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LOW-ASPECT-RATIO WING IN A BOUNDED INVISCID FLOW (KRYLO MALOGO --ETC(U)
AUG 76 V I KHOLYAVKO, Y F USIK
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LOW-ASPECT-RATIO WING IN A BOUNDED INVISCID FLOW

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The aerodynamic characteristics of a thin planar wing moving near a solid /3* or free surface (hydrofoil) are determined. Use is made of the thin body theory, whereby the three-dimensional flow past a thin body elongated in the direction of motion is approximately replaced by two-dimensional flow in transverse planes. In the case under consideration, the problem is reduced to the study of the motion of a flat plate near an interface (Fig. 1).

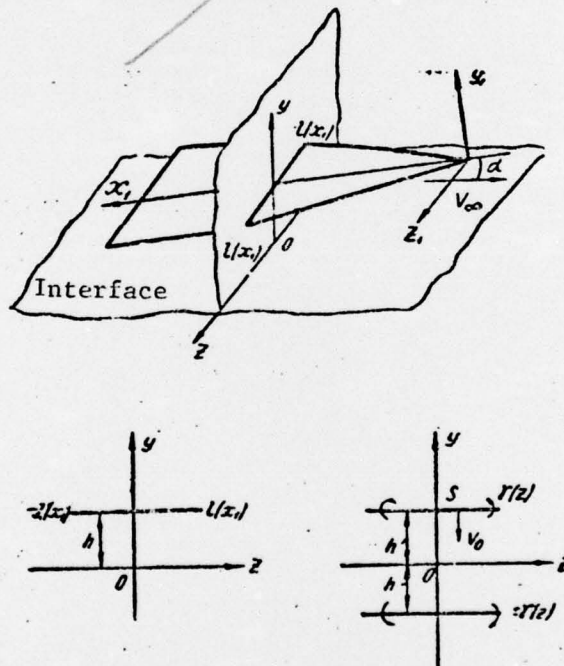


FIGURE 1

Let a planar wing of low aspect ratio, whose maximum span coincides with the trailing edge, move at a small angle of attack and a constant velocity V_∞ . A portion of the wing $dx_1 = V_\infty dt$ will pass through a fixed transverse plane ZOY in time dt . In the equivalent two-dimensional problem, this motion of the wing corresponds to a vertical displacement of a flat plate moving at constant velocity $V_0 = V_\infty \alpha$ by an amount $V_0 dt$ and to a change in the width of the plate from $l(x_1)$ to

* Numbers in the right margin indicate pagination in the original text.

$$l(x_1) + \frac{dl(x_1)}{dx_1} dx_1 = l(x_1) + \frac{dl(x_1)}{dx_1} V_\infty dt,$$

where $l(x_1)$ is the local wing semispan.

The flow in the ZOY plane, caused by the vertical displacement of the plate and by the change in its width, will cause the apparent mass of the plate to change by an amount corresponding to the wing lift increment over the length dx_1 , i.e.,

$$\frac{dY}{dx_1} = \frac{d(mV_0)}{dt} = V_0 \frac{dm(x_1)}{dt} = V_0 \frac{dm(x_1)}{dx_1} \frac{dx_1}{dt} = V_\infty^2 \alpha \frac{dm(x_1)}{dx_1}.$$

Integration of this expression over the chord length $0 \leq x_1 \leq b$ gives the wing lift

$$Y = \int_0^b \frac{dY}{dx_1} dx_1 = V_\infty^2 \alpha m(b), \quad (1)$$

where $m(b)$ is the apparent mass of the plate, determined in the wing section along the maximum span.

The transverse aerodynamic moment relative to the OZ_1 axis passing through the wing apex is calculated from the formula

$$M_{z_1} = - \int_0^b x_1 \frac{dY}{dx_1} dx_1 = - V_\infty^2 \alpha \int_0^b x_1 dm(x_1) = - V_\infty^2 \alpha \times \\ \times [m(b)b - \int_0^b m(x_1) dx_1]. \quad (2)$$

The pressure drag force (induced drag force) of the wing, allowing for the suction at the leading edges, is determined from the relation

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$$X_t = \frac{1}{2} Y \alpha. \quad (3)$$

Instead of the forces and moment (1) - (3), we will consider the following coefficients:

$$c_y = \frac{Y}{q_\infty S}; \\ c_{x_t} = \frac{X_t}{q_\infty S}; \quad m_{z_1} = \frac{M_{z_1}}{q_\infty S b},$$

where S is the wing area in the plane;

$$S = 2 \int_0^b l(x_1) dx_1;$$

q_∞ is the velocity head;

$$q_\infty = \frac{\rho V_\infty^2}{2}.$$

Then the aerodynamic characteristics of a wing of low aspect ratio may be represented as follows:

$$c_y = 2 \frac{m(b)}{\rho S} \alpha; \quad c_{x_t} = \frac{1}{2} c_y \alpha = \frac{m(b)}{\rho S} \alpha^2; \\ m_{z_1} = -2 \frac{m(b)}{\rho S} \alpha \left[1 - \frac{1}{bm(b)} \int_0^b m(x_1) dx_1 \right]. \quad (4)$$

If the lift and rolling moment coefficients are known, the position of the wing pressure center relative to the wing apex in fractions of the root chord can be calculated from the formula

$$x_p = -\frac{m_{z_1}}{c_y} = 1 - \frac{1}{bm(b)} \int_0^1 m(x_1) dx_1. \quad (5)$$

Formulas (4) and (5) were obtained from general relationships of the thin body theory and are applicable in cases where the assumptions of this theory are valid. We will note two distinctive features resulting from formulas (4) and (5):

1. According to Eq. (4), the lift and moment coefficients are linear functions of the angle of attack. The results of experimental studies confirm the linear nature of these relationships for wings of low aspect ratio up to $\alpha \leq 6^\circ$.

2. The lift coefficient is independent of the shape of the wing in the plane $z_1 = \ell(x_1)$ and is determined solely by the wing cross section on the trailing edge. Experimental data and calculations using exact theories also confirm this characteristic for wings with aspect ratio $\lambda \leq 1.5$.

Thus, despite the limitation to the case of $\lambda \rightarrow 0$, the thin body theory leads to qualitatively accurate results for wings of finite aspect ratio. It may be postulated that under certain conditions, this theory will also yield satisfactory quantitative results. /5

It follows from Eqs. (3) and (4) that the problem of determination of the aerodynamic characteristics of a low-aspect-ratio wing in a bounded flow is reduced to finding the apparent mass of the plate near the interface. To calculate the apparent mass, we will use the method of singularities and distribute a vortex layer of continuous strength $\gamma(z)$ over the length of the plate within the limits $-\ell \leq z \leq \ell$. Obviously, in this case $\gamma(z) = -\gamma(-z)$. The influence of the interface will be taken into account by using the method of specular reflection (Fig. 1).

The distribution $\gamma(z)$ must satisfy the following condition on the plate: a fluid particle adhering to the plate at any point S must have a constant vertical velocity V_0 (Fig. 1). Using this boundary condition, we obtain an integral equation for determining the unknown function $\gamma(z)$:

$$\int_{-\ell}^{\ell} \frac{\gamma(z) dz}{z - z_s} \pm \int_{-\ell}^{\ell} \frac{\gamma(z)(z - z_s) dz}{(z - z_s)^2 + 4h^2} = 2\pi V_0. \quad (6)$$

Here the plus sign pertains to the motion of the wing under a free surface (hydrofoil), and the minus sign pertains to the motion of the wing near a solid surface (ground). The values of ℓ and h are determined in the transverse flow plane ZOY as a function of the x_1 coordinate, which enters into Eq. (6) as a parameter. To calculate the lift and induced drag coefficients of the wing, the values of ℓ and h must be determined in the maximum span section.

We introduce a new variable θ from the relation $z = \ell \cos\theta$ (when $-\ell \leq z \leq \ell$, we have $\pi < \theta < 0$) and make the substitution $\gamma(z) = \gamma(\ell \cos\theta) = \gamma(\theta)$, having set $\bar{h} = \frac{h}{\ell}$. Eq. (6) is then written

$$\int_0^{\pi} \frac{\gamma(\theta) \sin \theta d\theta}{\cos \theta - \cos \theta_0} \pm \int_0^{\pi} \frac{\gamma(\theta) (\cos \theta - \cos \theta_0) \sin \theta}{(\cos \theta - \cos \theta_0)^2 + 4h^2} d\theta = 2\pi V_0. \quad (7)$$

We note the particular solution of Eqs. (6) and (7) which corresponds to the motion of the wing in a boundless fluid ($h = \infty$). We obtain from Eqs. (6) and (7), when $h = \infty$,

$$\int_{-i}^i \frac{\gamma(z) dz}{z - z_0} = \int_0^{\pi} \frac{\gamma(\theta) \sin \theta}{\cos \theta - \cos \theta_0} d\theta = 2\pi V_0.$$

whence*

$$\gamma = 2V_0 \frac{z}{\sqrt{z^2 - z_0^2}} = 2V_0 \operatorname{ctg} \theta. \quad (8)$$

To solve Eq. (7) in general form for $h \neq \infty$, we represent the unknown function $\gamma(\theta)$ as a series

$$\gamma(\theta) = 2V_0 [A_0 \operatorname{ctg} \theta + \sum_{n=1}^{\infty} A_{2n} \sin 2n\theta]. \quad (9)$$

in which the first term when $A_0 = 1$ is determined by solution (8), and the second term and $A_0 \neq 1$ characterize the additional load on the plate due to the influence of the flow boundary. It is obvious that when $h \rightarrow \infty$, the coefficient $A_0 \rightarrow 1$, and $A_{2n} \rightarrow 0$ ($n = 1, 2, \dots$). /6

Series (9) was written by using the condition

$$\gamma(\theta) = -\gamma(\pi - \theta).$$

If solution (9) is known, the apparent mass is calculated as follows. The values of the velocity potential function on the surface of the plate at $V_0 = 1$ are determined to within a constant which later is not significant:

$$\varphi_{u, \ell} = \pm \frac{1}{2} \int \gamma(z) dz = \mp \frac{1}{2} \int \gamma(\theta) \sin \theta d\theta,$$

where the upper** symbol pertains to the upper surface (ϕ_u) and the lower** symbol, to the lower surface (ϕ_ℓ). The general formula for determining the apparent mass is¹

$$m = -\rho \int \varphi \frac{\partial \varphi}{\partial n} dS.$$

Since in this case, on the lower surface of the plate $\frac{\partial \phi}{\partial n} = +1$, and on the upper $\frac{\partial \phi}{\partial n} = -1$, and in addition $dS = dz$, then

* Translator's Note: In the display equations, read ctg as cotan .

** sic (translator)

$$\begin{aligned}
 m &= -4\rho \int_0^l \varphi_\ell(z) dz = -4\rho l \int_0^{\frac{\pi}{2}} \varphi_\ell(\theta) \sin \theta d\theta = \\
 &= 2\rho l^2 \int_0^{\frac{\pi}{2}} \sin \theta d\theta \int \gamma(\theta) \sin \theta d\theta.
 \end{aligned}$$

Substituting the expression for $\gamma(\theta)$ from (9) into this formula, we obtain

$$m = \pi\rho l^2 \left(A_0 + \frac{A_2}{2} \right). \quad (10)$$

Thus, to determine the apparent mass of the plate, it is necessary to know the first two coefficients A_0 and A_2 of expansion (9). The remaining coefficients A_{2n} ($n = 2, 3, 4, \dots$) characterize the distribution of the load over the plate span and do not directly affect the overall aerodynamic characteristics of the wing.

When $h = \infty$, it follows from solution (8) that $A_0 = 1$ and $A_{2n} = 0$, and therefore by (10)

$$m_\infty = \pi\rho l^2. \quad (11)$$

The aerodynamic characteristics of the wing, allowing for Eq. (11), are determined from formulas (4) and (5)

$$\begin{aligned}
 C_{y_\infty} &= \frac{\pi\lambda}{2} \alpha; \quad C_{x_i} = \frac{1}{2} C_{y_\infty} \alpha = \frac{1}{2\lambda} C_{y_\infty}^2; \\
 x_p &= 1 - \frac{1}{b^2(b)} \int_0^b l^2(x_1) dx_1.
 \end{aligned} \quad (12)$$

Here $\lambda = \frac{4l^2(b)}{S}$ is the wing aspect ratio,

$z_1 = l(x_1)$ is the equation of the wing planform.

Formulas (12) are known relationships for a wing of low aspect ratio in an unbounded fluid.²

Let us now calculate the expansion coefficients A_{2n} ($n = 0, 1, 2, \dots$). In view of (9), Eq. (7) is transformed as follows:

$$A_0 - \sum_{n=1}^{\infty} A_{2n} \cos 2n\theta_s \pm I(\bar{h}, \theta_s) = 1, \quad (13)$$

where the following symbols have been introduced

$$\begin{aligned}
 I(\bar{h}, \theta_s) &= A_0 \left(\frac{1}{2} J_0 - \cos \theta_s J_1 + \frac{1}{2} J_2 \right) - \\
 &- \frac{1}{2} \cos \theta_s \sum_{n=1}^{\infty} A_{2n} (J_{2n-1} + J_{2n+1}) + \frac{1}{4} \sum_{n=1}^{\infty} A_{2n} (J_{2n-2} - J_{2n+2}); \\
 J_m &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos m\theta d\theta}{(\cos \theta - \cos \theta_s)^2 + 4\bar{h}^2}.
 \end{aligned}$$

After calculation of integrals J_m and further transformations of formula (13), we finally obtain equations for the expansion coefficients A_{2n} ($n = 0, 1, 2, \dots$)

1. For the motion of a wing near a solid surface

$$A_0 \zeta_0(\bar{h}, \vartheta_s) + 1 = \sum_{n=1}^{\infty} A_{2n} [\zeta_{2n}(\bar{h}, \vartheta_s) - \cos 2n \vartheta_s]. \quad (14)$$

2. For the motion of a wing above a free surface (hydrofoil)

$$A_0 [2 + \zeta_0(\bar{h}, \vartheta_s)] - 1 = \sum_{n=1}^{\infty} A_{2n} [\zeta_{2n}(\bar{h}, \vartheta_s) - \cos 2n \vartheta_s]. \quad (15)$$

The following symbols have been introduced into Eqs. (14) and (15):

$$\zeta_0(\bar{h}, \vartheta_s) = -\frac{u}{V} |\cos \vartheta_s| - 2\bar{h} \frac{u}{V};$$

$$\zeta_{2n}(\bar{h}, \vartheta_s) = \left(\frac{u + \cos \vartheta_s}{t} \right)^{2n} T_{2n}(t);$$

$$u = \frac{1}{2} \sqrt{V - (\sin^2 \vartheta_s + 4\bar{h}^2)};$$

$$\bar{u} = \frac{1}{2} \sqrt{V + \sin^2 \vartheta_s + 4\bar{h}^2};$$

$$V = \sqrt{(\sin^2 \vartheta_s + 4\bar{h}^2)^2 + 16\bar{h}^2 \cos^2 \vartheta_s};$$

$$t = \frac{u}{\sqrt{u^2 + 4\bar{h}^2}}; \quad T_{2n}(t) = \cos(2n \arccos t),$$

where $T_{2n}(t)$ is a Chebyshev polynomial of the first kind.

If the series of ϑ_s values are fixed in relation (14) or (15), systems of algebraic equations are obtained for determining the expansion coefficients A_{2n} ($n = 0, 1, \dots$).

The number of fixed points ϑ_{si} ($i = 1, 2, \dots$) determines the number of equations in systems (14) and (15), and hence, the number of desired coefficients A_{2n} ($n = 0, 1, 2, \dots, i - 1$). The remaining coefficients ($n = i; i + 1, \dots$) must be taken to be equal to zero.

Equations (14) and (15) were used to calculate the coefficients when a single point (approximation I), three points (approximation II) and five points (approximation III) are fixed.

Part of the calculations for the motion of a wing near a solid surface are shown in Fig. 2. From known coefficients A_0 and A_2 and formula (10), one can find the apparent mass of the plate, and from formulas (4) and (5), the aerodynamic characteristics of the wing. /8

Figure 3 shows values of the derivative of the lift coefficient with respect to the angle of attack C_y^α , referred to the value of $C_{y_\infty}^\alpha$ in an unbounded flow (12). Points denote average values \bar{C}_y^α , obtained from the theory of a vortical lifting surface for rectangular wings with $\lambda = 0.4-1.6$.⁴

Analysis of the calculations performed shows that satisfactory results up to $\bar{h} \geq 0.1$ can already be obtained in approximation III for a wing near a solid surface and in II for a hydrofoil.

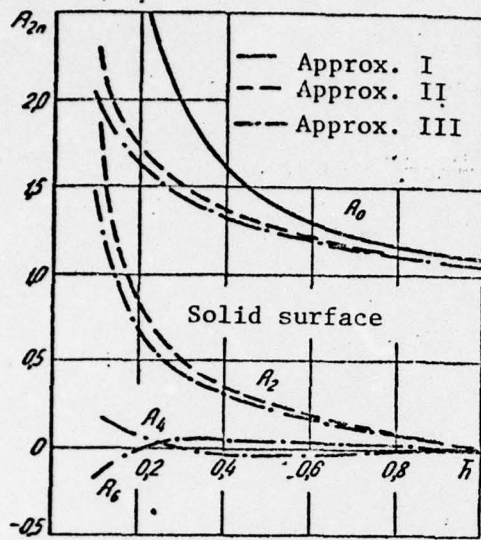


FIGURE 2

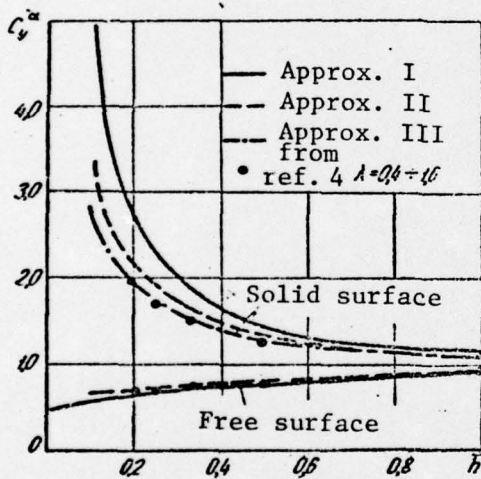


FIGURE 3

If the treatment is limited to the values $\bar{h} \geq 0.4$ (wing near the ground) and $\bar{h} \geq 0.25$ (hydrofoil), the aerodynamic characteristics of a wing of low-aspect-ratio can be determined from approximation I. The simple analytical relations which follow from (14) and (15) hold in this case when $\theta_s = \frac{\pi}{2}$ and $A_{2n} = 0$ ($n = 1, 2, 3, \dots$).

1. For a wing near a solid surface

$$A_0 = \frac{V\sqrt{1+4\bar{h}^2}}{2\bar{h}}, \quad m = \pi\rho l^2 \frac{V\sqrt{1+4\bar{h}^2}}{2\bar{h}}$$

Since

$$\bar{C}_v^a = \frac{m}{m_\infty},$$

then

$$\bar{C}_v^a = \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}.$$

2. For a hydrofoil

$$A_0 = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}; \quad m = \pi\rho l^2 \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}; \quad (17)$$

$$\bar{C}_\mu^c = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}.$$

Calculations made by using formulas (16) and (17) are indicated by solid lines in Fig. 2. /9

Let us note that if 2 is replaced by a higher number (for example, 3) in the expression $\frac{\sqrt{1+(2\bar{h})^2}}{2\bar{h}}$, formulas (16) and (17) give satisfactory agreement with the calculations made in approximation III up to $\bar{h} \geq 0.10$.

Thus, the following approximate formulas can be recommended for calculating the apparent mass of a plate in a bounded flow for $\bar{h} \geq 0.10$:

1. Near a solid surface

$$m = \pi\rho \frac{\sqrt{1+9\bar{h}^2}}{3\bar{h}} l^2. \quad (18)$$

2. Under a free surface

$$m = \frac{\pi\rho l^2}{2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}}}. \quad (19)$$

Formulas (18) and (19) are convenient for calculating the moment characteristics and position of wing pressure centers. For example, for a wing near a solid surface, we find from (5) and (18)

$$x_p = 1 - \frac{\bar{h}}{bl^2(b)\sqrt{1+9\bar{h}^2}} \int_0^l l^2(x_1) \sqrt{\frac{\bar{P}(x_1) + 9\bar{h}^2(x_1)}{h(x_1)}} dx_1.$$

Outside the integral $\bar{h} = \frac{h}{l(b)}$, h is the position of the wing trailing edge relative to the interfaces, and $l(b)$ is the wing semispan.

In the integrand, h and l depend on the x_1 coordinate. At small angles of attack

$$h(x_1) = h + (b - x_1)\alpha = l(b) \left[\bar{h} + \frac{b}{l(b)} (1 - x_1)\alpha \right].$$

Under the assumptions of the thin body theory, $\frac{b}{l(b)} \sim \frac{1}{\lambda}$, and under certain conditions, the second term on the right-hand side may be of the order of the first term; therefore, the position of the pressure center on a planar wing near the interface depends on the angles of attack.

We will confine the discussion to the case of $\alpha \rightarrow 0$; then $h(x_1) = l(b)\bar{h} = \text{const}$, and the position of the pressure center is determined from the formula

$$x_p = 1 - \frac{1}{\sqrt{1+9\bar{h}^2}} \int_0^1 \bar{l}^2(x_1) \sqrt{\bar{l}^2(x_1) + 9\bar{h}^2} dx_1, \quad (20)$$

where $\bar{l}(x_1) = \frac{l(x_1)}{l(b)}$ is the relative local wing semispan; $\bar{x}_1 = \frac{x_1}{b}$.

We will consider a family of wings whose planforms are given by the equation $\bar{l}(x_1) = \bar{x}_1^m$. /10

The exponent m changes within the limits $0 < m < \infty$. For $m = 1$, we obtain a triangular wing, and for $m \rightarrow 0$, a rectangular one.

By (20), we have

$$x_p = 1 - \frac{1}{\sqrt{1+9\bar{h}^2}} \int_0^1 \bar{x}_1^{2m} \sqrt{\bar{x}_1^{2m} + 9\bar{h}^2} d\bar{x}_1. \quad (21)$$

If such transformations are made for a hydrofoil, it follows from (5) and (19) when $\alpha \rightarrow 0$ that

$$x_p = 1 - \left(2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}} \right) \int_0^1 \frac{\bar{l}^2(x_1) \sqrt{\bar{l}^2(x_1) + 9\bar{h}^2}}{2\sqrt{\bar{l}^2(x_1) + 9\bar{h}^2 - 3\bar{h}}} dx_1. \quad (22)$$

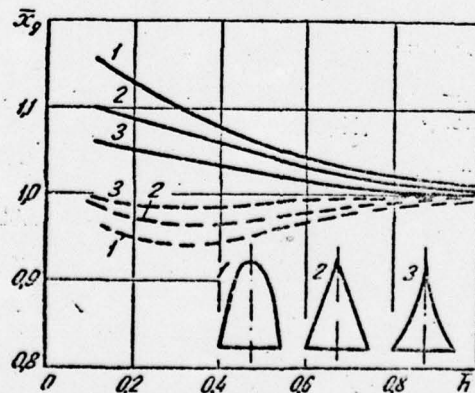


FIGURE 4

For the family of wings $\bar{\ell}(\bar{x}_1) = \bar{x}_1^m$

$$x_p = 1 - \left(2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}} \right) \int_0^1 \frac{\bar{x}_1^{2m} \sqrt{\bar{x}_1^{2m} + 9\bar{h}^2}}{2\sqrt{\bar{x}_1^{2m} + 9\bar{h}^2} - 3\bar{h}} d\bar{x}_1. \quad (23)$$

for $\bar{h} = \infty$ (flow of unbounded fluid), by (21) and (23)

$$x_{p\infty} = \frac{2m}{2m+1}. \quad (24)$$

This result can also be determined from formula (12).

In the general case, the integrals in formulas (21) and (23) are expressed by hypergeometric functions.³ Figure 4 shows the calculations of $x_p = \frac{x_p}{x_{p\infty}}$ for wings with $m = \frac{1}{2}, 1, 2$ (solid lines - for a wing near a solid surface; dashed lines - for a hydrofoil).

Analysis of the results obtained shows that as the wing approaches a solid surface (ground), the pressure center shifts toward the trailing edge, and this shift is greater the smaller the exponent m . In particular, the greatest shift should be observed for a rectangular wing ($m \rightarrow 0$).

The influence of the free surface boundary for a hydrofoil is the opposite of the ground effect. /11

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