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(9) TECHNICAL NOTE ON THE STATISTICS INVOLVED IN AVAILABILITY CALCULATIONS.

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(10) W. A. Youngblood

by

W. A. Youngblood

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I. GLOSSARY OF TERMS

A - fractional availability, defined as

$$\frac{MTBF}{MTBF + MTTR}$$

MTBF - true (population) mean time between failures

MTTR - true (population) mean time to repair a failure

ERT - true (population) median time to repair a failure

F - failure rate, equal to $(MTBF)^{-1}$

f - the number of failures measured in an interval of operating time T

T - an interval of time during which an equipment is in operation to count failures

R_p - measured time to repair any individual failure

μ_r - true (population) mean value for the distribution of $\log R_p$

σ_r - true (population) standard deviation for the distribution of $\log R_p$

m_r - measured (sample) mean for a set of n values of $\log R_p$

s_r - measured (sample) standard deviation for a set of n values of $\log R_p$

P - per cent confidence in a statement of the range in which a parameter is included

t_o - the value which is exceeded in P per cent of a large group of tests having a Student's distribution

n - the number of measurements in a sample set

II. EQUIPMENT AVAILABILITY

The availability of an equipment can be defined as the average fraction of desired operating time that the equipment is actually available for use.[1] If the average or mean time between failures, MTBF, and the mean time to repair the failures, MTTR, are known, the fractional availability is given as

$$A = \frac{MTBF}{MTBF + MTTR} \quad . \tag{1}$$

The individual intervals between failures and the times to repair these failures in practice have random lengths and may therefore be considered as random variables governed by probability distributions having certain forms. Since the true MTBF and MTTR are parameters of the distributions, their values can only be estimated from actual measured samples of finite size. In fact, we can state only that any parameter lies within a specified range and then give the probability, or confidence, that the statement is true. The range and confidence will, of course, depend upon the number of measurements used in calculating the parameter estimate.

In view of the situation which arises with regard to experimental evaluation of such parameters as MTBF and MTTR, the problem of estimation becomes even more difficult for the case of the availability A, which depends upon both MTBF and MTTR, neither of which can be calculated exactly with perfect confidence from a finite number of measurements. Therefore let us focus our attention on a procedure for estimating MTBF and MTTR before continuing our consideration of availability.

III. MEAN TIME BETWEEN FAILURES

Events which occur at random in time and which are statistically independent generally follow a Poisson distribution

[1] NAVSHIPS 94324, p I-2-2.

for the number of events per interval of time. Such a situation arises for the occurrence of failures in an equipment per unit time if we consider truly independent failures after the equipment has reached a stable failure rate after its initial break-in period.

First, let us define a failure rate F, the average number of failures per unit time, as the reciprocal of MTBF. Then the average number of failures in any time interval T will be FT. Finally, if we let f be the measured number of failures in any interval T, the probability distribution for f is

$$\Phi(f/T) = \frac{(FT)^f}{f!} e^{-FT} . \qquad (2)$$

Note that f is an integer and a random variable. From Eq. 2 we can get the probability of occurrence of any specific number of failures during an interval T for a given failure rate F. What is desired is an estimate of the value of the MTBF (the reciprocal of F) for a measured number of failures f_m.

Suppose we choose a particular value for the product FT, say FT_a. We can then calculate from Eq 2 a value f_a which f will exceed in P per cent of a very large number of tests. Such a point is indicated in Fig. 1. Similarly, values of f which will be exceeded in P per cent of tests can be calculated from Eq 2 for all values of FT, and a curve C_f is obtained.

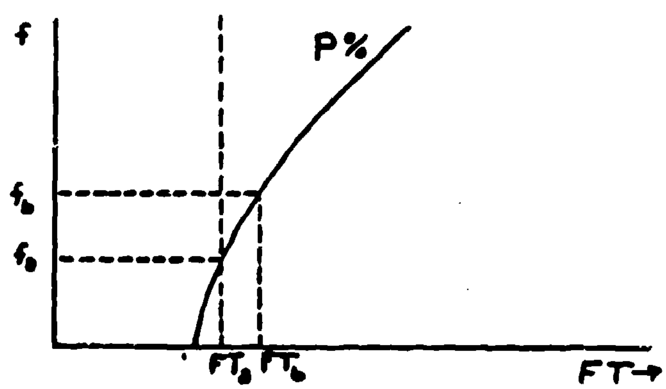

Figure 1

Now suppose we are testing a particular equipment. Though we cannot know the true value of FT, we assume that it does have some fixed mean number of failures FT_a associated with its failure distribution during our tests. From the curve of Fig. 1, we find that the number of failures f will exceed f_a in P per cent of tests. For example, we might measure f_b in one test. A horizontal line through f_b intersects the curve C_f at a point whose abscissa is FT_b; and FT_b will exceed the true value FT_a in P per cent of a large group of tests. Because of the way in which the curve C_f was constructed, the preceding statement is true for any large group of tests even if the true value FT_a varies from test to test (but remains constant during any single test).

We are now in position to make the following statements: "For any measured number of failures f_b in an interval T, the true value of FT lies between zero and a value FT_b associated with f_b by the curve C_f. Since C_f was obtained on a P per cent basis, our preceding statement will be correct in P per cent of a very large group of tests, and thus we have a confidence of P per cent in the first statement."

For practical applications, it is more convenient to plot values of f_a versus the reciprocal of FT, which is MTBF/T as indicated in Fig. 2. Curves are shown here for values of P equal to 90, 75, and 50 per cent. Thus we are now in a position to estimate, with a designated confidence coefficient, the minimum value which MTBF might assume, given a certain number of failures f in a test period T.

IV. MEAN AND MEDIAN TIME TO REPAIR

Experience has shown that the random length time intervals R_p required to repair an equipment are distributed according to the lognormal distribution function.[2] In other words, the

[2] NAVSHIPS 94324, p I-3-63

logarithms of the repair times, log R_p,[3] are distributed gaussianly with mean μ_r and standard deviation σ_r.[3] Since the median of the distribution of R_p is the antilog of the median of the distribution of log R_p and since the median and the mean of the distribution of log R_p are the same, we have the following relations involving the median equipment repair time ERT:

$$\log \text{ERT} = \mu_r, \tag{3a}$$

$$\text{ERT} = 10^{\mu_r} . \tag{3b}$$

The mean time to repair MTTR can be shown to fit the following relations[4]:

$$\log \text{MTTR} = \mu_r + \frac{1}{2}\sigma_r^2 \log_e 10, \tag{4a}$$

$$\text{MTTR} = 10^{(\mu_r + \frac{1}{2}\sigma_r^2 \log_e 10)} \tag{4b}$$

$$= 10^{(\mu_r + 1.15\,\sigma_r^2)} \tag{4c}$$

Incidentally, from equations 3a and 4 a, we find that

$$\log \text{MTTR} = \log \text{ERT} + 1.15\,\sigma_r^2 . \tag{5}$$

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3 Logarithms to the base 10 are used throughout this work unless otherwise indicated.

4 J. Aitchison and J.A. C. Brown, THE LOGNORMAL DISTRIBUTION, Cambridge, p 6-9.

Determination of a confidence interval and confidence coefficient for MTTR depends upon calculation of a multiplicity of confidence intervals and associated confidence coefficients for the mean μ_r and the standard deviation σ_r. The procedure is time consuming and far from straightforward. Fortunately, it is possible to estimate on a confidence interval basis the mean μ_r of the distribution of log R_p. Via equation 3b we then have an equivalent estimate of the median equipment repair time ERT.

It can be shown that if a random variable x is distributed gaussianly (μ, σ), then a transformed variable

$$t = \frac{\bar{x} - \mu}{s} \sqrt{n - 1}\ , \tag{6}$$

where $\bar{x}$ is the sample mean and s is the sample standard deviation of n measurements of x, has the Student's distribution with (n-1) degrees of freedom. Since our log R_p are taken to be gaussianly distributed (μ_r, σ_r),we can write

$$t = \frac{m_r - \mu_r}{s_r} \sqrt{n - 1}, \tag{7}$$

where

$$m_r = \frac{1}{n} \sum_{i=1}^{n} \log R_{p_i} \tag{8a}$$

and

$$s_r^2 = \frac{1}{n} \sum_{i=1}^{n} (\log R_{p_i})^2 - m_r^2\ . \tag{8b}$$

Suppose we choose a value t_o from a table of the t-distribution for (n-1) degrees of freedom which will be exceeded in P per cent of the tests. We can then state: "$t_o \leq t$ with a probability of P per cent." The variable t is related, however, to our μ_r and measured averages by equation (7). Thus we have

P per cent confidence in the statements:

$$t_o \le t = \frac{m_r - \mu_r}{s_r} \sqrt{n - 1} \qquad (9a)$$

$$-\infty \le \mu_r \le m_r - \frac{s_r}{\sqrt{n-1}} \quad t_o \qquad (9b)$$

(NOTE: t_o will be negative for any $P > 50$ per cent).

Now by equation 3b, we say

$$0 \le ERT \le 10^{\left(m_r - \frac{t_o}{\sqrt{n-1}} s_r\right)} \qquad (10)$$

with P per cent confidence, where, it will be remembered, m_r and s_r are the sample mean and sample variance, respectively, of measured log R_p values. They are calculated according to equations 8.

V. CONCLUSIONS

(Mean Time Between Failures)

1. The availability A is defined in terms of MTBF and MTTR. (Mean Time To Repair) The best statement which can be made about either of these, based on limited measurements, is the confidence with which we can depend upon stating correctly a range of values which will include the true value of the quantity in question. To convert such statements into equivalent information about A is difficult and far from straightforward arithmetically. On the other hand, a qualitative judgment concerning A can be obtained from quantitative information concerning ranges for MTBF and MTTR.

2. ~~We have a~~ A very good procedure is available for obtaining confidence intervals with associated confidence coefficients for MTBF. Such information is included in Figure 2.

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(Equipment Repair Time)

cont.

3. The MTTR depends upon the ERT and σ_r (sigma(r)) ~~as indicated in equation 5 repeated here~~, which is the standard deviation of the measured repair time.

$$\log MTTR = \log ERT + 1.15 \sigma_r^2 . \qquad (5)$$

We have ~~a~~ good procedure for establishing confidence intervals and associated confidence coefficients for ERT exists. ~~via equation 10.~~ Unfortunately, calculation of MTTR from ERT depends on a knowledge of the true (population) σ_r, ~~according to equation 5.~~ This true value cannot be known from a restricted set of measurements. Even using an estimate leads to difficulties in arithmetic computation similar to those mentioned in connection with finding A from MTBF and MTTR. However, it can be assumed that σ_r generally lies between 0.4 and 0.7.[5] Therefore one can obtain a qualitative feel for the range of MTTR from quantitative confidence interval information about ERT and from the historically estimated range for σ_r from equipments in general.

sigma(r)

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[5] NAVSHIPS 94324, I-2-13