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PIECEWISE APPROXIMATION OF PICTURES USING MAXIMAL NEIGHBORHOODS--ETC(U)
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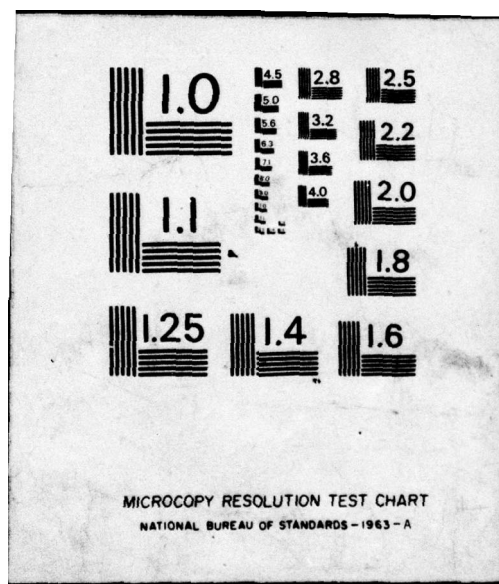
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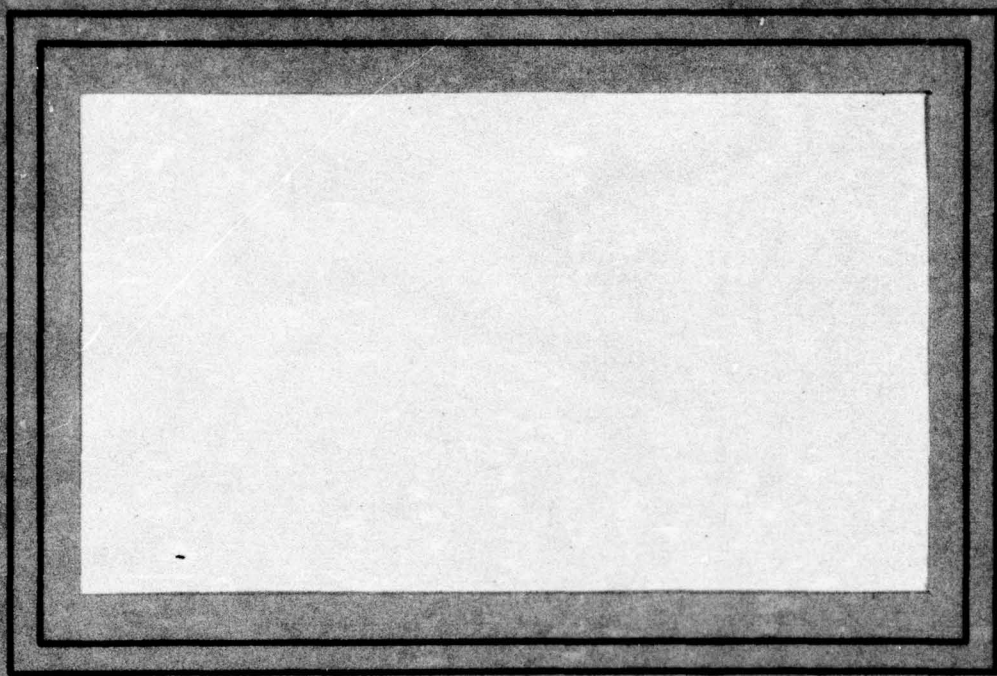
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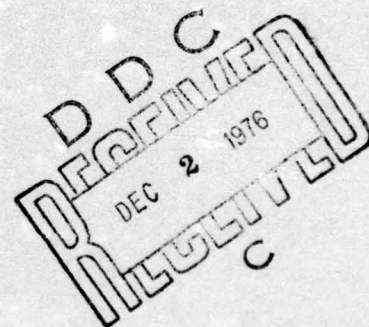
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PIECEWISE APPROXIMATION OF PICTURES
USING MAXIMAL NEIGHBORHOODS

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ABSTRACT

Suppose that we are given a picture having approximately piecewise constant gray level. Each point P has a largest neighborhood $N(P)$ that is entirely contained in one of the constant regions, and the set of maximal $N(P)$'s (i.e., $N(P)$'s not contained in other $N(P)$'s) constitutes an economical description of the picture, generalizing the Blum "skeleton" or medial axis transformation. The picture can be smoothed, without excessive blurring, by averaging over each $N(P)$. By taking differences between pairs of touching maximal $N(P)$'s, the edges between the regions can be detected; since this edge detection scheme is not based on symmetrical detection operators, it is not handicapped when two adjacent regions differ greatly in size.

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1. Introduction

This paper develops a general method of constructing piecewise approximations to a picture. The picture is assumed to be composed of a set of regions, each having approximately constant gray level (possibly noisy). Some examples of such pictures are shown in Figure 1.

The approximations are defined by sets of neighborhoods, each of which is contained in one of the regions, but is not contained in any other such neighborhood. (A more precise definition is given in Section 3, and the implementation of the method is described in Section 4.) For brevity, we shall refer to this type of approximation from now on as a SPAN (for Spatial Piecewise Approximation by Neighborhoods).

SPANs have several important applications in picture processing and analysis:

- a) The SPAN is a generalization of the Blum [1] "skeleton" or "medial axis transformation" (MAT). Previously, the MAT has been defined only for pictures that are explicitly segmented into objects and background; see Sections 2-3. Thus the SPAN, like the MAT, provides a compact representation of the given picture. It also provides a basis for describing the shapes of the regions that comprise the picture -- e.g., branches on the SPAN skeleton correspond to lobes on the regions.

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- b) In constructing the SPAN, a maximal neighborhood $N(P)$ of each point P is first found that is contained in the region to which P belongs. Thus the picture can be smoothed, without blurring the edges of the regions, by replacing the gray level at every point P by the average gray level taken over $N(P)$. Examples of smoothings obtained in this way are given in Section 5.
- c) At the edges of the regions, pairs of maximal SPAN neighborhoods -- contained in the regions on the two sides of the edge -- will touch. Thus if we compute the difference between the average gray levels taken over a pair of such touching neighborhoods, and output this difference at the point where the neighborhoods touch, we will obtain outlines of the region edges. This method of edge detection, unlike most previous methods, is not based on the use of symmetrical edge detection operators; hence it is not handicapped when attempting to detect the edge between two regions that differ greatly in size. Examples of region edges detected by this approach are shown in Section 6.

Other approaches to piecewise approximation of pictures have been investigated by Pavlidis and his students [2-5]. Typically, such approaches begin with an initial partition of the picture into cells, and modify the parti-

tion by merging and splitting cells, and adjusting cell boundaries, while insuring that a given approximation criterion remains satisfied on each piece of the partition. These approaches too can be used to smooth the picture without blurring the region edges, and to detect these edges. But the SPAN approach provides an alternative which is of potential interest for several reasons:

- 1) By constructing an approximation to the picture out of neighborhoods, the SPAN provides a basis for obtaining structural descriptions of the regions in the picture, similar to those provided by the MAT, but applicable directly to unsegmented, noisy pictures.
- 2) The SPAN is constructed by a "parallel" order-independent process of neighborhood growing and nonmaximum suppression. This process could be implemented very efficiently on a parallel array-processing computer.
- 3) Because of the simplicity of the operations used to construct the SPAN, it can be regarded as a possible computational model for visual perception of region shapes in piecewise constant scenes.

2. The medial axis transformation (MAT)

Over ten years ago, Blum [1] proposed a method of representing a shape S in terms of the set of maximal disks that are contained in S . Specifically, for each point (x,y) in S , let $N_r(x,y)$ be a disk (i.e., a circular neighborhood) of radius r centered at (x,y) . For small values of r , $N_r(x,y)$ will be entirely contained in S (if (x,y) is on the border of S , this will only be true for $r=0$), but for larger r 's it will extend outside S . Let $N(x,y)$ be the largest disk centered at (x,y) that is entirely contained in S . We call $N(x,y)$ maximal if it is not contained in any other $N(u,v)$. It is easily seen that the union of the maximal disks $N(x,y)$ is exactly S . [Indeed, each $N(x,y)$ is contained in S ; but every point of S is contained in at least one of the $N(x,y)$'s.] Thus the set of centers and radii of the maximal disks completely determines S . Blum called this representation of S its medial axis transformation (MAT). Note that the set of centers of the maximal disks constitutes a sort of "skeleton" of S .

The "disks" used in the above definition need not be circular; any family of shapes can be used. When S is an object in a digital picture, it is more convenient to use squares rather than circles. A discussion of the digital MAT, and its use to represent objects in digital pictures, can be found in [6-7]. A simple example of a digital MAT is shown in Figure 2.

Blum's definition of the MAT applies only to objects

that have been explicitly extracted from a picture -- or equivalently, to two-valued pictures, in which we can call the set of points having one value the "object", and the remaining points the "background". The definition is not directly applicable to pictures in which there are many gray levels. A generalization to the grayscale case was given by Levi and Montanari [8], extending earlier work by Rutovitz [9]. This generalization is based on the relationship between Blum's MAT and the distances from points of S to the outside of S . Specifically, let $N(x,y)$, the largest disk centered at (x,y) that is entirely contained in S , have radius $r(x,y)$. Then $r(x,y)$ is the shortest distance from (x,y) to the border of S . (In the digital case, if we use square "disks", this is still true for a suitably modified notion of "distance", e.g., "city block" or "chess-board" distance; see [10].) If (x,y) is a point of the MAT, i.e., the center of one of the maximal disks, it is easily seen that it is a local maximum of the function $r(x,y)$, and conversely.

We can now define a "gray-weighted distance", in a grayscale picture f , as follows: Let S be any subset of the picture, and let ρ be any path (lying inside S) from a point (x,y) of S to the border of S . We can define a "gray-weighted length" of ρ by integrating the gray levels of f along ρ ; this integral grows rapidly if ρ passes through points of high gray level, and slowly if it passes through low-level points. Note that if $f \equiv 1$ inside S , the gray-weighted length becomes the length in the ordinary

sense. (In the digital case, the integral becomes a discrete sum.) Let us define the gray-weighted distance from (x,y) to the border of S as the shortest gray-weighted path length from (x,y) to the border. We can then define the "gray-weighted MAT" (GMAT) as the set of points of S whose gray-weighted distances to the border of S are local maxima, together with their associated distances.

The two-valued MAT is very sensitive to noise, particularly noise that is located near the center of the object S . This is illustrated in Figure 3, where we see how the presence of a single noise point at the center of an object can drastically alter the MAT. The GMAT would be less sensitive to noise, since the effects of gray level fluctuations on the gray-weighted length of a path should tend to cancel out. However, the presence of noise should create many "noisy" local maxima in the GMAT. Also, the GMAT is defined only when a subset of the given picture has been specified; it is not defined directly for the picture itself.

In this paper we develop a somewhat different approach to defining a GMAT for arbitrary grayscale pictures. Our approach assumes that the picture's gray level is approximately piecewise constant (except for noise), but does not require us to specify the region whose GMAT is to be constructed, nor to explicitly segment the picture into the regions. It yields GMATs for all the constant regions simultaneously, thus providing a representation for the

entire picture. At the same time, it is designed to be relatively insensitive to noise.

3. Approximation of piecewise constant pictures using maximal neighborhoods: The SPAN

Suppose that we are given a picture whose gray level is approximately piecewise constant; in other words, the picture can be partitioned into a set of regions R_i , in each of which the gray level is approximately constant. Some examples of such pictures were shown in Figure 1.

Let (x,y) be a point of one of the constant regions R , and let $N_r(x,y)$ be the disk of radius r centered at (x,y) , as in Section 2. We would like to find the largest $r = r(x,y)$ such that $N_r(x,y)$ is entirely contained in R . Our approach will be to apply some simple statistical tests to the gray level population in $N_r(x,y)$, in order to decide whether $N_r(x,y)$ is contained in a single constant region or overlaps several of the regions.* In designing these tests, we will assume that the gray level in each of the regions R is normally distributed (e.g., that R has constant gray level corrupted by Gaussian noise).

We began by calculating the mean μ_r and standard deviation σ_r of the gray levels in the neighborhood $N_r(x,y)$, for each r . Using these, we can compute confidence intervals around the "true" mean -- i.e., for a given probability p , we can determine a neighborhood I_r of μ_r (in units of σ_r) such that, with probability p , the mean of the

*A related approach was used in [11] in an attempt to determine an optimal degree of smoothing to use at each point of a picture. In [12], an analogous method was used to find natural piecewise constant approximations of one-dimensional strings of data.

gray level population of which $N_r(x,y)$ is a sample lies inside I_r . If $N_r(x,y)$ lies within a single region R , this I_r should be small, since $N_r(x,y)$ is a sample of a normally distributed gray level population. On the other hand, if $N_r(x,y)$ overlaps more than one region, its gray level distribution will tend to become multimodal, and the internal I_r will become larger. Thus we can assume that N_r is contained in R as long as the length of I_r remains below some threshold t . (The details will be given in Section 4.)**

For a given choice of confidence p and maximum confidence interval length t , we can thus decide which neighborhoods $N_r(x,y)$ are contained in a single region R . Let $N(x,y)$ be the largest such neighborhood, as in Section 2. We call $N(x,y)$ q -maximal if it does not overlap any other $N(u,v)$ by fraction q or more of its own area. For example, if $q=1$, $N(x,y)$ is q -maximal if it is not contained in any other (larger) $N(u,v)$. The examples in the next section used $q=1$, but any $q \geq 0.8$ would have given the same results (see the end of Section 4).

We shall call the set of q -maximal neighborhoods the q -SPAN of the given picture f , and the set of their centers the " q -skeleton" of f . Note that these depend on the values of p and t , as well as q . In the next Section we describe a specific implementation of this definition, for the case

**Another possible approach might have been to use a test for multimodality to detect that $N_r(x,y)$ overlaps more than one region. Such tests were used by Chow and Kaneko [13] to determine local thresholds for segmenting a picture.

of a digital picture, and give examples of the results obtained.

4. Implementation of the SPAN

In the present digital implementation, we used upright square neighborhoods of sizes $2k+1$ by $2k+1$ (corresponding to a chessboard distance measure). Thus the neighborhood of radius k contains $(2k+1)^2$ points.

Using Student's t distribution, for a given confidence p , we can find a value t_p such that

$$\text{Prob} \left\{ \left| \frac{\mu_k - \mu}{s_k / (2k+1)} \right| < t_p \right\} = p$$

-- i.e., such that the absolute difference between the neighborhood mean μ_k and the population mean μ , measured in units of $s_k / (2k+1)$, is less than t_p with probability p . Here we assume that $k \geq 1$, and we define s_k to be the corrected sample standard deviation

$$s_k = \sigma_k (2k+1) / \sqrt{(2k+1)^2 - 1}$$

where σ_k is the standard deviation of the gray levels in the neighborhood. We divide s_k by $2k+1$ to reflect the fact that the variability of the mean μ_k should decrease with the square root of the sample size [14].

Thus the p confidence interval about the sample mean (= the interval within which the true mean lies, with probability p) is

$$I_k = (\mu_k - t_p s_k / (2k+1), \mu_k + t_p s_k / (2k+1))$$

At each point (x,y) , we choose the largest k such that the length $|I_k|$ (i.e., $2t_p s_k / (2k+1)$) is less than a given threshold t . This defines the neighborhood $N(x,y)$. If $|I_k| \geq t$ for $k \geq 1$, we take $k=0$ as a default option, i.e., we define $N(x,y)$ to be (x,y) itself. We then find maximal $N(x,y)$'s as described in Section 3.

SPANs were computed for the ten pictures in Figure 1, using $p = .95$, $t = .85$, and $q = 1$. In these examples, only three neighborhood sizes were used: 1×1 , 3×3 , and 5×5 (i.e., radii of 0, 1, and 2). The radii of the neighborhoods $N(x,y)$ for these ten pictures are displayed in Figure 4, with radius values 0, 1, and 2 represented by gray levels 20, 40, and 60, respectively. Figure 5 shows the results of suppressing (= setting to zero) the points whose $N(x,y)$'s are nonmaximal.

In these examples, since radii greater than 2 were not allowed, we obtain a thick area of points having gray level 60 (corresponding to radius 2) wherever a region is more than 5 points wide. This could have been avoided by allowing larger radii; we used only a few radii here in order to avoid excessive computational cost in developing and testing the SPAN programs. The computer time required to produce each of the pictures in Figure 5 was 12 seconds on a Univac 1108.

The results obtained for $q=1$ would be the same for any q in the interval $(.8, 1)$. This is because the largest neighborhood size used was 5×5 , and if two 5×5 upright squares overlap by more than 80%, they must be identical. If larger neighborhood sizes are allowed, the results become more sensitive to the choice of q . Allowing $q < 1$ should reduce the sensitivity of the SPAN to noise.

The results of varying t are shown in Figure 6. (The effects of changing p would be similar.) It is seen that taking $t = .6$, rather than $.85$, yields thicker skeletons, but is capable of detecting lower contrasts between regions.

5. Application of the SPAN to smoothing

Since each neighborhood $N(x,y)$ is contained within one of the regions R , we should be able to smooth the picture without blurring the edges of the regions by replacing the gray level at (x,y) by the average gray level measured over $N(x,y)$. An early discussion of smoothing by averaging over regions of variable size, which could grow as long as they did not cross over edges, was given by Roetling [15]. This approach should, in principle, yield optimal smoothings, since it averages over the entire region containing each point; but this requires that the picture be explicitly segmented into the regions, which is a computationally costly process. A simpler approach is to use a neighborhood of each point, but to allow each neighborhood to be as large as possible, as long as it does not go outside the region containing that point -- i.e., to use the $N(x,y)$'s as neighborhoods. An unsuccessful attempt to automatically determine an averaging neighborhood size at each point of a picture was reported in [11].

The results of smoothing the ten pictures of Figure 1 using the $N(x,y)$'s as averaging neighborhoods are shown in Figure 7. For comparison purposes, the results of averaging over a 3×3 square neighborhood at every point are shown in Figure 8. It is seen that the 3×3 smoothings blur the region edges, while the SPAN smoothings do not. Note, however, that the SPAN smoothings do not smooth out the noise near the region edges, where the $N(x,y)$ neighborhoods have zero radius.

6. Application of the SPAN to edge detection

A problem with most methods of edge detection is that they make use of symmetrical edge detection operators, and so cannot take full advantage of the uniformity of the regions between which an edge is to be detected, if these regions are of very different sizes. For example, suppose that regions A and B have widths a and b , where $a < b$, and that we detect edges by taking differences of averages computed over neighborhoods of size c . If $c > a$, our detector will be too big for region A, so that parts of the picture lying beyond A will be included in the A average. But if $c \leq a < b$, our detector sees only part of region B, and cannot take full advantage of B's uniformity. For a discussion of this problem see [16], and compare [12].

Here again, an optimal approach would be to use the regions themselves as averaging neighborhoods. (Once we have extracted the regions, we know where their edges are; but we still need to do the averaging in order to determine the strengths of these edges.) However, explicitly determining the regions is computationally costly. A simpler idea is to use the maximal $N(x,y)$'s as averaging neighborhoods. If P is a point where two such $N(x,y)$'s touch, we can take the difference of averages over the $N(x,y)$'s as the edge value at P . Note that P may be interior to a region (e.g., if a region is rectangular, there will be many maximal $N(x,y)$'s inside it); but in such a case the difference of averages will be close to zero. On the other hand, if P is on an edge between two contrasting

regions, there should be maximal $N(x,y)$'s contained in the two regions and meeting at P, and the difference of their averages will be high. Thus the edge values computed in this way should be high along region edges, and low or zero elsewhere. (In the pictures shown below, edge values less than 6 have been suppressed.)

The results of applying this edge detection scheme to the pictures of Figure 1, using pairs of adjacent, non-overlapping maximal $N(x,y)$'s, are shown in Figure 9. For comparison, the output of a Roberts-like gradient ($\max(|p-s|, |q-r|)$ in the neighborhood $\begin{smallmatrix} pq \\ rs \end{smallmatrix}$) for the same pictures is shown in Figure 10. The Roberts gradient gives good results when the edges are sharp and the regions are small, but for larger regions with blurred edges, the SPAN edge detector gives better results.

7. Discussion and conclusions

In this paper we have defined a technique for piecewise approximation of pictures, based on maximal neighborhoods, that can be applied to pictures which are approximately piecewise constant. As the examples in Figure 1 show, several real-world classes of pictures can be treated as piecewise constant for purposes of SPAN construction. On the other hand, for a picture that contains a major gray level ramp, the SPAN method breaks down, since it will attempt to approximate the ramp by a staircase.

It should be possible, in principle, to generalize the SPAN approach to a wider class of pictures, e.g., pictures that are approximately piecewise linear, rather than piecewise constant, in gray level. Here, for each neighborhood $N_r(x,y)$, we would test the hypothesis that its gray level population is a good fit to a ramp, say in the least squares sense. The largest r for which this fit is sufficiently good would define the neighborhood $N(x,y)$, and we could then find the maximal $N(x,y)$'s as above.

It should also be possible to generalize the SPAN to pictures containing textured regions. Suppose that a region consists of small constant-value elements on a constant value background. Then for small values of the radius k in Section 4, we will obtain small maximal neighborhoods contained in individual texture elements and spaces. As k gets larger, the neighborhoods will overlap both texture elements and background, so that s_k will become large and

the size $|I_k|$ of the confidence interval will exceed the threshold t . But as k gets still larger, s_k stops increasing (as long as the neighborhood stays within the region), while the $2k+1$ in the denominator continues to increase, so that $|I_k|$ may again drop below t . Thus if we consider only k 's that are large compared to the texture element size, we should still be able to find maximal neighborhoods corresponding to the textured regions.

The practical utility of the SPAN approach is somewhat limited, at present, by its computational cost. This limitation could be overcome if a suitable parallel processing capability were available. In any case, the SPAN approach is a useful conceptual contribution, as a generalization of Blum's MAT concept to noisy, unsegmented pictures. Like the MAT, it provides natural, concise approximations to such pictures that can be used for purposes of encoding, recognition, and description.

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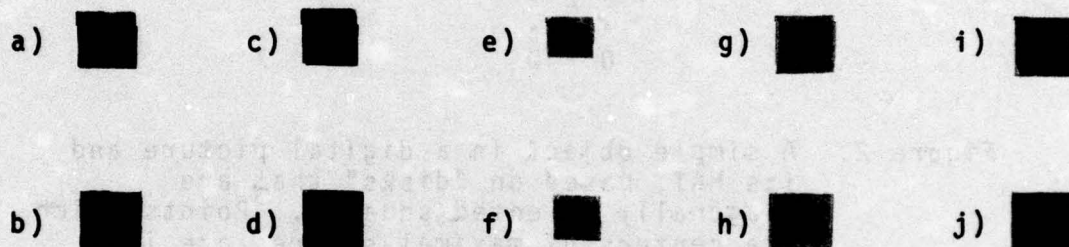


Figure 1. Some examples of pictures that are approximately piecewise constant. Each picture is a 32x32 pixel array having 64 possible gray levels. The noisy versions have had normally distributed noise added with a mean of zero and a standard deviation of 4 gray levels.

a) Portion of a LANDSAT image of the Monterey, California area; b) Noisy version of (a); c) White blood cell; d) Noisy version of (c); e) Chromosome; f) Noisy version of (e); g) Disk; h) Noisy version of (g); i) U-shaped object; j) Noisy version of (i).

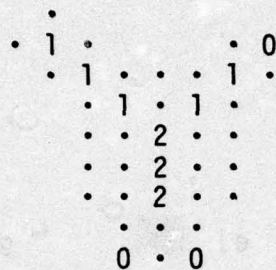


Figure 2. A simple object in a digital picture and its MAT, based on "disks" that are diagonally oriented squares. Points which are centers of maximal squares are indicated by integers that give the radii of these squares; the other points of the object are indicated by dots.



(a)

(b)

Figure 3. Sensitivity of the MAT to noise. a) The MAT of a diagonally oriented square is a single point at the center of the square. b) If the center point of the square is deleted, the MAT of what remains consists of all the points, since for any of the remaining points, the maximal "disk" now has radius zero.

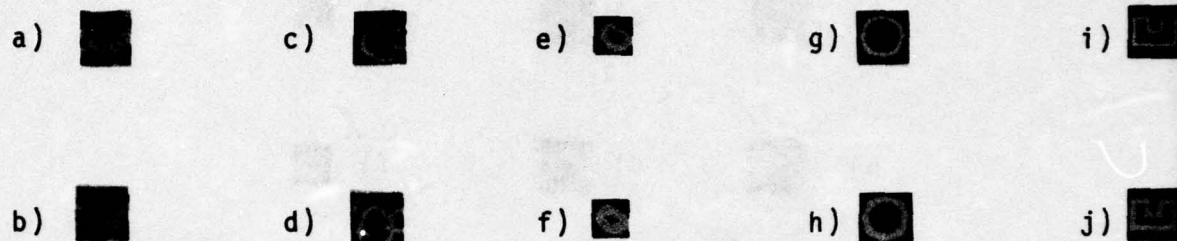


Figure 4. Radii of the neighborhoods $N(x,y)$ for the pictures in Figure 1, using $p = .95$ and $t = .85$. Only the values 0, 1, and 2 were allowed; they are represented by gray levels 20, 40, and 60, respectively.

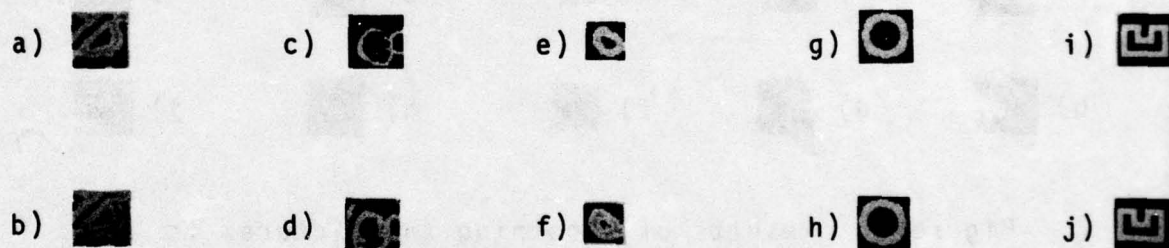


Figure 5. Results of suppressing nonmaximal neighborhoods for the pictures in Figure 1. The gray level at (x,y) is set to zero if the neighborhood $N(x,y)$ is contained in a larger neighborhood $N(u,v)$; otherwise, the same gray levels as in Figure 4 are used.

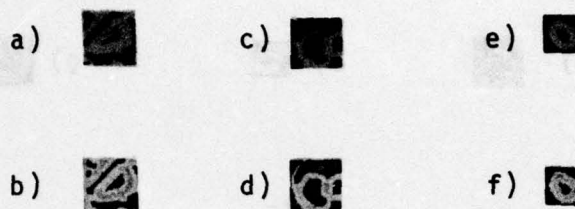


Figure 6. Results of taking $t = .6$, rather than $.85$, in constructing SPANs for Figures 1a, c, e. Parts a, c, e show the radii, and parts b, d, f the results of suppressing nonmaxima.

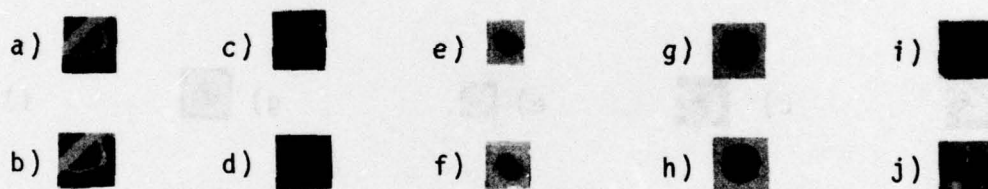


Figure 7. Results of smoothing the pictures in Figure 1 by averaging over the neighborhoods $N(x,y)$.

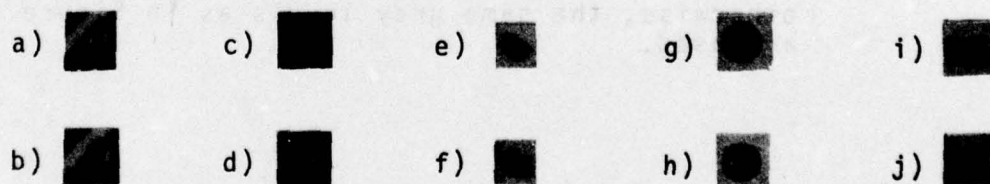


Figure 8. Results of smoothing the same pictures by averaging over a 3×3 neighborhood of each point.

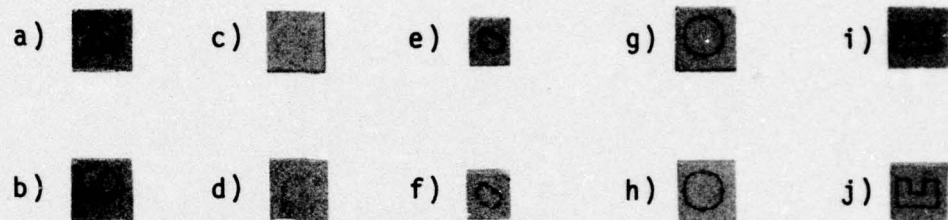


Figure 9. Results of detecting edges on the pictures in Figure 1 using differences of averages over the maximal neighborhoods $N(x,y)$.

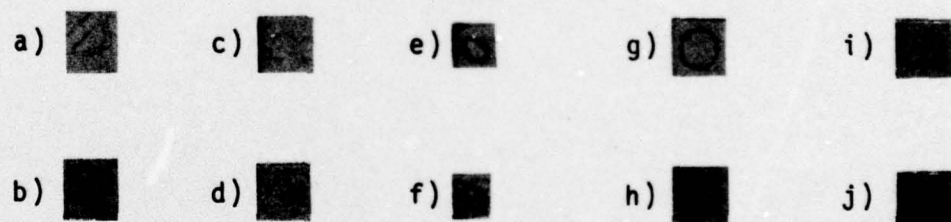


Figure 10. Results of detecting edges on the same pictures using a Roberts-like gradient operator.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Suppose that we are given a picture having approximately piecewise constant gray level. Each point P has a largest neighborhood $N(P)$ that is entirely contained in one of the constant regions, and the set of maximal $N(P)$'s (i.e., $N(P)$'s not contained in other $N(P)$'s) constitutes an economical description of the picture, generalizing the Blum "skeleton" or medial axis transformation. The picture can be smoothed, without excessive blurring, by averaging over each $N(P)$.			

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20 Abstract

By taking differences between pairs of touching maximal N(P)'s, the edges between the regions can be detected; since this edge detection scheme is not based on symmetrical detection operators, it is not handicapped when two adjacent regions differ greatly in size.

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