

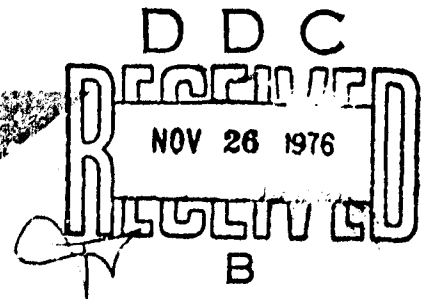
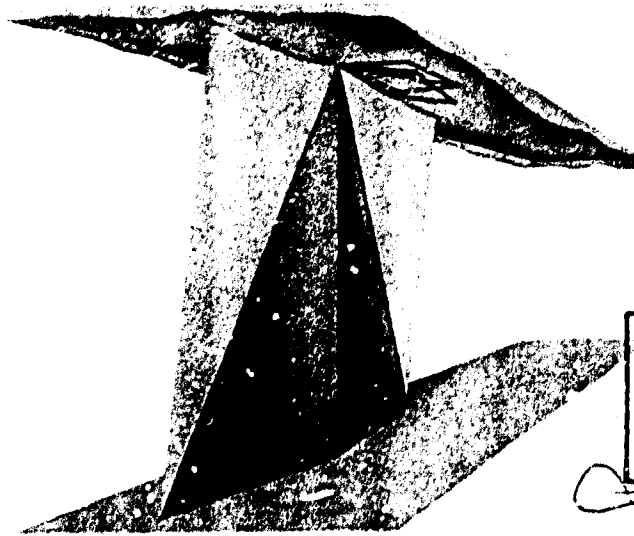
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"MODESRCH", AN IMPROVED COMPUTER PROGRAM FOR OBTAINING ELF/VLF/LF MODE CONSTANTS IN AN EARTH-IONOSPHERE WAVEGUIDE

Interim Report No. 77T

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by

D. G. MORFITT

and

C. H. SHELLMAN

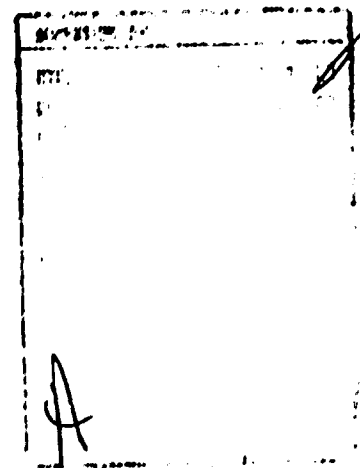


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ABSTRACT:

MODESRCH is a FORTRAN computer program designed to obtain rapidly, economically, and accurately, the mode constants needed to describe propagation in the earth-ionosphere waveguide at ELF, VLF and LF (10 Hz to 60 kHz). It is suggested that MODESRCH, along with required mode summing procedures, be used to replace "waveguide" and "wave-hop" codes currently used in network analysis programs which cover the ELF to LF frequency bands. This report contains a discussion of the analytical approach taken in MODESRCH, a FORTRAN listing of the program, instructions for using the program and some sample calculations.

INTRODUCTION

Radio field strengths at ELF, VLF and LF (10 Hz to about 60 kHz) are usually calculated using procedures based on either normal mode theory (reference 1) or on a semi-geometrical optics approach (reference 2).

Normal mode theory treats the field as being composed of one or more discrete families (modes) of plane waves confined to the natural waveguide bounded by the earth and the ionosphere. The number of waveguide modes needed to adequately represent the field is proportional to the earth-ionosphere separation measured in radio wavelengths. Thus, only a single mode is required below 3 kHz, about ten to fifteen modes at 30 kHz, and as many as twenty-five modes may be important at 60 kHz. The propagation parameters for each mode depend on the characteristics of the earth and the ionosphere, and are independent of the relative positions of the transmitter and receiver. For a homogeneous waveguide, once all of the important modes have been found, the radio fields can be determined as a function of distance by a simple mode summing calculation.

Computer programs based on waveguide mode theory have been brought to a high degree of sophistication and versatility in recent years, making it possible to calculate fields for any antenna configuration at any altitude, for both vertically and horizontally polarized waves, and for inhomogeneous waveguides. Unfortunately, solutions to the modal equation cannot be determined explicitly. Instead, the complex angles of incidence at the waveguide boundaries, which uniquely identify each mode, must be determined by iterative techniques. At frequencies above about 30 kHz, where there is a large number of modes to be found, the mode finding procedure of reference 1 may miss some solutions and it has taken human judgment to determine if the set of modes

found was complete. If the set of modes was judged incomplete, new initial conditions had to be resubmitted to the computer in an attempt to compute the missing modes, resulting in additional computer and manpower costs. Systems analysis codes do not allow human intervention, and so they have not been able to rely on mode theory when a sizeable number of modes is needed to represent the radio fields.

Previous practice has been to compute radio fields at frequencies below 30 kHz using waveguide mode theory and to use a wave-hop theory, such as described in reference 2, for computing fields at and above 30 kHz. The wave-hop procedure considers the total field at the receiver to be composed of a groundwave component and a number of skywaves following unique geometric ray paths connecting the transmitter and receiver with intermediate reflections off the earth and the ionosphere. Calculations of radio field strength, using the wave-hop procedure of reference 2, agree well with waveguide mode calculations for horizontally homogeneous normal ionospheres. They do not agree for severely depressed ionospheres over low conductivity ground.

The wave-hop approach has features which make it somewhat inconvenient for system analyses. For each separation distance between transmitter and receiver there is a unique ray geometry. Thus, for calculations of fields as a function of distance; e.g., for aircraft operations, certain inherent calculations must be repeated many times. Also, for inhomogeneous waveguides such as across day-night ionospheric gradients or across nuclear-induced ionospheric depressions, the geometry is ill-defined and the calculations tend toward approximations.

Aside from the limitations of either the waveguide approach or the wave-hop approach to long-wave propagation prediction, it would be advantageous to

use a single method across the entire ELF, VLF, LF frequency bands. Systems codes used for network analyses would be considerably reduced in size, saving computer costs. Comparisons of systems performance as a function of frequency would be more meaningful if a single propagation theory were invoked.

The MODESRCH computer program described in this report has the required ability to compute waveguide mode constants rapidly, economically and accurately over the frequency band of interest. MODESRCH contains all of the basic features of the previously reported waveguide mode computational procedures of reference 1 and several important additions. The new additions virtually guarantee that all waveguide modes of importance will be found at any frequency up to at least 60 kHz. MODESRCH, used with presently available mode summing techniques, will provide the radio fields required for systems analysis.

This report describes the MODESRCH theory. It also contains a listing of the FORTRAN program, instructions for the user, and examples of MODESRCH calculations.

II. DISCUSSION OF MODESRCH PROCEDURE

A. General Background

In the propagation of ELF, VLF and LF terrestrial radio waves to great distances, the waves are confined within the space between the earth and the ionosphere. This space acts as a waveguide, and the "waveguide concept" is applicable for characterizing the propagated fields as a function of distance.

The energy within the waveguide is considered to be partitioned among a series of modes. Each mode represents a resonant condition; i.e., for a discrete set of angles of incidence of the waves on the ionosphere, resonance occurs and energy will propagate away from the source. The complex angles (θ) for which this occurs are called the eigenangles. They may be obtained using the 'full-wave' procedures described in reference 1 by solving the determinantal equation

$$F_1(\theta) = | \bar{R}_d(\theta) R_d(\theta) - \underline{1} | = 0 \quad (1)$$

$$\text{where } R_d(\theta) = \begin{bmatrix} {}_{\parallel}R_{\parallel d}(\theta) & {}_{\perp}R_{\parallel d}(\theta) \\ {}_{\parallel}R_{\perp d}(\theta) & {}_{\perp}R_{\perp d}(\theta) \end{bmatrix} \quad (2)$$

is the complex ionospheric reflection coefficient matrix looking up into the ionosphere from height "d" and

$$\bar{R}_d(\theta) = \begin{bmatrix} {}_{\parallel}\bar{R}_{\parallel d}(\theta) & 0 \\ 0 & {}_{\perp}\bar{R}_{\perp d}(\theta) \end{bmatrix} \quad (3)$$

is the complex reflection matrix looking down from height "d" towards the ground.

Note: Any variable "A" denoted by $\underline{A}$ in this report implies a matrix.

The complex angle, θ , is the angle of incidence at height $z = H$ where the modified index of refraction given by

$$n^2 = 1 - \alpha(H - Z) = 1 - \frac{2}{r_e} (H - Z) \quad (4)$$

is unity. Here r_e is the radius of the earth.

The notation $\parallel$ for the R 's and $\bar{R}$'s denotes vertical polarization while the notation, $\perp$, denotes horizontal polarization. The first subscript on the R 's refers to the polarization of the incident wave while the second applies to the polarization of the reflected wave.

To find solutions of (1) for the eigenangles, θ , the well known Newton-Raphson method may be used. The procedure is to guess a solution θ_0 to the equation $F_1(\theta) = 0$. The function $F_1(\theta)$ is then reevaluated for $\theta_0 + \delta\theta$ and the correction to θ_0 found from the equation

$$\Delta\theta = - \frac{F_1(\theta_0)\delta\theta}{F_1(\theta_0 + \delta\theta) - F_1(\theta_0)} \quad (5)$$

The correction determined by (5) is then evaluated and the process repeated until the quantities $|\Delta\theta_r|$ and $|\Delta\theta_i|$ are reduced to within a preassigned tolerance. The subscripts r and i denote the real and imaginary parts, respectively. The final solution for mode " n " is then given by

$$[\theta_n]_j = [\theta_n]_{j-1} + \Delta\theta_n \quad j = 1, 2, 3, \dots \quad (6)$$

B. Introduction to MODESRCH Theory

The "MODESRCH" procedure also obtains the full-wave solution of equation 1 in terms of the eigenangles. In this case; however, the $\bar{R}$ and $\bar{R}$ terms are modified in order to make simpler equations.

The basic premise of the MODESRCH theory is to modify the function $F_1(\theta)$ so that it contains no poles and then in this form, apply a newly-developed root-finding algorithm to obtain the complete set of eigenangles.

In particular, the $F_1(\theta)$ equation of reference 1 becomes upon modification (see Appendix A):

$$F_3(\theta) = (\|n_{\parallel} - \|\bar{X}_{\parallel}\|d_{\parallel}) (\perp n_{\perp} - \perp \bar{X}_{\perp} \perp d_{\perp}) - \|\bar{X}_{\perp} \perp \bar{X}_{\parallel}\| d_{\perp} \perp d_{\parallel} \quad (7)$$

$$\text{where } \bar{X} = (\bar{R} + 1)/c \quad (8)$$

and $c = \cos\theta$ (θ = complex angle of incidence)

$$\text{or } \|\bar{X}_{\parallel}\| = (\|R_{\parallel}\| + 1)/c$$

$$\|\bar{X}_{\perp}\| = \|R_{\perp}\|/c \quad (9)$$

$$\perp \bar{X}_{\parallel} = \perp R_{\parallel}/c$$

$$\perp \bar{X}_{\perp} = (\perp R_{\perp} + 1)/c$$

$$\text{and } \bar{R}^{-1} = (\bar{R}^{-1} + 1)/c$$

$$\text{or } \frac{\|n_{\parallel}\|}{\perp d_{\parallel}} = \left(\frac{\|R_{\parallel}\| + 1}{\perp R_{\parallel}} \right) / c \quad (10)$$

$$\frac{\perp n_{\perp}}{\perp d_{\perp}} = \left(\frac{\perp R_{\perp} + 1}{\perp R_{\perp}} \right) / c$$

The modified variables:

$$\left(\frac{\bar{R}+1}{c} \right) \quad \left(\frac{\bar{R}^{-1}+1}{c} \right)$$

are related to making simpler equations in the 'MODESRCH' procedure and in removing the zeros in $F_3(\theta)$ at $\theta = 90^\circ$. The functions $F_1(\theta)$ and $F_3(\theta)$ are not the same, but except for the point where $\theta=90^\circ$, they have the same zeros.

The solutions to the modal equation; (i.e., equation 7) for $F_3(\theta)=0$ are the complex eigenangles (θ) needed to obtain the mode constants (e.g., phase velocity, attenuation rate, and excitation factor) which

Note: From this point on, let $F_3(\theta)$ be denoted by $F(\theta)$ for convenience.

are the parameters of interest in computing field strengths. The total field at any point along the guide may be computed from the vector sum of the contributions from each individual mode.

As in reference 1, the MODESRCH procedure requires that the upper boundary of the waveguide be assigned arbitrary electron and ion density distributions with height as well as variable collision frequency profiles with height. Allowance is also made for the anisotropy of the ionosphere. The lower boundary of the guide is taken to be a smooth homogeneous earth characterized by an adjustable conductivity and permittivity. Earth curvature is also included via the artifice of the modified refractive index (e.g., Equation (4)).

The (X,Y,Z) coordinate system used is such that Z is directed into the ionosphere, the X-Z plane is the plane of propagation, and invariance in the Y-axis is assumed.

C. General Steps in the Solution of the Modal Equation

(1) For VLF/LF cases the input electron and ion density profiles vs height are modified by eliminating those heights above which and below which the corresponding particle densities will have negligible effects on the modal solutions. For ELF cases, profiles are not cutoff at the top or the bottom heights but instead the complete input profile is used in subsequent calculations.

(2) The modal solutions of Equation (7), the complex eigenangles ($\theta = \theta_{\text{real}} + i\theta_{\text{imaginary}}$), are determined within a bounded region of the complex "THETA" plane defined by a set of limiting THETA values. This bounded region will be called the "contour rectangle" and the limiting THETA values are denoted by the corners of the rectangle (e.g., A,B,C, and D). These parameters are portrayed in Figure 1.

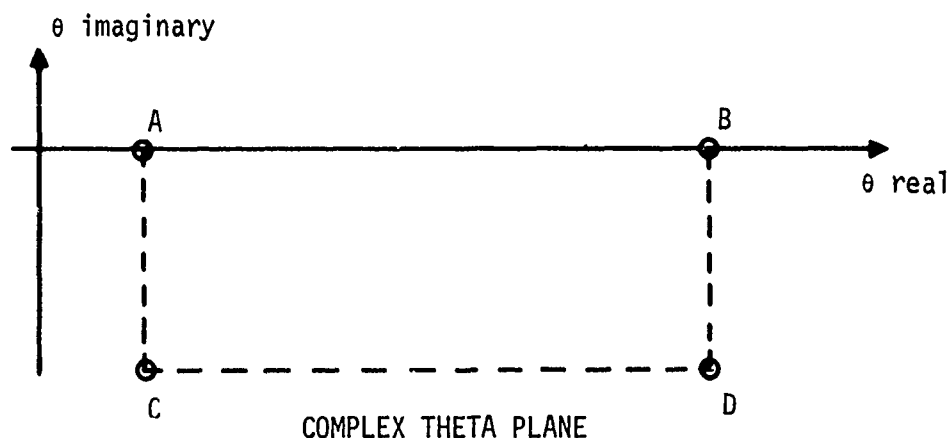


Figure 1. The Contour Rectangle ABCD

(3) Four fixed angles (θT_i , $i=1,2,3,4$) are chosen in the complex THETA plane. These angles are somewhat outside the corners A,B,C, and D of the contour rectangle. For VLF and LF calculations these angles are the given values of θ at which full-wave solutions are to be carried out for use in a Lagrange interpolation procedure. At ELF (e.g., < 1000 Hz) the purpose of these angles is to identify the location of the contour rectangle in the THETA plane.

(4) At VLF and at LF, the differential equations for the ionospheric terms $\bar{X}(\theta)$ are integrated at the angles θT_i , to give full-wave values of $\bar{X}(\theta T_i)$ at the base of the modified electron-ion density profiles. The results obtained by integrating to the bottom of the profile (e.g., $Z=Z_b$) will be designated as $\bar{X}_b(\theta T_i)$.

(5) For VLF/LF cases a further upward integration of the $\bar{X}(\theta T_i)$ equations from the height Z_b to a reference height 'd' is then performed assuming a free space environment over a curved earth. This reference height may be taken to be an effective reflection height. At this height there will be

no poles in the function $\bar{X}(\theta)$ for the region of interest in the complex THETA plane. The determination of this height is important because the general root-finding algorithm used to locate the zeros of $F(\theta)$ requires that there be no poles in the vicinity of the zeros to be found. This reference height is also appropriate for performing Lagrange interpolation on the function $\bar{X}(\theta)$.

(6) At the reference height 'd', for VLF and LF, the contour rectangle is subdivided into small mesh squares. The value of $F(\theta)$ at each corner of each mesh square is determined by Lagrange interpolation of $\bar{X}(\theta)$ based on the given values of $\bar{X}(\theta_{T_i})$.

Recall from Equation (7) that the function $F(\theta)$ is computed from $\bar{X}(\theta)$, $n(\theta)$ and $d(\theta)$ so that for any angle, θ_j the value of $\bar{X}(\theta_j)$ is found from the Lagrangian interpolation of $\bar{X}(\theta_{T_i})$, $i=1,2,3,4$ whereas $n(\theta_j)$ and $d(\theta_j)$ are calculated directly.

(7) For ELF (e.g., <1000 Hz), the angles used in the full-wave integration of the $\bar{X}(\theta)$ equation consists of the usual corners of the contour rectangle plus all additional angles formed from the four corners of each mesh square into which the contour rectangle is also divided. In this case the integration is extended downward to the earth's surface (e.g., $Z=0$) and no further integration back up to the height 'd' is done. No Lagrangian interpolation is involved since the full-wave solution is computed for all angles of interest to the problem. It is important to realize at this point that the MODESRCH procedure has not been formulated so as to eliminate the poles for ELF cases. This may result in no solution being found to the modal equation due to the presence of poles. Because of this result, it may be advisable to compute modes for the ELF cases by the methods of reference 1. This problem has not; however, occurred in practice.

(8) The zeros of $F(\theta)$ at 'd' for VLF/LF (or at '0' for ELF) are then found by applying the root-finder algorithm to the mesh squares.

(9) Since $F(\theta)$, as determined from step (8) is approximate, the exact solution of $F(\theta)$ is then obtained by applying the usual Newton-Raphson procedures where the full-wave solution is used to obtain $\bar{X}(\theta)$

(10) With the determination of the exact solutions of the θ 's (eigenangles), the required mode constants and excitation factors, may be calculated.

D. Optimization of Input Ionospheric Density Profiles

Computing efficiency for VLF and LF may be improved by cutting off the top and bottom heights of the input profile.

(1) Exponential Profiles (electrons only)

An exponential, electrons only, ionospheric profile may be specified as input to the MODESRCH procedure. The profile is computed as described in Wait's NBS TN 300, reference 3, where

$$\omega_p(z) = 2.5 \times 10^5 \exp[\beta(z-h')] = 3.18254 \times 10^9 N(z)/v(z) \quad (11)$$

and β is in inverse kilometres, z and h' are in kilometres, $N(z)$ is the electron density in electrons per cubic centimetre and $v(z)$ is the collision frequency in collisions per second. The exponential collision frequency is given in reference 3 as

$$v(z) = 1.86 \cdot 10^{11} \cdot \exp(-0.15z) \quad (12)$$

The top of the profile may be modified by choosing a value for the variable SCLHTS. The relationship is that:

$$\text{Top Height} = \text{Integer} \left(h' + \frac{\text{SCLHTS}}{\beta} + 0.5 \right) \quad (13)$$

The nearest integral value of altitude as given by 'Top Height' is used to set the top of the profile. All heights above this value are neglected in the MODESRCH computations.

The bottom of the exponential profile is set by assigning a minimum value (i.e., ENMIN) of electron density in electrons/cm³. All altitudes, at which the corresponding density is less than the value of ENMIN, will be neglected.

(2) VLF/LF - Tabular Electron-Ion Density Profiles

When the particle density profiles are in tabular form, the top and bottom of the profile may be cut off using the following criteria, as presented in reference 4. Here the particle profiles may consist of ions as well as of electrons.

The (isotropic) squared index of refraction is written as:

$$n^2(z) = 1 - A - jB \quad (14)$$

where

$$A = A_e + A_+ + A_-$$

$$B = B_e + B_+ + B_-$$

or

$$A = \underbrace{\frac{N_e e^2}{M_e \epsilon_0 [\omega^2 + \nu_e^2]}}_{A_e} + \underbrace{\frac{N_+ e^2}{M_+ \epsilon_0 [\omega^2 + \nu_+^2]}}_{A_+} + \underbrace{\frac{N_- e^2}{M_- \epsilon_0 (\omega^2 + \nu_-^2)}}_{A_-} \quad (15)$$

and

$$B = \frac{\nu_e}{\omega} A_e + \frac{\nu_+}{\omega} A_+ + \frac{\nu_-}{\omega} A_- \quad (16)$$

with the identification that

ν_e = electron-neutral particle collision frequency (s⁻¹)

ν_+ = positive ion-neutral particle collision frequency (s⁻¹)

ν_- = negative ion-neutral particle collision frequency (s⁻¹)

N_e = electron density (cm⁻³)

N_+ = positive ion density (cm⁻³)

N_- = negative ion density (cm⁻³)

ω = frequency (radians s^{-1})

m_e = mass of the electron (kg)

m_+ = mass of the positive ions (kg)

m_- = mass of the negative ions (kg)

ϵ_0 = permittivity of free space

The top part of the ionospheric profile which may be neglected in the calculations is defined as that altitude(z) above which:

$$B(z) > \text{Maximum} \left(\text{TEMPO-B}, 20 \frac{\omega}{v_e} \right) \quad (17)$$

where 'TEMPO-B' is assigned a value (e.g., 2.0).

An additional requirement for cutting off the top of the profile is that the electron density must be as great as 'TOPEN'. The variable 'TOPEN' is assigned a value (e.g., 1000 electrons/cm²). The top of the profile is taken to be the higher of the two heights corresponding to 'B' (equation (17)) and to 'TOPEN'.

The bottom of the input ionospheric profile is neglected (for VLF/LF cases) at those altitudes for which the computed value of B in equation (16) is less than 'CUTOFF'. The variable 'CUTOFF' is assigned a value (e.g., 0.0001). The relationship for cutoff is:

$$B(z) < \text{CUTOFF} \quad (18)$$

(3) ELF Tabular Electron-Ion Density Profiles

In the case of ELF the profile top and bottom are not cut off and the complete profile is used in the calculations.

E. Integration of the Differential Equations for $\bar{X} = (\underline{R} + \underline{1})/C$

Through the Ionosphere

The differential equations for the ionosphere reflection matrix, $\underline{R}$, are given in reference 5.

The differential equations are of the form

$$dR/dz = - ik/2 \left\{ \underline{\underline{S}}^{(21)} + \underline{\underline{S}}^{(22)} \underline{\underline{R}} - \underline{\underline{RS}}^{(11)} - \underline{\underline{RS}}^{(12)} \underline{\underline{R}} \right\} \quad (19)$$

where z is the height variable and $\underline{\underline{S}}^{(kl)}$ are 2×2 partitions of a 4×4 matrix given by:

$$\underline{\underline{S}} = \begin{bmatrix} T_{11} + T_{44} + T_{14}/C + CT_{41} & -T_{12}/C - T_{42} & -T_{11} + T_{44} + T_{14}/C - CT_{41} & -T_{12}/C - T_{42} \\ -T_{31} - T_{34}/C & C + T_{32}/C & T_{31} - T_{34}/C & -C + T_{32}/C \\ \hline -T_{11} + T_{44} - T_{14}/C + CT_{41} & T_{12}/C - T_{42} & T_{11} + T_{44} - T_{14}/C - CT_{41} & T_{12}/C - T_{42} \\ T_{21} + T_{34}/C & C - T_{32}/C & -T_{31} + T_{34}/C & -C - T_{32}/C \end{bmatrix} \quad (20)$$

where

$$C = \cos \theta$$

(21)

θ = complex angle of incidence (deg).

The intermediate matrix T relates the electromagnetic field components to their derivatives. In turn, T is defined in terms of the susceptibility matrix M :

where T is defined as

$$\underline{\underline{T}} = \begin{bmatrix} \left(-\frac{SM_{31}}{1+M_{33}} \right) & \left(\frac{SM_{32}}{1+M_{33}} \right) & (0) & \left(\frac{C^2 + M_{33}}{1+M_{33}} \right) \\ (0) & (0) & (1) & (0) \\ \left(\frac{M_{23}M_{31}}{1+M_{33}} - M_{21} \right) & \left(C^2 + M_{22} - \frac{M_{23}M_{32}}{1+M_{33}} \right) & (0) & \left(\frac{SM_{23}}{1+M_{33}} \right) \\ \left(\frac{M_{13}M_{31}}{1+M_{33}} - M_{11} \right) & \left(\frac{M_{13}M_{32}}{1+M_{33}} - M_{12} \right) & (0) & \left(\frac{SM_{13}}{1+M_{33}} \right) \end{bmatrix} \quad (22)$$

where $S = \sin(\theta)$.

Finally, the susceptibility $\tilde{M}$ matrix is defined as

$$\tilde{M} = \frac{-P}{U(U^2 - Y^2)} \begin{bmatrix} (U^2 - \ell^2 Y^2 - 2(H-z)/r_e) & (-jU\ell Y - \ell m Y^2) & (jUmY - \ell n Y^2) \\ (jU\ell Y - \ell m Y^2) & (U^2 - m^2 Y^2 - 2(H-z)/r_e) & (-jU\ell Y - mn Y^2) \\ (-jUmY - \ell n Y^2) & (jU\ell Y - mn Y^2) & (U^2 - n^2 Y^2 - 2(H-z)/r_e) \end{bmatrix} \quad (23)$$

where

$$j = -1$$

$$P = (\omega_p/\omega)^2$$

$$U = 1 - j(\nu/\omega)$$

$$Y = \mu_0 \omega_m / \omega \text{ henrys}$$

$$\omega_m = \text{magnetic gyrofrequency (s}^{-1}\text{)}$$

$$\omega_p = \text{plasma frequency (s}^{-1}\text{)} \quad (24)$$

$$\ell = \cos \theta_D \cos \varphi_a$$

$$m = \cos \theta_D \sin \varphi_a$$

$$n = -\sin \theta_D$$

$$\nu = \text{collision frequency (s}^{-1}\text{)}$$

$$\omega = \text{wave frequency (s}^{-1}\text{)}$$

$$\mu_0 = \text{free space permeability (henrys/m)}$$

$$\theta_D = \text{magnetic dip angle (deg)}$$

$$r_e = \text{earth's radius (km)}$$

$$H = \text{altitude where the index of refraction is unity (km)}$$

$$\varphi_a = \text{direction of propagation east of magnetic north (deg)}$$

$$z = \text{altitude (km)}$$

The matrix $\tilde{M}$ described above is general in that it represents a single ionospheric constituent. Three species are considered by this model: electrons and positive and negative ions. Thus for each species there will

be a matrix $\underline{M}_i$, and the matrix elements needed for computing the $\underline{I}$ matrix elements are found from:

$$\underline{M} = \sum_{i=1}^3 \underline{M}_i \quad (25)$$

Earth curvature terms have been added to the diagonal elements of $\underline{M}$, by the terms $-2(z-z)/r_e$.

Equation (19) may be rearranged to form the following differential equations for $\underline{\bar{X}} = (\underline{R} + 1)/C$, (see Equation (8)), giving:

$$\frac{d\underline{\bar{X}}}{dz} = \frac{-ik}{2} \underline{A} + \underline{B} \underline{\bar{X}} + \underline{\bar{X}} \underline{C} + \underline{\bar{X}} \underline{D} \underline{\bar{X}} \quad (26)$$

where k is the wave number, z is height, and

$$\underline{A} = (\underline{S}^{21} - \underline{S}^{22} + \underline{S}^{11} - \underline{S}^{12})/C = \begin{pmatrix} 4T_{41} & 0 \\ 0 & 4 \end{pmatrix} \quad (27)$$

$$\underline{B} = \underline{S}^{22} + \underline{S}^{12} = 2 \begin{pmatrix} T_{44} = CT_{41} & -T_{42} \\ 0 & -C \end{pmatrix} \quad (28)$$

$$\underline{C} = -\underline{S}^{11} + \underline{S}^{12} = 2 \begin{pmatrix} T_{11} - CT_{41} & 0 \\ T_{31} & -C \end{pmatrix} \quad (29)$$

$$\underline{D} = -C \underline{S}^{12} = \left(\begin{array}{c|c} C(T_{11} - T_{14}) - T_{14} + C^2 T_{41} & T_{12} + CT_{42} \\ \hline -CT_{31} + T_{34} & C^2 - T_{32} \end{array} \right) \quad (30)$$

Equation (26) may be integrated numerically by the Runge-Kutta method. The integration is carried out in the direction of decreasing ionospheric height. Error testing is based on comparing the results of integrating through a height interval with one Runge-Kutta step with the result of integrating through the same height interval using two steps. If the difference is greater than a specified tolerance, the step size in that interval is cut in half. The initial set of integration steps is taken to be the input profile segments. However, the integration steps used in one integration are taken to be the initial set of steps for the next integration.

Note that since the elements of $\underline{\underline{X}}(\theta)$ are complex, the integration is that of eight real coupled differential equations for the real and imaginary parts of the elements of $\underline{\underline{X}}(\theta)$.

The classical procedure of assuming a semi-infinite homogeneous ionosphere at the top of the profile is followed. The solution for reflection from this semi-infinite medium is taken from reference 6 and is used as the starting solution for the integration.

This method of integration is identified as the "full-wave" procedure for obtaining ionospheric reflection coefficients. The values obtained from the integration are denoted as $\underline{\underline{X}}(\theta T_i)_b$, where $z=b$ is the lower cutoff height of the integration.

F. Integration of $\underline{\underline{X}} = (\underline{\underline{R}} + \underline{\underline{1}})/C$ Through Free Space Over a Curved Earth

As was stated in Section II, C-5, the general root-finding algorithm used to locate the zeros of $F(\theta)$ requires that there be no poles in the vicinity of the zeros to be found. This requires that there be no poles in $\underline{\underline{X}} = (\underline{\underline{R}} + \underline{\underline{1}})/C$ in this region of the θ -plane. This can be assured by integrating $\underline{\underline{X}}$ from the bottom of the profile upward through free-space over a curved earth to an effective "reflection height", defined by the condition

$$\sum_{\substack{4 \\ \theta\text{-points}}} \sum_{\substack{4 \\ \text{elements}}} \left| \frac{d\underline{\underline{X}}(\theta T_i)}{dC} \right|^2 \rightarrow \text{Minimum} \quad (31)$$

where the summation is over the four elements of $\underline{\underline{X}}$ and over a set of points (e.g., θT_i $i = 1, 2, 3, 4$) in the θ -plane distributed over the region in which zeros of $F(\theta)$ are to be located. This condition is also appropriate for defining a level at which Lagrange interpolation can best be performed.

In practice the set of points (θT_i) are used in defining the "effective reflection height" and these are taken to be the "given" points in the θ -plane used for Lagrange interpolation.

As in previous waveguide formulations, e.g., reference 1, the earth curvature effect is approximated by assuming a dielectric medium which varies linearly in the square of the refractive index over a flat earth.

Considering the usual right handed cartesian coordinate system, where $+Z$ is taken vertically upward, formulation for such an integration from the bottom of the profile at " Z_b " to another height " Z_t ", can be obtained following reference 7. The derivation of the equations is given in Appendix B. The final equations are listed and discussed as follows.

Just below the free space-ionosphere boundary (e.g., Z_b), let the total electric wave field be composed of two upward (e.g., $+Z$ direction) traveling incident components and two downward traveling reflected components. They are defined by $E_{11,y}^i$ and $E_{11,y}^r$ respectively. E_{11} is in the X - Z plane and E_y is parallel to the y -axis. The total-field components just below the boundary are given in reference 7, page 118 as

$$\left. \begin{aligned} E_x &= (E_{11}^i - E_{11}^r) \cos \theta_I \\ E_y &= E_y^i + E_y^r \\ H_x &= (E_y^r - E_y^i) \cos \theta_I \\ H_y &= E_{11}^i + E_{11}^r \end{aligned} \right\} \quad (32)$$

where θ_I is the angle in the x - y plane which the incident wave normal makes with the vertical z axis. The following identifications are made:

$$\left. \begin{aligned} {}_{\perp}R_{\perp} &= E_y^r/E_y^i \\ {}_{\parallel}R_{\perp} &= E_y^r/E_{11}^i \\ {}_{\parallel}R_{\parallel} &= E_{11}^r/E_{11}^i \\ {}_{\perp}R_{\parallel} &= E_{11}^r/E_y^i \end{aligned} \right\} \quad (33)$$

Equations (32) and (33) are combined with the differential equations to be satisfied by the wave fields at oblique incidence (reference 7, pp. 140), to give two independent sets of relationships. Either of these sets of field variables may be set equal to zero without affecting the other, so that the corresponding waves are propagated independently.

For the first set of equations the electric field is everywhere parallel to the y-axis and the waves are said to be "horizontally polarized". For the second set of equations the electric field is parallel to the X-Z plane and the waves are said to be vertically polarized, although the electric vector is not in general vertical.

The equations become (as derived in Appendix B):

For Horizontal Polarization:

$$(-A/C)h_1(\zeta) + (-B/C)h_2(\zeta) + E_y^i \left(\frac{{}_{\perp}R_{\perp} + 1}{C} \right) + E_{11}^i \left(\frac{{}_{\parallel}R_{\perp}}{C} \right) = 0 \quad (34)$$

$$(-A/C) \left\{ ch_1(\zeta) + Kh_1'(\zeta) \right\} + (-B/C) \left\{ ch_2(\zeta) + Kh_2'(\zeta) \right\} + 2E_y = 0 \quad (35)$$

For Vertical Polarization:

$$(-Q/C)h_1(\zeta) + (-G/C)h_2(\zeta) + E_{11}^i \left(\frac{{}_{\parallel}R_{\parallel} + 1}{C} \right) + E_y^i \left(\frac{{}_{\perp}R_{\parallel}}{C} \right) = 0 \quad (36)$$

$$\begin{aligned} &(-Q/C) \left\{ ch_1(\zeta) + [Kh_1'(\zeta) + Lh_1(\zeta)]/n^2 \right\} + \\ &(-G/C) \left\{ ch_2(\zeta) + [Kh_2'(\zeta) + Lh_2(\zeta)]/n^2 \right\} + 2E_{11}^i = 0 \end{aligned} \quad (37)$$

The coefficients A, B, Q, and G are arbitrary constants to be determined.

The functions $h_1(\zeta)$ and $h_2(\zeta)$ satisfy the differential equation

$$\frac{d^2 w}{d\zeta^2} + \zeta w = 0 \quad (38)$$

and are the modified Hankel functions of order 1/3 (e.g., $h_1(\zeta)$ and $h_2(\zeta)$) as defined by the Computation Laboratory at Cambridge, Massachusetts (reference 8). The primes on these quantities denote derivatives with respect to the argument. Also

$$\zeta = (k/\alpha)^{2/3} [C^2 + \alpha(Z-H)] \quad (39)$$

$$C = \cos(\theta)$$

$$\alpha = 2/r_e$$

$$k = 2\pi/\lambda \quad (40)$$

$$K = i(\alpha/k)^{1/3}$$

$$L = i(\alpha/2K)$$

$$i = \sqrt{-1}$$

where

H: is the altitude where the index of refraction is unity.

λ : is the free space wave length.

n: is the index of refraction of the medium.

r_e : is the radius of the earth.

Note that the $\underline{R}$ terms may be written as:

$$\underline{\bar{X}}_{\parallel} = \frac{\underline{\bar{R}}_{\parallel} + 1}{C}$$

$$\underline{\bar{X}}_{\perp} = \frac{\underline{\bar{R}}_{\perp} + 1}{C}$$

$$\underline{\bar{X}}_{\parallel} = \frac{\underline{\bar{R}}_{\parallel}}{C} \quad (41)$$

$$\underline{\bar{X}}_{\perp} = \frac{\underline{\bar{R}}_{\perp}}{C}$$

The "effective reflection height" is found by integrating $\bar{X}(\theta)$ upward to heights at, say one kilometer intervals above the bottom of the profile. That height at which the minimizing condition, Equation (31), is satisfied is the "effective reflection height". This height is chosen as the reference height, "d", used in classical waveguide theory as the height at which R and $\bar{R}$ (i.e., $\bar{X}$, n, and d) are computed and as the height at which Lagrange interpolation is performed. For evaluations of $F(\theta)$, Z_t is then always taken to be at $z=d$.

The upward free space integration of the ionospheric $\bar{X}(\theta)$ parameters is carried out in the following steps.

STEP I:

Two sets of boundary conditions (B.C.) are applied to equations (34), (35), (36) and (37) at the Z_b level. These sets are

$$\text{SET I: } E_{11}^i = 1, E_y^i = 0$$

$$\text{SET II: } E_{11}^i = 0, E_y^i = 1$$

Here $\bar{X}_b(\theta)$ is the result of the Runge-Kutta integration of Equation (26) at what will be denoted as the "from level", ("f"). Where

$$\zeta_f = (k/\alpha)^{2/3} [C^2 + \alpha(Z_b - H)] \quad (42)$$

For Horizontal Polarization at the "from level" (Z_b), the B.C. give:

For B.C. SET I:

$$\left(\frac{A}{C}\right)_I = \bar{X}_{-1}^f \cdot \left(\frac{a_{22}}{\Delta_1}\right) \quad (43)$$

$$\left(\frac{B}{C}\right)_I = -\bar{X}_{-1}^f \cdot \left(\frac{a_{21}}{\Delta_1}\right) \quad (44)$$

For B.C. SET II:

$$\left(\frac{A}{C}\right)_{II} = (\underline{1}\bar{X}_1^f \cdot a_{22} - 2a_{12})/\Delta_1 \quad (45)$$

$$\left(\frac{B}{C}\right)_{II} = (2a_{11} - \underline{1}\bar{X}_1^f \cdot a_{21})/\Delta_1 \quad (46)$$

where

$$\left. \begin{aligned} a_{11} &= h_1(\zeta_f), \quad a_{12} = h_2(\zeta_f) \\ a_{21} &= Ch_1(\zeta_f) + Kh_1'(\zeta_f) \\ a_{22} &= Ch_2(\zeta_f) + Kh_2'(\zeta_f) \\ \Delta_1 &= a_{11} a_{22} - a_{21} a_{12} \end{aligned} \right\} \quad (47)$$

For Vertical Polarization at the "from level" (Z_b), the B.C. give:

For B.C. SET I:

$$(Q/C)_I = (\underline{\parallel}\bar{X}_\parallel^f \cdot a_{22} - 2a_{12})/\Delta_2 \quad (48)$$

$$(G/C)_I = (2a_{11} - \underline{\parallel}\bar{X}_\parallel^f \cdot a_{21})/\Delta_2 \quad (49)$$

For B.C. SET II:

$$(Q/C)_{II} = \underline{1}\bar{X}_\parallel^f \cdot \frac{a_{22}}{\Delta_2} \quad (50)$$

$$(G/C)_{II} = -\underline{1}\bar{X}_\parallel^f \cdot \frac{a_{21}}{\Delta_2} \quad (51)$$

where:

$$\left. \begin{aligned} a_{11} &= h_1(\zeta_f), \quad a_{12} = h_2(\zeta_f) \\ a_{21} &= Ch_1(\zeta_f) + [Kh_1'(\zeta_f) + Lh_1(\zeta_f)]/n_f^2 \\ a_{22} &= Ch_2(\zeta_f) + [Kh_2'(\zeta_f) + Lh_2(\zeta_f)]/n_f^2 \\ \Delta_2 &= a_{11} \cdot a_{22} - a_{21} a_{12}, \quad n_f^2 = 1 + \alpha(Z_b - H) \end{aligned} \right\} \quad (52)$$

STEP II:

The values of E_{11}^i and E_y^i at the "to" level (e.g., at the height Z_t) are determined from the values of the coefficients; A/C , B/C , Q/C , and G/C , for the SET I and SET II boundary conditions.

Here the ζ parameter at the "to" level will be denoted by:

$$\zeta_t = (k/\alpha)^{2/3} [C^2 + \alpha (Z_t \cdot H)] \quad (53)$$

For Horizontal Polarization at the Z_t "to" level the results are

$$((E_y^i)_t)_I = ((\frac{A}{C})_I \cdot a_{21} + (\frac{B}{C})_I \cdot a_{22})/2 \quad (54)$$

$$((E_y^i)_t)_{II} = ((\frac{A}{C})_{II} \cdot a_{21} + (\frac{B}{C})_{II} \cdot a_{22})/2 \quad (55)$$

where

$$\left. \begin{aligned} a_{21} &= Ch_1(\zeta_t) + Kh_1'(\zeta_t) \\ a_{22} &= Ch_2(\zeta_t) + Kh_2'(\zeta_t) \\ K &= i(\alpha/k)^{1/3} \end{aligned} \right\} \quad (56)$$

For Vertical Polarization at the Z_t "to" level the results are:

$$((E_{11}^i)_t)_I = ((Q/C)_I \cdot a_{21} + (G/C)_I \cdot a_{22})/2 \quad (57)$$

$$((E_{11}^i)_t)_{II} = (Q/C)_{II} \cdot a_{21} + (G/C)_{II} \cdot a_{22})/2 \quad (58)$$

where

$$\left. \begin{aligned} a_{21} &= Ch_1(\zeta_t) + [Kh_1'(\zeta_t) + Lh_1(\zeta_t)]/n_t^2 \\ a_{22} &= Ch_2(\zeta_t) + [Kh_2'(\zeta_t) + Lh_2(\zeta_t)]/n_t^2 \\ L &= i(\frac{\alpha}{2k}), \quad n_t^2 = 1 + \alpha(Z_t - H) \end{aligned} \right\} \quad (59)$$

STEP III:

Knowing the values of

$$((E_{11}^i)_t)_{I,II} \text{ and } ((E_y^i)_t)_{I,II}$$

at the "to" level from sets of the coefficients A/C , B/C , Q/C and G/C the elements of the ionospheric $\bar{X}$ (i.e., $\bar{R}$) can be found at the "to" level.

For Horizontal Polarization at the "to" level the results are:

$$(\bar{X}_I)_t = [V_1((E_y^i)_t)_{II} - V_2((E_{11}^i)_t)_I]/W_1 \quad (60)$$

$$(\bar{X}_{II})_t = [V_2((E_{11}^i)_t)_I - V_1((E_{11}^i)_t)_{II}]/W_1 \quad (61)$$

where:

$$\left. \begin{aligned} W_1 &= ((E_{11}^i)_t)_I \cdot ((E_y^i)_t)_{II} - ((E_{11}^i)_t)_{II} \cdot ((E_y^i)_t)_I \\ V_1 &= (\frac{A}{C})_I h_1(\zeta_t) + (\frac{B}{C})_I h_2(\zeta_t) \\ V_2 &= (\frac{A}{C})_{II} h_1(\zeta_t) + (\frac{B}{C})_{II} h_2(\zeta_t) \end{aligned} \right\} \quad (62)$$

For Vertical Polarization at the "to" level the results are:

$$(\underline{\underline{X}}_{\parallel})_t = [V_1((E_y^i)_t)_{II} - V_2((E_y^i)_t)_I]/W_2 \quad (63)$$

$$(\underline{\underline{X}}_{\perp})_t = [V_2((E_{\parallel}^i)_t)_I - V_1((E_{\parallel}^i)_t)_{II}]/W_2 \quad (64)$$

where

$$\left. \begin{aligned} W_2 &= ((E_{\parallel}^i)_t)_I \cdot ((E_y^i)_t)_{II} - ((E_{\parallel}^i)_t)_{II} \cdot ((E_y^i)_t)_I \\ V_1 &= (Q/C)_I h_1(\zeta_t) + (G/C)_I h_2(\zeta_t) \\ V_2 &= (Q/C)_{II} h_1(\zeta_t) + (G/C)_{II} h_2(\zeta_t) \end{aligned} \right\} \quad (65)$$

Thus having the values of the ionospheric reflection coefficients $\underline{\underline{X}}(\theta)$ at Z_b the resulting values of these terms at the height $Z_t=d$ are obtained by applying the minimizing condition, e.g., so that the function $F(\theta)$ may be computed in a form where poles are absent.

The values of the terms $\parallel R_{\parallel}$, $\perp R_{\perp}$, $\parallel R_{\perp}$ and $\perp R_{\parallel}$ at height Z may be obtained from the results of the above equation in terms of $\underline{\underline{X}}(\theta)$ by the relations of Equation (41).

G. The Ground Reflection Coefficient Matrices

The ground reflection coefficients (e.g., $\parallel \bar{R}_{\parallel}$ and $\perp \bar{R}_{\perp}$) are needed in the computation of the $F(\theta)$ equation. The particular form of these coefficients, as used in the MODESRCH procedure, is identified by the ratios given in Appendix A by the equations A-9 and A-10:

$$\frac{_{||}n_{||}}{_{||}d_{||}} = \frac{_{||}\bar{R}_{||} + 1}{_{||}\bar{R}_{||}C} \quad (66)$$

$$\frac{_{\perp}n_{\perp}}{_{\perp}d_{\perp}} = \frac{_{\perp}\bar{R}_{\perp} + 1}{_{\perp}\bar{R}_{\perp}C} \quad (67)$$

The equations needed to determine the values of $_{||}\bar{R}_{||}$, $_{\perp}\bar{R}_{\perp}$ (in terms of $_{||}n_{||}$, $_{\perp}n_{\perp}$, $_{||}d_{||}$, and $_{\perp}d_{\perp}$) at a reference height "d" may be obtained from equations of Section F.

Needed substitutions to those equations are:

$$Z = 0 \text{ for } Z = Z_b \quad (68)$$

$$(_{\perp}\bar{X}_{||})^f = (_{||}\bar{X}_{\perp})^f = 0$$

$$\frac{_{||}\bar{R}_{||}^0 + 1}{C_{||}\bar{R}_{||}} = \left(\frac{_{||}n_{||}}{_{||}d_{||}} \right) \Big|_{Z=0} \text{ for } (_{||}\bar{X}_{||})_b \quad (69)$$

$$\frac{_{\perp}\bar{R}_{\perp}^0 + 1}{C_{\perp}\bar{R}_{\perp}} = \left(\frac{_{\perp}n_{\perp}}{_{\perp}d_{\perp}} \right) \Big|_{Z=0} \text{ for } (_{\perp}\bar{X}_{\perp})_b \quad (70)$$

Where the Fresnel reflection coefficients for the ground-free space interface are

$$_{||}\bar{R}_{||}^0 = \frac{C n_g^2 - \sqrt{n_g^2 - S^2}}{C n_g^2 + \sqrt{n_g^2 - S^2}} \quad (71)$$

and

$$_{\perp}\bar{R}_{\perp}^0 = \frac{C - \sqrt{n_g^2 - S^2}}{C + \sqrt{n_g^2 - S^2}} \quad (72)$$

with

$$n_g^2 = (\epsilon - j \sigma/\omega)/\epsilon_0 \quad (73)$$

and

ϵ = dielectric constant of the ground.

σ = conductivity of the ground.

ω = angular wave frequency.

$C = \cos\theta$, $S = \sin\theta$.

Well behaved forms of $\left(\frac{\parallel n \parallel}{\parallel d \parallel}\right) \Big|_{z=0}$ and $\left(\frac{\perp n \perp}{\perp d \perp}\right) \Big|_{z=0}$ at the "from" level (e.g. $Z = 0$), are derived in appendix C, and are given as:

$$\left. \begin{aligned} \parallel n \parallel &= 1, & \parallel d \parallel &= \frac{1}{2} \left[C - \left(\frac{\sqrt{n_g^2 - S^2}}{n_g^2} \right) \right], \\ \perp n \perp &= \frac{1}{\sqrt{n_g^2 - S^2}}, & \perp d \perp &= \frac{1}{2} \left[\left(C / \sqrt{n_g^2 - S^2} \right) - 1 \right] \end{aligned} \right\} \quad (74)$$

At the "from" level (e.g., $z=0$), Equations (45) and (46) become:

For Horizontal Polarization:

$$(A/C)_{II} = (\perp n \perp \cdot a_{22} - 2 \perp d \perp \cdot a_{12}) / (\Delta_1 \cdot \perp d \perp) \quad (75)$$

$$(B/C)_{II} = (2 \cdot \perp d \perp \cdot a_{11} - \perp n \perp \cdot a_{21}) / (\Delta_1 \cdot \perp d \perp) \quad (76)$$

where:

$$\left. \begin{aligned} a_{11} &= h_1(\zeta_0), \quad a_{12} = h_2(\zeta_0) \\ a_{21} &= Ch_1(\zeta_0) + Kh_1'(\zeta_0) \quad ; \quad \zeta_0 = (k/\alpha)^{2/3} [C^2 - \alpha H] \\ a_{22} &= Ch_2(\zeta_0) + Kh_2'(\zeta_0) \quad ; \quad \alpha = 2/r_e \\ \Delta_1 &= a_{11} \cdot a_{22} - a_{21} \cdot a_{12} \end{aligned} \right\} \quad (77)$$

For Vertical Polarization:

Equations (48) and (49) become:

$$(Q/C)_I = (n_{\parallel} \cdot a_{22} - 2 \cdot d_{\parallel} \cdot a_{12}) / (\Delta 2 \cdot d_{\parallel}) \quad (78)$$

$$(G/C)_I = (2 \cdot d_{\parallel} \cdot a_{11} - n_{\parallel} \cdot a_{21}) / (\Delta 2 \cdot d_{\parallel}) \quad (79)$$

where:

$$\left. \begin{aligned} a_{11} &= h_1(\zeta_0), \quad a_{12} = h_2(\zeta_0) \\ a_{21} &= Ch_1(\zeta_0) + ((Kh_1'(\zeta_0) + Lh_1(\zeta_0))/n_0^2) \\ a_{22} &= Ch_2(\zeta_0) + ((Kh_2'(\zeta_0) + Lh_2(\zeta_0))/n_0^2) \\ \Delta 2 &= a_{11} \cdot a_{22} - a_{21} \cdot a_{12}, \quad n_0^2 = 1 - \alpha H \end{aligned} \right\} \quad (80)$$

At the "to" level (e.g., $z=d$), Equation (55) becomes, for Horizontal Polarization:

$$((E_y^i)_d)_{II} = ((A/C)_{II} \cdot a_{21} + (B/C)_{II} \cdot a_{22})/2 \quad (81)$$

where

$$\left. \begin{aligned} a_{21} &= Ch_1(\zeta_d) + Kh_1'(\zeta_d) \\ \zeta_d &= (k/\alpha)^{2/3} [C^2 + \alpha(d - H)] \\ a_{22} &= Ch_2(\zeta_d) + Kh_2'(\zeta_d) \end{aligned} \right\} \quad (82)$$

and Equation (57) becomes, for Vertical Polarization:

$$((E_{11}^i)_d)_I = ((Q/C)_I \cdot a_{21} + (G/C)_I \cdot a_{22})/2 \quad (83)$$

where

$$\left. \begin{aligned} a_{21} &= Ch_1(\zeta_d) + [Kh_1'(\zeta_d) + Lh_1(\zeta_d)]/n_d^2 \\ a_{22} &= Ch_2(\zeta_d) + [Kh_2'(\zeta_d) + Lh_2(\zeta_d)]/n_d^2 \\ n_d^2 &= 1 + \alpha (d - H) \end{aligned} \right\} \quad (84)$$

At the "to" level (e.g. $Z = d$) define:

$$\left. \frac{N_{\parallel}}{D_{\parallel}} \right|_{z=d} = \frac{N_{\parallel}}{D_{\parallel}} \quad (85)$$

and

$$\left. \frac{N_{\perp}}{D_{\perp}} \right|_{z=d} = \frac{N_{\perp}}{D_{\perp}} \quad (86)$$

Equations (61) and (62) for Horizontal Polarization give:

$$N_{\perp} = (A/C)_{II} h_1(\zeta_d) + (B/C)_{II} h_2(\zeta_d) \quad (87)$$

$$D_{\perp} = ((E_y^i)_d)_{II} \quad (88)$$

and Equations (64) and (65) for Vertical Polarization give:

$$N_{\parallel} = (Q/C)_I h_1(\zeta_d) + (G/C)_I h_2(\zeta_d) \quad (89)$$

$$D_{\parallel} = ((E_{11}^i)_d)_I \quad (90)$$

The results of Equations (87) - (90) can be rearranged by reference to Equation (10)

where

$$\frac{N_{\parallel}}{D_{\parallel}} = \left(\frac{1}{\bar{R}_{\parallel}} + 1 \right) \frac{1}{C}$$

and

$$\frac{N_{\perp}}{D_{\perp}} = \left(\frac{1}{\bar{R}_{\perp}} + 1 \right) \frac{1}{C}$$

to give explicit expressions for $(\bar{R}_{\parallel})_d$ and $(\bar{R}_{\perp})_d$ as

$$(\bar{R}_{\parallel})_d = \frac{1}{C \left(\frac{N_{\parallel}}{D_{\parallel}} \right) - 1} = \frac{D_{\parallel}}{C N_{\parallel} - D_{\parallel}} \quad (91)$$

and

$$(\bar{R}_{\perp})_d = \frac{1}{C \left(\frac{N_{\perp}}{D_{\perp}} \right) - 1} = \frac{D_{\perp}}{C N_{\perp} - D_{\perp}} \quad (92)$$

Inspection of the equations will reveal that $_{\parallel}N_{\parallel}$, $_{\parallel}D_{\parallel}$, $_{\perp}N_{\perp}$, and $_{\perp}D_{\perp}$ contain no poles for any finite value of θ .

The resulting equations of this section are equivalent to those given in reference 9.

H. Excitation Factors, Height Gains and Auxiliary Functions

(1) Excitation Factors

The modal excitation factor and the modal height gain functions are two parameters needed in computing electric field strengths. The excitation factor formulas, as presented in reference 10, are summarized in the table 1. The column headings only apply to excitation of the electric field components E_z , E_y and E_x and the row headings apply to excitation by a vertical dipole (λ_v), horizontal dipole end on (λ_E) and a horizontal dipole broadside (λ_B). The direction of z is taken positive into the ionosphere. Positive x is the direction of propagation and y is normal to the plane of propagation.

Table 1 - EXCITATION FACTORS

Field Component	E_z	E_y	E_x
Exciter ↓			
λ_V	$B_1 \frac{(1 + {}_{II}\bar{R}_{II})^2 (1 - {}_I\bar{R}_I {}_I R_I)}{{}_{II}\bar{R}_{II} D_{II}}$	$-\frac{B_1}{S} \frac{{}_{II}R_I (1 + {}_{II}\bar{R}_{II}) (1 + {}_I\bar{R}_I)}{D_{I2}}$	$\frac{B_1}{S} \frac{(1 + {}_{II}\bar{R}_{II})^2 (1 - {}_I\bar{R}_I {}_I R_I)}{{}_{II}\bar{R}_{II} D_{II}}$
λ_E	$B_2 \frac{(1 + {}_{II}\bar{R}_{II})^2 (1 - {}_I\bar{R}_I {}_I R_I)}{{}_{II}\bar{R}_{II} D_{II}}$	$-\frac{B_2}{S} \frac{{}_{II}R_I (1 + {}_{II}\bar{R}_{II}) (1 + {}_I\bar{R}_I)}{D_{I2}}$	$\frac{B_2}{S} \frac{(1 + {}_{II}\bar{R}_{II})^2 (1 - {}_I\bar{R}_I {}_I R_I)}{{}_{II}\bar{R}_{II} D_{II}}$
λ_B	$B_2 \frac{{}_I R_I (1 + {}_I\bar{R}_I) (1 + {}_{II}\bar{R}_{II})}{D_{I2}}$	$-\frac{B_2}{S} \frac{(1 + {}_I\bar{R}_I)^2 (1 - {}_{II}\bar{R}_{II} {}_{II} R_{II})}{{}_I\bar{R}_I D_{22}}$	$\frac{B_2}{S} \frac{{}_I R_I (1 + {}_I\bar{R}_I) (1 + {}_{II}\bar{R}_{II})}{D_{I2}}$

The R and $\bar{R}$'s are the elements of the reflection matrix looking into the ionosphere and towards the ground from the reference level "d" within the guide.

The reference height "d" is that value where solutions to $F(\theta) = 0$ are determined.

B_1 and B_2 are given by:

$$B_1 = S^{5/2} \left/ \frac{\partial F}{\partial \theta} \right|_{\theta=\theta_n} \quad (93)$$

$$B_2 = -B_1/S \quad (94)$$

where S is the sine of the eigenangle and the denominator is the derivative of the modal equation (i.e. eq. 7) at an eigenangle, θ .

The definitions of the D_{ij} 's are given in the next section.

To compute electric field strengths the excitation factors must be supplemented with definitions of the height gains.

(2) Height Gain Terms

The height gain relationships at height Z are described below.

From Equation (39):

$$\zeta = (k/\alpha)^{2/3} [c^2 + \alpha(Z - H)] \quad (95)$$

At any height Z .

The height gain for the vertical electric field (E_z) is $f_{\parallel}(z)$ and is given by:

$$f_{\parallel}(Z) = [(Q/C)_I h_1(\zeta) + (G/C) h_2(\zeta)] \exp[(Z - d)/r_e] \quad (96)$$

where d is the reference height for solving $F(\theta) = 0$ for the eigenangle and where $(Q/C)_I$ and $(G/C)_I$ are given by Equations (48) and (49) respectively. The term " r_e " is the radius of the earth.

Also, the height gain for the horizontal electric field component (E_y) normal to the plane of propagation at height Z is given by:

$$f_{\perp}(Z) = [(A/C)_{II} h_1(\zeta) + (B/C)_{II} h_2(\zeta)] \quad (97)$$

where $(A/C)_{II}$ and $(B/C)_{II}$ are given by Equations (45) and (46) respectively.

The height gain for the horizontal electric field components (E_x) in the plane of propagation at height Z is given by:

$$\begin{aligned} g(Z) &= \frac{1}{ik} \frac{d}{dZ} (f_{\parallel}(Z)) \text{ or} \\ g(Z) &= -i \exp\left(\frac{Z-d}{a}\right) \left\{ \left(\frac{2}{ak}\right)^{1/3} [(Q/C)_I h_1'(\zeta) + (G/C)_I h_2'(\zeta)] \right. \\ &\quad \left. + \left(\frac{2}{ak}\right) [(Q/C)_I h_1(\zeta) + (G/C)_I h_2(\zeta)] \right\} \quad (98) \end{aligned}$$

At height $Z = d$ (i.e. the reference height) the argument is:

$$\zeta_d = (k/\alpha)^{2/3} [c^2 + \alpha(d - H)] \quad (99)$$

and

$$D_{11} = f_{\parallel}^2(\zeta_d), \quad D_{12} = f_{\parallel}(\zeta_d) f_{\perp}(\zeta_d) \quad (100)$$

$$D_{22} = f_{\perp}^2(\zeta_d)$$

(3) Modified Excitation Terms

Various combinations of the modal excitation factor and the height gain function are required for computing field strengths by different mode-sum programs. Reference 11 is an example.

Table 2 illustrates the case where the excitation factors as described in Table 1 and the height gains from Equations (96), (97) and (98) are lumped together and defined as modified excitation factors. Table 2 illustrates combinations obtained for computing the vertical electric field (E_z). These particular combinations will be discussed further in Section V-C-2 for the option NPUNCH = 7.

Table 2
Excitation Factors for Computing the
Vertical Electrical Field (E_z)

Vertical source:

$$X_{VZ} = B_1 \frac{(1 + \bar{R}_1)^2 (1 - \bar{R}_1 R_1)}{\bar{R}_1 \cdot D_{11}} \cdot f_1(Z_t) \cdot f_1(Z_r)$$

Horizontal broadside source:

$$X_{BZ} = \frac{\bar{R}_1 (1 + \bar{R}_1) (1 + \bar{R}_1) \cdot f_1(Z_t) \cdot f_1(Z_r)}{D_{12}}$$

Horizontal end-on source:

$$X_{EZ} = \frac{(1 + \bar{R}_1)^2}{\bar{R}_1 \cdot D_{11}} (1 - \bar{R}_1 R_1) \cdot g(Z_t) \cdot f_1(Z_r)$$

Z_t is height of the transmitter and

Z_r is the height of the receiver

The excitation factor usually considered is the vertical component of the electric field vector for vertical receiving and transmitting point dipoles, where both receiver and transmitter are at the ground (e.g., $z = 0$). The excitation factor X_{VZ} is denoted in this case by X_0 . Wait's excitation factor is computed as $\Delta_0 = -.5ikh_r X_0$ where k is the wave number in km^{-1} , h_r is REFLHT and $i = \sqrt{-1}$.

(4) Fast Mode Conversion

Reference 12 presents equations and a computer program for calculating the vertical electric field strength at a receiver by the procedure of mode conversion. An input parameter required for these calculations

is denoted as 'FOVR'. The mode conversion program requires that the reference height used for computing ionospheric and ground reflection coefficients be at the ground (i.e., $z=0$).

The reference height used in the MODESRCH computer program is at some height "d" ($d \neq 0$). Therefore the equation needed to obtain the required 'FOVR' term is given by"

$$\text{FOVR} = \left\{ \frac{(1 + \underline{R}_1)(1 - \underline{R}_1 \underline{R}_1)}{(1 + \underline{R}_1)(1 - \underline{R}_1 \underline{R}_1)} \right\} \bigg|_{z=d} \cdot \frac{f_1(d)}{f_1(d)} \quad (101)$$

or

$$\left\{ \frac{(1 + \underline{R}_1)(1 - \underline{R}_1 \underline{R}_1)}{(1 + \underline{R}_1)(1 - \underline{R}_1 \underline{R}_1)} \right\} \bigg|_{z=d} \cdot \frac{f_1(d)}{f_1(d)}$$

I. Use of Lagrange Interpolation

The general root-finding algorithm, (section III), used to locate the zeros of a function, $f(\theta)$, requires that values of $f(\theta)$ and its derivatives wrt θ be available upon call for any value of θ in the region of interest. In Section F a criterion was given for choosing a reference height, 'd', so as to minimize the variation of $\underline{X} = (\underline{R} + 1)/C$ wrt θ thereby placing the dominant waveguide effects into $\underline{n}$ and $\underline{d}$ (i.e., into $\underline{R}$). (See Equation 7.) On the other hand, far more computation is required to evaluate $\underline{X}$ than to evaluate $\underline{n}$ and $\underline{d}$. This suggests the utilization of an approximate representation of $\underline{X}$ to be used within the general root-finder, and then following the zeros of $f(\theta)$ (see Equation 7) to their locations which will be consistent with the exact formulation for $\underline{X}$.

The general root-finder requires that the function $f(\theta)$ be analytic with no poles and therefore an approximate representation of $\underline{X}$ must be analytic with no poles. A polynomial representation has been chosen, which can be represented by the Lagrange interpolation formula

$$\bar{X}(\theta)_{\text{approx.}} = \sum_{i=1}^n \left\{ \prod_{\substack{k=1 \\ k \neq i}}^n (\theta - \theta_k) \right\} p_i \quad \text{and where } p_i = \frac{\bar{X}(\theta_i)}{\prod_{\substack{k=1 \\ k \neq i}}^n (\theta_i - \theta_k)} \quad (102)$$

where $\bar{X}(\theta_i)$ are evaluations of $\bar{X}$ using the exact formulation for $\bar{X}$ at n selected angles, θ_i , distributed over the θ -plane in the region of interest.

Since $F(\theta)$ (see Equation 7) in both approximate and exact (complete) forms is analytic with no poles, each zero of $F(\theta)_{\text{approx.}}$ may be followed uniquely to a corresponding zero of $F(\theta)_{\text{exact}}$. The continuous transition from one to the other may be represented, for example, by

$$F(\theta)_{\text{comb.}} = F(\theta)_{\text{approx.}} \cos^2 \beta + F(\theta)_{\text{exact}} \sin^2 \beta \quad (103)$$

with β varying continuously from 0 to 90° . In practice β is incremented in finite steps with a Newton-Raphson iteration at each step. How far the zero of $F(\theta)$ "travels" in the θ -plane as β is varied from 0 to 90° depends; however, on how well $F(\theta)$ (i.e., $\bar{X}(\theta)$) is represented by the Lagrange interpolation. It is possible, in principle, that a zero may travel from the region of interest to infinity, or vice-versa. There are no guarantees and although, in most cases in the VLF and LF range, the use of Lagrange interpolation may save on computation by an order of magnitude, it nevertheless represents the "weakest link" in formulating a self-contained mode search. In some cases, especially in the ELF range, it is necessary to use the (exact) formulation for F .

J. Comments of the Differentiation of $F(\theta)$ wrt θ

The excitation factor components given in Section H require the computation of the derivative $F(\theta)$ wrt θ for each eigenvalue of θ . In

addition, the general root-finder, (section III), requires, in its later stages, some evaluations of $dF/d\theta$, and the following of zeros by the use of Newton-Raphson iteration described in Section I requires computation of $dF/d\theta$. Traditionally this derivative has been evaluated by computing F at two closely-spaced values of θ , using a large number of significant digits in the computation. An alternative is to analytically differentiate F , which implies analytic differentiation of $\mathbb{X}$, η and $\underline{d}$. This alternative was chosen for the mode search program.

Note that differentiation of the numerical (Runge-Kutta) integration described in Section E is performed by the same type numerical integration, with the integrand analytically differentiated wrt θ .

In certain sections of MODESRCH where $F(\theta)$ is computed in terms of Lagrange interpolation, the formulation of the derivative wrt θ is given by

$$\frac{dF(\theta)}{d\theta} = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left\{ \prod_{\substack{k=1 \\ k \neq i \\ k \neq j}}^n (F(\theta) - F_k(\theta)) \right\} \cdot \left\{ P_i + P_j \right\} \quad (104)$$

where

$$P_\ell = \frac{F(\theta_\ell)}{\prod_{\substack{k=1 \\ k \neq \ell}}^n (X_\ell - X_k)}$$

and n is the number of points.

III. THE GENERAL ROOT-FINDER

There is a need to formulate a procedure to determine the complex zeros of a complex function. For the method presented here, the search for such zeros is based on the fact that the lines of constant phase of any complex function, $F(\theta)$, may be discontinuous only at points where $F(\theta) = 0$ or $F(\theta) \rightarrow \infty$. The function $F(\theta)$, to be discussed, is such that it contains no poles and only $F(\theta) = 0$ is considered. Also, the lines of constant phase around $F(\theta) = 0$ progress only in a counter-clockwise direction.

Let:

$$F(\theta) = F_R(\theta_r, \theta_i) + j F_I(\theta_r, \theta_i) \quad (106)$$

where

$$\theta = \theta_r + j\theta_i$$

$$\text{Also } F(\theta) = [F_R(\theta_r, \theta_i)^2 + F_I(\theta_r, \theta_i)^2]^{1/2} e^{j\phi} \quad (107)$$

where

$$\phi = \tan^{-1} \left(\frac{F_I(\theta_r, \theta_i)}{F_R(\theta_r, \theta_i)} \right) \quad (108)$$

and

$F_R(\theta)$ is the real part of the complex function $F(\theta)$

$F_I(\theta)$ is the imaginary part of the complex function $F(\theta)$

θ_r is the real part of the complex number (θ)

θ_i is the imaginary part of the complex number (θ) .

If $F(\theta)$ has no poles, this implies that the line of any particular constant phase value $\phi = \phi_c$, radiating from a zero of $F(\theta)$, must cross a closed contour containing that zero at least once. Furthermore, no other zero of $F(\theta)$ may be on this phase line. These relations are illustrated in Figure 2.

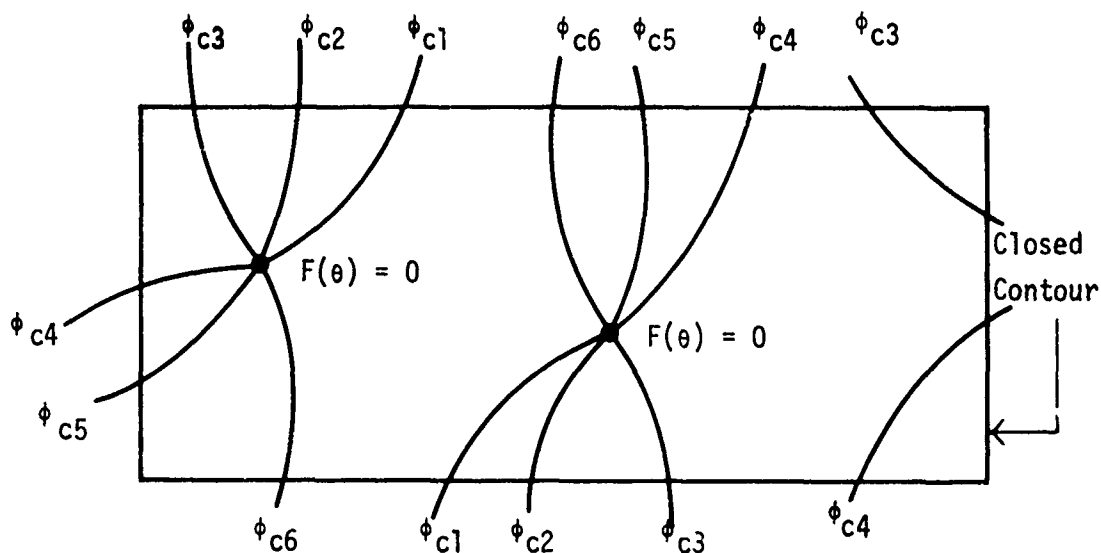


Figure 2. Constant Phase Lines of $F(\theta)$.

A line of constant phase (e.g., $\phi = \phi_c$) which crosses the contour may be followed inward until it leads to a zero of $F(\theta)$ or until the line again reaches the contour.

The procedure is as follows.

(A) Let $\phi_c = 0^\circ$ (or 180°), this implies that (109)

$$F_I(\theta_r, \theta_i) = 0$$

Also let $\phi_c = 90^\circ$ (or 270°), this implies that

$$F_R(\theta_r, \theta_i) = 0 \quad (110)$$

(B) Construct a "contour rectangle" in the complex θ plane. Denote the corners of this rectangle by:

TLEFT - value of real part of Theta at left edge of rectangle.

TRIGHT - value of real part of Theta at right edge of rectangle.

TBOT - value of imag. part of Theta at bottom edge of rectangle

TTOP - value of imag. part of Theta at top edge of rectangle.

Define the following parameters:

TMESH - equal to about half the average spacing between zeros within the rectangle.

TOL - tolerance to which zeros are to be found.

If two zeros are closer than 'TOL', the zeros will not be found by this method.

(C) Divide the 'contour rectangle' up into small mesh squares where the dimensions of the squares is such that the length of a side is smaller than the average separation of the zeros of $F(\theta)$ (e.g., size of a mesh side = 'TMESH').

(D) Normalize the value of the corners of the "contour rectangle" into mesh units. The corners are then denoted as JLT, JRT, JTOP, and JBOT. That is θ 's for the "contour rectangle" are:

θ_1 at left edge: $JLT = TLEFT/TMESH$

θ_2 at right edge: $JRT = TRIGHT/TMESH$

θ_3 at top edge: $JTOP = TTOP/TMESH$

θ_4 at bottom edge: $JBOT = TBOT/TMESH$.

(E) Figure 3 illustrates the rectangular region of the complex θ -plane within which the roots of $F(\theta) = 0$ are to be found.

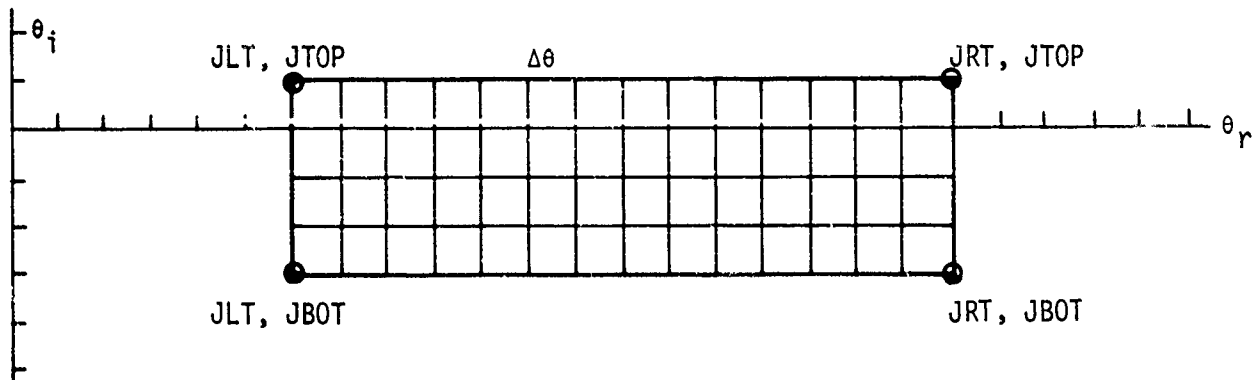


Figure 3. "Contour Rectangle" with "MESH" Squares.

In the above example all the axes are marked with tic marks, with spacing $\Delta\theta = \text{TMESH}$. The mesh size within the rectangle is equal to TMESH .

In the above figure, $JLT = 6$, $JRT = 20$, $JTOP = 1$, $JBOT = -3$.

Since some "slop" occurs at the rectangle boundary, the limits of the rectangle are made one TMESH larger on all sides than actually desired.

TMESH should be no larger than about one-half the average spacing between roots.

(F) The procedure then is to examine the character of $F(\theta)$ in a counter clockwise direction around the contour rectangle (see Figure 4). Intermediate steps of investigating mesh squares are also carried out in a counterclockwise direction. Again see Figure 4.

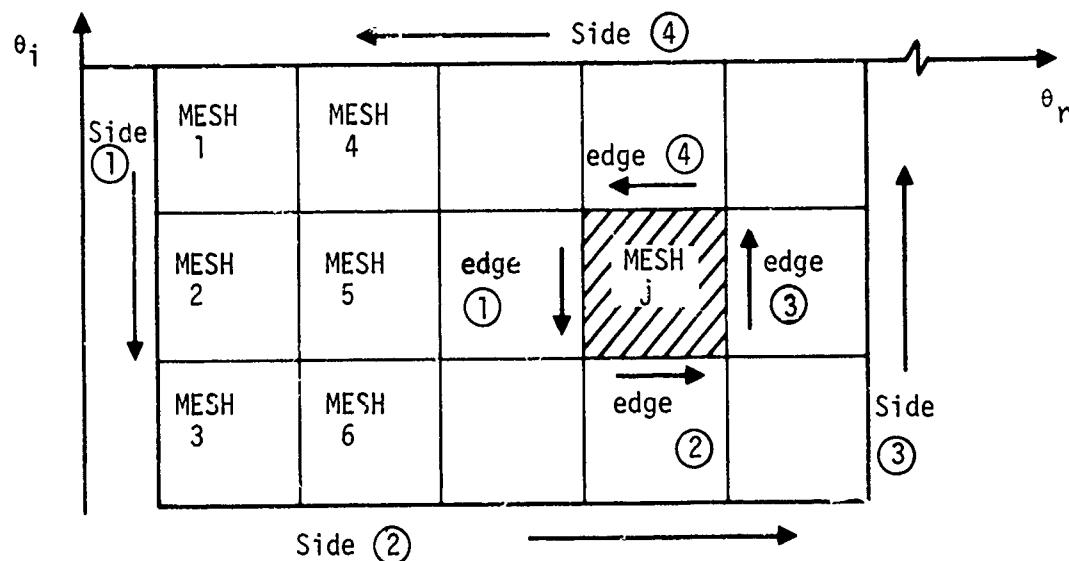


Figure 4. Directions for Determining Zeros of $F(\theta)$.

(G) Consider an individual mesh square (e.g., mesh no. j). The corners of the square may be labeled in terms of $F(\theta_r, \theta_i)$ where each edge is one-mesh unit in length. Figure 5 illustrates normalized coordinates of the mesh.

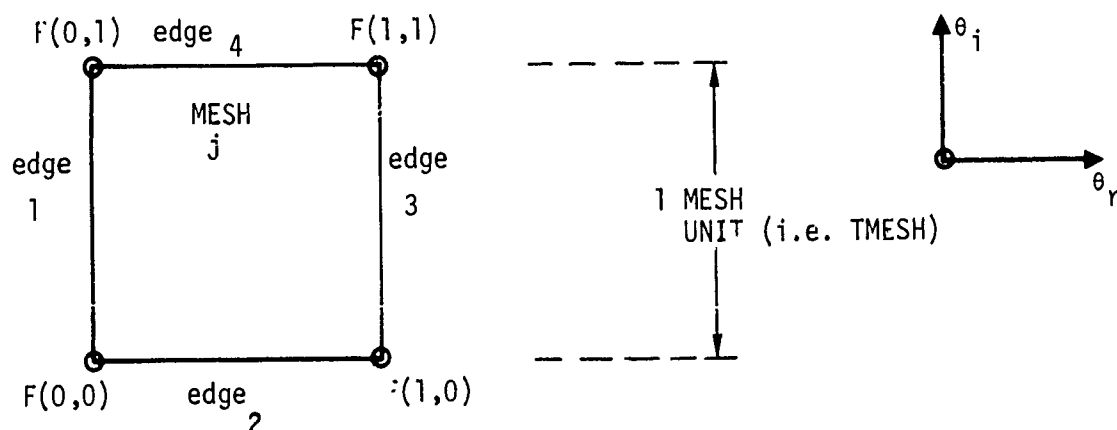


Figure 5. Normalized Coordinates of Each Mesh Square for $F(\theta_r, \theta_i)$.

Note that for each unit mesh square the lower left hand corner is taken as the reference position.

A basic assumption put on $F(\theta)$ is that $F_I(\theta)$ and $F_R(\theta)$ are to be linear along each edge of a mesh square.

The coordinates of the corners of mesh square No. 1 (see Figure 4) are given by:

$$\begin{aligned} F(0,1) &= F(JLT, JTOP) \\ F(0,0) &= F(JLT, JTOP-1) \\ F(1,0) &= F(JLT+1, JTOP-1) \\ F(1,1) &= F(JLT+1, JTOP) \end{aligned} \tag{111}$$

All other squares are identified by increasing or decreasing the θ_r and/or θ_i coordinates by increments of 1-mesh unit.

(H) The root-finder procedure continues as follows. First, mesh square No. 1 is examined. The values of $F_I(0,1)$ and $F_I(0,0)$ for mesh edge No. 1 are computed. If the signs of these two values are identical, there can be no line with constant value of $F_I(\theta) = 0$ passing through this edge. Next, investigate mesh square No. 2 (see Figure 4). Compute the values of $F_I(0,1)$ and $F_I(0,0)$ for this mesh square. The value of $F_I(0,1)$ for mesh square No. 2 is identical to that of $F_I(0,0)$ for mesh square No. 1, however, the value of $F_I(0,0)$ for mesh square No. 2 will be a new value. The coordinates of mesh square No. 2 are identified by:

$$\begin{aligned} F(0,i) &= F(JLT, JTOP-1) \\ F(0,0) &= F(JLT, JTOP-2) \\ F(1,0) &= F(JLT+1, JTOP-2) \\ F(1,1) &= F(JLT+1, JTOP-1) \end{aligned} \tag{112}$$

If the signs of $F_I(0,1)$ and $F_I(0,0)$ for mesh square No. 2 are opposite, this implies that the line $F_I(\theta) = 0$ enters mesh square No. 2 somewhere along edge No. 1. Because of the linearity assumption of $F_I(\theta)$ along

the edge of a mesh square, the intersection between the edge and the constant phase line $F_I(\theta)=0$ is (as shown in Appendix D, eq. 13) to be at:

$$\theta_r = 0, \theta_i = \left(\frac{-F_I(0,0)}{F_I(0,1)-F_I(0,0)} \right) \quad (113)$$

θ_r is identified as ENTER-R

θ_i is identified as ENTER-I

(I) It has now been determined that the constant line $F_I(\theta) = 0$ has entered mesh square No. 2. The problem is to find where this line exits from the square. Also it must be determined whether a zero to $F(\theta)$ lies within the mesh square. Several tests must be made:

- 1) Computations are made to obtain the values of $F_I(1,0)$ and $F_I(1,1)$ for mesh square No. 2. At this point values are known for $F_I(\theta)$ at all four corners of mesh square No. 2.
- 2) Test for there being two (hyperbolic) lines entering and leaving the mesh square along each line of which $F_I(\theta) = 0$.
 - (a) If only one line, the parameter LTWO is set to zero.
 - (b) If the values of $F_I(0,0)$ and $F_I(1,1)$ are each greater than zero; and the values of $F_I(1,0)$ and $F_I(0,1)$ are each less than zero then there are two hyperbolic lines of $F_I(\theta) = 0$ entering and leaving the mesh square. This condition is illustrated in Figure 6.

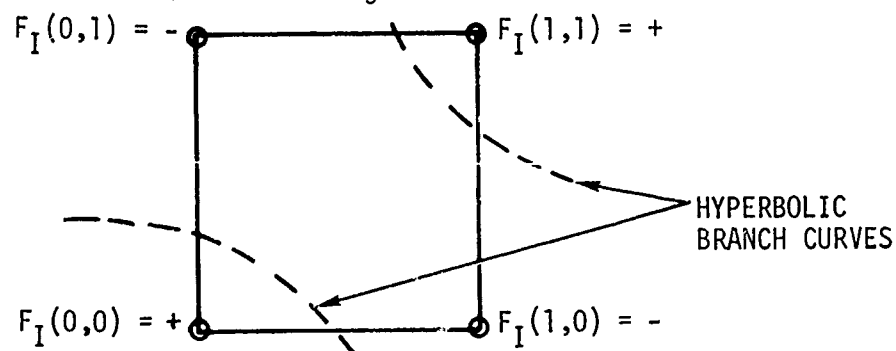


Figure 6. Hyperbolic Branch Curves of $F_I(\theta)$ Within a Mesh Square.

For this case, the parameter "LTWO" is set equal to 1.

- (c) Also, if the values of $F_I(0,0)$ and $F_I(1,1)$ are each less than zero, and the values of $F_I(1,0)$ and $F_I(0,1)$ are each greater than zero, the result is identical to (b) above.
- 3) Test for there being at least one hyperbolic branch line entering and leaving the mesh square along which $F_R(\theta) = 0$.
 - (a) If the values of $F_R(0,0)$, $F_R(0,1)$, $F_R(1,0)$ and $F_R(1,1)$ are all of the same sign, then there is no line of $F_R(\theta) = 0$ passing through the mesh square and the variable "I90" is set to zero.
 - (b) If the condition of (a) above, is not met, then there is at least one branch line entering and leaving the mesh and the variable 'I90' is set equal to 1.
- 4) Test to see if there exists the conditions for the presence of hyperbolic lines at both $F_I(\theta) = 0$ and $F_R(\theta) = 0$ within the mesh square.
 - (a) If the condition is not present (e.g., "LTWO" is equal to 0 and "I90" is equal to 0) then determine which mesh edge (e.g., 2,3, or 4) where the original entry line exits and proceed with an analysis of the appropriate mesh square. For example, as can be observed from Figure 4, edge No. 2 leads to mesh square No. 3, edge No. 3 leads to mesh square No. 5 and edge No. 4 leads to mesh square No. 1.

The line $F_I(\theta) = 0$ is followed continuously from mesh to mesh until the line exits from the "contour rectangle". When this occurs, the value of the leading edge (counter clockwise direction) of the mesh square,

through which the $F_I(\theta) = 0$ line exits, is noted. By identifying the mesh square, in which this line exit occurs, it is possible to later avoid re-entering this mesh square through this exit edge while following along the outer edge of the "contour rectangle". Figure 7 illustrates this situation. Note that it is possible that a solution to $F(\theta) = 0$ will be found as the line $F_I(\theta) = 0$ progresses through the various mesh squares. This case will be discussed in the following Section (b).

The next step after exit from mesh square No. k is to examine mesh square No. 3 for intersections of the line $F_I(\theta) = 0$ on edge No. 1 of mesh square No. 3.

- (b) If hyperbolic lines of both $F_I(\theta) = 0$ and $F_R(\theta) = 0$ do exist within mesh square No. 2, then the following hyperbolic relations are examined.

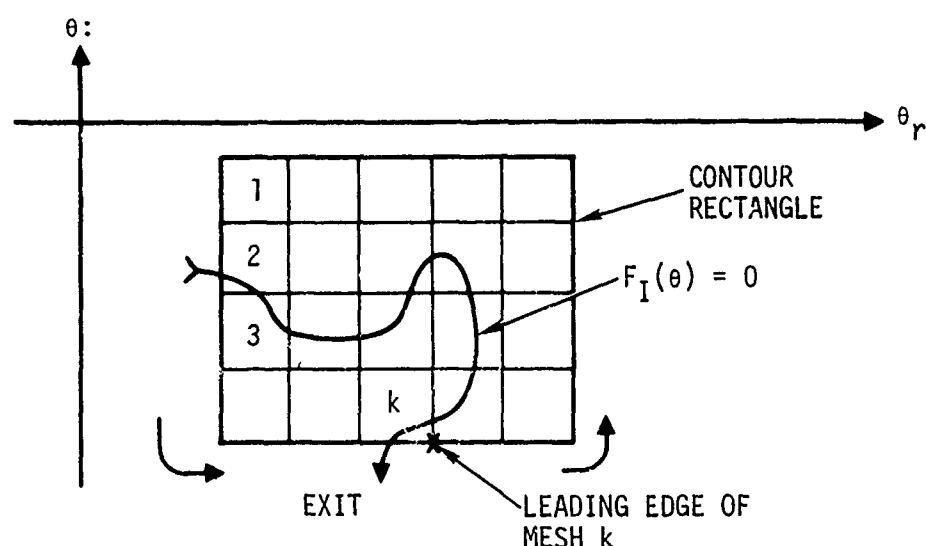


Figure 7. Contour Rectangle, Mesh Squares and the line $F_I(\theta) = 0$.

It is shown in Appendix D that the relationship for $F(\theta)$ based on a knowledge of $F(\theta)$ at each of the corners of a mesh square is given by

$$F(\theta) = a + b\theta_i + c\theta_r + d\theta_i\theta_r \quad (114)$$

where

$$\begin{aligned} a &= F(0,0) \\ b &= F(0,1) - F(0,0) \\ c &= F(1,0) - F(0,0) \\ d &= F(0,0) + F(1,1) - F(0,1) - F(1,0) \end{aligned} \quad (115)$$

These coefficients describe the variation of $F(\theta)$ within the mesh square given the values of $F(\theta)$ at the corners of the square and that a linear variation exists between values of $F(\theta)$ along the edges of the square.

When $F_R(\theta) = 0$ (or $F_I(\theta) = 0$), Equation 114 is the equation of an equilateral hyperbola with vertical and horizontal asymptotes.

Now consider the constant phase line $F_I(\theta) = 0$. From Equation 114 this is

$$(a_i + b_i\theta_i) + (c_i + d_i\theta_i)\theta_r = 0 \quad (116)$$

or

$$\theta_r = - \frac{a_i + b_i\theta_i}{c_i + d_i\theta_i}$$

The following equation is also obtained from Equation 114.

$$(a_i + c_i\theta_r) + (b_i + d_i\theta_r)\theta_i = 0 \quad (117)$$

or

$$\theta_i = - \frac{a_i + c_i\theta_r}{b_i + d_i\theta_r}$$

The position of the crossings of the hyperbolic asymptotes (which are parallel to the sides of the square) for the lines $F_I(\theta) = 0$ are computed since both lines (hyperbolic branches) may enter and leave the square.

For the function $F_I(\theta) = 0$ the line of the vertical asymptote is given in Appendix D as

$$\theta_r = -b_i/d_i \quad (118)$$

where θ_r is identified as CENTER-R, while the line of the horizontal asymptote is given by

$$\theta_i = -c_i/d_i \quad (119)$$

where θ_i is identified as CENTER-I.

There are two possible points at which there may be crossings of the (hyperbolic) lines $F_R(\theta) = 0$ and $F_I(\theta) = 0$. A crossing point is chosen to be a zero of the function if it lies within the current mesh square and if it lies on the hyperbolic branch of $F_I(\theta) = 0$ currently being followed. Note that in the example being examined the hyperbolic branch line being examined enters mesh square No. 2 along edge No. 1 and exits along edge No. 2.

Both Equation (116) and Equation (117) must be examined since in any given case either form (but not both forms) may be indeterminate.

Equation (116) may be considered as two equations, that is for real and imaginary parts of the constants a , b , c , and d :

$$\theta_r = -\frac{(a_r + b_r \theta_i)}{(c_r + d_r \theta_i)} = -\frac{a_i + b_i \theta_i}{c_i + d_i \theta_i} \quad (120)$$

and, equating the two right hand equations the result is

$$P_1 \theta_i^2 + Q_1 \theta_i + W_1 = 0 \quad (121)$$

where

$$\begin{aligned} P_1 &= b_r d_i - b_i d_r \\ Q_1 &= a_r d_i + b_r c_i - a_i d_r - b_i c_r \\ W_1 &= a_r c_i - a_i c_r \end{aligned} \quad (122)$$

This equation is solved (see Appendix E) for θ_i . Then, knowing θ_i , θ_r can be determined from Equation (116). In the same manner Equation (117) gives

$$\theta_i = -\frac{a_r + c_r \theta_r}{b_r + d_r \theta_r} = -\frac{a_i + c_i \theta_r}{b_i + d_i \theta_r} \quad (123)$$

and equating the two right hand equations gives

$$P_2 \theta_r^2 + Q_2 \theta_r + W_2 = 0 \quad (124)$$

where

$$P_2 = c_r d_i - c_i d_r$$

$$Q_2 = a_r d_i + c_r b_i - a_i d_r - c_i b_r \quad (125)$$

$$W_2 = a_r c_i - a_i c_r$$

this equation may be solved for θ_r (see Appendix E). Then, knowing θ_r , θ_i can be determined from Equation (117).

If

$$|b_r d_i - b_i d_r| < |c_r d_i - c_i d_r| \quad (126)$$

Equations (116) and (121) are solved for θ_r and θ_i . If the inequality of Equation (126) is not true then Equations (117) and (124) are solved for θ_r and θ_i .

A solution of Equations (116) and (121) (or Equations (117) and (124)) is accepted if it lies within the current mesh square. That is, if the solution is denoted θ_s or $(\theta_{rs} + j\theta_{is})$, then θ_s must satisfy the relationships

$$0 \geq \theta_{rs} \leq 1 \text{ and } 0 \geq \theta_{is} \leq 1 \quad (127)$$

to be a solution.

Also the solution must lie on the hyperbolic branch of $F_I(\theta) = 0$ which is currently being followed. The relationship is that (using Equation (113) and Equations (118) and (119):

$$\text{if } (\text{ENTER R} - \text{CENTER R}) \cdot (\theta_{rs} - \text{CENTER R}) > 0 \quad (128)$$

and

$$\text{if } (\text{ENTER I} - \text{CENTER I}) \cdot (\theta_{is} - \text{CENTER I}) > 0$$

then the solution of $F(\theta)$ is on the current branch line. If Equation (128) is not true then the solution θ_s is not on the current branch; however, the improper solution will be selected later when following another constant phase line of $F_I(\theta) = 0$.

Figure 8 illustrates the case where θ_s is a proper solution to $F(\theta) = 0$.

Figure 9 illustrates the case where θ_s is not a proper solution to $F(\theta) = 0$.

If the value θ_s is a solution to $F(\theta) = 0$ the line $F_I(\theta) = 0$ is followed out of mesh square No. 2 through edge No. 2 into mesh square No. 3 through its edge No. 4 (see Figure 4). The line $F_I(\theta) = 0$ is then followed from mesh to mesh until the line $F_I(\theta) = 0$ exits from the contour rectangle (see Figure 10).

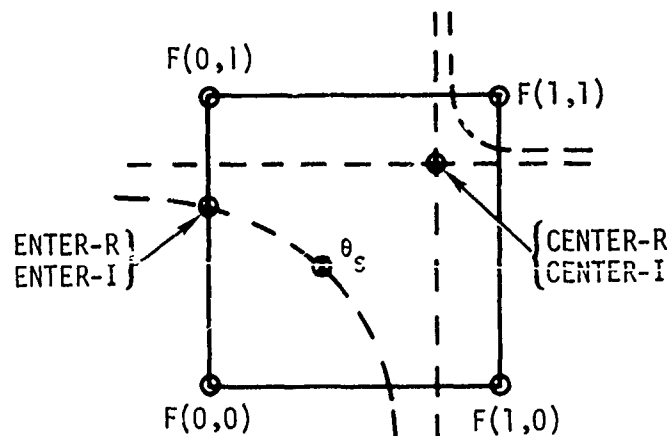


Figure 8. θ_s is a Proper Solution to $F(\theta) = 0$.

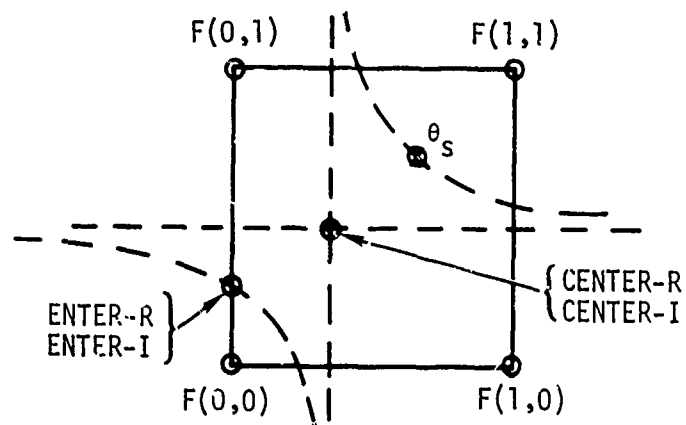


Figure 9. θ_s is not the Proper Solution to $F(\theta) = 0$.

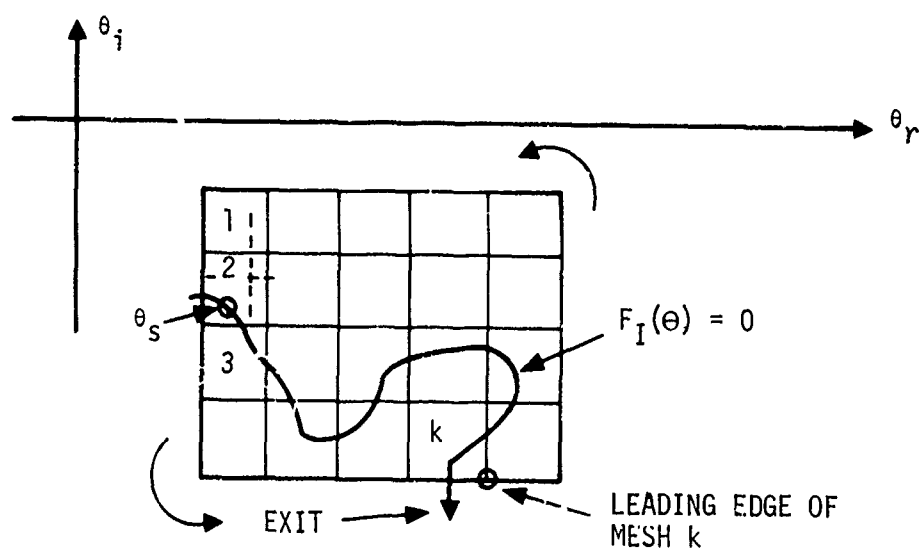


Figure 10. Contour Rectangle, Mesh Squares and Hyperbolic Lines of $F_I(\theta) = 0$. θ_s is a Solution of $F(\theta) = 0$.

At this point the value of the leading edge (counter clockwise direction) of the mesh square, through which the $F_I(\theta) = 0$ line exits, is noted. This mesh edge will also be along one of the contour rectangle sides. By identifying the mesh square in which this line exits occurs, it is then possible to later avoid re-entering this mesh through this exit edge while following the sides of the contour rectangle. This is important because if the search were allowed to re-enter at this location, the same zero of $F(\theta)$ would be found twice.

The next step is to examine mesh square No. 3 for intersections of the line $F_I(\theta)$ and edge No. 1 of that mesh.

(J) The entire contour rectangle is now searched, by the methods described in Section I, so that finally all of the zeros of the function $F(\theta) = 0$, which lie within the contour rectangle, are known. These solutions must be considered approximate; however, in that the hyperbolic approximation has been used in solving for $F(\theta) = 0$.

(K) To alleviate this problem, use is made of the Newton-Raphson iteration method to remove the mesh approximation where:

$$\Delta\theta = - \frac{F(\theta)}{\left. \frac{dF(\theta)}{d\theta} \right|_{\theta = \text{mesh}}} \quad (129)$$

If two approximate solutions are nearly equal, they may converge to a single value after application of the Newton-Raphson procedure.

In order to resolve the two solutions further analysis is needed. Let the two approximate solutions be $\theta_{s,1}$ and $\theta_{s,2}$. Consider a set of 9 mesh squares arranged in a 3x3 formation with the one containing the solution $\theta_{s,1}$ at the center. There are 16 corner points associated with the corners of these squares (e.g. $(\theta_x)_i$, $i=1,2,3,\dots,16$). The functions $F((\theta_x)_i)$ are then computed

for each of the 16 corners. These values will be denoted as $F_{\text{mesh}}((\theta_x)_i)$. The set of functions given by the following equation are also computed,

$$F_{\text{no mesh}}((\theta_x)_i) = F(\theta_{s,1}) + \left. \frac{dF}{d\theta} \right|_{\theta_{s,1}} (\theta_{xi} - \theta_{s,1}) \quad (130)$$

($i=1,2,3,\dots,16$)

The next step is to compute the following function at each of the 16 corners:

$$F_{\text{total}}((\theta_x)_i) = F_{\text{mesh}}((\theta_x)_i) \cos^2 \beta + F_{\text{no mesh}}((\theta_x)_i) \sin^2 \beta \quad (131)$$

where β is incremented from 0° to 90° in whatever increments are required to resolve the zeros of $F(\theta)$. Considering the 9 squares one at a time the hyperbolic equation (Eq. 114) is solved to get 0,1, or 2 solutions in each of the squares. Only those solutions $(\theta_{q,j})$ which exist inside of a given square are kept. The final solution $(\theta_{p,1})$ is chosen to be that one solution from the set $(\theta_{q,j})$ which is closest to the original $\theta_{s,1}$.

The same type of analysis is then carried out for the solution $\theta_{s,2}$. The final solutions $\theta_{p,1}$ and $\theta_{p,2}$ are iterated by the Newton-Raphson procedure to give the exact solutions. If these two solutions again converge to a single number, the value of β in Equation 131 is incremented and the entire procedure is again carried through with the values $\theta_{p,1}$ and $\theta_{p,2}$ replacing the values of $\theta_{s,1}$ and $\theta_{s,2}$. Note that completeness (in that the full set of zeros has been determined) is assured in Steps I and J whereas exact values must await Step K.

IV STRUCTURE AND DESCRIPTION OF THE MODESRCH COMPUTER PROGRAM

(A) Program Structure

The MODESRCH computer program finds the waveguide mode constants of the earth-ionosphere waveguide for radio propagation at ELF-VLF-LF. These mode constants are determined in the form of modal: eigenangles, attenuation rates, phase velocities and excitation factors.

The individual subroutines of the MODESRCH program are listed, in Table 3. In this table, those subroutines are divided into special sub-groups which are in accordance with their function in the total program structure.

Figures 11 through 14 illustrate the structure of the MODESRCH computer program. In particular, Figure 11 presents the general flow of the program showing the order in which the various subroutines and sub-groups are executed. This sequence is indicated, in the figure, by the numerals 1 through 6. Figure 12 shows the structure of the Ionospheric Full-Wave Integration procedure. Figure 13 gives the Free Space Integration Structure along with the structure of the routines for computing the values of the function $F(\theta)$ and the derivative of $F(\theta)$ with respect to θ . Figure 14 illustrates the structure of the Root-Finder procedure.

TABLE 3
LIST OF PROGRAM
ROUTINES FOR MODESRCH

MAIN FLOW

MAIN
WVGUID
PRFLIN
SETRH
NEOUT
NPOUT

ROOT-FINDER

FZERUS
NOMESH
FINDF
QUAD

F-FUNCTION

FCTVAL
FDFDT
FINAL
LAGRNG
RBARS
MDHNKL

FULL WAVE INTEGRATION

INTEG
INITLR
QUARTC
TMTRX
SMTRX
ENNU
DIFFEQ
ROUT

FREE SPACE INTEGRATION

FSINTG
MDHNKL

AUXILIARY ROUTINES

CASIN
CANG
XFER

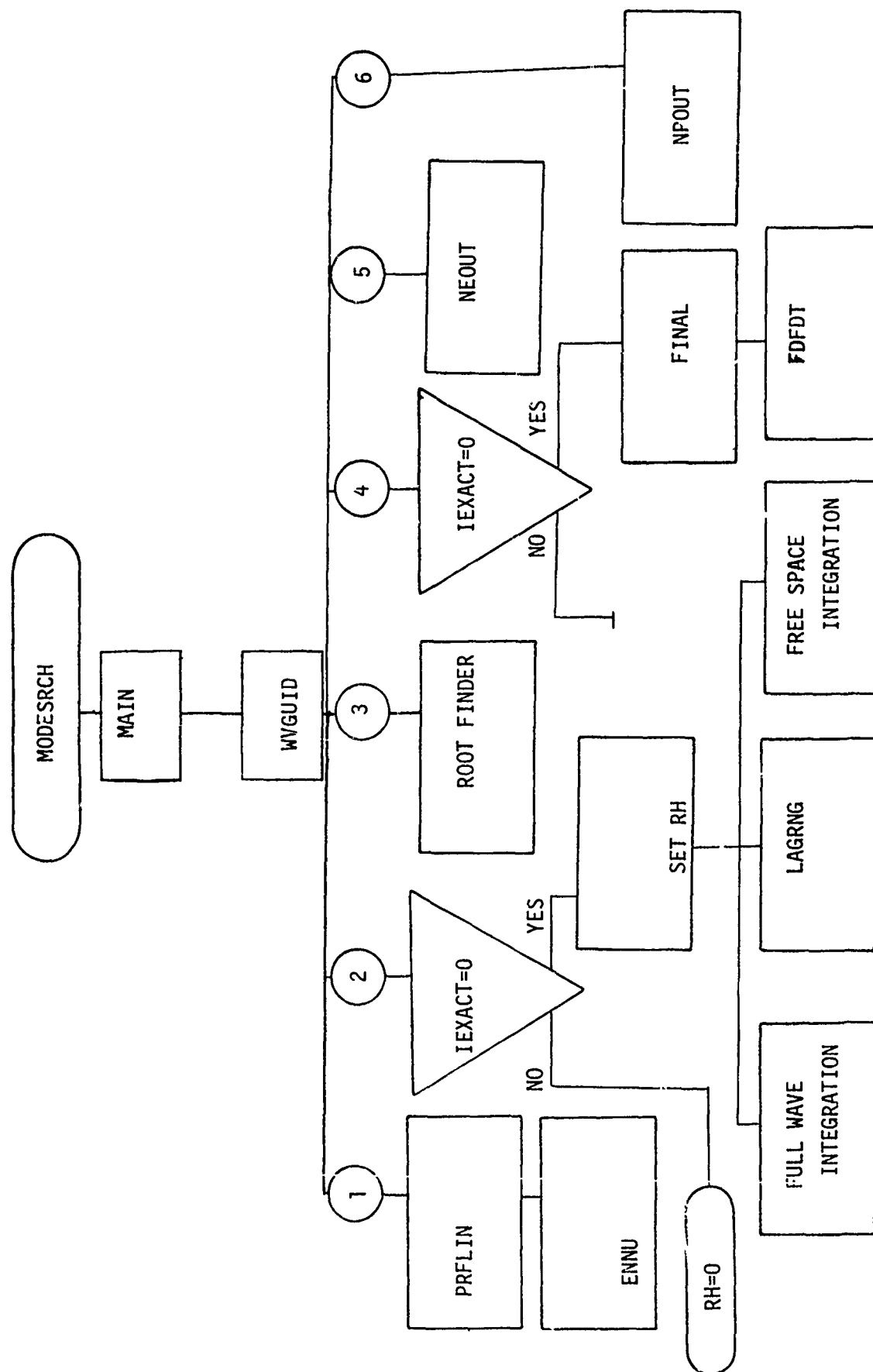


Figure 11. Structure of MODESRCH Computer Program.

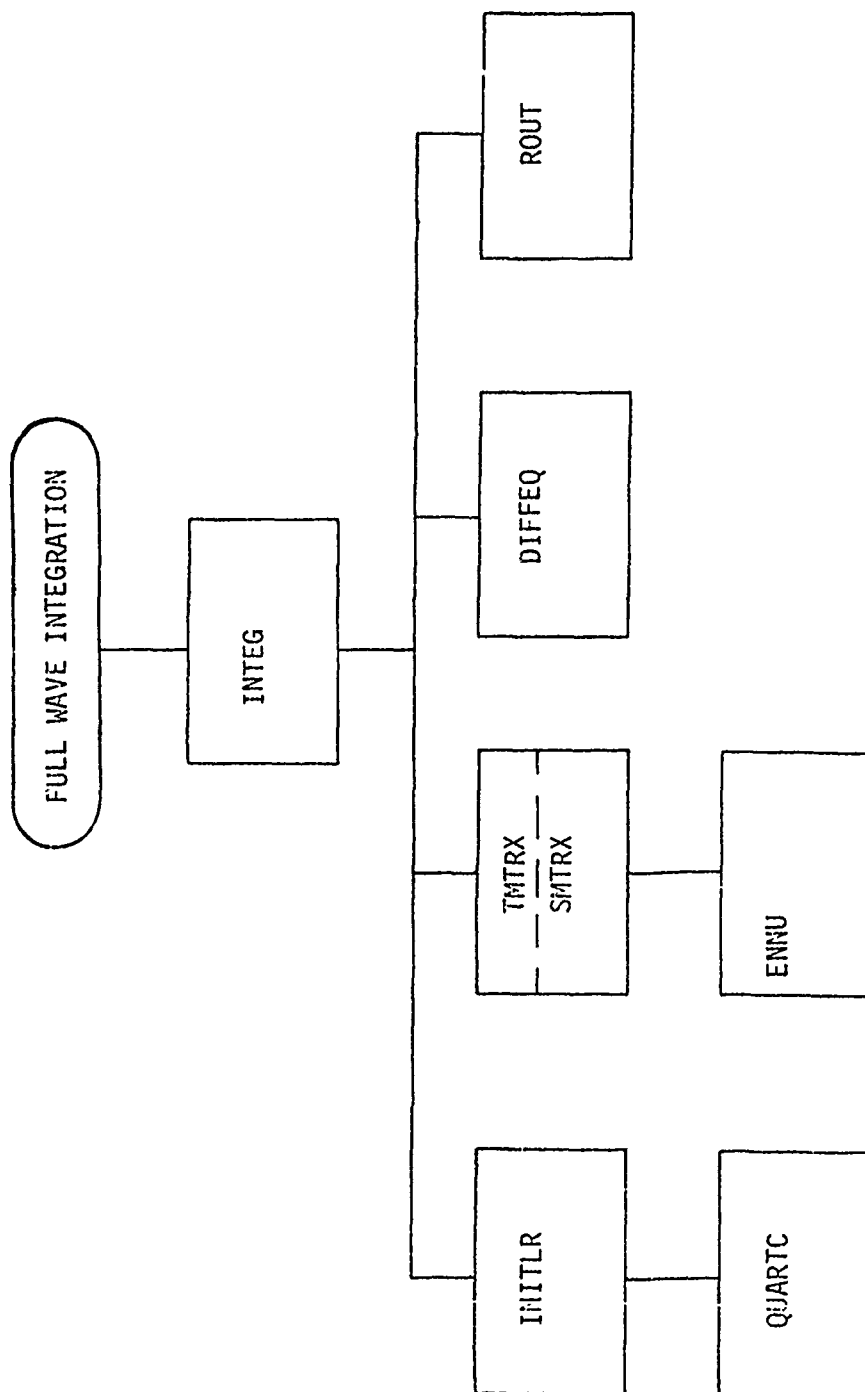


Figure 12. Structure of the Ionospheric Full-Wave Integration Procedure
for the MODESRCH Computer Program.

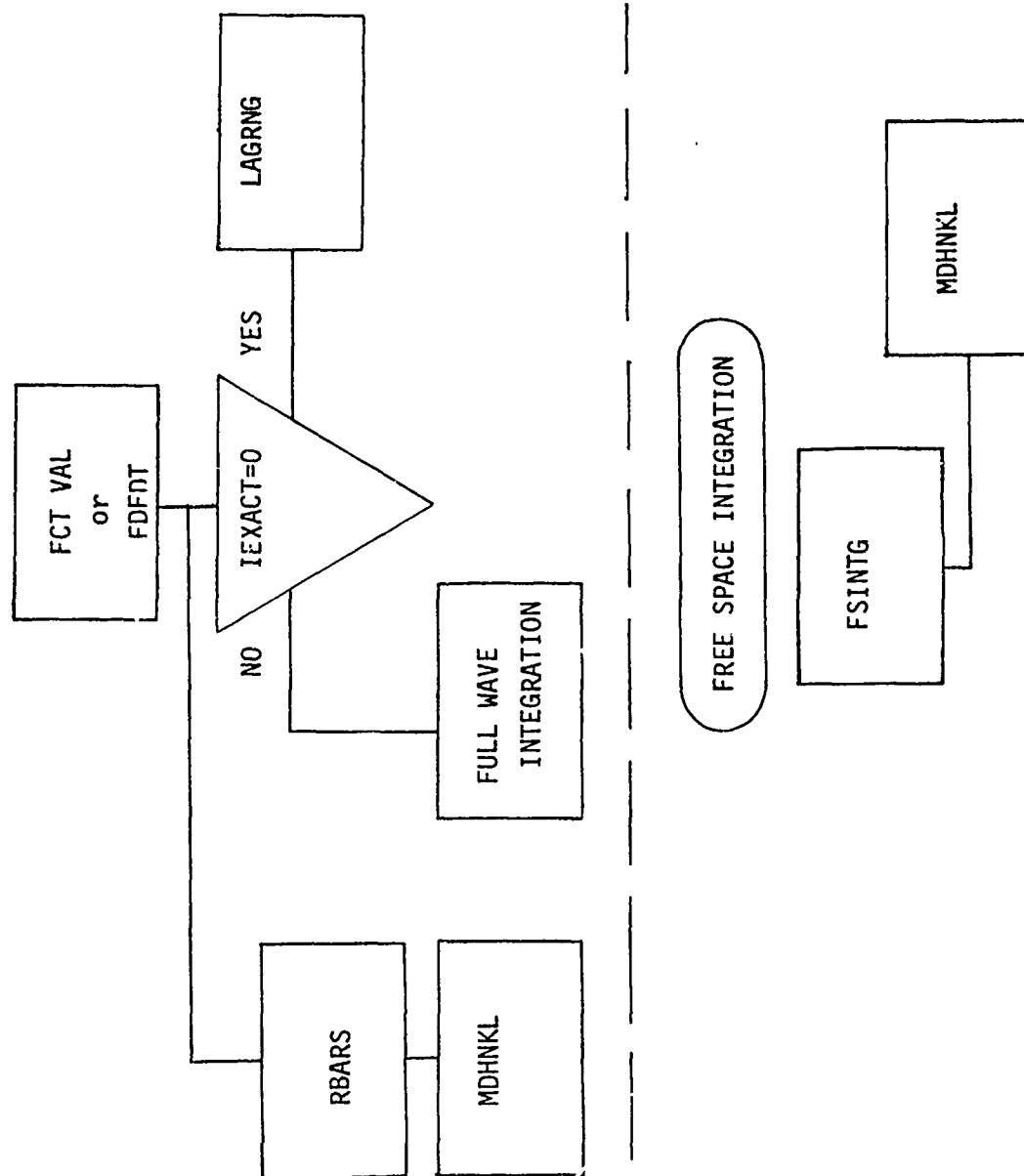


Figure 13. Structure of the Free Space Integration Procedure and Structure of the Function Value Routine and Derivative wrt θ of the Function Value Routine.

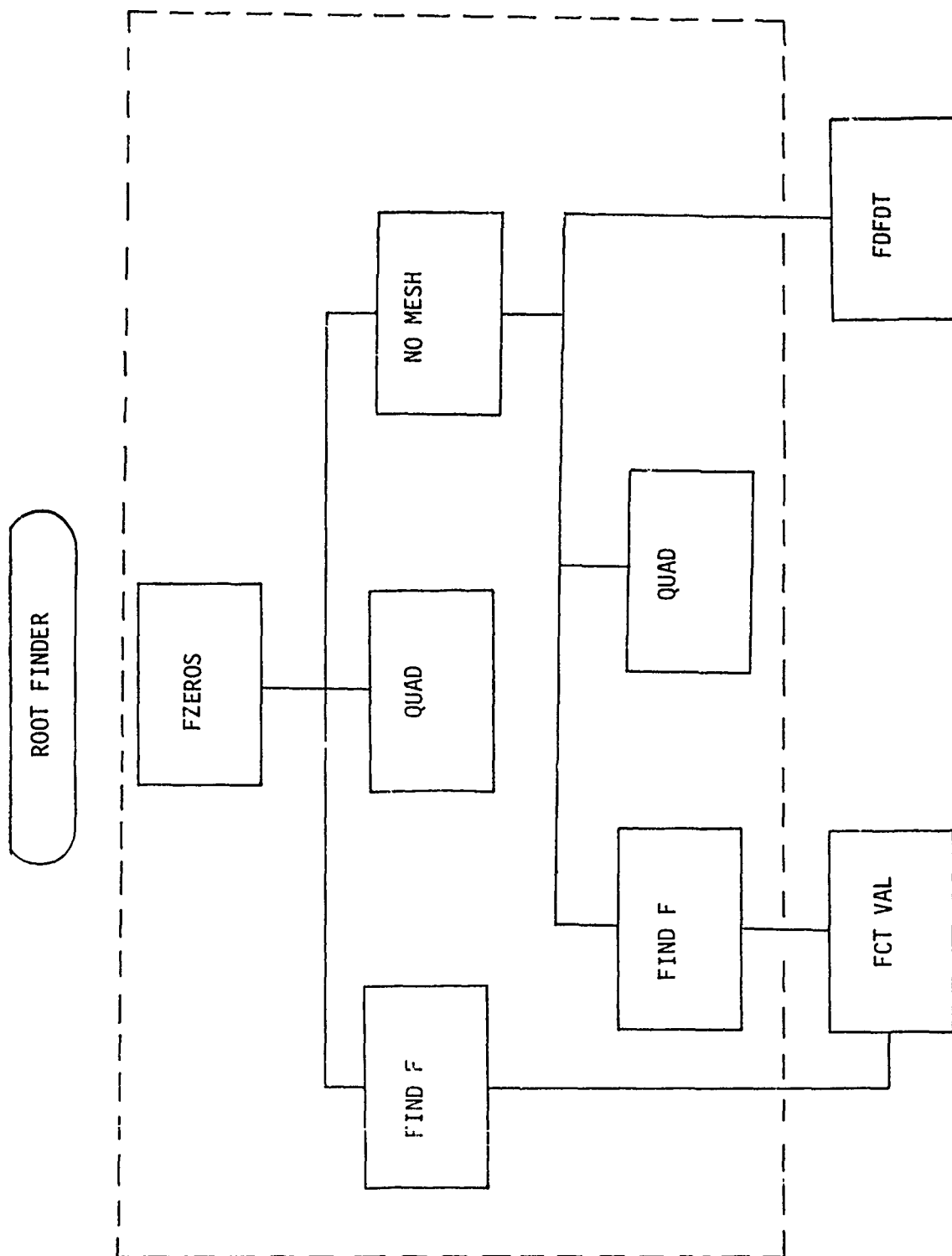


Figure 14. Structure of the Root Finder Procedure for the MODESRCH Computer Program.

(B) Computer Program Description

The function of each program subroutine is listed below:

(1) MAIN ROUTINE:

This routine controls the sequence of input and execution. It provides for NAMELIST input.

(2) SUBROUTINE PRFLIN:

This routine provides for input of electron density and collision frequency profile cards and for limiting the extent of the top and bottom of the electron density profile. Input of ion profiles is also provided for.

(3) SUBROUTINE WVGUID:

This routine controls the running sequence of the total computer program. Specific operations are to:

- (a) Initialize the full-wave solution of the ionospheric integration routines.
- (b) If the propagation frequency is greater than 1000 Hz and the variable 'IEXACT' is set equal to zero, then a reference height, (RK), is set through the routine SET RK. If either of the above is not true, the reference height is taken as zero.
- (c) Initialize the ground reflection coefficient equations.
- (d) Initialize the free-space integration calculations.
- (e) Find the zeros (i.e., eigenangles) of the function $F(\theta)$ (see Equation 7), through the routine FZEROS. Lagrange interpolation is used if the frequency is greater than 1000 Hz or if the variable IEXACT is greater than zero. Otherwise, full wave solutions of the ionospheric reflection coefficients are computed.

- (f) If the variable IEXACT is equal to zero or the frequency is greater than 1000 Hz, then the use of the routine FINAL is used to iterate on the eigenangle solutions of $F(\theta) = 0$ as the function is changed from that using Lagrange Interpolation to that using exact full-wave solutions of the ionospheric reflection coefficients.
- (g) Order the eigenangles according to the value at the real part of the eigenangles.
- (h) Compute and punch the eigenangles if the variable 'NEIGEN' is set equal to one. This is done through the routine NEOUT.
- (i) Compute and punch the mode constant parameters if the variable 'NPUNCH' is set greater than zero. This is done through the routine NPOUT.

(4) SUBROUTINE SETRH:

- (a) This routine selects the values at THETA, (θ) at which full-wave solutions of the ionospheric reflection coefficients are to be carried out for use in Lagrange interpolation.
- (b) This routine also selects the height, to the nearest whole kilometer, at which the function:

$$\sum_{\theta\text{-points}} \left(\sum_{R\text{-elements}} \left\{ \left| \frac{d(\frac{R+1}{c})}{d \cdot c} \right|^2 \right\} \right) \quad (31)$$

is minimized. In the above expression $C = \cos(\theta)$, R is the reflection coefficient matrix and the summations are over all θ -points at which full-wave solutions are

to be carried out for use in Lagrange interpolation. The summation is also over all elements of the R reflection matrix. Derivatives of the full-wave solutions are not computed; rather, derivatives of the Lagrange interpolation formula are used. Note that the full-wave solutions are carried out to the bottom of the profile, using the ionospheric reflection coefficient integration procedures (i.e., subroutine INTEG) after which, the full wave solution is then carried out in an upward direction through free space over a curved earth (by use of the free space integration routine FSINTG) to various heights which are increased successively by intervals of one kilometer. That height at which the minimum occurs is printed out as the 'REFLECTION HEIGHT' and is used; thereafter, as the Reference Height at which the function, $F(\theta)$ is to be computed.

(5) SUBROUTINE NEOUT:

This routine provides for punched card output of eigenangles.

(6) SUBROUTINE NPOUT:

This routine provides for computing and printing tables and punched card output associated with the 'NPUNCH' option.

(7) SUBROUTINE INTEG:

This subroutine performs an integration of the differential equations for the ionosphere reflection matrix using Runge-Kutta integration formulas. The integration variables are

the elements of the matrix $(R+1)/C$ where R is the reflection matrix described in Equation 2. If 'IDERIV' is set non-zero, the derivatives of the $(R+1)/C$ elements wrt $C = \cos(\text{THETA})$ are also integration variables. The set of heights at which pairs of integration steps begin and end is stored in the array 'RKHTS'. Sizes of the steps are determined by comparing the values of the elements of $(R+1)/C$ after a pair of steps and values of the same variables after a single double-size comparison step. The $(R+1)/C$ variables used in the comparison step are denoted as 'X'. If the difference is too large, the step size pairs are cut in half by adding a new height to the list in 'RKHTS'. In order to enhance stability in other parts of the program, no heights are deleted from the list unless the list is reset by a call to the SET RK subroutine.

(8) SUBROUTINE ENNU:

Computation of electron density and collision frequency (and ion density if appropriate) as a function of height. If profile was input from formatted cards, logarithmic interpolation is used. See also notes in subroutine PRFLIN.

(9) SUBROUTINE INITLR:

This routine computes the coefficients of the Booker Quartic equation and solves for its roots. The routine then computes the value of $(R+1)/C$. For reflection from a sharply bounded anisotropic ionosphere of semi-infinite extent where R is the reflection coefficient matrix. The solution is used as the initial condition for Runge-Kutta integration.

The solution is based on the material of reference 6. Derivatives wrt $C = \cos(\theta)$ are also computed if the variable 'IDERIV' is set non-zero.

(10) SUBROUTINE QUARTIC:

This routine obtains the solution for the roots of a fourth-order polynomial (QUARTIC Equation). A summary of pertinent equations is given in reference 6.

(11) SUBROUTINE TMTRX:

This routine computes the elements of the 'M' matrix of Equation 23. Those combinations of 'M' matrix elements used in the 'I' matrix which do not include use of θ are the final output of this routine. Computations of quantities which are not functions of height or of θ are carried out in the routine INIT T.

(12) SUBROUTINE SMTRX:

This routine computes the coefficients used in the differential equations for $(R+1)/C$. These are stored in common area 'S MTX' and are analogous to 'S' matrix elements given by Equation 20. Derivatives of these coefficients wrt $C = \cos(\theta)$ are also computed if 'IDERIV' is set non-zero.

(13) SUBROUTINE DIFFEQ:

This routine computes the derivative of the differential equations for $(R+1)/C$, where R is the ionospheric reflection matrix. The equations are derived from the differential equations for 'R' as given in reference 5. Storage into the array 'R MTRX' is made so that the equations may be used with either the main set of integration variables,

or with the comparison set, X. Differential equations for the derivatives of $(R+1)/C$ wrt $C=\cos(\theta)$ are also used if 'IDERIV' is set non-zero. Note that the variable called 'R' in this routine is actually $(R+1)/C$.

(14) SUBROUTINE RBARS:

This subroutine computes values of variables which may be used to form the elements of the RBAR matrix, where RBAR represents reflection of an ELM wave from the earth's surface. Namely, $RBAR11=DEN11/(C*NUM11-DEN11)$, or $(1.0/RBAR11+1.0)/C=NUM11/DEN11$, and $RBAR22=DEN22/(C*NUM22-DEN22)$, or $(1.0/RBAR22+1.0)/C=NUM22/DEN22$. Derivatives wrt $C=\cos(\theta)$ are also computed if 'IDERIV' is set non-zero. Note that the equations are formulated in such a way that a smooth transition is made from the 'CURVED EARTH' form to the 'FLAT EARTH' form. See also notes in subroutine MDHKNL regarding definitions of hankel function parameters. The value of earth's radius is such as to make $2.0/RE=3.14E-4$. Computation at $\theta=90^\circ$ is not excluded.

(15) SUBROUTINE MDHKNL:

This subroutine computes values of variables which may be combined to form modified hankel functions as described in 'TABLES OF THE MODIFIED HANKEL FUNCTIONS OF ORDER ONE THIRD AND OF THEIR DERIVATIVES', by the staff of the computation laboratory, Harvard University Press, 1945. The notation is such that the actual $H1=H1*EXP(-E)$. The actual $H2=H2*EXP(E)$. Actual derivatives wrt argument, Z, are $H1P*EXP(-E)$ and $H2P*EXP(E)$.

(16) SUBROUTINE FSINTG:

This routine performs an integration of the differential equations for the ionosphere reflection matrix through a free space region over a curved earth. The integration may be performed in either a positive or negative height direction, but in this program the integration is always upward. The integration variables are the elements of the matrix $(R+1)/C$ where R is the reflection matrix. The solution is based on Budden, reference 7, in particular the material on pp.118, 327-329, 336-338, and 343-345. If 'IDERIV' is set non-zero, the derivatives of $(R+1)/C$ elements wrt $C=\cos(\theta)$ are also integration variables. Computation at $\theta=90$ is not excluded. Also, the equations are formulated in such a way that a smooth transition is made from the 'CURVED EARTH' form to the 'FLAT EARTH' form for appropriate values of θ . The initial and final values of the integration variables are stored in the common area 'INTEGR'. This routine carries out the computation of height-gain coefficients Q, G, A, B for two conditions on the upgoing wave at each height $=Z_f$, namely $E_{11}=1, E_Y=0$ and $E_{11}=0, E_Y=1$. Each of the coefficients Q, G, A, B is divided by $C=\cos(\theta)$. Computation of the upgoing fields E_{11} and E_Y at height $=Z_t$ for the two conditions described above are also done. The integration proceeds from height Z_b to height Z_t when 'FSINTG' is called.

(17) SUBROUTINE LAGRNG:

This routine performs Lagrange interpolation of reflection coefficients in the COS (THETA) plane. Also to obtain derivatives of interpolated values WRT COS (THETA) the Lagrange interpolation formula itself is differentiated WRT COS (THETA).

(18) SUBROUTINE FINAL:

The routine for following the eigen-angles as the F function is changed from that using Lagrange interpolation to that using exact full-wave solution values of reflection coefficients.

(19) SUBROUTINE FZEROS: (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL, MPRINT,ZEROS,NR Z)

For Root-Finder FZEROS is a routine for finding the zeros of a complex function, F, which lie within a specified rectangular region of the complex (THETA) plane, provided the function has no poles in the vicinity of the rectangle.

EXPLANATION OF PARAMETERS--

- TLEFT - value of real part of THETA at left edge of rectangle.
- TRIGHT - value of real part of THETA at right edge of rectangle.
- TBOT - value of imag part of THETA at bottom edge of rectangle.
- TTOP - value of imag part of THETA at top edge of rectangle.

- TMESH - set equal to about half the average spacing between zeros within the rectangle. A smaller value may be used as a safety measure, but too small a value will result in excessive run time.
- TOL - tolerance to which zeros are to be found. If two zeros are closer than 'TOL', the root-finder will stop with an error message.
- MPRINT - normally set to zero. A non-zero value leads to print-out for debugging.
- ZEROS - output list of (complex) values of THETA at which zeros are found.
- NR Z - the number of zeros found.
- SUBROUTINES TO BE PROVIDED--
- FCT VAL - (THETA,F) - to return the value of the function, F, at the point in the complex plane specified by 'THETA'.
- F DFDT - (THETA,F,DFDT) > same as 'FCT VAL' except that the derivative, DFDT, of the function WRT THETA must also be returned.

(20) SUBROUTINE QUAD:

For Root-Finder finds the solution for the real roots of a Quadratic Equation of the form $A \cdot X^2 + 2.0 \cdot B \cdot X + C = 0.0$, where X is called 'SOL' in this routine. The number of real roots found is given by 'NR SOL'. A value of 1 for 'NR SOL' results from the Quadratic Equation approaching linearity. Used by subroutines FZEROS and NO MESH.

(21) SUBROUTINE NO MESH:

For Root-Finder the routine for finding exact (in the sense of NO MESH approximation) locations of zeros of the function, F , for which a complete, but approximate, set was found in subroutine FZEROS.

(22) SUBROUTINE FINDF:

This routine calls upon the routine FCT VAL to compute $F(\theta)$.

(23) SUBROUTINE FCT VAL:

This routine is for computing F function values at mesh points, called on principally by subroutines FZEROS and NO MESH. This is the modified F function which has no poles and which has no zeros at $\text{THETA}=90$.

(24) SUBROUTINE F DFDT:

This routine for computing function $F(\theta)$ values and their derivatives WRT THETA at arbitrary values of THETA . Called on principally by subroutines NO MESH and FINAL. This is the modified F function which has no poles and which has no zeros at $\text{THETA}=90$.

(25) SUBROUTINE XFER:

Routine for transferring one array into another.

(26) SUBROUTINE R OUT

Output of Runge-Kutta integration variables, used only for debugging.

(27) FUNCTION CASIN

Routine for computing the complex sine function of a complex argument.

(28) FUNCTION CANG:

Routine for computing the arc tangent of a complex angle.

V. RUNNING THE PROGRAM

A. Control Cards

These cards contain mnemonic words denoting the program operations to be executed. These words will be assumed to begin in column 1 of the card. In the text of this outline, these words will be enclosed in apostrophes. Blanks will be denoted by a space indicated by '_' to clarify positioning of blank columns. These cards are summarized in Table 4.

(1) 'PROFILE-i' initiates reading of the ionospheric profile. An alternative method for specifying an ionospheric profile is presented in Section B.

The value of i indicates the number of species to be used. It may have one of two possible values: 1 or 3. If the position for i is left blank, $i = 1$ is assumed.

The 'PROFILE_i' card must be followed by an alphanumeric card which may be used to identify the profile.

The profile cards contain the following parameters punched according to the format (F7.2,4X,5(1X,E9.2): height in kilometers and the specie densities in number per cubic centimeter. The first specie is electrons. The cards are input in order of descending altitude and the profile is terminated with a dummy height with value less than zero. A maximum of 101 profile cards may be used.

In the case of $i = 3$, the program assumes that the first two species read are electrons, N_e and singly charged positive ions, N_i^+ . The required third specie is thus singly charged negative ions, N_i^- , and their density is computed and printed by the program as $N_i^- = N_i^+ - N_e$.

In the integration of the ionospheric reflection elements through the ionosphere, the program interpolates exponentially between input values. Thus, for example, an exponential profile can be input with only the top and bottom densities specified.

(2) 'COLFREQ' initiates reading of an ionospheric collision frequency profile. An alternative form for specifying exponential collision frequencies is discussed in Section B. This profile, if used, must follow the 'PROFILE_i' input because i specifies the number of species.

The 'COLFREQ' card is immediately followed by the profile. The input format and order is the same as used for 'PROFILE_i', except that collision frequencies in collisions per second for electrons, and when i is greater than 1 for positive ions and negative ions, must be input. The profile is input in order of descending height and is terminated by a dummy height of value less than zero. A maximum of 50 collision frequency profile cards may be used. As with the ionospheric specie profiles, the program interpolates exponentially between input values and there need be no correspondence between the altitudes used to define the two profiles.

When 'COLFREQ' is used, the collision frequency profile can be over-ridden later in the job by the exponential collision frequency specification described in Section B-1. Note that in this case the exponential parameters, 'COEFNU' and 'EXPNU', must be input through NAMELIST.

(3) 'ID' indicates that the next card is identification for the data being run. This identifying card is merely read and printed. It may contain any alphanumeric information desired.

(4) '&DATUM' and '&END' initiate and terminate reading of Namelist. The form of this input is described below.

All cards appearing between '&DATUM' and '&END' must have a blank in column 1. In the sample data decks this will be indicated by '''.

Variable names are punched with no imbedded blanks. The name is followed by an equal sign (=) and the value of the variable punched in any valid format. Integer variables may only be input with integer numbers and real variables may be input with either integer numbers or floating point numbers. The value of the variable must be followed by a comma. All blanks in the values punched are treated as zeros. A comma is the only valid delimiter. For example, 'I=1' would result in I=10. If a variable name is repeated, the second value will be used.

If the variable defines an array, the array may be filled by a string of values separated by commas. There need not be as many values specified in the string as there are in the dimension of the array. The string of variables may be continued on additional cards by not repeating the variable name. Alternatively, the value of single elements may be specified by subscripting the variable name before the equal sign such as 'ARRAY(3) = -6.E3,'.

Complex variables must be input in order of real and imaginary parts. Arrays of complex variables are input the same as described above except that twice as many values are required. For example, 'CARRAY = 1., 3., -1., -3.,' is used to input CARRAY(1) = (1., 3.) and CARRAY(2) = (-1., -3.).

(5) 'QUIT' signals normal termination of a job. This is not required provided a /* card is used at the end of the input deck.

(6) A reasonable order of input is shown in Table 4.

TABLE 4

SUMMARY OF CONTROL CARDS

PROFILE_i	read ionospheric profiles (i=1 or 3)*
COLFREQ_i	read ionospheric collision frequency profiles
ID	read and print identification card(80 card columns)
_&DATUM	begin Namelist read
_&END	end Namelist read and begin Waveguide calculations
QUIT	end of job (optional)
/*	

*Note: If i not specified, 1 is assumed.

B. Input Variables(1) General Input variables needed for computations of the modal solutions.

'FREQ' is the frequency in kilohertz.

'MRATIO' is the ratio of the mass of the specie to the mass of an electron. Default values are for an electron, 'MRATIO = 1'. For positive or negative ions 'MRATIO = 58000'.

An exponential collision frequency may be specified for each specie. 'C0EFNU' is the collision frequency, in collisions per second, at the ground. 'EXPNU' is the height variation in inverse kilometers. The collision frequency at an altitude of z kilometers in this case is defined by:

$$v(z)_j = C0EFNU(j) * \exp(ENPNU(j) * z)$$

where j=1 is for electron-neutral particle collisions, j=2 is for positive ion-neutral particle collisions and j=3 is for negative ion-neutral particle collisions.

The parameter 'NUFLAG' allows for a choice of exponential collision frequency profiles to be used in the calculations. In this instance values are not assigned to COEFNU(j) and EXPNU(j); instead default values, which are built into the MODESRCH program are used in the calculations. With 'NUFLAG = 0', the values of the exponential parameters 'COEFNU' and 'EXPNU' (as described in Wait's TN 300, Reference 3) are used. If 'NUFLAG = 1', the values of these exponential parameters (as computed by Moler, Reference 13) are used. The special values are:

'NUFLAG = 0' (Wait):

COEFNU(1) = 1.816E+11, coll/sec

COEFNU(2) = 4.540E+9, coll/sec

COEFNU(3) = 4.540E+9, coll/sec

EXPNU(1) = -0.15 km⁻¹

EXPNU(2) = -0.15 km⁻¹

EXPNU(3) = -0.15 km⁻¹

'NUFLAG = 1' (Moler):

COEFNU(1) = 4.303E+11 coll/sec

COEFNU(2) = 1.076E+10 coll/sec

COEFNU(3) = 1.076E+10 coll/sec

EXPNU(1) = -0.1622 km⁻¹

EXPNU(2) = -0.1622 km⁻¹

EXPNU(3) = -0.1622 km⁻¹

If an exponential electron collision frequency profile is to be used other than those described by Wait or Moler, 'NUFLAG' must be set = 0 and the proper values assigned to 'COEFNU(j)' and 'EXPNU(j)'.

It is important to note that if the collision frequency profile is entered as tabulated values via the control card 'COLFREQ' and then in the same job step, additional calculations are wanted for new collision

frequency values, as input through the Namelist variables 'COEFNU(j)' and 'EXPNU(j)'; then, 'NUFLAG' must be set equal to zero and new values assigned to 'COEFNU(j)' and 'EXPNU(j)'.

An exception to this procedure occurs when the new values of 'COEFNU(j)' and 'EXPNU(j)' are those denoted as "Moler's values". In this case the parameter 'NUFLAG' is set equal to 1 and no values need be assigned to 'COEFNU(j)' or 'EXPNU(j)' in that this is done automatically within the program.

'RHO' is the distance in Mm from the transmitter. This variable is used only in punched output. It is useful to further identify punched data that is being generated along a propagation path.

The geomagnetic field is specified by three variables. 'AZIM' is the clockwise angle between magnetic north and the horizontal propagation direction in degrees east of north. 'CODIP' is the magnetic co-dip angle in degrees from the vertical down. The magnetic equator is specified by 'CODIP = 90'. 'MAGFLD' is the magnetic field intensity in webers per square meter.

The ground conditions are specified by two variables. 'SIGMA' is the conductivity in mhos per meter and 'EPSR' is the relative dielectric constant.'

'H' is the height in kilometers at which the modified refractive index is unity. This is the height to which the eigenangles are referred.

'REFLHT' is the height in kilometers which is used to normalize the excitation factor defined by Wait. 'REFLHT' must not be zero.

'GMAX' is the maximum separation (in degrees) of the theta angles at which full wave solutions, to be used in Lagrange interpolation, are carried out.

'SEP' is a real variable which defines the minimum separation between eigen solutions.

'RTOL' is the variable which determines the precision of the Runge-Kutta integration for the ionospheric reflection elements.

'NPUNCH' determines the specific parameters to be punched on output cards.

'CARDPN' is an option for punching or not punching on cards the outputs from the 'NPUNCH' option.

'CARDPT' is an option for printing, or not printing, the values computed using the 'NPUNCH' option.

'NEIGEN' is the option for punching eigenangles on the cards.

'TALT' is the height of the transmitting antenna in km.

'RALT' is the height of the receiving antenna in km.

Note: Input of TALT and RALT is only applicable to the calculation of modal excitation factors as described in Section V-C-2. Computed output values, as a function of 'RALT' and 'TALT', are obtained only through the option NPUNCH = 7.

'TMESH' is the angular dimension of the mesh squares contained in the contour rectangle. This value is in degrees. 'TMESH' should always be smaller than the average separation of the eigenangles. Also 'TMESH' should be chosen smaller than the separation between the values of 'RANGER' (maximum) and 'RANGER' (minimum) or the values of 'RANGEI' (maximum) and 'RANGEI' (minimum). Unless 'TMESH' is assigned a value via NAMELIST, it is computed within the program as:

$$TMESH = \sqrt{3.75/freq_{kHz}} \quad (132)$$

'LUB' is a real variable used to terminate the iterative process.

'LUB' is used to test the iterative change in the magnitude of the complex

eigenangles. The iteration is stopped when the change in the magnitude of the complex eigenangle is less than or equal to 'LUB'. Unless 'LUB' is assigned a value via NAMELIST, it is computed within the program as:

$$LUB = \sqrt{15.0/\text{freq}_{\text{kHz}}} \cdot (0.01) \quad (133)$$

'IEXACT' is an option that determines whether Lagrangian interpolation or the "full wave" solution is used in initial stages of the program. IEXACT = 0 is for Lagrangian. IEXACT = 1 is for "full wave".

(2) Exponential profiles (electrons only)

An exponential, electrons only, ionospheric profile may be specified. The profile is computed as described in Wait's NBS TN 300, Reference 3, where

$$\omega_r(z) = 2.5 \times 10^5 \exp[\beta(z-h')] = 3.18254 \times 10^9 N(z)/\nu(z) \quad (11)$$

and β is in inverse kilometers, z and h' are in kilometers, $N(z)$ is the electron density in electrons per cubic centimeter and $\nu(z)$ is the collision frequency in collisions per second. The values used to compute $\nu(z)$ are 'COEFNU(1)' and 'EXPNU(1)' and are given in Section V-B-1. Note that the term 'BETA' is β and 'HPRIME' is h' .

The equation for computing the electron density ($N(z)$, electrons/cm³) at height z is given by:

$$N(z) = 7.8554 \times 10^{-5} \cdot \text{COEFNU}(1) \cdot [\exp(((\text{BETA} + \text{EXPNU}(1))z) - \text{BETA} \cdot \text{HPRIME})] \quad (134)$$

'SCLHTS' is a real variable which indicates the number of scale heights above h' which will be used to denote the top of the profile when the β, h' exponential electron density profile is used.

The top of the profile may be modified by choosing a value for the variable SCLHTS. The relationship is that:

$$\text{Top Height} = \text{Integer} \left(\text{HPRIME} + \frac{\text{SCLHTS}}{\text{BETA}} + 0.5 \right) \quad (135)$$

The nearest integral value of altitude as given by 'Top Height' is used to set the top of the profile. All heights above this value are neglected in the MODESRCH computations.

The bottom of the exponential profile is set by assigning a minimum value (i.e. ENMIN) of electron density in electrons/cm³. All altitudes (z), at which the corresponding density is less than 'ENMIN' will be neglected.

(3) VLF/LF - Tabular Electron-Ion Density Profiles.

Computing efficiency for VLF and LF may be improved by cutting off the top and bottom heights of the input ionospheric profile.

When the particle density profiles are input to the program via 'PROFILE_i' (i = 1 implies electrons only) (i = 3 implies electrons and ions), the top and bottom of the profile may be cut off using the criteria, as presented in Section II, D-2. In that section it is stated that the top part of the input ionospheric profile which may be neglected in the calculations is defined as that altitude(z) above which:

$$B(z) > \text{Maximum} \left(\text{TEMPO-B}, 20 \frac{\omega}{v_e} \right) \quad (17)$$

where 'TEMPO-B' is assigned a value (e.g. 2.0) and B(z) is given by eq. 16. Also an additional requirement for cutting off the top of the profile is that the electron density must be as great as 'TOPEN'. The variable 'TOPEN' is assigned a value (e.g. 1000 electrons/cm²). The top of the profile is taken to be the higher of the two heights corresponding to B(z), (equation 17), and to 'TOPEN'.

The bottom of the input ionospheric profile is neglected (for VLF/LF cases) at those altitudes for which the computed value of B in equation 16 is less than 'CUTOFF'. The variable 'CUTOFF' is assigned a value (e.g. 0.0001). The relationship for Cutoff is:

$$B(z) < \text{CUTOFF} \quad (18)$$

If it is not desired to cut off the profile bottom for certain calculations then the Namelist parameter 'ICUT' must be set equal to zero.

(4) ELF Tabular Electron-Ion Density Profiles

The same criteria for the variable 'PROFILE_i' as was presented for the VLF/LF case is applicable to ELF, (i.e. i = 1 or blank implies electrons only and i = 3 corresponds to electrons and ions).

In the case of ELF the profile top and bottom are not cut off and the Namelist variable 'ICUT' is not applicable.

(5) Namelist Variables with Initial Values.

Initial values of the Namelist variables are presented in Table 5.

TABLE 5
NAMELIST VARIABLES AND INITIAL VALUES

<u>NAME</u>	<u>VALUE</u>	<u>UNITS</u>	<u>NAME</u>	<u>VALUE</u>	<u>UNITS</u>
NPROF	None	-	ICUT	1	
FREQ	0.0	kHz	NUFLAG	0	
H	50.0	km	COEFNU(3)	1.816+11, 4.540E9, 4.540E9	cell/sec " "
REFLHT	50.0	km	EXPNU(3)	-0.15, -0.15, -0.15	km ⁻¹ "
RHO	0.0	Mm			
AZIM	None	degrees	SEP	0.1	degrees
CODIP	None	degrees	RTOL	0.001	degrees
MAGFLD	None	WEBERS/m ²	GMAX	5.0	degrees
SIGMA	4.64	mho/m			
EPSR	81.0	-			
BETA	None	km ⁻¹	TOPEN	1000.	electrons/cm ³
HPRIME	-99.0	km	ENMIN	0.1	electrons/cm ³
SCLHTS	3		RANGER	None	degrees
MRATIO(3)	1,58000.0 58000.0	-	RANGEI	None	degrees
			TEMPOB	2.0	-
CUTOFF	0.0001	-	TALT	0.0	km
NEIGEN	0	-	RALT	0.0	km
NPUNCH	1	-	IEXACT	0	-
CARDPT	0	-	TMESH	-1.0	degrees
CARDPN	1	-	LUB	-1.0	-

Notes: Numbers in parenthesis after Name indicate dimensions.
Lack of decimal point indicates integer variables.

(6) The Eigenangle (θ)

The modal solutions of equation 1, which are denoted as complex eigenangles ($\theta = \theta_{\text{real}} + i\theta_{\text{imaginary}}$), are determined within a bounded region of the complex "THETA" plane defined by a set of limiting THETA values. This bounded region will be called the "contour rectangle" and the limiting THETA values are denoted by "RANGER" (minimum and maximum) and 'RANGEI' (maximum and minimum). These parameters are portrayed in Figure 15.

Where Point:

- A: RANGER (minimum), RANGEI (maximum)
- B: RANGER (maximum), RANGEI (maximum)
- C: RANGER (minimum), RANGEI (minimum)
- D: RANGER (maximum), RANGEI (minimum)

The small squares inside the "contour rectangle" are called the "Mesh Squares". 'RANGER' is the real part in degrees of the minimum and maximum values assigned to the contour rectangle.

'RANGEI' is the imaginary part in degrees of the minimum and maximum values assigned to the contour rectangle.

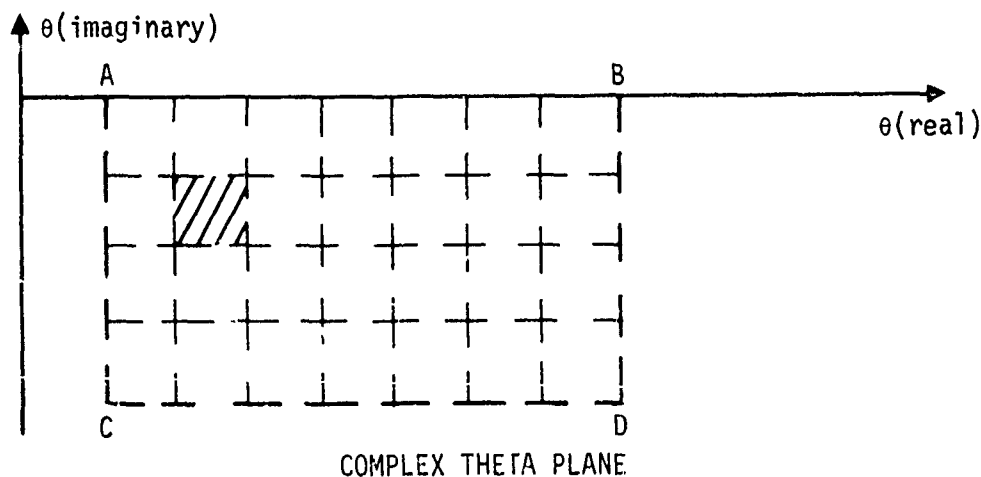


Figure 15. Showing the Contour Rectangle ABCD and Mesh Squares

i.e. $RANGER = \text{Minimum}, \text{maximum},$

$RANGEI = \text{Maximum}, \text{minimum}$

Note: The minimum value must be given before the maximum value for RANGER while the maximum value must be given first for RANGEI.

During program execution, the contour rectangle, as described by RANGER and RANGEI, is automatically divided into small sub-rectangles. This is done to increase computation efficiency. The division process is applied only to the RANGER values while the RANGEI values remain the same as those input through NAMELIST.

The algorithm is that the number of segments 'NS' into which the original contour rectangle is divided is given by:

$$NS = \text{TRUNCATION} \left\{ \left[\frac{RANGER(\text{max}) - RANGER(\text{min}) - LUB}{GMAX} \right] + 1 \right\} \quad (136)$$

The number of degrees in each segment is given by:

$$\Delta^\circ = \frac{RANGER(\text{max}) - RANGER(\text{min})}{NS}$$

C. Program Outputs

For purposes of clarity in describing program outputs, the following symbols will be used:

k = wave number in km^{-1}

h_r = reference height for Wait's excitation factor (REFLHT)

i = $\sqrt{-1}$

θ = (θ_r, θ_i) = final mode solution referenced to H

θ' = (θ'_r, θ'_i) = θ referenced to the ground

α = $-8686 k \text{Im}(\sin\theta')$ = attenuation rate in dB/Mm

v/c = $1/\text{Re}(\sin\theta')$ = phase velocity/speed of light

Note: The above procedure for sub-dividing the RANGER value is done only for VLF/LF calculations and is not applied for calculations at ELF.

λ_{mn} = excitation factor

X_{mn} = modified excitation factor

Λ_{mn} = normalized excitation factor = $-.5ikh_r X_{mn}$

The terms, X_{mn} , Λ_{mn} and λ_{mn} , are elements of 3 by 3 complex matrices. The individual elements are subscripted first by $m = V, E$ or B to specify a vertical, end fire or broadside dipole and second by $n = Z, Y$ or X to specify the field component excited. Thus, X_{VZ} is the z component of the electric field excited by a vertical dipole source. The terms X_{mn} and Λ_{mn} are also functions of the transmitter and receiver altitudes (TALT and RALT). These are the excitation factors mentioned in Section II, H. The terms X_0 and Λ_0 specify X_{VZ} and Λ_{VZ} for both transmitter and receiver at the ground. In both printed and punched output, each X_0 , X_{mn} and λ_{mn} is given as magnitude and phase in radians and each Λ_0 and Λ_{mn} is given as $20 \log$ (magnitude) and phase in radians. Note that in the MODESRCH computer program all the λ_{mn} terms are used whereas only X_{VZ} , X_{EZ} , X_{BZ} , X_0 and Λ_0 of the other excitation terms are utilized. The output from the computer program consists of the following:

(1) Computer Listing Output"

- (a) ID Information
- (b) PROFILE information
- (c) NAMELIST information
- (d) Values of 'TMESH' and 'LUB'

Also, heights and electron densities of top and bottom of profile.

- (e) Value of TEMPO-B at top of the profile, value of Wait's OMEGA-R at top of the profile or height of the profile where Wait's $\text{OMEGA-R} = 2.5 \times 10^5$

- (f) Reflection height -- height at which ionospheric reflection coefficients are interpolated and the height at which the ground reflection coefficients are computed (e.g., the height where $F(\theta)$ is computed).
 - (g) Theta range for each 'contour rectangle'.
 - (h) H: the height at which the eigenangle solutions are computed
 - (i) Mode No. n
 - (j) THETA (θ_r, θ_i) The final mode solution referenced to H (θ_r and θ_i are in degrees)
 - (k) $\alpha(\theta'_n)$: Attenuation rate dB/Mm where $\alpha = -8686 \cdot K \cdot \text{Imag}(\text{SIN } \theta')$ and where K is the wave number in km^{-1}
 - (l) $V/C(\theta'_n)$: The phase velocity/speed of light
- $$V/C = \frac{1}{\text{Real}(\text{SIN } \theta')}$$
- (m) Magnitude and Phase of the modal excitation factor as defined by Wait. ($\Lambda_0(\text{dB}), \phi_{\Lambda_0}(\text{radians})$)
 - (n) THETAP (θ'_r, θ'_i) The θ eigenangle referenced to the ground (θ'_r, θ'_i are in degrees)
 - (o) Polarization mixing ratio:

$$\rho = \left| \frac{(1 - \bar{I}R_L \ IR_L)}{(\parallel R_L \ \bar{I}R_L)} \right| = \left| \frac{(\bar{I}R_{\parallel} \ \parallel R_{\parallel})}{(1 - \parallel R_{\parallel} \ \bar{I}R_{\parallel})} \right| \quad (138)$$

When NEIGEN = 1 that ionospheric height where the electron density is equal either to the value of the input variable ENMIN (for β, h' electron-density profiles) or to the lowest height of the input profile (for tabular electron density profiles) is printed. This parameter is

denoted as "D" and corresponds to the variable "D" in the reference 1 WAVEGUIDE program where D is defined as the height in kilometers below which ionospheric integration is terminated.

Also when NEIGEN = 1, individual eigenangles (referenced to the height H) are printed.

If NEIGEN = 0, the above variables are not printed.

(2) Punched Output

When NEIGEN = 1 the value of "D" as described above is punched and also the individual eigenangle solutions are punched. These parameters are punched in the format of usual NAMELIST variables. When NEIGEN = 0, these variables are not punched.

Additional punched output from the program is obtained with the variable NPUNCH.

NPUNCH options for obtaining output cards:

NPUNCH=1*** gives output cards for horizontally homogeneous mode-sum or WKB-sum in terms of (T'S).

NPUNCH=2*** gives output cards in terms of SNVLF output.

NPUNCH=7*** gives output cards for input into ELF-WKB mode-sums.

NPUNCH=8*** gives output cards for input to mode conversion in terms of (FOVR).

NPUNCH=9*** gives output cards for input into mode conversion in terms of both (T'S) and (FOVR).

NPUNCH=0*** the above variables are not computed.

CARDPN=0*** gives usual mode summary print out but no cards.

CARDPN=1*** gives usual mode summary print and cards.

CARDPT=0*** causes the output cards to not be listed.

CARDPT=1*** causes the output cards to be listed.

When NPUNCH equals 1, 2, 7, 8, or 9, each entry into '&DATUM' results in a card containing RHO, FREQ, AZIM, CODIP, MAGFLD, SIGMA and EPSR in the format:

('R',F7.3,' F',F8.4', A',F8.3,' C',F8.3,' M',E10.3,' S',E10.3,'
E',F5.1)

and each exit via '&END' results in a blank card. In other program descriptions, the first card is referred to as the RFACMSE header card.

NPUNCH = 1 generates two cards per mode in the order

(1,θ',T₁,T₂) and (2,θ',T₃,T₄)

with format (I1,2F9.5,4E15.8).

Where:

$$T_1 = \frac{S^{\frac{1}{2}} (1 + \bar{R}_{\parallel})^2 (1 - \bar{R}_{\perp} \bar{R}_{\parallel})}{\left. \frac{\partial F}{\partial \theta} \right|_{\theta=\theta_n} \bar{R}_{\parallel} D_{11}}$$

$$T_2 = \frac{S^{\frac{1}{2}} (1 + \bar{R}_{\perp})^2 (1 - \bar{R}_{\parallel} \bar{R}_{\parallel})}{\left. \frac{\partial F}{\partial \theta} \right|_{\theta=\theta_n} \bar{R}_{\perp} D_{22}}$$

$$T_3 = \frac{S^{\frac{1}{2}} (1 + \bar{R}_{\parallel}) (1 + \bar{R}_{\perp}) \bar{R}_{\perp}}{\left. \frac{\partial F}{\partial \theta} \right|_{\theta=\theta_n} D_{12}}$$

$$T_4 = \frac{\bar{R}_{\parallel}}{\bar{R}_{\perp}} \quad (139)$$

θ' = complex eigenangle referenced to the ground and is punched as θ_r (degrees) and θ_i (degrees). Examination of Table 1 and equation 139 shows that the excitation " λ " values can be obtained from the "T" values. The "T" values are punched as complex numbers.

NPUNCH = 2 generates a single card per mode containing $(\alpha, V/C, |\Lambda_0|, \phi_{\Lambda_0})$ with format (4F10.5). Here Λ_0 is in dB above one $\mu V/\text{meter}$ and ϕ_{Λ_0} is in radians. This format is denoted as SNVLF.

NPUNCH = 7 generates two cards per mode. The first card contains θ' with a comma after the real and imaginary parts. Here θ_r and θ_i are in degrees. The second card contains X_{VZ}, X_{BZ} and X_{EZ} for transmitter and receiver heights of RALT and TALT. There is a comma after each term. This form of output is suitable for a Namelist type input.

These values are punched as magnitude and phase of the complex excitation number. Two comments which are pertinent to the NPUNCH = 7 output are:

- (a) The values of the X_{mz} 's are computed as functions of the transmitter and receiver heights (TALT and RALT).
- (b) If the punched output is to be used as input in the WDB-ELF Field Strength computer program (reference 14), then the NAMELIST terms TALT and RALT, input to the MODESRCH program must be zero for compatibility.

NPUNCH = 8 generates one card per mode. The card contains the variables $(\theta', |X_0|, \phi_{X_0}, \text{FOVR}, \text{HOFWR})$ using the format (0P2F9.5, 1PE14.5, 0PF9.5, 1P2E16.8, 0PF7.2). Here θ' is punched as θ_r and θ_i each in degrees. ϕ_{X_0} is in radians and 'FOVR' is a complex number. The variable 'HOFWR' is the profile height (km) which corresponds to a value of Wait's $\text{OMEGA-R} = 2.5 \times 10^{+5}$.

D. Examples of Program Inputs and Outputs

Example I illustrates some of the options for data input. This example is for an exponential electron density profile defined by 'BETA' and 'HPRIME' parameters. In this case the exponential collision frequency profile, corresponding to that described by Wait, (Reference 3), is included by setting the 'NUFLAG' variable to zero. Also, putting NPUNCH = 1 or greater causes the mode constants to be printed in the output listing and if 'CARDPN=1', (which is the default value) functions of the excitation factors are punched. The value NEIGEN = 1 gives both a listing and punched cards for the eigenangles at a height of 50 km. The printed output is also shown in example I.

Example II lists the output cards as computed from 'NPUNCH = 1' as presented in example I. Note that the first card gives the values for RHO, FREQ, AZIM, CODIP, MAGFLD, SIGMA and EPSR as used in the modal calculations. The card will be denoted as the RFACMSE card. Also the complex eigenangles at the ground are also punched on the cards as θ_r^i and θ_i^i . There are two cards per mode corresponding to $T_1(\text{Real})$, $T_1(\text{Imaginary})$ and $T_2(\text{Real})$ and $T_2(\text{Imaginary})$ on card No. 1. Card No. 2 contains $T_3(\text{Real})$ and $T_3(\text{Imaginary})$, $T_4(\text{Real})$, and $T_4(\text{Imaginary})$.

Where 'NEIGEN = 1' as in example I, the values of the eigenangles at a height of $H = 50$ km are also punched one per card as $\theta(\text{real})$ and $\theta(\text{imaginary})$. The value of D as punched corresponds to the bottom of the electron density profile.

Example III lists the punched card output obtained from the input parameters of example I when 'NPUNCH = 2'. Here the first card is the usual RFACMSE card. The second card gives the propagation frequency. The following cards give, for each mode, the values of the attenuation

rate (dB/Mm); the ratio V/C ; the magnitude of Wait's excitation factor (dB); and the phase of Wait's excitation factor (radians).

Example IV lists the punched card output obtained from the input parameters of example I where 'NPUNCH = 8'. This output is suitable for input into the mode conversion computer program described in reference 12. The first output card is the RFACMSE card. The following cards give for each mode the values of θ' (real) (degrees), θ' (imaginary) (degrees); the excitation component from a vertical exciter for the vertical electric field at $z=0$, (magnitude (not dB) and phase (radians)); the real and imaginary parts of the complex number 'FOVR'; and the height, in km, used for mode conversion, 'HOFWR'.

Example V presents input data where the electron density and positive ion density are entered in tabular form. Note that in this instance the collision frequencies for electrons, positive and negative ions are all entered automatically within the program through the variable 'NUFLAG = 1'. This corresponds to the use of these terms as computed by Møller (reference 13).

Example VI shows the printed output obtained from the input of example V. Note that for this case the profile top is not cutoff whereas the bottom is cutoff through the variable 'CUTOFF'.

Example VII lists the punched card output obtained from example V for NPUNCH = 9. Note that this punch option gives a combination of NPUNCH = 1 and NPUNCH = 8. There are three cards per mode.

Example VIII presents input data similar to example V. In this case the variable 'TOPEX' has been set to a small number so as to illustrate the output messages obtained when the top of the profile is cutoff.

Example IX shows the input when both particle density (electrons and positive ions) and collision frequency profiles (electron, positive ions and negative ions) are input in tabular form. This procedure will generally be the case for computations at ELF.

Example X illustrates the usual printed output obtained for NPUNCH greater than 0. Note that since this is an ELF case only one mode is found.

Example XI lists the output obtained for NPUNCH = 7. This output is compatible with a program which computes WKB-Fields at ELF (reference 14). Note however in this regard the NAMELIST variables 'TALT' and 'RALT' must both be zero. The punched card output for NPUNCH = 7 contains the following: the RFACMSE card; a card with θ_r' (degrees) and θ_i' (degrees); and a single card with the magnitude (not dB) and phase (radians) of the vertical excitation factors for vertical, end-on and broadside exciters.

Example XII shows the output obtained at 10 kHz for propagation over sea water at right. Whereas Examples XIII and XIV illustrate the results for propagation for the same conditions at 60 kHz. Note that considerably more modes are required to compute the total field at 60 kHz than at 10 kHz.

&DATUM
 BETA=0.5,HPRIME=87.0,SCLHTS=6.0,ENMIN=0.1,FREQ=27.0,
 NUFLAG=0,NPUNCH=1,RANGEK=70.0,87.0,RANGEI=0.0,-1.0,
 AZIM=283.0,CUDIP=21.0,MAGFLD=5.44E-5,EPSK=15.0,SIGMA=1.0E-2,
 &END

INPUT:

IMESH = 0.373 LUB = 0.0075

PRINTED OUTPUT:

COMPUTED PROFILE PARAMETERS:

H	NE	X	Y	Z	WR
99.0	2.045E 03	6.454E 04	-5.640E 01	3.805E-01	1.009E 08
70.6	1.000E-01	4.545E 00	-5.640E 01	2.678E-01	7.005E 01

OMEGA-R = 2.5E05 AT TOP HEIGHT = 87.00

REFLECTION HT SET TO	THETA RANGE =
90.64 KM	82.750 TO 87.000
90.64 KM	78.500 TO 82.750
90.64 KM	74.250 TO 78.500
91.64 KM	70.000 TO 74.250

MODE	H = 50.00	THETA	ATTEN	VUVERC	WAIT MAG	WAIT ANG	THETAP	POL
1	65.889	-0.342	2.117	0.99469	-25.537	0.188	89.762	2.29915
2	63.364	-0.167	1.664	0.99884	-9.550	-0.448	89.002	0.83315
3	62.750	-0.214	2.335	1.00014	-0.858	0.256	88.555	0.76029
4	60.304	-0.214	4.265	1.00051	-6.558	0.256	88.555	0.76029
5	79.520	-0.293	3.528	1.00897	-9.021	-0.396	83.703	1.15905
6	76.820	-0.249	5.300	1.01898	-11.287	0.508	82.346	0.78515
7	76.005	-0.349	7.298	1.02448	-17.501	-0.721	77.956	0.76962
8	73.277	-0.224	5.580	1.03593	-17.624	-0.407	74.658	1.28896
9	72.312	-0.507	13.330	1.04134	-5.372	0.236	73.792	0.50270
10								2.03806

MODE	H = 70.64	THETA	ATTEN	VUVERC
1	85.889	-0.342	2.117	0.99469
2	83.364	-0.167	1.664	0.99884
3	82.750	-0.214	2.335	1.00014
4	80.304	-0.214	4.265	1.00051
5	79.520	-0.293	3.528	1.00897
6	76.820	-0.249	5.300	1.01898
7	76.005	-0.349	7.298	1.02448
8	73.277	-0.224	5.580	1.03593
9	72.312	-0.507	13.330	1.04134
10				

EXAMPLE I

TIME REQUIRED FOR THIS PROFILE = 83.20 SECONDS

[illegible]

```

D = 70.64,
EIGEN =
35.869,
33.364,
82.750,
82.750,
83.304,
73.520,
73.820,
76.005,
73.277,
72.312,
3.0,
-6.342,
-0.167,
-0.214,
-0.214,
-0.293,
-0.224,
-0.269,
-0.349,
-0.224,
-0.507,
0.0,

```

7

12

71

14

①

○

α	v/c	$1/\Lambda_0$	Φ_{Λ_0}
27.000	0.99489	-25.53743	0.18805
2.11708	0.99334	-5.54964	-0.44820
1.06405	1.00014	-6.85628	0.25077
2.33522	1.00014	-6.85628	0.25077
2.33522	1.00014	-6.85628	0.25077
4.252649	1.00051	-6.43642	-0.39581
3.522649	1.00897	-9.02146	-0.50844
2.99503	1.01898	-11.28732	-0.72139
7.29815	1.02248	-6.50111	0.40091
5.38048	1.03395	-17.02431	-1.09743
13.32971	1.04154	-5.37150	0.23563

EXAMPLE III

[illegible]

EXAMPLE IV

PROFILE 3	NFLCNUCL	NITE	40000	SEC	6.00 MM	DNA NIGHTTIME
100.00	7.74E	02	1.04E	03		
95.00	3.14E	02	6.05E	02		
90.00	3.69E	02	4.03E	02		
85.00	1.50E	02	3.15E	02		
80.00	5.10E	01	4.47E	02		
75.00	3.80E	00	4.50E	02		
70.00	3.40E	02	5.65E	02		
65.00	1.77E	03	7.05E	03		
60.00	3.40E	03	1.05E	03		
55.00	2.00E	03	1.54E	03		
50.00	1.71E	03	2.11E	03		
45.00	9.73E	04	3.75E	03		
40.00	3.46E	04	3.55E	03		
35.00	1.67E	04	3.68E	03		
30.00	7.70E	05	3.65E	03		
25.00	3.43E	05	3.65E	03		
20.00	1.50E	05	3.72E	03		
15.00	9.58E	06	4.48E	03		
10.00	3.43E	06	5.50E	03		
5.00	2.17E	06				
0.00						

INPUT:

```

&DATUM
&AZIM=283.0, &CODIP=21.0, &MAGFLD=5.44E-5,
&FRW=27.0, &EPSK=15.0, &SIGMA=1.0E-2, &NUFLAG=1, &NPUNCH=9,
&RANG=70.0, &R7=87.0, &RANGEI=0.0, &R1=1.0,
&END

```

EXAMPLE V

PRINTED OUTPUT:

TMESH = 0.373 LUB = 0.0075
 WARNING--THE TOP OF THE PROFILE CAN NOT BE CUT OFF. THE TOP OF THE PROFILE MAY BE TO LOW.
 TEMPO-B AT TOP = 2.344E 01 WAITS OMEGA-R AT TOP = 7.962E 07
 THE BOTTOM PROFILE HEIGHT IS = 35.414 AT B(CUTOFF) = 0.1J0E-03

OMEGA-R EQUALS 2.5E05 AT TOP HEIGHT = 80.98

REFLECTION HT SET TO	32.41 KM	THETA RANGE =	82.750 TO	87.000
REFLECTION HT SET TO	83.41 KM	THETA RANGE =	78.500 TO	82.750
REFLECTION HT SET TO	83.41 KM	THETA RANGE =	74.250 TO	78.500
REFLECTION HT SET TO	84.41 KM	THETA RANGE =	70.000 TO	74.250

MODE	H = 50.00	ATTEN	VOVERC	WAIT MAG	WAIT ANG	THETAP	PUL
1	88.408	3.143	0.95399	-45.101	0.036	89.068	0.44411
2	35.320	2.148	0.95345	-20.244	0.169	89.739	2.03634
3	32.849	3.784	0.59991	-4.393	-0.065	88.201	1.66707
4	32.110	2.666	1.00161	-12.809	-0.046	86.710	0.45272
5	79.574	6.803	1.00870	-5.471	-0.072	82.714	1.55826
6	76.624	4.098	1.01202	-12.795	-0.120	81.155	0.61951
7	75.819	9.774	1.02330	-9.170	-0.240	77.733	1.03780
8	74.993	6.504	1.02765	-9.408	-0.267	76.074	0.97222
9	71.923	11.798	1.04346	-13.849	-0.611	73.399	0.67283
10	70.957	11.417	1.04930	-6.698	0.275	72.340	1.53786

TIME REQUIRED FOR THIS PROFILE = 165.08 SECONDS

EXAMPLE VI

PUNCHED OUTPUT (NPUNCH=9):

1	1.2	F	7.2	2.2	A	63.2	11.4	6	61.0	12.0	M	5.4	4.2	E	-0.5	8	1.0	10.0	-2.6	F	15.0		
1	89.66	03	-6.3	06402	6.86	12.3	4402	-0.0	-1.8	4566	285E	-0.4	-9.0	012	00E	-10	-4.5	83	7980	E	-10		
2	89.69	108	-6.3	08402	-4.2	427	810	1E	-2.7	3.78	1.3	3.99E	-0.7	9.58	233	74E	-0.1	1.8	55	7581	E	-02	
1	87.09	708	-6.3	0840	3.2	231	5E	-0.4	1.0	068	1.9	98	65	527E	-0.3	-1.5	33	54	18.5E	-0.3	-80	-98	
1	89.73	074	-5.4	83711	5.46	352	60E	-0.4	-3.1	937	016	E	-0.3	5.3	527	7810E	-1.0	3.6	35	9604	E	-11	
2	89.73	074	-5.4	83711	8.93	194	35E	-0.1	2.7	14	358	38E	-0.7	9.5	76	693	E	-0.1	1.7	43	7776	E	-02
1	87.73	074	-5.4	8371	6.8	726	7E	-0.2	1.2	536	-6.4	903	751	E	-0.4	3.2	22	18	25E	-0.4	80	98	
1	88.57	098	-1.6	86141	-1.2	874	148E	-1.1	2.0	265	32E	-0.2	3.7	91	990	35E	-0.9	-1.0	22	2291	3E	-09	
2	88.57	098	-1.6	86141	-5.7	1078	44E	-1.0	-4.0	010	227E	-0.8	3.9	04	202	7E	-0.1	1.3	00	2548	E	-02	
1	88.20	098	-1.6	8614	4.2	027	1E	-0.2	1.3	557	23	653	475E	-0.4	-2.5	33	39	8E	-0.4	80	98		
1	88.71	025	-0.2	41544	3.4	172	138E	-1.4	-7.0	003	256E	-0.3	-1.8	75	271	0E	-0.8	-1.8	75	365	76E	-09	
2	88.71	025	-0.2	41544	4.3	435	74E	-0.4	0.7	087	755	6E	-0.6	9.9	56	101	40E	-0.1	1.2	32	66	67E	-02
1	88.71	025	-0.2	4154	1.6	502	2E	-0.2	1.3	231	0	-1.8	293	68E	-0.3	1.2	28	4	34	50E	-0.3	80	98
1	82.41	203	-0.0	05841	1.2	550	95E	-0.3	-1.0	107	280	7E	-0.2	-1.8	88	346	50E	-0.4	-4.6	33	377	E	-09
2	82.41	203	-0.0	05841	1.2	204	275E	-0.5	-1.4	421	63	74E	-0.5	9.9	36	226	2E	-0.1	5.8	40	26	36E	-03
1	82.41	203	-0.0	0584	3.7	653	8E	-0.2	1.4	950	7	8	352	86	54E	-0.4	-0.1	33	78	59E	-0.4	80	98
1	81.15	297	-0.3	10324	9.5	703	89E	-1.1	1.0	27	504	15E	-0.3	5.7	29	374	0E	-0.9	7.1	94	270	51E	-11
2	81.15	297	-0.3	10324	1.4	134	974E	-1.1	1.5	553	231	8E	-0.3	9.7	13	152	86E	-0.1	3.1	94	270	51E	-11
1	77.73	752	-0.2	33641	1.8	234	5E	-0.2	1.0	937	1	78	90	590	E	-0.3	2.7	13	73E	-0.3	80	98	
2	77.73	752	-0.2	33641	2.3	329	40E	-0.3	-1.1	191	437	62E	-0.2	-7.0	17	143	7E	-0.4	-2.3	87	235	39E	-03
1	77.73	752	-0.2	33641	5.4	233	7E	-0.2	1.3	124	2	77	088	47E	-0.3	-1.1	62	69	12E	-0.3	80	98	
1	70.07	305	-0.5	32932	3.1	797	554E	-1.3	-1.1	150	974	26E	-0.2	-8.4	58	354	43E	-0.8	-1.8	75	223	89E	-08
2	70.07	305	-0.5	32932	2.2	631	635E	-1.5	2.3	021	304E	-0.4	9.8	01	202	3E	-0.1	-1.2	19	545	14E	-08	
1	70.07	305	-0.5	3258	2.3	642E	-0.2	1.8	574	-1.4	123	33	584E	-0.2	2.3	15	23	32E	-0.3	80	98		
1	73.25	524	-0.4	01372	4.2	307	053E	-1.0	-2.0	109	301	00E	-0.3	-1.6	02	386	5E	-0.1	-5.0	37	63	86E	-02
2	73.25	524	-0.4	01372	3.4	277	05E	-1.0	-2.2	146	873	3E	-0.3	9.8	09	293	32E	-0.1	-3.0	23	51	788E	-02
1	72.34	846	-0.4	38911	4.7	348	051E	-0.3	-1.0	46	003	13E	-0.2	1.0	46	003	13E	-0.2	1.0	46	003	13E	-02
2	72.34	846	-0.4	38911	3.2	227	01E	-0.3	-2.3	16	107E	-0.5	9.7	84	967	93E	-0.1	-3.7	64	0547	8E	-02	
1	72.34	846	-0.4	3891	5.2	690	6E	-0.2	1.8	455	-7.9	7520	39E	-0.4	-2.1	73	22	84E	-0.5	80	98		

EXAMPLE VII

PROFILE 3
SATELLITE NIGHT AMBIENT PROFILE

250.00	1.00E 05	1.00E 05
225.00	5.00E 03	5.00E 03
220.00	5.20E 03	5.20E 03
210.00	1.00E 03	1.00E 03
200.00	1.00E 03	1.00E 03
190.00	6.50E 02	6.50E 02
180.00	4.50E 02	4.50E 02
170.00	3.15E 02	3.45E 02
160.00	2.82E 02	2.82E 02
155.00	2.60E 02	2.60E 02
150.00	2.50E 02	2.50E 02
145.00	2.55E 02	2.55E 02
140.00	2.80E 02	2.80E 02
130.00	3.70E 02	3.70E 02
120.00	5.80E 02	5.80E 02
112.00	1.10E 03	1.10E 03
110.00	1.30E 03	1.30E 03
106.00	1.70E 03	1.70E 03
104.00	1.90E 03	1.90E 03
102.00	1.98E 03	1.98E 03
100.00	2.00E 03	2.00E 03
99.00	1.95E 03	1.95E 03
95.00	5.62E 02	6.30E 02
90.00	3.15E 02	4.11E 02
85.00	1.97E 02	4.11E 02
80.00	5.09E 01	4.11E 02
75.00	3.14E 00	4.11E 02
70.00	3.27E -02	4.11E 02
65.00	1.00E -02	5.35E 02
60.00	5.73E -03	7.06E 02
55.00	3.59E -03	1.09E 03
50.00	2.06E -03	1.54E 03
45.00	1.22E -03	2.11E 03
40.00	6.74E -04	2.73E 03
35.00	3.47E -04	3.25E 03
30.00	1.67E -04	3.55E 03
25.00	7.70E -05	3.69E 03
20.00	3.45E -05	3.65E 03
15.00	1.50E -05	3.65E 03
10.00	6.56E -06	3.72E 03
5.00	3.43E -06	4.40E 03
0.0	0.0	0.0

INPUT:

-99.99
COL FREQ 3

250.00	1.05E 02	4.50E -01	4.50E -01
225.00	3.50E 01	9.00E -01	9.00E -01
220.00	5.00E 01	1.00E 00	1.00E 00
210.00	5.30E 01	1.30E 00	1.30E 00
200.00	4.50E 01	2.00E 00	2.00E 00
150.00	1.00E 03	4.50E 01	4.50E 01
120.00	1.00E 04	3.00E 02	3.00E 02
100.00	5.90E 04	8.00E 03	8.00E 03
0.00	4.30E 11	1.07E 10	1.07E 10

-99.99
&DATUM

RANGER=78.0,85.0,RANGE I=-30.0,-40.0,

NUFLAG=0,

AZIM = 90.0, COL IP = 16.0, MAGFLD = 5.50E -05,

NUFLAG=0,AZIM=90.0,COLIP=16.0,MAGFLD=5.5E -05,EP SR=10.0,SIGMA=3.0E -04,
REFLHT=60.,FREQ=0.075,NFUNCH=7,
&END

EXAMPLE IX

INPUT:

```
ID HAWAII TC CALIFORNIA (NIGHTTIME)
&DATUM
A/IM=58.5,CURIP=39.0,MAGFLD=4.31E-5,
BETA=0.0,HPXIMC=30.0,SCLHTS=0.0,
FREJ=00.0,
RANGER=60.0,0.0,RANGEI=0.0,-2.0,
&END
```

PRINTED OUTPUT (PARTIAL):

```
TMFSH = 0.250 LUR = 0.0050
COMPUTED PROFILE PARAMETERS:
H NE X Y Z
90.0 4.705E 03 1.012E 05 1.074E 02 -2.011E 01 2.085E-01 1.505E 08
79.4 1.000E-01 1.217E 06 2.239E-03 -2.011E 01 3.228E 00 2.615E 02

OMEGA-R = 2.5E05 AT TOP HEIGHT = 88.00

REFLECTION HT SET TO 89.42 KM THETA RANGE = 85.000 TO 90.000
REFLECTION HT SET TO 89.42 KM THETA RANGE = 80.000 TO 85.000
REFLECTION HT SET TO 89.42 KM THETA RANGE = 75.000 TO 80.000
REFLECTION HT SET TO 89.42 KM THETA RANGE = 70.000 TO 75.000
REFLECTION HT SET TO 89.42 KM THETA RANGE = 65.000 TO 70.000
REFLECTION HT SET TO 89.42 KM THETA RANGE = 60.000 TO 65.000
```

EXAMPLE XIII

PRINTED OUTPUT CONTINUED:

MODE	H	THETA	ATTEN	VOVERC	WAIT MAG	WAIT ANG	THETA	POL
1	60.739	-0.030	0.325	0.99376	-79.481	0.107	69.985	0.64230
2	30.043	-0.109	2.150	0.99451	-0.344	0.317	69.991	1.45497
3	0.017	-0.017	0.306	0.99660	-0.518	0.064	69.981	0.66656
4	0.110	-0.110	0.146	0.99733	-27.700	0.239	69.940	1.34101
5	0.024	-0.024	0.543	0.99911	-7.019	-0.117	69.933	0.97130
6	0.005	-0.005	2.099	0.99985	-0.946	0.286	69.929	0.71320
7	0.000	-0.000	1.261	1.00019	-1.170	-0.147	69.921	1.35340
8	0.003	-0.003	2.001	1.00031	-11.490	0.427	85.407	0.63947
9	0.083	-0.083	2.002	1.00098	-2.233	-0.427	85.404	1.28110
10	0.091	-0.091	3.384	1.00073	-11.742	0.439	83.904	0.73214
11	0.055	-0.055	4.021	1.00105	-11.010	-0.259	81.730	1.15225
12	78.482	-0.105	4.021	1.00125	-11.043	0.426	80.972	0.85126
13	77.904	-0.100	4.163	1.00160	-8.500	-0.327	79.736	1.02989
14	75.902	-0.106	5.215	1.00189	-10.000	0.408	79.921	0.98782
15	75.302	-0.106	4.954	1.00229	-10.000	-0.409	77.937	0.92150
16	74.292	-0.116	5.609	1.00257	-19.244	0.382	77.143	1.14250
17	72.866	-0.111	5.704	1.00306	-11.529	-0.503	75.993	0.82648
18	72.020	-0.115	8.195	1.00338	-8.401	0.352	75.906	1.31816
19	70.370	-0.120	6.507	1.00393	-13.243	-0.610	74.185	1.74457
20	68.697	-0.120	7.506	1.00423	-17.912	0.319	73.591	0.51927
21	67.020	-0.124	11.889	1.00491	-7.922	-0.731	72.592	1.06745
22	65.370	-0.125	13.492	1.00533	-7.297	0.286	71.987	0.75372
23	63.697	-0.132	13.902	1.00601	-10.956	-0.867	70.910	1.06158
24	62.020	-0.132	19.002	1.00649	-18.411	0.254	69.880	0.62821
25	60.900	-0.135	16.170	1.00722	-10.000	-1.023	68.929	2.05682
26	60.000	-0.139	10.880	1.00792	-20.000	0.222	68.080	2.35314
27	60.263	-0.139	10.453	1.00800	-20.157	-1.203	67.944	0.55159
28	64.291	-0.148	12.371	1.00923	-6.451	0.191	66.200	2.73990
29	63.513	-0.144	20.990	1.10115	-21.847	-1.409	65.249	9.59895
30	61.553	-0.160	14.129	1.10849	-0.350	0.101	64.438	0.49134
31	61.737	-0.157	23.361	1.11799	-23.434	4.638	63.439	0.04913
32	60.138	-0.173	16.208	1.12643	-6.342	0.130	62.592	3.73157
33		-0.173		1.13671	-24.053	4.369	61.009	6.48751

TIME REQUIRED FOR THIS PROFILE = 350.80 SECONDS

EXAMPLE XIV

E. Error Messages

- (1) EIGENANGLES AT θ_1 AND θ_2 ARE TO BE RESOLVED (program execution stops).

Solution: Assign to RANGER and RANGEI values which constitute a small "contour rectangle" which incloses the above two eigenangles and re-execute the program. An alternative is to decrease 'GMAX'.

- (2) ERROR IN CONTROL CARD (Program execution stops).

Solution: Check control cards (i.e., PROFILE, COLFREQ, &DATUM, or &END)

- (3) TOP OF PROFILE CANNOT BE SET -- At the bottom of the PROFILE, B is greater than the value of TEMPO-B (program execution stops).

Solution: Problem is due to incompatibilities between values of TEMPO-B and the actual electron and ion density values.

- (4) WARNING -- THE TOP OF THE PROFILE CANNOT BE CUTOFF. THE TOP OF THE PROFILE MAY BE TOO LOW (program execution continues).

Reason: In top card of electron density profile the computed B value is less than the input value of TEMPO-B.

- (5) THE PROFILE BOTTOM CANNOT BE CUTOFF. ALL B-VALUES (AS A FUNCTION OF HEIGHT) ARE LESS THAN CUTOFF. THIS IMPLIES THAT THE PROFILE IS VERY CLOSE TO FREE SPACE (Program execution stops).

Solution: Check values of 'CUTOFF', and individual values of electron and ion densities. If o.k., put 'ICUT' = 0 and re-execute.

- (6) THE BOTTOM OF THE PROFILE CANNOT BE CUTOFF USING
'CUTOFF' = XXX AT HEIGHT = YYY BECAUSE THE VALUE OF B AT
THE BOTTOM OF THE PROFILE IS LARGER THAN 'CUTOFF' (Program
execution continues).
- (7) NO MODES (program execution continues).
There are no eigenangle solutions located inside the con-
tour rectangle as chosen. Solutions: Change RANGER and
RANGEI values and re-execute the program.
- (8) INVALID MODE AT θ (program execution continues).
Solution: Cut down the size of the contour rectangle in
the region of θ .
- (9) STEP TOO SMALL IN INTEG OR TOO MANY STEPS IN INTEG (pro-
gram execution stops).
Solution: Examine PROFILE (e.g. too dense at top or too
sparse at bottom) and other ionospheric parameters such
as magnetic field, collision frequency, etc. First decrease
'GMAX' and re-execute the program. If problem persists,
increase 'ENMIN' and again re-execute.
- (10) PROBLEM IN SORTING Q VALUES OR Q FAILS TO CONVERGE IN
QUARTIC (program execution stops).
Solution: Again look at ionospheric parameter values.
RANGER (minimum) is probably too small. Increase the
value of RANGER (minimum) and re-execute program.
- (11) X MODES FOUND ON SAME PHASE LINE (program execution
continues).
Solution: Information only.

(12) WARNING X MODES FOUND ON Y PHASE LINES (program execution continues).

Solution: Information only.

(13) If run time is very long it is probably related to #8 above and thus solution is the same as for #8.

(14) NO EXIT FROM MESH SQUARE (program execution stops)

Solution: The value of TMESH should be changed slightly (1%) and the program rerun. If message continues - STOP.

(15) ONLY X MODES FOUND ON Y PHASE LINES (program execution stops).

Solution: Reduce TMESH by 1/2 and re-execute program.

(16) PROBLEMS IN SUBROUTINE NO MESH (program execution stops).

Solutions: Reduce TMESH by 1/2 and re-execute program.

If message continues - STOP.

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APPENDIX A

Modification of the Modal Equation

Given:

$$F_1(\theta) = (1 - \parallel \bar{R} \parallel \parallel R \parallel) (1 - \perp \bar{R} \perp \perp R \perp) - \parallel R \perp \perp R \parallel \parallel \bar{R} \parallel \perp \bar{R} \perp \quad (A-1)$$

where $R = R(\theta)$ and $\bar{R} = \bar{R}(\theta)$.

Let

$$\parallel \bar{R} \parallel = \frac{\parallel N \parallel}{\parallel D \parallel}, \quad \perp \bar{R} \perp = \frac{\perp N \perp}{\perp D \perp} \quad (A-2)$$

where $\parallel N \parallel$, $\parallel D \parallel$, $\perp N \perp$ and $\perp D \perp$ are in such form that they contain no poles in the region of interest in the θ -plane.

Then

$$F_1(\theta) = (1 - \frac{\parallel N \parallel}{\parallel D \parallel} \parallel R \parallel) (1 - \frac{\perp N \perp}{\perp D \perp} \perp R \perp) - \parallel R \perp \perp R \parallel \cdot \frac{\parallel N \parallel}{\parallel D \parallel} \cdot \frac{\perp N \perp}{\perp D \perp} \quad (A-3)$$

and

$$F_2(\theta) = \parallel D \parallel \cdot \perp D \perp F_1(\theta) = (\parallel D \parallel - \parallel N \parallel \parallel R \parallel) (\perp D \perp - \perp N \perp \perp R \perp) - \parallel R \perp \perp R \parallel \parallel N \parallel \perp N \perp \quad (A-4)$$

Also

$$F_2(\theta) = \left[\begin{array}{l} \parallel D \parallel - \parallel N \parallel \cdot \frac{\parallel R \parallel + 1}{c} + \parallel N \parallel \\ \perp D \perp - \perp N \perp \cdot \frac{\perp R \perp + 1}{c} + \perp N \perp \\ - [c^2 \frac{\parallel R \perp}{c} \cdot \frac{\perp R \parallel}{c} \cdot \parallel N \parallel \cdot \perp N \perp] \end{array} \right] \quad (A-5)$$

Then

$$F_3(\theta) = \frac{F_2(\theta)}{C^2} = \frac{D_{\parallel} D_{\perp} F_1(\theta)}{C^2} = \left\{ \begin{aligned} & \left[\frac{N_{\parallel} + D_{\parallel}}{C} - N_{\parallel} \frac{(R_{\parallel} + 1)}{C} \right] \cdot \left[\frac{N_{\perp} + D_{\perp}}{C} - N_{\perp} \frac{(R_{\perp} + 1)}{C} \right] \\ & - \left[N_{\parallel} \cdot N_{\perp} \cdot \frac{R_{\perp}}{C} \frac{R_{\parallel}}{C} \right] \end{aligned} \right\} \quad (A-6)$$

Let $\bar{X} = \frac{R + 1}{C}$

$$\frac{(N_{\parallel} + D_{\parallel})}{C} = n_{\parallel} \frac{(N_{\perp} + D_{\perp})}{C} = n_{\perp} \quad (A-7)$$

$$N_{\parallel} = d_{\parallel} \quad N_{\perp} = d_{\perp}$$

Then

$$F_3(\theta) = \frac{D_{\parallel} D_{\perp} F_1(\theta)}{C^2} = [n_{\parallel} - d_{\parallel} \bar{X}_{\parallel}] [n_{\perp} - d_{\perp} \bar{X}_{\perp}] - d_{\parallel} d_{\perp} \bar{X}_{\perp} \bar{X}_{\parallel} \quad (A-8)$$

Note that:

$$\frac{n_{\parallel}}{d_{\parallel}} = \frac{N_{\parallel} + D_{\parallel}}{C N_{\parallel}} = \frac{D_{\parallel} / N_{\parallel} + 1}{C} = \left(\frac{1}{\bar{R}_{\parallel}} + 1 \right) 1/C = \frac{\bar{R}_{\parallel} + 1}{C \bar{R}_{\parallel}} \quad (A-9)$$

$$\frac{n_{\perp}}{d_{\perp}} = \frac{N_{\perp} + D_{\perp}}{C N_{\perp}} = \frac{D_{\perp} / N_{\perp} + 1}{C} = \left(\frac{1}{\bar{R}_{\perp}} + 1 \right) 1/C = \frac{\bar{R}_{\perp} + 1}{C \bar{R}_{\perp}} \quad (A-10)$$

Also note that no poles in N or D implies that there are no poles in n or d.

APPENDIX B

Derivation of the Equations for Free Space Integration

In the MODESRCH routine the differential equations for the ionospheric and ground reflection coefficient matrices have to be integrated through a free space region over a curved earth. In the following derivation of the necessary differential equations for the ionospheric reflection matrices a number of references are made to Budden's work, reference 7.

A right handed Cartesian system is assumed with the X and Y axes lying in a horizontal plane boundary between the bottom of the ionosphere and the free space below. The positive Z direction is upward. A plane wave is incident upon the ionosphere from below with the wave normal in the X-Z plane at an angle θ_1 to the Z axis.

The differential equations describing wave fields at oblique incidence (Op. cit., P. 140, eqns. 9.49-9.54) may be separated into two independent sets. Either of these sets of field variables may be set equal to zero without affecting the other, so that the corresponding waves are propagated independently.

For the first set the electric field is everywhere parallel to the Y-axis, and the waves are said to be 'horizontally polarized'. The equations are:

$$\frac{\partial E_y}{\partial z} = i k H_x \quad (B-1)$$

$$i k S E_y = -i k H_z \quad (B-2)$$

$$\frac{\partial H_x}{\partial z} + i k S H_z = i k n^2 E_y \quad (B-3)$$

where $i = \sqrt{-1}$

$$S = \sin \theta_I$$

$$k = 2 \pi / \lambda$$

λ = free space wavelength

n = index of refraction of the medium

and $H = Z_0 H$

where $Z_0 = E_x / H_y$

H_z may be eliminated from eqns. 2 and 3 to give

$$\frac{\partial H_x}{\partial Z} = i k (n^2 - S^2) E_y \quad (B-4)$$

Elimination of H_x between eqns. 1 and 4 gives

$$\frac{d^2 E_y}{dZ^2} + k^2 q^2 E_y = 0 \quad (B-5)$$

where $q^2 = (n^2 - S^2)$

For the second set of equations the electric field is parallel to the X-Z plane and the waves are said to be 'vertically polarized' even though the electric vector is not, in general, vertical.

The equations are:

$$\frac{\partial E_x}{\partial Z} + i k S E_z = -i k H_y \quad (B-6)$$

$$\frac{\partial H_y}{\partial Z} = -i k n^2 E_x \quad (B-7)$$

$$-i k S H_y = i k n^2 E_z \quad (B-8)$$

The field component E_z may be eliminated from eqns. (6) and (8) to give

$$\frac{\partial E_x}{\partial Z} = -i k \frac{q^2}{n^2} H_y \quad (B-9)$$

Elimination of E_x between eqns. 7 and 9 gives

$$\frac{d^2 H_y}{dz^2} - \frac{1}{n^2} \frac{d(n^2)}{dz} \frac{d H_y}{dz} + k^2 q^2 H_y \quad (B-10)$$

Just below the free space-ionosphere boundary the total electric wave field is composed of two upward traveling incident components and two downward traveling reflected components. They are $E_{\parallel,y}^i$ and $E_{\parallel,y}^r$ respectively, where $E_{\parallel}$ is in the X-Y plane. Following Budden (Op Cit. Pg. 118 eq. 8.72) the equations for free space integration of reflection at oblique incidence may be derived.

The total field components just below the boundary are

$$E_x = (E_{\parallel}^i - E_{\parallel}^r) \cos \theta_I \quad (B-11,a)$$

$$E_y = E_y^i + E_y^r \quad (B-11,b)$$

$$H_x = (E_y^r - E_y^i) \cos \theta_I \quad (B-11,c)$$

$$H_y = E_{\parallel}^i + E_{\parallel}^r \quad (B-11,d)$$

Define:

$$_{\perp}R_{\perp} = E_y^r/E_y^i \quad (B-12,a)$$

$$_{\parallel}R_{\perp} = E_y^r/E_{\parallel}^i \quad (B-12,b)$$

$$_{\parallel}R_{\parallel} = E_{\parallel}^r/E_{\parallel}^i \quad (B-12,c)$$

$$_{\perp}R_{\parallel} = E_{\parallel}^r/E_y^i \quad (B-12,d)$$

where the R's are functions of θ_I .

For Horizontal Polarization:

Combine eqns. (11,b), (12,a) and (12,b) to get

$$E_y = E_y^i + E_y^i {}_{\perp}R_{\perp} + E_{\parallel}^i {}_{\parallel}R_{\perp}$$

or

$$E_y = E_y^i (R_{\perp} + 1) + E_{\parallel}^i R_{\perp} \quad (B-13)$$

The wave equation for Horizontal Polarization (e.g., eq 5) is:

$$\frac{d^2 E_y}{dz^2} + k^2 q^2 E_y = 0 \quad (B-5)$$

where $q^2 = n^2 - S^2$

and $S = \sin \theta_I$

To include a curved earth let

$$n^2 = 1 + \alpha (Z-H)$$

where:

n is the index of refraction of the medium,

$\alpha = 2/r_e$, where r_e is the radius of the earth.

H is that height where $n = 1$.

then $q^2 = 1 + \alpha (Z-H) - S^2 = C^2 + \alpha (Z-H)$

where $C = \cos \theta_I$

so that

$$\frac{d^2 E_y}{dz^2} + k^2 [C^2 + \alpha (Z-H)] E_y = 0 \quad (B-14)$$

to get standard form:

Let $\rho = \left(\frac{k}{\alpha}\right)^{2/3} [C^2 + \alpha (Z-H)]$ (B-15)

then $\frac{d\rho}{dz} = \left(\frac{k}{\alpha}\right)^{2/3} \alpha = \left(\frac{k}{\alpha}\right)^{2/3} \alpha^{1/3}$

$$\frac{d^2 \rho}{dz^2} = 0$$

and then expanding gives

$$\begin{aligned}\frac{d^2 E_y}{dZ^2} &= \frac{d}{dZ} \left\{ \frac{d E_y}{dZ} \right\} = \frac{d}{dZ} \left\{ \frac{d E_y}{dZ \rho} \frac{d \rho}{dZ} \right\} \\ &= \frac{d^2 E_y}{d\rho^2} \left(\frac{d\rho}{dZ} \right)^2 + \frac{d E_y}{d\rho} \frac{d^2 \rho}{dZ^2}\end{aligned}$$

now, substituting the above results into eq. B-14 gives

$$\frac{d^2 E_y}{dZ^2} = (k^2 \alpha)^{2/3} \frac{d^2 E_y}{d\rho^2} \quad (B-16)$$

Also, since

$$(k^2 \alpha)^{2/3} \left(\frac{k}{\alpha} \right)^{2/3} = k^2$$

then

$$k^2 q^2 = (k^2 \alpha)^{2/3} \left(\frac{k}{\alpha} \right)^{2/3} [C^2 + \alpha (z-h)]$$

or

$$k^2 q^2 = (k^2 \alpha)^{2/3} \rho$$

so that the wave equation (eq. 5) becomes upon substitution of the above:

$$(k^2 \alpha)^{2/3} \left\{ \frac{d^2 E_y}{d\rho^2} + \rho E_y \right\} = 0$$

or

$$\frac{d^2 E_y}{d\rho^2} + \rho E_y = 0 \quad (B-17)$$

The general solution to this equation is

$$E_y(\rho) = A h_1(\rho) + B h_2(\rho) \quad (B-18)$$

where A and B are arbitrary constants and $h_1(\rho)$ and $h_2(\rho)$ are modified Hankel functions of order 1/3. (See reference 8).

To relate E_y and H_x :

equation (1) is:

$$\frac{d E_y}{dz} = i k H_x$$

or

$$\begin{aligned} H_x &= -\frac{i}{k} \frac{d E_y}{dz} \\ &= -\frac{i}{k} \frac{d E_y}{dp} \frac{dp}{dz} = -\frac{i}{k} (k^2 \alpha)^{1/3} \frac{d E_y}{dp} \\ H_x &= -k \left(\frac{\alpha}{k}\right)^{1/3} \frac{d E_y}{dp} \end{aligned}$$

$$\text{let } K = i \left(\frac{\alpha}{k}\right)^{1/3} \quad (\text{B-19})$$

$$\text{then } H_x = -K \frac{d E_y}{dp}$$

or

$$H_x = -K \{ A h_1'(\rho) + B h_2'(\rho) \} \quad (\text{B-20})$$

The primes on the above quantities denote derivatives with respect to the argument.

From Eq. 13,

$$E_y = C E_y^i \left(\frac{1 R_{\perp} + 1}{C} \right) + C E_y^i \left(\frac{1 R_{\perp}}{C} \right) \quad (\text{B-21})$$

and from eq. 11c

$$H_x = C E_y^r - C E_y^i$$

also from eq. 11,b

$$E_y^r = E_y - E_y^i$$

Substitution for E_y^r into eq. (11,c) gives

$$C E_y - C E_y^i = H_x + C E_y^i$$

or

$$C E_y - H_x = 2C E_y^i \quad (B-22)$$

Substitutions into eq. 21 for E_y from eq. 18 gives

$$A h_1(\rho) + B h_2(\rho) = C E_y^i \left(\frac{1R_{\perp}+1}{C} \right) + C E_{\parallel}^i \left(\frac{1R_{\perp}}{C} \right)$$

and substitution into eq. 22 for E_y and H_x from equations 18 and 20 gives

$$C \{A h_1(\rho) + B h_2(\rho)\} + K \{A h_1'(\rho) + B h_2'(\rho)\} = 2 C E_y^i$$

or

$$\left(-\frac{A}{C} \right) h_1(\rho) + \left(-\frac{B}{C} \right) h_2(\rho) + E_y^i \left(\frac{1R_{\perp}+1}{C} \right) + E_{\parallel}^i \left(\frac{1R_{\perp}}{C} \right) = 0 \quad (B-23)$$

$$\left(-\frac{A}{C} \right) \{C h_1(\rho) + K h_1'(\rho)\} + \left(-\frac{B}{C} \right) \{C h_2(\rho) + K h_2'(\rho)\} + 2E_y^i = 0 \quad (B-24)$$

Equations 23 and 24 are the equations to be used for 'horizontal polarization' in the free space integration procedure.

For Vertical Polarization:

Applying previous equations (e.g., equations 11 and 12) gives:

$$H_y = E_{\parallel}^i + E_{\parallel}^r \quad (B-11,d)$$

$$E_x = (E_{\parallel}^i - E_{\parallel}^r) \cos \theta_1 \quad (B-11,a)$$

$$1R_{\parallel} = E_{\parallel}^r / E_{\parallel}^i \quad (B-12,c)$$

$$\perp R_{\parallel} = E_{\parallel}^r / E_y^i \quad (B-12,d)$$

Combine eqns. (11,d), (12,c) and (12,d) to get

$$H_y = E_{\parallel}^i + E_{\parallel}^i R_{\parallel} + E_y^i R_{\perp}$$

also combining eqns. (11,a), (12,c) and (12,d)

$$E_x = (E_{\parallel}^i - E_{\parallel}^i R_{\parallel} - E_y^i R_{\perp}) C$$

In different form:

$$H_y = C E_{\parallel}^i \left(\frac{R_{\parallel} + 1}{C} \right) + C E_y^i \left(\frac{R_{\perp}}{C} \right) \quad (B-25)$$

where $C = \cos \theta_I$

and from eqns. (11,a) and (11,d)

$$E_x = E_{\parallel}^i C - H_y C + E_{\parallel}^i C$$

or

$$C H_y + E_x = 2E_{\parallel}^i C \quad (B-26)$$

To obtain the wave equation for vertical polarization, follow Budden (op. cit. pg. 343). Note that in this case the electric vector has both vertical (E_z) and horizontal (E_y) components.

The wave equation for vertical polarization is given by equation 10 as:

$$\frac{d^2 H_y}{dZ^2} - \frac{1}{n^2} \frac{d(n^2)}{dZ} \frac{d H_y}{dZ} + k^2 q^2 H_y = 0 \quad (B-10)$$

Any linear second-order differential equation with a term containing the first derivative (e.g., $d H_y/dZ$) can be reduced to its 'normal form', that is a form without a first derivative term, by a change in the dependent variable.

$$\text{Let } V = H_y/n \quad (B-27)$$

$$\text{or } H_y = nV \quad (B-28)$$

then equation 10 becomes:

$$\frac{d^2V}{dZ^2} + k^2 \left[q^2 + \frac{1}{2k^2 n^2} \frac{d^2(n^2)}{dZ^2} - \frac{3}{4k^2 n^4} \left(\frac{d(n^2)}{dZ} \right)^2 \right] V = 0 \quad (B-29)$$

where for a curved earth:

$$q^2 = c^2 + \alpha(Z-H)$$

Note that:

$$\frac{d(n^2)}{dZ} = \frac{d}{dZ} (1 + \alpha(Z-H)) = \alpha \quad (B-30)$$

and

$$\frac{d^2(n^2)}{dZ^2} = 0 \quad (B-31)$$

The wave equation, Eq. 29, then becomes, after substitution of eqns. 30, and 31:

$$\frac{d^2V}{dZ^2} + k^2 \left[c^2 + \alpha(Z-H) - \underbrace{\frac{3\alpha^2}{4k^2 n^4}}_{\text{neglect this term}} \right] V = 0 \quad (B-32)$$

let

$$\rho = \left(\frac{k}{\alpha} \right)^{2/3} [c^2 + \alpha(Z-H)] \quad (B-33)$$

Compare the resulting form of eq. 32 with eq. 17

Therefore the solution is

$$V = Q h_1(\rho) + G h_2(\rho) \quad (B-34)$$

and since

$$H_y = nV \quad (B-28)$$

$$H_y = n \{ Q h_1(\rho) + G h_2(\rho) \} \quad (B-35)$$

Note:

$$\frac{d(n^2)}{dz} = 2 n \frac{dn}{dz} \quad \text{or} \quad \frac{dn}{dz} = \frac{1}{2n} \frac{d(n^2)}{dz}$$

but from eq. 30

$$\frac{d(n^2)}{dz} = \alpha$$

$$\therefore \frac{dn}{dz} = \frac{\alpha}{2n} \quad (\text{B-36})$$

Also from eq. 33,

$$\frac{dp}{dz} = \left(\frac{k}{\alpha}\right)^{2/3} \alpha = \left(\frac{k^2 \alpha^3}{\alpha^2}\right)^{1/3} = (k^2 \alpha)^{1/3} \quad (\text{B-37})$$

To relate H_y and E_x :

From eq. (7):

$$\frac{d H_y}{dz} = -i k n^2 E_x \quad (\text{B-7})$$

or

$$E_x = \frac{i}{k n^2} \frac{d H_y}{dz} = \frac{i}{k n^2} \frac{d}{dz} (nV)$$

or

$$E_x = \frac{i}{k n^2} \left\{ n \frac{dV}{dz} + \frac{dn}{dz} V \right\}$$

and

$$E_x = \frac{i}{k n^2} \left\{ n \frac{dV}{dp} \frac{dp}{dz} + \frac{dn}{dz} V \right\} \quad (\text{B-38})$$

Substituting eqns. 36 and 37 into eq. 38 gives

$$E_x = \frac{i}{k n^2} \left\{ n (k^2 \alpha)^{1/3} \frac{dV}{dp} + \frac{\alpha}{2n} V \right\}$$

or

$$E_x = \frac{1}{n} \left\{ i \left(\frac{\alpha}{k}\right)^{1/3} \frac{dV}{dp} + i \frac{\alpha}{2k} \frac{V}{n^2} \right\} \quad (\text{B-39})$$

Now define

$$K = i \left(\frac{\alpha}{k} \right)^{1/3} \quad (B-40)$$

and

$$L = i \left(\frac{\alpha}{2k} \right) \quad (B-41)$$

Then

$$E_x = \frac{1}{n} \left\{ K \frac{dV}{dz} + L \frac{V}{n^2} \right\} \quad (B-42)$$

Since L is small let $n^2 \approx 1$

Then

$$\frac{1}{n} E_x = \frac{1}{n^2} \left\{ K \frac{dV}{dp} + L V \right\} \quad (B-43)$$

Substituting for V from Eq. (34) gives:

$$\begin{aligned} \frac{1}{n} E_x = \frac{1}{n^2} \{ & K [Q h_1'(\rho) + G h_2'(\rho)] \\ & + L [Q h_1(\rho) + G h_2(\rho)] \} \end{aligned} \quad (B-44)$$

Substituting eq. 35 into eq. 25 for H_y gives

$$n \left\{ Q h_1(\rho) + G h_2(\rho) \right\} = C E_{\parallel} i \left(\frac{\parallel R_{\parallel} + 1}{C} \right) + C E_y i \left(\frac{\perp R_{\parallel}}{C} \right) \quad (B-45)$$

Substituting eqns. 35 and 44 into eq. (26) for H_y and E_x gives:

$$\begin{aligned} n \left[C \{ Q h_1(\rho) + G h_2(\rho) \} + \frac{1}{n^2} \{ K [Q h_1'(\rho) + G h_2'(\rho)] + \right. \\ \left. \frac{1}{n^2} \{ L [Q h_1(\rho) + G h_2(\rho)] \} \right] = 2 E_{\parallel} i C \end{aligned} \quad (B-46)$$

Now neglect the multiplier n so that $\perp$ and $\parallel$ will be alike for coupling to get from equation 45:

$$\left(-\frac{Q}{C} \right) h_1(\rho) + \left(-\frac{G}{C} \right) h_2(\rho) + E_{\parallel} i \left(\frac{\parallel R_{\parallel} + 1}{C} \right) + E_y i \left(\frac{\perp R_{\parallel}}{C} \right) = 0$$

(B-47)

and from equation 46:

$$\begin{aligned} & \left(-\frac{Q}{C}\right) \{C h_1(\rho) + [K h_1'(\rho) + L h_1(\rho)]/n^2\} \\ & + \left(-\frac{G}{C}\right) \{C h_2(\rho) + [K h_2'(\rho) + L h_2(\rho)]/n^2\} + 2 E_{\parallel}^i = 0 \end{aligned}$$

B-48

Equations 47 and 48 are the equations to be used for 'vertical polarization' in the free space integration procedure.

The four equations for ionospheric reflection at oblique incidence (e.g., eqns. 23, 24, 47 and 48) may be used to accomplish the full-wave integration of the four ionospheric reflection coefficients ($\tilde{R}$) through a free space medium over a curved earth. The integration is to be carried out from some starting height " Z_f ", where the values of the R 's are known, to some other level " Z_t " where the values of the R 's are to be determined.

The free space integration procedure consists of three steps (I, II and III) which are as follows:

I. At the "from" level (i.e., $Z = Z_f$) the known values of the elements of $\tilde{R}$ may be substituted into equations (23, 24, 47, and 48) and then two sets of boundary conditions, B. C., applied successfully. That is for Set I assign ($E_{\parallel}^i = 1, E_y^i = 0$) and solve for the coefficients A_I and B_I from equations 23 and 24; and for the coefficients Q_I and G_I from equations 47 and 48. Next, for Set II assign ($E_y^i = 1, E_{\parallel}^i = 0$) and solve for the coefficients A_{II} and B_{II} from equations 23 and 24; and for the coefficients Q_{II} and G_{II} from equations 47 and 48.

The procedure includes a need for solving two simultaneous equations. That is if:

$$a_{11}\bar{A} + a_{12}\bar{B} = V_1 \quad (B-49,a)$$

$$a_{21}\bar{A} + a_{22}\bar{B} = V_2 \quad (B-49,b)$$

then

$$\bar{A} = \frac{(V_1 a_{22} - V_2 a_{12})}{(a_{11} a_{22} - a_{21} a_{12})} \quad (B-50,a)$$

$$\bar{B} = \frac{(V_2 a_{11} - V_1 a_{21})}{(a_{11} a_{22} - a_{21} a_{12})} \quad (B-50,b)$$

For Horizontal Polarization at the Z_f (from) level, the following identifications are made:

Identify terms of eq. 23 with eq. (49,a) and terms of eq. 24 with (49,b). That is let:

$$\bar{A} = -\left(\frac{A}{C}\right), \quad \bar{B} = -\left(\frac{B}{C}\right);$$

where

$$C = \cos \theta_I, \quad \rho_f : \text{implies a function of}$$

o at the "from" level.

$$\rho_f = \left(\frac{k}{\alpha}\right)^{2/3} [C + \alpha(Z_f - H)]$$

$$a_{11} = h_1(\rho_f), \quad a_{12} = h_2(\rho_f)$$

$$a_{21} = C h_1(\rho_f) + K h_1'(\rho_f)$$

$$a_{22} = C h_2(\rho_f) + K h_2'(\rho_f)$$

$$V_1 = -E_y i \left(\frac{\perp R_I^f + 1}{C} \right) - E_{\parallel} i \left(\frac{\parallel R_I^f}{C} \right)$$

$$V_2 = -2 E_y i$$

where from (19)

$$K = i \left(\frac{\alpha}{k} \right)^{1/3}$$

(B-51)

For Set I B.C.: ($E_{11}^i = 1, E_y^i = 0$) Applying these boundary conditions and the relations of equation 51 to equation 23 and to equation 24 gives:

$$\left\{ \begin{array}{l} \left(-\frac{A}{C}\right)_I a_{11} + \left(-\frac{B}{C}\right)_I a_{12} = \left(\frac{R_1^f}{C}\right) \\ \left(-\frac{A}{C}\right)_I a_{21} + \left(-\frac{B}{C}\right)_I a_{22} = 0 \end{array} \right\}$$

Solving these equations simultaneously by the application of equation 50(A,B) gives:

$$\left(\frac{A}{C}\right)_I = \frac{R_1^f}{C} \cdot \frac{a_{22}}{\Delta_1} \quad (B-52,a)$$

$$\left(\frac{B}{C}\right)_I = -\frac{R_1^f}{C} \cdot \frac{a_{21}}{\Delta_1} \quad (B-52,b)$$

where

$$\Delta_1 = (a_{11}a_{22} - a_{21}a_{12}) \quad (B-53)$$

with a_{ij} as defined in equation 51

For Set II B.C.: ($E_{11}^i = 0, E_y^i = 1$) Applying these B.C. conditions and the relations of equation 51 to equation 23 and equation 24 gives:

$$\left(-\frac{A}{C}\right)_{II} a_{11} + \left(-\frac{B}{C}\right)_{II} a_{12} = -\frac{(R_1^f + 1)}{C}$$

$$\left(-\frac{A}{C}\right)_{II} a_{21} + \left(-\frac{B}{C}\right)_{II} a_{22} = -2$$

Solving these equations simultaneously by the application of equations

50(A,B) gives:

$$\left(\frac{A}{C}\right)_{II} = \left(\frac{(1R_1^f + 1)}{C} a_{22} - 2a_{12}\right) \frac{1}{\Delta_1} \quad (B-54,a)$$

$$\left(\frac{B}{C}\right)_{II} = \left(2a_{11} - \left(\frac{(1R_1^f + 1)}{C}\right) a_{21}\right) \frac{1}{\Delta_1} \quad (B-54,b)$$

Where Δ_1 is again defined in equation 53.

For Vertical Polarization at the Z_f , "from", level the following identifications are made.

Identify terms of equation 47 with equation (49,a) and terms of equation 48 with (49,b). That is let:

$$\bar{A} = \left(-\frac{Q}{C}\right), \bar{B} = \left(-\frac{G}{C}\right)$$

$$a_{11} = h_1(\rho_f), a_{12} = h_2(\rho_f)$$

$$a_{21} = C h_1(\rho_f) + [K h_1'(\rho_f) + L h_1(\rho_f)]/n_f^2$$

$$a_{22} = C h_2(\rho_f) + [K h_2'(\rho_f) + L h_2(\rho_f)]/n_f^2$$

Where from equation 40 and 41

$$K = i\left(\frac{\alpha}{k}\right)^{1/3}, L = i\left(\frac{\alpha}{2k}\right)$$

and

$$V_1 = -E_{\parallel}^i \left(\frac{(1R_{\parallel}^f + 1)}{C}\right) - E_y^i \left(\frac{(1R_{\parallel}^f)}{C}\right)$$

$$V_2 = -2E_{\parallel}^i$$

(B-55)

For Set I B.C.: ($E_I^i = 1, E_y^i = 0$) Applying these B.C. conditions and the relations of equation 55 to equations 47 and 48 gives:

$$\left(\frac{Q}{C}\right)_I a_{11} + \left(\frac{G}{C}\right)_I a_{12} = \left(\frac{R_I^f + 1}{C}\right)$$

$$\left(\frac{Q}{C}\right)_I a_{21} + \left(\frac{G}{C}\right)_I a_{22} = 2$$

Solving these equations simultaneously by the application of equations 50(A,B) gives:

$$\left(\frac{Q}{C}\right)_I = \left[\left(\frac{R_I^f + 1}{C} \right) a_{22} - 2a_{12} \right] \frac{1}{\Delta_2} \quad (B-56,a)$$

$$\left(\frac{G}{C}\right)_I = \left[2a_{11} - \left(\frac{R_I^f + 1}{C} \right) a_{21} \right] \frac{1}{\Delta_2} \quad (B-56,b)$$

Where

$$\Delta_2 = (a_{11}a_{22} - a_{21}a_{12})$$

with a_{ij} as defined in equation 55

For Set II B.C.: ($E_I^i = 0, E_y^i = 1$) Applying these boundary conditions and the relations of equation 55 to equations 47 and 48 gives:

$$\left(\frac{Q}{C}\right)_{II} a_{11} + \left(\frac{G}{C}\right)_{II} a_{12} = \frac{R_{II}^f}{C}$$

$$\left(\frac{Q}{C}\right)_{II} a_{21} + \left(\frac{G}{C}\right)_{II} a_{22} = 0$$

Solving this equation simultaneously by the application of equations 50(A,B) gives:

$$\left(\frac{Q}{C}\right)_{II} = \frac{R_{II}^f}{C} \frac{a_{22}}{\Delta_2} \quad (B-57,a)$$

$$\left(\frac{G}{C}\right)_{II} = - \frac{R_{II}^f}{C} \frac{a_{21}}{\Delta_2} \quad (B-57,b)$$

Where Δ_2 is again defined in equation 55

II. The values of the electric field terms $E_{\parallel}^i$ and E_y^i at the "to" level (e.g., at height = Z_t) may be determined from the values of the coefficients (A, B, F and G) as determined in Step I for the Set I and Set II boundary conditions.

For Horizontal Polarization at the Z_t "to" level, the following identifications are made.

Identify terms of equation 24 with equation 49,b. That is:

$$\left. \begin{aligned} \bar{A} &= \left(-\frac{A}{C}\right), \quad \bar{B} = \left(-\frac{B}{C}\right) \\ a_{21} &= C h_1(\rho_t) + K h_1'(\rho_t); \quad v_2 = -2 \left(E_y^i\right)_t \\ a_{22} &= C h_2(\rho_t) + K h_2'(\rho_t) \\ \text{Where} \\ \rho_t &= \left(\frac{k}{\alpha}\right)^{2/3} [C^2 + \alpha(Z_t - H)] \\ \rho_t &: \text{ implies a function of } \rho \text{ at "to" level} \\ K &= i \left(\frac{\alpha}{k}\right)^{1/3} \end{aligned} \right\} \quad (B-58)$$

For Set I B.C.: (e.g. $(E_{\parallel}^i)_f = 1$; $(E_y^i)_f = 0$)

$$\begin{aligned} \left(-\frac{A}{C}\right) a_{21} + \left(-\frac{B}{C}\right) a_{22} &= -2 \left(E_y^i\right)_t \bigg|_I \\ \text{or} \quad \left(E_y^i\right)_t \bigg|_I &= \left(\frac{\left(\frac{A}{C}\right)_I a_{21} + \left(\frac{B}{C}\right)_I a_{22}}{2} \right) \end{aligned} \quad (B-59)$$

Where the values of $(A/C)_I$ and $(B/C)_I$ were determined by equation 52 and the a_{ij} are given in equation 58.

For Set II B.C.: (e.g. $(E_{\parallel}^i)_f = 0$, $(E_y^i)_f = 1$)

$$\left(-\frac{A}{C}\right)_{II} a_{21} + \left(-\frac{B}{C}\right)_{II} a_{22} = -2 \left(\left(E_y^i\right)_t\right)_{II}$$

or

$$\boxed{\left(\left(E_y^i\right)_t\right)_{II} = \frac{\left(\frac{A}{C}\right)_{II} a_{21} + \left(\frac{B}{C}\right)_{II} a_{22}}{2}}$$
(B-60)

Where the values of $(A/C)_{II}$ and $(B/C)_{II}$ were determined by equation 54 and the a_{ij} are given by equation 58.

For Vertical Polarization at the Z_t "to" level, the following identifications are made.

Identify terms of equation 48 with equation (49,b). That is:

$$\left. \begin{aligned} \bar{A} &= \left(-\frac{Q}{C}\right); \bar{B} = \left(-\frac{G}{C}\right) \\ a_{21} &= \left\{ C h_1(\rho_t) + [K h_1'(\rho_t) + L h_1(\rho_t)]/h^2 \right\} \\ a_{22} &= \left\{ C h_2(\rho_t) + [K h_2'(\rho_t) + L h_2(\rho_t)]/h^2 \right\} \\ v_2 &= -2(E_{\parallel}^i)_t \\ L &= i\left(\frac{\alpha}{2k}\right) \text{ from equation 41} \end{aligned} \right\} \quad (B-61)$$

For Set I B.C.: (e.g. $(E_{\parallel}^i)_f = 1$; $(E_y^i)_f = 0$)

$$\left(-\frac{Q}{C}\right)_I a_{21} + \left(-\frac{G}{C}\right)_I a_{22} = -2 \left(\left(E_{\parallel}^i\right)_t\right)_I$$

$$\boxed{\text{or } \left(\left(E_{\parallel}^i\right)_t\right)_I = \frac{\left[\left(\frac{Q}{C}\right)_I a_{21} + \left(\frac{G}{C}\right)_I a_{22}\right]}{2}}$$
(B-62)

Where $(A/C)_I$ and $(G/C)_I$ are given by equation 56 and the a_{ij} are given by equation 61.

For Set II B.C.: $\left(\text{e.g. } (E_{\parallel}^i)_f = 0; (E_y^i)_f = 1 \right)$

$$\left(-\frac{Q}{C} \right)_{II} a_{21} + \left(-\frac{G}{C} \right)_{II} a_{22} = -2 \left((E_{\parallel}^i)_t \right)_{II}$$

$$\boxed{\text{or } \left((E_{\parallel}^i)_t \right)_{II} = \frac{\left(\frac{Q}{C} \right)_{II} a_{21} + \left(\frac{G}{C} \right)_{II} a_{22}}{2}} \quad (\text{B-63})$$

Where $(Q/C)_{II}$ and $(G/C)_{II}$ are given by equation 57 and the a_{ij} are given by equation 61.

III. Knowing the values of

$$\left((E_{\parallel}^i)_t \right)_I, \left((E_{\parallel}^i)_t \right)_{II}, \left((E_y^i)_t \right)_I \text{ and } \left((E_y^i)_t \right)_{II}$$

at the "to" level, from both sets of coefficients A, B, Q and G, as determined in Step II, the elements of the ionospheric R's can be found at the "to" level.

For Horizontal Polarization equation 23 at the "to" level is:

$$\boxed{\begin{aligned} &\left(-\frac{A}{C} \right) h_1(\rho_t) + \left(-\frac{B}{C} \right) h_2(\rho_t) + \left[E_y^i \right]_t \left(\frac{1R_{\perp} + 1}{C} \right)_t \\ &+ \left[E_{\parallel}^i \right]_t \left(\frac{1R_{\parallel}}{C} \right)_t = 0 \end{aligned}} \quad (\text{B-64})$$

Applying the B.C. of Set I for $(A/C)_I$ and $(B/C)_I$ to equation (64) and applying the B.C. of Set II for $(A/C)_{II}$ and $(B/C)_{II}$ to equation (64) gives two equations.

$$\begin{aligned} & \left(\left(E_y^i \right)_t \right)_I \left(\frac{1R_1 + 1}{C} \right)_t + \left(\left(E_{\parallel}^i \right)_t \right)_I \left(\frac{\parallel R_1}{C} \right)_t \\ & + \left(-\frac{A}{C} \right)_I h_1(\rho_t) + \left(-\frac{B}{C} \right)_I h_2(\rho_t) = 0 \end{aligned} \quad (B-65)$$

and

$$\begin{aligned} & \left(\left(E_y^i \right)_t \right)_{II} \left(\frac{1R_1 + 1}{C} \right)_t + \left(\left(E_{\parallel}^i \right)_t \right)_I \left(\frac{\parallel R_1}{C} \right)_t \\ & + \left(-\frac{A}{C} \right)_{II} h_1(\rho_t) + \left(-\frac{B}{C} \right)_{II} h_2(\rho_t) = 0 \end{aligned} \quad (B-66)$$

Identifying terms of equations 65 and 66 with equations 49,a and 49,b respectively gives:

$$\left. \begin{aligned} \bar{B} &= \left(\frac{1R_1 + 1}{C} \right)_t & \bar{A} &= \left(\frac{\parallel R_1}{C} \right)_t \\ a_{11} &= \left(\left(E_{\parallel}^i \right)_t \right)_I & a_{12} &= \left(\left(E_y^i \right)_t \right)_I \\ a_{21} &= \left(\left(E_{\parallel}^i \right)_t \right)_{II} & a_{22} &= \left(\left(E_y^i \right)_t \right)_{II} \\ v_1 &= \left(\frac{A}{C} \right)_I h_1(\rho_t) + \left(\frac{B}{C} \right)_I h_2(\rho_t) \\ v_2 &= \left(\frac{A}{C} \right)_{II} h_1(\rho_t) + \left(\frac{B}{C} \right)_{II} h_2(\rho_t) \end{aligned} \right\} \quad (B-67)$$

Then from equation (50,a):

$$\left(\frac{\parallel R_1}{C} \right)_t = \frac{v_1 \left(\left(E_y^i \right)_t \right)_{II} - v_2 \left(\left(E_y^i \right)_t \right)_I}{\left(\left(E_{\parallel}^i \right)_t \right)_I \left(\left(E_y^i \right)_t \right)_{II} - \left(\left(E_{\parallel}^i \right)_t \right)_{II} \left(\left(E_y^i \right)_t \right)_I} \quad (B-68)$$

And from equation (50,b):

$$\left(\frac{1R_1 + 1}{C} \right)_t = \frac{v_2 \left(\left(E_{\parallel}^i \right)_t \right)_I - v_1 \left(\left(E_{\parallel}^i \right)_t \right)_{II}}{\left(\left(E_{\parallel}^i \right)_t \right)_I \left(\left(E_y^i \right)_t \right)_{II} - \left(\left(E_{\parallel}^i \right)_t \right)_{II} \left(\left(E_y^i \right)_t \right)_I} \quad (B-69)$$

For Vertical Polarization equation 47 at the "to" level is:

$$\left[\begin{aligned} & \left(-\frac{Q}{C} \right) h_1(\rho_t) + \left(-\frac{G}{C} \right) h_2(\rho_t) + \left(E_{\parallel}^i \right)_t \left(\frac{\parallel R_{\parallel} + 1}{C} \right)_t \\ & + \left(E_y^i \right)_t \left(\frac{\perp R_{\parallel}}{C} \right)_t = 0 \end{aligned} \right] \quad (B-70)$$

Applying the conditions of Set I for $(Q/C)_I$ and $(G/C)_I$ to equation 70 and applying the conditions of Set II for $(Q/C)_{II}$ and $(G/C)_{II}$ to equation (70) gives the two equations:

$$\begin{aligned} & \left(\left(E_{\parallel}^i \right)_t \right)_I \left(\frac{\parallel R_{\parallel} + 1}{C} \right)_t + \left(\left(E_y^i \right)_t \right)_I \left(\frac{\perp R_{\parallel}}{C} \right)_t \\ & + \left[\left(-\frac{Q}{C} \right)_I h_1(\rho_t) + \left(-\frac{G}{C} \right)_I h_2(\rho_t) \right] = 0 \end{aligned} \quad (B-71)$$

and

$$\begin{aligned} & \left(\left(E_{\parallel}^i \right)_t \right)_{II} \left(\frac{\parallel R_{\parallel} + 1}{C} \right)_t + \left(\left(E_y^i \right)_t \right)_I \left(\frac{\perp R_{\parallel}}{C} \right)_t \\ & + \left[\left(-\frac{Q}{C} \right)_{II} h_1(\rho_t) + \left(-\frac{G}{C} \right)_{II} h_2(\rho_t) \right] = 0 \end{aligned} \quad (B-72)$$

Identifying terms of equation 65 with equation 49,a

$$\left. \begin{aligned} \bar{A} &= \left(\frac{\parallel R_{\parallel} + 1}{C} \right)_t & \bar{B} &= \left(\frac{\perp R_{\parallel}}{C} \right)_t \\ a_{11} &= \left(\left(E_{\parallel}^i \right)_t \right)_I & a_{12} &= \left(\left(E_y^i \right)_t \right)_I \\ a_{21} &= \left(\left(E_{\parallel}^i \right)_t \right)_{II} & a_{22} &= \left(\left(E_y^i \right)_t \right)_{II} \\ v_1 &= \left(\frac{Q}{C} \right)_I h_1(\rho_t) + \left(\frac{G}{C} \right)_I h_2(\rho_t) \\ v_2 &= \left(\frac{Q}{C} \right)_{II} h_1(\rho_t) + \left(\frac{G}{C} \right)_{II} h_2(\rho_t) \end{aligned} \right\} \quad (B-67)$$

Then from equation (50,a)

$$\left(\frac{R_{\parallel} + 1}{C}\right)_t = \frac{v_1 \left(\left(E_y^i\right)_t\right)_{II} - v_2 \left(\left(E_y^i\right)_t\right)_I}{\left(\left(E_{\parallel}^i\right)_t\right)_I \left(\left(E_y^i\right)_t\right)_{II} - \left(\left(E_{\parallel}^i\right)_t\right)_{II} \left(\left(E_y^i\right)_t\right)_I} \quad (B-73)$$

and from equation (50,b)

$$\left(\frac{1}{C}\right)_t = \frac{v_2 \left(\left(E_{\parallel}^i\right)_t\right)_I - v_1 \left(\left(E_{\parallel}^i\right)_t\right)_{II}}{\left(\left(E_{\parallel}^i\right)_t\right)_I \left(\left(E_y^i\right)_t\right)_{II} - \left(\left(E_{\parallel}^i\right)_t\right)_{II} \left(\left(E_y^i\right)_t\right)_I} \quad (B-74)$$

APPENDIX C

MODIFIED GROUND REFLECTION COEFFICIENTS

Given: The Fresnel Reflection Coefficients at the ground (i.e., $z = 0$)

$$R_{\parallel}^{\circ} = \frac{CN_g^2 - W}{CN_g^2 + W} \quad (C-1)$$

$$R_{\perp}^{\circ} = \frac{C - W}{C + W} \quad (C-2)$$

where

$$W = \sqrt{N_g^2 - S^2} \quad (C-3)$$

with

$$S = \sin \theta$$

$$N_g = \text{index at refraction of ground}$$

Define modified reflection coefficients as:

$$A = \left(\frac{1}{R_{\parallel}^{\circ}} + 1 \right) / C \quad (C-4)$$

$$B = \left(\frac{1}{R_{\perp}^{\circ}} + 1 \right) / C \quad (C-5)$$

where $C = \cos \theta$

substituting equations 1 and 2 into equations 4 and 5 gives:

$$A = \left[\frac{1}{\left(\frac{CN_g^2 - W}{CN_g^2 + W} \right)} + 1 \right] 1/C \quad (C-6)$$

$$B = \left[\frac{1}{\left(\frac{C - W}{C + W} \right)} + 1 \right] 1/C \quad (C-7)$$

then with further manipulation:

$$\begin{aligned}
 A &= \left[\frac{CN_g^2 + W}{CN_g^2 - W} + 1 \right] / C = \left[\frac{CN_g^2 + W + CN_g^2 - W}{CN_g^2 - W} \right] / C \\
 &= \left[\frac{2 CN_g^2}{CN_g^2 - W} \right] / C = \frac{1}{1/2[C - W/N_g^2]} \quad (C-8)
 \end{aligned}$$

and

$$\begin{aligned}
 B &= \left[\frac{C + W}{C - W} + 1 \right] / C = \left[\frac{C + W + C - W}{C - W} \right] / C \\
 &= \left[\frac{2C}{C - W} \right] 1/C = \frac{2 \cdot 1/W}{(C - W)} \cdot 1/W = \frac{1}{\frac{1}{2} \left[\frac{C}{W} - 1 \right]} \quad (C-9)
 \end{aligned}$$

then define:

$$A = \frac{\|n\|}{\|d\|} \quad \text{so that:} \quad \left. \begin{aligned} \|n\| &= 1 \\ \|d\| &= \frac{1}{2}[C - W/N_g^2] \end{aligned} \right\} \quad (C-10)$$

and

$$B = \frac{1n_1}{1d_1} \quad \text{So that:} \quad \left. \begin{aligned} 1n_1 &= 1/W \\ 1d_1 &= \frac{1}{2} \left[\frac{C}{W} - 1 \right] \end{aligned} \right\} \quad (C-11)$$

The final forms of A and B as presented above are well behaved functions as $N_g \rightarrow \infty$ and $\theta \rightarrow 90^\circ$.

APPENDIX D

MESH SQUARE ANALYSIS

Consider an individual mesh square. See figure D-I. Any point in the complex θ plane which lies within (or on an edge of) the square is described by the complex function $F(\theta_r, \theta_i)$. That is:

$$F(\theta) = F_R(\theta_r, \theta_i) + j F_I(\theta_r, \theta_i) \quad (D-1)$$

and

$$\theta = \theta_r + j\theta_i \quad (D-2)$$

where

$F_R(\theta)$ is the real part of the complex function $F(\theta)$

$F_I(\theta)$ is the imaginary part of the complex function $F(\theta)$

θ_r is the real part of the complex number (θ)

θ_i is the imaginary part of the complex number (θ) .

The lower left hand corner of the square is taken as the reference position and each side of the square is one-mesh unit in length. The geometry of the mesh square is shown below where the $F(\theta_r, \theta_i)$ values of the corners of the square are identified.

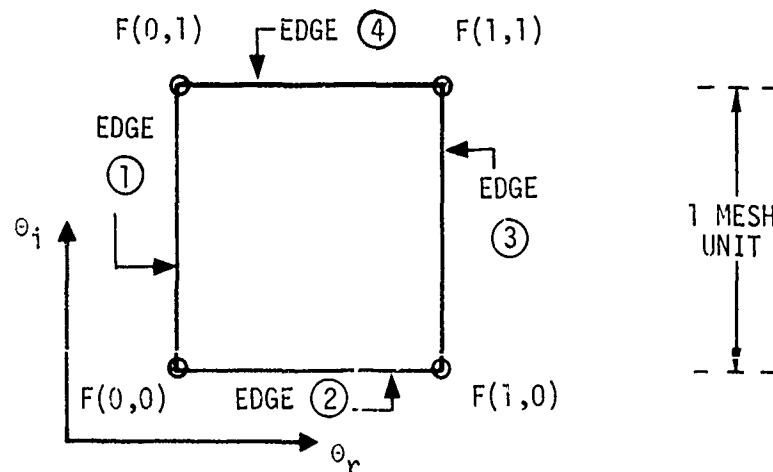


Figure D-I. Mesh Square Geometry

A basic assumption put on $F(\theta_r, \theta_i)$ is that $F_R(\theta_r, \theta_i)$ and $F_I(\theta_r, \theta_i)$ are to be linear along each edge of the mesh square.

The function of $F(\theta_r, \theta_i)$ at any point within the square is given by:

$$\begin{aligned} F(\theta_r, \theta_i) = & F(0,0)(1 - \theta_r - \theta_i + \theta_r \theta_i) \\ & + F(0,1)(\theta_i - \theta_r \theta_i) \\ & + F(1,0)(\theta_r - \theta_r \theta_i) \\ & + F(1,1)(\theta_r \theta_i) \end{aligned} \quad (D-3)$$

or expanding:

$$\begin{aligned} F(\theta_r, \theta_i) = & F(0,0) + \theta_i(F(0,1) - F(0,0)) + \theta_r(F(1,0) - F(0,0)) \\ & + \theta_r \theta_i(F(0,0) + F(1,1) - F(0,1) - F(1,0)) \end{aligned} \quad (D-4)$$

which can be written as

$$F(\theta_r, \theta_i) = a + b\theta_i + c\theta_r + d\theta_r \theta_i \quad (D-5)$$

(a) Hyperbolic and Asymptotic Lines:

Consider the imaginary part of $F(\theta_r, \theta_i)$ and $F_I(\theta_r, \theta_i) = 0$. This gives (from equation D-5):

$$a_i + b_i \theta_i + c_i \theta_r + d_i \theta_r \theta_i = 0 \quad (D-6)$$

Also, in the same way the real part of $F(\theta_r, \theta_i)$ gives:

$$a_r + b_r \theta_i + c_r \theta_r + d_r \theta_r \theta_i = 0 \quad (D-7)$$

Equation D-6 (or equation D-7) is the equation of an "equilateral" or "rectangular" hyperbola with vertical and horizontal asymptotes.

To determine the center of the asymptotes for the hyperbolic curve of equation D-6, that equation must be solved for θ_r and also for θ_i .

That is, (from equation D-6):

$$(a_i + b_i \theta_i) + (c_i + d_i \theta_i) \theta_r = 0$$

or

$$\theta_r = - \left(\frac{a_i + b_i \theta_i}{c_i + d_i \theta_i} \right) \quad (D-8)$$

Also from equation D-6:

$$(a_i + c_i \theta_r) + (b_i + d_i \theta_r) \theta_i = 0$$

or

$$\theta_i = - \left(\frac{a_i + c_i \theta_r}{b_i + d_i \theta_r} \right) \quad (D-9)$$

Now let $\theta_r \rightarrow \infty$ in equation (D-8) so that:

$$c_i + d_i \theta_i = 0$$

or

$$\theta_i = - \frac{c_i}{d_i} \quad (D-10)$$

The line described by equation (D-10) is the horizontal asymptote to the equilateral hyperbola.

From equation D-9 let $\theta_i \rightarrow \infty$ so that:

$$b_i + d_i \theta_r = 0$$

or

$$\theta_r = - \frac{b_i}{d_i} \quad (D-11)$$

The line described by equation (D-11) is the vertical asymptote to the equilateral hyperbola. These asymptotic lines in relation to the mesh square are illustrated in the following figure:

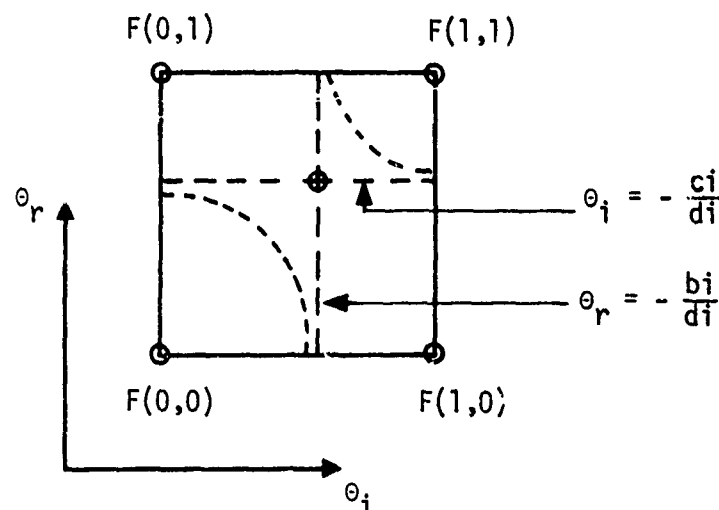


Figure D-II. Vertical and Horizontal Asymptotes for an equilateral hyperbola.

(b) Intersection of Hyperbolic Lines with the Edges of the Mesh Square:

Identify the edges of the mesh square, as shown in figure D-I, as edges 1, 2, 3 and 4. Also, consider only the imaginary part of $F(\theta_r, \theta_i)$ of equation D-4 and equate $F_I(\theta_r, \theta_i)$ to 0. Equation D-4 becomes:

$$\begin{aligned}
 & F_I(0,0) + \theta_i(F_I(0,1) - F_I(0,0)) + \theta_r(F_I(1,0) \\
 & - F_I(0,0)) + \theta_r\theta_i(F_I(0,0) + F_I(1,1) - F_I(0,1) \\
 & - F_I(1,0)) = 0
 \end{aligned} \tag{D-12}$$

To find the intersection of a branch of the equilateral hyperbola, described by equation D-12, with edge No. 1 of the square, set $\theta_r = 0$ and

note that θ_i varies between 0 and 1 along edge No. 1. Equation D-12 becomes:

$$F_I(0,0) + \theta_i(F_I(0,1) - F_I(0,0)) = 0$$

or

$$\theta_i = \frac{-F_I(0,0)}{F_I(0,1) - F_I(0,0)} \quad (D-13)$$

Therefore the point of intersection for edge No. 1 is:

$$\theta_r = 0, \theta_i \text{ is as given by equation D-13.}$$

The point of intersection between the hyperbola and the remaining three sides may be computed in a similar manner. The results are as follows:

The point of intersection for edge No. 2 is:

$$\theta_r = -\frac{F_I(0,0)}{F_I(1,0) - F_I(0,0)}, \theta_i = 0 \quad (D-14)$$

The point of intersection edge No. 3 is:

$$\theta_r = 1, \theta_i = -\frac{F_I(1,0)}{F_I(1,1) - F_I(1,0)} \quad (D-15)$$

The point of intersection edge No. 4 is:

$$\theta_r = -\frac{F_I(0,1)}{F_I(1,1) - F_I(0,1)}, \theta_i = 1 \quad (D-16)$$

APPENDIX E
SOLUTION OF THE QUADRATIC EQUATION
FOR THE GENERAL ROOT-FINDER.

Given the Quadratic equation as used in the Root-Finder procedure:

$$aX^2 + 2bX + C = 0 \quad (E-1)$$

$$X = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a} = \frac{b}{a} \left[-1 \pm \sqrt{1 - \frac{ac}{b^2}} \right]$$

let $\epsilon = -ac/b^2$

and then $(1+\epsilon)^{1/2} = 1 + \frac{1}{2}\epsilon + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \frac{\epsilon^2}{2} + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right) \frac{\epsilon^3}{3 \cdot 2} + \dots$

Now $X = \frac{b}{a} \left[-1 \pm \sqrt{1 + \epsilon} \right]$

so that

$$\begin{aligned} X_1 &= \frac{b}{a} \left\{ \frac{1}{2} \left(\frac{-ac}{b^2} \right) + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \left(\frac{-ac}{b^2} \right)^2 + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right) \frac{\left(\frac{-ac}{b^2} \right)^3}{3 \cdot 2} + \dots \right\} \\ X_1 &= -\frac{1}{2} \frac{C}{b} \left\{ 1 + \frac{\left(\frac{1}{2} - 1 \right)}{2} \left[-\frac{ac}{b^2} \right] + \frac{\left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right)}{3 \cdot 2} \left[\frac{-ac}{b^2} \right]^2 + \dots \right\} \quad (E-2) \end{aligned}$$

let $t_0 = 1$

$$t_1 = \frac{\left(\frac{1}{2} - 1 \right)}{2} \left[-\frac{ac}{b^2} \right]$$

$$t_2 = \frac{\left(\frac{1}{2} - 1 \right) \left(\frac{1}{2} - 2 \right)}{3 \cdot 2} \left[\frac{ac}{b^2} \right]^2$$

$$\text{or } t_i = t_{i-1} \left(\frac{\frac{1}{2} - 1}{1 + 1} \right) \left[-\frac{ac}{b^2} \right]$$

$$\text{or } t_i = t_{i-1} \left(\frac{i - 1/2}{1 + 1} \right) \left[\frac{ac}{b^2} \right]$$

$$\text{and } x_1 = -\frac{1}{2} \frac{c}{b} \left\{ t_0 + t_1 + t_2 + \dots + t_N \right\} \quad (\text{E-3})$$

$$\begin{aligned} \text{Now } x_1 &= \frac{b}{a} \left[-1 + \sqrt{1 + \epsilon} \right] \\ \text{also } x_2 &= b/a \left[-1 - \sqrt{1 + \epsilon} \right] \end{aligned} \quad (\text{E-4})$$

$$\text{Then: } -x_1 = \frac{b}{a} \left[1 - \sqrt{1 + \epsilon} \right]$$

$$\text{and } \frac{-2b}{a} - x_1 = \frac{b}{a} - \frac{b}{a} \sqrt{1 + \epsilon} - \frac{-2b}{a}$$

$$\text{or } \frac{-2b}{a} - x_1 = -\frac{b}{a} - \frac{b}{a} \sqrt{1 + \epsilon}$$

$$\text{or } \frac{-2b}{a} - x_1 = \frac{b}{a} \left[-1 - \sqrt{1 + \epsilon} \right]$$

such that:

$$\frac{-2b}{a} - x_1 = x_2 \quad (\text{E-5})$$

Equations E-3 and E-5 give the solutions of the Quadratic equation (i.e., Equation E-1) as solved in the Root-Finder routine.

APPENDIX F
FORTRAN LISTING OF
THE MODESRCH COMPUTER
PROGRAM

```

C PROGRAM ***MODESRCH***
C THIS PROGRAM FINDS THE WAVEGUIDE MODES FOR THE EARTH-IONOSPHERE
C WAVEGUIDE FOR ELF-VLF-LF
C MAIN
C FOR MS
C PROGRAM FOR COMPUTING EARTH-IONOSPHERE WAVEGUIDE EIGEN-ANGLES AND
C EXCITATION FACTORS WHICH ARE OUTPUT ON PUNCHED CARDS
C FOR USE IN A PROGRAM WHICH COMPUTES FIELD STRENGTHS.
C
C THIS ROUTINE CONTROLS THE SEQUENCE OF INPUT AND EXECUTION AND
C PROVIDES FOR NAMELIST INPUT. A DESCRIPTION OF THE
C INPUT PARAMETERS IS GIVEN IN A DNA REPORT.
C
C

```

```

COMMON/IC COM/IDENT(20)
COMMON/PTALT/ TALT,RALT
COMMON/BPRT/ BPRINT
COMMON /FP/HDFWK
COMMON /RTOL C/RTOL
COMMON/IXACT C/IXACT
COMMON/FLD COM/AZIM,CODIP,MAG FLD
COMMON/FREQ COM/FREQ
COMMON/GND COM/EPSP,SIGMA
COMMON/BFP COM/BETA,HPRIME,SCLHTS,ENMIN
COMMON/WR COM/OMEGA,TEMPUB
COMMON/TOPE C/TOPE
COMMON/EXPNU C/COEFNU(3),EXPNU(3)
COMMON/ICN COM/IONS,MRATIO(3)
COMMON/LIMS C/RANGER(2),RANGE1(2)
COMMON/GMAX C/GMAX
COMMON/PARAM C/TMESH
COMMON/SEP COM/SEP
COMMON/TCL COM/LUB
COMMON/NFIGN C/NEIGEN
COMMON/NFNCH C/NPUNCH
COMMON/CRD PRT/CARDPT
COMMON/CRD PNC/CARDPN
COMMON/CLT OF/ ICUT,CUTOFF
COMMON/RHO COM/RHO,REFLHT
COMMON/H COM/H
COMMON/MFRNT/MPRINT
COMMON/XFRNT/XPRINT
COMMON/EXTRA9/IBUFFR(500)

```

```

C
C
NAMELIST/DATUM/AZIM,CODIP,MAG FLD,
$      FREQ,TALT,RALT,
$      LPSP,SIGMA,
$      BETA,HPRIME,SCLHTS,ENMIN,
$      TOPEN,TEMPUB,
$      COEFNU,EXPNU,NUFLAG,
$      MRATIO,TMESH,LUB,
$      RANGER,RANGE1,

```

```

$      GMAX,SEP,RTOL,IEXACT,
$      NEIGEN,
$      NPUNCH,RHO,REFLHT,
$      H,
$      ICUT,CUTOFF,
$      BPRINT,NPBUF,
$      MPRINT,XPRINT,CARDPT,CARDPN

```

C
C

```

      INTEGER PROF,COLF,ADAT,QUIT
      INTEGER CARDPT,CARDPN
      INTEGER XPRINT
      INTEGER BPRINT
      REAL MAGFLD,MRATIO,LUB
      DIMENSION IBCD(20)
      DATA PROF/'PROF'//,COLF/'COLF'//,IDEN/'ID '//,
$      WAVE/'WAVE'//,ADAT/' &DA'//,IEND/' &EN'//,
$      QUIT/'QUIT'//
      DATA IBLANK/' '//
      DATA ITHREE/'3 '//

```

C
C

```

      HOFWR=0.0
      OMEGAR=2.5E05

```

C
C
C

INITIAL VALUES OF NAMELIST PARAMETERS.

```

      IEXACT=0
      ICUT = 1
      CUTOFF=0.0001
      TGPEN = 1.0E3
      SCLHTS = 3.0
      EAMIN = 1.0E-1
      TEMPOR=2.0
      HPRIME=-99.0
      FREQ = 0.0
      RHO = 0.0
      REFLHT = 50.0
      H = 50.0
      TALT=0.0
      RALT=0.0

```

C

```

      GMAX = 5.0
      RTOL=0.001
      SEP = 0.1
      TMESH=-1.0
      LUB=-1.0

```

C

```

      NUFLAG = 0
      COEFNU(1) = 1.816E11
      COEFNU(2) = 4.540E9
      COEFNU(3) = 4.540E9
      EXPNU(1) = -0.15

```



```

C
C
C NAMELIST INPUT.
C
20 IBCD(1) = IBCD(1)
   IBCD(2) = IBCD(2)
   J = 3
   DO 21 I=1,25
     READ 901,IBCD
     PRINT 902,IBCD
     IF (IBCD(1) .EQ. IEND) GO TO 22
     DO 21 I=1,25
       IBCD(J) = IBCD(I)
       J = J+1
21 CONTINUE
22 IBCD(J) = IBCD(1)
   J = J+1
   IBCD(J) = IBCD(2)
   CALL CDRK (IBCD,4*J)
   READ(1,CATUM)
   HCFWK=C.C
C
   IF (TMESH .LT. 0) TMESH=SQRT(3.75/FREQ)
   IF (LUB .LT. 0) LUB=SQRT(15.0/FREQ)*0.01
   PRINT 205, TMESH, LUB
205 FORMAT('C', 'TMESH = ', F6.3, 4X, 'LUB = ', F6.4)
   IF (FREQ .LE. 1.0) IFXACT=1
C
   IF (NUFLAG .EQ. 0) GO TO 40
   COEFNU(1) = 4.303E11
   COEFNU(2) = 1.076E10
   COEFNU(3) = 1.076E10
   EXPNU(1) = -0.1622
   EXPNU(2) = -0.1622
   EXPNU(3) = -0.1622
C
C MISCELLANEOUS INPUT OPERATIONS AND CALL TO CONTROL ROUTINE.
40 IF (HPRIME .GT. 0) CALL SET EN
   IF (HPRIME .LT. 0 .AND. FREQ .GT. 1.0) CALL CUT TOP
   IF (RANGER(1) .LT. RANGER(2)) GO TO 41
   TEMP = RANGER(1)
   RANGER(1) = RANGER(2)
   RANGER(2) = TEMP
41 IF (RANGEI(1) .GT. RANGEI(2)) GO TO 42
   TEMP = RANGEI(1)
   RANGEI(1) = RANGEI(2)
   RANGEI(2) = TEMP
42 CONTINUE
   INCR=ITIME(ICK)
   CALL WVGLID
   INCR=ITIME(ICK)-INCR
   TIME=FLCAT(INCR)/100.
   PRINT 615, TIME

```

```

615  FORMAT('0',/' TIME REQUIRED FOR THIS PROFILE = ',F10.2,' SECONDS')
      PRINT 980
980  FORMAT('1')
      IF(HPRIME .LT. 0 ) CALL RLPEAT
      GO TO 10
C
901  FORMAT(2CA4)
902  FORMAT(' ',20A4)
C
      END

```

```

      SUBROUTINE WVGUID
C     FOR MS
C     CONTROL ROUTINE.
C
      COMMON/TOL COM/TOL
      COMMON/IXACT C/IXACT
      COMMON/MFRNT/MFPRINT
      COMMON/MPNCH C/MPNCH
      COMMON/NEIGN C/NEIGN
      COMMON/LIMS C/RANGER(2),RANGEI(2)
      COMMON/PARAM C/TMESH
      COMMON/GMAX C/GMAX
      COMMON/NR OF AS/NR A
      COMMON/HTS COM/HTS(101)
      COMMON/SIDES C/TLEFT,TRIGHT,TBOT,TTOP
      COMMON/THETA C/THETA
      COMMON/FIG COM/EIGEN(100),NR E
      COMMON/KXACT C/ KEXACT
      COMMON/D COM/D
      COMPLEX THETA,EIGEN,
      *          ELIST(100),
      *          TEMP
C
C
      IF (MPNCH .EQ. 0 .AND. NEIGN .EQ. 0) GO TO 90
C
C     INITIALIZE FULL-WAVE SOLUTION.
      CALL SET RK
      CALL INIT 1
C
C     SET UP SMALL RECTANGLES.
      NR EL = 0
      NR BOXS = (RANGER(2)-RANGER(1)-TOL)/GMAX+1.0
      IF(IXACT .NE. 0) NR BOXS = 1
      RLSPAN = (RANGER(2)-RANGER(1))/NR BOXS
      IBOX = 1
      TTOP = RANGEI(1)
      TBOT = RANGEI(2)
      TLEFT = RANGER(2)
C
C     ZO TRIGHT = TLEFT
      TLEFT = TRIGHT-RLSPAN
C
C     SET REFERENCE HEIGHT AND INITIALIZE LAGRANGE INTERPOLATION.
      D = 0.0
      IF(IXACT .EQ. 0) CALL SET RH
C
C     INITIALIZE RBAR SOLUTION AND FREE-SPACE INTEGRATION.
      CALL INIT RB
      CALL INIT FS(HTS(NRA),D)
C
C     FIND ZEROS OF F FUNCTION - LAGRANGE INTERPOLATION USED IF NOT ELF.
      KEXACT=0

```

```

      CALL FZEROS (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL,MPRINT,
$          EIGEN,NR E)
      IF(NR F .EQ. 0) GO TO 77
C
C  ORDER EIGEN ANGLES ACCORDING TO VALUE OF REAL PART.
      IF (NR E .EQ. 1) GO TO 64
      NKEM1 = NR E-1
      DO 63 J=1,NKEM1
      JIP1 = JE+1
      DO 62 J=JIP1,NR F
      IF (REAL(EIGEN(J)) .LT. REAL(EIGEN(JE))) GO TO 62
      TEMP = EIGEN(J)
      EIGEN(J) = EIGEN(JE)
      EIGEN(JE) = TEMP
62 CONTINUE
63 CONTINUE
64 CONTINUE
C
C  FOLLOW EIGEN ANGLES AS F FUNCTION IS CHANGED FROM THAT USING
C  LAGRANGE INTERPOLATION TO THAT USING EXACT
C  FULL-WAVE SOLUTION VALUES OF REFLECTION COEFFICIENTS.
      IF(IEEXACT .EQ. 0) CALL FINAL
      IF(NR E .EQ. 0) GO TO 77
C
C  DISCARD EIGEN-ANGLES WHICH ARE OUTSIDE CURRENT SMALL RECTANGLE.
      J=0
      DO 71 JE=1,NR F
      IF (REAL(EIGEN(JE)) .LT. TLEFT-2.0*TOL .OR.
$          REAL(EIGEN(JE)) .GT. TRIGHT+2.0*TOL) GO TO 71
      J = J+1
      EIGEN(J) = EIGEN(JE)
71 CONTINUE
      NR F = J
C
C  STORAGE OF EIGEN ANGLES.
      IF(NR E .EQ. 0) GO TO 77
      DO 76 JE=1,NR E
      NR EL = NR EL+1
76 ELIST(NR EL) = EIGEN(JE)
77 IBOX = IBOX+1
      IF (IBOX .LE. NR BOXS) GO TO 20
      NR E = NR EL
      IF (NR E .EQ. 0) GO TO 85
      DO 78 JE=1,NR E
78 EIGEN(JE) = ELIST(JE)
C
C  COMPUTE AND PUNCH NPUNCH OUTPUT AND PRINT TABLE.
      IF (NPUNCH .NE. 0) CALL NP OUT
C
C  PRINT AND PUNCH EIGEN ANGLES IF NEIGEN SET TO 1.
      IF (NEIGEN .NE. 0) CALL NE OUT
      RETURN
C

```

```
85 PRINT 985  
985 FORMAT ('0','N) MODES')  
RETURN
```

```
C  
90 PRINT 990  
990 FORMAT ('0','SET NPUNCH OR NEIGEN')  
STOP
```

```
C  
END
```

```

SUBROUTINE SET RH
C FOR AS
C ROUTINE FOR SELECTING GIVEN POINTS IN THE LAGRANGE INTERPOLATION
C AND FOR SELECTING THE REFERENCE HEIGHT.
C
COMMON/MPRINT/MPRINT
COMMON/PARAM C/TMESH
COMMON/HTS COM/HTS(101)
COMMON/NR OF AS/NK A
COMMON/SIDES C/TLEFT,TRIGHT,TBOT,TTOP
COMMON/THETA C/THETA
COMMON/IDERV C/IDERV
COMMON/INTEGR/R(4),DRDC(4)
COMMON/D CCM/D
DIMENSION PART(2)
COMPLEX THETA,R,DRDC,
$ PARTS,
$ THETAG(20),
$ FBOT(4,20),R GIVEN(4,20),R TEMP(4,20),R SAVE(4,20)
EQUIVALENCE (PART(1),PARTS)
DATA HT INC/1.0/
DATA HMARGN/20.0/

C
C
C SELECT VALUES OF THETA AT WHICH FULL-WAVE SOLUTIONS ARE TO BE
C CARRIED OUT FOR USE IN LAGRANGE INTERPOLATION.
C
JLT = TLEFT/TMESH-1.0
JKT = TRIGHT/TMESH+2.0
JBOT = TBOT/TMESH-2.0
JTOP = TTOP/TMESH
JTUP = JTOP+1

C
PART(1) = (JKT+1)*TMESH
PART(2) = (JTOP+1)*TMESH
THETAG(1) = PARTS
PART(1) = (JLT-1)*TMESH
PART(2) = (JTOP+1)*TMESH
THETAG(2) = PARTS
PART(1) = (JKT+1)*TMESH
PART(2) = (JBOT-1)*TMESH
THETAG(3) = PARTS
PART(1) = (JLT-1)*TMESH
PART(2) = (JBOT-1)*TMESH
THETAG(4) = PARTS
NG = 4
CALL INIT LG (THETAG,NG)
NG8 = NG*8

C
C SELECT THE HEIGHT (TO NEAREST WHOLE KILOMETER) AT WHICH
C THE MINIMUM OCCURS IN
C THE SQUARE OF THE MAGNITUDE OF THE DERIVATIVE OF
C  $(R+1.0)/C$  WRT  $C=\cos(\text{THETA})$ , SUMMED OVER ALL FOUR
C ELEMENTS OF R=REFLECTION MATRIX AND SUMMED OVER ALL

```

```

C          POINTS AT WHICH FULL-WAVE SOLUTIONS ARE CARRIED OUT
C          FOR USE IN LAGRANGE INTERPOLATION. DERIVATIVES OF THE
C          FULL-WAVE SOLUTIONS ARE NOT COMPUTED. RATHER,
C          DERIVATIVES OF THE LAGRANGE INTERPOLATION FORMULA
C          ARE USED. NOTE THAT THE FULL-WAVE SOLUTIONS ARE
C          CARRIED OUT TO THE BOTTOM OF THE PROFILE AFTER WHICH
C          THE FULL-WAVE SOLUTION IS CARRIED OUT IN AN UPWARD
C          DIRECTION THROUGH FREE SPACE OVER A CURVED EARTH TO
C          HEIGHTS AT ONE KILOMETER INTERVALS. THE HEIGHT AT
C          WHICH THE MINIMUM OCCURS IS PRINTED OUT AS THE
C          'REFLECTION HT' AND IS THEREAFTER USED AS THE
C          REFERENCE HEIGHT AT WHICH THE F FUNCTION IS COMPUTED.
          IF (MPRINT .NE. 0) PRINT 911
911  FORMAT ('1','INTEGRATION TO DETERMINE REFLECTION HT--')
          IDERIV = 0
          RHTEMP = 9.9E9
          RFSAVE = 9.9E9
          LIMIT = NG
          GB = 0.0
          DO 12 JG=1,NG
          THETA = THETAG(JG)
          CALL INTEG
12  CALL XFER (R,R GIVEN(1,JG),8)
C
          CALL XFER (R GIVEN,RBOT,NG8)
          CALL XFER (R GIVEN,R TEMP,NG8)
          CALL XFER (R GIVEN,R SAVE,NG8)
          CALL SET LAG (NG,R GIVEN)
          DO 13 JG=1,LIMIT
          THETA = THETAG(JG)
          CALL LAG DER
          DO 13 I=1,4
13  GB = GB+CABS(DRDC(I))*2
          GB MIN = GB
          IF (MPRINT .NE. 0) PRINT 912,HTS(NR A),GB
          HTRST = HTS(NR A)
14  HTLAST = HTRST+HT INC
          CALL INIT FS (HTS(NR A),HTLAST)
          GB = 0.0
          DO 15 JG=1,NG
          THETA = THETAG(JG)
          CALL XFER (RBOT(1,JG),R,8)
          CALL FS INTG
15  CALL XFER (R,R GIVEN(1,JG),8)
          CALL SET LAG (NG,R GIVEN)
          DO 16 JG=1,LIMIT
          THETA = THETAG(JG)
          CALL LAG DER
          DO 16 I=1,4
16  GB = GB+CABS(DRDC(I))*2
          IF (MPRINT .NE. 0) PRINT 912,HTLAST,GB
912  FORMAT ('1','AT HT = ',F7.2,1X,'GB = ',1PE9.2)
          IF (GB .GT. GB MIN) GO TO 17

```

```

      RHTEMP = HTLAST
      CB MIN = GB
      CALL XFER (R GIVEN,R TEMP,NG8)
      GO TO 16
17  KHSAVE = RHTEMP
      CALL XFER (R TEMP,R SAVE,NG8)
18  HTERST = HTLAST
      IF (HTERST .LT. (HTS(1)+ HMARGN) ) GO TO 14
      IF(RHSAVE .GT. 9.0F9) GO TO 90
      D = RHSAVE
      CALL XFER (R SAVE,R GIVEN,NG8)
      CALL SET LAG (NG,R GIVEN)
C
      PRINT 913,D,TLEFT,TRIGHT
913  FORMAT ('0','REFLECTION HT SET TO',F7.2,1X,'KM',
           $ 5X,'THETA RANGE = ',F8.3,' TO ',F8.3)
C
      IF (MPRINT .EQ. 0) GO TO 39
      PRINT 931
931  FORMAT ('0','REFLECTION COEFFICIENTS AT THE GIVEN POINTS--')
      DO 32 JG=1,NG
      32  PRINT 932,THETAG(JG),(R GIVEN(I,JG),I=1,4)
932  FORMAT ('0','AT THETA = ',2F9.3,5X,'R = ',4(2X,2F9.5))
      39  CONTINUE
      RETURN
C
      90  PRINT 900,TLEFT,TRIGHT
900  FORMAT('C','REFLECTION HT CANNOT BE SET FOR THETA RANGE = ',
           $F8.3,' TO ',F8.3)
      STOP
C
      END

```

SUBROUTINE PRFL IN

C FOR MS
C THIS ROUTINE PROVIDES FOR INPUT OF ELECTRON DENSITY AND COLLISION
C FREQUENCY PROFILE CARDS AND FOR LIMITING THE EXTENT
C OF THE TOP AND BOTTOM OF THE ELECTRON DENSITY PROFILE.
C INPUT OF ION PROFILES IS ALSO PROVIDED FOR. FURTHER
C DESCRIPTION OF THE INPUT IS GIVEN IN A DNA REPORT.
C

COMMON/HTLOGN/NR NU,HTS NU(50),ALCNU(50,3)
COMMON/HPRT/ BPRINT
COMMON/CLT OF/ ICUT,CUTOFF
COMMON/FLD COM/AZIM,CODIP,MAG FLD
COMMON/FRQ COM/ FREQ
COMMON/NR OF AS/NR A
COMMON/HTS COM/HTS(101)
COMMON/ALOGN C/ALOGN(101,3)
COMMON/GRP COM/BETA,HPRIME,SCLHTS,ENMIN
COMMON/EXPNU C/COEFNU(3),EXPNU(3)
COMMON/ION COM/IONS,MRATIO(3)
COMMON/EN COLL/HT,EN(3),CNU(3)
COMMON/WR COM/OMEGA,TEMPOB
COMMON/HP/HJFWR
COMMON/TCPEN C/TOPEM
COMMON/JAY COM/JAY
REAL MAGFLD,MRATIO
DIMENSION IBCD(20),ENS(3),
*ITITLE(20),SAVHT(101),SAVEN(101,3)

C
C INTEGER BPRINT

C *****

C INPUT OF AN ELECTRON DENSITY PROFILE, IONS OPTIONAL.
C

READ 901,IBCD
PRINT 902,IBCD
DO 113 K = 1,20
113 ITITLE(K)=IBCD(K)
J = 1
11 READ 904,HT,ENS(1),ENS(2)
904 FORMAT (F7.2,5X,E9.2,E10.2)
IF (HT .LT. 0.0) GO TO 15
HTS(J) = HT
IF (J .NE. 1 .AND. HTS(J) .GE. HTS(J-1)) STOP
ENS(3) = ENS(2)-ENS(1)
NTUT=J
SAVHT(J)=HT
SAVEN(J,1)=ENS(1)
SAVEN(J,2)=ENS(2)
SAVEN(J,3)=ENS(3)
PRINT 916,HT,ENS(1),ENS(2)
916 FORMAT(' ',F7.2,5X,1PE9.2,E10.2,E10.2)
DO 13 K=1,3

```

      IF (ENS(K) .LT. 1.0E-10) ENS(K) = 1.0E-10
13  ALOGEN(J,K) = ALOG(ENS(K))
      NR A = J
      J = J+1
      GO TO 11
15  PRINT 906,HT
      RETURN

```

```

C
C*****
C

```

```

      ENTRY REPEAT

```

```

C
C      THIS SECTION IS USED WHEN THE ORIGINAL UNMODIFIED PROFILE
C      IS TO BE USED IN THE CALCULATIONS
C

```

```

      NR A=NTOT
      DO 134 K = 1,3
      DO 134 KJ=1,NTOT
      HTS(KJ) = SAVHT(KJ)
      IF(SAVEN(KJ,K) .LT. 1.0E-10) SAVEN(KJ,K) = 1.0E-10
134  ALOGEN(KJ,K)=ALOG(SAVEN(KJ,K))
      RETURN

```

```

C
C*****
C

```

```

      ENTRY CCLF IN

```

```

C
C      INPUT OF COLLISION FREQUENCY PROFILE CARDS
C

```

```

      J = 1
21  READ 901,(IBCD(L),L=1,20)
      PRINT 902,(IBCD(L),L=1,20)
      CALL CORE (IBCD,80)
      READ (69,905) HT,CNU
905  FORMAT (F7.2,4X,3(1X,E9.2) )
      IF (HT .LT. 0.0) GO TO 23
      HTS NU(J) = HT
      DO 22 K=1,3
      IF(CNU(K) .LT. 1.0E-10) CNU(K) = 1.0E-10
22  ALGNU(J,K) = ALOG(CNU(K))
      J = J+1
      GO TO 21
23  NF NU=J-1
      RETURN

```

```

C
C
C*****
C

```

```

      ENTRY SET EN

```

```

C
C      USE OF VARIOUS NAMELIST PARAMETERS TO SET UP A DESCRIPTION OF AN-
C      EXPONENTIAL ELECTRON DENSITY PROFILE IN TERMS OF
C      VALUES AT THE TOP AND BOTTOM OF THE PROFILE.

```

```

C
HTS(1) = AINT(HPRIME+SCLHTS/BETA+0.5)
FNA= 7.85535E-5*COEFNU(1)*EXP((EXPNU(1)+BETA)*HTS(1)-BETA*HPRIME)
ALOGEN(1,1) = ALOG(FNA)
HTS(2) = (ALOG(ENMIN/(7.85535E-5*COEFNU(1)))+BETA*HPRIME)/
$      (EXPNU(1)+BETA)
IF (HTS(2) .GT. 0.0) GO TO 501
HTS(2) = 0.0
ENO= 7.85535E-5*COEFNU(1)*EXP(-BETA*HPRIME)
ALOGEN(2,1) = ALOG(ENO)
GO TO 502
501 ALOGEN(2,1) = ALOG(ENMIN)
502 NK A = 2
951 FORMAT ('.',F7.2,5X,1PE9.2)

```

```

C
C      -----COMPUTED PROFILE PARAMETERS-----
C

```

```

POW=6.282185307D03*FREQ
POY=-1.758796D11/POW*MAGFLD
PRINT 213
213 FORMAT(/' COMPUTED PROFILE PARAMETERS: '/
$      T5,'H',T15,'NE',T27,'NU',T39,'X',T51,'Y',T63,'Z',T75,'WR')
DO 25 L=1,2
PONU=COEFNU(1)* EXP(EXPNU(1)*HTS(L))
PONE=7.85535D-05*PONU* EXP(BETA*(HTS(L)-HPRIME))
POWR=3.182357D09*PONE/PONU
PCX = 3.182357D09*PONE/(POW*POW)
POZ=PONU/POW
25- PRINT 214, HTS(L), PONE, PONU, PCX, POY, POZ, POWR
214 FORMAT(F7.1,1P6E12.3)

```

```

C
C      DETERMINATION OF HEIGHT WHERE OMEGA-R EQUALS 2.5E05
HOFWR=HPRIME
PRINT 952,HOFWR
952 FORMAT('C',5X,'OMEGA-R = 2.5E05 AT TOP HEIGHT = ',F7.2)

```

```

C
C      RETURN
C

```

```

C *****
C

```

```

C      ENTRY CUT TOP
C

```

```

C      USE OF TEMPC-B AND TOPEN TO LIMIT THE EXTENT OF THE TOP OF AN
C      ELECTRON DENSITY PROFILE WHICH HAS BEEN INPUT FROM
C      PROFILE CARDS. APPLICABLE ALSO TO THE ION PROFILES
C      IF IONS ARE AN INPUT.
C

```

```

C      IN THIS SECTION OF THE PROGRAM THE TOP OF THE PROFILE IS DETERMINE
C      THE NOTATION USED IS THAT WAITS ('OMEGA-R') IS DENOTED BY 'WR'.
C      THE VALUE OF 'B' AS DESCRIBED IN G.E. TEMPO REPORT 75-TMP-9,V.7,
C      PAGES 11 AND 12 IS DENOTED AS 'BCUT'
C      AND IS INPUT IN TO THIS PROGRAM AS THE NAMELIST VARIABLE 'TEMPOB'
C

```

```

C
C
C      TO DETERMINE THE VALUE OF 'BCUT' TO BE USED
      AND TO CUT OFF THE TOP OF THE PROFILE
      OMEGA=6.28319E3*FREQ
      PLASCN=3.182357E9
      DO 560 JREV=1, NR A
      J=NR A-JREV+1
      HT=HTS(J)
      JAY=J
      IF(J .EQ. NR A) JAY = NR A-1
      CALL EN NU
      OMOVNU=10.0*OMEGA/CNU(1)
      BCUT=2.0*OMOVNU
      IF(BCUT .LT. TEMPOB) BCUT = TEMPOB
      L=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
      IF(IONS .NE. 0) B=B+
      + (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
      + (PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
      IF(3 .GT. BCUT .AND. EN(1) .GT. TOPEN) GO TO 561
580 BHOT=B
      WK=PLASCN*EN(1)/CNU(1)
      IF(IONS .NE. 0) WR=WR+
      $ PLASCN*EN(2)/(CNU(2)*MRATIO(2))
      $ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
      PRINT 60B
60B FORMAT('O', 'WARNING--THE TOP OF THE PROFILE CAN NOT BE CUT OFF.',
      $ ' THE TOP OF THE PROFILE MAY BE TOO LOW.' )
      PRINT 61C, B, WR
61C FORMAT('C', ' TEMPO-B AT TOP = ', 1PE11.3, 'WAITS OMEGA-B AT ',
      $ 'TOP = ', 1PE11.3)
      GO TO 900

C
C
C      HERE THE VALUE OF ---B--- AT THIS HEIGHT IS GREATER THAN TEMPO-
581 IF(J .NE. NR A) GO TO 583
C
C      IF THE HEIGHT IS THE BOTTOM OF THE PROFILE
      PRINT 603, B, TEMPOB
603 FORMAT('O', 'THE TOP OF THE PROFILE CAN NOT BE SET. ', /2X,
      $ 'AT THE BOTTOM OF THE PROFILE, B = ', 1PE13.3, ', WHICH',
      $ ' IS GREATER THAN THE VALUE OF TEMPO-B ( = ', 1PE13.3, ')', /
      $ 5X, 'CHECK THE INPUT VALUES OF --TEMPOB--AND--TOPEN--.', /
      $ 5X, 'ALSO CHECK THE VALUES OF THE ELECTRON AND ION DENSITIES ',
      $ 'AT THE BOTTOM OF THE PROFILE.', /
      $ 30X, '***PROGRAM EXECUTION STOPS***')

C
      STOP 1111

C
C
C      ***INTERPOLATION***
583 BTOP = B
      HTTOP=HTS(J)

```

```

      HTBOT=HTS(J+1)
54  HT=(HTTOP+HTBOT)/2.0
      CALL FN NU
      B=(PLASCN*EN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
      IF(IONS .EQ. 0) B2=0.0
      IF(IONS .NE. 0) B2=
      $ (PLASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
      $ +(PLASCN*EN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
      B=B+B2
      IF(HTTOP-HTBOT .LT. 0.10) GO TO 566
      IF(B .GT. FCUT .AND. EN(1) .GT. TUPEN) GO TO 565
      BROT =B
      HTBOT = HT
      GO TO 64
565  BTUP=B
      HTTOP=HT
      GO TO 64
566  WR=PLASCN*EN(1)/CNU(1)
      IF(IONS .NE. 0) WR=WR+
      $ PLASCN*EN(2)/(CNU(2)*MRATIO(2))
      $ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
      PRINT 600
600  FORMAT('C','THE TOP OF THE PROFILE IS SET TO B1--')
      IF(IONS .EQ. 0) PRINT 906,HT,EN(1)
906  FORMAT(' ',F7.2,5X,1PE9.2,1PE9.2,1PE9.2)
      IF(IONS .NE. 0) PRINT 906,HT,EN(1),EN(2),EN(3)
      PRINT 614,BCUT,WR
614  FORMAT('C',' AT THE TOP OF THE PROFILE B = ',1PE11.3,
      $ ' OMEGA-R = ',1PE11.3)
      HTS(1) = HT
      ALOGEN(1,1) = ALOG(EN(1))
      IF(IONS .EQ. 0) GO TO 67
      ALOGEN(1,2) = ALOG(EN(2))
      ALOGEN(1,3) = ALOG(EN(3))
67  CONTINUE
      IF(J .EQ. 1) GO TO 900
      JN=1
      JP1 = J+1
      DO 68 JJ = JP1,NR A
      JN=JN+1
      HTS(JN) = HTS(JJ)
      DO 68 K = 1,3
68  ALOGEN(JN,K) = ALOGEN(JJ,K)
      NR A = JN
900  CONTINUE

```

C
C
C
C
C
C
C
C

BOTTOM CUT OFF

THIS SECTION CUTS OFF THE BOTTOM OF THE PROFILE.

```

C      IF ( N**2 = 1-A-JB), THAN THOSE PROFILE DENSITIES WHICH COMPUTE
C      TO "B" LESS THEN "CUTOFF" ARE NEGLECTED
C
C      IF THE PROFILE BOTTOM IS TO BE CUTOFF THEN "ICUT= ",
C      IF NOT THEN ICUT=0
C      IF(ICUT .LT. 1) GO TO 702
C
C      DO 590 JRFV = 1,NR A
C      J=NR A-JRFV+1
C      HT=HTS(J)
C      JAY=J
C      IF(JAY .EQ. NR A) JAY =NR A-1
C      CALL EN NU
C      B1=(PLASCN*FN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
C      IF(IONS .EQ. 0) B2=0.0
C      IF(IONS .NE. 0) B2 =
C      $ (PLASCN*FN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
C      $ +(PLASCN*FN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
C      B=B1+B2
C      IF(P .GT. CUTOFF) GO TO 6300
C      BROT=B
C      590 CONTINUE
C      PRINT 6030
C      6030 FORMAT('0',' THE PROFILE BOTTOM CAN NOT BE CUT OFF.',/
C      $ ' ALL B-VALUES (AS A FUNCTION OF HEIGHT) ARE LESS THAN CUTOFF.',/
C      $ ' THIS IMPLIES THAT THE PROFILE IS VERY CLOSE TO FREE SPACE.'//
C      $ 30X, '***PROGRAM EXECUTION STOPS***')
C
C      STOP 2222
C
C      6300 IF(J .NE. NR A) GO TO 6301
C      PRINT 704,CUTOFF,HTS(NR A),B
C      704 FORMAT('C',' THE BOTTOM OF THE PROFILE CAN NOT BE CUT OFF',
C      $ ' USING CUTOFF = ',E13.3,' AT HEIGHT = ',F10.3,/
C      $ ' BECAUSE THE VALUE OF B AT THE BOTTOM OF THE PROFILE',
C      $ ' WHERE R = ',1PE13.3,' IS LARGER THAN CUTOFF.'//)
C      IF(IPRINT .EQ. 0) GO TO 4111
C      PRINT 4112,B1,B2,B
C      4112 FORMAT(' THE VALUES ARE B-ELECTRONS = ',1PE11.4,' B-IONS = ',
C      $ 1PE11.4,' B-TOTAL = ',1PE11.4//)
C      4111 CONTINUE
C      GO TO 702
C
C      ***INTERPOLATION***
C
C      6301 BTOP=B
C      HTTOP=HTS(J)
C      HTBOT=HTS(J+1)
C      304 HT=(HTTOP+HTBOT)/2.0
C      CALL EN NU
C      B1=(PLASCN*FN(1)*CNU(1))/(OMEGA*(OMEGA**2+CNU(1)**2))
C      IF(IONS .EQ. 0) B2=0.0
C      IF(IONS .NE. 0) B2 =

```

```

      S (PI ASCN*EN(2)*CNU(2))/(OMEGA*(OMEGA**2+CNU(2)**2)*MRATIO(2))
      S +(PI ASCN*FN(3)*CNU(3))/(OMEGA*(OMEGA**2+CNU(3)**2)*MRATIO(3))
      R=R1+R2
      IF (HTTOP-HTBOT .LT. 0.10) GO TO 366
      IF (B .GT. CUTOFF) GO TO 365
      BOUT = B
      HTBOT = HT
      GO TO 364
365  BTOP=R
      HTTOP=HT
      GO TO 364
366  CONTINUE
      A1=EN(1)
      IF (IONS .EQ. 0) GO TO 6661
      A2=FN(2)
      A3=EN(3)
      GO TO 6662
6661 A2 = 0.0
      A3 = 0.0
6662 CONTINUE
      PRINT 703,HT,CUTOFF
703  FORMAT('0','THE BOTTOM PROFILE HEIGHT IS = ',F10.3,
      $' AT B(CUTOFF) = ',F12.3,/)
      IF (BPRINT .EQ. 0) GO TO 4113
      PRINT 4114,A1,A2,A3,B1,B2,3
4114 FORMAT(' ',
      $ ' THE ELECTRON, POSITIVE ION AND NEGATIVE ION DENSITIES',
      $ ' ARE EQUAL TO ',// 1X,1PE10.3,3X,1PE10.3,3X,1PE10.3,//
      $ 1X,'B-ELECTRONS = ',1PE11.4,' B-IONS = ',1PE11.4,' B-TOTAL = ',
      $ 1PE11.4/)
4113 CONTINUE
      NR A = J+1
      HTS(NR A) = HT
      ALOGEN(NR A,1) = ALOG(EN(1))
      IF (IONS .EQ. 0) GO TO 667
      ALOGEN(NR A,2) = ALOG(EN(2))
      ALOGEN(NR A,3) = ALOG(EN(3))
667  CONTINUE
C
702  CONTINUE
C
C
      PRINT 913
913  FORMAT('1')
C
C *****
C
C THIS SECTION IS USED TO DETERMINE THE PROFILE HEIGHT WHERE
C THE VALUE OF OMEGA-R EQUALS 2.5E05
C THE VARIABLE 'WR' IS USED AS WAIT'S OMEGA-P
C
      DO 750 JREV=1,NR A
      J=NR A-JREV+1

```

```

HT=HTS(J)
JAY=J
IF(J .FC. NR A) JAY = NR A-1
CALL EN NU
WR=PLASCN*EN(1)/CNU(1)
IF(IUNS .NE. 0) WR=WR+
$ PLASCN*EN(2)/(CNU(2)*MRATIO(2))
$ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
IF(WR .GT. OMEGAR) GO TO 761
DBOT=B
790 CONTINUE
HCFWR=HTS(1)
PRINT 3608,HCFWR
3608 FORMAT('C',5X,'WARNING THE TOP OF THE PROFILE IS TO LOW',
*' FOR OMEGA-R TO BE EQUAL TO 2.5E05.',/5X,'THE VALUE OF',
*' TOP-HEIGHT IS THEREFORE TAKEN AS THE HEIGHT AT THE TOP OF ',
*' THE PROFILE',//10X,'TOP HEIGHT = ',F7.2)
GL TO 9000

C
761 IF(J .NE. NR A) GO TO 763
HCFWR=HTS(NR A)
PRINT 3603,HCFWR
3603 FORMAT('C',5X,'THE VALUE OF THE COMPUTED --OMEGA-R-- -- AT THE ',
$ 'BOTTOM OF THE PROFILE --HOFWR = ',F7.2,' -- ',/5X,'IS ',
*'GREATER THAN THE INPUT LIMIT OF --OMEGA-R--')
GL TO 9000

C
C
C
C
***INTERPOLATION***

733 BTOP=B
HTTOP=HTS(J)
HTBOT=HTS(J+1)
764 HT=(HTTOP+HTBOT)/2.0
CALL EN NU
WR=PLASCN*EN(1)/CNU(1)
IF(IUNS .NE. 0) WR=WR+
$ PLASCN*EN(2)/(CNU(2)*MRATIO(2))
$ +PLASCN*EN(3)/(CNU(3)*MRATIO(3))
IF(HTTOP-HTBOT .LT. 0.10) GO TO 766
IF(WR .GT. OMEGAR) GO TO 765
BBOT=B
HTBOT=HT
GO TO 764
765 BTOP=B
HTTOP=HT
GO TO 764
766 HOFWR=HT
PRINT 3600,HOFWR
3600 FORMAT('C',5X,'OMEGA-R EQUALS 2.5E05 AT TOP HEIGHT = ',F7.2)
9000 CONTINUE
RETURN
C

```

901 FORMAT (20A4)
902 FORMAT (' ',20A4)

C

END

```

      SUBROUTINE FINAL
C   FOR MS
C
C   ROUTINE FOR FOLLOWING THE EIGEN-ANGLES AS THE F FUNCTION IS CHANGED
C   FROM THAT USING LAGRANGE INTERPOLATION TO THAT USING
C   EXACT FULL-WAVE SOLUTION VALUES OF REFLECTION
C   COEFFICIENTS.
C
      COMMON/KXACT C/ KEXACT
      COMMON/THETA C/THETA
      COMMON/SEP COM/SEP
      COMMON/TOL COM/TOL
      COMMON/EIG COM/EIGEN(100),NR E
      COMMON/MPRINT/MPRINT
      COMMON/IDERV C/IDERV
      DIMENSION FRACTS(32)
      COMPLEX
      $          THETA,
      $          SAVE T,
      $          EIGEN,
      $          F,DFDT,DELT,
      $          FI,DFIDT
      DATA MAX NR/6/
C
C
      IF (MPRINT .NE. 0) PRINT 900
900 FORMAT ('1','ITERATIONS FOR EXACT SOLUTION')
      IDERV = 1
C
      DO 19 JE=1,NR E
10 FRACTS(1) = 0.0
      FRACTS(2) = 1.0
      NR STPS = 2
      NS = 2
      IF (MPRINT .NE. 0) PRINT 901,JE
901 FORMAT (' ',18)
      THETA = EIGEN(JE)
11 SAVE T = THETA
      NCOUNT = 0
      FE FRCT = FRACTS(NS)
      FI FRCT = 1.0-FE FRCT
12 IF (MPRINT .NE. 0) PRINT 902,THETA
902 FORMAT (' ',10X,'THETA =',2F9.4)
      KEXACT=1
      CALL F DFDT (THETA,F,DFDT)
      IF (MPRINT .NE. 0) PRINT 104,F,DFDT
104 FORMAT (' ',60X,'EXACT F =',2E11.3,3X,2E11.3)
      IF (NS .EQ. NR STPS) GO TO 14
C
13 KEXACT=0
      CALL F DFDT (THETA,FI,DFIDT)
      IF (MPRINT .NE. 0) PRINT 105,FI,DFIDT
105 FORMAT (' ',60X,'APPRX F =',2E11.3,3X,2E11.3)

```

```

F = F1 FRCT*FI+FE FRCT*F
DFDT = F1 FRCT*DFIDT+FE FRCT*DFDT
IF (MPRINT .NE. 0) PRINT 106,F,DFDT
106 FORMAT (' ',00X,'COMBN F =',2E11.3,3X,2E11.3)
14 DELT = -F/DFDT
THETA = THETA+DELT
NCOUNT = NCOUNT+1
IF (CABS(DELT) .GT. SEP .AND. NCOUNT .GE. 2) GO TO 20
IF (CABS(DELT) .LT. 0.3*TOL) GO TO 16
IF (NS .NE. NR STPS .AND. CABS(DELT) .LT. SEP) GO TO 16
GO TO 12

```

C

```

20 IF (MPRINT .NE. 0) PRINT 906
906 FORMAT ('0','RESET')
IF (NR STPS .GE. MAX NR) GO TO 17
THETA = SAVE T
DO 22 NREV=NS,NR STPS
N = NR STPS-NREV+NS
22 FRACTS(N+1) = FRACTS(N)
FRACTS(NS) = (FRACTS(NS-1)+FRACTS(NS+1))/2.0
NR STPS = NR STPS+1
GO TO 11

```

C

```

16 IF (MPRINT .NE. 0) PRINT 907
907 FORMAT ('0')
NS = NS+1
IF (NS .LE. NR STPS) GO TO 11
EIGEN(JE) = THETA
IF (MPRINT .NE. 0) PRINT 902,THETA
GO TO 19
17 PRINT 109,THETA
109 FORMAT ('0','INVALID MODE AT',2F9.4, 'TRY SMALLER RECTANGLE AROUND
$ THIS AREA WITH APPROPRIATE RANGER AND RANGE1 VALUES AND ',
$ 'RE-EXECUTE')
NR E = NR E-1
IF (NR E .EQ. 0) GO TO 33
DO 18 JJ=JE,NR E
18 EIGEN(JJ) = EIGEN(JJ+1)
GO TO 10
19 CONTINUE

```

C

```

IF (NR E .LE. 1) GO TO 33
!STOP = 0
NREM1 = NR E-1
DO 32 J=1,NREM1
JP1 = J+1
DO 32 JJ=JP1,NR E
IF (CABS(EIGEN(J)-EIGEN(JJ)) .GT. TOL) GO TO 32
PRINT 905,J,JJ
905 FORMAT ('0','EIGENS NR',I3,1X,'AND',I3,1X,'ARE TO BE RESOLVED',/
$ 'TRY SMALLER RECTANGLE AROUND
$ THIS AREA WITH APPROPRIATE RANGER AND RANGE VALUES AND RE-EXECUTE
$')

```

```

      PRINT 302,EIGEN(J),EIGEN(JJ)
302 FORMAT ('0','EIGEN ANGLES AT',2F9.4,1X,'AND',2F9.4,1X,'ARE TO BE R
      ESOLVED')
      ISTOP = 1
      32 CONTINUE
      IF (ISTOP .NE. 0) STOP
      33 RETURN
C
      90 PRINT 990
990 FORMAT ('0','PROBLEMS IN SUBROUTINE FINAL')
      STOP
C
      END

```

```

      SUBROUTINE NE OUT
C   FOR MS
C   ROUTINE PROVIDING FOR PUNCHED OUTPUT OF EIGEN ANGLES
C
      COMMON/EIG COM/EIGEN(100),NR E
      COMMON/HTS COM/HTS(101)
      COMMON/NR OF AS/NR A
      COMMON/H COM/H
      COMMON/FRO COM/FREQ
      COMPLEX EIGEN,
$         THETA,
$         S,SO
      DATA PI/3.141593/
      DATA RTD/57.29578/
      DATA VEL LT/2.997928E5/
      DATA RE/6369.427/
C
C
      WAVE NR = 2.0*PI*FREQ*1000.0/VEL LT
      ACDEF = -8.6859*WAVE NR*1000.0
      CAP K = 1.0/(1.0-H/RE)
C
      PUNCH 971,HTS(NR A)
971  FORMAT (1X,'D = ',F7.2,',')
      PRINT 972,HTS(NR A)
972  FORMAT ('0',1X,'D = ',F7.2,',')
      PUNCH 975
975  FORMAT (1X,'EIGEN = ')
      PRINT 976
976  FORMAT (' ',1X,'EIGEN = ',20X,'ATTEN   VOVERC')
      DO 77 JE=1,NR E
      THETA = EIGEN(JE)
      PUNCH 977,THETA
977  FORMAT (1X,2(F8.3,','))
      S = C SIN(THETA/RTD)
      SO = S*CAP K
      ATTEN = ACDEF*AIMAG(SO)
      VOVERC = 1.0/REAL(SO)
      PRINT 978,THETA,ATTEN,VOVERC
      77 CONTINUE
978  FORMAT (' ',1X,2(F8.3,','),F15.3,F9.5)
      THETA = 0.0
      PUNCH 977,THETA
      PRINT 978,THETA
      RETURN
C
      END

```

SUBROUTINE NP OUT

C FOR MS

C ROUTINE FOR COMPUTING AND PRINTING TABLE AND PUNCHED CARD OUTPUT
C ASSOCIATED WITH NPUNCH OPTION. FURTHER DESCRIPTION
C OF OUTPUT IS GIVEN IN A DNA REPORT.
C

COMMON/IC COM/IDENT(20)
COMMON/MPRINT/MPRINT
COMMON/NPUNCH C/NPUNCH
COMMON/CRD PST/CARDPT
COMMON/CRD PNC/CARDPN
COMMON/RHO COM/RHO, WAIT RH
COMMON/FLD COM/AZIM, CODIP, MAGFLD
COMMON/GAD COM/EPSP, SIGMA
COMMON/FREQ COM/FREQ
COMMON/H COM/H
COMMON/WN/WAVE NR
COMMON/HF/HOFWR
COMMON/THETA C/THETA
COMMON/EIG COM/EIGEN(100), NR E
COMMON/INTEGR/X11, X21, X12, X22
COMMON/RB COM/NUM11, NUM22, DEN11, DEN22
COMMON/FLF2 C/F1, F2, HG, F1ZT, F1ZR, F2ZT, F2ZR, GF1ZT
COMPLEX

\$ THETA, EIGEN,
\$ X11, X21, X12, X22,
\$ NUM11, NUM22, DEN11, DEN22,
\$ F1, F2, HG, F1ZT, F1ZR, F2ZT, F2ZR, GF1ZT,
\$ C,
\$ R11, R21, R12, R22,
\$ RBAR11, RBAR22, D11, D22,
\$ THETAP, TERM1, TERM2, TERM3, TERM4,
\$ RB11P1, RB22P1,
\$ FM, DFMDT,
\$ UFDT, FACTOR,
\$ S, CP, CASIN,
\$ Q1, Q2, Q3, FOVRD, FOVRO,
\$ XVZ, XEZ, XBZ,
\$ SP, STURE1, ECUMP, EXLOG

REAL MAGFLD
INTEGER CARDPT, CARDPN
DATA RE/6369.427/
DATA RTD/57.29578/
DATA ALN TEN/2.302585/
DATA PI/3.141593/
DATA RTD/57.29578/

C
C
C

PRINT/PUNCH 'ID' INFORMATION
IF(MPRINT .NE. 0) PRINT 901, IDENT
901 FORMAT('C', 20A4)
IF(CARDPN .GT. 0) PUNCH 902, IDENT
902 FORMAT(20A4)

```

C
C PUNCH KHO CARD, PRINT H VALUE AND TABLE HEADINGS.
  IF (MPRINT.NE. 0) PRINT 971,RHO,FREQ,AZIM,CODIP,MAGFLD,SIGMA,EPSR
971 FORMAT ('O',F7.3,' F',F8.4,' A',F8.3,' C',F8.3,' M',1PF10.3,
  ' S',E10.3,' E',0PF5.1)
  IF (ICARDPN.GT. 0)
    SPUNCH 972,RHO,FREQ,AZIM,CODIP,MAGFLD,SIGMA,EPSR
972 FORMAT ('R',F7.3,' F',F8.4,' A',F8.3,' C',F8.3,' M',1PE10.3,
  ' S',E10.3,' E',0PF5.1)
  PRINT 974,H
974 FORMAT ('O',9X,'H =',F6.2)
  PRINT 975
975 FORMAT (' ',1X,'MODE',6X,'THETA',6X,'ATTEN',3X,'VUVERC',3X,
  ' WAIT MAG',2X,'WAIT ANG',8X,'THETAP',13X,'POL')
  IF (NPUNCH.EQ. 2 .AND. CARDPN.EQ. 1) PUNCH 939,FREQ
939 FORMAT(F8.3)
C
  ACUSEI = -8.6858896*WAVE NR*1000.0
  MODE = C
  IDERIV = 1
C
  DO 79 JE=1,NR E
    THETA = EIGEN(JE)
C
C FORMATION OF THE USUAL REFLECTION MATRIX, R, FROM (R+1.0)/C AND
C FORMATION OF THE USUAL RBAR MATRIX VALUES FROM
C FORM USED TO FIND EIGEN ANGLES. ALSO, COMPUTATION OF
C DERIVATIVES FOR USE IN COMPUTING DFDT.
  CALL F DFDT (THETA,FM,DFMDT)
  C = CCOS(THETA/RTD)
  R11 = C*X11-1.0
  R21 = C*X21
  R12 = C*X12
  R22 = C*X22-1.0
  D11 = C*NUM11-DEN11
  D22 = C*NUM22-DEN22
  RBAR11 = DEN11/D11
  RBAR22 = DEN22/D22
  RB11P1 = C*NUM11/D11
  RB22P1 = C*NUM22/D22
C
  DFDT = DFMDT*C**2/(D11*D22)*RTD
  S = CSIN(THETA/RTD)
  FACTOR = CSQRT(S)/DFDT
C
  ITEM=1
  IF (CABS(1.0-R11*RBAR11) .GE. CABS(1.0-R22*RBAR22) ) ITEM=2
C
C VLF-LF EXCITATION TERMS (E.G. THE 'T' TERMS TO BE USED FOR
C NPUNCH=1 OR NPUNCH = 9 ).
  TERM1 = RB11P1**2*(1.0-RBAR22*R22)*FACTOR/(RBAR11*F1*F1)
  TERM2 = RB22P1**2*(1.0-RBAR11*R11)*FACTOR/(RBAR22*F2*F2)
  TERM3 = RB11P1*RB22P1*FACTOR*R21/(F1*F2)

```

```

      TERM4 = F12/R21
C
C   ELF EXCITATION TERMS (E.G. THE TERMS TO BE USED FOR NPUNCH = 7 ).
      XVZ=TERM1*S*S*F1ZT*F1ZK
      XEZ=-TERM1*S*GF1ZT*F1ZK
      XBZ=-TERM3*TERM4*S*F2ZT*F1ZK
C   IN TERMS OF MAGNITUDE AND PHASE THESE TERMS ARE GIVEN BY
      XVZM=CABS(XVZ)
      XVZA=CANG(XVZ)
      XEZM=CABS(XEZ)
      XEZA=CANG(XEZ)
      XBZM=CABS(XBZ)
      XBZA=CANG(XBZ)
C
C   EIGEN ANGLE REFERRED TO GROUND LEVEL.
      RATIO = F/RE
      CP = CSQRT(C**2+(C**2-1.0)*(2.0*RATIO+3.0*RATIO**2))
      THETAP = 90.0-CASIN(CP)*RTD
C
C   ATTENUATION AND PHASE VELOCITY REFERRED TO GROUND LEVEL.
      SP = S/(1.0-H/RE)
      ATTEN = ACOEF*AIMAG(SP)
      VOVERC = 1.0/REAL(SP)
C
C   WAIT'S EXCITATION.
      STORE1 = RB11P1**2*(1.0-RBAR22*R22)
      *FACTOR/RBAR11
      ECOMP = H*G**2*STORE1*S**2
      EXLOG = CLOG(ECOMP*0.5*WAVE NR*WAIT RH)
      WM = 20.0*REAL(EXLOG)/ALN TEN
      WA = AIMAG(EXLOG)-PI/2.0
      IF (WA .LT. -PI/2.0) WA = WA+2.0*PI
C
C   MODE CONVERSION TERMS TO BE USED FOR NPUNCH=8 AND NPUNCH=9.
      XM= CABS(ECOMP)
      XA= CANG(ECOMP)
      Q1=1.0-R11*RBAR11
      Q2=1.0-R22*RBAR22
      Q3= RB22P1/RB11P1
      IF( CABS(Q1) .GE. CABS(Q2) ) FOVRD=Q3*Q1/(R12*RBAR22)
      IF( CABS(Q2) .GE. CABS(Q1) ) FOVRD = Q3*(R21*RBAR11)/Q2
      FOVRD=(FOVRD*F1)/F2
C
C   POLARIZATION.
      D1 = CABS(R21*RBAR22)
      D2 = CABS(1.0-RBAR11*R11)
      IF (D1 .GE. D2) POL = CABS(1.0-RBAR22*R22)/D1
      IF (D2 .GE. D1) POL = CABS(R12*RBAR11)/D2
C
C   PRINT VALUES FOR TABLE.
C
      MODE = MODE+1
      PRINT 932,MODE,THETA,ATTEN,VOVERC,WM,WA,THETAP,POL

```

932 FORMAT (' ',I4,2F8.3,F6.3,F9.5,F10.3,F9.3,3X,2F8.3,F12.5)

NPUNCH OPTION FOR OBTAINING OUTPUT CARDS

NPUNCH=1*** GIVES OUTPUT CARDS FOR HORIZONTALLY HOMOGENOUS
MODE-SUM OR WKB-SUM IN TERMS OF (T'S)

NPUNCH=2*** GIVES OUTPUT CARDS IN TERMS OF SNVLF OUTPUT

NPUNCH=7*** GIVES OUTPUT CARDS FOR INPUT INTO ELF-WKB MODE-SUMS

NPUNCH=8*** GIVES OUTPUT CARDS FOR INPUT TO MODE CONVERSION
IN TERMS OF (FOVR)

NPUNCH=9*** GIVES OUTPUT CARDS FOR INPUT INTO MODE CONVERSION
IN TERMS OF BOTH (T'S) AND (FOVR)

CARD PN=0*** GIVES USUAL MODE SUMMARY PRINT OUT BUT NO CARDS

CARD PN=1*** GIVES USUAL MODE SUMMARY PRINT AND CARDS

CARD PT=0*** CAUSES THE OUTPUT CARDS TO NOT BE LISTED

CARD PT=1*** CAUSES THE OUTPUT CARDS TO BE LISTED

PUNCH 'NPUNCH' OPTION CARDS----'MPRINT' USED ONLY FOR DEBUGGING

IF (MPRINT .NE. 0) PRINT 933,THETAP,TERM1,TERM2,

THETAP,TERM3,TERM4

933 FORMAT ('0','1',OP2F9.5,1X,1P4E15.8/' ', '2',OP2F9.5,1X,1P4E15.8)

IF (MPRINT .NE. 0) PRINT 930

930 FORMAT (' ')

IF (NPUNCH .EQ. 1 .AND. CARDPT .EQ. 1) PRINT 930

IF (NPUNCH .EQ. 1 .AND. CARDPT .EQ. 1)

* PRINT 931,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4

931 FORMAT (' ', '1',OP2F9.5,11,1P4E15.8/' ', '2',OP2F9.5,11,1P4E15.8/)

IF (NPUNCH .EQ. 1 .AND. CARDPN .EQ. 1)

* PUNCH 934,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4

934 FORMAT ('1',OP2F9.5,11,1P4E15.8,/, '2',OP2F9.5,11,1P4E15.8)

IF (NPUNCH .EQ. 1) GO TO 600

IF (NPUNCH .EQ. 2 .AND. CARDPT .EQ. 1)

*PRINT 937,ATTEN,VOVERC,WM,WA

937 FORMAT ('0',4F10.5)

IF (NPUNCH .EQ. 2 .AND. CARDPN .EQ. 1)

*PUNCH 938,ATTEN,VOVERC,WM,WA

938 FORMAT (4F10.5)

IF (NPUNCH .EQ. 2) GO TO 600

IF (NPUNCH .EQ. 7 .AND. CARDPT .EQ. 1) PRINT 930

IF (NPUNCH .EQ. 7 .AND. CARDPT .EQ. 1)

* PRINT 626,THETAP,XVZM,XVZA,XEZM,XEZA,XBZM,XBZA

626 FORMAT (2(F10.5,' '),/5X,3(1PE13.4,' ', 'OPF8.3,' ')/)

IF (NPUNCH .EQ. 7 .AND. CARDPN .EQ. 1)

* PUNCH 627,THETAP,XVZM,XVZA,XEZM,XEZA,XBZM,XBZA

627 FORMAT (5X,2(F10.5,' '),/5X,3(1PE13.4,' ', 'OPF8.3,' '))

IF (NPUNCH .EQ. 7) GO TO 600

IF (NPUNCH .EQ. 8 .AND. CARDPT .EQ. 1) PRINT 930

IF (NPUNCH .EQ. 8 .AND. CARDPT .EQ. 1)

*PRINT 935, THETAP,XM,XA,FOVRO,HOFWR

```

935 FORMAT(    OP2F9.5,1PE14.5,OPF9.5,1P2E16.8,OPF7.2/)
      IF(NPUNCH .EQ. 8 .AND. CARDPN .EQ. 1 )
        *PUNCH 936, THETAP,XM,XA,FUVRO,HOFWR
936 FORMAT(OP2F9.5,1PE14.5,OPF9.5,1P2E16.8,OPF7.2)
      IF(NPUNCH .EQ. 8) GO TO 600
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 ) PRINT 930
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 )
        * PRINT 931,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
      IF(NPUNCH .EQ. 9 .AND. CARDPT .EQ. 1 )
        *PRINT 935, THETAP,XM,XA,FUVRO,HOFWR
      IF(NPUNCH .EQ. 9 .AND. CARDPN .EQ. 1 )
        * PUNCH 934,THETAP,ITERM,TERM1,TERM2,THETAP,ITERM,TERM3,TERM4
      IF(NPUNCH .EQ. 9 .AND. CARDPN .EQ. 1 )
        *PUNCH 936, THETAP,XM,XA,FUVRO,HOFWR
600 CONTINUE
79 CONTINUE

C      IF(CARD PN .GT. 0) PUNCH 700
700 FORMAT (' ')
      RETURN

C      END

```

```

      FUNCTION CASIN(S)
C ROUTINE FOR COMPUTING THE COMPLEX SINE FUNCTION OF A COMPLEX
C ARGUMENT.
C FOR MS
C
      COMPLEX CASIN,S,SSQ,SPOWER,SUM,TERM,PARTS,C,CPLX I
      DATA CPLX I/(0.0,1.0)/
      DIMENSION PART(2)
      EQUIVALENCE (PART(1),PARTS)
C
C
      IF (CABS(S) .GT. 0.7) GO TO 20
      FACTOR = 1.0
      SPOWER = S
      SSQ = S**2
      SUM = S
      IF (CABS(SUM) .LT. 1.0E-7) GO TO 12
      DO 11 K=3,99,2
      FACTOR = FACTOR*(K-2.0)/(K-1.0)
      SPOWER = SPOWER*SSQ
      TERM = FACTOR*SPOWER/K
      SUM = SUM+TERM
      PARTS = TERM/SUM
      IF (ABS(PART(1))+ABS(PART(2))) .LT. 1.0E-10) GO TO 12
11 CONTINUE
12 CASIN = SUM
      RETURN
C
20 C = CSQRT(1.0-S**2)
      CASIN = -CPLX I*CLOG(C+CPLX I*S)
      RETURN
C
      END

```

```

FUNCTION CANG(ARG)
  IMPLICIT REAL (A-H,O-Z)
  COMPLEX ARG, MINUSI/(0.,-1.)
  ARG1=ARG
  ARG2=MINUSI*ARG
  CANG=ATAN2(ARG1,ARG2)
  IF (ARG1 .LT. 0.) CANG= CANG+6.283185
  RETURN
END

```

```

SUBROUTINE INTEG
C FOR MS
C THIS SUBROUTINE PERFORMS AN INTEGRATION OF THE DIFFERENTIAL
C EQUATIONS FOR THE IONOSPHERE REFLECTION MATRIX
C USING RUNGE-KUTTA INTEGRATION FORMULAS. THE
C INTEGRATION VARIABLES ARE THE ELEMENTS OF THE MATRIX
C (R+1.0)/C WHERE R IS THE REFLECTION MATRIX DESCRIBED
C BY R/DOEN. IF IDERIV IS SET NON-ZERO, THE DERIVATIVES
C OF (R+1.0)/C ELEMENTS WRT C=COS(THETA) ARE ALSO
C INTEGRATION VARIABLES. THE SET OF HEIGHTS AT WHICH
C PAIRS OF INTEGRATION STEPS BEGIN AND END IS STORED
C IN THE ARRAY 'RKHTS'. SIZES OF THE STEPS ARE
C DETERMINED BY COMPARING THE VALUES OF THE ELEMENTS
C OF (R+1.0)/C AFTER A PAIR OF STEPS AND VALUES OF THE
C SAME VARIABLES AFTER A SINGLE DOUBLE-SIZE COMPARISON
C STEP. THE (R+1.0)/C VARIABLES USED IN THE COMPARISON
C STEP ARE DENOTED AS 'X'. IF THE DIFFERENCE IS TOO
C LARGE, THE STEP SIZE PAIRS ARE CUT IN HALF BY ADDING
C A NEW HT TO THE LIST IN 'RKHTS'. IN ORDER TO
C ENHANCE STABILITY IN OTHER PARTS OF THE PROGRAM, NO
C HEIGHTS ARE DELETED FROM THE LIST UNLESS THE LIST IS
C RESET BY A CALL TO 'SET RK'.

```

```

COMMON/NR OF AS/NR A
COMMON /RTOL C/RTOL
COMMON/HTS COM/HTS(101)
COMMON/XPRNT/XPRINT
COMMON/INTEGR/R(16),DRDH(16)
COMMON/X INTEGR/X(8),DXDH(8)
COMMON/EN COLL/HT
COMMON/WN/WAVE NR
COMMON/IDERV C/IDERIV
COMMON/N CCM/N
COMMON/CVRFLO/IOVFLO
COMMON/JAY COM/JAY
DIMENSION
$      RO(16),HDELRO(16),DELR1(16),DELR2(16),
$      XO(8),HDELXO(8),DELX1(8),DELX2(8),
$      P SAVE(16),
$      RKHTS(201)
INTEGR XPRINT
DATA MAX NR/201/
DATA DHMIN/0.01/

```

```

C
C
N = 8
IF (IDERIV .NE. 0) N = 16
IOVFLO = 0
IF(XPRINT .NE. 0) CALL R COLS
THIRD = 1.0/3.0
CALL INIT S
JAY = 1
JRK = 1

```

```

HT = HTS(1)
CALL S MTRX
CALL INITL R
C
CALL XFER (R,X,8)
IF(XPRINT .NE. 0) PRINT 901
901 FORMAT ('0')
IF(XPRINT .NE. 0) CALL PRINT R
C
NRAM1 = NR A-1
DO 79 J=1,NRAM1
JAY = J
30 CALL XFER(R,R SAVE,N)
HT = RKHTS(JRK)
CALL XFER (R,X,8)
31 DELH = (RKHTS(JRK+1)-RKHTS(JRK))*0.5
IF (ABS(DELH) .LT. DHMIN) GO TO 90
DH = DELH*WAVE NR/2.0
HDM = 0.5*DH
TDM = 2.0*DH
C
C BEGIN FIRST RUNGE-KUTTA STEP OF THE PAIR AND THE COMPARISON STEP.
CALL R DERIV
IF (IOVFLC .NE. 0) GO TO 60
C
DO 32 I=1,N
PO(I) = R(I)
HDELRO(I) = DRDH(I)*HDM
32 R(I) = RO(I)+HDELRO(I)
C
DO 33 I=1,8
XU(I) = X(I)
HDELXO(I) = HDELPO(I)*2.0
33 X(I) = XO(I)+HDELXO(I)
C
HT = HT+C.5*DELH
CALL S MTRX
CALL R DERIV
IF (IOVFLC .NE. 0) GO TO 60
C
DO 34 I=1,N
DELRL(I) = DRDH(I)*DH
34 P(I) = RO(I)+0.5*DELRL(I)
C
CALL R DERIV
IF (IOVFLC .NE. 0) GO TO 60
C
DO 35 I=1,N
DELRL2(I) = DRDH(I)*DH
35 R(I) = RO(I)+DELRL2(I)
C
HT = HT+C.5*DELH
CALL S MTRX

```

```

CALL R DERIV
CALL X DERIV
IF(IQVFLC .NE. 0) GO TO 60

C
DO 36 I=1,N
HDEL3 = DRDH(I)*MDH
DEL4 = (HDEL0(I)+DEL1(I)+DEL2(I)+HDEL3)*THIRD
36 R(I) = RC(I)+DEL4

C
C BEGIN SECOND RUNGE-KUTTA STEP OF THE PAIR.
DO 37 I=1,8
DELX1(I) = DXDH(I)*TDH
37 X(I) = X0(I)+0.5*DELX1(I)

C
CALL R DERIV
CALL X DERIV
IF(IQVFLC .NE. 0) GO TO 60

C
DO 42 I=1,N
RO(I) = R(I)
HDEL0(I) = DRDH(I)*MDH
42 R(I) = RO(I)+HDEL0(I)

C
DO 43 I=1,8
DELX2(I) = DXDH(I)*TDH
43 X(I) = XC(I)+DELX2(I)

C
HT = HT+C.5*DELH
CALL S MTRX
CALL R DERIV
IF(IQVFLC .NE. 0) GO TO 60

C
DO 44 I=1,N
DEL1(I) = DRDH(I)*DH
44 R(I) = RO(I)+0.5*DEL1(I)

C
CALL R DERIV
IF(IQVFLC .NE. 0) GO TO 60

C
DO 45 I=1,N
DEL2(I) = DRDH(I)*DH
45 R(I) = RO(I)+DEL2(I)

C
HT = HT+C.5*DELH
CALL S MTRX
CALL R DERIV
CALL X DERIV
IF(IQVFLC .NE. 0) GO TO 60

C
DO 46 I=1,N
HDEL3 = DRDH(I)*MDH
DEL4 = (HDEL0(I)+DEL1(I)+DEL2(I)+HDEL3)*THIRD
46 R(I) = RO(I)+DEL4

```

```

C
DO 47 I=1,8
HDELX3 = DXDH(I)*DH
DELX4 = (HDELX0(I)+DELX1(I)+DELX2(I)+HDELX3)*THIRD
47 X(I) = X0(I)+DELX4

C
C ESTIMATE TRUNCATION ERROR AND CUT STEP SIZE IF APPROPRIATE.
IF(XPRINT .NE. 0) CALL PRINT R
DO 51 I=1,7,2
ERRSQ = (R(I)-X(I))**2+(R(I+1)-X(I+1))**2
RSQ = R(I)**2+R(I+1)**2
IF (PSQ .GT. 1.0) ERPSQ = ERRSQ/RSQ
IF (ERRSQ .GT. RTOL**2) GO TO 61
51 CONTINUE
GO TO 70

C
60 IF (XPRINT .NE. 0) CALL PRINT R
61 IF (XPRINT .NE. 0) PRINT 960
960 FORMAT (' ', 'OVRFLO')
IF (NR RK .GE. MAX NR) GO TO 95
IOVFLO = 0
CALL XFER(R SAVE,R,N)
CALL XFER(R SAVE,X,8)
HT = RKHTS(JRK)
CALL S MTRX

C
NRRKM1 = NR RK-1
DO 62 JCOUNT=JRK,NRRKM1
JJ = NRRKM1-JCOUNT+JRK
62 RKHTS(JJ+2) = RKHTS(JJ+1)
RKHTS(JRK+1) = (RKHTS(JRK)+RKHTS(JRK+2))*0.5
NR RK = NR RK+1
GO TO 31

C
70 JRK = JRK+1
IF (ABS(RKHTS(JRK)-HTS(J+1)) .GT. 0.001) GO TO 30
79 CONTINUE
RETURN

C
C
C RESET THE INTEGRATION STEP PAIRS TO BE THE INPUT PROFILE SEGMENTS.
ENTRY SET RK
DO 81 J=1,NR A
81 RKHTS(J) = HTS(J)
NR RK = NR A
RETURN

C
90 PRINT 990
990 FORMAT ('0','STEP TOO SMALL IN INTEG')
STOP

C
95 PRINT 995
995 FORMAT ('0','TOO MANY STEPS IN INTEG')

```

C STOP
END

```

SUBROUTINE EN NU
C FOR MS
C COMPUTATION OF ELECTRON DENSITY AND COLLISION FREQUENCY (AND ION
C DENSITY IF APPROPRIATE) AS A FUNCTION OF HEIGHT.
C IF PROFILE WAS INPUT FROM FORMATTED CARDS, LOGARITHMIC
C INTERPOLATION IS USED. SEE ALSO NOTES IN SUBROUTINE
C PREFIN.
C
COMMON/NR OF AS/NR A
COMMON/ICN COM/IONS
COMMON/JAY COM/J
COMMON/HTS COM/HTS(101)
COMMON/HTLOGN/NK NU,HTS NU(50),ALCNU(50,3)
COMMON/ALOGN C/ALOGEN(101,3)
COMMON/EXPNU C/COEFNU(3),EXPNU(3)
COMMON/EN COLL/HT,EN(3),CNU(3)
C
DATA JNU/1/
C
FACTR = 1.0/(HTS(J)-HTS(J+1))
NK SPEC = 1
IF (IONS .NE. 0) NR SPEC = 3
F1 = (HT-HTS(J+1))*FACTR
F2 = (HTS(J)-HT)*FACTR
DO 11 K=1,NR SPEC
11 EN(K) = EXP(ALOGEN(J,K)*F1+ALOGEN(J+1,K)*F2)
IF(COEFNU(1) .LT. 0.0) GO TO 15
DO 12 K=1,NR SPEC
12 CNU(K) = COEFNU(K)*EXP(EXPNU(K)*HT)
RETURN
C
15 IF(JNU .GT. NR NU-1) JNU=1
16 IF (HT .LE. HTS NU(JNU) .OR. JNU .EQ. 1) GO TO 17
JNU = JNU-1
GO TO 16
17 IF (HT .GE. HTS NU(JNU+1) .OR. JNU .EQ. NR NU-1) GO TO 18
JNU = JNU+1
GO TO 17
18 DO 19 K = 1, NR SPEC
ALN = ALCNU(JNU,K)+(ALCNU(JNU+1,K)-ALCNU(JNU,K))
$ * (HT-HTS NU(JNU))/(HTS NU(JNU+1)-HTS NU(JNU))
19 CNU(K) = EXP(ALN)
RETURN
C
END

```

```

      SUBROUTINE R OUT
C FOR MS
C OUTPUT OF RUNGE-KUTTA INTEGRATION VARIABLES, USED ONLY FOR DEBUGGING.
C
      COMMON/INTEGR/R(8)
      COMMON/X INTGR/X(8)
      COMMON/EA COLL/HT,EC OMIT(2)
C
C
      ENTRY R COLS
      PRINT 900
900 FORMAT('C',8X,'HT',10X,'11R11',16X,'11R1',16X,'1R11',17X,'1R1')
      RETURN
C
C
      ENTRY PRINT R
      PRINT 901,HT,(X(I),I=1,8)
      PRINT 901,HT,(R(I),I=1,6)
901 FORMAT(' ',F10.2,4(2X,F4.5,F9.5))
      PRINT 902
902 FORMAT('0')
      RETURN
C
      END

```

```

      SUBROUTINE XFER (A,B,N)
C   FOR MS
C   ROUTINE FOR TRANSFERRING ONE ARRAY INTO ANOTHER.
C
      DIMENSION A(1),b(1)
C
C
      DO 11 J=1,N
11  B(J) = A(J)
      RETURN
C
      END

```

```

      SUBROUTINE INITL R
C   FOR MS
C   COMPUTATION OF (R+1.0)/C FOR REFLECTION FROM A SHARPLY BOUNDED
C   ANISOTROPIC IONOSPHERE OF SEMI-INFINITE EXTENT
C   WHERE R IS THE REFLECTION COEFFICIENT MATRIX AS DEFINED
C   BY BUDDEN. SOLUTION IS USED AS INITIAL CONDITION
C   FOR RUNGE-KUTTA INTEGRATION. THE SOLUTION IS BASED
C   ON RADIO SCIENCE PAPER VOL. 5, AUG. 1968, P.792.
C   DERIVATIVES WRT C=COS(THETA) ARE ALSO COMPUTED IF
C   IDERIV IS SET NON-ZERO.
C
      COMMON/THETA C/THETA(2)
      COMMON/XPRINT/XPRINT
      COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
      COMMON/INTEGR/R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC
      COMMON/C CCM/C
      COMMON/S CCM/S
      COMMON/IDERV C/IDERIV
      COMPLEX
      $      C,CSQ,S,
      $      M11,M21,M31,M12,M22,M32,M13,M23,M33,
      $      R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
      $      QTEMP(4),EIP104,Q(2),
      $      B4,B3,B2,B1,B0,
      $      D11,D12,D13,D31,D32,D33,
      $      P(2),T(2),
      $      DEN,FACTOR,
      $      DSDC,DQDC(2),DDENDC,
      $      DB3DC,DB2DC,DB1DC,DB0DC,
      $      DD11DC,DD13DC,DD31DC,DD33DC,
      $      UPDC(2),DTDC(2)
      INTEGER XPRINT
      DATA EIP104/(0.707107,0.707107)/
C
C   CSQ = C**2
C
C   BOOKER QUARTIC COEFFICIENTS AND ITS FOUR ROOTS.
      B4 = 1.0+M33
      B3 = S*(M13+M31)
      B2 = -(CSQ+M33)*(1.0+M11)+M13*M31-(1.0+M33)*(CSQ+M22)+M23*M32
      B1 = S*(M12*M23+M21*M32-(CSQ+M22)*(M13+M31))
      B0 = (1.0+M11)*(CSQ+M22)*(CSQ+M33)+M12*M23*M31+M13*M21*M32
      $      -M13*(CSQ+M22)*M31-(1.0+M11)*M23*M32-M12*M21*(CSQ+M33)
C
      CALL QUARTC(B4,B3,B2,B1,B0,QTEMP)
C
C   SELECTION OF THE TWO ROOTS CORRESPONDING TO UPWARD TRAVELLING
C   WAVES AS BEING THOSE OF ANGLE LESS THAN 45 BUT
C   GREATER THAN -135 DEGREES IN THE COMPLEX PLANE.
      K = 0
      DO 21 KT=1,4
      IF (REAL(EIP104*QTEMP(KT)) .LT. 0.0) GO TO 21

```

```

      K = K+1
      IF (K .GE. 3) GO TO 90
      Q(K) = CTEMP(KT)
21  CONTINUE
      IF (K .NE. 2) GO TO 90

C
      IF(XPRINT .NE. 0) PRINT 901,THETA,(Q(J),J=1,2)
901  FORMAT('C', 'THETA = ',2F6.3,5X, 'Q = ',2(E15.4,E13.4),/)

C
C  REMAINDER OF THE SOLUTION AND ITS DERIVATIVES.
      DO 31 J=1,2
      D11 = 1.0+M11-Q(J)**2
      D12 = M12
      D13 = M13+Q(J)*S
      D31 = M31+Q(J)*S
      D32 = M32
      D33 = C5C+M33
      DEN = D11*D33-D13*D31
      P(J) = (-D12*D33+D13*D32)/DEN
31  T(J) = U(J)*P(J)-S*(-D11*D32+D12*D31)/DEN

C
      IF (IDERIV .EQ. 0) GO TO 50
      DSDC = -C/S
      DB3DC = DSDC*(M13+M31)
      DB2DC = -2.0*C*(1.0+M11+1.0+M33)
      DB1DC = (DSDC/S)*B1-S*2.0*C*(M13+M31)
      DB0DC = 2.0*C*((1.0+M11)*(C5C+M22+C5C+M33)-M13*M31-M12*M21)

C
      DO 41 K=1,2
      DQDC(K) = -(((DB3DC*Q(K)+DB2DC)*Q(K)+DB1DC)*Q(K)+DB0DC)
      /(((4.0*B4*Q(K)+3.0*B3)*Q(K)+2.0*B2)*Q(K)+B1)
      DD11DC = -2.0*Q(K)*DQDC(K)
      DD13DC = Q(K)*DSDC
      DD31DC = Q(K)*DSDC
      DD33DC = 2.0*C
      DDENDC = D11*DD33DC+DD11DC*D33-D13*DD31DC-DD13DC*D31
      DPDC(K) = -DDENDC*P(K)/DEN+(-D12*DD33DC+DD13DC*D32)/DEN
41  DTDC(K) = Q(K)*DPDC(K)+DQDC(K)*P(K)
      $      -((-D11*D32+D12*D31)*(DSDC-S*DDENDC/DEN)/DEN
      $      -S*(-DD11DC*D32+D12*DD31DC)/DEN

C
50  DEN = (T(1)*C+P(1))*(C+Q(2))-(T(2)*C+P(2))*(C+Q(1))
      FACTOR = 2.0/DEN
      R11 = (T(1)*(C+Q(2))-T(2)*(C+Q(1)))*FACTOR
      R22 = ((T(1)*C+P(1))-(T(2)*C+P(2)))*FACTOR
      R12 = -(T(1)*P(2)+T(2)*P(1))*FACTOR
      R21 = -(C(1)-Q(2))*FACTOR

C
      IF (IDERIV .EQ. 0) RETURN
      DDENDC = (T(1)*C+P(1))*(1.0+DQDC(2))
      $      +(T(1)+DTDC(1)*C+DPDC(1))*(C+Q(2))
      $      -(T(2)*C+P(2))*(1.0+DQDC(1))
      $      -(T(2)+DTDC(2)*C+DPDC(2))*(C+Q(1))

```

```

DR11DC = -R11*DDENDC/DEN
$      +(T(1)*(1.0+DQDC(2))+DTDC(1)*(C+Q(2))
$      -T(2)*(1.0+DQDC(1))-DTDC(2)*(C+Q(1)))*FACTOR
DR22DC = -R22*DDENDC/DEN
$      +((T(1)+DTDC(1)*C+DPDC(1))
$      -(T(2)+DTDC(2)*C+DPDC(2)))*FACTOR
DR12DC = -R12*DDENDC/DEN
$      -(T(1)*DPDC(2)+DTDC(1)*P(2)
$      -T(2)*DPDC(1)-DTDC(2)*P(1))*FACTOR
DR21DC = -R21*DDENDC/DEN
$      -(DQDC(1)-DQDC(2))*FACTOR
RETURN

```

C

```

90 PRINT 900
900 FORMAT('C', 'PROBLEM IN SORTING Q VALUES')
PRINT 995, THETA, QTEMP
995 FORMAT ('O', 'THETA = ', 2F8.3, 5X, 'Q = ', 4(1PE13.3, 1PE11.3))
STOP

```

C

END

```

SUBROUTINE QUARTC(B4,FOUR B3,SIX B2,FOUR B1,ONE B0,Q)
C FOR MS
C SOLUTION FOR THE ROOTS OF A FOURTH-ORDER POLYNOMIAL (QUARTIC
C EQUATION) TAKEN FROM BURNSIDE AND PANTON (1904) THE
C THEORY OF EQUATIONS. A SUMMARY OF PERTINENT EQUATIONS
C IS GIVEN IN RADIO SCIENCE VOL. 3, AUG. 1968,PP.792-795.
C
C COMPLEX FOUR B3,SIX B2,FOUR B1,ONE B0,Q,B4,
$ H,I,G,H PRIME,G PRIME,
$ SQ ROOT,P POS,P,LOG P,
$ CUB RTO,CUB RT1,CUB RT2,OMEGA1,OMEGA2,
$ ROOT P,ROOT Q,ROOT R,
$ TWLV B3,TWLV B2,F,DFDQ,DEL Q,FB4
COMPLEX*16 B3,B2,B1,B0,B3 SQ
REAL MAG POS,MAG NEG
DIMENSION Q(4),P RI(2)
DATA OMEGA1/(-0.5,0.8660254038)/,OMEGA2/(-0.5,-0.8660254038)/
EQUIVALENCE(P,P RI(1))
C
C
B3 = FOUR B3/(4.0*B4)
B2 = SIX B2/(6.0*B4)
B1 = FOUR B1/(4.0*B4)
B0 = ONE B0/B4
C
B3 SQ = B3**2
H = B2-B3 SQ
I = B0-4.0*B3*B1+3.0*B2**2
G = B1+B3*(-3.0*B2+2.0*B3 SQ)
H PRIME = -I/12.0
G PRIME = -G**2/4.0-H*(H**2+3.0*H PRIME)
C
SQ ROOT = CSQRT(G PRIME**2+4.0*H PRIME**3)
P = (-G PRIME+SQ ROOT)*0.5
MAG POS = ABS(P RI(1))+ABS(P RI(2))
P POS = P
P = (-G PRIME-SQ ROOT)*0.5
MAG NEG = ABS(P RI(1))+ABS(P RI(2))
IF(MAG POS .GT. MAG NEG) P = P POS
LOG P = CLOG(P)
CUB RTO = CEXP(LOG P/3.0)
CUB RT1 = OMEGA1*CUB RTO
CUB RT2 = OMEGA2*CUB RTO
C
ROOT P = CSQRT(CUB RTO-H PRIME/CUB RTO-H)
ROOT Q = CSQRT(CUB RT1-H PRIME/CUB RT1-H)
ROOT R = CSQRT(CUB RT2-H PRIME/CUB RT2-H)
IF(CABS(G) .LT. 1.0E-50) GO TO 21
SIGN = -ROOT P*ROOT Q*ROOT R*2.0/G
IF(SIGN .LT. 0.0) ROOT R = -ROOT R
21 Q(1) = +ROOT P+ROOT Q+ROOT R-B3
Q(2) = +ROOT P-ROOT Q-ROOT R-B3
Q(3) = -ROOT P+ROOT Q-ROOT R-B3

```

```

SUBROUTINE QUARTC(B4,FOUR B3,SIX B2,FOUR B1,ONE B0,Q)
C FOR MS
C SOLUTION FOR THE ROOTS OF A FOURTH-ORDER POLYNOMIAL (QUARTIC
C EQUATION) TAKEN FROM BURNSIDE AND PANTON (1954) THE
C THEORY OF EQUATIONS. A SUMMARY OF PERTINENT EQUATIONS
C IS GIVEN IN RADIO SCIENCE VOL. 3, AUG. 1966,PP.792-795.
C
C COMPLEX FOUR B3,SIX B2,FOUR B1,ONE B0,Q,B4,
$ H,I,G,H PRIME,G PRIME,
$ SQ ROOT,P POS,P,LOG P,
$ CUB RTO,CUB RT1,CUB RT2,OMEGA1,OMEGA2,
$ ROOT P,ROOT Q,ROOT R,
$ TWLV B3,TWLV B2,F,DFDQ,DEL Q,FB4
COMPLEX*16 B3,B2,B1,B0,B3 SQ
REAL MAG POS,MAG NEG
DIMENSION Q(4),P RI(2)
DATA OMEGA1/(-0.5,0.8660254038)/,OMEGA2/(-0.5,-0.8660254038)/
EQUIVALENCE(P,P RI(1) )
C
C
B3 = FOUR B3/(4.0*B4)
B2 = SIX B2/(6.0*B4)
B1 = FOUR B1/(4.0*B4)
B0 = ONE B0/B4
C
B3 SQ = B3**2
H = B2-B3 SQ
I = B0-4.0*B3*B1+3.0*B2**2
G = P1+B3*(-3.0*B2+2.0*B3 SQ)
H PRIME = -I/12.0
G PRIME = -G**2/4.0-H*(H**2+3.0*H PRIME)
C
SQ ROOT = CSQRT(G PRIME**2+4.0*H PRIME**3)
P = (-G PRIME+SQ ROOT)*0.5
MAG POS = ABS(P RI(1))+ABS(P RI(2))
P POS = P
P = (-G PRIME-SQ ROOT)*0.5
MAG NEG = ABS(P RI(1))+ABS(P RI(2))
IF(MAG POS .GT. MAG NEG) P = P POS
LOG P = CLOG(P)
CUB RTO = CEXP(LOG P/3.0)
CUB RT1 = OMEGA1*CUB RTO
CUB RT2 = OMEGA2*CUB RTO
C
ROOT P = CSQRT(CUB RTO-H PRIME/CUB RTO-H)
ROOT Q = CSQRT(CUB RT1-H PRIME/CUB RT1-H)
ROOT R = CSQRT(CUB RT2-H PRIME/CUB RT2-H)
IF(CABS(G) .LT. 1.0E-50) GO TO 21
SIGN = -ROOT P*ROOT Q*ROOT R*2.0/G
IF(SIGN .LT. 0.0) ROOT R = -ROOT R
21 Q(1) = +ROOT P+ROOT Q+ROOT R-B3
Q(2) = +ROOT P-ROOT Q-ROOT R-B3
Q(3) = -ROOT P+ROOT Q-ROOT R-B3

```

```

SUBROUTINE T MTRX
C FOR MS
C COMPUTATION OF M MATRIX ELEMENTS AS DEFINED BY BUDDEN IN PROC.
C ROY. SOC. (LONDON), A227, PP.516-537 (1955). THOSE
C COMBINATIONS OF M MATRIX ELEMENTS USED IN THE
C T MATRIX WHICH DO NOT INCLUDE USE OF THETA ARE THE
C FINAL OUTPUT OF THIS ROUTINE. THE DUMMY INDEXING
C ON L IS USED IN VIEW OF POSSIBLE STORAGE (AS FUNCTIONS
C OF HEIGHT) AND RE-USE OF THESE COMBINATIONS FOR
C DIFFERENT VALUES OF THETA.
C
COMMON/FREQ COM/FREQ
COMMON/FLD COM/AZIM,COOIP,MAG FLD
COMMON/H COM/H
COMMON/ICN COM/IONS,MRATIO(3)
COMMON/WN/WAVE NR
COMMON/EN COLL/HT,EN(3),CNU(3)
COMMON/M MTX/M11,M21,M31,M12,M22,M32,M13,M23,M33
COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
$ X31(001),X42(001),X32(001),X41(001)
COMPLEX
$ U,USQ,D,IUD,
$ USQD,TA,TR,
$ D TEMP,CPLX I,UDTEMP,
$ M13D,M31D,M23D,M32D,
$ M2332D,M1331D,M2331D,M3213D,
$ M11,M21,M31,M12,M22,M32,M13,M23,M33,
$ X11,X44,X12,X34,X14,X31,X42,X32,X+1
REAL MAG FLD,LY,MY,NY,IUD PRT,MRATIO,
$ LSQYSQ,MSQYSQ,NSQYSQ,LMYSQ,LNYSQ,MNYSQ
DATA CPLX I/(0.0,1.0)/
DATA PI/3.141592653/
DATA OTR/0.01745329252/
DATA COEFF X/3.182357F03/,COEFF Y/1.758796E11/
DATA VEL LT/2.997928E05/
DATA RE/6369.427/
DIMENSION U PARTS(2),USQ PRT(2),D PARTS(2),IUD PRT(2)
EQUIVALENCE (U,U PARTS(1) ),(USQ,USQ PRT(1) ),(D,D PARTS(1) ),
$ (IUD,IUD PRT(1) )
C
C
C CALL EN NU
C
IF(IONS .NE. 0) GO TO 20
X = (UEF EN*EN(1)
Z = CNU(1)*OV OMGA
U PARTS(1) = 1.0
U PARTS(2) = -Z
USQ PRT(1) = 1.0-Z**2
USQ PRT(2) = -Z-Z
D = -Y**U*(USQ-YSQ)
IUD PR(1) = Z*D PARTS(1)-D PARTS(2)
IUD PRT(2) = D PARTS(1)+Z*D PARTS(2)

```

```

      USQD = USQ*D
      GO TO 30
C
20  C = 1.0
      USQD = 0.0
      IUD = 0.0
      DO 22 K = 1,3
        X = COEF LN*FN(K)/MRATIO(K)
        Z = CHU(K)/OMEGA
        U = 1.0-CPLX I*Z
        USQ = U**2
        UTEMP = -X/(U*(USQ-YSQ/MRATIO(K)**2))
        U = U+UTEMP/MRATIO(K)**2
        UTEMP = U*CTEMP/MRATIO(K)
        IF (K.EQ. 2) UTEMP = -UTEMP
        IUD = IUD +CPLX I*UTEMP
22  USQD = USQD+USQ*UTEMP
C
30  CRVTRM = 2.0*(HT-H)/RE
C
      M11 = JSQD-LSQYSQ*D+CRVTRM
      M22 = JSQD-MSQYSQ*D+CRVTRM
      M33 = USQD-NSQYSQ*D+CRVTRM
      TA = NY*IUD
      TB = LMYSQ*D
      M21 = +TA-TB
      M12 = -TA-TB
      TA = MY*IUD
      TB = LNYSQ*D
      M13 = +TA-TB
      M31 = -TA-TB
      TA = LY*IUD
      TB = MNYSQ*D
      M32 = +TA-TB
      M23 = -TA-TB
C
      D = 1.0/(1.0+M33)
      M13D = M13*D
      M31D = M31*D
      M23D = M23*D
      M32D = M32*D
      M2332D = M32*M23D
      M1331D = M31*M13D
      M2331D = M31*M23D
      M3213D = M32*M13D
C
      L = 1
      X11(L) = -M31D
      X44(L) = -M13D
      X12(L) = M32D
      X34(L) = M23D
      X14(L) = D
      X31(L) = M2331D-M21

```

```

X42(L) = M3213D-M12
X32(L) = M22-M2332D
X41(L) = 1.0+M11-M1331D
RETURN

```

C
C
C
C

COMPUTATION OF QUANTITIES WHICH ARE NOT FUNCTIONS OF HEIGHT
OR OF THETA.

```

ENTRY INIT T
OMEGA = 2.0*PI*FREQ*1000.0
WAVE NR = OMEGA/VEL LT
COEFF EN = COEFF X*1.0E06/OMEGA**2
UV OMEGA = 1.0/OMEGA
SIN DIP = SIN(CCDIP*DTR)
DIR CS L = SIN DIP*COS(AZIM*DTR)
DIR CS M = SIN DIP*SIN(AZIM*DTR)
DIR CS N = -COS(CDIP*DTR)
Y = -COEFF Y*MAG FLD/OMEGA
YSQ = Y**2
LY = DIR CS L*Y
MY = DIR CS M*Y
NY = DIR CS N*Y
LSQYSQ = DIR CS L**2*YSQ
MSQYSQ = DIR CS M**2*YSQ
NSQYSQ = DIR CS N**2*YSQ
LMYSQ = DIR CS L*DIR CS M*YSQ
LNYSQ = DIR CS L*DIR CS N*YSQ
MNYSQ = DIR CS M*DIR CS N*YSQ
RETURN

```

C

END

```

      SUBROUTINE S MTRX
C   FOR MS
C   COMPUTATION OF COEFFICIENTS USED IN THE DIFFERENTIAL EQUATIONS
C   FOR  $(k+1.0)/C$ . THESE ARE STORED IN COMMON AREA 'S MTRX'
C   AND ARE ANALOGOUS TO S MATRIX ELEMENTS GIVEN BY
C   BUDDEN (1955). DERIVATIVES OF THESE COEFFICIENTS
C   WRT  $C=\cos(\theta)$  ARE ALSO COMPUTED IF IDERIV
C   IS SET NON-ZERO.
C
      COMMON/THETA C/THETA
      COMMON/C CLM/C
      COMMON/S CLM/S
      COMMON/STORE X/X11(001),X44(001),X12(001),X34(001),X14(001),
      $             X31(001),X42(001),X32(001),X41(001)
      COMMON/S MTRX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
      $         DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
      COMMON/IDERIV C/IDERIV
      COMPLEX
      $       THETA,
      $       C,S,CSQ,CS,
      $       CSDC,DCSDC,
      $       X11,X44,X12,X34,X14,X31,X42,X32,X41,
      $       TEMP,TWOC,
      $       A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
      $       DB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
      DATA RTD/57.29575/

C
C
      CALL T MTRX
C
      L = 1
      TEMP = X41(L)+X41(L)
      A11 = TEMP+TEMP
      A22 = 4.C
      TEMP = S*X44(L)-C*X41(L)
      B11 = TEMP+TEMP
      B12 = -X42(L)-X42(L)
      TWOC = C+C
      B22 = -TWOC
      TEMP = -S*X11(L)-C*X41(L)
      C11 = TEMP+TEMP
      C21 = X31(L)+X31(L)
      C22 = -TWOC
      D11 = CS*(X11(L)-X44(L))-(1.0-X14(L))+CSQ*(X41(L)-X14(L))
      D21 = -C*X31(L)+S*X34(L)
      D12 = S*X12(L)+C*X42(L)
      D22 = -X32(L)

C
      IF (IDERIV .EQ. 0) RETURN
      TEMP = CSDC*X44(L)-X41(L)
      DB11DC = TEMP+TEMP
      DB22DC = -2.0
      TEMP = -CSDC*X11(L)-X41(L)

```

```

DC110C = TEMP+TEMP
DC220C = -2.0
DD110C = DCSDC*(X11(L)-X44(L))
      + 2.0*(X41(L)-X14(L))
DD210C = -X31(L)+DSDC*X34(L)
DD120C = DSDC*X12(L)+X42(L)
RETURN

```

C
C

C INITIALIZATION FOR GIVEN VALUE OF THETA.

```

ENTRY INIT S
C = CCOS(THETA/PTD)
CSQ = C**2
S = CSIN(THETA/RTD)
CS = C*S

```

C

```

IF (IDERIV .EQ. 0) RETURN
DSDC = -C/S
DCSDC = S-CSQ/S
RETURN

```

C

```

END

```

SUBROUTINE DIFF EQ

```

C FOR MS
C DIFFERENTIAL EQUATIONS FOR (R+1.0)/C WHERE R IS THE REFLECTION
C MATRIX AS DEFINED BY BUDDEN. THE EQUATIONS ARE
C DERIVED FROM THE DIFFERENTIAL EQUATIONS FOR R
C GIVEN BY BUDDEN IN DC. RCY. SOC. (LONDON), A27,
C PP.516-537 (1955). STORAGE INTO THE ARRAY 'R MTRX'
C IS MADE SO THAT THE EQUATIONS MAY BE USED WITH
C EITHER THE MAIN SET OF INTEGRATION VARIABLES, OR
C WITH THE COMPARISON SET, X. DIFFERENTIAL EQUATIONS
C FOR THE DERIVATIVES OF (R+1.0)/C WRT C=COS(THETA) ARE
C ALSO USED IF IDERIV IS SET NON-ZERO. NOTE THAT THE
C VARIABLE CALLED 'R' IN THIS ROUTINE IS ACTUALLY
C (R+1.0)/C. MULTIPLICATION BY -1 IS EFFECTED BY
C APPROPRIATE TRANSFER OF VALUES FROM 'DERIV' TO
C COMMON AREA VARIABLES.

COMMON/INTG/R(16),DRDH(16)
COMMON/X INTG/X(8),DXDH(8)
COMMON/S MTRX/A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
$ CB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC
COMMON/EXTRA/R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
$ DR11DH,DR21DH,DR12DH,DR22DH,
$ ER11CH,DR21CH,DR12CH,DR22CH
COMMON/IDERV C/IDERV
COMMON/M CM/N
COMMON/CONVFLD/ICVFLD
COMPLEX
$ R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC,
$ DR11DH,DR21DH,DR12DH,DR22DH,
$ ER11CH,DR21CH,DR12CH,DR22CH,
$ A11,A22,B11,B12,B22,C11,C21,C22,D11,D21,D12,D22,
$ CB11DC,DB22DC,DC11DC,DC22DC,DD11DC,DD21DC,DD12DC,
$ D11R11,D12R21,D21R12,D22R22,
$ F11R22,R12R21,
$ G11R11,G12R21,G21R12,
$ D21R11,D12R11,D11R21,D22R21,D11R12,D22R12,D21R22,D12R22
DIMENSION R MTRX(16),DERIV(16)
EQUIVALENCE(R11,R MTRX(1)),(DR11DH,DERIV(1))

C
C
ENTRY P DERIV
CALL XFFP (R,R MTRX,N)
IXFLAG = 0
20 DO 21 I=1,8
IF(ABS(R MTRX(1))) .GT. 1.0E4) GO TO 90
21 CONTINUE

C
D11R11 = D11*R11
D12R21 = D12*R21
D21R12 = D21*R12
D22R22 = D22*R22
R11R22 = R11*R22

```

```

P12R21 = R12*P21
C
DR11DH = A11*(B11+C11+D11R11+D12R21+D21R12)*R11
+ B12*R21+C21*R12+D22*R12R21
DR21DH = (B22+C11+D11R11+D12R21+D22R22)*R21
+ C21*R22+D21*R11R22
DR12DH = (B11+C22+D11R11+D21R12+D22R22)*R12
+ B12*R22+D12*R11R22
DR22DH = A22*(B22+C22+D12R21+D21R12+D22R22)*R22
+ D11*R12R21
IF (IXFLAG .NE. 0) GO TO 40
C
IF (IDERIV .EQ. 0) GO TO 22
G11R11 = D11DC*R11
G12R21 = D12DC*R21
G21R12 = D21DC*R12
C
D21R11 = D21*R11
D12R11 = D12*R11
D11R21 = D11*R21
D22R21 = D22*R21
D11R12 = D11*R12
D22R12 = D22*R12
D21R22 = D21*R22
D12R22 = D12*R22
C
DR11CH = (LB11DC+DC11DC+G11R11+G12R21+G21R12)*R11
$ + (B11+C11+D11R11+D11R11+D12R21+D21R12)*DR11DC
$ + (D12R11+B12+D22R12)*DR21DC
$ + (D21R11+C21+D22R21)*DR12DC
DR21CH = (LB22DC+DC11DC+G11R11+G12R21)*R21+DD21DC*R11R22
$ + (D11R21+D21R22)*DR11DC
$ + (B22+C11+D11R11+D12R21+D12R21+D22R22)*DR21DC
$ + (D22R21+C21+D21R11)*DR22DC
DR12CH = (LB11DC+DC22DC+G11R11+G21R12)*R12+DD12DC*R11R22
$ + (D11R12+D12R22)*DR11DC
$ + (B11+C22+D11R11+D21R12+D21R12+D22R22)*DR12DC
$ + (D22R12+B12+D12R11)*DR22DC
DR22CH = (LB22DC+DC22DC+G12R21+G21R12)*R22+DD11DC*R12R21
$ + (D12R22+D11R12)*DR21DC
$ + (D21R22+D11R21)*DR12DC
$ + (B22+C22+D12R21+D21R12+D22R22+D22R22)*DR22DC
C
22 DO 24 I=1,N,2
   DRDH(I) = DERIV(I+1)
24 DRDH(I+1) = -DERIV(I)
   RETURN
C
C
ENTRY X DERIV
CALL XFEF (X,R MTRX,8)
IXFLAG = 1
GO TO 20

```

```

C
40 DO 42 I=1,7,2
   EXCH(I) = DERIV(I+1)
42 EXCH(I+1) = -DERIV(I)
   RETURN

C
90 IDVFLD = 1
   RETURN

C
   END

```

```

SUBROUTINE RBAR
C FOR MS
C THIS SUBROUTINE COMPUTES VALUES OF VARIABLES WHICH MAY BE USED TO
C FORM THE ELEMENTS OF THE RBAR MATRIX, WHERE RBAR
C REPRESENTS REFLECTION OF AN ELM WAVE FROM THE EARTH'S
C SURFACE. NAMELY, RBAR11=DEN11/(C*NUM11-DEN11),
C OR (1.0/RBAR11+1.0)/C=NUM11/DEN11,
C AND RBAR22=DEN22/(C*NUM22-DEN22),
C OR (1.0/RBAR22+1.0)/C=NUM22/DEN22. DERIVATIVES
C WRT C=COS(THETA) ARE ALSO COMPUTED IF IDERV
C IS SET NON-ZERO. NOTE THAT THE
C EQUATIONS ARE FORMULATED IN SUCH A WAY THAT A SMOOTH
C TRANSITION IS MADE FROM THE 'CURVED EARTH' FORM TO
C THE 'FLAT EARTH' FORM. SEE ALSO NOTES IN SUBROUTINE
C MUHNKL REGARDING DEFINITIONS OF HANKEL FUNCTION
C PARAMETERS. THE VALUE OF EARTH'S RADIUS IS SUCH AS
C TO MAKE 2.0/RE=3.14E-4. COMPUTATION AT THETA=90
C IS NOT EXCLUDED.
C
COMMON/RTALT/ TALT,RALT
COMMON/THETA C/THETA
COMMON/C CCM/C
COMMON/RB COM/NUM11,NUM22,DEN11,DEN22,DNM1DC,DNM2DC,DDN1DC,DDN2DC
COMMON/F1F2 C/F1,F2,HG,F1ZT,F1ZR,F2ZT,F2ZR,GF1ZT
COMMON/FRC COM/FREQ
COMMON/H CCM/H
COMMON/D CCM/D
COMMON/GND COM/EPREL,SIGMA
COMMON/IDERV C/IDERV
COMPLEX
$ EZ,FZ,
$
$ THETA,C,
$ NUM11,NUM22,DEN11,DEN22,
$ DNM1DC,DNM2DC,DDN1DC,DDN2DC,
$ F1,F2,HG,HG FCTR,F1ZT,F1ZR,F2ZT,F2ZR,DF1ZT,GF1ZT,
$ CSQ,SSQ,
$ NGSQ,I,
$ SQR,
$ L,DZDC,
$ H1,H1P,H1PP,
$ H2,H2P,H2PP,
$ E,E0,EXPON,
$ KK,KX,
$ A1,A2,A3,A4,B1,B2,B3,B4,
$ DA1DC,DA2DC,DA3DC,DA4DC,DB1DC,DB2DC,DB3DC,DB4DC
PEAL KVRANT,KVRATT,NDSQ,NDSQ
DATA RTE/57.29578/
DATA I/(C.0,1.0)/
DATA EPSLNO/8.85434E-12/
DATA VEL LT/2.997923E5/
DATA PI/3.141593/
DATA RE/6369.427/

```

```

C
C = COS(THETA/RTD)
CSQ = C**2
SSQ = 1.0-CSQ

C
C
HEIGHT GAIN FOR Z=0
SQR = CSQRT(AGSQ-SSQ)
NUM11 = 1.0
DEN11 = (C-SQR/AGSQ)/2.0
NUM22 = 1.0/SQR
DEN22 = (C/SQR-1.0)/2.0
IF (IDERIV .EQ. 0) GO TO 15
DNM1DC = 0.0
DEN1DC = (1.0-C/(SQR*AGSQ))/2.0
DNM2DC = -C/SQR**3
DEN2DC = -(C**2/SQR**3+1.0/SQR)/2.0

C
15 Z = KVRATT*(CSQ+EMO)
CALL MDPNKL (Z,H1,H2,H1P,H2P,E0)
B1 = C*H1+(KK*H1P+KX*H1)/NOSQ
B2 = C*H2+(KK*H2P+KX*H2)/NOSQ
B3 = C*H1+KK*H1P
B4 = C*H2+KK*H2P
A1 = NUM11*B1-2.0*DEN11*H1
A2 = NUM11*B2-2.0*DEN11*H2
A3 = NUM22*B3-2.0*DEN22*H1
A4 = NUM22*B4-2.0*DEN22*H2
IF (IDERIV .EQ. 0) GO TO 20
DZDC = KVRATT*2.0*C
H1PP = -Z*H1
H2PP = -Z*H2
DB1DC = B1+(C*H1P+(KK*H1PP+KX*H1P)/NOSQ)*DZDC
DB2DC = B2+(C*H2P+(KK*H2PP+KX*H2P)/NOSQ)*DZDC
DB3DC = B1+(C*H1P+KK*H1PP)*DZDC
DB4DC = B2+(C*H2P+KK*H2PP)*DZDC
DA1DC = NUM11*DB1DC+DNM1DC*B1-2.0*(DEN11*H1P*DZDC+DDN1DC*H1)
DA2DC = NUM11*DB2DC+DNM1DC*B2-2.0*(DEN11*H2P*DZDC+DDN1DC*H2)
DA3DC = NUM22*DB3DC+DNM2DC*B3-2.0*(DEN22*H1P*DZDC+DDN2DC*H1)
DA4DC = NUM22*DB4DC+DNM2DC*B4-2.0*(DEN22*H2P*DZDC+DDN2DC*H2)
HG FC TR = (A2*H1-A1*H2)/(-KK)

C
C
HEIGHT GAINS FOR ELEVATED TRANSMITTER AND ELEVATED RECEIVER
Z = KVRATT*(CSQ+EMZT)
CALL MDPNKL (Z,H1,H2,H1P,H2P,EZ)
EXPON = CEXP(EZ-E0)
H1 = H1/EXPON
H2 = H2/EXPON
H1P=H1P/EXPON
H2P=H2P/EXPON
F1ZT = (A2*H1-A1*H2)/(-KK)*EXP((TALT-D)/RE)
F2ZT = (A4*H1-A3*H2)*(-SQR/KK)

C
C
PART OF THE EQUATION OF THE DERIVATIVE OF HEIGHT GAIN FOR
C
C
PERAL AT HEIGHT Z

```

```

C DF1Z1 = (A2*H1P-A1*H2P)/(-KK)*EXP((TALT-D)/RE)
C THE DERIVATIVE OF THE HEIGHT GAIN FOR FPERL AT HEIGHT Z
C EXCLUSIVE OF THE D11 TERM
C DF1Z1 = -1*(AVRKUT*DF1Z1+ARG*F1Z1/2.0)

C Z = KVRATT*(CSQ+FMZR)
CALL MDHNKL(Z,H1,H2,H1P,H2P,EZ)
EXPON = C*EXP(EZ-E0)
H1 = H1/EXPON
H2 = H2*EXPON
F1ZP = (A2*H1-A1*H2)/(-KK)*EXP((TALT-D)/RE)
F2ZR = (A4*H1-A3*H2)*(-SQR/KK)

C HEIGHT GAINS AT Z=D
C 20 Z = KVRATT*(CSQ+EMD)
CALL MDHNKL(Z,H1,H2,H1P,H2P,E)
EXPON = C*EXP(E-E0)
H1 = H1/EXPON
H2 = H2*EXPON
H1P = H1F/EXPON
H2P = H2F*EXPON

C B1 = C*H1+(KK*H1P+KX*H1)/NDSQ
B2 = C*F2+(KK*H2P+KX*H2)/NDSQ
B3 = C*F1+KK*H1P
B4 = C*H2+KK*H2P
DEN11 = A2*B1-A1*B2
DEN22 = A4*B3-A3*B4
NUM11 = 2.0*(A2*H1-A1*H2)
NUM22 = 2.0*(A4*H1-A3*H2)
IF (IDFIV.EQ.0) RETURN
H1PP = -Z*H1
H2PP = -Z*H2
DB1DC = F1+(C*H1P+(KK*H1PP+KX*H1P)/NDSQ)*DZDC
DB2DC = F2+(C*H2P+(KK*H2PP+KX*H2P)/NDSQ)*DZDC
DB3DC = F1+(C*H1P+KK*H1PP)*DZDC
DB4DC = F2+(C*H2P+KK*H2PP)*DZDC
DDN1DC = A2*DB1DC+DA2DC*B1-A1*DB2DC-DA1DC*B2
DDN2DC = A4*DB3DC+DA4DC*B3-A3*DB4DC-DA3DC*B4
DNM1DC = 2.0*(A2*H1P*DZDC+DA2DC*H1-A1*H2P*DZDC-DA1DC*H2)
DNM2DC = 2.0*(A4*H1P*DZDC+DA4DC*H1-A3*H2P*DZDC-DA3DC*H2)

C FOLLOWING FOUR STATEMENTS NEEDED ONLY FOR COMPUTING EXCITATION
C FACTORS.
C HEIGHT GAIN FOR FPERL AT Z=D
C F1 = (A2*H1-A1*H2)/(-KK)
C HEIGHT GAIN FOR FPERP AT Z=D
C F2 = (A4*H1-A3*H2)*(-SQR/KK)
C HEIGHT GAIN FOR FPERL AT Z=0 DIVIDED BY
C HEIGHT GAIN FOR FPERL AT Z=D
C HG = EXP(-D/RE)*HG FCTR/F1
C RETURN

```

```

C
C  COMPUTATION WHICH IS INDEPENDENT OF THETA.
  ENTRY INIT RB
  OMEGA = 2.0*PI*FREQ*1000.0
  WAVE NR = (OMEGA/VEL LT
  NOSQ = EPSKEL-1*SIGMA/OMEGA/EPSLNO

C
  ARG = 2.0/(WAVE NR*RE)
  AVRKOT = EXP(ALOG(ARG)/3.0)
  KVRATT = 1.0/AVRKOT
  KVRATT = 1.0/AVRKOT**2
  KK = 1*AVRKOT
  KX = 1/(WAVE NR*RE)

C
  EMZK = 2.0*(KALT-H)/RE
  EMZT = 2.0*(TALT-H)/RE

C
  EMD = 2.0*(D-H)/RE
  NOSQ = 1.0+EMD
  EMJ = -2.0*H/RE
  NOSQ = 1.0-2.0*H/RE
  RETURN

C
  END

```

```

      SUBROUTINE MDHNKL (Z,H1,H2,H1P,H2P,E)
C   FOR MS
C   THIS SUBROUTINE COMPUTES VALUES OF VARIABLES WHICH MAY BE COMBINED
C   TO FORM MODIFIED HANKEL FUNCTIONS AS DESCRIBED
C   IN 'TABLES OF THE MODIFIED HANKEL FUNCTIONS OF ORDER
C   ONE THIRD AND OF THEIR DERIVATIVES', BY THE STAFF OF
C   THE COMPUTATION LABORATORY, HARVARD UNIVERSITY
C   PRESS, 1945. THE NOTATION IS SUCH THAT THE
C   ACTUAL  $H_1 = H_1 * \exp(-E)$ , THE ACTUAL  $H_2 = H_2 * \exp(E)$ .
C   ACTUAL DERIVATIVES WRT ARGUMENT, Z, ARE  $H_{1P} * \exp(-E)$ 
C   AND  $H_{2P} * \exp(E)$ .
C
      COMPLEX Z,H1,H2,H1P,H2P,E,
$         R,
$         I,PARTS,
$         ZPOWER,TERM,
$         SUMA,SUMB,SUMC,SUMD,
$         MZ3,
$         F,G,FP,GP,T,TP,
$         IPI16T,I2THRD,IPI512,
$         SUM1,SUM2,SUM1P,SUM2P,
$         RTZ,OVRZ,OVRM42,Z3H,M3HOVZ,IRTZ,MIZ3H,MIOV23,
$         CSUM,ESUM,OSUMP,ESUMP,
$         EDIFF
      DIMENSION A(22),B(22),CAP(14),PART(2)
      EQUIVALENCE (PART(1),PARTS)
      DATA AO/C.93043671693/
      DATA BO/C.67829672514/
      DATA CAP/
$    1.04166666666667D-01,8.35503472222222D-02,1.28226574556327D-01,
$    2.91849026464140D-01,8.81627267443758D-01,3.32140828186277D 00,
$    1.49957629868626D 01,7.89230130115870D 01,4.74451538866000D 02,
$    3.20749009100000D 03,2.40865496000000D 04,1.93923120000000D 05,
$    1.79190200000000D 06,1.74843770000000D 07/
      DATA I/(C.C,1.0)/
      DATA ROOT3/1.73205080756888/
      DATA ALPHA/0.853667218838951/
      DATA PI/3.14159265358979/
      DATA I1ST/1/
      DATA IFIRST/1/
C
C
      ZMAG=CABS(Z)
      IF (ZMAG .GT. 4.2) GO TO 50
C
C   MACLAURIN'S SERIES FORM FOR MAGNITUDE(Z) NOT GREATER THAN 4.2
      IF (I1ST .EQ. 0) GO TO 12
      A(1) = 1.0/6.0
      B(1) = 2.0/24.0
      DO 11 M=2,22
      A(M) = A(M-1)*(3.0*M-2.0)/((3.0*M)*(3.0*M-1.0)*(3.0*M-2.0))
      B(M) = B(M-1)*(3.0*M-1.0)/((3.0*M+1.0)*(3.0*M)*(3.0*M-1.0))
11 CONTINUE

```

```

      I1ST = 0
12  SUMA = 1.0
      SUMB = 1.0
      SUMC = 0.0
      SUMD = 0.0
      ZPOWER = 1.0
      MZ3 = -Z**3
      DO 15 M=1,22
      ZPOWER = ZPOWER*MZ3
      TERM = A(M)*ZPOWER
      SUMA = SUMA+TERM
      SUMC = SUMC+M*TERM
      PARTS = TERM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-10) GO TO 16
      TERM = B(M)*ZPOWER
      SUMB = SUMB+TERM
      SUMD = SUMD+M*TERM
      PARTS = TERM
      IF (ABS(PART(1))+ABS(PART(2)) .LT. 1.0E-10) GO TO 16
15  CONTINUE
16  F = A0*SUMA
      G = Z*B0*SUMB
      FP = 3.0*A0*SUMC/Z
      GP = (3.0*SUMD+SUMB)*B0
      T = 1*(G-F-F)/ROOT3
      TP = 1*(GP-FP-FP)/ROOT3
      H1 = G+T
      H1P = GP+TP
      H2 = G-T
      H2P = GP-TP
      E = 0.0
      RETURN

```

C
C ASYMPTOTIC SERIES FOR MAGNITUDE(Z) GREATER THAN 4.2

```

50  IF (IFIRST .EQ. 0) GO TO 51

```

```

      IPI16T = 16.0*PI*I/12.0
      I2THFD = 1*2.0/3.0
      IPI512 = 5.0*PI*I/12.0
      ALPHSQ = ALPHA**2
      IFIRST = 0

```

```

51  RTZ = CSQRT(Z)
      OVRZ = 1.0/Z
      OVRM4Z = -0.25*OVRZ
      Z3H = Z*RTZ
      M3HOVL = -1.5*OVRZ
      IRTZ = I*RTZ
      M1Z3H = -I*Z3H
      M1OVL3 = -OVRZ**3
      F = -I2THFD*Z3H+IPI512
      R = CSQRT(ALPHSQ/RTZ)

```

C
 OSUM = 0.0
 OSUMF = 0.0

```

    ESUM = 0.0
    ESUMP = 0.0
    ZPOWER = 1.0
    DO 52 M=2,14,2
    ZPOWER = ZPOWER*M10VZ3
    TERM = CAP(M-1)*ZPOWER
    USUM = CSUM+TERM
    USUMP = CSUMP+(M-1)*TERM
    TERM = CAP(M)*ZPOWER
    LSUM = ESUM+TERM
    ESUMP = ESUMP+M*TERM
    PARTS = TERM
    IF (ABS(PART(1))+ABS(PART(2))) .LT. 1.0E-8) GO TO 53
52 CONTINUE
53 CONTINUE
    USUM = M1Z3H*OSUM
    USUMP = M1Z3H*M3HOVZ*OSUMP
    FSUMP = M3HOVZ*ESUMP
    SUM1 = 1.0-OSUM+ESUM
    SUM2 = 1.0+OSUM+FSUMP
    SUM1P = -OSUMP+ESUMP
    SUM2P = CSUMP+ESUMP
C
    H1 = SUM1
    H2 = SUM2
    H1P = (CVRM4Z+IKTZ)*SUM1+SUM1P
    H2P = (CVRM4Z-IKTZ)*SUM2+SUM2P
    PARTS = 2
    IF (PART(1) .GT. 0.0) GO TO 70
C
    IF (PART(2) .GT. 0.0) GO TO 63
    EDIFF = CEXP(E+E-IP116T)
    H1 = H1+EDIFF*H2
    H1P = H1P+EDIFF*H2P
    GO TO 70
63 EDIFF = CEXP(-E-E+IP116T)
    H2 = H2+EDIFF*H1
    H2P = H2P+EDIFF*H1P
C
70 H1 = F*H1
    H2 = R*H2
    H1P = R*H1P
    H2P = P*H2P
    RETURN
C
END

```

```

SUBROUTINE FS INTG
C FOR MS
C THIS SUBROUTINE PERFORMS AN INTEGRATION OF THE DIFFERENTIAL
C EQUATIONS FOR THE IONOSPHERE REFLECTION MATRIX
C THROUGH A FREE SPACE REGION OVER A CURVED EARTH.
C THE INTEGRATION MAY BE PERFORMED IN EITHER A POSITIVE
C OR NEGATIVE HEIGHT DIRECTION, BUT IN THIS PROGRAM THE
C INTEGRATION IS ALWAYS UPWARD. THE INTEGRATION VARIABLES
C ARE THE ELEMENTS OF THE MATRIX  $(R+1.0)/C$  WHERE K IS THE
C REFLECTION MATRIX DESCRIBED BY BUDDEN. THE SOLUTION
C IS BASED ON BUDDEN, RADIO WAVES IN THE IONOSPHERE,
C IN PARTICULAR THE MATERIAL ON PP.118,327-329,
C 330-338, AND 343-345. IF IDERV
C IS SET NON-ZERO, THE DERIVATIVES OF  $(R+1.0)/C$  ELEMENTS
C WRT  $C=\cos(\theta)$  ARE ALSO INTEGRATION VARIABLES.
C COMPUTATION AT  $\theta=90$  IS NOT EXCLUDED. ALSO,
C THE EQUATIONS ARE FORMULATED IN SUCH A WAY THAT A
C SMOOTH TRANSITION IS MADE FROM THE 'CURVED EARTH' FORM
C TO THE 'FLAT EARTH' FORM FOR APPROPRIATE VALUES OF
C  $\theta$ . THE INITIAL AND FINAL VALUES OF
C THE INTEGRATION VARIABLES ARE STORED IN THE COMMON
C AREA 'INTEGR'.

```

```

COMMON/XPRINT/XPRINT
COMMON/INTEGR/R11,R21,R12,R22,DR11DC,DR21DC,DR12DC,DR22DC
COMMON/THETA C/THETA
COMMON/C CCM/C
COMMON/H CCM/H
COMMON/WA/WAVE NR
COMMON/IDERV C/IDERV
COMMON/THETA,C,
$      R11,R21,R12,R22,
$      DR11DC,DR21DC,DR12DC,DR22DC,
$      I,KK,KX,CSQ,
$      EXPUN,
$      Z,H1,H2,H1P,H2P,E0,E,
$      H1PP,H2PP,DZDC,
$      A11,A12,A21,A22,
$      DA11DC,DA12DC,DA21DC,DA22DC,
$      F1,G1,A1,B1,F2,G2,A2,B2,
$      DF1DC,DG1DC,DA1DC,DB1DC,DF2DC,DG2DC,DA2DC,DB2DC,
$      EP1,EP2,EY1,EY2,
$      DEP1DC,DEP2DC,DEY1DC,DEY2DC,
$      DEN,V1,V2,
$      DDENDC,DV1DC,DV2DC
REAL KVRATT,NOSQ,NZSQ
INTEGER XPRINT
DATA I/(C.C,1.0)/
DATA RE/6369.427/
DATA RTE/57.29573/

```

```

IF (XPRINT .NE. 0) PRINT 100,R11,R21,R12,R22

```

```

100 FORMAT ('0', 'AT HT=Z0 R =', 4(2X, 2F12.5))
      IF (IDERIV .NE. 0 .AND. XPRINT .NE. 0)
      $      PRINT 101, DR11DC, DR21DC, DR12DC, DR22DC
101 FORMAT ('0', 'DERIVS =', 4X, 4(2X, 1P2E12.5))

C
      C = COS(THETA/RTD)
      CSQ = C**2

C
C      COMPUTATION OF HEIGHT-GAIN COEFFICIENTS F,G,A,B FOR TWO CONDITIONS
C      ON THE UPGOING WAVE AT HEIGHT=Z0, NAMELY, E11=1, EY=0
C      AND E11=0, EY=1. EACH OF THE COEFFICIENTS F,G,A,B IS
C      DIVIDED BY C=COS(THETA).

      Z = KVRATT*(CSQ+EMO)
      CALL MDHAKL (Z, H1, H2, H1P, H2P, EU)
      A11 = H1
      A12 = H2
      A21 = C*F1+(KK*H1P+KX*H1)/NOSQ
      A22 = C*F2+(KK*H2P+KX*H2)/NOSQ
      DEN = A11*A22-A21*A12
      F1 = (R11*A22-2.0*A12)/DEN
      G1 = (2.0*A11-R11*A21)/DEN
      F2 = R12*A22/DEN
      G2 = -R12*A21/DEN

C
      IF (IDERIV .EQ. 0) GO TO 25
      DZDC = KVRATT*2.0*C
      H1PP = -Z*H1
      H2PP = -Z*H2
      DA11DC = H1P*DZDC
      DA12DC = H2P*DZDC
      DA21DC = H1+(C*H1P+(KK*H1PP+KX*H1P)/NOSQ)*DZDC
      DA22DC = H2+(C*H2P+(KK*H2PP+KX*H2P)/NOSQ)*DZDC
      DDENDC = A11*DA22DC+DA11DC*A22-A21*DA12DC-DA21DC*A12
      DF1DC = (R11*DA22DC+DR11DC*A22-2.0*DA12DC)/DEN-F1*DDENDC/DEN
      DG1DC = (2.0*DA11DC-R11*DA21DC-DR11DC*A21)/DEN-G1*DDENDC/DEN
      DF2DC = (R12*DA22DC+DR12DC*A22)/DEN-F2*DDENDC/DEN
      DG2DC = (-R12*DA21DC-DR12DC*A21)/DEN-G2*DDENDC/DEN

C
25 A11 = H1
   A12 = H2
   A21 = C*F1+KK*H1P
   A22 = C*F2+KK*H2P
   DEN = A11*A22-A21*A12
   A1 = R21*A22/DEN
   B1 = -R21*A21/DEN
   A2 = (R22*A22-2.0*A12)/DEN
   B2 = (2.0*A11-R22*A21)/DEN

C
      IF (IDERIV .EQ. 0) GO TO 30
      DA11DC = H1P*DZDC
      DA12DC = H2P*DZDC
      DA21DC = H1+(C*H1P+KK*H1PP)*DZDC
      DA22DC = H2+(C*H2P+KK*H2PP)*DZDC

```

$DDENDC = A11*DA22DC + DA11DC*A22 - A21*DA12DC - DA21DC*A12$
 $DA1DC = (R21*DA22DC + DR21DC*A22)/DEN - A1*DDENDC/DEN$
 $DB1DC = (-R21*DA21DC - DR21DC*A21)/DEN - B1*DDENDC/DEN$
 $DA2DC = (R22*DA22DC + DR22DC*A22 - 2.0*DA12DC)/DEN - A2*DDENDC/DEN$
 $DB2DC = (2.0*DA11DC - R22*DA21DC - DR22DC*A21)/DEN - B2*DDENDC/DEN$

C
 C COMPUTATION OF UPCOMING FIELDS E11 AND EY AT HEIGHT=ZZ FOR THE TWO
 C CONDITIONS DESCRIBED ABOVE.

30 Z = KVRATT*(CSQ+EMZ)
 CALL MEHKL (Z,H1,H2,H1P,H2P,E)
 EXPON = CEXP(E-E0)
 H1 = H1/EXPON
 H2 = H2*EXPON
 H1P = H1P/EXPON
 H2P = H2P*EXPON

C
 $A21 = C*F1 + (KK*H1P + KX*H1)/NZSQ$
 $A22 = C*F2 + (KK*H2P + KX*H2)/NZSQ$
 $EP1 = (F1*A21 + G1*A22)/2.0$
 $EP2 = (F2*A21 + G2*A22)/2.0$

C
 IF (IDERIV .EQ. 0) GO TO 35
 $H1PP = -Z*H1$
 $H2PP = -Z*H2$
 $DA21DC = H1 + (C*H1P + (KK*H1PP + KX*H1P)/NZSQ)*DZDC$
 $DA22DC = H2 + (C*H2P + (KK*H2PP + KX*H2P)/NZSQ)*DZDC$
 $DFP1DC = (F1*DA21DC + DF1DC*A21 + G1*DA22DC + DG1DC*A22)/2.0$
 $DFP2DC = (F2*DA21DC + DF2DC*A21 + G2*DA22DC + DG2DC*A22)/2.0$

C
 35 $A21 = C*F1 + KK*H1P$
 $A22 = C*F2 + KK*H2P$
 $EY1 = (A1*A21 + B1*A22)/2.0$
 $EY2 = (A2*A21 + B2*A22)/2.0$

C
 IF (IDEPV .EQ. 0) GO TO 40
 $DA21DC = H1 + (C*H1P + KK*H1PP)*DZDC$
 $DA22DC = H2 + (C*H2P + KK*H2PP)*DZDC$
 $DEY1DC = (A1*DA21DC + DA1DC*A21 + B1*DA22DC + DB1DC*A22)/2.0$
 $DEY2DC = (A2*DA21DC + DA2DC*A21 + B2*DA22DC + DB2DC*A22)/2.0$

C
 C COMPUTATION OF REFLECTION COEFFICIENTS AT HEIGHT=ZZ.

40 $V1 = F1*H1 + G1*H2$
 $V2 = F2*H1 + G2*H2$
 $DEN = EP1*EY2 - EP2*EY1$
 $R11 = (V1*EY2 - V2*EY1)/DEN$
 $R12 = (V2*EP1 - V1*EP2)/DEN$

C
 IF (IDERIV .EQ. 0) GO TO 45
 $DV1DC = (F1*H1P + G1*H2P)*DZDC + DF1DC*H1 + DG1DC*H2$
 $DV2DC = (F2*H1P + G2*H2P)*DZDC + DF2DC*H1 + DG2DC*H2$
 $DDENDC = EP1*DEY2DC + DEP1DC*EY2 - EP2*DEY1DC - DEP2DC*EY1$
 $DR11DC = (V1*DEY2DC + DV1DC*EY2 - V2*DEY1DC - DV2DC*EY1)/DEN$
 $-R11*DDENDC/DEN$

```

DR12DC = (V2*DEP1DC+DV2DC*EP1-V1*DEP2DC-DV1DC*EP2)/DEN
$
-R12*DDENDC/DEN
C
42 V1 = A1*F1+B1*H2
V2 = A2*F1+B2*H2
R21 = (V1*EY2-V2*EY1)/DEN
R22 = (V2*EP1-V1*EP2)/DEN
C
IF (IDERIV .EQ. 0) GO TO 50
DV1DC = (A1*H1P+B1*H2P)*DZDC+DA1DC*H1+DB1DC*H2
DV2DC = (A2*H1P+B2*H2P)*DZDC+DA2DC*H1+DB2DC*H2
DR21DC = (V1*DEY2DC+DV1DC*EY2-V2*DEY1DC-DV2DC*EY1)/DEN
$
-R21*DDENDC/DEN
DR22DC = (V2*DEP1DC+DV2DC*EP1-V1*DEP2DC-DV1DC*EP2)/DEN
$
-R22*DDENDC/DEN
C
50 IF (XPRINT .NE. 0) PRINT 500,R11,R21,R12,R22
500 FORMAT ('0','AT HT=ZZ H =',4(2X,2F12.5))
IF (IDERIV .NE. 0) .AND. XPRINT .NE. 0
$
PRINT 501,DR11DC,DR21DC,DR12DC,DR22DC
501 FORMAT ('0','DERIVS = ',4X,4(2X,1P2E12.5))
RETURN
C
C
C COMPUTATION WHICH IS INDEPENDENT OF THETA. THE INTEGRATION WILL
C PROCEED FROM HEIGHT Z0 TO HEIGHT ZZ WHEN FSINTG IS
C CALLED.
ENTRY INIT FS(Z0,ZZ)
ARG = 2.0/(WAVE NR*RE)
AVRKOT = EXP(ALOG(ARG)/3.0)
KVRATT = 1.0/AVRKOT**2
KY = 1*AVRKOT
KX = 1/(WAVE NR*RE)
C
EMO = 2.0*(Z0-H)/RE
NUSQ = 1.0+EMO
EMZ = 2.0*(ZZ-H)/RE
NZSQ = 1.0+EMZ
RETURN
C
END

```

```

SUBROUTINE FCT VAL (THETAZ,F)
C FOR MS.
C ROUTINE FOR COMPUTING F FUNCTION VALUES AT MESH POINTS, CALLED ON
C PRINCIPALLY BY SUBROUTINES FZEPOS AND NO MESH. THIS
C IS THE MODIFIED F FUNCTION WHICH HAS NO POLES AND WHICH
C HAS NL ZEROS AT THETA=90.
C
COMMON/IDERV C/IDERIV
COMMON/IXACT C/IEXACT
COMMON/THETA C/THETA
COMMON/INTEGR/R11,R21,R12,R22
COMMON/RB COM/NUM11,NUM22,DEN11,DEN22
COMPLEX THETAZ,F,
$      THETA,
$      R11,R21,R12,R22,
$      NUM11,NUM22,DEN11,DEN22
C
C
      THETA = THETAZ
      IDERIV=0
      IF(IEXACT .EQ. 0) GO TO 11
      CALL INTEG
      CALL FS INTG
      GO TO 12
11 CALL LACRNG
12 CALL PBARS
      F = (NUM11-R11*DEN11)*(NUM22-R22*DEN22)
$      -R12*R21*DEN11*DEN22
      RETURN
C
      END

```

```

SUBROUTINE F DFDT (THETAZ,F,DFDT)
C FOR MS
C ROUTINE FOR COMPUTING F FUNCTION VALUES AND THEIR DERIVATIVES
C WRT THETA AT ARBITRARY VALUES OF THETA. CALLED ON
C PRINCIPALLY BY SUBROUTINES NO MESH AND FINAL. THIS
C IS THE MODIFIED F FUNCTION WHICH HAS NO POLES AND WHICH
C HAS NO ZEROS AT THETA=90.
C

```

```

COMMON/IDERV C/IDERIV
COMMON/IXACT C/IXACT
COMMON/KXACT C/ KEXACT
COMMON/THETA C/THETA
COMMON/INTEGR/R11,R21,R12,R22,
$ DR11DC,DR21DC,DR12DC,DR22DC
COMMON/RB COM/NUM11,NUM22,DEN11,DEN22,
$ DNM1DC,DNM2DC,DDN1DC,DDN2DC
COMPLEX
$ THETAZ,F,DFDT,
$ THETA,
$ DFDC,S,
$ R11,R21,R12,R22,
$ DR11DC,DR21DC,DR12DC,DR22DC,
$ NUM11,NUM22,DEN11,DEN22,
$ DNM1DC,DNM2DC,DDN1DC,DDN2DC,
$ FACTR1,FACTR2,R12R21,UN1DN2,
$ DFDR11,DFDR21,DFDR12,DFDR22,
$ DFDM1,DFDM2,DFDM1,DFDM2
DATA RTD/57.295 /

```

```

C
C
C THETA = THETAZ
C IDERV=1
C IF(KEXACT.EQ. 0 .AND. IXACT.EQ. 0) GO TO 11
C CALL INTEG
C CALL FS INTG
C GO TO 12
11 CALL LAG DER
12 CALL RBARS

```

```

C
FACTR1 = NUM11-R11*DEN11
FACTR2 = NUM22-R22*DEN22
R12R21 = R12*R21
DN1DN2 = DEN11*DEN22
F = FACTR1*FACTR2-R12R21*DN1DN2

```

```

C
DFDM1 = FACTR2
DFDM2 = FACTR1
DFDM1 = -R11*FACTR2-R12R21*DEN22
DFDM2 = -R22*FACTR1-R12R21*DEN11
DFDR11 = -DEN11*FACTR2
DFDR21 = -R12*DN1DN2
DFDR12 = -R21*DN1DN2
DFDR22 = -DEN22*FACTR1

```

```

DFDC = DFDR11*DR11DC+DFDR21*DR21DC
      +DFDR12*DR12DC+DFDR22*DR22DC
      +DFDNM1*DNM1DC+DFDNM2*DNM2DC
      +DFDDN1*DDN1DC+DFDDN2*DDN2DC

```

```

S = L SIN(THETA/RTD)

```

```

DFDI = -S*DFDC/RTD

```

```

RETURN

```

```

END

```

```

SUBROUTINE LAGNG
C FOR MS
C ROUTINE FOR PERFORMING LAGRANGE INTERPOLATION OF REFLECTION
C COEFFICIENTS IN THE COS(THETA) PLANE.
C

```

```

COMMON/THETA C/THETA
COMMON/INTEGR/R(4),DRDC(4)
COMPLEX THETAG(1),R GIVEN(4,1),
$ THETA,R,DRDC,
$ C,PROD,SUM(4),COEF(4,100),CG(100),DEN(100)
DATA RTD/57.29578/

```

```

C
C
C LAGRANGE INTERPOLATION.
C C = CCOS(THETA/RTD)
21 DO 22 I=1,4
22 SUM(I) = 0.0
DO 24 JG=1,NG
PROD = 1.0
DO 23 J=1,NG
IF (J .EQ. JG) GO TO 25
PROD = PROD*(C-CG(J))
23 CONTINUE
DO 24 I=1,4
24 SUM(I) = SUM(I)+COEF(I,JG)*PROD
DO 25 I=1,4
25 R(I) = SUM(I)
RETURN

```

```

C
C
C INITIALIZATION OF LAGRANGE INTERPOLATION USING INFORMATION ON
C LOCATION OF GIVEN VALUES IN COS(THETA) PLANE.

```

```

ENTRY INIT LG (THETAG,NG)
DO 31 JG=1,NG
31 CG(JG) = CCOS(THETAG(JG)/RTD)

```

```

C
DO 34 JG=1,NG
PROD = 1.0
DO 33 J=1,NG
IF (J .EQ. JG) GO TO 33
PROD = PROD*(CG(JG)-CG(J))
33 CONTINUE
34 DEN(JG) = PROD
RETURN

```

```

C
C
C FURTHER INITIALIZATION OF LAGRANGE INTERPOLATION USING INFORMATION
C ON GIVEN VALUES.

```

```

ENTRY SET LAG (NG,R GIVEN)
DO 41 JG=1,NG
DO 41 I=1,4
41 COEF(I,JG) = R GIVEN(I,JG)/DEN(JG)
RETURN

```

```

C
C
C      ENTRY LAG DER
C      COMPUTATION OF DERIVATIVES OF INTERPOLATED VALUES WRT COS(THETA).
C      NO EXACT VALUES OF DRDC ARE USED.
C      THE LAGRANGE INTERPOLATION FORMULA ITSELF
C      IS DIFFERENTIATED WRT COS(THETA).
      C = CCOS(THETA/RTD)
      DO 62 I=1,4
62  SUM(I) = 0.0
      NGM1 = NG-1
      DO 64 JA=1,NGM1
      JAP1 = JA+1
      DO 64 JB=JAP1,NG
      PROD = 1.0
      DO 63 J=1,NG
      IF (J .NE. JA .OR. J .EQ. JB) GO TO 63
      PROD = PROD*(C-CC(J))
63  CONTINUE
      DO 64 I=1,4
64  SUM(I) = SUM(I)+PROD*(COEF(I,JA)+COEF(I,JB))
      DO 65 I=1,4
65  DRDC(I) = SUM(I)
      GO TO 21
C
      END

```

```

SUBROUTINE FZEROS (TLEFT,TRIGHT,TBOT,TTOP,TMESH,TOL,MPRINT,
$              ZEROS,NR Z)
C FOR ROOT-FINDER
C FZEROS IS A ROUTINE FOR FINDING THE ZEROS OF A COMPLEX FUNCTION, F,
C WHICH LIE WITHIN A SPECIFIED RECTANGULAR REGION OF THE
C COMPLEX (THETA) PLANE, PROVIDED THE FUNCTION HAS NO
C POLES IN THE VICINITY OF THE RECTANGLE.
C
C EXPLANATION OF PARAMETERS--
C TLEFT - VALUE OF REAL PART OF THETA AT LEFT EDGE OF RECTANGLE.
C TRIGHT - VALUE OF REAL PART OF THETA AT RT EDGE OF RECTANGLE.
C TBOT - VALUE OF IMAG PART OF THETA AT BOTTOM EDGE OF RECTANGLE.
C TTOP - VALUE OF IMAG PART OF THETA AT TOP EDGE OF RECTANGLE.
C TMESH - SET EQUAL TO ABOUT HALF THE AVERAGE SPACING BETWEEN
C ZEROS WITHIN THE RECTANGLE. A SMALLER VALUE MAY BE USED
C AS A SAFETY MEASURE, BUT TOO SMALL A VALUE WILL RESULT
C IN EXCESSIVE RUN TIME.
C TOL - TOLERANCE TO WHICH ZEROS ARE TO BE FOUND. IF TWO
C ZEROS ARE CLOSER THAN 'TOL', THE ROOT-FINDER WILL STOP
C WITH AN ERROR MESSAGE.
C MPRINT - NORMALLY SET TO ZERO. A NON-ZERO VALUE LEADS TO
C PRINT-OUT FOR DEBUGGING.
C ZEROS - OUTPUT LIST OF (COMPLEX) VALUES OF THETA AT WHICH
C ZEROS ARE FOUND.
C NR Z - THE NUMBER OF ZEROS FOUND.
C SUBROUTINES TO BE PROVIDED--
C FCT VAL (THETA,F) - TO RETURN THE VALUE OF THE FUNCTION, F,
C AT THE POINT IN THE COMPLEX PLANE SPECIFIED BY 'THETA'.
C F DFDT (THETA,F,DFDT) - SAME AS 'FCT VAL' EXCEPT THAT THE
C DERIVATIVE, DFDT, OF THE FUNCTION WRT THETA MUST ALSO
C BE RETURNED.
C
C COMMON/ZLS COM/NR ZT,NR ZLS(100)
C COMMON/TMC COM/TMESH
C DIMENSION ZEROS(2,1)
C COMPLEX F,PREV F,F00,F10,F01,F11,PARTS
C DIMENSION PART(2),SOL(2),THETA(2)
C DIMENSION K EDGE1(50),K EDGE2(50),K EDGE3(50),K EDGE4(50)
C EQUIVALENCE (PART(1),PARTS)
C
C IF (MPRINT .NE. 0) PRINT 906
906 FORMAT ('1')
C
C TMESH = TMESH
C
C SIDES OF RECTANGLE IN TMESH UNITS
C JLT = TLEFT/TMESH
C IF (TLEFT .GE. 0.0) JLT = JLT-1
C IF (TLEFT .LT. 0.0) JLT = JLT-2
C
C JRT = TRIGHT/TMESH
C IF (TRIGHT .GT. 0.0) JRT = JRT+2
C IF (TRIGHT .LE. 0.0) JRT = JRT+1

```

```

C      JBOT = TPOT/TMESH
      IF (TPOT .GE. 0.0) JBOT = JBOT-1
      IF (TPOT .LT. 0.0) JBOT = JBOT-2

C      JTOP = ITOP/TMESH
      IF (ITOP .GT. 0.0) JTOP = JTOP+2
      IF (ITOP .LE. 0.0) JTOP = JTOP+1

C
C      INITIALIZATION OF PARAMETERS.
      KI = JTOP
      KR = JLT
      KEDGE = 1
      CALL FIND F (KR,KI,F)
      NR E1 = 0
      NR E2 = C
      NR E3 = C
      NR E4 = C
      NR ZT = C
      NR ZL = C
      NR Z = 0
10  IF (NR ZL .EQ. 0) GO TO 20
      NR ZLS(NR Z) = NR ZL
      NR ZL = 0

C
20  PREV F = F
      IF (NR ZL .GT. 1) PRINT 920,NR ZL
920  FORMAT ('0',13,1X,'MODES FOUND ON SAME PHASE LINE')
      NR ZL = C
      GO TO (21,26,31,36),KEDGE

C
C      SEARCH ALONG LEFT EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
21  IF (KI .EQ. JBOT) KEDGE = 2
      IF (KEDGE .EQ. 2) GO TO 26
      KI = KI-1
      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
&      .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
&      GO TO 20
      IF (NR E1 .EQ. 0) GO TO 23
      DO 22 K=1,NR E1
      IF (KI .EQ. K EDGE1(K)) GO TO 20
22  CONTINUE
23  F01 = PREV F
      F00 = F
      LI = KI
      LP = JLT
      GO TO 43

C
C      SEARCH ALONG BOTTOM EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
26  IF (KR .EQ. JRT) KEDGE = 3
      IF (KEDGE .EQ. 3) GO TO 31
      KR = KR+1

```

```

      CALL FIND F (KR,KI,F)
      IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
$       .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
$       GO TO 20
      IF (NR E2 .EQ. 0) GO TO 28
      DO 27 K=1,NR E2
      IF (KR .EQ. K EDGE2(K)) GO TO 20
27 CONTINUE
28 F00 = PREV F
   F10 = F
   LI = JBT-1
   LR = KR-1
   GO TO 48

C
C SEARCH ALONG RIGHT EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
31 IF (KI .EQ. JTOP) KEDGE = 4
   IF (KEDGE .EQ. 4) GO TO 36
   KI = KI+1
   CALL FIND F (KR,KI,F)
   IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
$       .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
$       GO TO 20
   IF (NR E3 .EQ. 0) GO TO 33
   DO 32 K=1,NR E3
   IF (KI .EQ. K EDGE3(K)) GO TO 20
32 CONTINUE
33 F10 = PREV F
   F11 = F
   LI = KI-1
   LR = JRT-1
   GO TO 53

C
C SEARCH ALONG TOP EDGE OF RECTANGLE FOR SIGN CHANGES IN IMAG(F).
36 IF (KR .EQ. JLT) GO TO 80
   KR = KR-1
   CALL FIND F (KR,KI,F)
   IF ((AIMAG(PREV F) .GT. 0.0 .AND. AIMAG(F) .GT. 0.0)
$       .OR. (AIMAG(PREV F) .LT. 0.0 .AND. AIMAG(F) .LT. 0.0))
$       GO TO 20
   IF (NR E4 .EQ. 0) GO TO 38
   DO 37 K=1,NR E4
   IF (KR .EQ. K EDGE4(K)) GO TO 20
37 CONTINUE
38 F11 = PREV F
   F01 = F
   LI = JTOP-1
   LR = KR
   GO TO 58

C
C ENTER MESH SQUARE FROM LEFT SIDE OR EXIT RECTANGLE AT RIGHT EDGE.
41 LR = LR+1
   IF (LR .LE. JRT-1) GO TO 42
   NR E3 = NR E3+1

```

```

      K EDGE3(NR E3) = LI+1
      IF (MPRINT .NE. 0) PRINT 990
      GO TO 10
42 F01 = F11
   F00 = F10
43 CALL FIND F (LR+1,LI+1,F11)
   CALL FIND F (LR+1,LI,F10)
   LEU3 = 1
   ENTER R = 0.0
   ENTER I = -AIMAG(F00)/AIMAG(F01-F10)
   GO TO 60
C
C ENTER MESH SQUARE FROM BOTTOM SIDE OR EXIT RECTANGLE AT TOP EDGE.
46 LI = LI+1
   IF (LI .LE. JTOP-1) GO TO 47
   NR E4 = NR E4+1
   K EDGE4(NR E4) = LR
   IF (MPRINT .NE. 0) PRINT 990
   GO TO 10
47 F00 = F01
   F10 = F11
48 CALL FIND F (LR,LI+1,F01)
   CALL FIND F (LR+1,LI+1,F11)
   LEU4 = 2
   ENTER R = -AIMAG(F00)/AIMAG(F10-F00)
   ENTER I = 0.0
   GO TO 60
C
C ENTER MESH SQUARE FROM RIGHT SIDE OR EXIT RECTANGLE AT LEFT EDGE.
51 LR = LR-1
   IF (LR .GE. JLT) GO TO 52
   NR E1 = NR E1+1
   K EDGE1(NR E1) = LI
   IF (MPRINT .NE. 0) PRINT 990
   GO TO 10
52 F11 = F01
   F10 = F00
53 CALL FIND F (LR,LI+1,F01)
   CALL FIND F (LR,LI,F00)
   LEU5 = 3
   ENTER R = 1.0
   ENTER I = -AIMAG(F10)/AIMAG(F11-F10)
   GO TO 60
C
C ENTER MESH SQUARE FROM TOP SIDE OR EXIT RECTANGLE AT BOTTOM EDGE.
56 LI = LI-1
   IF (LI .GE. JBOT) GO TO 57
   NR E2 = NR E2+1
   K EDGE2(NR E2) = LR+1
   IF (MPRINT .NE. 0) PRINT 990
   GO TO 10
57 F01 = F00
   F11 = F10

```

```

58 CALL FIND F (LR,LI,F00)
   CALL FIND F (LR+1,LI,F10)
   LEDGE = 4
   ENTER R = -AIMAG(F01)/AIMAG(F11-F01)
   ENTER I = 1.0
   GO TO 60

C
C FOR DEBUGGING ONLY, PRINT CO-ORDINATE OF LOWER LEFT CORNER
C OF CURRENT MESH SQUARE. RESULTING SET OF PRINTED
C CO-ORDINATES GIVES TRACE OF EACH LINE ALONG WHICH
C IMAG(F)=0.0 FROM ITS ENTRY ON THE EDGE OF THE
C RECTANGLE TO ITS EXIT AT ANOTHER POINT ON THE
C RECTANGLE.
   60 IF (MPRINT .NE. 0) PRINT 960,LR,LI
960 FORMAT (' ',20x,2I5)
C TEST FOR THERE BEING TWO (HYPERBOLIC) LINES ENTERING AND LEAVING
C THE MESH SQUARE ALONG EACH OF WHICH IMAG(F)=0.0
C IF SO, SET 'LTWO' NON-ZERO.
   LTWO = 0
   IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0
$      .AND. AIMAG(F10) .LT. 0.0 .AND. AIMAG(F01) .LT. 0.0)
$      .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0
$      .AND. AIMAG(F10) .GT. 0.0 .AND. AIMAG(F01) .GT. 0.0))
$      LTWO = 1
C TEST FOR THERE BEING AT LEAST ONE (HYPERBOLIC) LINE ENTERING AND
C LEAVING THE MESH SQUARE ALONG WHICH REAL(F)=0.0
C IF NOT, SET 'I90' TO ZERO.
   I90 = 1
   IF ((REAL(F00) .GT. 0.0 .AND. REAL(F10) .GT. 0.0
$      .AND. REAL(F01) .GT. 0.0 .AND. REAL(F11) .GT. 0.0)
$      .OR. (REAL(F00) .LT. 0.0 .AND. REAL(F10) .LT. 0.0
$      .AND. REAL(F01) .LT. 0.0 .AND. REAL(F11) .LT. 0.0))
$      I90 = 0
   IF (LTWO .EQ. 0 .AND. I90 .EQ. 0) GO TO 70

C
C COMPUTATION OF COEFFICIENTS TO BE USED IN DESCRIBING THE VARIATION
C OF THE FUNCTION WITHIN A MESH SQUARE GIVEN THE
C VALUES AT THE CORNERS OF THE SQUARE AND LINEAR
C VARIATION ALONG ITS EDGES. ALSO, THE POSITION OF
C CROSSING OF THE HYPERBOLIC ASYMPTOTES (WHICH ARE
C PARALLEL TO THE SIDES OF THE SQUARE) FOR THE LINES
C IMAG(F)=0.0 ARE COMPUTED IF BOTH LINES (BRANCHES)
C ENTER AND LEAVE THE SQUARE.
   PARTS = F00
   AR = PART(1)
   AI = PART(2)
   PARTS = F01-F00
   BR = PART(1)
   BI = PART(2)
   PARTS = F10-F00
   CR = PART(1)
   CI = PART(2)
   PARTS = F00+F11-F01-F10

```

```

DR = PART(1)
DI = PART(2)
C
IF (LTWC .EQ. 0) GO TO 61
CENTR R = -BI/DI
CENTR I = -CI/DI
IF (I90 .EQ. 0) GO TO 70
C
61 AB = AR*BI-AI*BR
AL = AR*CI-AI*CR
AD = AR*DI-AI*DR
EC = BR*CI-BI*CF
BD = BR*DI-BI*DR
CD = CR*DI-CI*DR
C
IF (ABS(RD) .LT. ABS(CD)) GO TO 64
C SOLUTION FOR TWO POSSIBLE POINTS AT WHICH THERE MAY BE CROSSINGS
C OF THE (HYPERBOLIC) LINES  $\text{REAL}(F)=0.0$  AND  $\text{IMAG}(F)=0.0$ .
C A CROSSING POINT IS CHOSEN TO BE A ZERO OF THE
C FUNCTION IF IT LIES WITHIN THE CURRENT MESH SQUARE
C AND IF IT LIES ON THE BRANCH OF  $\text{IMAG}(F)=0.0$  CURRENTLY
C BEING FOLLOWED. MULTIPLE CROSSINGS ALONG THIS BRANCH
C (ACTUALLY A SERIES OF CONNECTING HYPERBOLIC BRANCHES)
C IN THE SAME OR IN DIFFERENT SQUARES ARE NOTED SINCE
C THESE MUST LATER BE RESOLVED.
A = CD
b = (AD-BC)*0.5
C = AB
CALL QUAD (A,B,C,SOL,NR SOL)
IF (NR SOL .EQ. 0) GO TO 70
DO 63 N=1,NR SOL
UREAL = SOL(N)
IF (UREAL .LT. 0.0 .OR. UREAL .GT. 1.0) GO TO 63
D1 = BR+CR*UREAL
D2 = BI+CI*UREAL
IF (ABS(D1) .GE. ABS(D2)) UIMAG = -(AR+CR*UREAL)/D1
IF (ABS(D2) .GE. ABS(D1)) UIMAG = -(AI+CI*UREAL)/D2
IF (UIMAG .LT. 0.0 .OR. UIMAG .GT. 1.0) GO TO 63
IF (LTWC .EQ. 0) GO TO 62
IF ((ENTER R-CENTR R)*(U REAL-CENTR R) .LT. 0.0
$ .OR. (ENTER I-CENTR I)*(U IMAG-CENTR I) .LT. 0.0)
$ GO TO 63
62 THETA(1) = (LR+U REAL)*TMESH
THETA(2) = (LI+U IMAG)*TMESH
IF (MPPINT .NE. 0) PRINT 962,LR,LI,THETA(1),THETA(2)
962 FORMAT (' ',2I4,3X,'THETA =',2F9.4)
NR ZT = NR ZT+1
ZEROS(1,NR ZT) = THETA(1)
ZEROS(2,NR ZT) = THETA(2)
IF (NR ZL .EQ. 0) NR Z = NR Z+1
NR ZL = NR ZL+1
63 CONTINUE
GO TO 67

```

```

C
C ALTERNATE SOLUTION FOR THE ABOVE. TWO FORMS ARE NEEDED SINCE,
C IN ANY GIVEN CASE, EITHER FORM (BUT NOT BOTH FORMS)
C MAY BE INDETERMINATE.
64 A = BD
   B = (AD+BC)*0.5
   C = AC
   CALL QUAD (A,B,C,SOL,NR SOL
   IF (NR SOL .EQ. 0) GO TO 70
   DO 66 N=1,NR SOL
   UIMAG = SOL(N)
   IF (UIMAG .LT. 0.0 .OR. UIMAG .GT. 1.0) GO TO 66
   D1 = CR+CR*UIMAG
   D2 = CI+CI*UIMAG
   IF (ABS(D1) .GE. ABS(D2)) UREAL = -(AR+BR*UIMAG)/D1
   IF (ABS(D2) .GE. ABS(D1)) UREAL = -(AI+BI*UIMAG)/D2
   IF (UREAL .LT. 0.0 .OR. UREAL .GT. 1.0) GO TO 66
   IF (LTWO .EQ. 0) GO TO 65
   IF ((ENTER R-CENTR R)*(U REAL-CENTR R) .LT. 0.0
   $      .OR. (ENTER I-CENTR I)*(U IMAG-CENTR I) .LT. 0.0)
   $      GO TO 66
65 THETA(1) = (LR+U REAL)*TMESH
   THETA(2) = (LI+U IMAG)*TMESH
   IF (IMPRINT .NE. 0) PRINT 965,LR,LI,THETA(1),THETA(2)
965 FORMAT (1' ',2I4,3X,'THETA =',2F9.4)
   NR ZT = NR ZT+1
   ZEROS(1,NR ZT) = THETA(1)
   ZEROS(2,NR ZT) = THETA(2)
   IF (NR ZL .EQ. 0) NR Z = NR Z+1
   NR ZL = NR ZL+1
66 CONTINUE
67 CONTINUE
C
70 CONTINUE
C TEST FOR EXIT FROM LEFT EDGE OF MESH SQUARE.
   IF (LEDGE .EQ. 1) GO TO 72
   IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F01) .GT. 0.0)
   $      .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F01) .LT. 0.0))
   $      GO TO 72
   IF (LTWO .EQ. 0) GO TO 51
   EXIT R = 0.0
   EXIT I = -AIMAG(F00)/AIMAG(F01-F00)
   IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
   $      .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
   $      GO TO 72
   GO TO 51
C
C TEST FOR EXIT FROM BOTTOM EDGE OF MESH SQUARE.
72 IF (LEDGE .EQ. 2) GO TO 73
   IF ((AIMAG(F00) .GT. 0.0 .AND. AIMAG(F10) .GT. 0.0)
   $      .OR. (AIMAG(F00) .LT. 0.0 .AND. AIMAG(F10) .LT. 0.0))
   $      GO TO 73
   IF (LTWO .EQ. 0) GO TO 50

```

```

EXIT R = -AIMAG(F00)/AIMAG(F10-F00)
EXIT I = 0.0
IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
$      .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
$      GO TO 73
GO TO 56

C
C TEST FOR EXIT FROM RIGHT EDGE OF MESH SQUARE.
73 IF (LEDGE .EQ. 3) GO TO 74
IF ((AIMAG(F10) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0)
$      .OR. (AIMAG(F10) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0))
$      GO TO 74
IF (LTWO .EQ. 0) GO TO 41
EXIT R = 1.0
EXIT I = -AIMAG(F10)/AIMAG(F11-F10)
IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
$      .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
$      GO TO 74
GO TO 41

C
C TEST FOR EXIT FROM TOP EDGE OF MESH SQUARE.
74 IF (LEDGE .EQ. 4) GO TO 90
IF ((AIMAG(F01) .GT. 0.0 .AND. AIMAG(F11) .GT. 0.0)
$      .OR. (AIMAG(F01) .LT. 0.0 .AND. AIMAG(F11) .LT. 0.0))
$      GO TO 90
IF (LTWC .EQ. 0) GO TO 46
EXIT R = -AIMAG(F01)/AIMAG(F11-F01)
EXIT I = 1.0
IF ((ENTER R-CENTR R)*(EXIT R-CENTR R) .LT. 0.0
$      .OR. (ENTER I-CENTR I)*(EXIT I-CENTR I) .LT. 0.0)
$      GO TO 90
GO TO 46

C
90 IF (NR 2 .NE. 0) CALL NO MESH (TMESH,TOL,MPRINT,ZERUS,NR 2)
RETURN

C
90 PRINT 909,LR,LI
909 FORMAT ('C', 'NO EXIT FROM MESH SQUARE',216)
STOP

C
990 FORMAT('O')

C
END

```

```

      SUBROUTINE QUAD (A,B,C,SOL,NR SOL)
C   FOR ROOT-FINDER
C
C   SOLUTION FOR THE REAL ROOTS OF A QUADRATIC EQUATION OF THE
C   FORM  $A \cdot X^2 + 2.0 \cdot B \cdot X + C = 0.0$ , WHERE X IS CALLED 'SOL'
C   IN THIS ROUTINE. THE NUMBER OF REAL ROOTS FOUND IS
C   GIVEN BY 'NR SOL'. A VALUE OF 1 FOR 'NR SOL' RESULTS
C   FROM THE QUADRATIC EQUATION APPROACHING LINEARITY.
C   USED BY SUBROUTINES FZERDS AND NO MESH.
C
C   DIMENSION SOL(2)
C
C   ACBSQ = A*C/B**2
C   IF (ABS(ACBSQ) .LT. 0.3) GO TO 20
C
C   ARG = B**2-A*C
C   NR SOL = 0
C   IF (ARG .LT. 0.0) RETURN
C   NR SOL = 2
C   ROOT = SQRT(ARG)
C   SOL(1) = (-B+ROOT)/A
C   SOL(2) = (-B-ROOT)/A
C   RETURN
C
20  TERM = 1.0
C   SUM = 1.0
C   DO 21 K=1,50
C     TERM = TERM*((K-0.5)/(K+1.0))*ACBSQ
C     SUM = SUM+TERM
C     IF (ABS(TERM) .LT. 1.0E-10) GO TO 22
21  CONTINUE
22  SOL(1) = -C/(2.0*B)*SUM
C   NR SOL = 1
C   IF (ABS(A/(2.0*B)) .LT. 1.0E-30) RETURN
C   NR SOL = 2
C   SOL(2) = -2.0*B/A-SOL(1)
C   RETURN
C
C   END

```

```

      SUBROUTINE FIND F (JR,JI,F)
C   FOR FOOT-FINDER
C
      COMMON/1MC COM/TMESH
      DIMENSION PART(2)
      COMPLEX F,THETA,PARTS
      EQUIVALENCE (PART,PARTS)
C
C
      PART(1) = JR*TMESH
      PART(2) = JI*TMESH
      THETA = PARTS
      CALL FCT VAL (THETA,F)
      RETURN
C
      END

```

```

      SUBROUTINE NO MESH (TMESH,TOL,MPRINT,ZEROS,NR Z)
C   FOR FOOT-FINDER
C   ROUTINE FOR FINDING EXACT (IN THE SENSE OF NO MESH APPROXIMATION)
C   LOCATIONS OF ZEROS OF THE FUNCTION, F, FOR WHICH
C   A COMPLETE, BUT APPROXIMATE, SET WAS FOUND IN
C   SUBROUTINE FZEROS.
C   TWO DEFICIENCIES REMAIN IN THIS ROUTINE. ONE IS
C   THAT ZEROS CLOSER THAN THE FINITE VALUE 'TOL' CANNOT
C   BE RESOLVED. THIS HAS CAUSED A PROBLEM A FEW TIMES
C   IN USAGE TO DATE, BUT HAS ALWAYS BEEN RESOLVED BY
C   USING A SMALLER VALUE OF 'TOL'. THE SECOND PROBLEM
C   IS THAT THE RESOLUTION OF 'MULTIPLE CROSSINGS',
C   WHICH OCCUR BECAUSE OF THE NON-ANALITICITY OF THE
C   MESH APPROXIMATION, IS SUCH THAT IN PRINCIPLE A
C   ZERO CAN BE MISSED. THERE IS NO EVIDENCE THIS HAS
C   EVER OCCURRED IN PRACTICE. IT MAY OR MAY NOT BE
C   COST EFFECTIVE TO DEVELOP THE PROGRAM FURTHER TO
C   CORRECT THESE TWO PROBLEMS.
C
      COMMON/ZLS COM/NR ZT,NR ZLS(100)
      DIMENSION PART(2),SOL(2)
      COMPLEX ZEROS(1),
$          THETA,
$          F,DFDT,DELT,
$          PARTS,
$          ZEROSO(100),TMIN,
$          ATEMP,BTEMP,CTEMP,DTEMP,
$          FAKRAY(4,4),FOO,F01,F10,F11
      EQUIVALENCE (PART(1),PARTS)
C
C   IF (MPRINT .NE. 0) PRINT 900
900 FORMAT ('1','ITERATIONS TO REMOVE MESH')
C
      NS = 1
C
C   NEWTON-RAPHSON (FIRST-ORDER) ITERATION TO FIND EXACT LOCATIONS
C   OF THE ZEROS OF F.
      DO 29 JZ=1,NR ZT
      ZEROSO(JZ) = ZEROS(JZ)
      IF (MPRINT .NE. 0) PRINT 901,JZ
901 FORMAT (' ',I8)
      21 IF (MPRINT .NE. 0) PRINT 902,ZEROS(JZ)
902 FORMAT (' ',10X,2(F9.4))
      THETA = ZEROS(JZ)
      CALL F DFDT (THETA,F,DFDT)
      DELT = -F/DFDT
      ZEROS(JZ) = ZEROS(JZ)+DELT
      IF (CABS(DELT) .GT. J.3*TOL) GO TO 21
      29 CONTINUE
C
C   NUMBER OF UNIQUE ZEROS COMPARED TO TOTAL NUMBER MINUS
C   MULTIPLICITY DUE TO MULTIPLE CROSSINGS.

```

```

30 IF (NR ZT .EQ. 1) RETURN
   JZ = 1
   DO 32 J=2,NR ZT
   MATCH = 0
   JMI = J-1
   DO 32 JJ=1,JMI
   IF (ABS(ZEROS(J)-ZEROS(JJ)) .GT. TOL) GO TO 32
   IF (MPRINT .NE. 0) PRINT 905,J,JJ
905 FORMAT ('0','ZEROS NR',I3,1X,'AND',I3,1X,
   $        'ARE THE SAME')
   MATCH = 1
32 CONTINUE
   IF (MATCH .NE. 0) GO TO 33
   JZ = JZ+1
   ZEROS(JZ) = ZEROS(J)
33 CONTINUE
C
   IF (JZ .EQ. NR Z) RETURN
   IF (JZ .LT. NR Z) GO TO 50
   PRINT 947,JZ,NR Z
947 FORMAT ('0','WARNING--',I3,1X,'ZEROS FOUND ON',I3,1X,
   $        'PHASE LINES')
   NR Z = JZ
   RETURN

```

C THE FOLLOWING LOGIC IS NEEDED ONLY OCCASIONALLY, NAMELY, WHEN
C TEST INDICATES THAT A ZERO HAS BEEN MISSED IN THE
C INITIAL NEWTON-RAPHSON ITERATION. IT ALLOWS FOR
C STEP-WISE CHANGE IN THE FORM OF THE FUNCTION FROM
C THE MESH APPROXIMATION TO THE EXACT FORM.

```

50 NS = NS*2
   IF (NS .GT. 16) GO TO 90
C
   DO 59 JZ=1,NR ZT
   IF (MPRINT .NE. 0) PRINT 901,JZ
   ZEROS(JZ) = ZEROS0(JZ)
   DO 58 K=1,NS
   FN FRCT = FLOAT(K)/FLOAT(NS)
   FM FRCT = 1.0-FN FRCT
51 THETA = ZEROS(JZ)
   IF (MPRINT .NE. 0) PRINT 902,ZEROS(JZ)
   CALL F DFDT (THETA,F,DFDT)
   IF (K .EQ. NS) GO TO 50
C
   JRO = REAL(THETA)/TMESH
   IF (REAL(THETA) .LT. 0.0) JRO = JRO-1
   JIO = AIMAG(THETA)/TMESH
   IF (AIMAG(THETA) .LT. 0.0) JIO = JIO-1
   DO 52 JRLOOP=1,4
   DO 52 JILOOP=1,4
   JR = JRO+JRLOOP-2
   JI = JIO+JILOOP-2
52 CALL FIND F (JR,JI,FARRAY(JRLOOP,JILOOP))

```

C

```

DMIN = 9.9E9
DO 67 JRLCOP=1,3
DO 67 JILCOP=1,3
FOO = FARRAY(JRLCOP,JILCOP)
FOI = FARRAY(JRLCOP,JILCOP+1)
FIO = FARRAY(JRLCOP+1,JILCOP)
FII = FARRAY(JRLCOP+1,JILCOP+1)
JR = JR0+JFLCOP-2
JI = JI0+JILCOP-2
REFR = REAL(ZEROS(JZ))/TMESH-JR
REFI = AIMAG(ZEROS(JZ))/TMESH-JI
ATEMP = FOO
BTEMP = FOI-FJO
CTEMP = FIO-FJO
DTEMP = FOO+FII-FOI-FIO
PARTS = (ATEMP+BTEMP*REFI+CTEMP*REFR+DTEMP*REFR*REFI)*FM FRCT
        +F*FN FRCT
AF = PART(1)
AI = PART(2)
PARTS = (BTEMP+DTEMP*REFR)*FM FRCT+DFDT*(0.0,1.0)*TMESH*FN FRCT
BK = PART(1)
BI = PART(2)
PARTS = (CTEMP+LTEMP*REFI)*FM FRCT+DFDT*TMESH*FN FRCT
CR = PART(1)
CI = PART(2)
PARTS = CTEMP*FM FRCT
DR = PART(1)
DI = PART(2)

```

C

```

AB = AR*BI-AI*BK
AC = AR*CI-AI*CR
AD = AR*DI-AI*DR
BC = BR*CI-BI*CK
BD = BR*DI-BI*DR
CD = CR*CI-CI*DR

```

C

```

IF (ABS(BD) .LT. ABS(CD)) GO TO 64
A = CD
F = (AD-BC)*0.5
C = AB
CALL QUAD (A,B,C,SOL,NR SOL)
IF (NR SOL .EQ. 0) GO TO 67
DO 62 N=1,NR SOL
UREAL = SOL(N)
IF (UREAL+REFR .LT. -0.030 .OR. UREAL+REFR .GT. 1.030) GO TO 62
D1 = BR+CR*UREAL
D2 = BI+CI*UREAL
IF (ABS(D1) .GE. ABS(D2)) UIMAG = -(AR+CR*UREAL)/D1
IF (ABS(D2) .GE. ABS(D1)) UIMAG = -(AI+CI*UREAL)/D2
IF (UIMAG+REFI .LT. -0.003 .OR. UIMAG+REFI .GT. 1.003) GO TO 62
PART(1) = (JR+UREAL+REFR)*TMESH
PART(2) = (JI+UIMAG+REFI)*TMESH

```

```

      THETA = PARTS
      DIST = CABS(THETA-ZEROS(JZ))
      IF (DIST .GT. DMIN) GO TO 62
      DMIN = DIST
      TMIN = THETA
62  CONTINUE
      GO TO 67

C
64  A = BD
      B = (AD+BC)*0.5
      C = AC
      CALL QUAD (A,B,C,SOL,NR SOL)
      IF (NR SOL .EQ. 0) GO TO 67
      DO 66 N=1,NR SOL
      UIMAG = SOL(N)
      IF (UIMAG+REFI .LT. -0.030 .OR. UIMAG+REFI .GT. 1.030) GO TO 66
      D1 = CR+CR*UIMAG
      D2 = CI+CI*UIMAG
      IF (ABS(D1) .GE. ABS(D2)) UREAL = -(AR+BR*UIMAG)/D1
      IF (ABS(D2) .GE. ABS(D1)) UREAL = -(AI+BI*UIMAG)/D2
      IF (UREAL+REFR .LT. -0.003 .OR. UREAL+REFR .GT. 1.003) GO TO 66
      PART(1) = (JR+UREAL+REFR)*TMESH
      PART(2) = (JI+UIMAG+REFI)*TMESH
      THETA = PARTS
      DIST = CABS(THETA-ZEROS(JZ))
      IF (DIST .GT. DMIN) GO TO 66
      DMIN = DIST
      TMIN = THETA
66  CONTINUE
67  CONTINUE

C
      IF (DMIN .GT. 9.0E9) GO TO 95
      DELT = TMIN-ZEROS(JZ)
      ZEROS(JZ) = TMIN
      IF (CABS(DELT) .LT. 0.03*TMESH) GO TO 57
      GO TO 51
50  DELT = -F/DFDT
      ZEROS(JZ) = ZEROS(JZ)+DELT
      IF (CABS(DELT) .LT. 0.3*TUL) GO TO 57
      GO TO 51
57  IF (MPRINT .NE. 0) PRINT 907
907  FORMAT ('0')
58  CONTINUE
59  CONTINUE
      GO TO 30

C
90  PRINT 990,JZ,NR Z
990  FORMAT ('0','ONLY',I3,1X,'ZERUS FOUND ON',I3,1X,'PHASE LINES')
      STOP

C
95  PRINT 995
995  FORMAT ('0','PROBLEMS IN SUBROUTINE NO MESH')
      STOP

```

C

END

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