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PERMUTATION TYPE SCHEDULES ON A SINGLE MACHINE UNDER COST CRITERIA

Research Report No. 76-17

by

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July, 1976

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Optimal permutation type schedules are obtained for a class of n jobs one machine type problems using as criteria i. the total processing cost, ii. the total deferral cost, and iii. the total processing and deferral cost. The set of seven models developed include cases when the machine deteriorates with usage, and when resetting of the machine is possible at the completion of each job. Particular attention is devoted to accounting for the time value of money, an important factor in the case of large tasks extending over prolonged periods of time such as construction projects. Use is made of a

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Abstract

Optimal permutation type schedules are obtained for a class of n jobs one machine type problems using as criteria i. the total processing cost, ii. the total deferral cost, and iii. the total processing and deferral cost. The set of seven models developed include cases when the machine deteriorates with usage, and when resetting of the machine is possible at the completion of each job. Particular attention is devoted to accounting for the time value of money, an important factor in the case of large tasks extending over prolonged periods of time such as construction projects. Use is made of a theorem in underlying the structural equivalence of all the models considered. Finally, an application related to a modular configuration is discussed.

1. Introduction

Consider a set $J = \{1, 2, \ldots, n\}$ of jobs to be processed on a single machine which can handle only one job at a time. At time origin all jobs are available for processing. The processor is continuously utilized until all jobs are terminated.

Associated with each job i is its processing time or service time p_i known with certainty. The completion time or flow time will be denoted by $F_i(\beta)$ where β is a permutation of J. An optimal permutation β may sometimes exist under an appropriate economic objective function. In particular, using cost as a criterion, one may consider:

1. The minimization of the total processing cost.
2. The minimization of the total deferral or delay cost.
3. The minimization of the sum total of processing and deferral cost.

Assume that associated with job i there is a <u>linear</u> deferral or delay cost h_i so that when following a permutation schedule β, the flow time is $F_i(\beta)$ and the delay cost is $h_i F_i(\beta)$. The permutation analysis of Smith's [20] using the technique of adjacent pairwise interchange, provides necessary and sufficient conditions for β to be optimal under objective function 2. For any i and j, job i precedes job j iff $(p_i/h_i) \leq (p_j/h_j)$.

Let $G_i(t)$, $i = 1, 2, \ldots, n$ be the cost associated if job i finishes at time t. The problem consisting in determining a schedule that minimizes the function $\sum_{i=1}^{i=n} G_i(t_i)$ has been investigated by several authors. The general problem formulation can be found in the works of Held and Karp [11] and Lawler [12]. Generalizations of the linear functions involving precedence relations were considered by Conway et al [4] and Sidney [19]. Other variations involving mean flow time at least as a partial criterion appears in the works of Emmons [7], [8] and Heck and Roberts [10]. Another popular type of criterion has been the so-called weighted tardiness cost function in which

$$G_i(t) = \beta_i \max \{0, t - p_i\} + \gamma_i t$$

where β_i and γ_i are constants. The works of Schild and Fredman [17], Elmaghraby [6], Gelders and Kleindorfer [9], Lawler [13], Srinivasan [21] and Schwimer [18] are to be noted. Additional references appear in Conway et al [4] and Baker [1]. Rinnooy Kan et al [16] have considered the case when $G_i(t)$ is a non-decreasing cost function and proven a set of theorems and corollaries to implement a number of elimination criteria in developing computational results for the weighted tardiness problem.

In the present work, we restrict ourselves only to permutation type schedules which minimize an objective function in the form of a cost criterion. A set of 7 models is considered in which the minimization of the following cost is involved: total processing cost, total deferral cost, and finally, the sum total of processing cost and deferral cost. All these models are solved by an appeal to a particular theorem.

2. Minimization of the Total Processing Cost

It is evident that the criterion based on the minimization of the total delay cost is customer biased and neglects the production aspect of the problem. In other words, the scheduling defined under the prestated criterion does not account for the cost associated with the processing phase of the problem, indeed an important characteristic. One in fact could easily make the case for developing an optimal sequence from among the possible set of permutations of J which is strictly production oriented, rather than service oriented, and consider the minimization of the total processing cost. This would fall at the other end of the spectrum of cost objectives that one may select in a legitimate fashion in the same way as selecting the total delay cost.

In its simplest form, one may assume that the cost of processing job i on the machine is proportional to its processing time p_i and is independent of the state of the machine or the sequence selection. Under this linearity assumption, the total cost of processing all jobs on the machine is $\sum_{i=1}^{i=n} a_i p_i$ with a_i being the cost of processing job i for one unit of time on the machine All sequences selected are obviously optimal.

Model 1. Minimization of the Total Processing Cost for a Class of Nonlinear Costs

In practice, the cost associated in the operation of a machine is more complex and usually involves terms which increase with the length of time the machine is in use. This fact has been well recognized for a long time by persons involved in the study of equipment replacement and maintenance problems. References in the area abound starting with the classical work of Terborgh [22]. (For literature review see Dean [5] Barlow and Proschan [2] and more recently Pierskalla and Voelker [15]).

Typically the machine deteriorates with age, that is, the longer the machine is used, the higher the cost of operating it due to increased costs of maintenance, repair and operations, the later being experienced due to a decrease in the efficiency of operation of the machine. Thus, the action of the machine may worsen as it ages, hence needing more adjustments. We explicitly assume that any repair adjustments and maintenance have negligible time compared to the processing time of any of the jobs.

Alternatively, one could conceive the processor to be a human operator performing well-defined tasks on the jobs. Human fatigue is a limiting factor in the performance level of man-machine systems. McFarland [14] defines fatigue as "... a group of phenomena associated with impairment, or loss of efficiency and skill, and the development of anxiety, frustration or boredom." The causes of fatigue involve the interaction of physiological and psychological factors. In industry, fatigue is the chief factor in limiting production, because it decreases efficiency. This causes a corresponding decrease in the rate of production or work output [3]. The state of the operator changes as work progresses on the jobs, and one may consider this worsening condition in his state as an aging process.

Let u represent the age of the machine at time t. Assume that initially the machine age is x and that as jobs are processed on it the machine ages at the rate (du/dt) = 1. Without loss in generality, we shall set u to be the state of the system at time t with x being the initial state. Let $c_i(u)$, i = 1, 2, ..., n be a continuous function of u, $u \geq 0$, representing the cost per unit time of processing job i when the machine has age u. Let for i = 1, 2, ..., n

$$\begin{aligned} T_0 &= x \\ T_{i-1} &= x + p_1 + p_2 + \dots + p_{i-1} \end{aligned} \tag{1}$$

Thus, T_{i-1} represents the state of the system just prior to processing job i on the machine. T_n is the final state of the system at the time when the processing of the last job has been completed.

We now prove the following

Lemma:

Sequence* {1, 2, ..., i - 1, i, i + 1, ..., n} is preferred to sequence {1, 2, ..., i - 1, i + 1, i, ...} if $[c_i(u) - c_{i+1}(u)]$ is a non-decreasing function of u in the interval $[T_0, T_n]$.

---

*No special symbolism is used to denote position in sequence in a permutation schedule as this will be clear from the content..

Proof. The total cost associated when processing all of job i with processing time p_i is

$$\int_{x+p_1+p_2+\ldots+p_{i-1}}^{x+p_1+p_2+\ldots+p_{i-1}+p_i} c_i(u)du = C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_i) - C_i(x + p_1 + p_2 + \ldots + p_{i-1})$$

where

$$\int_0^u c_i(v)dv = C_i(u)$$

For a given sequence 1, 2, ..., i - 1, i, i + 1, ..., n, the total cost of operating the machine starting with a machine age x at time origin is

$$\sum_{j=1}^{j=n} [C_j(x + p_1 + p_2 + \ldots + p_{j-1} + p_j) - C_j(x + p_1 + p_2 + \ldots + p_{j-1})]$$

Call the previous schedule by the symbol S. We formulate a schedule S' by interchanging the positions of the jobs in the ith and (i + 1)th position. If now schedule S is better, then necessarily the following inequality must be satisfied:

$$\begin{aligned} & C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_i) - C_i(x + p_1 + p_2 + \ldots + p_{i-1}) \\ & + C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1} + p_i + p_{i+1}) - C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1} + p_i) \\ & \leq C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1} + p_{i+1}) - C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1}) \\ & + C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_{i+1} + p_i) - C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_{i+1}) \end{aligned}$$

This relation may be written alternatively as

$$\begin{aligned} & C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_i) - C_i(x + p_1 + p_2 + \ldots + p_{i-1}) \\ & - [C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1} + p_i) - C_{i+1}(x + p_1 + p_2 + \ldots + p_{i-1})] \\ & \leq C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_i + p_{i+1}) - C_i(x + p_1 + p_2 + \ldots + p_{i-1} + p_{i+1}) \end{aligned}$$

$$- [C_{i+1}(x + p_1 + p_2 + \dots + p_{i-1} + p_i + p_{i+1}) - C_{i+1}(x + p_1 + p_2 + \dots + p_{i-1} + p_{i+1})]$$

Using notation (1), the above inequality may be written as

$$\int_{T_{i-1}}^{T_{i-1}+p_i} [c_i(u) - c_{i+1}(u)]du \leq \int_{T_{i-1}+p_{i+1}}^{T_{i-1}+p_i+p_{i+1}} [c_i(u) - c_{i+1}(u)]du \quad (2)$$

Inequality (2) is satisfied if the function $[c_i(u) - c_{i+1}(u)]$ is non-decreasing in the interval $[T_0, T_n]$. Using this lemma and the technique of adjacent pairwise interchange, the following theorem can be proven.

Theorem 1

Let f(u) be a non-decreasing (non-increasing) function of u in the interval $[T_0, T_n]$, and let $a_i(x)$, $b_i(x)$ be given quantities independent of u for i = 1, 2, ..., n. Let

$$c_i(u) = a_i(x)f(u) + b_i(x)$$

Then an optimal schedule results in the sense of minimizing total cost iff the jobs are sequenced in the order of descending (ascending) $a_i(x)$'s.

This theorem can be proven easily by showing that the necessary, sufficiency and transitivity conditions hold. Note that the quantities $a_i(x)$ and $b_i(x)$ depend only on the initial state of the system and the characteristic of job i, i = 1, 2, ..., n. These quantities are independent of the state of the system when starting the processing of job i.

The function $c_i(u)$ is often termed the operating cost rate function. An interesting special case arises when $c_i(u)$ is the power function

$$c_i(u) = a_i(x)u^\lambda + b_i(x)$$

For $\lambda > 0$, it is optimal to sequence the jobs in the order $a_1(x) \geq a_2(x) \geq \cdots \geq a_3(x)$. For $\lambda < 0$, the sequencing rule is anthetical. For $\lambda = 0$, any sequence is optimal.

Model 2 - Minimization of the Total Discounted Linear Processing Cost

In many situations involving large complex tasks extending over prolonged periods of time (long makespan) such as construction projects, the time value of money cannot be neglected. We assume that at time origin the machine age is x and the interest rate over continuous time is $\alpha \geq 0$. We again set a_i to be the cost of processing job i for one unit of time on the machine. The total discounted cost of processing job i on the machine is

$$e^{\alpha x} \int_{x+p_1+\cdots+p_{i-1}}^{x+p_1+\cdots+p_{i-1}+p_i} a_i e^{-\alpha u} du$$

This is equivalent to stating that there is an operating cost rate function associated in the processing of job i on the machine which is equal to

$$c_i(u) = a_i e^{\alpha x} e^{-\alpha u}$$

The function $e^{-\alpha x}$ is a strictly decreasing function of u. Hence from Theorem 1, the optimal sequence which will minimize the total discounted cost in the processing of all n jobs is given by the ordering rule

$$a_1 \leq a_2 \leq \cdots \leq a_n$$

that is the least expensive jobs are processed first.

3. Fixed Resetting of Machines under Possible Deteriorating Conditions

When the state of the machine deteriorates with usage, it is quite conceivable that it be reset through appropriate adjustments to a given level. We assume here that the machine is uninterrupted during the processing of any job and that it may be subject to deterioration. At the completion of each job, the machine is reset to state x = 0 prevailing at time origin. The retooling of a machine prior to the processing of any of the jobs would be a special case. Similarly, in the case of human fatigue a rest period would correspond to a resetting condition. We further assume that resetting time is negligible compared to the processing time of each job, and further that the resetting of the machine does not affect the processing time of any of the jobs. We again let $c_i(u)du$ to be the cost of operating the machine when aging from u to u + du when processing job i on it, the rate of aging when in operation being unity. In addition, we assume that associated with the resetting of the machine prior to the processing of job i, there is a cost K_i which depends on job i, but which does not depend on its predecessor (sequence independent). Again, our objective is to investigate simple permutation type schedules which minimizes total cost of resetting and processing. The case when minimizing the total undiscounted processing and resetting costs is trivial. The discounted cost case is not so obvious.

Model 3: Minimization of the Total Discounted Processing and Resetting Costs

Here, $c_i(u)du$ is again the operating cost rate function for job i, i = 1, 2, ..., n. The interest rate is α. The total discounted cost associated with job i (resetting the machine plus processing the job) is

$$e^{-\alpha(p_1+p_2+\ldots+p_{i-1})}[K_i + \int_0^{p_i} e^{-\alpha u} c_i(u)du]$$

This expression can be alternatively written as

$$\alpha \frac{K_i + \int_0^{p_i} e^{-\alpha y} c_i(y)dy}{1 - e^{-\alpha p_i}} \int_{p_1+\ldots+p_{i-1}}^{p_1+\ldots+p_{i-1}+p_i} e^{-\alpha u}du$$

This is again equivalent to stating that associated with the processing of job i on the machine, there exists an operating cost rate function equal to

$$\alpha \frac{K_i + \int_0^{p_i} e^{-\alpha y} c_i(y)dy}{1 - e^{-\alpha p_i}} e^{-\alpha u}$$

Since $e^{-\alpha u}$ is strictly decreasing in u, from Theorem 1 the minimization of the total discounted processing and resetting costs is achieved iff the jobs are sequenced, so that

$$\frac{K_1 + \int_0^{p_1} e^{-\alpha y} c_1(y)\,dy}{1 - e^{-\alpha p_1}} \leq \frac{K_2 + \int_0^{p_2} e^{-\alpha y} c_2(y)\,dy}{1 - e^{-\alpha p_2}} \leq \cdots \leq \frac{K_n + \int_0^{p_n} e^{-\alpha y} c_n(y)\,dy}{1 - e^{-\alpha p_n}}$$

4. <u>Minimization of the Total Deferral or Delay Cost</u>

We shall not expound on the justification of this type of objective function since excellent references exist in this regard. We assume that associated with job i, i = 1, 2, ..., n, there is a linear deferral or delay cost h_i. We consider the following model in which cost is discounted

<u>Model 4</u> - <u>Minimization of the Total Discounted Linear Delay Cost</u>

Let α be the interest rate. The discounted deferral cost associated in processing the job in the position i is

$$\int_0^{p_1+p_2+\ldots+p_i} h_i e^{-\alpha u} du = \frac{h_i}{\alpha} - \frac{h_i}{\alpha} e^{-\alpha p_i} e^{-\alpha(p_1+p_2+\ldots+p_{i-1})}$$

$$= \frac{h_i}{\alpha} - \frac{h_i e^{\alpha x} e^{-\alpha p_i}}{1 - e^{-\alpha p_i}} \int_{x+p_1+p_2+\ldots+p_{i-1}}^{x+p_1+p_2+\ldots+p_{i-1}+p_i} e^{-\alpha u} du$$

Since the term h_i/α will not affect the minimization problem, we may simply state that the delay cost associated with job in position i is equivalent to an operating cost rate function $c_i(u)$ which is equal to

$$c_i(u) = -\frac{h_i e^{\alpha x} e^{-\alpha p_i}}{1 - e^{-\alpha p_i}} e^{-\alpha u}$$

and from Theorem 1, an optimal sequence minimizing the total discounted deferral cost is obtained iff the jobs are ordered so that

$$-\frac{h_1 e^{\alpha x} e^{-\alpha p_1}}{1 - e^{-\alpha p_1}} \leq -\frac{h_2 e^{\alpha x} e^{-\alpha p_2}}{1 - e^{-\alpha p_2}} \leq \cdots \leq -\frac{h_n e^{\alpha x} e^{-\alpha p_n}}{1 - e^{-\alpha p_n}}$$

This reduces to Rothkopf's [23] results.

$$\frac{e^{\alpha p_1} - 1}{h_1} \leq \frac{e^{\alpha p_2} - 1}{h_2} \leq \cdots \leq \frac{e^{\alpha p_n} - 1}{h_n} \tag{2}$$

As an interesting special case, we note that the undiscounted deferral cost in processing the job in position i is

$$\lim_{\alpha \to 0} \int_0^{p_1+p_2+\cdots+p_{i-1}+p_i} h_i e^{-\alpha u} du$$

The optimal sequence may be obtained by multiplying each term in (2) by $1/\alpha$ and using l'Hospital's rule to compute the limit as $\alpha \to 0$. Smith's result [20]

$$\frac{p_1}{h_1} \leq \frac{p_2}{h_2} \leq \cdots \leq \frac{p_n}{h_n}$$

is immediately obtained.

5. Minimization of the Sum Total of Processing and Deferral Costs

Our final class of criterion assumes that processing costs and deferral costs are jointly prevailing. In a strict sense, this type of prevalence comes closest to reality and views the combined machine-job configuration as an aggregate system.

The objective here is to determine whether a simple permutation type schedule exists which balances the joint contribution of the total processing costs of jobs on the machine and the total deferral costs associated with each job. This would be similar to the set of objective functions encountered in classic inventory problems where production or procurement cost is balanced against the holding cost.

Model 5: Minimization of the Total Discounted Linear Production and Deferral Costs

In the special case when for each job production cost is proportional to production time and deferral cost is proportional to the flow time, the discounted cost associated in the processing of job in position i is from Models 2 and 4:

$$e^{\alpha x} \int_{x+p_1+\ldots+p_{i-1}}^{x+p_1+\ldots p_{i-1}+p_i} a_i e^{-\alpha u} du + \int_0^{p_1+p_2+\ldots+p_{i-1}+p_i} h_i e^{-\alpha u} du$$

This last expression can be written as:

$$\frac{h_i}{\alpha} + e^{\alpha x}\left(a_i - \frac{h_i}{e^{\alpha p_i} - 1}\right) \int_{x+p_1+\ldots+p_{i-1}}^{x+p_1+\ldots+p_{i-1}+p_i} e^{-\alpha u} du$$

Using arguments similar to the previous models and applying Theorem 1, it is easily verified that it is optimal to order the jobs according to the sequence

$$a_1 - \frac{h_1}{e^{\alpha p_1} - 1} \le a_2 - \frac{h_2}{e^{\alpha p_2} - 1} \le \cdots \le a_n - \frac{h_n}{e^{\alpha p_n} - 1}$$

Model 6: Minimization of the Total Discounted Processing, Resetting and Deferral Costs

Using the same symbols as in Models 3 and 4, the expression for the total discounted cost associated with job i, including resetting the machine, processing the job and deferral, is:

$$e^{-\alpha(p_1+p_2+\ldots+p_{i-1})}\left[K_i + \int_0^{p_i} e^{-\alpha u} c_i(u) du\right] + \int_0^{p_1+p_2+\ldots+p_{i-1}+p_i} h_i e^{-\alpha u} du$$

This expression can be written as:

$$\frac{h_i}{\alpha} + A_i \int_{p_1+p_2+\ldots+p_{i-1}}^{p_1+p_2+\ldots+p_{i-1}+p_i} e^{-\alpha u} du$$

where

$$A_i = \frac{\alpha K_i + \alpha \int_0^{p_i} e^{-\alpha y} c_i(y) dy - h_i e^{-\alpha p_i}}{1 - e^{-\alpha p_i}}$$

By a direct application of Theorem 1, it is easy to verify that it is optimal to sequence the jobs so that

$$A_1 \le A_2 \le \cdots \le A_n$$

<u>Model 7:</u> <u>Minimization of the Total Undiscounted Production and Deferral Costs under Linear Operating Cost Rate Function</u>

In this particular model, we assume that $c_i(u) = a_i u + b_i$, for i = 1, 2, ..., n. We denote by x *the state of the machine at time origin.* The total cost of processing job i on the machine plus its associated deferral cost is

$$\int_{x+p_1+p_2+\ldots+p_{i-1}}^{x+p_1+p_2+\ldots+p_{i-1}+p_i} (a_i u + b_i) du + h_i(p_1 + p_2 + \ldots + p_{i-1} + p_i)$$

$$= a_i p_i (x + p_1 + p_2 + \ldots + p_{i-1} + p_i) - \frac{a_i}{2} p_i^2 + b_i p_i$$

$$+ h_i(p_1 + p_2 + \ldots + p_{i-1} + p_i)$$

Using the technique of adjacent pairwise interchange, it can easily be verified that it is optimal to sequence the jobs so that

$$\frac{h_1}{p_1} + a_1 \ge \frac{h_2}{p_2} + a_2 \ge \cdots \ge \frac{h_n}{p_n} + a_n$$

One could have applied Theorem 1 by associating to job i the following cost function

$$\int_0^{p_1+p_2+\ldots+p_{i-1}+p_i} (h_i + a_i p_i) e^{-\alpha u} du$$

$$+\int_{x+p_1+p_2+\dots+p_{i-1}}^{x+p_1+p_2+\dots+p_{i-1}+p_i} (a_i x - \frac{a_i}{2} p_i + b_i)du$$

$$= \frac{h_i + a_i p_i}{\alpha} + \int_{x+p_1+\dots+p_{i-1}}^{x+p_1+\dots+p_{i-1}+p_i} [- \frac{(h_i + a_i p_i)e^{\alpha x}e^{-\alpha p_i}}{1 - e^{-\alpha p_i}} e^{-\alpha u} + (a_i x - \frac{a_i}{2} p_i + b_i)]du$$

(Note that this cost function reduces to the true cost associated with job i for $\alpha \to 0$). Since the term $(h_i + a_i p_i)/\alpha$ will not affect the minimization problem, we simply associate with job i the following operating cost rate function

$$- \frac{(h_i + a_i p_i)e^{\alpha x}e^{-\alpha p_i}}{1 - e^{-\alpha p_i}} e^{-\alpha u} + (a_i x - \frac{a_i}{2} p_i + b_i)$$

A straightforward application of Theorem 1 will verify that it is optimal to sequence the jobs so that

$$\frac{h_1 + a_1 p_1}{e^{\alpha p_1} - 1} \geq \frac{h_2 + a_2 p_2}{e^{\alpha p_2} - 1} \geq \dots \geq \frac{h_n + a_n p_n}{e^{\alpha p_n} - 1}$$

Multiplying each term in the above inequality by α and using l'Hospital's rule to compute limits as $\alpha \to 0$, we obtain the desired result.

6. An Application to Modular Systems

The long range solution for reliable, low cost, high-performance electronic systems is the functional module usually appearing in the form of an electronic package. In standardized digital electronic modular subsystems common in avionics, it is important to implement an effective fault detection, isolation and repair procedure to a failed digital subsystem and to the defective module.

For a digital subsystem consisting of n modules, assume that one or more of these modules have failed with no apriori information on the faulty modules. All modules are to be fully tested by a single processor for fault detection, isolation and repair (or replacement). It is evident that each module can be viewed as a job. The problem of identifying a proper test sequencing pattern to meet a given criterion can clearly be related to the class of problems which has been investigated. The effectiveness of a testing program can be quite significant in the productivity of a test/repair facility specializing in this type of operation.

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