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THE CONFIDENCE IN MEASUREMENTS OF MTBF, MTTR AND AVAILABILITY.(U)

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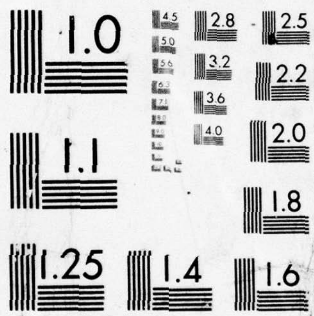

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(14) ~~Document Number~~
TRACOR-64-180-U

FG

TRACOR, INC. 1701 Guadalupe St. Austin 1, Texas

(1)

(15) Contract NObsr-91039
Project Serial No. SF041-001, Task 8100
TRACOR Project 51
(16) (17) SF041001

MOST Project -9

(9) TECHNICAL NOTE ON THE CONFIDENCE IN MEASUREMENTS OF MTBF, MTTR AND AVAILABILITY. (6)

(10) G. T./Kemp

by

G. T. Kemp

(11) 26 March 1964

(12) 29p.

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NOTE ON THE CONFIDENCE IN MEASUREMENTS OF MTBF, MTTR AND AVAILABILITY

I. INTRODUCTION

The purpose of this technical note is to outline the steps necessary for determination of confidence factors in Mean-Time-Between-Failures (MTBF), Mean-Time-To-Repair (MTTR), and Availability (A). In addition, some statistical background information will be given to aid in understanding the reasons for the various steps taken in the calculations.

In this note and as generally used, confidence or confidence factor will be associated with the probability that a given statement is true. For example, if one makes an MTBF measurement on an equipment and from the measurements determines a time T such that there is 90% probability that the true MTBF is less than T, then he can say, with 90% confidence, that the true MTBF is less than T. This means that if one took a large number of sets of measurements and made the above statement for each set, he would be correct about 90% of the time, and incorrect about 10% of the time.

The raw test data from the reliability test specified in memo serial 688E-040 of 2 March 1964 required for the calculations outlined in this note are simply (1) a list of the times between failures for each of the three applicable modes and (2) a list of the times to repair the equipment for each applicable mode. From these data, one can estimate MTBF and MTTR, and therefore equipment availability for each mode. The following two sections of this technical note discuss the calculation of these three quantities and point up two problem areas and alternative solutions for arriving at availability with specified confidence.

One other purpose of this note is to give the buyer sufficient information to allow a reasonable prediction (prior to the test) of the measured values required to achieve specified values with a given confidence. The required design goals of the contractor will be discussed in Section IV.

II. CONFIDENCE IN MTBF

The basic assumption in the calculation of confidence limits on MTBF (which will be designated by a subscript of the percent confidence; e.g. $MTBF_{90}$) is that the times between failures follow a Poisson distribution.

Intuitively we know that if we tested one equipment over a very long period or a large number of equipments over a shorter period. that we would find the true MTBF. However, in the practical case at hand, the test is necessarily quite limited in extent. If only one or two failures occur during the test, one has less confidence that the measured average is close to the true value than if there are many failures during the test. However, by assuming the distribution to be of a certain type (Poisson), one can take the actual test measurement of times-to-failure and calculate a lower limit on the true MTBF for any specified confidence. A family of three curves has been plotted in Fig. 1 for three confidence factors and up to 21 failures. The abcissa is normalized so that one simply multiplies it by the test interval, T, in hours to obtain MTBF. This has been done in Fig. 2 for a 2000 hour test interval. An example will illustrate the proper interpretation of this graph. If the equipment fails 12 times during the 2000 hour test, one can say with 90% confidence that the true MTBF is not less than 113 hours. Of course, this means that the true MTBF is most likely greater than this value. In fact, from the 50% curve, we can say with 50% confidence that the true value is not less than 156 hours, which means that the probability of the true value being greater than 156 hours is the same as for it being less.

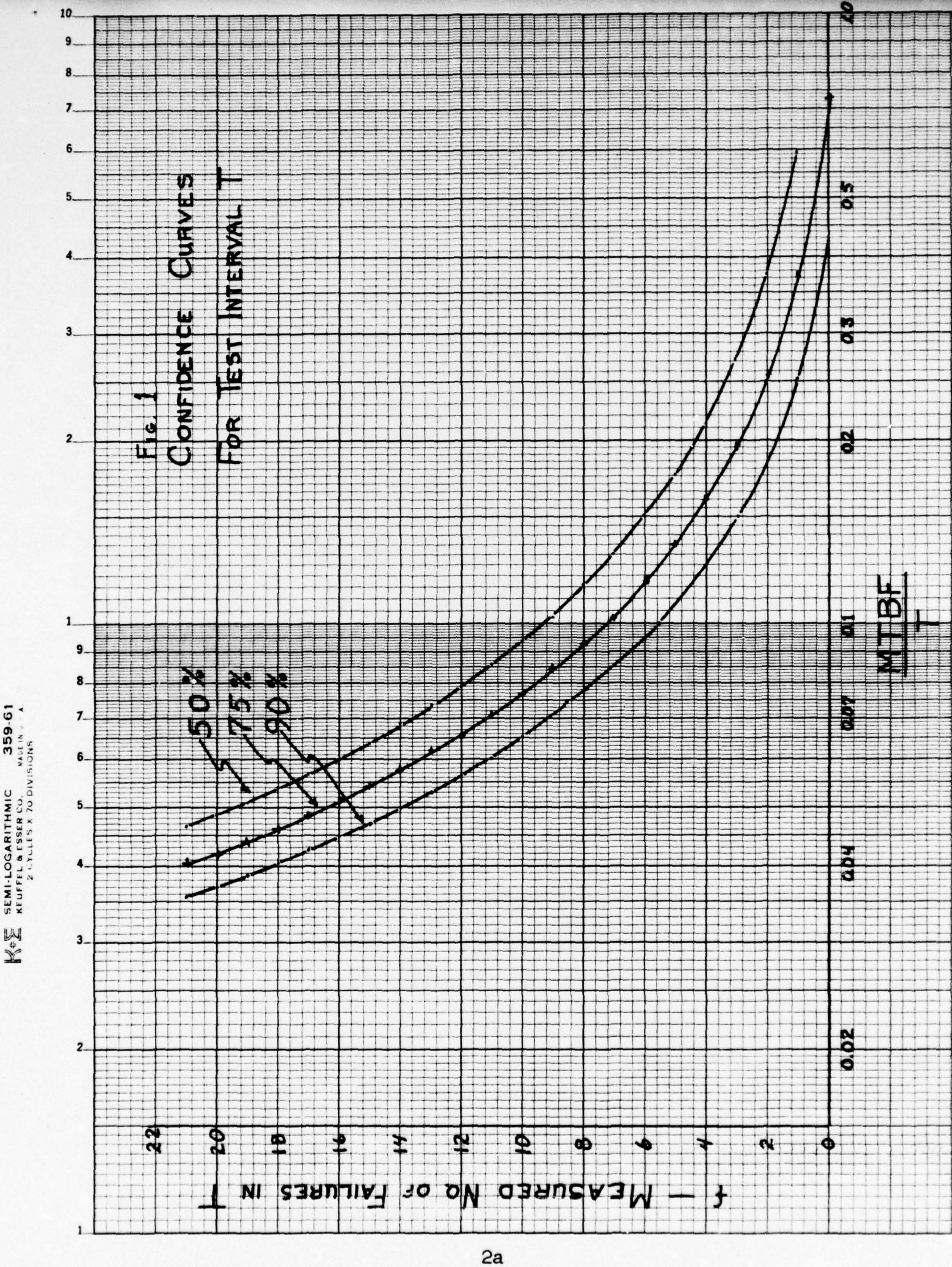


Fig. 1
Confidence Curves
For Test Interval T
50%
75%
90%
f — Measured No. of Failures in T
MTBF/T
0
2
4
6
8
10
12
14
16
18
20
22
0.02
0.04
0.07
0.1
0.2
0.3
0.5
1.0
SEMI-LOGARITHMIC 359-61
KEUFFEL & ESSER CO.
2 CYCLES X 70 DIVISIONS

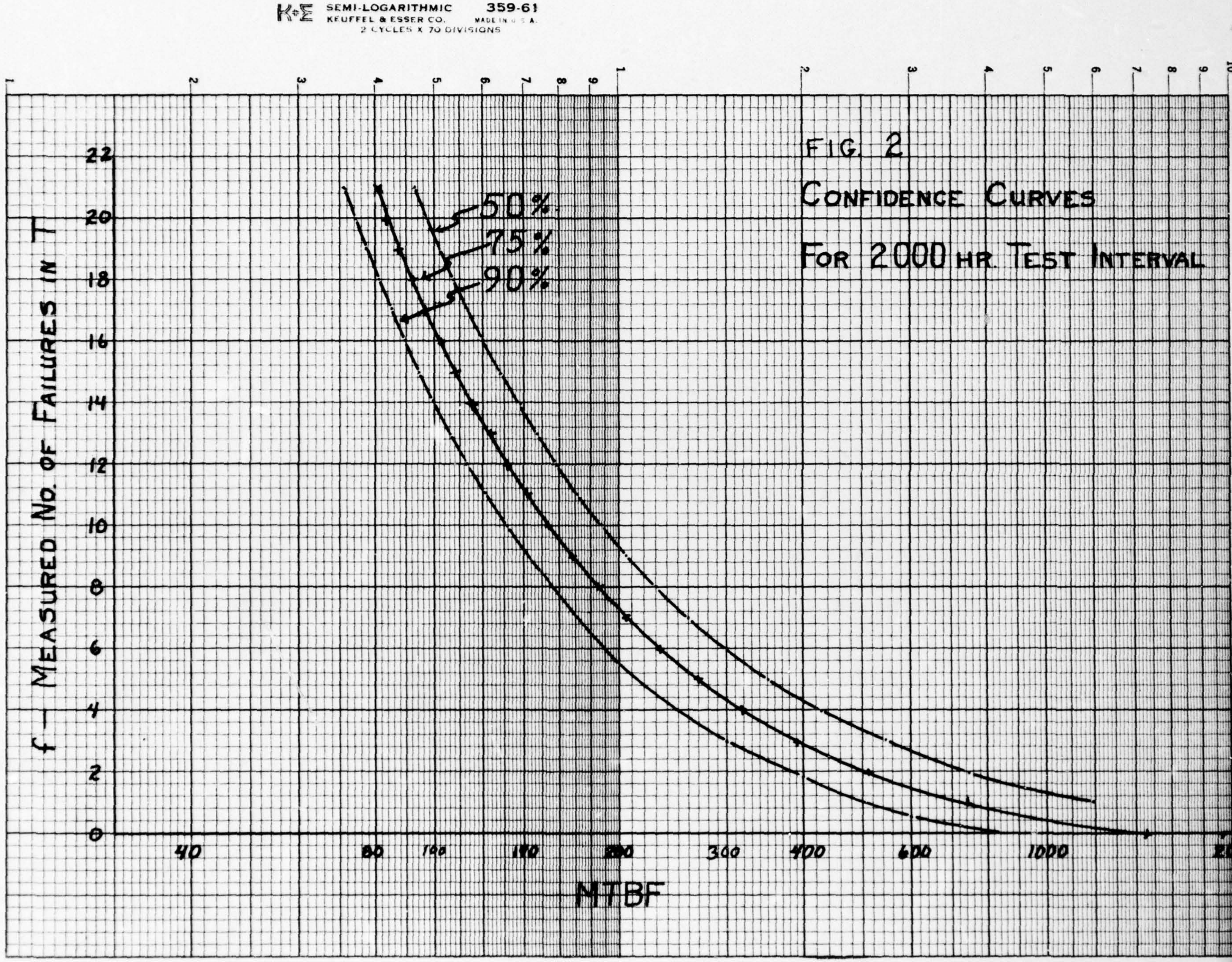


FIG. 2

CONFIDENCE CURVES

FOR 2000 HR. TEST INTERVAL

Table I was obtained from Figure 2 for specific MTBF values of interest. The $MTBF_{meas}$ values listed under each confidence factor are those required to be able to have the specified confidence that the true value is not less than the specified MTBF. Note that Table I contains non-integral numbers of failures, which of course is not a realistic situation.

The proper interpretation of this table can be best illustrated by the following example: For an $MTBF_{Meas}$ = 143 hrs., one can say with 90% confidence that the true MTBF is greater than 100 hrs (the specified value, which we will label $MTBF_{90}$, etc). If one chooses simply to accept the measured average, it appears from the table that he could have about 40% to 45% confidence that the true MTBF is not less than the measured value in the range of interest. Of course, integral values of the number of failures will be used in actual practice, but fractional values are used here just to show the relationship of the various confidence factors.

Table II is similar to Table I, except that only integral values of the number of failures are considered. This table lists the number of failures allowed in order to claim the particular confidence in the specified MTBF. For example, if an equipment has 13 failures during the 2000 hours test interval, we could say, with 90% confidence, that the true MTBF is greater than 100 hours, or, with 75% confidence that the true MTBF is greater than 120 hours, etc. The values in Table II are based on the assumption that for any measured value which yields an MTBF between two specified values, the lower limit will prevail. For example, with 12 failures, the true MTBF is greater than 112 hours, with 90% confidence, but credit is given only on the 100 hour value. In two cases where the failure numbers were very close to a higher category, they were put into the higher category.

TABLE I

MEASURED MTBF VALUES REQUIRED TO ACHIEVE SPECIFIED MTBF WITH SPECIFIED CONFIDENCE

For 2K hr. Test Interval

Specified MTBF	90% Confidence		75% Confidence		50% Confidence	
	Failures	$MTBF_{Meas}$	Failures	$MTBF_{Meas}$	Failures	$MTBF_{Meas}$
100 hrs	(14)	143 hrs	(16.4)	122 hrs	(19.2)	104 hrs
120	(11.2)	178	(13.2)	152	(16)	125
135	(9.7)	206	(11.6)	172	(14)	143
150	(8.5)	235	(10.2)	196	(12.5)	160
175	(6.9)	290	(8.6)	233	(10.6)	189
200	(5.6)	358	(7.2)	278	(9.3)	215
250	(3.9)	512	(5.5)	364	(7.3)	274

TABLE II

Number of failures during a 2000 hr. test allowed to achieve the specified MTBF with specified confidence.

Specified MTBF	90% Confidence	75% Confidence	50% Confidence
	Failures	Failures	Failures
100	14, 13, 12	16, 15, 14	19, 18, 17
120	11, 10	13, 12	16, 15
135	9,	11	14, 13
150	8	10, 9	12, 11
175	6,7	8	10
200	5	7,6	9,8
250	4	5	7

III. CONFIDENCE IN MEAN-TIME-TO-REPAIR (MTTR)

It seems appropriate here to re-emphasize the statement in the introduction that this note has a two-fold purpose -- one, to allow the buyer to predict the approximate range of measurements required to meet his specifications (including confidence factor) and two, to outline the calculations to be made on the test data to determine to what extent the specifications have been met.

As shown in Section II, there is no complication in the calculation of MTBF from test data for any specified confidence, nor in predicting the range of required measurements to meet the specifications.

However, the calculation of availability involves both MTBF and MTTR as

$$A = \frac{MTBF}{MTBF + MTTR} \quad . \tag{1}$$

We will avoid, for the moment, the problem of relating confidence on MTBF and MTTR to a confidence on A, but will simply specify confidence on MTBF and MTTR and accept A as calculated from these values. Therefore for particular values of A (as in the contract specification) we can relate MTTR and MTBF.

For the prediction aspect, the problem at hand is to determine what value of $MTTR_{meas}$ will allow us to say, with specified confidence, that the true MTTR is less than a value determined from eqn. (1), for a given value of A. For a confidence on both MTTR and MTBF of, say 90%, we have,

$$MTTR_{90} = \frac{1-A}{A} MTBF_{90} \tag{2}$$

The $MTBF_{90}$ values are those in Table I or II under the heading, "specified MTBF." Now, from eqn. (2), we can calculate a required $MTTR_{90}$ for specified values of $MTBF_{90}$ and A.

The prediction of a measured value, $MTTR_{meas}$, to correspond to an $MTTR_{90}$ requires two assumptions. The first assumption is based on considerable Navy experience in the repair of shipboard equipment and is that individual repair times are distributed log-normally; this means that if an equipment were tested and repaired a large number of times, or if many equipments with the same true median equipment repair time were repaired, the logarithms of the repair times would yield a normal (or Gaussian) distribution curve.

The second assumption which must be made in working with MTTR is a value of the standard deviation of the normal curve. For the lognormal distribution, the arithmetic mean (MTTR) is not the same as the median (ERT), but they are related through the standard deviation, σ, in the equation

$$\log MTTR = \log ERT + 1.15\,\sigma^2 \quad . \tag{3}$$

Measured values of σ range from .4 to .7, with an average of about .55. The .55 value is the one used in NAVSHIPS-94324, so it will also be used here for prediction, which yields the equation

$$ERT = .45\ MTTR \quad . \tag{4}$$

It must be remembered that equation (4) is an approximation which is only as good as our estimate of σ. This will bring up a problem later when we try to obtain a confidence value on MTTR from measured data, but for prediction purposes we assume that equation 4 is a good approximation.

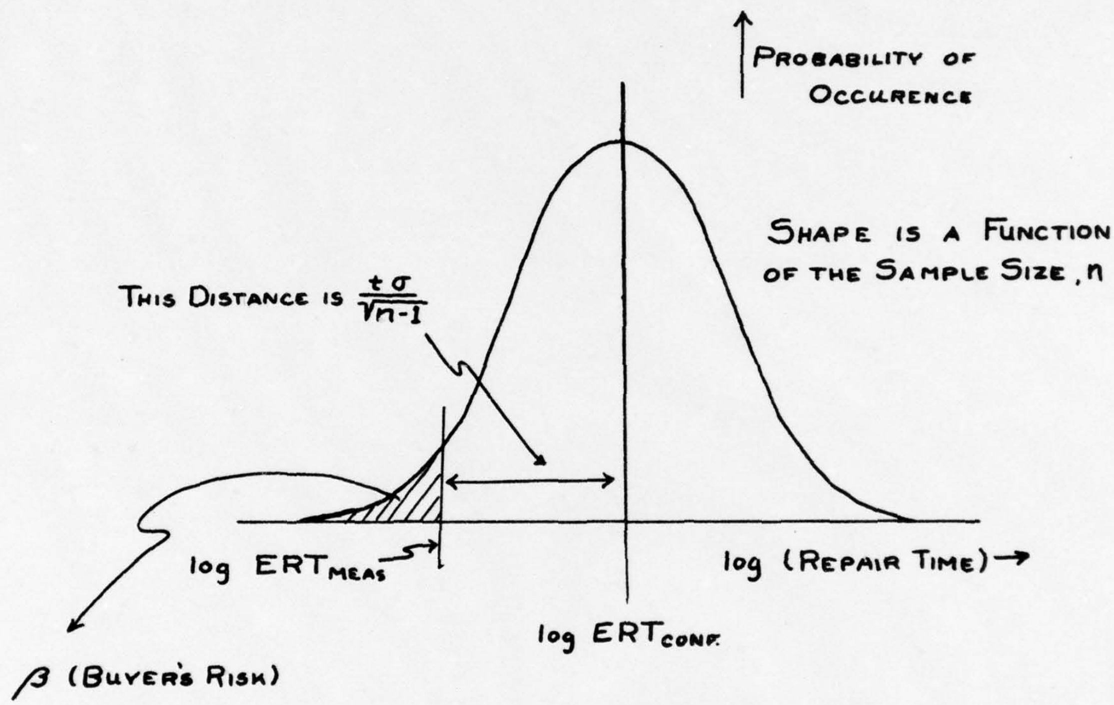
Now, the mean value of the lognormal distribution of repair times is the median of a large number of equipment repair times, ERT. Thus, if we knew the true ERT and σ for an equipment, we could predict the probability that a repair time would fall between specified limits. Unfortunately, we do not know the true ERT, but we are forced into making a statement, with some specified confidence, that the true ERT is less than some specific value, based on a relatively small sample of repair times. This is where the "Student's t" distribution comes into play. The "t" distribution is similar to a Gaussian distribution, but its density function is slightly broader (and consequently lower for unit area). Of course, the amount of broadening is a function of the sample size. The "t" table is simply a convenient way of picking off a suitable abcissa of the "t" distribution for which a specified percent of the total area under the curve lies on one side of the chosen abcissa. Thus, the value of "t" is determined by the sample size, n, and the desired confidence factor (= 1 - risk).

Another way of looking at the relationship of the ERT's is shown in Figure 3. First we will define ERT_{conf} as the value of ERT for which we have a specified confidence (1 - risk), that the true ERT is less than ERT_{conf}. Then for a given sample size, n, a specified risk and a specified value of ERT_{conf}, we can determine a value of ERT_{meas}. For our present prediction problem, ERT_{meas} is the value which would have to be measured to satisfy the specified requirements. This can be expressed analytically, assuming a lognormal distribution of repair times, as

$$\frac{t\sigma}{\sqrt{n-1}} = \log ERT_{conf} - \log ERT_{meas} \quad , \qquad (5)$$

FIG. 3

RELATIONSHIP OF ERT VALUES



where the left side of the equality is the abcissa value between log ERT_{conf} and log ERT_{meas} as shown in Figure 3. (Incidentally, ERT_{meas} is the same as $MTTR_G$ as used in NAVSHIPS-94324, p. I-3-63).

Of course, if we turn the problem around and have a set of measurements yielding an ERT_{meas}, then by sliding the distribution curve along the log (repair time) axis, we can find the log ERT_{conf} which satisfies all of the other parameters. We can state, with the specified confidence that the true ERT is less than or equal to the ERT_{conf}; and we will be correct $(1 - \beta)$ fraction of the time and incorrect β fraction of the time.

Now, back to the determination of the ERT_{meas} which corresponds to an ERT_{conf}. From eqn. (5),

$$ERT_{meas} = \frac{ERT_{conf}}{\text{antilog} \frac{\sigma t}{\sqrt{n-1}}} \tag{6}$$

where t may be obtained from "t" tables (for example, see p. I-3-66 of NAVSHIPS-94324, except that we substitute β for α in this case. More complete tables are available in "Experimental Design in Psychological Research," by Allen L. Edwards, or "Biometrika Tables For Statisticians," Volume I, Table 9, p. 132). Having assumed a value of $\sigma = .55$ we can write, from eqns. (4) and (6) that

$$MTTR_{meas} = \frac{MTTR_{conf}}{\text{antilog} \frac{.55t}{\sqrt{n-1}}} \;. \tag{7}$$

First, we will assume a confidence of 90%. For a given value of $MTBF_{90}$ (we will choose the nominal value and the two end values from the pay schedule for numerical computations) there is an allowed number of failures (we will use the maximum number listed in Table II for each case considered). From this n, and t from the .10 column in the NAVSHIPS-94324 "t" table we obtain the following three equations:

for $MTBF_{90}$ = 100 hrs ; $MTTR_{meas} = (0.620)MTTR_{90}$, (8a)

for $MTBF_{90}$ = 150 hrs ; $MTTR_{meas} = (0.509)MTTR_{90}$, (8b)

for $MTBF_{90}$ = 250 hrs ; $MTTR_{meas} = (0.337)MTTR_{90}$. (8c)

Explaining eqn. (8a), if we have a measured value of MTTR ($=MTTR_{meas}$), then by dividing it by 0.62 we obtain a value of MTTR ($=MTTR_{90}$) about which we can say, with 90% confidence, that the true MTTR is less than $MTTR_{90}$. This is for a 90% confidence $MTBF_{90}$ = 100 hrs.

Using eqn. (2) and three values of A from the contract payment schedule, (nominal and two extreme) we can calculate $MTTR_{90}$ for various $MTBF_{90}$ from the following equations:

for A = .960 ; $MTTR_{90} = (0.0417)MTBF_{90}$ (9a)

for A = .9868 ; $MTTR_{90} = (0.01337)MTBF_{90}$ (9b)

for A = .995 ; $MTTR_{90} = (0.00502)MTBF_{90}$. (9c)

For example:

for $MTBF_{90}$ = 150 hr and A = 98.68%

$$MTTR_{90} = \frac{1-.9868}{.9868}(150)$$

$$= 2.00 \text{ hrs} .$$

Substituting this value into (8b), we get

$$MTTR_{meas} = (.509)\ (2.00)$$
$$= 1.02 \text{ hrs.}$$

This says that if the contractor had a measured MTTR of 1.02 hrs. with 8 failures, then he could state with 90% confidence that his true MTTR meets the requirements for an A of 98.68%.

The same procedure is followed for 75% confidence. Note, however, that the numerical values of the $MTBF_{75}$ are the same as for $MTBF_{90}$. This simply means that we are specifying the confidence values, but allowing more failures for the lower confidence. A listing follows of the relationships between measured and confidence values for 75%:

$$\text{for } MTBF_{75} = 100 \text{ hrs} \ ; \ MTTR_{meas} = (0.797) MTTR_{75} \quad (10a)$$

$$\text{for } MTBF_{75} = 150 \text{ hrs} \ ; \ MTTR_{meas} = (0.743) MTTR_{75} \quad (10b)$$

$$\text{for } MTBF_{75} = 250 \text{ hrs} \ ; \ MTTR_{meas} = (0.625) MTTR_{75} \ . \quad (10c)$$

The relationship between $MTTR_{75}$ and $MTBF_{75}$ is the same as for the 90% confidence values (see eqns. 9a, b and c) since it involves only A. Using these equations and equations 10a, b and c, we get the required measured values for 75% confidence.

Table III shows the measured $MTTR_{meas}$ which would be required to achieve the specified $MTTR_{spec}$ for the indicated confidence, A and the specified MTBF. This table is based on requiring the same confidence in MTBF and MTTR, whether 75% or 90%. The value of $MTBF_{spec}$ is the same for both confidence factors, but the number of allowed failures to achieve the confidence in MTBF is different and affects the calculation of $MTTR_{meas}$. We could have mixed confidence factors, but felt that nothing would be gained by such confusion.

TABLE III

The $MTTR_{meas}$ is the value which must be measured in order to achieve the corresponding $MTTR_{spec}$ for the indicated confidence and $MTBF_{spec}$. Note that this table is based on an assumed σ = .55 and is <u>for prediction only</u>.

	Specified A	MTTR* Spec.	Required $MTTR_{meas}$ to achieve $MTTR_{spec}$ with 90% Confidence	Required $MTTR_{meas}$ to achieve $MTTR_{spec}$ with 75% Confidence
$MTBF_{spec}$ = 100 hrs	.960	4.17	2.58	3.32
	.9868	1.34	.83	1.07
	.995	.50	.31	.40
$MTBF_{spec}$ = 150 hrs	.960	6.25	3.12	4.64
	.9868	2.00	1.02	1.48
	.995	.75	.38	.56
$MTBF_{spec}$ = 250 hrs	.960	10.40	3.50	6.50
	.9868	3.34	1.12	2.08
	.995	1.25	.42	.78

$$*MTTR_{spec} = \left(\frac{1-A}{A}\right) MTBF_{spec}$$

All times are in hours.

Note in Table III that the $MTTR_{meas}$ required to obtain the specified $MTTR_{conf}$ with 90% confidence changes only a little between 150 and 250 hours, which might make it a bit tedious in certain cases. This situation arises because with fewer samples (as is the case for 250 hr), one must choose a larger value to achieve the required confidence. This effect is not so pronounced for 75% confidence.

It must be emphasized that Table III is based on an assumed $\sigma = .55$. Therefore, the table is only as good as this assumption and is useful only as a guideline in setting up the specification. When the tests are complete, a sample standard deviation (called s) will be calculated from actual data. A new table would then be calculated using the actual standard deviation, s; this would be the "payoff" table.

This brings us back to the problem mentioned in connection with equation 4, regarding the confidence on MTTR. The fact is that we can relate only ERT values by the use of the "t" tables and not MTTR values unless we know σ. Since we do not know σ, but only a small sample estimate of it, s, then we cannot have 100% confidence in it. Thus, from equation 3, we cannot have as much confidence in MTTR as we have in ERT. There is a distribution assumed for s, based on the χ^2 distribution (see Biometrika Tables for Statisticians, Volume I, Table 35, p. 184), which employs the same basic approach as the "t" distribution; i.e., it takes into account the sample size. Thus a confidence range for σ, as estimated from s, can be obtained by computing a set of tables similar to the "t" tables over the range of interest.

However, the problem remains of how to combine the confidence factors of ERT and σ for use in equation 3 to arrive at a confidence factor for MTTR. There appears to be a method for doing this, but it is quite messy and would involve some computer time.

There are three major alternative solutions to the problem. One is to assume (or rather dictate) 100% confidence in s, or alternatively to simply specify a value of σ to be used in calculating MTTR. The second solution would be to put your specification on ERT and ignore confidence on MTTR. The third would be to do the necessary computations to determine confidence values on MTTR.

In the first case the MTTR value so determined would carry less confidence than on ERT, but on the other hand, if one chooses the third alternative, this effectively tightens the specifications. The second alternative is rather unsatisfactory in that it ignores availability. Incidentally, the rigorous computation of a confidence on MTTR would depend on the actual values of ERT and s as well as their confidence factors.

The first alternative seems like the best compromise, so long as it is clearly stated that the assumption of 100% confidence in s is being made.

The same kind of arguments can be made regarding the confidence in A. First, if a confidence is specified, it tightens the confidence on MTBF and/or MTTR (or ERT), and secondly, it is straightforward to specify a confidence in MTBF and MTTR and simply calculate A.

IV. CONTRACTOR REQUIREMENTS

A. MTBF

In the preceding sections, we have discussed the buyer's point of view and concluded that in order to have a relatively high confidence that the true MTBF value doesn't exceed a specified value, for a small sample, we must measure a considerably smaller MTBF. This takes into account the possibility of a small sample measurement yielding a value anywhere in the assumed Poisson distribution. The fraction of the time when the equipment tests good and is actually bad is the buyer's risk (= 1 - confidence).

The contractor faces essentially the same problem, in that he must design the equipment so that his risk of having a good equipment test bad is relatively small. His design must be such that the true MTBF is large enough so that if a Poisson distribution were fitted around the true mean value that only 10% or so (his risk) of the area under the distribution curve would be outside the "acceptable" range of MTBF values.

Because of the unsymmetrical nature of the Poisson distribution, the contractor can't use the curves of Figure 1. Reference to Figure 4 shows that the buyer's risk is cut off of the left side of the buyer's curve, while the contractor's risk is off the right end of his curve. Using a cumulative Poisson table (Biometrika Tables for Statisticians, Table 7) one can determine the contractor's MTBF design goal requirements for his desired risk. Of course, the contractor has the choice to design toward any one of the specified MTBF's and take his chances on the test measurements.

As an example, consider the nominal specified MTBF = 150 hours and a buyer's requirement of 75% confidence. Table II shows that the maximum number of allowed failures is 10 in this case. Using this value and an assumed contractor risk of 10%, and going into the Biometrika Table 7, we come out with a contractor's MTBF design goal of 286 hours. Tables IVa, IVb and IVc show the required contractor's design goals for 10% and 25% risk to him, for buyer's confidence factors of 90%, 75% and 50%.

In examining these tables, one must realize that the contractor might be lucky enough to get a "fair" (mean value) test measurement, in which case he would get more "payoff." Let's say he chooses to design toward the specified MTBF of 150 hours and that the buyer required 90% confidence. If he wants to take a 10% risk, he would design for a 371 hour MTBF. Now, if the equipment has a true MTBF of 371 hour, he has a 90% chance that it will

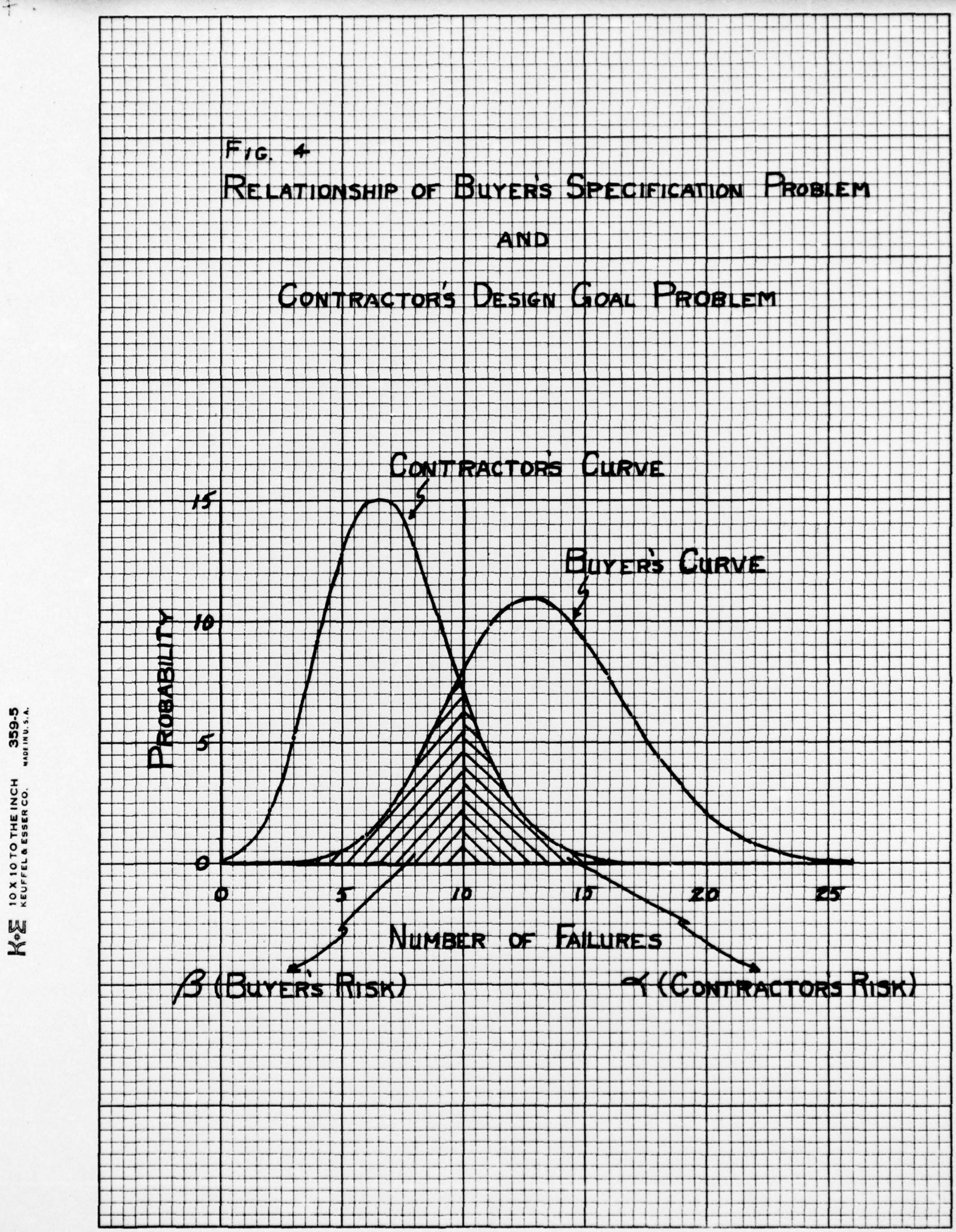


Fig. 4
Relationship of Buyer's Specification Problem
and
Contractor's Design Goal Problem
Contractor's Curve
Buyer's Curve
Probability
15
10
5
0
0
5
10
15
20
25
Number of Failures
β (Buyer's Risk)
α (Contractor's Risk)
10 X 10 TO THE INCH 359-5
KEUFFEL & ESSER CO.
MADE IN U.S.A.

90% BUYER's CONFIDENCE

Specified MTBF	Maximum Number of Allowed Measured Failures	Manufacturer's MTBF Design Goal	
		for 10% Risk	for 25% Risk
100	14	194	165
120	11	256	210
135	9	323	258
150	8	370	294
175	7	430	339
200	5	635	476
250	4	823	593

TABLE IVa

Manufacturer's MTBF Design Goals Required for 90% Buyer's Confidence in Specified MTBF Values

75% BUYER's CONFIDENCE

Specified MTBF	Maximum Number of Allowed Measured Failures	Manufacturer's MTBF Design Goal for 10% Risk	Manufacturer's MTBF Design Goal for 25% Risk
100	16	167	142
120	13	210	177
135	11	256	210
150	10	286	233
175	8	370	292
200	7	430	339
250	5	635	475

TABLE IVb

Manufacturer's MTBF Design Goals Required for 75% Buyer's Confidence in Specified MTBF Values

50% BUYER's CONFIDENCE

Specified MTBF	Maximum Number of Allowed Measured Failures	Manufacturer's MTBF Design Goal for 10% Risk	Manufacturer's MTBF Design Goal for 25% Risk
100	19	138	119
120	16	167	143
135	14	194	164
150	12	231	192
175	10	286	231
200	9	320	260
250	7	430	339

TABLE IVc

Manufacturer's MTBF Design Goals Required for 50% Buyer's Confidence in Specified MTBF Values

test out for the 150 hour "payoff" or better. He also has a 37% chance of getting paid on the 250 hour value. Incidentally, it isn't difficult to calculate the contractor's chance of attaining any particular "payoff" for any chosen design goal.

B. <u>MTTR (or ERT)</u>

The buyer requires that the test repair times (ERT_{meas}) be shorter than his maximum acceptable value so that he will be confident that he is receiving a good equipment. There is still a risk (= 1 - confidence) that an equipment is bad but tests good, but this is part of the game.

The contractor has the inverse problem, in that he takes a risk that a good equipment will test bad. In order to keep this risk to a reasonable value, he must design even better than the required test value. The relationship of the buyer's and contractor's curves and risks is shown in Figure 5 (taken from NAVSHIPS-94324, p. I-3-67, but with some notation changes). Actually, the accept-reject line is really a zone, graded for "payoff." For "payoff" purposes, the ERT_{AL} can be thought of as the measured value and the buyer's curve is slid to a position where the risk area is as desired. The contractor must decide which ERT_{AL} he will shoot for and slide his curve to conform to his desired risk, yielding his ERT_{design} value.

The calculation of the design ERT is done from the basic relation,

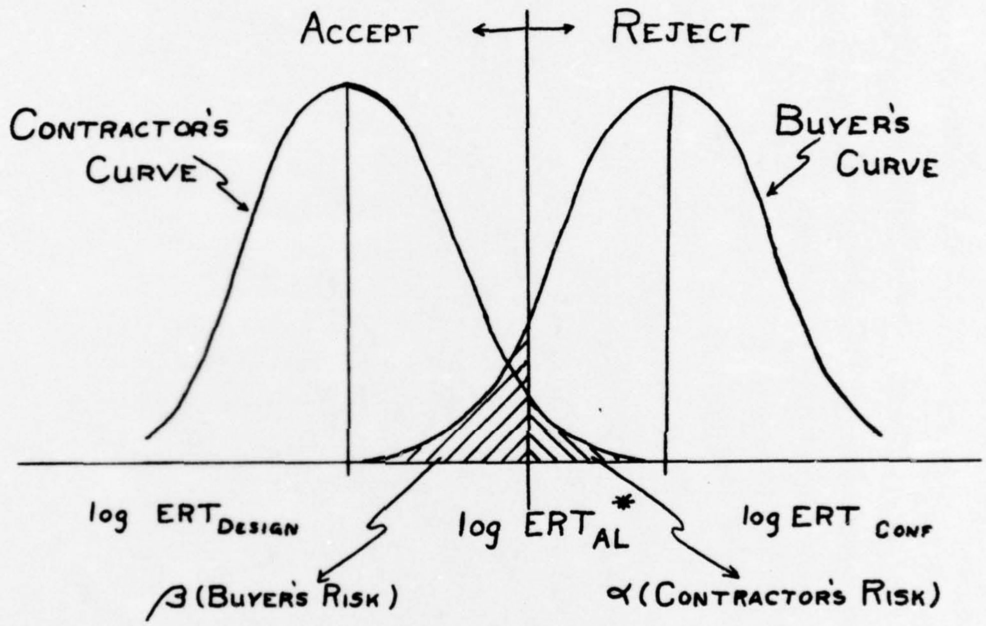
$$\log ERT_{AL} = \log ERT_{design} + \frac{\sigma t}{\sqrt{n-1}} ,$$

where ERT_{AL} is the same as ERT_{meas}. Rearranging and taking the antilog yields

$$ERT_{design} = \frac{ERT_{AL}}{\text{antilog } \frac{.55t}{\sqrt{n-1}}} .$$

FIG. 5

RELATIONSHIP OF BUYER'S SPECIFICATION PROBLEM AND CONTRACTOR'S DESIGN GOAL PROBLEM



* THIS IS THE ACCEPTANCE LEVEL, BUT WE HAVE ALSO CALLED IT ERT_{MEAS} FOR USE IN DETERMINING THE "PAYOFF."

As before, we will assume a value for σ of .55, which allows us to convert ERT to MTTR and get the relation

$$MTTR_{design} = \frac{MTTR_{AL}}{\text{antilog}\ \frac{.55t}{\sqrt{n-1}}},$$

from which we calculate the MTTR design goals.

Tables Va and Vb contain the required contractor MTTR design goals for 10% and 25% contractor risk. Table Va is for a 90% buyer's confidence on both MTBF and MTTR, and Table Vb is for a 75% buyer's confidence. Actually, these two tables contain all of the information contained in Table III, with the addition of the contractor's design goal values. The interpretation of Table Va can be illustrated with an example: We can say, with 90% confidence, that an equipment with an $MTTR_{meas}$ value of 1.02 hrs will have a true MTTR less than 2.0 hr. for an $MTBF_{meas}$ for which we have 90% confidence that the true MTBF is greater than 150 hrs. Also, the 90% confidence value of MTBF and the 90% confidence value of MTTR will yield an A = .9868. Now, let's assume that the contractor decides to design on A = .9868 and an appropriate MTBF to yield a 90% confidence value of 150 hrs (see Table IVa) with a 10% contractor risk. He would design on an MTTR of .52 hrs.

You will note some inversions in Tables Va and Vb, but they have been checked and appear to be correct. For example, in Table Va under the 25% risk column, note that the design goal is higher at MTBF = 150 than at the two extreme values. This comes about because of the number of failures tied to the MTBF values.

TABLE Va

Required Contractor MTTR Design Goals to achieve the $MTTR_{meas}$ for two indicated values of contractor risk. Both $MTTR_{spec}$ and $MTBF_{spec}$ are assumed at 90% buyer's confidence. This table is based on an assumed $\sigma = .55$ and is therefore for prediction purposes only.

90% Buyer's Confidence

	Specified A	$MTTR_{spec}$*	Required $MTTR_{meas}$ to achieve $MTTR_{spec}$ with 90% Confidence	Required Contractor MTTR Design Goals for 10% risk	Required Contractor MTTR Design Goals for 25% risk
$MTBF_{spec}$ = 100 hrs	.960	4.17	2.58	1.60	2.02
	.9868	1.34	.83	0.51	0.65
	.995	.50	.31	0.19	0.24
$MTBF_{spec}$ = 150 hrs	.960	6.25	3.12	1.59	2.22
	.9868	2.00	1.02	0.52	0.73
	.995	.75	.38	0.19	0.27
$MTBF_{spec}$ = 250 hrs	.960	10.40	3.50	1.06	2.01
	.9868	3.34	1.12	0.34	0.64
	.995	1.25	.42	0.13	0.24

$^*MTTR_{spec} = \left(\frac{1-A}{A}\right) MTBF_{spec}$

All times are in hours.

TABLE Vb

Required Contractor MTTR Design Goals to achieve the $MTTR_{meas}$ for two indicated values of contractor risk. Both $MTTR_{spec}$ and $MTBF_{spec}$ are assumed at 75% buyer's confidence. This table is based on an assumed $\sigma = .55$ and is therefore <u>for prediction</u> purposes only.

75% Buyer's Confidence

	Specified A	$MTTR_{spec}$*	Required $MTTR_{meas}$ to achieve $MTTR_{spec}$ with 75% confidence	Required Contractor MTTR Design Goals for 10% risk	Required Contractor MTTR Design Goals for 25% risk
$MTBF_{spec}$ = 100 hrs	.960	4.17	3.32	2.13	2.65
	.9868	1.34	1.07	0.69	0.85
	.995	.50	.40	0.25	0.32
$MTBF_{spec}$ = 150 hrs	.960	6.25	4.64	2.59	3.45
	.9868	2.00	1.48	0.83	1.10
	.995	.75	.56	0.31	0.42
$MTBF_{spec}$ = 250 hrs	.960	10.40	6.50	2.45	4.07
	.9868	3.34	2.08	0.78	1.30
	.995	1.25	.78	0.29	0.49

*$MTTR_{spec} = \left(\frac{1-A}{A}\right) MTBF_{spec}$.

All times are in hours.

As discussed in part A of this section, the contractor has a statistically predictable chance (provided his design is good) that the test will turn out to give him a "payoff" in any particular category. We have simply shown the 90% and 75% chance situations. Of course the statistics on the contractor's problem do not take into account the possibility that he may "goof" on the design, in which case he would have another variable. All the work we have done here is related simply to the statistics of test measurements with small samples.

Unfortunately, there doesn't seem to be any way to "fool" the statistics, so if you want to be sure (say 90%) that you are getting what you pay for (or more -- but not less) then you have to "stack the cards" this way. Of course you stand a good chance of getting a much better equipment than the minimum specified, provided the contractor will go along with it.

Incidentally, it seems obvious to me after going through all of this stuff, that the "payoff" to the contractor is about 20% up to his design efforts and about 80% to his luck in the test. Unfortunately, I have no better suggestion, unless you would be willing to make the "payoff" subject to revision by data from a larger sample, including barge tests, sea tests etc.