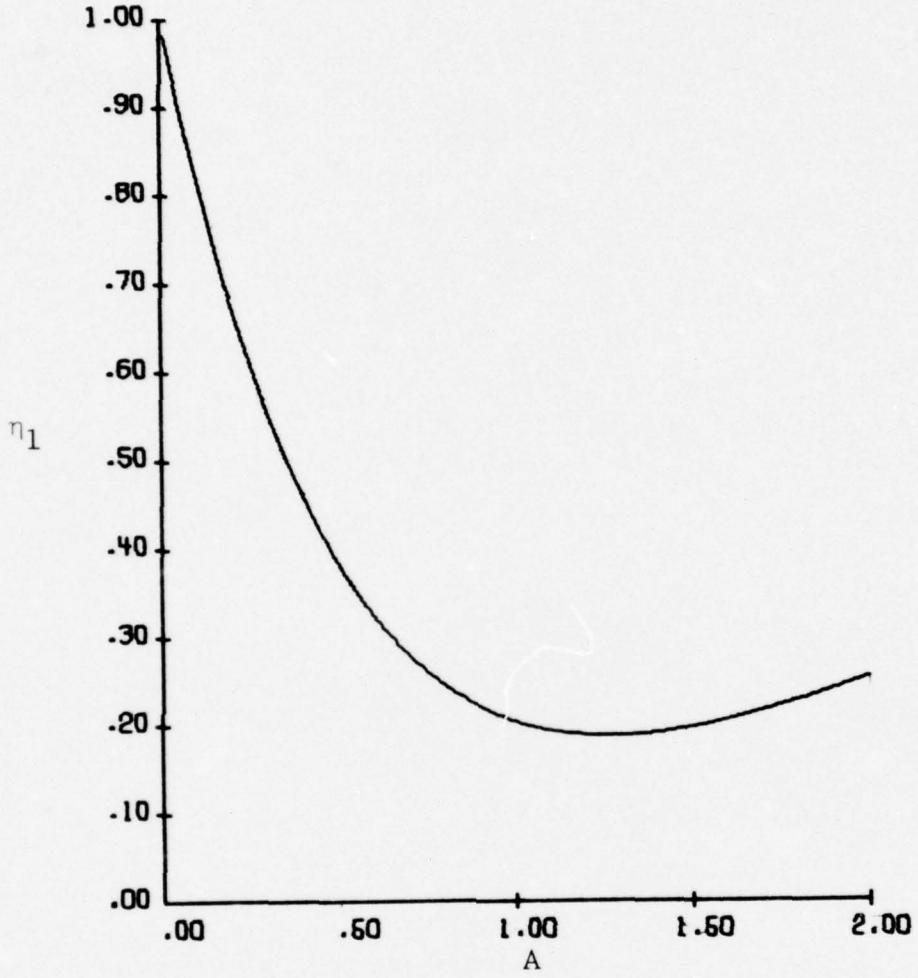


FIG. 7.6 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=1
ARBITRARY CASE

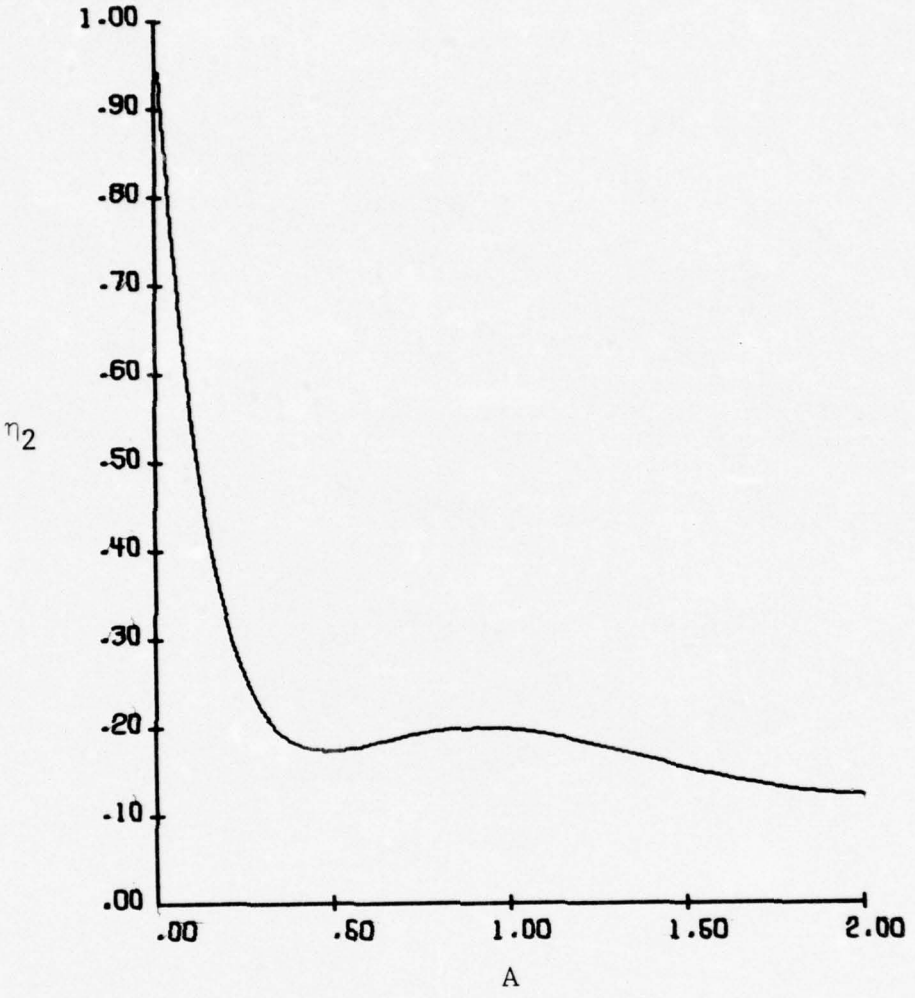


FIG. 7.7 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=1
ARBITRARY CASE

TRACOR

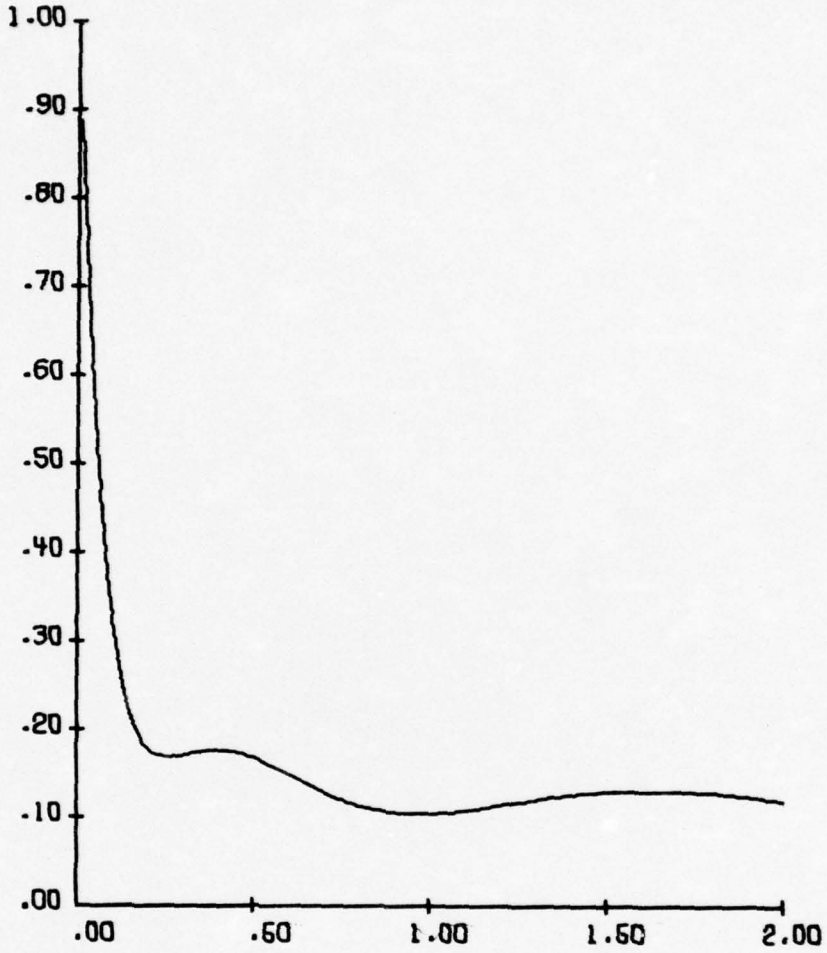


FIG. 7.8 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=1
ARBITRARY CASE

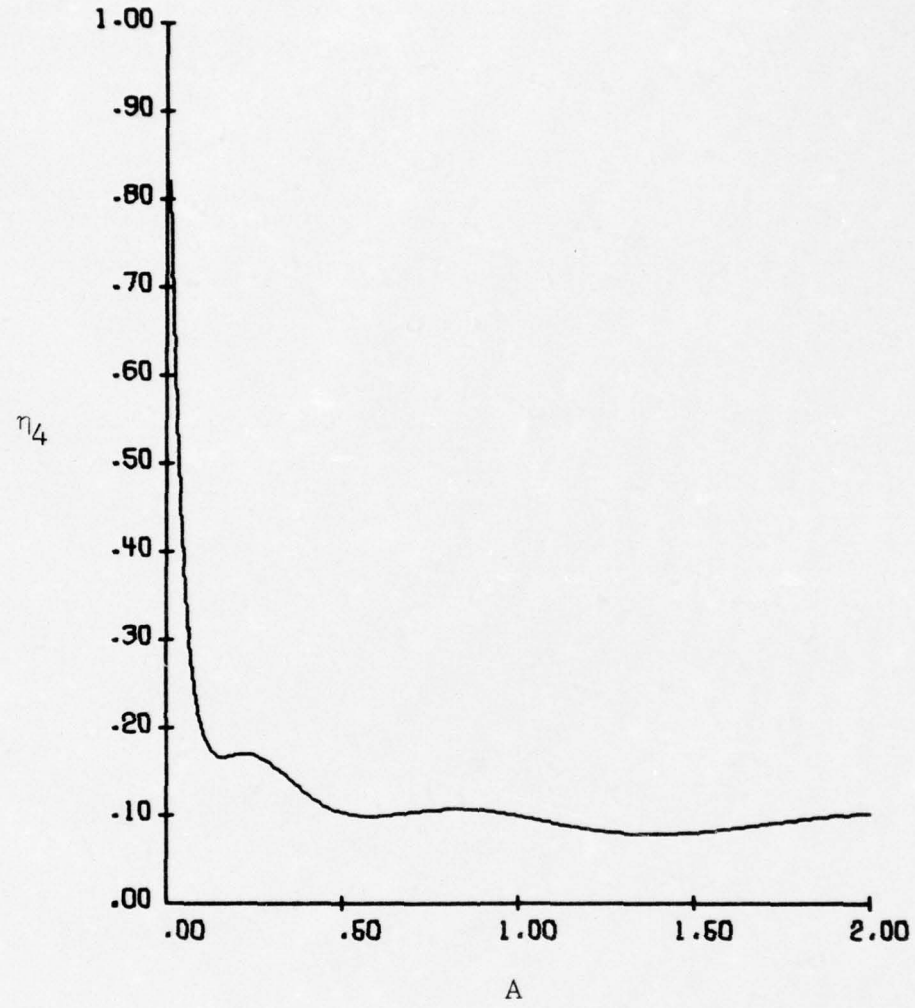


FIG. 7.9 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=1
ARBITRARY CASE

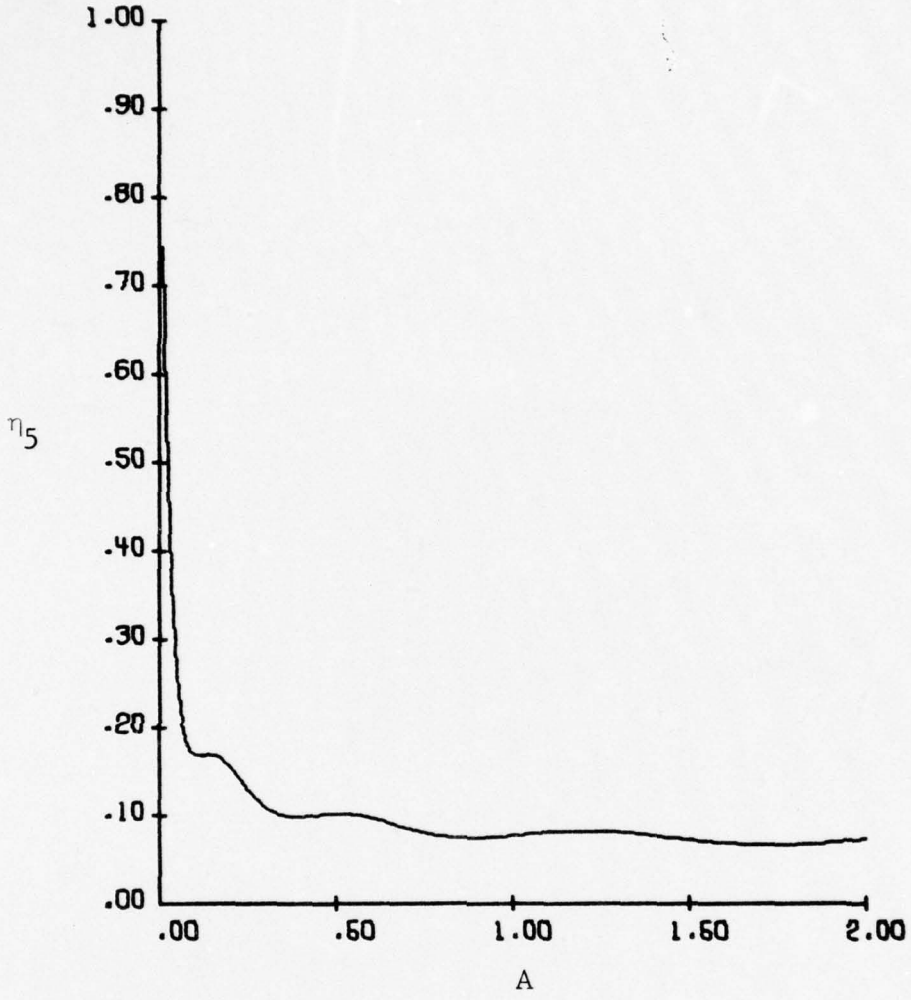


FIG. 7.10 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=1
ARBITRARY CASE

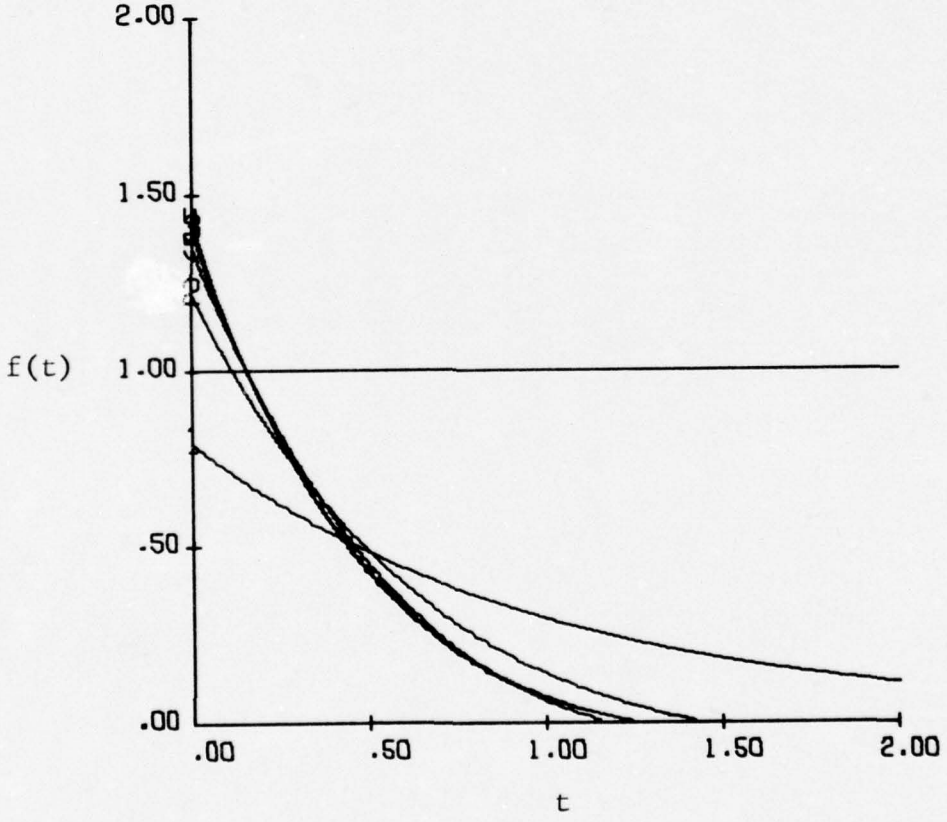


FIG. 7.11 F(T)=1 AND FIVE APPROXIMATIONS FOR A=.5
LAGUERRE CASE

TRACOR

6-70-93

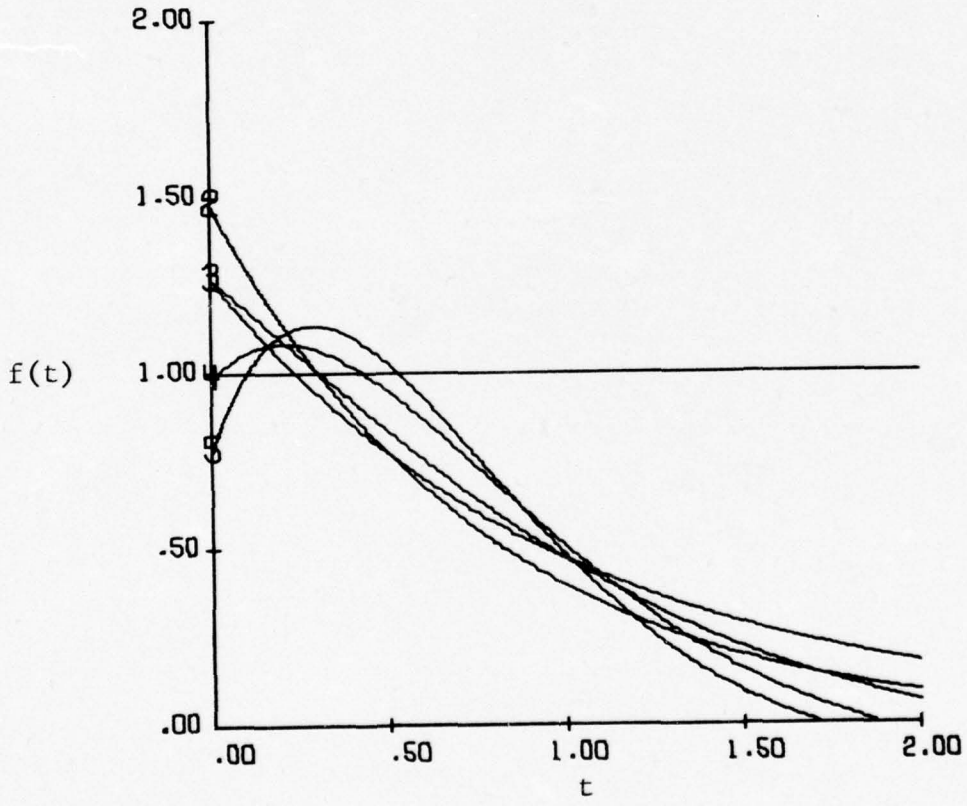


FIG. 7.12 F(T)=1 AND FIVE APPROXIMATIONS FOR A=1.0 LAGUERRE CASE

TRACOR

6-70-94

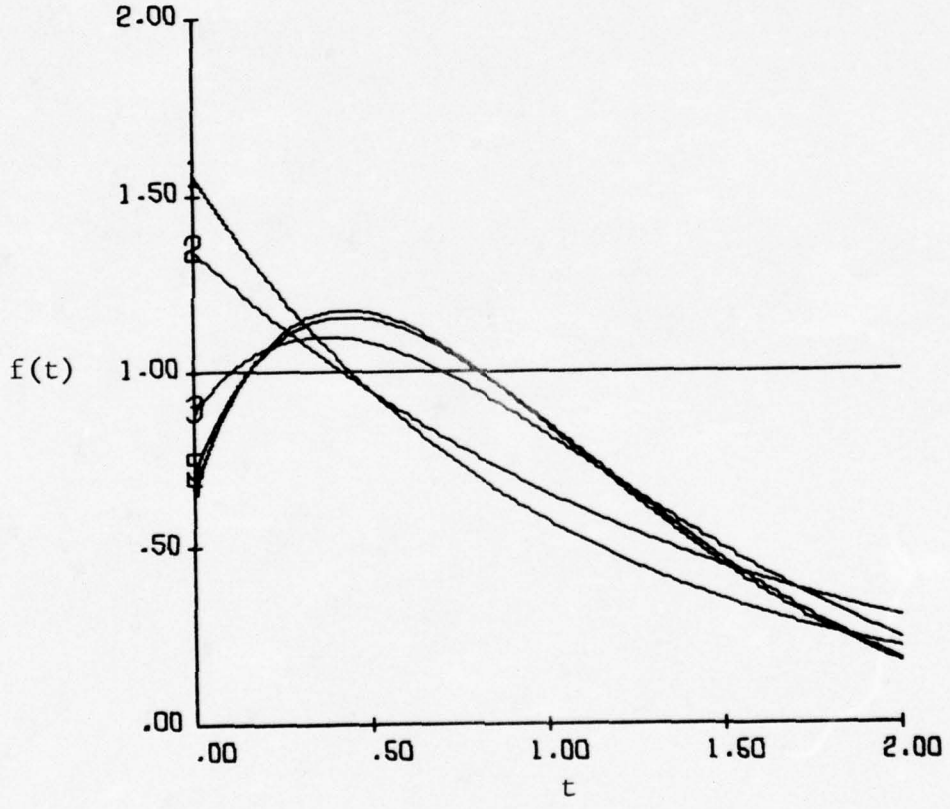


FIG. 7.13 F(T)=1 AND FIVE APPROXIMATIONS FOR A=1.5 LAGUERRE CASE

TRACOR

6-70-95

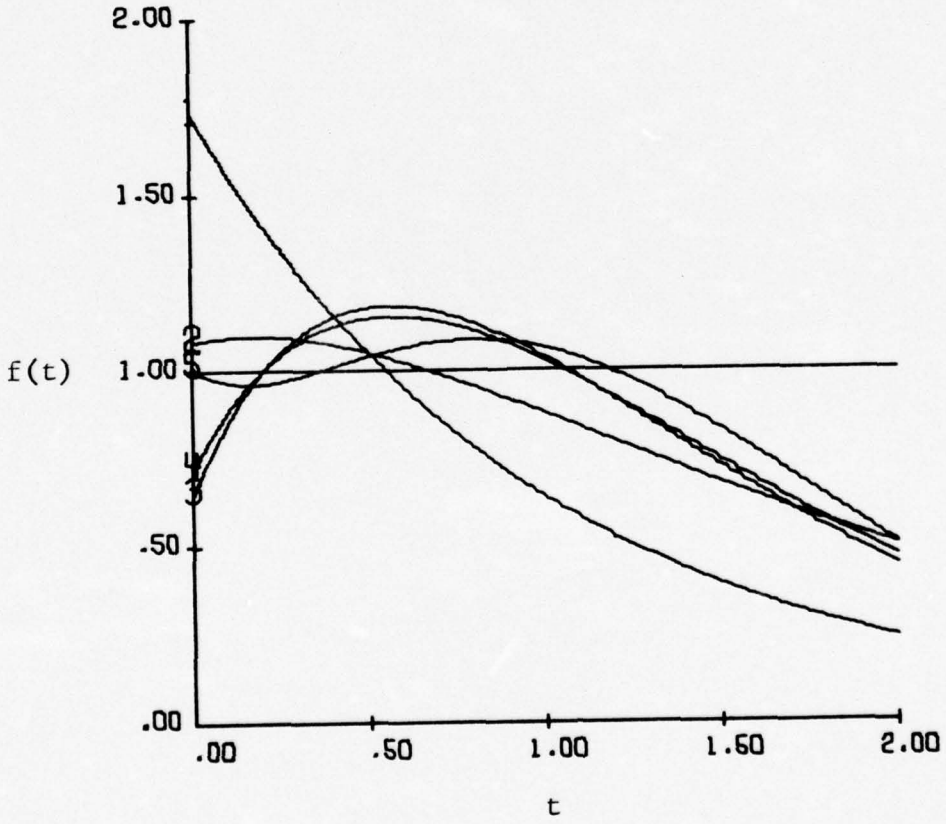


FIG. 7.14 F(T)=1 AND FIVE APPROXIMATIONS FOR A=2.0 LAGUERRE CASE

TRACOR

6-70-96

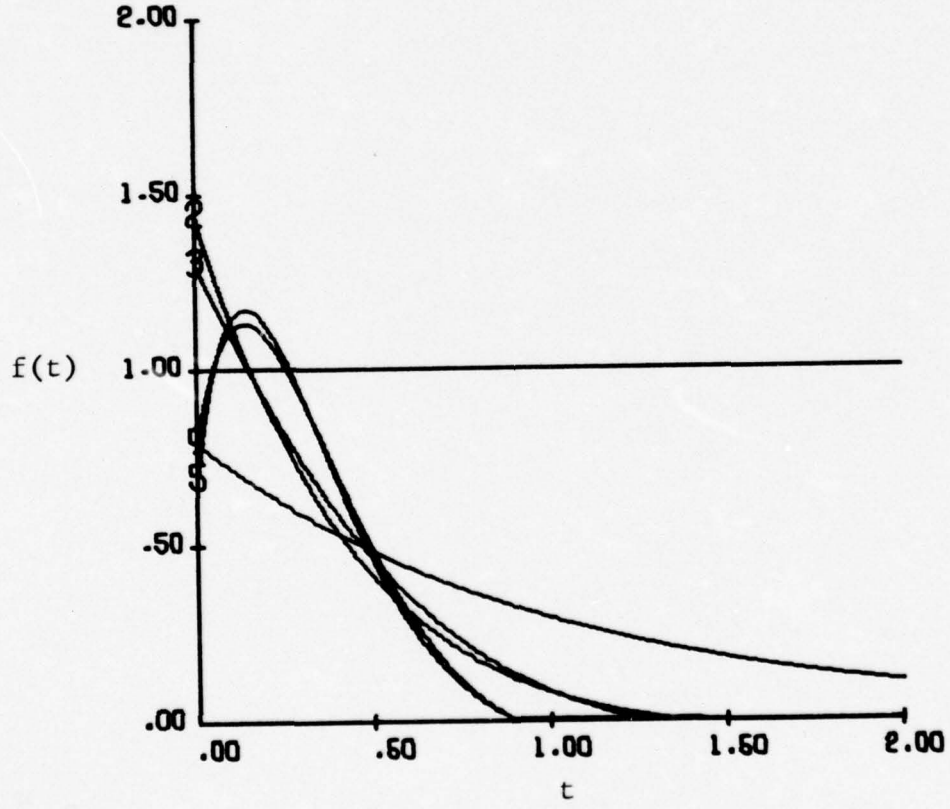


FIG. 7.15 F(T)=1 AND FIVE APPROXIMATIONS FOR A=.5
ARBITRARY CASE

TRACOR

6-70-97

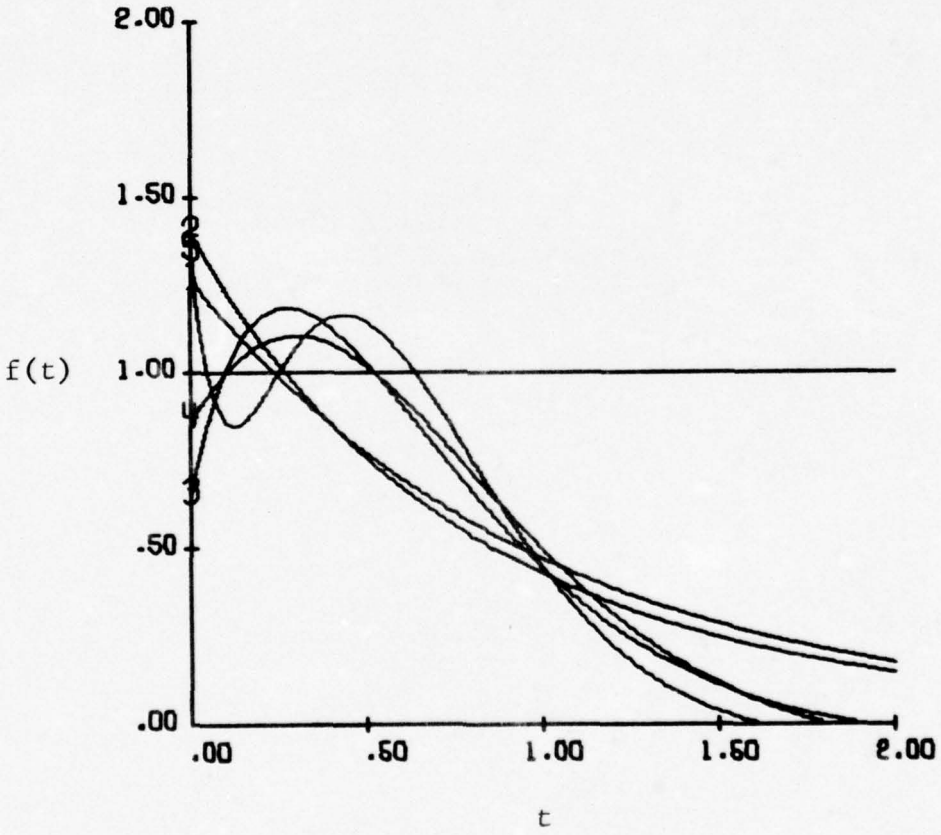


FIG. 7.16 F(T)=1 AND FIVE APPROXIMATIONS FOR A=1.0 ARBITRARY CASE

TRACOR
5-70-98

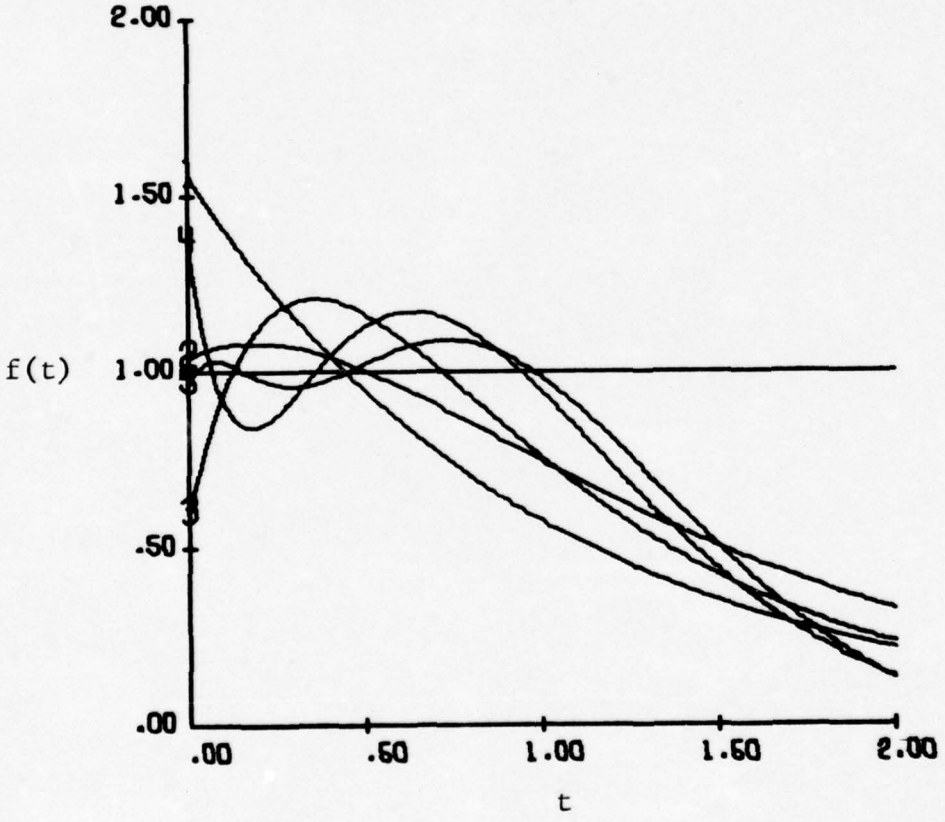


FIG. 7.17 F(T)=1 AND FIVE APPROXIMATIONS FOR A=1.5 ARBITRARY CASE

TRACOR

6-70-99

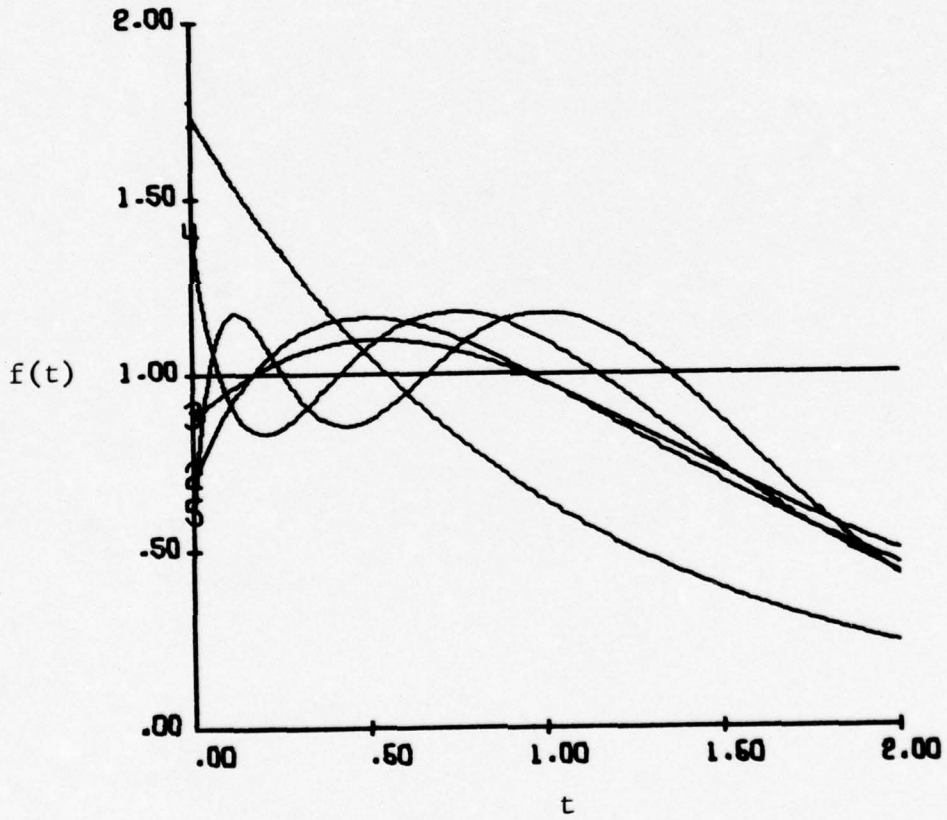


FIG. 7.18 F(T)=1 AND FIVE APPROXIMATIONS FOR A=2.0 ARBITRARY CASE

8. The function under consideration here is

$$f(t) = \frac{1}{A}(A-t),\ 0 < t < A\ .$$

The format of the plots for this function is the same as for the function f(t) = t which was described above.

8.1 $f(t) = 1/A(A-t)$ $0 < t < A$ Laguerre series

$$E_i = \frac{A}{3}$$

$$C_1 = \frac{\sqrt{2}}{A}\left[e^{-A} + A-1\right]$$

$$C_2 = \frac{\sqrt{2}}{A}\left[e^{-A}(2A+3) + A-3)\right]$$

$$C_3 = \frac{\sqrt{2}}{A}\left[e^{-A}(2A^2 + 4A + 5) + A-5\right]$$

$$C_4 = \frac{\sqrt{2}}{A}\left[e^{-A}(4/3A^3 + 2A^2 + 6A + 7) + A-7\right]$$

$$C_5 = \frac{\sqrt{2}}{A}\left[e^{-A}(2/3A^4 + 4A^2 + 8A + 9) + A-9\right]$$

8.2 $f(t) = \frac{1}{A}(A-t)$ Arbitrary series

$$E_i = \frac{A}{3}$$

$$C_1 = \frac{\sqrt{2}}{A}\left[e^{-A} + A - 1\right]$$

$$C_2 = \frac{2}{A}\left[2e^{-A} - 3/4e^{-2A} + \frac{1}{2}A - 5/4\right]$$

$$C_3 = \frac{\sqrt{6}}{A}\left[3e^{-A} - 3e^{-2A} + \frac{10}{9}e^{-3A} + \frac{A}{3} - \frac{10}{9}\right]$$

$$C_4 = \frac{2\sqrt{2}}{A}\left[4e^{-A} - \frac{15}{2}e^{-2A} + \frac{20}{3}e^{-3A} - \frac{35}{16}e^{-4A} + \frac{A}{4} - \frac{47}{48}\right]$$

$$C_5 = \frac{\frac{1}{2}\sqrt{10}}{A}\left[10e^{-A} - 30e^{-2A} + \frac{420}{9}e^{-3A} - 35e^{-4A} + \frac{252}{25}e^{-5A}\right.$$

$$\left. + \frac{2}{5}A - \frac{131}{75}\right]$$

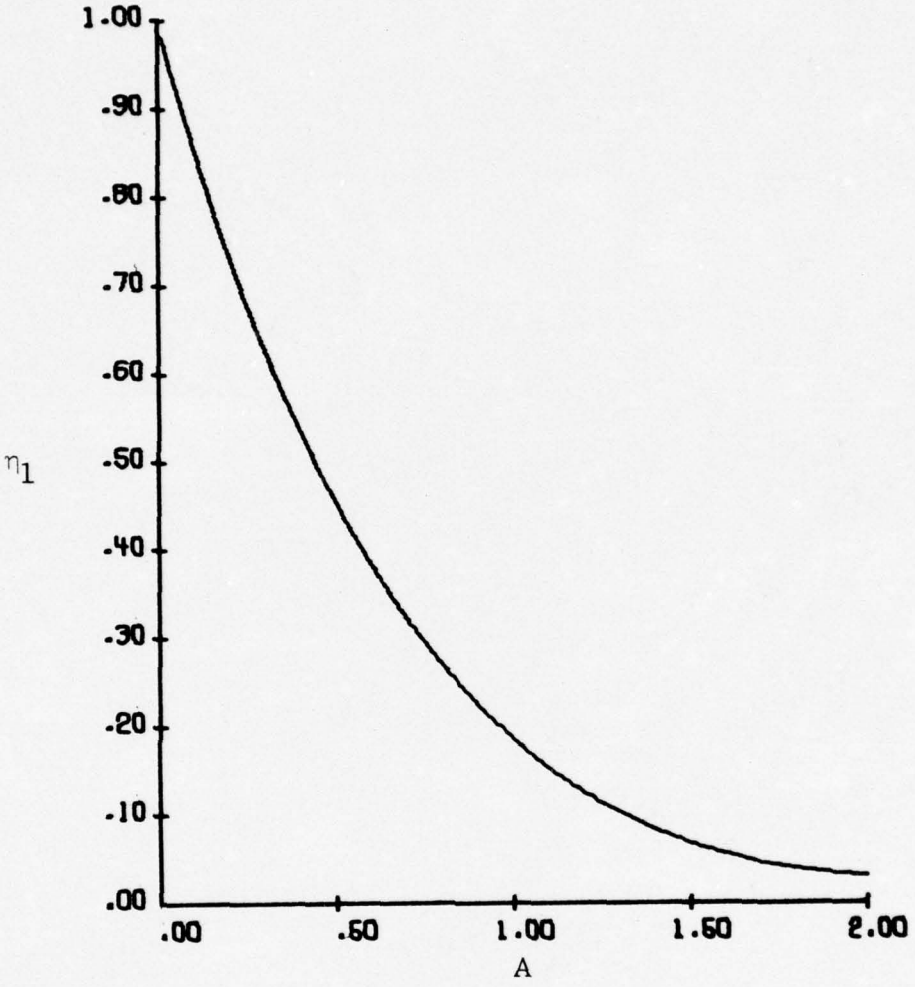


FIG. 8.1 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=1/A*(A-T)
LAGUERRE CASE

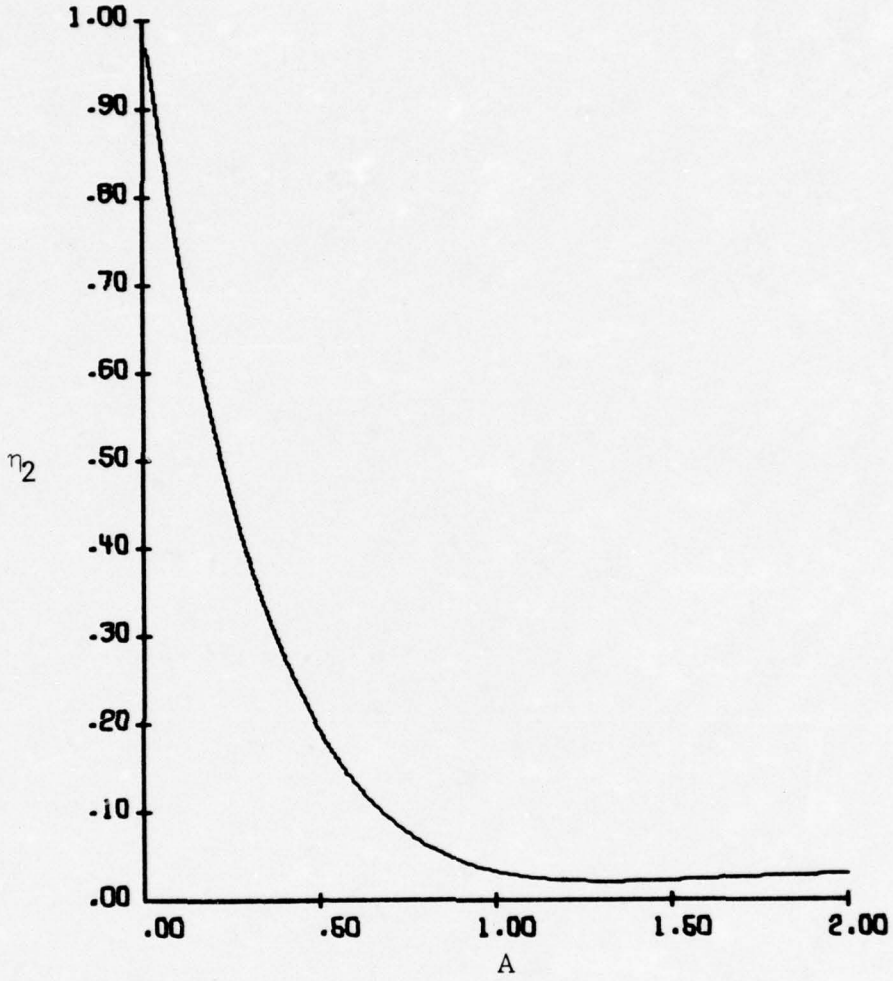


FIG. 8.2 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=1/A*(A-T)
LAGUERRE CASE

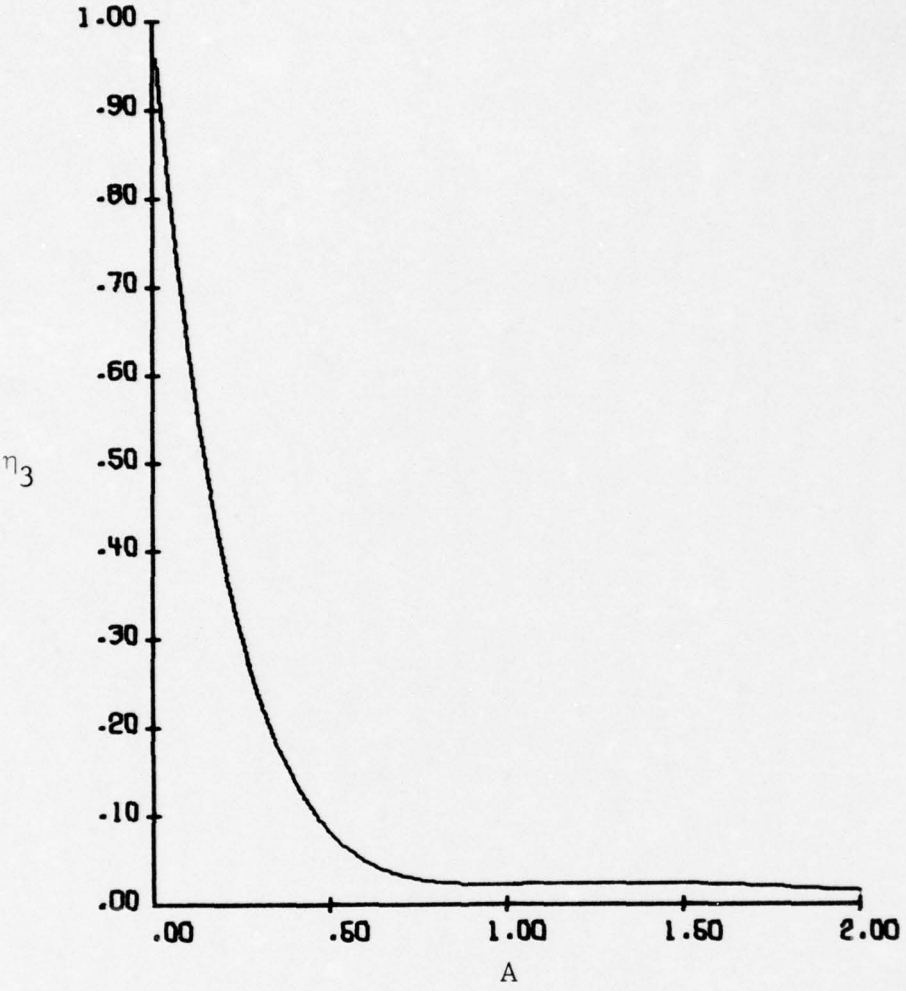


FIG. 8.3 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=1/A*(A-T)
LAGUERRE CASE

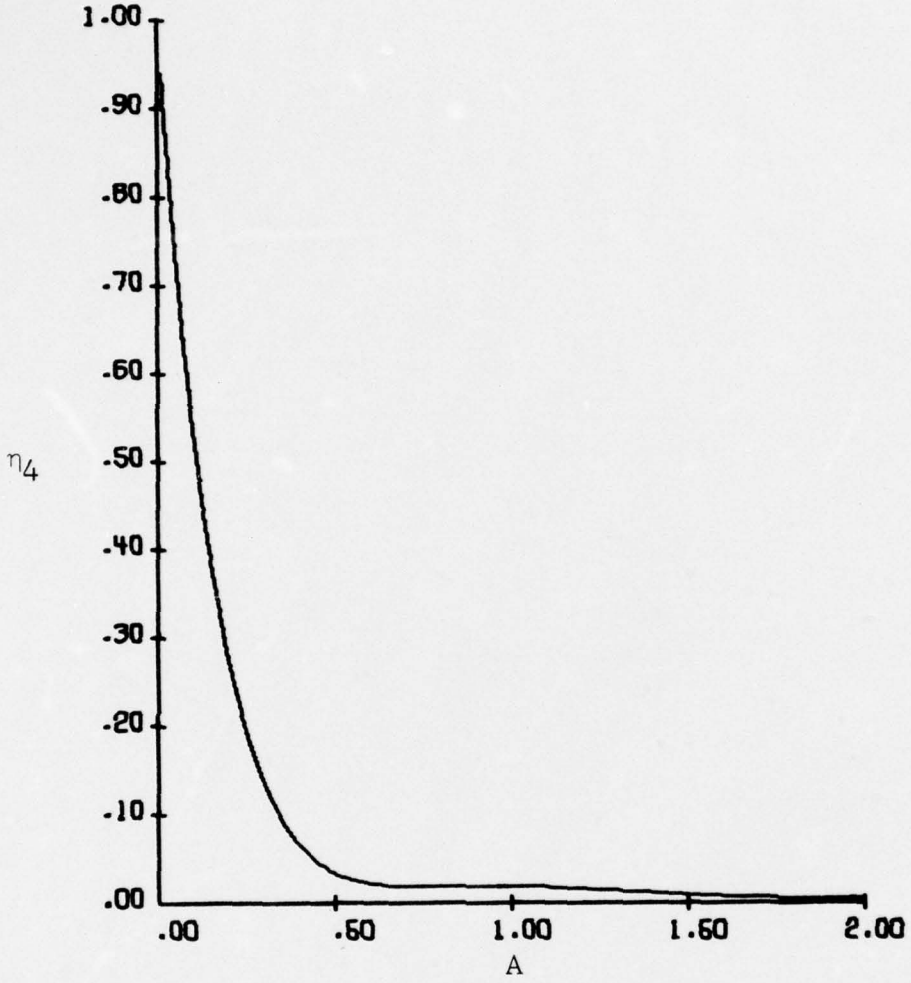


FIG. 8.4 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=1/A*(A-T)
LAGUERRE CASE

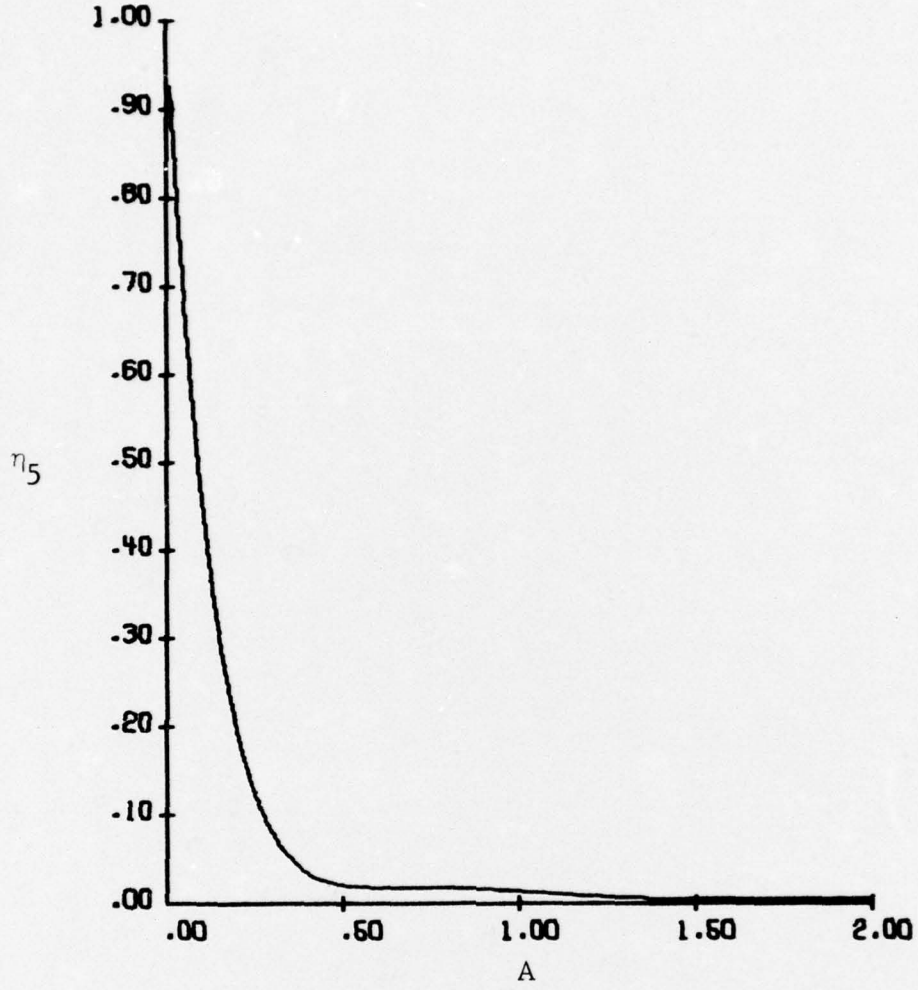


FIG. 8.5 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=1/A*(A-T)
LAGUERRE CASE

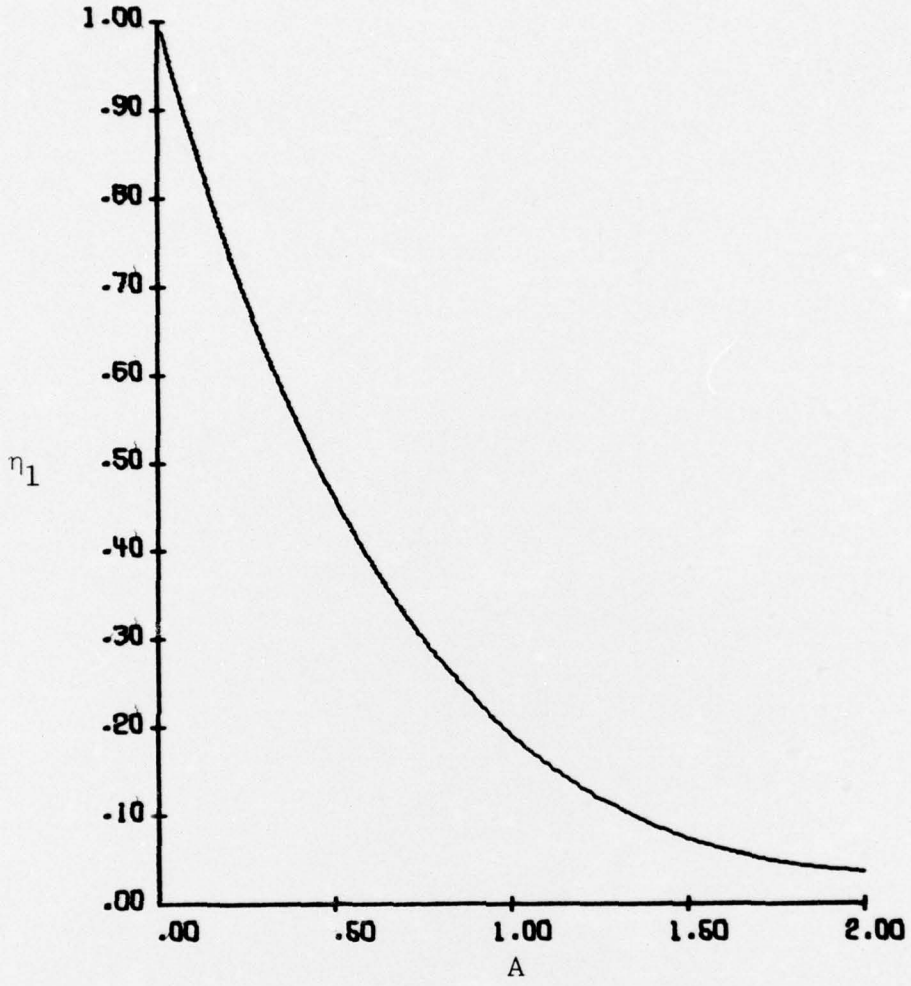


FIG. 8.6 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 1
F(T)=1/A*(A-T)
ARBITRARY CASE

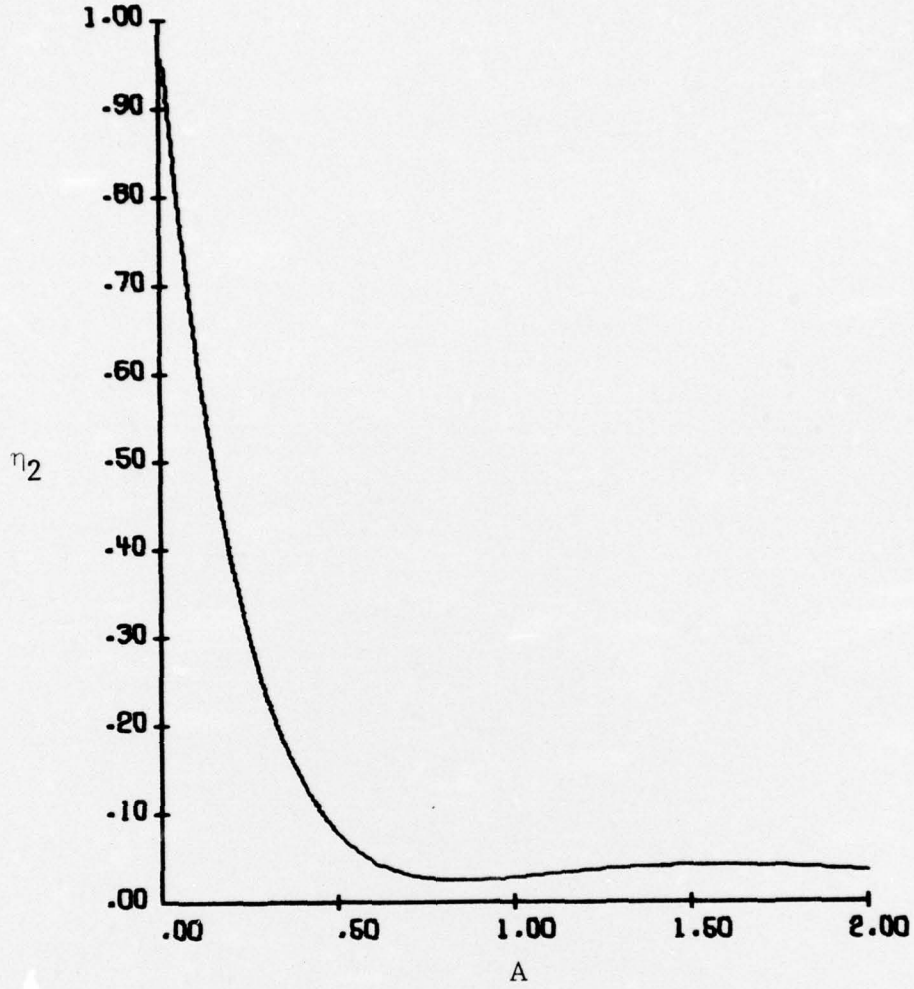


FIG. 8.7 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 2
F(T)=1/A*(A-T)
ARBITRARY CASE

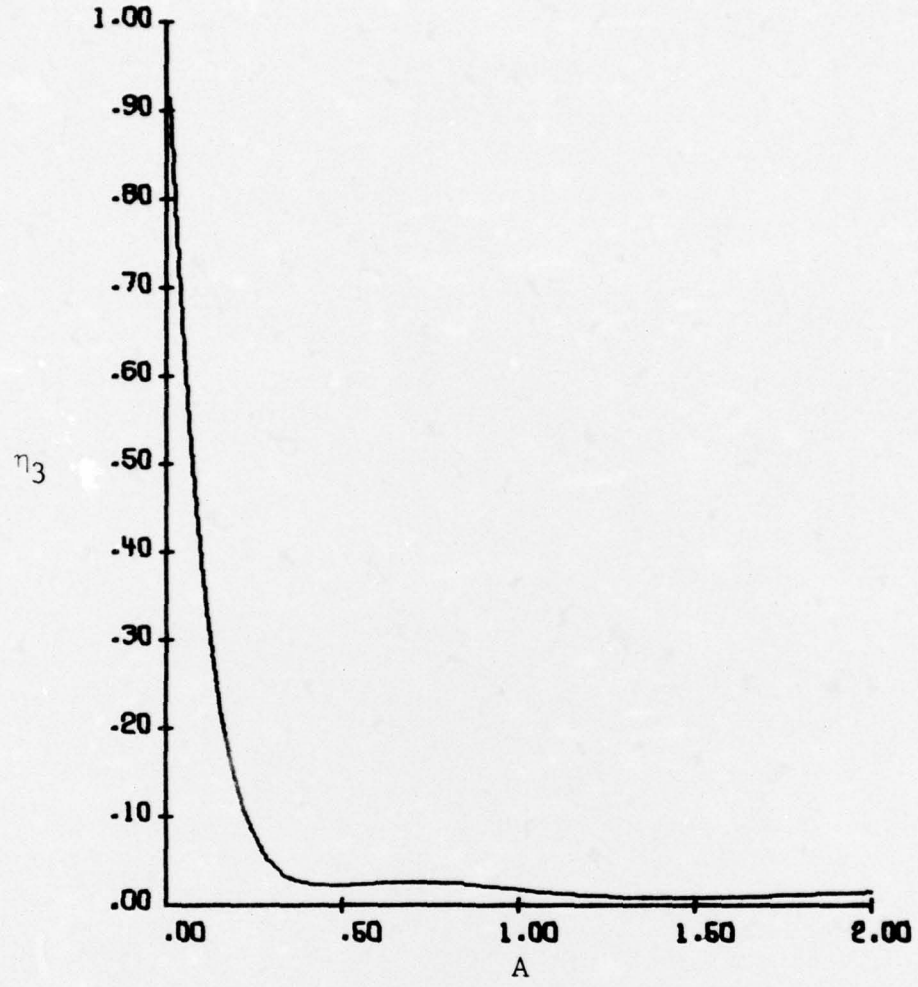


FIG. 8.8 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 3
F(T)=1/A*(A-T)
ARBITRARY CASE

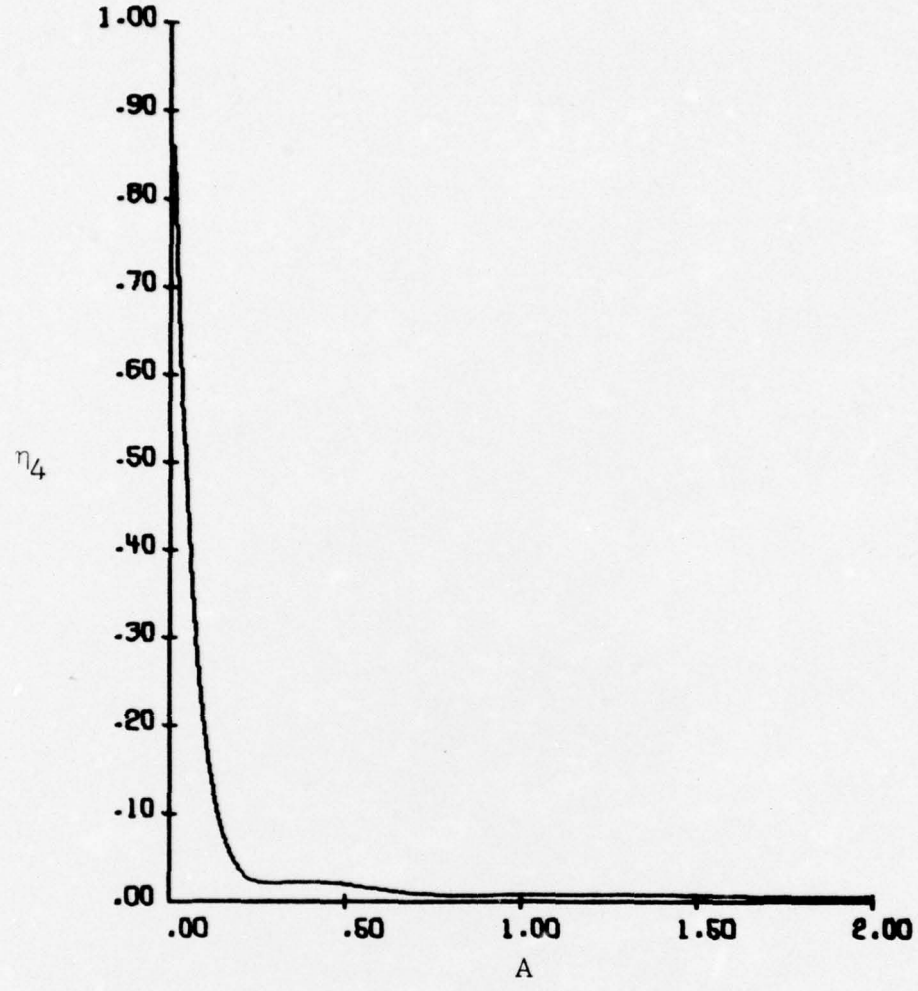


FIG. 8.9 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 4
F(T)=1/A*(A-T)
ARBITRARY CASE

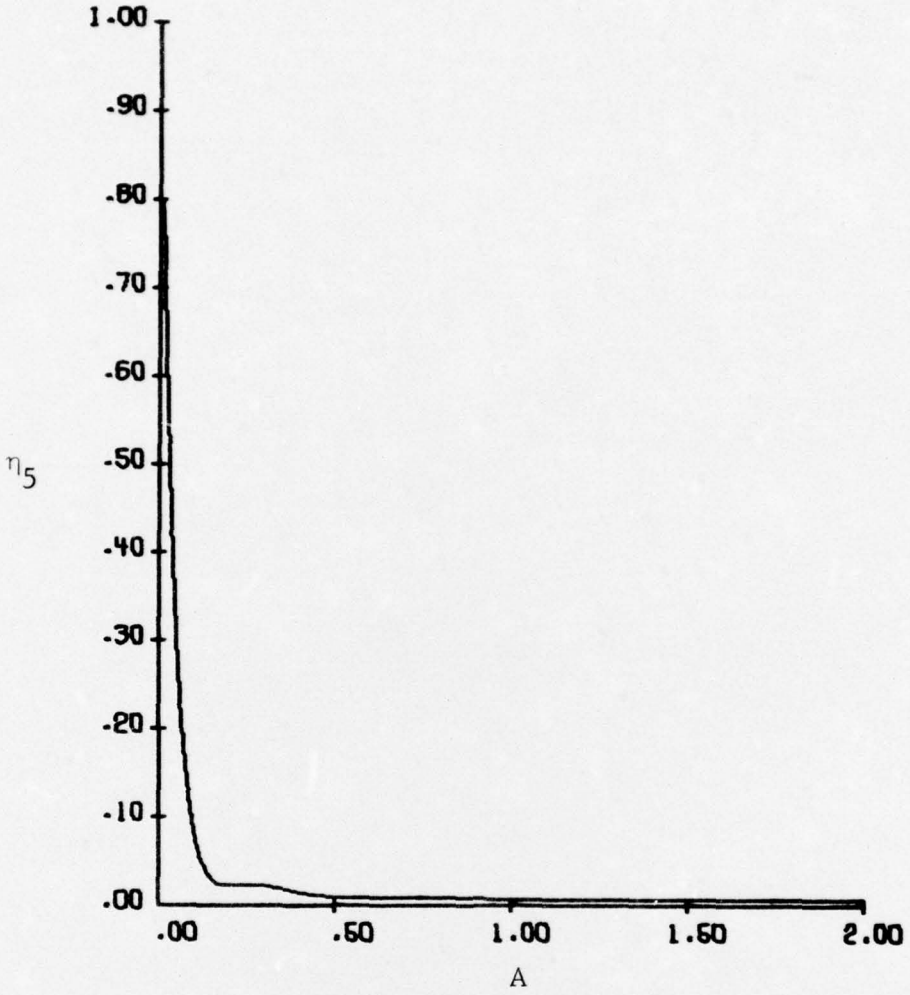


FIG. 8.10 PLOT OF
A VS. RELATIVE ERROR
NUMBER OF FILTERS = 5
F(T)=1/A*(A-T)
ARBITRARY CASE

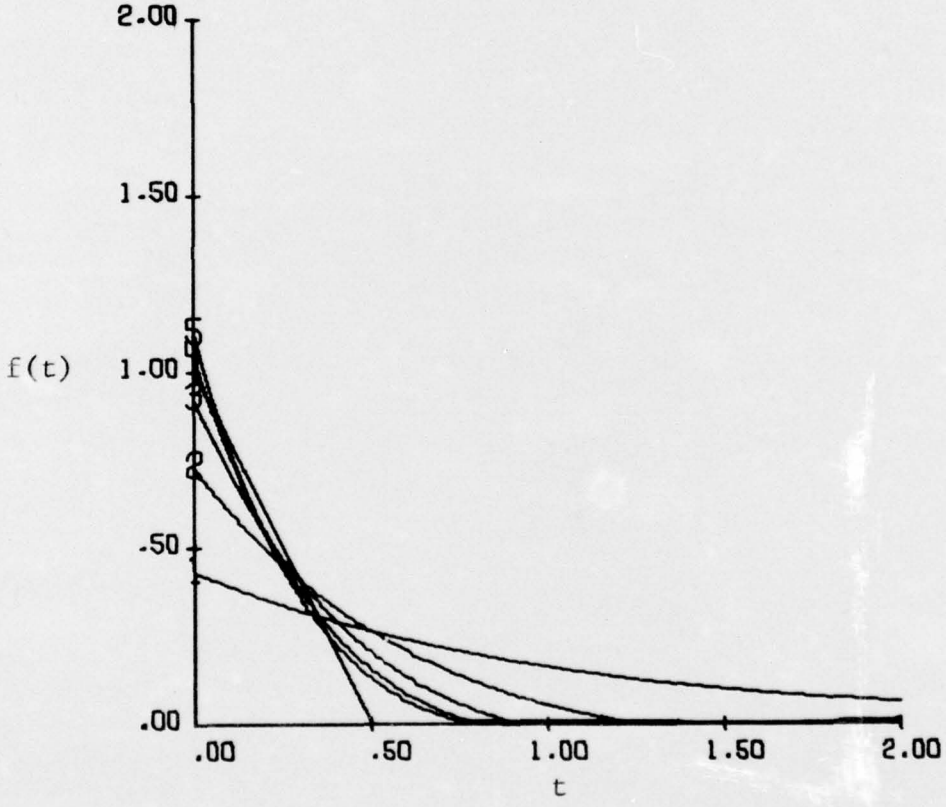


FIG. 8.11 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=.5
LAGUERRE CASE

TRACOR

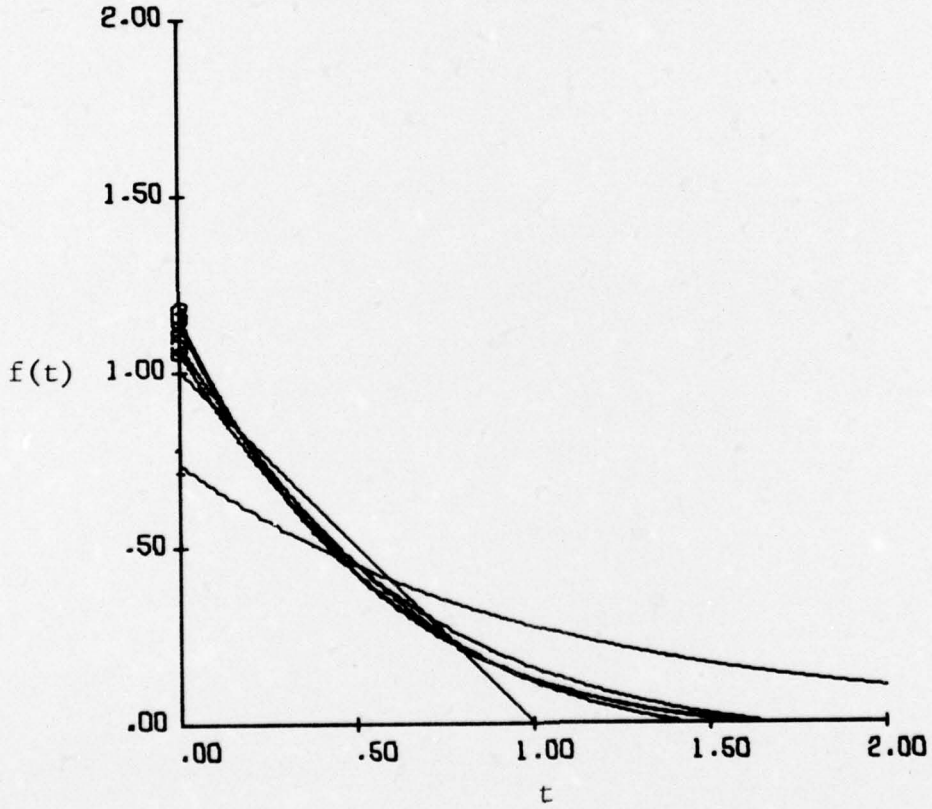


FIG. 8.12 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=1.0 LAGUERRE CASE

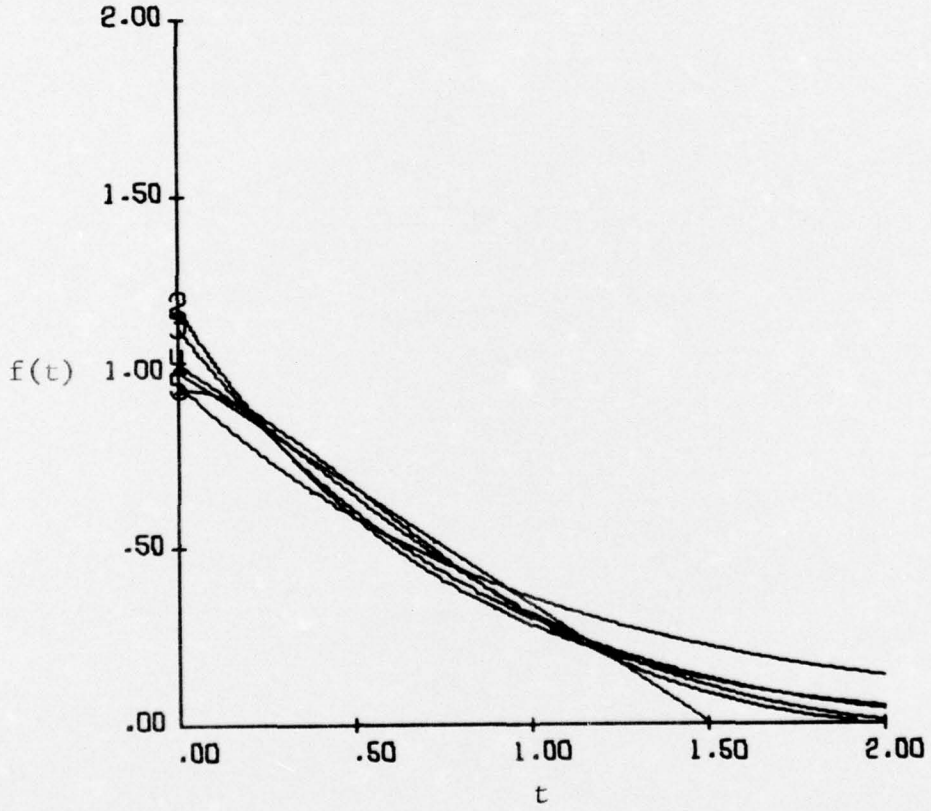


FIG. 8.13 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=1.5
LAGUERRE CASE

TRACOR

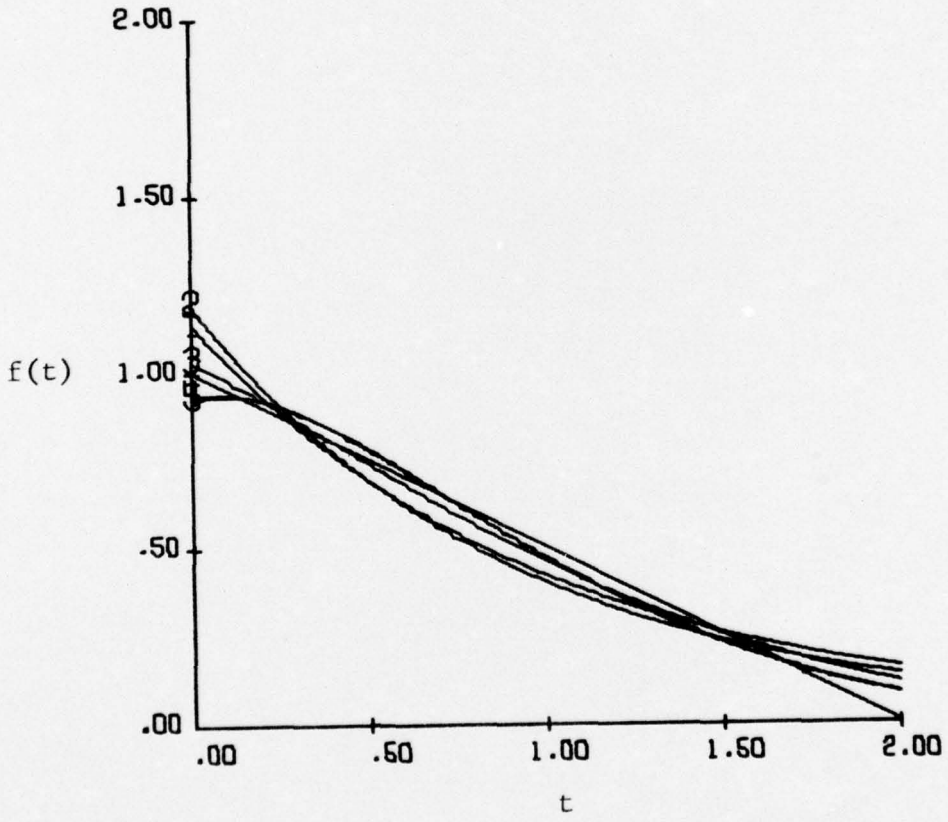


FIG. 8.14 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=2.0
LAGUERRE CASE

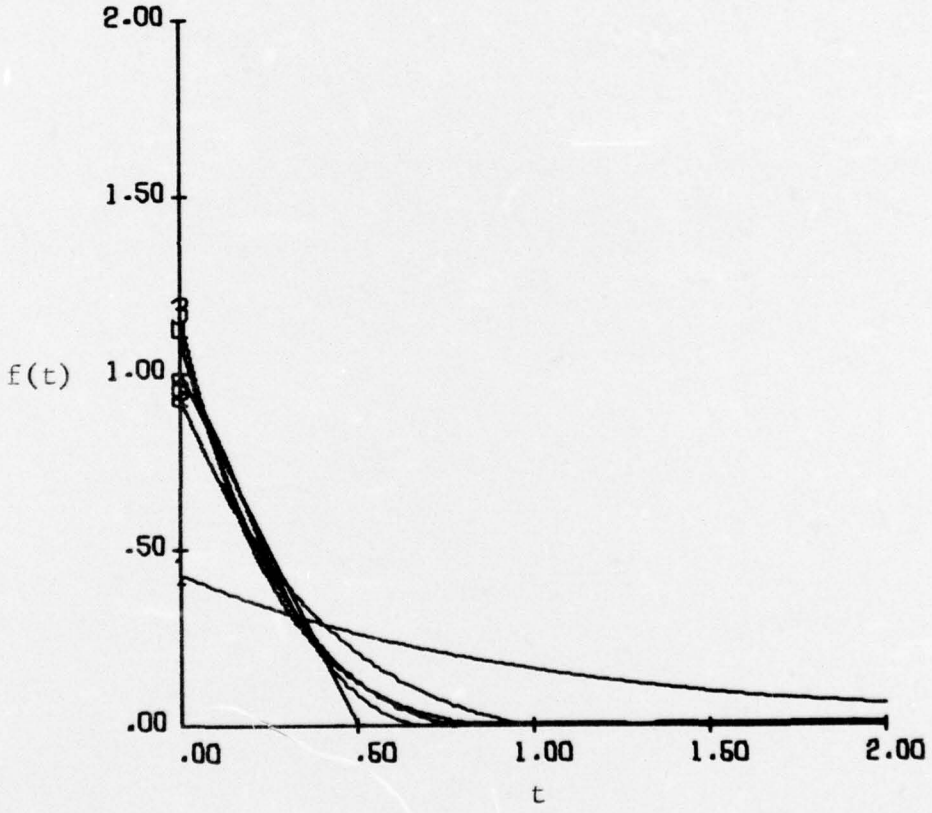


FIG. 8.15 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=.5 ARBITRARY CASE

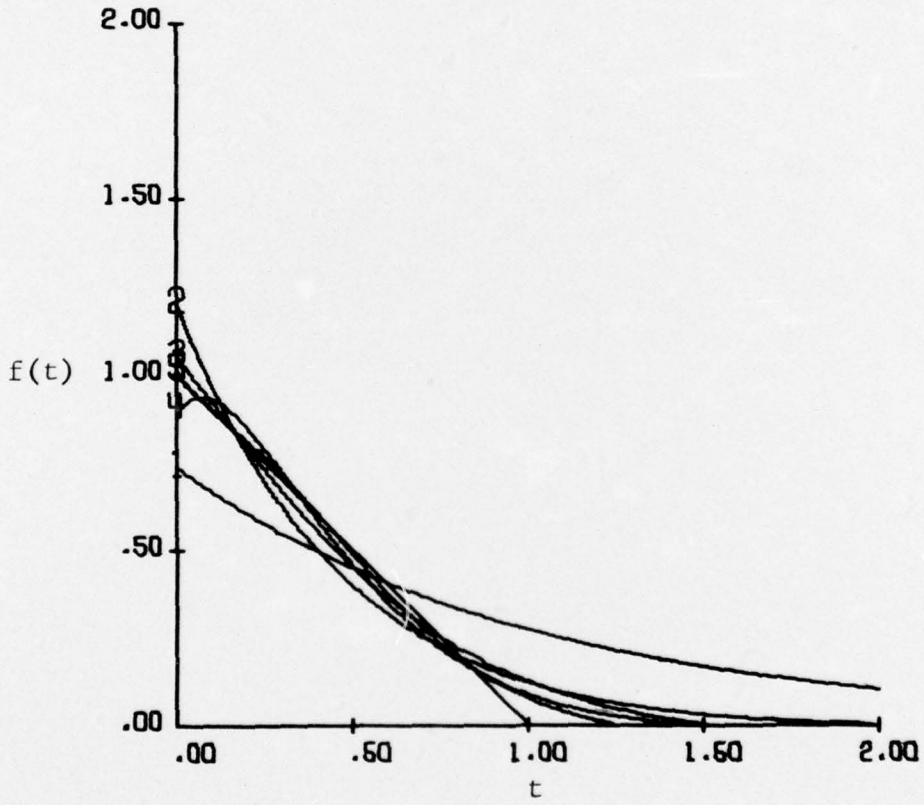


FIG. 8.16 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=1.0 ARBITRARY CASE

TRACOR

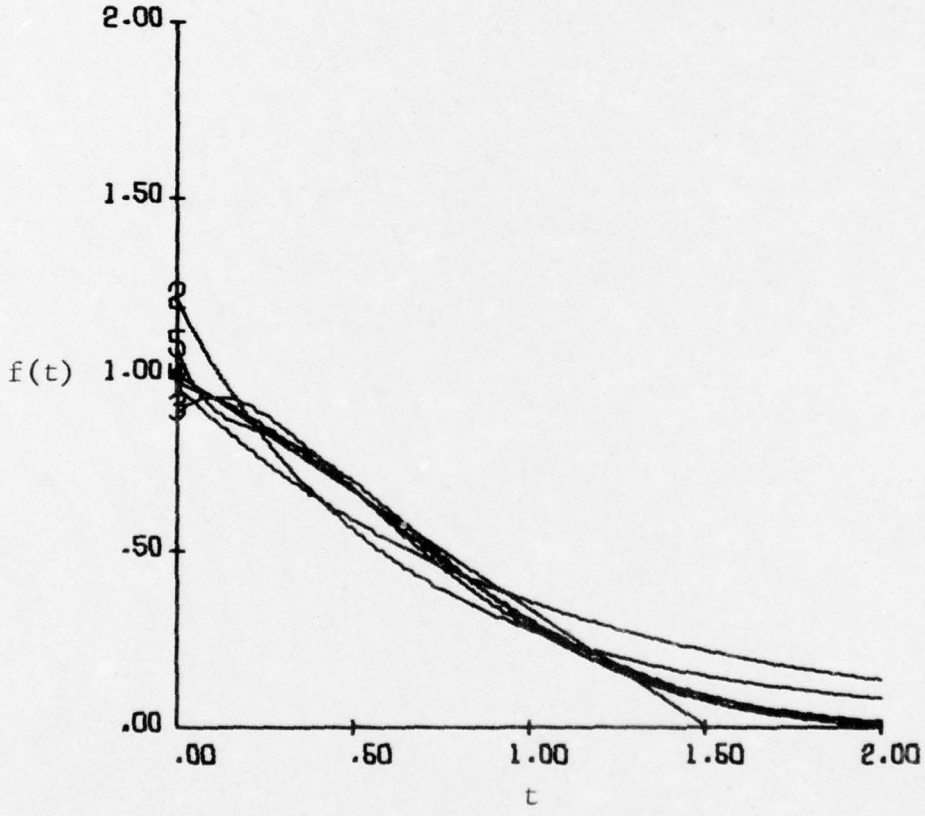


FIG. 8.17 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=1.5 ARBITRARY CASE

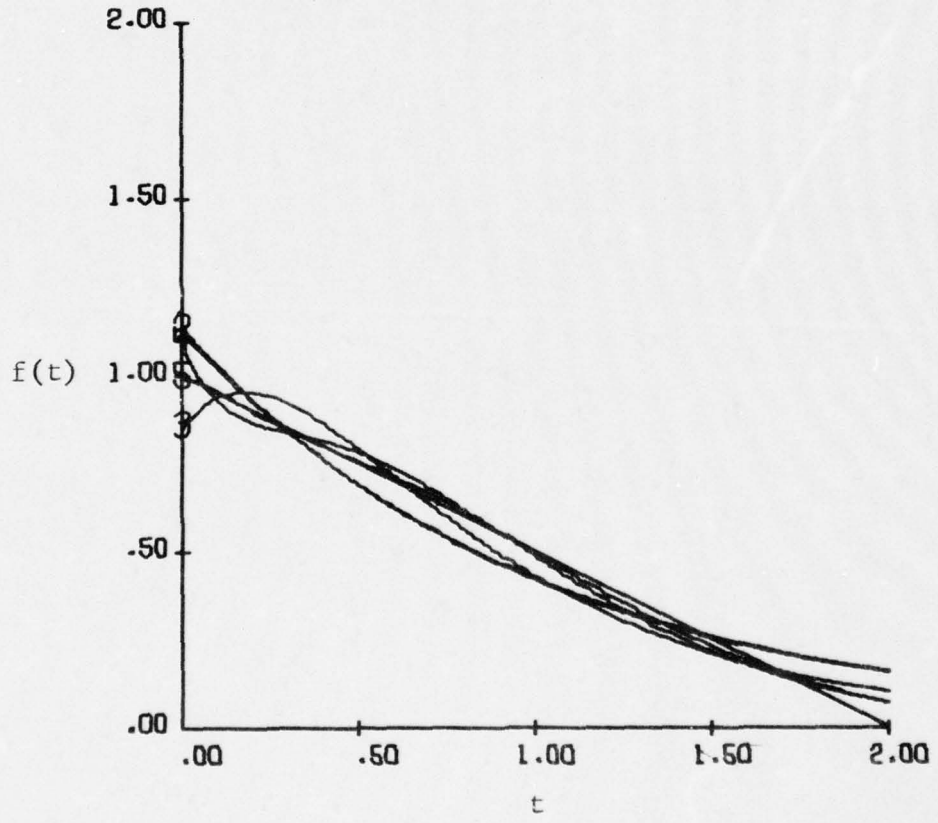


FIG. 8.18 F(T)=1/A(A-T) AND FIVE APPROXIMATIONS FOR A=2.0 ARBITRARY CASE

TRACOR

6-70-118