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NAVAL INTELLIGENCE SUPPORT CENTER
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WASHINGTON, D.C. 20390

AD A032527

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CLASSIFICATION: UNCLASSIFIED

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(6)

TITLE: The Effect of Wind Tunnel Flow Boundaries on the Flow Past a Wing of Low Aspect Ratio

(Vliyaniye granits potoka aerodinamicheskoy truby na obtekaniye kryla malogo udlineniya)

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PAGES: (21) Trans. of

SOURCE: Samoletostroyeniye i tekhnika vozdushnogo flota [redacted] 28, p3-9, 1972.
Khar'kov State University Publishing House, Khar'kov, 1972, pp. 3-9

(14) NISC-Trans-3836

ORIGINAL LANGUAGE: Russian

(10) Yu. F. /Usik
V.I. /Kholyavko

TRANSLATOR: C

NISC TRANSLATION NO. 3836

APPROVED P.T.K.

(11) DATE 24 Aug[redacted]t [redacted]976

(12) 13p.

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We will consider the motion of a thin plane wing of low aspect ratio in a cylindrical wind tunnel of circular cross section (Fig. 1). In accordance

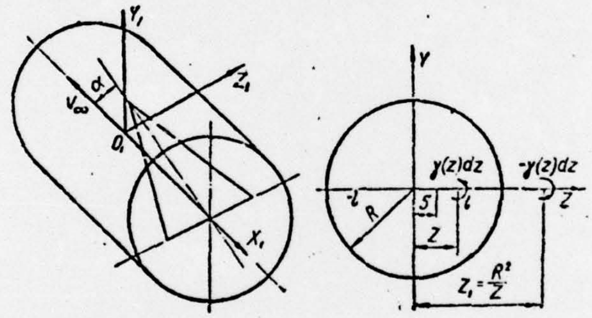

FIGURE 1

with the thin body theory, the three-dimensional flow in a wind tunnel caused by the motion of the wing will be replaced by a two-dimensional flow in a transverse plane fixed in the tunnel. The flow in this plane is caused by vertical displacement of the plate at velocity $V_0 = V_\infty \alpha$ (wing cross section wake) and by a change in its width. This flow will result in a change in the apparent mass of the plate, proportional to the increment of the wing lift on the length $dx_1 = V_\infty dt$:

$$\frac{dY}{dx_1} = \frac{d(mV_0)}{dt} = V_\infty^2 \alpha \frac{dm}{dx_1}. \qquad (1)$$

If (1) is summed up over the length of the root chord ($0 \leq x_1 \leq 1$), the total wing lift is obtained:

$$Y = V_\infty^2 \alpha m(1), \qquad (2)$$

where m(1) is the value of the apparent mass in section $x_1 = 1$, coinciding with the maximum wingspan.

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Using (1), one can determine the longitudinal wing moment relative to the o_1z_1 axis passing through the wing apex:

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We will introduce the lift and moment coefficient /4

$$C_Y = \frac{Y}{\frac{\rho V_\infty^2}{2} S}; \quad m_z = \frac{M_{z_1}}{\frac{\rho V_\infty^2}{2} S \cdot 1}$$

and determine the position of the wing pressure center relative to the apex in fractions of the root chord as $x_p = -\frac{m_z}{C_Y}$.

From (2) and (3), we obtain the following formulas:[1]

$$C_y^\alpha = 2\frac{m(1)}{\rho S}; \quad x_p = 1 - \frac{1}{m(1)}\int_0^1 m(x)\,dx. \quad (4)$$

According to (4), to determine the lift coefficient and position of the pressure center, it is necessary to know the chordwise distribution of the apparent mass of the wing cross sections.

To calculate the apparent mass of the plate moving perpendicularly to its plane in the region bounded by the circle, we will distribute a vortex sheet of variable strength $\gamma(z)dz$ over the length of the plate. A vortex element of $\gamma(z)dz$ will be placed at an arbitrary point of the plate $-\ell \leqslant z \leqslant \ell$. This vortex produces a vertical velocity of $\frac{1}{2\pi}\frac{\gamma(z)dz}{z - s}$ at a fixed point of the plate S. The action of all the vortices is determined by summing up over the length of the plate

$$\frac{1}{2\pi}\int_{-l}^{l} \frac{\gamma(z)\,dz}{z - s}.$$

The influence of the flow boundaries will be taken into account by means of an additional vortex with a strength of $\pm\gamma(z)dz$ (the plus sign refers to a wind tunnel with an open test section, and the minus sign, to a closed test section), placed at a distance $z_1 = \frac{R^2}{z}$ from the tunnel center (Fig. 1). /5

The effect of the wind tunnel on the plate is made up of the effects of all the additional vortices and is expressed by the fact that at point s of the wing, an additional vertical velocity is obtained, equal to

$$\pm\frac{1}{2\pi}\int_{-l}^{l} \frac{\gamma(z)\,dz}{\frac{R^2}{z} - s}.$$

Since the total velocity at point s should be equal to the displacement velocity of the plate V_0, then

$$\int_{-l}^{l} \frac{\gamma(z)\,dz}{z - s} + k\int_{-l}^{l} \frac{\gamma(z)\,dz}{\frac{R^2}{z} - s} = 2\pi V_0. \quad (5)$$

Here k = ±1, where the plus sign refers to a tunnel with an open test section (free jet), and the minus sign, to a tunnel with a closed test section (solid walls).

Relation (5) is an integral equation for determining the unknown strength $\gamma(z)$.

We will switch to new variables in Eq. (5) after performing the substitution $z = \ell\cos\vartheta$; $s = \ell\cos\theta$; $\gamma(z) \to \gamma(\vartheta)$; $0 \leqslant \vartheta \leqslant \pi$. After some simple transformations, we obtain ($\mu = \frac{\ell}{R}$)

$$\int_0^\pi \frac{\gamma(\vartheta)\sin\vartheta d\vartheta}{\cos\vartheta - \cos\theta} + k\int_0^\pi \frac{\gamma(\vartheta)\cos\vartheta\sin\vartheta d\vartheta}{\frac{1}{\mu^2} - \cos\vartheta\cos\theta} = 2\pi V_0. \quad (6)$$

Equation (5) or (6) has an exact solution corresponding to the motion of the wing in an unbounded flow. In this case R = ∞, and the second term in Eqs. (5) and (6), which allows for the boundary effect, vanishes, and Eq. (5) becomes

$$\int_{-l}^{l} \frac{\gamma_\infty(z)\,dz}{z - s} = 2\pi V_0.$$

Inverting this integral on the condition of unboundedness of $\gamma_\infty(z)$ on the ends of the [-ℓ, ℓ] interval, we obtain

$$\gamma_\infty = \frac{2V_0 z}{\sqrt{l^2 - z^2}} = 2V_0 \text{cotan}\,\vartheta \quad (7)$$

To find the solution of Eqs. (6) for finite wind tunnel radii R, the unknown function $\gamma(\vartheta)$ will be represented in the form of the following series with unknown coefficients: /6

$$\gamma(\vartheta) = 2V_0\left[A_0 \operatorname{ctg}\vartheta + \sum_{n=1}^{\infty} A_{2n}\sin n\vartheta\right]\ldots \quad (8)$$

Here the first term for $A_0 = 1$ is the solution in an unbounded flow, and the second term, when $A_0 \neq 1$, determines the boundary effect.

If the coefficients A_n (n = 0, 1, 2, ...) are known, the apparent mass of the plate is determined according to Ref. 1 by

$$m = \pi\rho l^2\left(A_0 + \frac{A_2}{2}\right). \quad (9)$$

In the special case of R = ∞ (unbounded flow), $A_0 = 1$, $A_2 = 0$, and

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To determine A_n (n = 0, 1, ...) in the general case with R ≠ ∞, we will substitute (8) into (6). After a series of transformations, we obtain

$$A_0\left[1 + k\frac{\mu^2}{\sqrt{1-\mu^4\cos^2\theta}\,(1+\sqrt{1-\mu^4\cos^2\theta})}\right] - \sum_{n=1}^{\infty} A_{2n}\left[\cos 2n\theta - k\frac{\mu^{2(2n-1)}\cos^{2(n-1)}\theta}{(1+\sqrt{1-\mu^4\cos^2\theta})^{2n}}\right] = 1. \quad (11)$$

If a series of values $\theta = \frac{\pi}{2p} s$ $(s = 0, 1, \ldots, p)$ is fixed in (11), a system of equations is obtained for determining the coefficients of the series A_0, $A_2, \ldots, A_{2p}$. The remaining coefficients must be set equal to zero.

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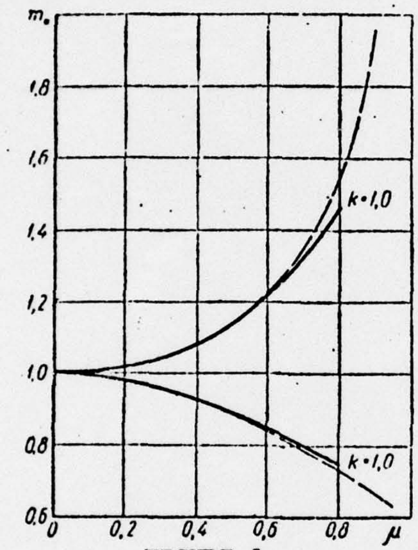

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The results obtained from calculations of the relative magnitude of the apparent mass of the plate ($m_* = \frac{m}{m_\infty}$) for p = 6 are given in Fig. 2 (dashed curve). /7

If we confine ourselves to the condition $\mu \ll 1$, we can obtain an approximate analytical solution of Eq. (5) or (6).

The denominator of the second term of Eq. (5) will be represented in the form

$$\frac{1}{\frac{R^4}{z} - s} = \frac{z}{R^2} \frac{1}{1 - \frac{sz}{R^2}} \approx \frac{z}{R^2}\left(1 + \frac{zs}{R^2} + \frac{z^2 s^2}{R^4} + \ldots\right) \approx \frac{z}{R^2}.$$

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If we now turn to the new variable ϑ $(z = \ell \cos \vartheta)$ and use expansion (8), we obtain from (12) in the accepted formulation

$$A_0 = \frac{1}{1 + \frac{k}{2}\mu^2}; \quad A_{2n} = 0 \, (n = 1, 2, \ldots). \qquad (13)$$

According to (9), the relative apparent mass will be written in this approximation as

$$m_* = \frac{1}{1 + \frac{k}{2}\mu^2}. \qquad (14)$$

Comparison of calculations made by means of formula (14) with the exact solution (Fig. 2) shows their satisfactory agreement for $\mu \leqslant 0.8$.

The approximate solution (14) is used below for analyzing the effect of the tunnel walls on the aerodynamic characteristics of a low-aspect-ratio wing.

The effect of the tunnel walls on the lift coefficient is established directly from (4) and (14). We will denote by $C^\alpha_{Y_\infty}$ and $C^\alpha_{Y_t}$ the values of the derivatives of the wing in an unbounded flow and in a wind tunnel. The relationship between them will be as follows:

$$C^\alpha_{y_\infty} = \frac{1}{m_*} C^\alpha_{Y_t} = \zeta_t C^\alpha_{Y_t},$$

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It is interesting to note that the correction factor in formula (15) is in complete agreement with the existing corrections in a circular tunnel[2] if in the latter corrections one takes $\lambda \to 0$.

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To allow for the effect of the tunnel walls on the position of the wing pressure center, it is necessary, according to (4), to know the distribution of the apparent mass over the length of the wing. This distribution depends on the planform of the wing.

We will consider a class of wings whose planform is given by the equation

$$z_1(x_1) = l(x_1) = Lx^p \quad (0 \leqslant x \leqslant 1), \qquad (16)$$

where L is the wing semispan.

The change in apparent mass along the length of the wing is determined from (14):

$$m(x) = \pi\rho L^2 \frac{x^{2p}}{1 \pm \frac{1}{2}\mu^2 x^{2p}}. \qquad (17)$$

The position of the pressure center of the wing relative to its apex in fractions of the root chord is calculated from formula (4):

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In an unbounded flow $\mu = 0$, and from (18) we obtain

$$x_{p_\infty} = \frac{2p}{2p+1}. \qquad (19)$$

We introduce the relative quantity $\delta = \frac{x_p}{x_{p_\infty}}$. We then obtain the following relation between the values of the pressure center in an unbounded flow x_{p_∞} and in a wind tunnel x_{p_t}:

$$x_{p_\infty} = \frac{1}{\delta} x_{p_t}. \qquad (20)$$

where according to (17) and (18), δ is determined as follows:

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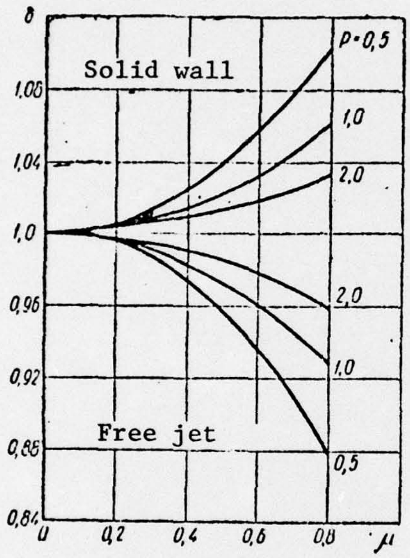
The magnitude of the correction factor δ for certain planforms (p values) of wings is shown in Fig. 3. /9



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The results of the calculations show that the effect of the tunnel walls on the position of the pressure center is less pronounced than that on the magnitude of the derivative C_Y^{α}.

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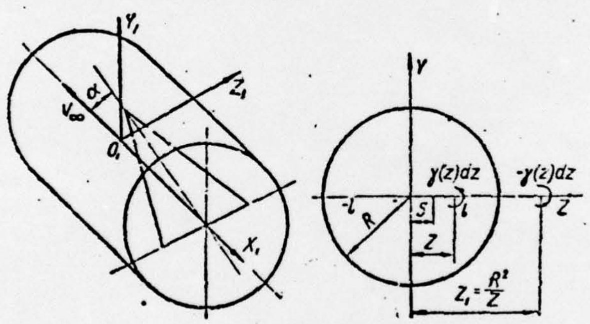

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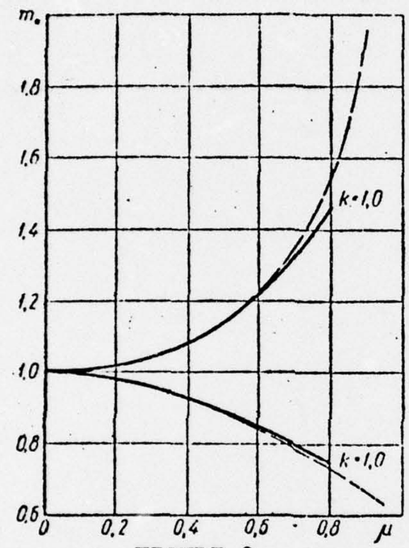

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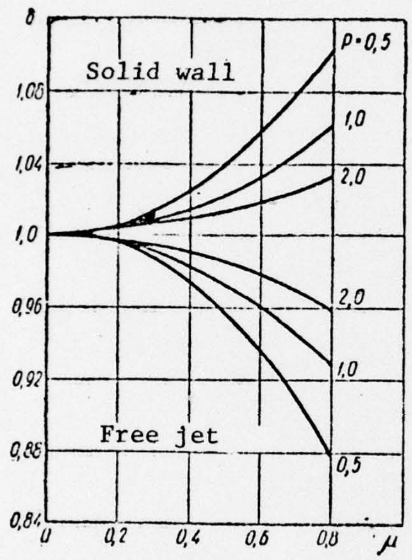

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The results of the calculations show that the effect of the tunnel walls on the position of the pressure center is less pronounced than that on the magnitude of the derivative C_Y^α.

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