

AD-A032 548

NAVAL INTELLIGENCE SUPPORT CENTER WASHINGTON D C TRA--ETC F/6 20/4  
WING HAVING MINIMUM INDUCED OPAG NEAR THE SURFACE OF THE EARTH --ETC(U)  
SEP 76 Y L ZHILIN  
NISC-TRANS-3844

UNCLASSIFIED

NL

1 OF 1

AD A032548



END

DATE  
FILMED

1 -77



DEPARTMENT OF THE NAVY  
 NAVAL INTELLIGENCE SUPPORT CENTER  
 TRANSLATION DIVISION  
 4301 SUITLAND ROAD  
 WASHINGTON, D.C. 20390

3  
 NW

AD A 032548

CLASSIFICATION: UNCLASSIFIED

APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED

TITLE:

6 Wing Having Minimum Induced Drag Near the Surface of the Earth  
 (Krylo minimal'nogo induktivnogo soprotivleniya vblizi poverkhnosti zemli)

AUTHOR(S):

10 Yu. L. Zhilin Yu. L.

PAGES:

6

SOURCE:

Izvestiya Akademii nauk SSSR, Mekhanika i mashinostroyeniye, No. 1, 1964  
 Pages 148-150

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION

DDC  
 RECEIVED  
 NOV 28 1978  
 C

ORIGINAL LANGUAGE: Russian

TRANSLATOR: DM

NISC TRANSLATION NO. 3844 ✓

14 NISC-Trans-3844

ADMISSION BY

WPS \_\_\_\_\_

DDC \_\_\_\_\_

UNANNOUNCED \_\_\_\_\_

JUSTIFICATION \_\_\_\_\_

BY \_\_\_\_\_

DISTRIBUTION/AVAILABILITY \_\_\_\_\_

Dist. \_\_\_\_\_

AKAW. DIST. \_\_\_\_\_

A

APPROVED P.T.K.

DATE 11 Sept 1976

12 7p.

407 682  
 689

## WING HAVING MINIMUM INDUCED DRAG NEAR THE SURFACE OF THE EARTH

[Zhilin, Yu. L. (Moscow); Krylo minimal'nogo induktivnogo soprotivleniya vblizi poverkhnosti zemli; Izvestiya Akademii nauk SSSR, Mekhanika i mashinostroyeniye, no. 1, 1964, pp. 148-150; Russian]

1. Let us examine the problem of a finite wing having minimum drag in the case of a given lift in the flow of an incompressible fluid near the flat surface of the earth. From the trailing edge of the wing runs a vortex sheet whose distance to the surface of the earth is considered constant. At a sufficiently great distance downstream from the wing the velocity induced by bound vortices may be ignored, and the flow be considered two-dimensional with great accuracy. Therefore, just as in the usual case of solving variation problems in the flow of an incompressible fluid, we shall introduce a plane (Trefitz plane) infinitely distant downstream from the wing. The potential of disturbed velocity  $\varphi$  in the Trefitz plane (Fig. 1) undergoes a break in the vortex sheet (AB) and heads towards zero as the distance from it increases. The lift  $Y$  and drag  $X$  acting on the wing, as is known, is dependent on potential in the following manner

$$Y = \rho V \int_{-l}^{+l} \Delta\varphi dx, \quad X = \frac{\rho}{2} \iint (\varphi_x^2 + \varphi_y^2) dx dy \quad (1.1)$$

Here  $V$  and  $\rho$  are the velocity and density of the incoming flow,  $l$  is

the wing span,  $\Delta\phi$  is the difference between the values  $\phi$  above and below the vortex sheet.

It is easy to show that the minimum drag value for a given lift is reached in the case where the potential  $\phi$  in the Trefftz plane satisfies the Laplace equation with a boundary condition of nonpassage at the surface of the earth

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{where } y = 0 \quad (1.2)$$

and with a condition of constant downwash in the vortex sheet

$$\frac{\partial\phi}{\partial y} = -\mu \quad \text{for AB} \quad (1.3)$$

where the constant  $\mu$  is expressed in terms of lift after solution of the problem. Let us introduce dimensionless variables (below the symbol \* for dimensionless values is dropped)

$$x^* = \frac{x}{1/2 l}, \quad y^* = \frac{y}{1/2 l}, \quad \phi^* = \frac{\phi}{1/2 \mu l} \quad (1.4)$$

The dimensionless potential  $\phi$  in the vortex sheet satisfies the condition

$$\frac{\partial\phi}{\partial y} = -1 \quad \text{where } y = h = \frac{2a}{l}, \quad -1 < x < 1 \quad (1.5)$$

where  $a$  is the distance from the wing to the surface of the earth.

Let us also introduce generally accepted coefficients of lift  $C_y$  and drag  $C_x$ . Using relationships (1.1), (1.3)-(1.4), it is easy to find that in the case of a wing of minimum drag

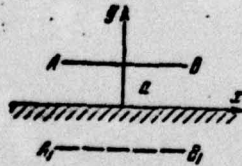


Fig. 1

$$k = \frac{C_x}{C_y^2} = \frac{1}{\lambda S}, \quad \Phi = \int_{-1}^1 \Delta\phi dx, \quad \lambda = \frac{l^2}{S} \quad (1.6)$$

where  $\lambda$  and  $S$  are wing aspect ratio and area, while integral  $\Phi$  depends

solely on  $h$ . If the wing is far from the surface of the earth, then, as

is known  $\phi = \pi$  and  $k = k_1 = (\pi\lambda)^{-1}$ . Let us introduce the value

$$K = \frac{k}{k_1} = \frac{\pi}{\phi} \quad (1.7)$$

which is equal to the ratio of the minimum wing drag far from the earth under similar  $C_y$  and  $\lambda$  values.

2. For determining the top view form of a wing of minimum drag we use the Zhukovskiy theorem and the hypothesis of plane cross sections. After designating the local angle of attack and length of chord of a particular wing cross section by  $\alpha(x)$  and  $b(x)$ , it is easy to obtain the following integral differential equation for circulation

$$\Gamma(x) = C_y \frac{Vb(x)}{2} \left[ \alpha(x) + \frac{v_1}{V} \right] \quad (2.1)$$

where  $\Gamma(x)$  is the velocity circulation in the cross section examined,  $C_y$  is the derivative of lift based on angle of attack of the profile,  $v_1$  is the downwash induced in the bound vortex. Velocity  $v_1$  is induced directly by the vortex sheet and its mirror reflection relative to the plane of the earth. This vortex system is formed by semiofinite straight line vortices extending along the flow beginning from the plane in which the bound vortex and its mirror reflection are located. Therefore velocity  $v_1$  is equal to half the downwash in the Trefftz plane, i.e.,  $2v_1 = -\mu = \text{const.}$  according to condition (1.3), while for a plane ( $\alpha = \text{const.}$ ) wing of minimum drag we obtain from equation (2.1) that the ratio  $\alpha(x)/b(x)$  is the same in all wing cross sections. Thus, it has been shown that a plane wing of minimum drag near the surface of the earth has a chord distribution along the wing span similar to the distribution of circulation or load. This theorem, which makes it possible to determine the form of a minimum drag wing if the circulation distribution

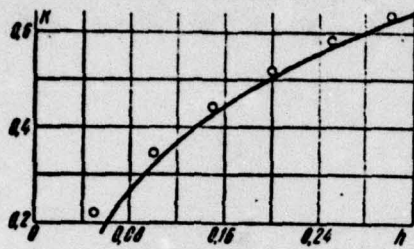


Fig. 2

is known, also remains valid in the case of a plane (uncambered) wing having minimum drag near a random cylindrical surface whose generating line is parallel to the velocity of the undisturbed flow.

In particular, if the wing is located far from the surface of the earth, then, as is known, it has minimum drag in the case of elliptical circulation distribution. In this case a plane wing of minimum drag has an elliptical form in top view.

3. A solution of the Laplace equation with boundary conditions (1.2) and (1.5) can be obtained, for example, by means of the parametric method /1/. Conformal depiction of the exteriors of segments  $AB$  and  $A_1B_1$  of plane  $z$  with sections along axis  $y$  from  $y = h$  to  $y = \infty$  respectively (fig. 1) to the rectangle in plane  $u$ , one side of which equals unity, is accomplished by means of the function /2/

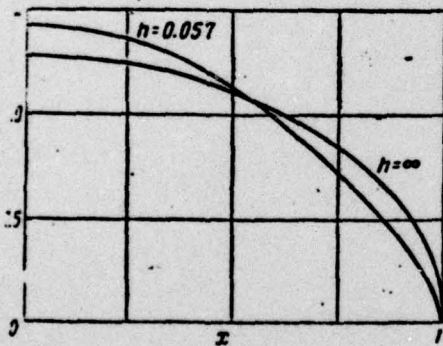


Fig. 3

$$z = -\frac{h}{\pi} \frac{\theta'(u)}{\theta(u)} - ih,$$

$$\theta(u) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^n \cos 2\pi n u$$

where  $\theta(y)$  is Jacobi's theta function. The essential parameter  $q$  entering the theta function is determined in terms of height  $h$ . Omitting the nonessential

details, we arrive at the final result of the solution of the boundary problem formulated in paragraph 1

$$\varphi(u) = \frac{h}{2\pi^2} \operatorname{Re} \left[ \frac{\phi''(u)}{\phi'(u)} + 2\pi i \frac{\phi'(u)}{\phi(u)} \right] \quad (3.2)$$

At sufficiently great distances from the surface of the earth the distribution of the difference of potentials along the wing span is described by the following approximate formula

$$\Delta\varphi = \frac{\sqrt{1+4h^2}}{h} \sqrt{1-x^2}$$

It follows from this that the circulation distribution will be elliptical. In this case  $\phi$  and  $K$  will be

$$\phi = \pi \sqrt{1 + \frac{1}{4h^2}}, \quad K = \left[ 1 + \frac{1}{4h^2} \right]^{-1/2}$$

The approximate relationships given above are valid for  $h$  values of the order of one and more. For lesser distances from the wing to the ground surface it was not possible to obtain obvious formulas. Consequently, appropriate calculations using (3.1)-(3.2) were made by computer.

Results of the calculation of  $K$  as a function of height  $h$  are given below:

$h =$	1.157	0.7035	0.4654	0.3165	0.2138
$K =$	0.927	0.8453	0.7481	0.6402	0.5246
$h =$	0.1385	0.108	0.08107	0.05708	
$K =$	0.403	0.3401	0.2695	0.181	

Fig. 2 gives a comparison of the value  $\bar{k}$  for a wing of minimum drag (solid curve) with the corresponding value for a wing with elliptical circulation distribution (dots) computed with the aid of reference /3/. As fig. 2 indicates, when the distance from the earth's surface are not great the drag of the optimum wing can be substantially less (up to 20%) than the wing with elliptical circulation distribution. Fig. 3 presents

the results of calculations of the distribution of dimensionless circulation  $\Gamma$  along the wing span at different  $h$  values. From these calculations and results of paragraph 2 it is clear that in the case of a wing of minimum drag a sharper reduction of the chord towards the end of the wing occurs than in the case of a wing with an elliptical form in top view.

Submitted on 2 August 1963