

AD-A032 855

HARVARD UNIV CAMBRIDGE MASS
SHARING RISKS OF DEFERRED PAYMENT.(U)
MAR 76 S SHAVELL

F/G 5/3

UNCLASSIFIED

TR-22

N00014-67-A-0298-0019
NL

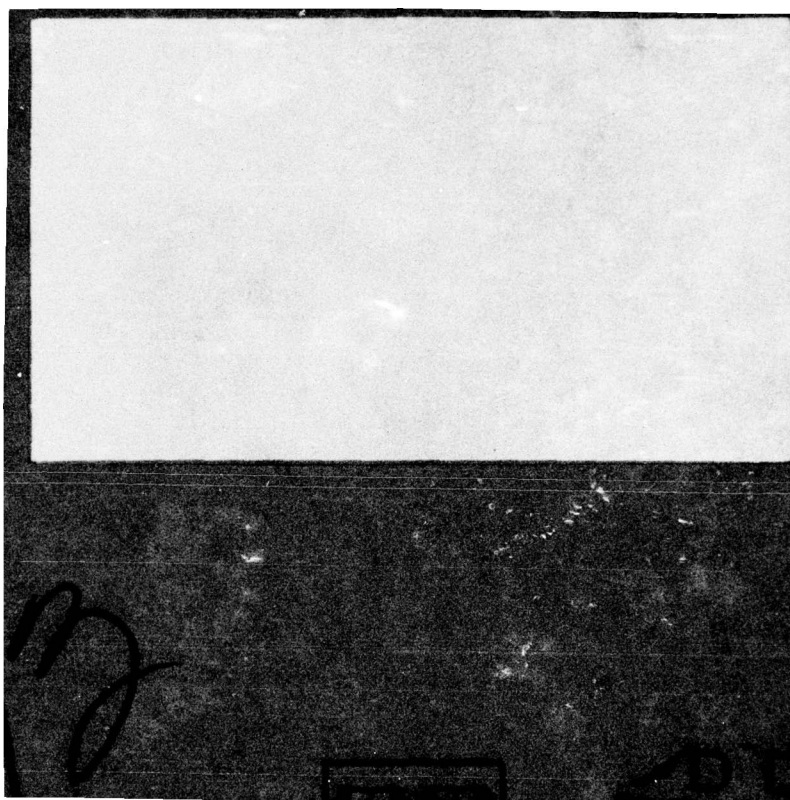
| OF |

AD
A032 855



END

DATE
FILMED
1 - 77



SHARING RISKS OF DEFERRED PAYMENT

by

Steven Shavell

Technical Report No. 22

Prepared under Contract No. N00014-67-A-0298-0019
Project No. NR 047-004
for the Office of Naval Research

This document has been approved for public release and sale;
its distribution is unlimited.

Reproduction in whole or part is permitted for any purpose
of the United States Government

Harvard University
1737 Cambridge Street, Room 404
Cambridge, Massachusetts 02138

March, 1976



Sharing Risks of Deferred Payment

Steven Shavell

ACQUISITION	RTIS	White Section	☒
	DDC	Deft Section	☐
	UNANNOUNCED		
	JUSTIFICATION		
BY	DISTRIBUTION/AVAILABILITY CODES		
	Dist. Avail. and Special		
A			

Uncertainty often enters into an economic transaction when payment is deferred.

In this case the risk to transactors can usually be reduced by agreement on a plan that makes the amount paid depend on the occurrence of uncertain events.

For example, if movements in the price level cannot be predicted, then the real value of a future payment is uncertain. In order to eliminate this risk, some economists have suggested that payment should be made according to a cost-of-living escalator. That is, payment should be a variable $(1 + \pi)N$ where π is the inflation in prices of the period in question and N is some nominal amount. In principle, however, payment could depend in an arbitrary way on inflation. The question then arises of what is a Pareto optimal payment plan--a plan designed so that no substitute arrangement could mutually benefit the transactors. Pareto-optimal payment plans are not, in general, cost-of-living escalators. They are determined jointly by transactors' attitudes toward risk, by opinions about the future rate of inflation, and by the correlation with inflation of income or outgo which is unrelated to the payment plan.

Another example in which deferred payment involves risk concerns contracts made between parties who transact in different currencies under conditions of uncertainty over the future rate of exchange. The usual type of contract, which fixes payment in one of the two currencies, is not Pareto optimal even if the parties engage in transactions in the forward exchange market. A Pareto-optimal contract, on the other hand, links payment to the exchange rate in a way

depending on a set of factors similar to those mentioned above for the case of inflation.

Before discussing the two examples, Pareto-optimal payment plans will be briefly considered in a slightly more general context. The section below on these plans is most closely related to Borch (1960, 1962) on reinsurance treaties and Arrow (1971) on optimal insurance policies but differs in its interpretation and in its emphasis on the effect of unrelated or "exogenous" changes in wealth on risk sharing.

Pareto-Optimal Payment Plans

Suppose that transactor U, who is to receive payment, and transactor V, who is to make payment, each acts as if he were maximizing the expected value of a function-giving utility of wealth.¹ Let the utility functions of U and V be $U(\cdot)$ and $V(\cdot)$, respectively. Denote by $d_U(\cdot)$ and $d_V(\cdot)$ the densities of their subjective probability distributions over the real numbers, to be interpreted as states of nature.² Assume also that $d_U(s) = 0$ if and only if $d_V(s) = 0$.

A payment plan is therefore a function $x(\cdot)$ giving the amount $x(s)$ which V is to pay U when s occurs.³ Suppose that $u(s)$ is the expected wealth of U exclusive of payment from V, given the occurrence of s , and that $v(s)$ is similarly defined. Then under a payment plan the expected utility of U is

$$(1) \quad \int U[u(s) + x(s)]d_U(s)d_s$$

and that of V is

$$(2) \quad \int V[v(s) - x(s)]d_V(s)d_s$$

(All these functions are assumed differentiable.) Accordingly, a Pareto-optimal payment plan $x(\cdot)$ maximizes (1) with (2) held constant.⁴

If neither transactor prefers risk, then a necessary and sufficient condition for Pareto optimality is that

$$(3) \quad d_v(s)V'[v(s) - x(s)] = kd_u(s)U'[u(s) + x(s)],$$

where k is a positive constant. This equation implies that the marginal rates of substitution between dollars in different states are the same for U and V . The equation follows easily from an argument used by Arrow (1971, p. 217) on optimal insurance policies or from the calculus of variations.⁵

Now differentiate (3) to obtain an equation in $x'(s)$,

$$\begin{aligned} d'_v(s)V'[v(s) - x(s)] + d_v(s)[v'(s) - x'(s)]V''[v(s) - x(s)] \\ = k\{d'_u(s)U'[u(s) + x(s)] + d_u(s)[u'(s) + x'(s)]U''[u(s) + x(s)]\} \end{aligned}$$

or, substituting for k and solving for $x'(s)$,

$$(4) \quad x'(s) = \frac{v'(s)[-V''(s)/V'(s)] - u'(s)[-U''(s)/U'(s)] + \{[d'_u(s)/d_u(s)] - [d'_v(s)/d_v(s)]\}}{[-V''(s)/V'(s)] + [-U''(s)/U'(s)]}$$

where $U'(s) = U'[u(s) + x(s)]$ and similarly for $U''(s)$, $V'(s)$, and $V''(s)$.

The slope of the payment schedule is therefore determined by the derivatives of wealth exclusive of payment, the levels of absolute risk aversion, and the difference in subjective beliefs as expressed by the proportional rate of change of the probability density. Other considerations, such as bargaining power and the market forces, place limits on the height of the payment schedule.

The effect of the difference in beliefs is easy to interpret. It is to insure that transactor in whose opinion the likelihood of higher s is growing most rapidly.

In order to interpret other terms of the equation, assume that risk-averse U and V have identical opinions. Then the equation implies that if $v'(s)$ and $u'(s)$ are of opposite sign, $x'(s)$ has the sign of $v'(s)$, offsetting movements in prepayment wealth for both transactors.⁶ But if $v'(s)$ and $u'(s)$ are of the same sign, $x'(s)$ is opposite in sign to the derivative of prepayment wealth for the transactor with the greater weighted level of risk aversion, offsetting movements in his prepayment wealth.

Finally, note two simple facts which follow from the equation (or directly from Jensen's inequality): (i) A payment plan for two risk-averse transactors with identical beliefs and constant expected wealth exclusive of payment is Pareto optimal if and only if payment is constant. (ii) A payment plan for a risk-neutral transactor and a risk averter with identical beliefs is Pareto optimal if and only if it leaves the risk averter with fixed final wealth.⁷

This second case is approached as the ratio of degrees of risk aversion of the transactors increases: Divide the numerator and denominator of (4) by $-V''(s)/V'(s)$ and let the ratio of degrees of risk aversion approach infinity. Then $x'(s)$ approaches $-u'(s)$. Similarly, if the ratio approaches zero, $x'(s)$ approaches $v'(s)$.

Two Examples

A. Indexing and Uncertainty over the Rate of Inflation

If s is the rate of inflation over a period and wealth is interpreted as real wealth, then a payment plan gives for any s the real payment $x(s)$; the nominal payment is $(1 + s)x(s)$. Consequently, a payment plan $x(s) \equiv c$, a constant, represents a cost-of-living escalator; a plan with $x'(s) > 0$ implies that real payments actually increase with inflation; a plan with $x'(s) < 0$, such as a nominally fixed payment, implies that real payments do not keep up with inflation.

If it were true that everyone shared the same beliefs about inflation and had no source of real income affected by it--that is, $u(\cdot)$ and $v(\cdot)$ constant--then a cost-of-living escalator would be Pareto-optimal (items i, ii). Hence, a situation in which all real income and wealth are protected against inflation is self-supporting in the sense that any new Pareto-optimal payment plan would be a cost-of-living escalator.

For most transactors, however, opinions over the future course of inflation differ and real income exclusive of payment is correlated with inflation. Therefore the results of the previous section show that a Pareto-optimal payment plan is rarely a cost-of-living escalator. From the discussion following (4), it becomes intuitively obvious that under a Pareto-optimal plan payments are made as if to compensate for other inflation losses or gains. For instance, if U 's weighted degree of risk aversion is sufficiently great compared with V 's, then real payment to U will rise with inflation if his real income from other sources is negatively correlated with inflation and fall if his income is positively correlated.

[The random variables $u(\cdot)$ and $v(\cdot)$ describing income from other sources may depend on $x(\cdot)$. For instance, the portfolio of an individual would probably change if his wage income were protected against inflation. Hours worked might also change. We do not consider explicitly such effects here⁸; we are concerned with risk sharing and therefore with what the final relationship must be among $u(\cdot)$, $v(\cdot)$, and $x(\cdot)$ for Pareto optimality.]

We now comment briefly on Pareto-optimal linkage for two important payment plans: wage contracts and bonds. It seems reasonable to argue that for many individuals real disposable income would have fallen recently even if their wages had been fully escalated. This is because average tax rates increase with nominal income and because, on balance, real income from asset portfolios--including cash, savings, and equity in insurance policies, pension funds, a home, or stocks--has often been negatively correlated with inflation. If this is expected in the future and if firms may be assumed risk neutral (or at least much less risk averse than wage earners), then according to a Pareto-optimal escalator clause real wages should rise with inflation. Of course, a firm would not agree to such a clause without getting something in return; wage earners ought to be willing to accept a lower expected real salary in order to enjoy extra inflation protection.

If such escalators are Pareto optimal, competition should enforce them. The way this would work is that, for example, a firm which offered a fixed nominal wage or even a cost-of-living escalator to an individual whose other income was negatively correlated with inflation would have to pay a higher expected wage

than a firm which offered an optimal escalator. Similar remarks obviously apply when the Pareto-optimal escalator does not involve a greater than cost-of-living increase.

Because wage earners differ, competition should encourage firms to offer a choice of escalators. Some escalators would have a relatively low expected real wage and a high degree of escalation, others a relatively high expected real wage and little escalation. A wage earner will wish for greater escalation the more averse he is to risk, the greater the amount of his nonwage income and the more negative its correlation with inflation, and the larger the degree to which his expectations of high rates of inflation exceed those of the firm (eq. [4]). The choice of escalators permitted by allowing wage earners to mix according to any desired ratio a nominally fixed and some highly escalated contract would likely be sufficient as a practical matter.

The case with bonds is similar, except that the predominant demand might not be for instruments as highly escalated as in the case of wage contracts. This claim is based on casual empiricism--that the correlation with inflation of other income and wealth is not as unfavorable for the average bondholder as for the average wage earner. In any event, to accommodate variable demand it would probably be enough for issuers to float only two bonds, one with a nominally fixed yield and one with linkage, for investors could purchase these in any proportion.

Although the theoretical arguments of the present section would suggest otherwise, widespread linkage of payments to the price level has not been observed in this country. One explanation, stressed by Blinder (1975), is that individuals are not sufficiently more risk averse than firms. (This explanation relies on

the existence of transaction costs [bargaining, legal, etc.], for without these any difference in attitude toward risk would make some kind of linkage mutually advantageous.) While there is obvious truth in this explanation and some evidence in its favor--for example, the "capping" of escalators in certain wage contracts--other not necessarily competing explanations also bear consideration. One of these has no doubt been the episodic nature and mild pace of inflation. Regarding wages, another explanation may be that most agreements in the labor market do not bind employees; presumably, employees could look for work elsewhere if payments under a plan turned out to be low.⁹ And, in the case of bonds, an explanation may be the unfavorable treatment under the tax laws of interest payment on indexed corporate debt.

Nevertheless, with the increased importance and awareness of inflation, a tendency toward linkage is to be expected. Whether this tendency is socially desirable is not clear. Our argument for Pareto optimality was distinctly microeconomic, concerned with only two transactors. It took no account of the influence which extensive use of linkage would have on the economy as a whole through effects on individual consumption, portfolio behavior, and expectations. When viewed in this larger context, linkage may not be in everyone's advantage.¹⁰

B. Payment in Foreign Currency and Uncertainty over the Exchange Rate

Consider a contract $x(\cdot)$ for deferred payment between risk-averse or risk-neutral parties U and V who transact in different currencies. For simplicity, assume that both agree over the probability distribution F describing the future rate of exchange. Let s be the number of units of U's currency purchasable with one

unit of V's currency at the time of payment. Then, if $x(s)$ is the payment in V's currency at the time of payment. Then, if $x(s)$ is the payment in V's currency, $sx(s)$ is the payment in U's currency. Consequently, the expected utility of U is

$$(5) \int U[u(s) + sx(s)]dF(s)$$

and that of V is

$$(6) \int V[v(s) - x(s)]dF(s)$$

Because U does not receive precisely what V gives up, the condition for a Pareto-optimal payment plan is a little different from that of the previous section. An argument analagous to Arrow's shows that the condition for Pareto optimality becomes

$$(7) \quad V'[v(s) - x(s)]/sU'[u(s) + sx(s)] = k$$

where k is a positive constant. The meaning of the equation is that the terms of trade in utility remain constant over exchange rates. Differentiation of (7) and solution for $x'(s)$ would yield an equation similar to (4) and with very much the same interpretation. It is clear from (7) that a contract specifying a fixed payment in the currency of either transactor is not in general Pareto optimal. Other things equal, a risk-averse U should receive less in his own currency when the exchange rate is in his favor. [By other things equal is meant that $u(\cdot)$ and $v(\cdot)$ are constant.] If $x'(s) > 0$, U's receipts of $sx(s)$ clearly increase with s ;¹¹ if $x'(s) \leq 0$, equality in (7) cannot be maintained unless $sx(s)$ rises with s . This is easy to understand, for it says that U receives less in his currency when it is costly for V in exchange for more when it is cheap.

It is assumed above that the transactors themselves share the risk of exchange rate fluctuation. When the forward rate or the implicit forward rate (associated with the spot exchange rate and the interest rates in the transactors' countries) make covering exchange rate risk very expensive, such an assumption may provide a good approximation to the truth.¹² Under these circumstances, one guesses that many potential contracts are never consummated just because their terms (payment fixed in one currency) would not allow adequate risk sharing.

When transactors use forward exchange markets, the common practice is for a contract to specify a fixed payment in, say, V's currency and for U to hedge against exchange rate fluctuation to the extent he desires by selling forward in V's currency. It is easy to see that in this case also (7) will rarely be satisfied. A complete analysis of Pareto-optimal contracts in the presence of forward exchange markets requires maximization of (5) over functions $x(\cdot)$ with (6) constant, where U and V each choose optimal forward transactions given $x(\cdot)$. The analysis, which will not be presented here, does not turn out to be interesting; the results amount to a complicated mathematical statement of the fact that Pareto-optimal contracts would make payment depend on the exchange rate not only to exploit the low "cost" of utility for U when s is high but also to offset variation of other income with changes in s . Forward markets alone are not a sufficiently discriminating device for such shifting of risk.

Concluding Comment

In the two examples above, payment plans were beneficial because insurance against price changes could not be purchased. More generally, of course, opportunity for improving the allocation of risk when payment is deferred often arises when markets for claims contingent on states of the world are not complete. However, it must be admitted that the scope for gains in welfare through use of payment plans is limited (though to a different degree) by the same two factors which frequently prevent formation of markets for contingent claims, namely, by transaction costs and moral hazard.

References

1. Of course, other arguments of the utility function (including what U transferred to V in the exchange preceding payment) are held constant and are suppressed in the notation.
2. Consideration of the case in which the states of nature are vectors is straightforward but does not seem to add insight.
3. Although as stated U and V have symmetric positions in the problem, we think of U as recipient, especially since in most real cases payment will be positive. If, as may sometimes be appropriate, it is assumed that payment is nonnegative, the consequent modifications (e.g., $\geq$ replaces $=$ in [3]) are, again, straightforward.
4. If, say, U believes some event is possible and V does not, then there are no Pareto-optimal plans. This is because V would then be willing to contract to pay U any amount if the event occurred. The problem obviously does not arise if either the condition $d(s) = 0$ if and only if $d^U(s) = 0$ or some constraint on borrowing is assumed. We chose to make the V former assumption in the interest of avoiding tedious consideration of cases in which constraints are or are not binding.
5. Borch (1960) first proved that an equation similar to (3) characterizes Pareto-optimal risk sharing.
6. This does not mean that the final wealth of both transactors is necessarily smoothed (in the sense, say, of a reduction in variance) although one might expect it to be the typical case.
7. Baily (1974) notes essentially this fact.
8. In unpublished notes, Blinder (1975) considers some of these effects in a context similar to that of this section.
9. One would therefore expect the following two factors to militate against indexed wage agreements: (a) low cost of changing jobs, (b) inability to negotiate similar contracts for a large fraction of employees in an industry.
10. Fischer (1975) has analyzed the effect on portfolio behavior of the opportunity to buy index-linked bonds and has constructed an example in which the consequence of their introduction is to make everyone worse off.
11. Here it is assumed that payment is positive.
12. The effective cost of forward transactions is likely to be higher the longer the duration of the contract and the larger the magnitude of the transaction.

References

Arrow, Kenneth J., "Uncertainty and the Welfare Economics of Medical Care." In Essays in the Theory of Risk-Bearing. Chicago: Markham, 1971

Baily, M. N., "Wages and Employment under Uncertain Demand." Rev. Econ. Studies 61 (1974): 37-50.

Blinder, A. S. "Notes on the Choice of an Escalator Clause." Unpublished notes. Princeton Univ., 1975.

Borch, Karl, "The Safety Loading of Reinsurance Premiums." Skandinavisk aktuarietidskrift (1960), pp. 163-84.

_____, "Equilibrium in a Reinsurance Market," Econometrica 30 (1962): 424-44.

Fischer, Stanley, "The Demand for Index Bonds." J.P.E. 83, no. 3 (June 1975): 509-34.

ent is part of the

DOCUMENT CONTROL DATA - R & D
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Project on Efficiency of Decision Making in Economic Systems, 1737 Cambridge St., #404, Harvard University, Cambridge, Mass. 02138	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP
3. REPORT TITLE 6 SHARING RISKS OF DEFERRED PAYMENT	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report, No. 22	
5. AUTHOR(S) (Last name, middle initial, last name) Steven Shavell 14 TR-22	
6. REPORT DATE March 1976 7a. CONTRACT OR GRANT NO. N00014-67-A-0298-0019 7b. PROJECT NO. NR -047 - 004	7c. TOTAL NO. OF PAGES 13 7d. NO. OF REFS 6 8a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 22 8b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its publication is unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.	
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Logistics & Mathematics Statistics Branch, Department of the Navy, Office of Naval Research, Wash. D. C.
13. ABSTRACT ➔ Because uncertainty often enters into economic transactions when payment is deferred, it may be advantageous to make the amount of payment depend on the occurrence of uncertain events. This general method of accomplishing risk-sharing is studied and its relevance is discussed in two cases: (1) uncertainty over the rate of inflation and cost-of-living escalators; (2) uncertainty over the exchange rate and foreign currency payment plans. ★	