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MODAL METHOD FOR FREE VIBRATION OF FINITE OVAL CYLINDRICAL SHELL WITH FREE ENDS

By Y.N. CHEN AND JOSEPH KEMPNER

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ABSTRACT

The modal method developed by the authors[1] applicable to the dynamic analysis of supported noncircular cylindrical shells is extended in the present work to deal with (but not restricted to) the free vibration problem of unsupported noncircular cylinders. The required modification includes the enforcement of the edge conditions at the free ends of a finite cylindrical shell which are unsatisfied by the modal functions owing to the presence of the variable curvature terms. Such conditions are posed as additional constraints by way of the well-known formalism of Lagrange multipliers. *In addition, it was found that the inclusion* of the lowest modes, commonly approximated by the Rayleigh-Love modes, is essential to the completeness of the eigen-function representation.

The validity of the proposed procedure of analysis is illustrated by its application to the solution of the free vibration problem of oval cylindrical shells with free ends.

Introduction

In an earlier article[1], the present authors presented an analysis of the natural vibration of noncircular cylincrical shells. In addition to the discussion on the contrast (as well as the similarity) of dynamic characteristics between circular and noncircular cylindrical shells, emphasis was placed on the development of a modal method suitable for application to problems in dynamics of noncircular, finite, cylindrical shells having various types of simple and clamped supports.

The most profound difficulties in analysis involving cylindrical shells with variable radius of curvature stem from the fact that the dependency of deformation (and velocity) of the shell element upon the two in-plane coordinates is, in general, not separable, as is the case of cylindrical shells of constant radius. Such complexity, compounded with the inherent difficulties accompanying the rigorous enforcement of the boundary conditions corresponding to various types of end conditions of a finite cylinder, nevertheless, can be handled by the aforementioned modal approach, as shown in Ref. 1. Such a procedure involves expanding the middle surface displacement parameters in series form on a functional basis constituted by the modal functions derived from the exact solution of the corresponding free vibration problem of a circular cylindrical shell. The validity of this procedure is justified by the fact that, within the framework of Donnel-type shell theory for the class of problems considered in Ref. 1, the boundary conditions prescribed for the oval shell are identically satisfied, inasmuch as the modal functions satisfy the required conditions of the circular cylinder. The same conclusion is also valid for <u>most</u> cases when Sanders' shell theory is employed. Even the exceptions may be overlooked, since the error induced proves to be negligibly small. Thus, this procedure permits the concentration of effort to cope with the problem of circumferential modal coupling.

This method of analysis, developed as a variation of the modal method, must still be modified further to be used in the dynamic analysis of oval cylindrical shells with free ends. In the present work, as a sequel to Ref. 1, such modifications, together with the characteristic behavior of the free-free oval cylindrical shell resulting from the application of the modified method, is discussed in detail. Primarily, the modification involves the posing of the unsatisfied boundary conditions as an additional restraint on the variational problem within the framework of the modal expansion.

The Energy Functional

In Ref. 1, a variational equation of motion based on Hamilton's principle was obtained in the form of

$$\delta F = 0 \tag{1}$$

in which

$$\begin{aligned}
\delta F = \int_x \int_s \{ & [u_{,x} + \nu(v_{,s} + w/r)]\delta u_{,x} + (1/2)(1-\nu)(v_{,x} + u_{,s})(\delta v_{,x} + \delta u_{,s}) \\
& + (\nu u_{,x} + v_{,s} + w/r)(\delta v_{,s} + \delta w/r) + k[w_{,xx} + \nu(w_{,s} - k_s v/r)_{,s}]\delta w_{,xx} \\
& + 2k(1-\nu)[w_{,xs} - (3k_s/4)v_{,x}/r + (k_s/4)u_{,s}/r][\delta w_{,xs} - (3k_s/4)\delta v_{,x}/r \\
& + (k_s/4)\delta u_{,s}/r] + k[\nu w_{,xx} + w_{,ss} - k_s(v/r)_{,s}][\delta w_{,ss} - k_s(\delta v/r)_{,s}] \\
& - \lambda(u\delta u + v\delta v + w\delta w)\}\, dx\, ds
\end{aligned}$$

In this expression, the middle surface displacement parameters u,v, and w, the coordinates x and s, and the radius of curvature $r = r(s)$ are all dimensionless quantities normalized with respect to the shell thickness h and an average radius r_o in the following manner (see Fig. 1):

$$(u, v, w) = (u^*, v^*, w^*)/h$$

$$(x, s, r) = (x^*, s^*, r^*)/r_o \qquad (2)$$

where $u^*,\ldots, x^*,\ldots$, etc. denote the corresponding physical parameters of the shell; r_o is defined to be $(L_o/2\pi)$, where L_o denotes the circumferential length of the cross-sectional contour. In addition, ν is Poisson's ratio, k_s is a tracing parameter which is equal to 1 for Sanders' shell theory and equal to zero for Donnell's theory. The constant k and the parameter λ are defined as

$$k = (1/12)(h/r_o)^2 \quad , \quad \lambda = \Omega^2 = \rho\omega^2 r_o^2(1-\nu^2)/E \qquad (3)$$

in which ω denotes the natural frequency of the oval shell, and ρ and E are the mass density and modulus of elasticity, respectively. The development of the preceding variational equation and the required modal functions were discussed in Ref. 1 and will not be repeated here.

It is worth noting that the variational functional will be maintained in the present form, which offers the optimal degree of operational symmetry. However, the usual integrations by parts, leading to the explicit determination of the equations of motion and the natural boundary conditions is needed to facilitate the discussion of the latter. To this end, the quantities δu, δv, δw, and $\delta w_{,x}$ are regarded as the chosen virtual displacements. It follows that, if the ends of the oval cylinder are free from traction (which is the primary concern of the present work), the following quantities are required to vanish at both ends of the shell; i.e.,

$$u_{,x} + \nu(v_{,s} + w/r) = 0$$

$$v_{,x} + u_{,s} - (3kk_s/4)(1/r)(4w_{,xs} - 3k_s\, v_{,x}/r + k_s u_{,s}/r) = 0$$

$$w_{,xx} + \nu(w_{,s} - k_s v/r)_{,s} = 0$$

$$w_{,xxx} + \nu(w_{,s} - k_s v/r)_{,xs} + (1/2)(1-\nu)(4w_{,xs} - 3k_s\, v_{,x}/r + k_s\, u_{,s}/r)_{,s} = 0 \tag{4}$$

The associated equations of motion, which represent a form of the well-known Sanders' equations, are not shown here.

Equations (4), in essence, state that the axial resultant force, the in-plane shear, the bending moment, and the effective transverse shearing force, respectively, must vanish at the boundary. Alternative boundary conditions for the various types of end supports (simple and clamped) were discussed in Ref. 1.

Modal Expansion and Kinematic Constraints

Similar to the cases of the supported cylinders analyzed in Ref. 1, the displacement parameters u, v, and w are assumed to be

$$u = \sum_{i=1}^{N} \alpha_i x_i U_n(x)\cos(ns)$$

$$v = \sum_{i=1}^{N} y_i V_n(x)\sin(ns)$$

$$w = \sum_{i=1}^{N} \alpha_i z_i W_n(x)\cos(ns) \tag{5}$$

where the subscripted indices i and n are related by

$$i = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd} \\ (n/2)+1 & \text{if } n \text{ is even} \end{cases} \tag{6}$$

and α_i is equal to $1/\sqrt{2}$ if $n = 0$ and is otherwise equal to 1. It should be noted that it has been assumed that u, v, and w are sinusoidal in time with circular frequency ω ; i.e., $u = \Sigma\, \alpha_i x_i U_n(x) \cos(ns)\sin(\omega t)$,..., etc. The time factor $\sin(\omega t)$ is deleted, and its presence is reflected by the introduction of the parameter λ , defined by Eq.(3), which appears in Eq.(1). In Eqs.(5) the deformations (as well as the corresponding velocities) are assumed to be symmetric with respect to the origin of the coordinate s ; i.e., s = 0. For the sake of brevity in identification, let the term "symmetric modes" be used in the ensuing discussion to describe the modes of deformation associated with the expansion given in Eqs.(5). Similarly, the term "antisymmetric modes" is used for modes of deformation given by similar expansions by replacing the functions sin(ns) by - cos(ns), and by replacing cos(ns) by sin(ns) in Eqs.(5). Such a distinction, of course, is trivial for the circular cylinders. Parallel to these designations, modes that are symmetric or antisymmetric with respect to the mid-span of the shell will be called "longitudinally symmetric" or "longitudinally antisymmetric", respectively.

In the present work, as in Ref. 1, the nondimensional radius of curvature is chosen to be

$$r = 1/(1 + \xi \cos 2s) \tag{7}$$

where ξ is the eccentricity parameter, the magnitude of which lies between zero and unity. Moreover, the quantities U_n, V_n, and W_n , introduced in the format of Eqs.(5) (for any value of n), represent the exact solution to the free vibration problem of a circular cylindrical shell posed by the variational problem of Eq.(1) for $r = 1$. These predetermined modal functions, by definition, satisfy the boundary conditions prescribed by Eqs.(4) for $r = 1$. The existence of such a set of exact solutions, of course, is well known. (See, for example, Ref. 2). Detailed discussions on the computational aspects of such a problem may be found in the works of Vronnay and Smith[3], and of Warburton[4], as well as in Ref. 1.

Upon substitution of Eqs.(5) (or their "antisymmetric" counterparts) into Eq.(1), it follows that, if the undetermined parameters x_i, y_i, and z_i are permitted to vary independently, a set of 3N algebraic equations can be obtained, the details of which may be found in Ref. 1. The presence of $r(s)$ as a function of the coordinate s given in Eq.(7) results in the coupling of these algebraic equations. Nevertheless, it is also clear that the "symmetric" and "antisymmetric" modes are not related, owing to the double symmetry possessed by $r(s)$ as given in Eq.(7). Within each of these two groups, the odd modes and the even modes are uncoupled as well. The circular cylinder problem can be treated as a degenerate case of this formulation. In such a case, coupling exists only among x_i, y_i, and z_i for a given value of the subscript n, the circumferential mode number.

As regards the boundary conditions, when Eqs.(5) (or their antisymmetric counterparts) are substituted into Eqs.(4), the presence of r as a function of s prevents the resulting algebraic form of the boundary conditions from being identically satisfied.

In this connection it may be advantageous to first discuss the simpler case corresponding to $k_s = 0$; i.e., the "Donnell" version of the equations involved, since most of the terms containing the variable radius of curvature vanish upon setting $k_s = 0$. In so doing, Eqs.(4) reduce to

$$u_{,x} + \nu(v_{,s} + w/r) = 0$$

$$v_{,x} + u_{,s} = 0$$

$$w_{,xx} + \nu w_{,ss} = 0$$

$$w_{,xxx} + (2-\nu)\, w_{,xss} = 0 \qquad (8)$$

for $x = x_e$, say, where x_e is the axial coordinate of a free end. Obviously, the third and fourth conditions of Eqs.(8) are identically satisfied by the deflection w in terms of $W_n(x)$, since each of the functions W_n satisfied such conditions. The second condition in Eqs.(8) is satisfied if x_i and y_i are related by

$$nU_n(x_e)\, x_i - V_n'(x_e) y_i = 0 \qquad (9)$$

for all values of i. In Eq.(9) a primed superscript denotes differentiation with respect to x. Finally, when the modal expansions are substituted into the first of Eqs.(8), it follows that x_i, y_i, and z_i must be related by

$$\sum_i [x_i U_n'(x_e) + \nu y_i n V_n(x_e)] \cos ns + \sum_i \nu z_i W_n(x_e)[\cos ns + (\xi/2)\cos(n+2)s$$
$$+ (\xi/2)\cos(n-2)s] = 0 \qquad (10)$$

for the symmetric modes. A similar condition is arrived at by employing the antisymmetric modes, and the result is identical to Eq.(10) with the cosine functions replaced by the corresponding sine functions. This condition can be satisfied provided

$$x_i U_n'(x_e) + \nu\{y_i n V_n(x_e) + z_i W_n(x_e) + (\xi/2)[z_{i+1} W_{n+2}(x_e) + z_{i-1} W_{n-2}(x_e)]\} = 0 \qquad (11)$$

for $i > 2$. Individual care must be administered for the irregular terms when $i \leq 2$; they can be derived from Eq.(10). For instance, the collection of coefficients of cos(s) in Eq.(10) yields the following constraint condition for $i = 1$ of the odd, symmetric group:

$$x_1 U_1'(x_e) + \nu\{y_1 V_1(x_e) + z_1 W_1(x_e) + (\xi/2)[z_1 W_1(x_e) + z_2 W_3(x_e)]\} = 0$$

The constant α_i has been deleted from Eqs.(10) and (11), but it must be restored to several terms whenever needed.

Hence, the variational problem posed by Eq.(1) must be solved with the kinematic constraint conditions represented by Eqs.(9) and (11), for all values of index i. Suppose that the modal expansions are truncated to retain the first N terms in each of the three series expansions; the unconstrained system of equations consists of 3N equations for the 3N undetermined parameters. (The mode V_0 in the symmetric case and U_0 and W_0 in the antisymmetric case may be retained to preserve formal symmetry but they are not coupled with the other modes). Equations (9) and (11) can then be utilized (for instance, with use of Lagrange multipliers) to reduce the system of equations to one that consists of N equations for the N undetermined parameters z_i. While such reduction is a welcome relief computationwise, the same reasoning indicates that the use of Sanders' equations in the analysis (and, perhaps, other shell theories as well) would lead to over restraint, since, upon retaining terms in Eqs.(4) that were dropped under k_s, the bending moment and the effective shearing force conditions produce two additional sets of kinematic constraints. This situation thus dictates the limitation of the present method of analysis. Nevertheless, based on the numerical verification in Ref. 1 that a similar degree of violation of the edge conditions when Sanders' equations are used is only a minor one, it is believed that the use of Sanders' equations in the present problem is also valid as long as the first two conditions of Eqs.(4) are enforced in the preceding manner. (Terms proportional to k in the second of Eqs.(4) are negligible.)

The Modal Functions

The modal functions U_n, V_n, and W_n, as mentioned in the foregoing sections, are the exact normal mode functions for the free vibration of the circular cylindrical shell. It can be shown that these functions, with subscripts deleted, must satisfy the following differential equations ($k_s = 0$):

$$U'' - 1/2(1-\nu)n^2U + \lambda U + 1/2(1+\nu)nV' + \nu W' = 0$$

$$1/2(1+\nu)nU' + (n^2 - \lambda)V - 1/2(1-\nu)V'' + nW = 0$$

$$\nu U' + nV + k(W^{iv} - 2n^2W'' + n^4W) + (1 - \lambda)W = 0 \qquad (12)$$

and the following boundary conditions:

$$U' + \nu(nV + W) = 0 \quad , \quad V' - nU = 0 \quad , \quad W'' - \nu n^2 W = 0 \quad ,$$

$$W''' - (2-\nu)n^2W' = 0 \quad , \qquad \text{for } x = \pm L/(2r_o) \qquad (13)$$

where L denotes the length of the shell.

To solve the above boundary value problem, the mode functions U, V, and W are assumed to have the exponential form

$$(U, V, W) = \text{Re} \sum_{k=1}^{8} (a_k, b_k, c_k) \exp(\beta_k x) \qquad (14)$$

The procedure to determine the amplitude ratios a_k/c_k, and b_k/c_k, and the axial wave numbers β_k has been extensively discussed[2]. In particular, a computational procedure proposed by Vronnay and Smith[3] is judged most convenient and is adopted in the present work. The natural frequencies and the associated characteristics can thus be determined numerically.

It is interesting to note that the issue of the lowest modes of free vibration is somewhat obscure in the literature. The so-called Rayleigh and Love modes, labeled as inextensional, are generally regarded as approximations, inasmuch as they satisfy neither the equations of motion nor the boundary conditions. As a consequence, the modes with frequencies higher than that of the "inextensional modes" are recognized as the first modes.

However, if the strain-displacement equations of the Sanders' shell theory are integrated for zero strains and zero edge forces, two, and only two, sets of exact solutions do exist, both occurring when the circumferential wave number n is equal to 1. They are

$$U_1 = 0 \quad , \quad V_1 = C_1 \quad , \quad W_1 = C_2$$

and

$$U_1 = C_3 \quad , \quad V_1 = C_4 x \quad , \quad W_1 = C_5 x \tag{15}$$

in which C's are constants. These expressions, being precisely the Rayleigh and Love modes, respectively, for circular cylindrical shells, actually represent rigid body translation and rotation. Although these modes are trivial solutions for $n = 1$, they are exact, nonetheless. This observation, together with the experimental evidence obtained by Sewall and Naumann[5], indicates that the "Rayleigh-Love" modes do exist, although they might be different from their original approximate version given by Eqs.(15) and they might not be inextensional in nature. In fact, the attempt herein to determine these modes numerically is successful.

The characteristic wave numbers β_k, determined from the secular equations for the amplitude ratios a_k/c_k and b_k/c_k for a given value of λ, may be categorized in the following fashion. The four roots for β_k^2

can be arranged in numerical order according to their absolute values. As λ increases from zero, $\beta_1^2, \beta_2^2, \beta_3^2, \beta_4^2$ form two complex conjugate pairs (hereby named zone 1). As λ increases further, β_1^2 and β_2^2 are real and positive, while β_3^2 and β_4^2 remain complex conjugates of each other (zone 2). As λ continues to increase, β_1^2 becomes real and negative while β_2^2 is real and positive; and β_3^2 and β_4^2 are complex conjugates (zone 3). When the analysis is based on Donnell's shell theory, the afore-mentioned secular equation is quite simple and such zonal behavior can be established without much difficulty. For the shell model selected for numerical study in the present work, Rayleigh modes (longitudinally symmetric) are found in zone 2, while Love modes (longitudinally antisymmetric) lie in zones 2 and 3. In any case, they are not rigid body motions, and they are exact. As λ increases beyond these modes, the modes commonly labeled as $m = 1,2, \ldots$ (m being the nominal axial wave number) are found in zone 3.

While Eqs.(15) are not exact for modes other than $n = 1$, they can be used to determine the approximate eigenvalues of the Rayleigh-Love modes to provide some reference for the numerical computation. Substitution of the mode shape functions given in Eqs.(15) into Eqs.(5) in place of those of the nth mode and the subsequent introduction of the resulting expressions into Eq.(1) with $r = 1$ yields the approximations:

$$\lambda_R \approx kn^6/(n^2 + 1)$$

$$\lambda_L \approx kn^6[1 + 24(1-\nu)/(nL/r_o)^2]/[n^2 + 1 + 12/(nL/r_o)^2] \qquad (16)$$

in which the subscripts "R" and "L" of λ_R and λ_L identify "Rayleigh" and "Love", respectively.

Discussion of Results

The geometric and physical parameters of a family of cylindrical shells selected for numerical study in the present work are as follows: $L/r_o = 5.33$, $r_o/h = 375$, $\nu = 0.37$. These values were chosen according to the data of models fabricated for a concurrent experimental study.

Similar to the procedure adopted in Ref. 1, the first phase of computation was carried out to determine the normal modes of vibration of a circular cylindrical shell. To this end, modes with circumferential wave number $0 \leq n \leq 29$ were obtained for the first four axial wave numbers. Here, the symbol $\bar{m}$ is chosen to denote such a number and $\bar{m}$ has the assigned values to 1 to 4. In addition, a second nominal axial wave number m is defined to me $m = \bar{m} - 2$. Thus, $\bar{m} = 1$ and 2 represent the first longitudinally symmetric (Rayleigh) modes and the first longitudinally antisymmetric (Love) modes, respectively; while $\bar{m} = 3$ and 4, respectively, commonly recognized as $m = 1$ and 2, and the second lowest longitudinally symmetric and longitudinally antisymmetric modes. It should be noted that the alternating alignment of symmetry and antisymmetry of modes has not been assumed. The question of circumferential symmetry, of course, does not arise for the case of circular cylinders.

As mentioned in the preceding section, the Rayleigh modes lie in zone 2 (β_1^2, $\beta_2^2 > 0$), and the Love modes were found in both zone 2 and 3 ($\beta_1^2 < 0$, $\beta_2^2 > 0$). Corresponding results are illustrated in Fig. 2, which displays the values of β_1^2 and β_2^2 versus the circumferential wave number n. The other two complex conjugate roots β_3^2 and β_4^2 are not shown, but they are very much greater than β_1^2 and β_2^2 in magnitude. Thus, β_1 and β_2 characterize the long wave form of the shell deformation, which combines with the short wave form characterized by β_3 and β_4.

Although an extremely high degree of accuracy is required for the eigenvalue λ in order to accurately determine the mode shape, the trial and error procedure[3] for the solution of the frequency equation can accomplish this end with relative ease. In all cases computed, the eigenvalue λ is determined up to 13 significant figures. With such accuracy easily attainable, the closeness of the first two eigenvalues, due to the narrowness of zone 2, does not cause insurmountable difficulties. The approximate values of λ_R and λ_L, given by Eqs.(16), provide valuable guidance to the determination of more accurate values of λ. For example, when $n = 9$, Eqs.(16) yield $\lambda_R \approx 3.84058\ldots\times10^{-3}$, and $\lambda_L \approx 3.8654\ldots\times10^{-3}$. The corresponding roots for $n = 9$ and $\bar{m} = 1,2$ are found to be $3.8283\ldots\times10^{-3}$ and $3.8424\ldots\times10^{-3}$, respectively. On the other hand, since the latter two values of λ, close as they are, yield entirely different mode shapes, the Rayleigh-Love approximate frequencies, i.e., λ_R and λ_L, can not be relied upon to provide serviceable mode functions. Since the roots of λ determined are so close to one another, plots such as that partially shown in Fig. 2 are quite valuable in the process of identifying $\bar{m}$.

Figures 3a and 3b show the mode shapes of $\bar{m} = 1$ and $\bar{m} = 2$, respectively, for some selected values of n, based upon solutions of Eqs.(12) to (14). They demonstrate that the deformation of the former is longitudinally symmetric and that of the latter antisymmetric. Such a property has not been assumed <u>a priori</u> in the analysis. Moreover, these mode functions turn out to be quite different from the Rayleigh-Love approximation. An examination of the numerical amplitudes of the functions U_n and V_n, although smaller than that of the corresponding W_n, suggests that the deformations of the first two families are not truly inextensional.

Upon the determination of 30 modal functions for each $\bar{m} = 1,2,3,4$ (15 even and 15 odd; i.e., $N = 15$), they are substituted into the variational functional δF, and the required numerical integrations with respect to x are performed. Lagrange multipliers are introduced to incorporate the 2N constraint conditions (Eqs.(9) and (11)) into F, and are subsequently eliminated. After all these straight-forward steps are completed, the system of equations is finally expressible in the form

$$Pz - \lambda Qz = 0 \tag{17}$$

of which the dimensionality is reduced to N. The eigenvalues and the corresponding eigenvectors are determined from Eq.(17) Results are shown in Figs. 4 to 6.

In Figs. 4 and 5 the nondimensional natural frequency Ω corresponding to the first four axial wave numbers $\bar{m}$ = 1,2,3,4 are shown versus the nominal circumferential wave number $\bar{n}$. The resulting variation of deflection along the circumference of a vibrating oval cylindrical shell can be substantially different from the simple, sinusoidal pattern of an equivalent circular cylindrical shell. Thus, $\bar{n}$ cannot be determined by the count of nodes along the circumference. Rather, the determination of $\bar{n}$ is made possible by performing the calculations for a series of values of the eccentricity parameter ξ with a small increment of ξ (chosen to be 0.1). The evaluation of the mode shape can then be traced by means of a plot of the frequency versus ξ exemplified by Fig. 6. The designation of a number for of a mode $\bar{n}$ is aided by the continuous variation of such a plot. It turns out, in the free-free case, that the values of $\bar{n}$ coincide with the index of the largest component (in absolute value) in the associated eigenvector z; such is not always the case for the supported shells studied in Ref. 1.

Figure 5 magnifies the region of low frequency and small $\bar{n}$ in which the contrast between the spectra of the circular and noncircular cylinders is most noticeable.

It is significant to note that, for any mode of deformation of an oval shell represents by a point in Figs. 4 or 5, a second pattern always exists. More precisely, for the same circumferential wave number $\bar{n}$, a higher frequency can always be found depending upon the status of symmetry about the principal axes. Similar characteristics were observed by Culberson and Boyd in their investigation[6] from the odd modes of a freely supported oval shell, and by the present authors[1] from both the odd and even modes of the seven other supported cases. However, unlike what was observed from most of the supported cases, a pattern seems to emerge from the present results in that a lower frequency is associated with the mode shape antisymmetric with respect to the major axis. In terms of the two sets of modal expansions (Eqs.(5) and their antisymmetric counterparts) and the sign of ξ which fixes the origin of the circumferential coordinate s, the lower mode possesses the following specification:

(a) Odd modes: The modal expansion (lower mode) is symmetric if the eccentricity parameter ξ is chosen to be negative; and it is antisymmetric if ξ is positive.

(b) Even modes: The modal expansion (lower mode) is antisymmetric regardless of the sign of ξ.

A visual version of these rules is given in Fig. 7. In Figs. 4 to 6, only the lower frequencies are shown.

Another interesting feature of the free-free shell is that the dependency of the natural frequency upon the eccentricity parameter is not as strong as in the supported cases. This weaker dependency is the direct result of a weaker coupling among the circumferential modes, which is reflected by the sharper domination by one component in any eigenvector. For this reason, only curves corresponding to the extreme values of the eccentricity parameter $|\xi| = 0$ and 1 are shown in Figs. 4 and 5, although computations were performed for all intermediate values with an increment of ξ equal to 0.1. Among the four groups, the Rayleigh modes ($\bar{m} = 1$) exhibit the strongest dependency upon ξ and the Love modes ($\bar{m} = 2$) display the weakest dependence. The variation of the natural frequency of the Rayleigh modes ($\bar{m} = 1$) versus the eccentricity parameter are shown in Fig.6. Similar curves for other values of $\bar{m}$, while not being shown here, exhibit both increasing and decreasing variations of the frequency as $|\xi|$ increases, but show much less variation with ξ.

In conclusion, it is observed that the rigorous enforcement of boundary conditions in the dynamic analysis of cylindrical shells of finite length is essential. Moreover, the present method of analysis succeeded in transforming a two-dimensional (spatial) problem into a one-dimensional one. It seems reasonable to predict that the convenient features of the present method are equally applicable to problems involving forced dynamic response of noncircular cylindrical shells. Finally, a remark of caution regarding the completeness of the modal functions in the functional basis must be made. For any given value of $\bar{m}$, granting truncation, the complete set of modal functions must be included in the modal expansion. A missing link would severely damage the accuracy of the neighboring modes as a result of coupling.

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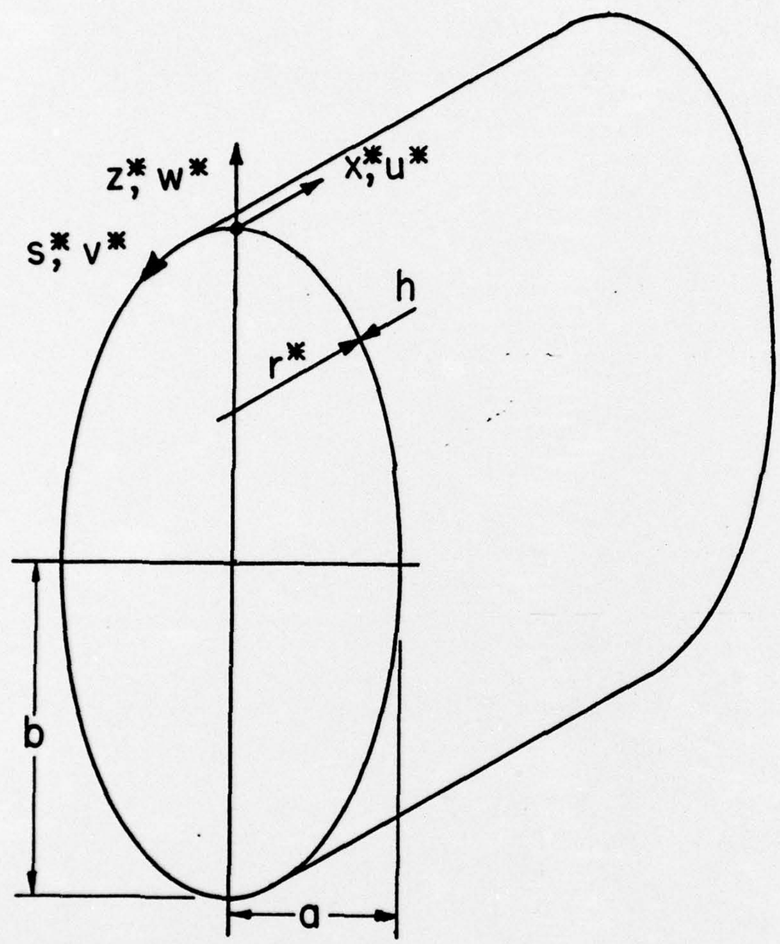

FIG. 1 COORDINATES AND SIGN CONVENTION.

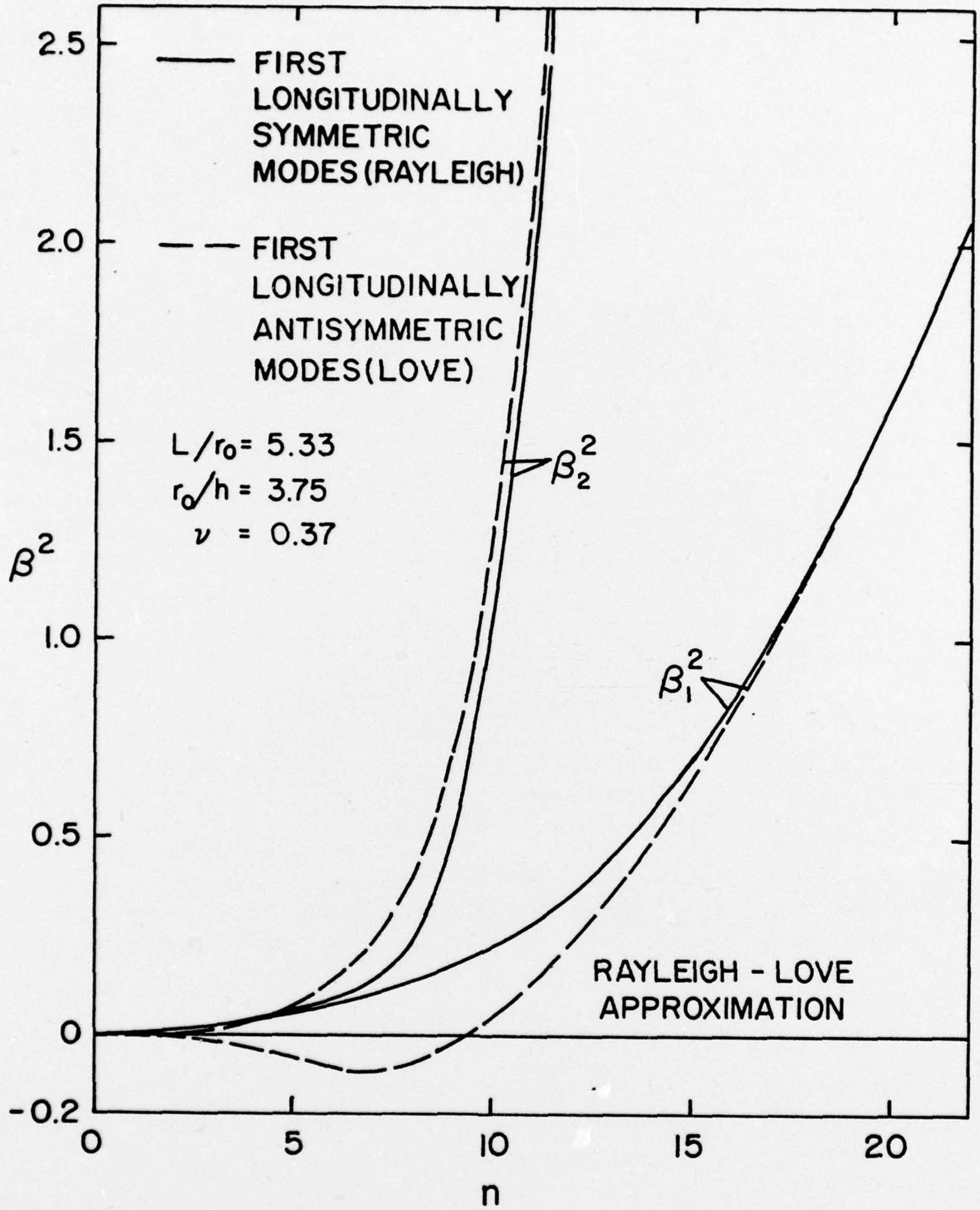

FIG. 2 CHARACTERISTIC WAVE NUMBERS (LONG WAVES) OF THE FIRST AND SECOND MODES OF A CIRCULAR CYLINDRICAL SHELL WITH FREE ENDS.

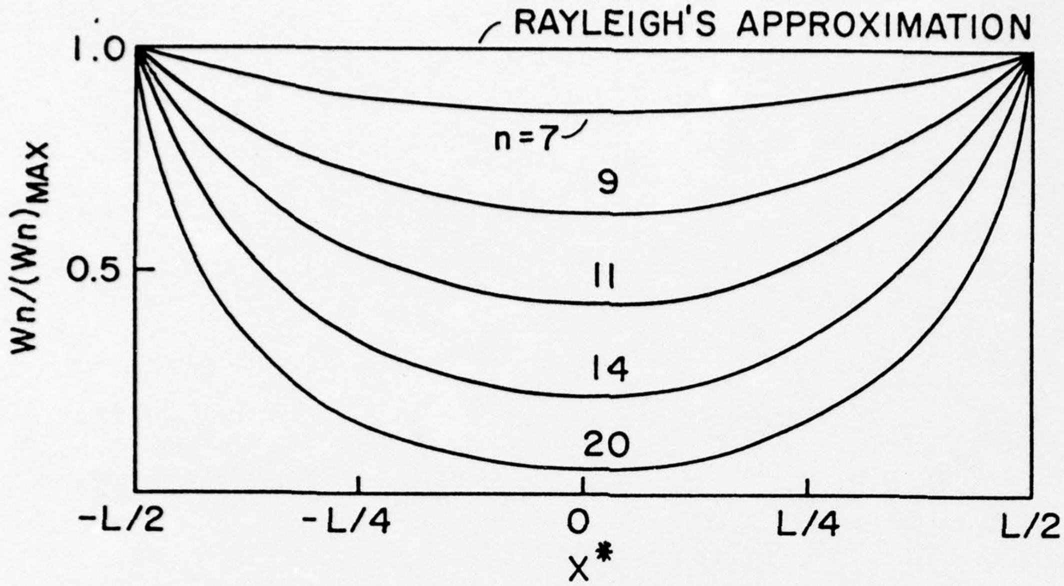

FIG. 3a DEFLECTION OF FIRST LONGITUDINALLY SYMMETRIC MODES OF A FREE-FREE CIRCULAR CYLINDRICAL SHELL FOR $\overline{m} = 1$ ($L/r_o = 5.33$, $r_o/h = 375$, $\nu = 0.37$).

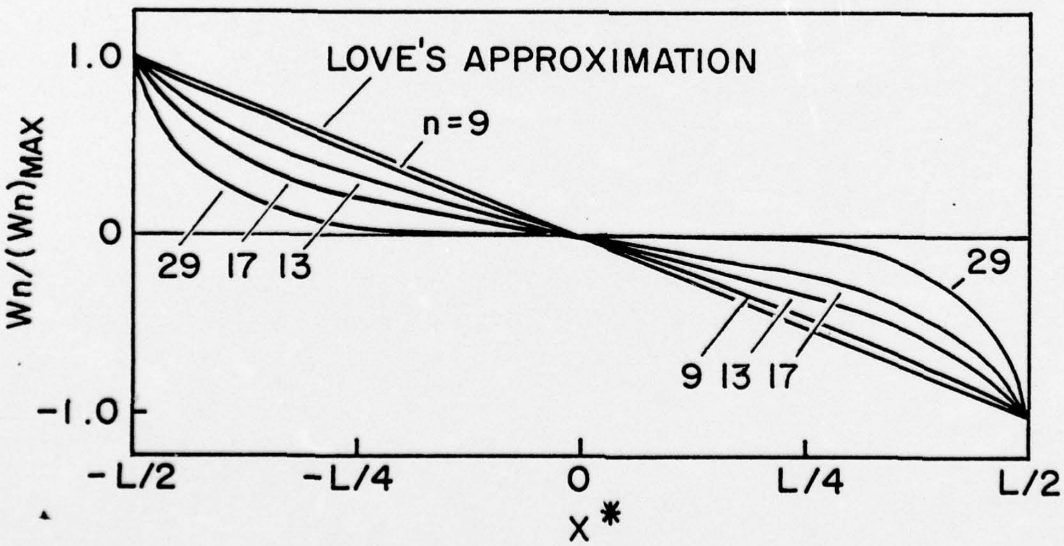

FIG. 3b DEFLECTION OF FIRST LONGITUDINALLY ANTISYMMETRIC MODES OF A FREE-FREE CIRCULAR CYLINDRICAL SHELL FOR $\overline{m} = 2$ ($L/r_o = 5.33$, $r_o/h = 375$, $\nu = 0.37$).

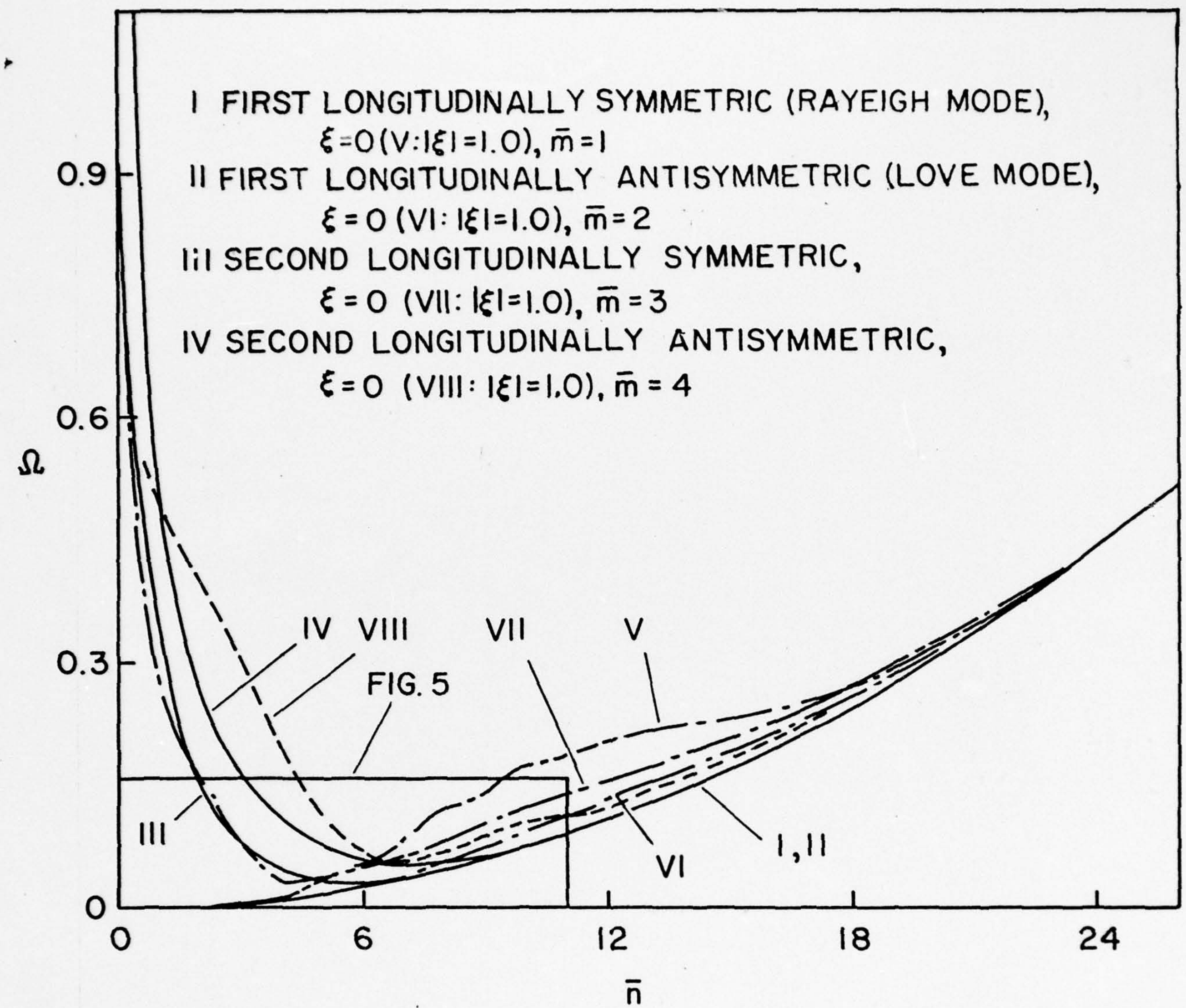

FIG. 4 NATURAL FREQUENCY OF FREE-FREE OVAL SHELLS VS. CIRCUMFERENTIAL WAVE NUMBER ($L/r_o = 5.33$, $r_o/h = 375$, $\nu = 0.37$).

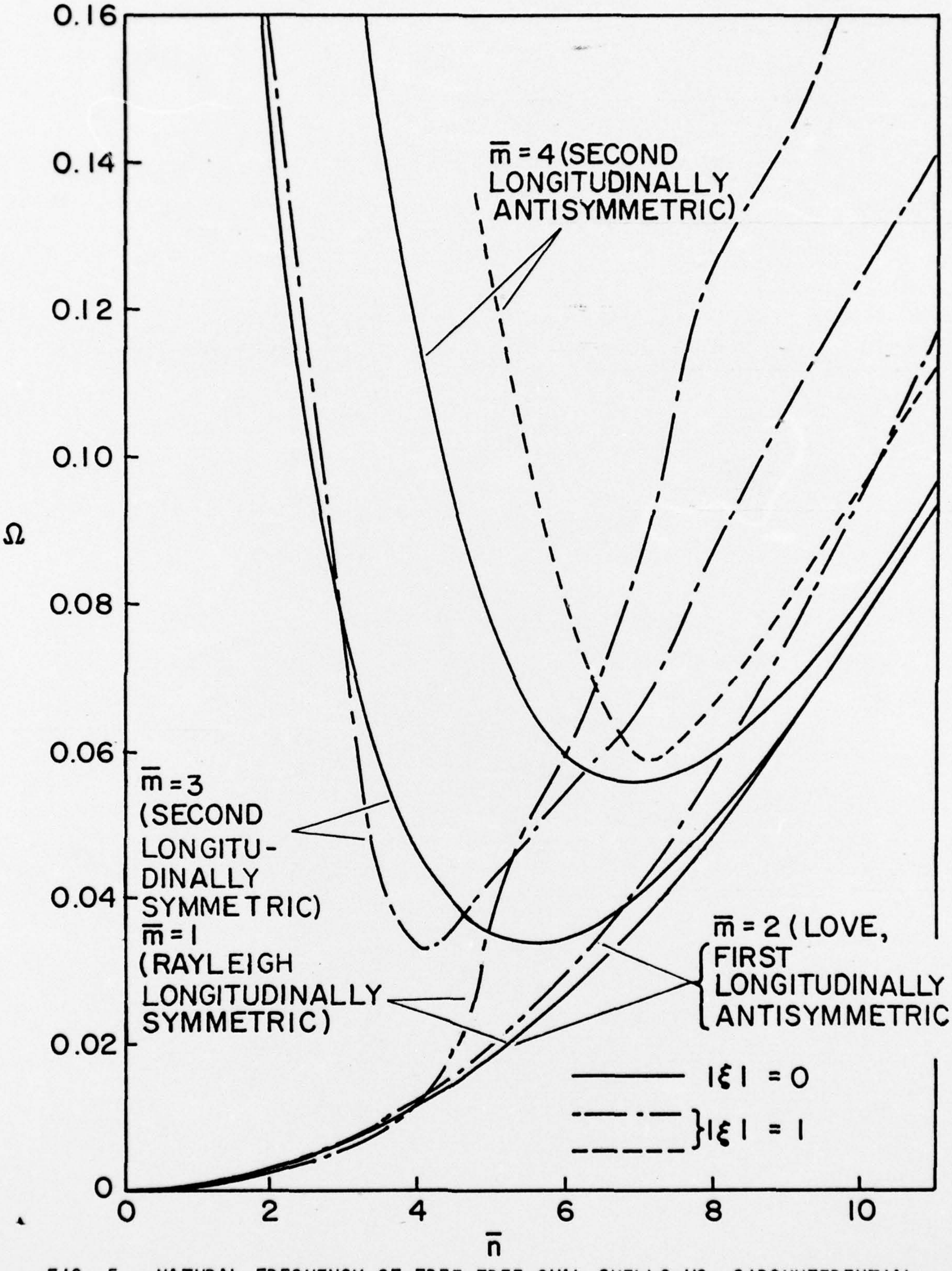

FIG. 5 NATURAL FREQUENCY OF FREE-FREE OVAL SHELLS VS. CIRCUMFERENTIAL WAVE NUMBER ($L/r_o = 5.33$, $r_o/h = 375$, $\nu = 0.37$).

F5.

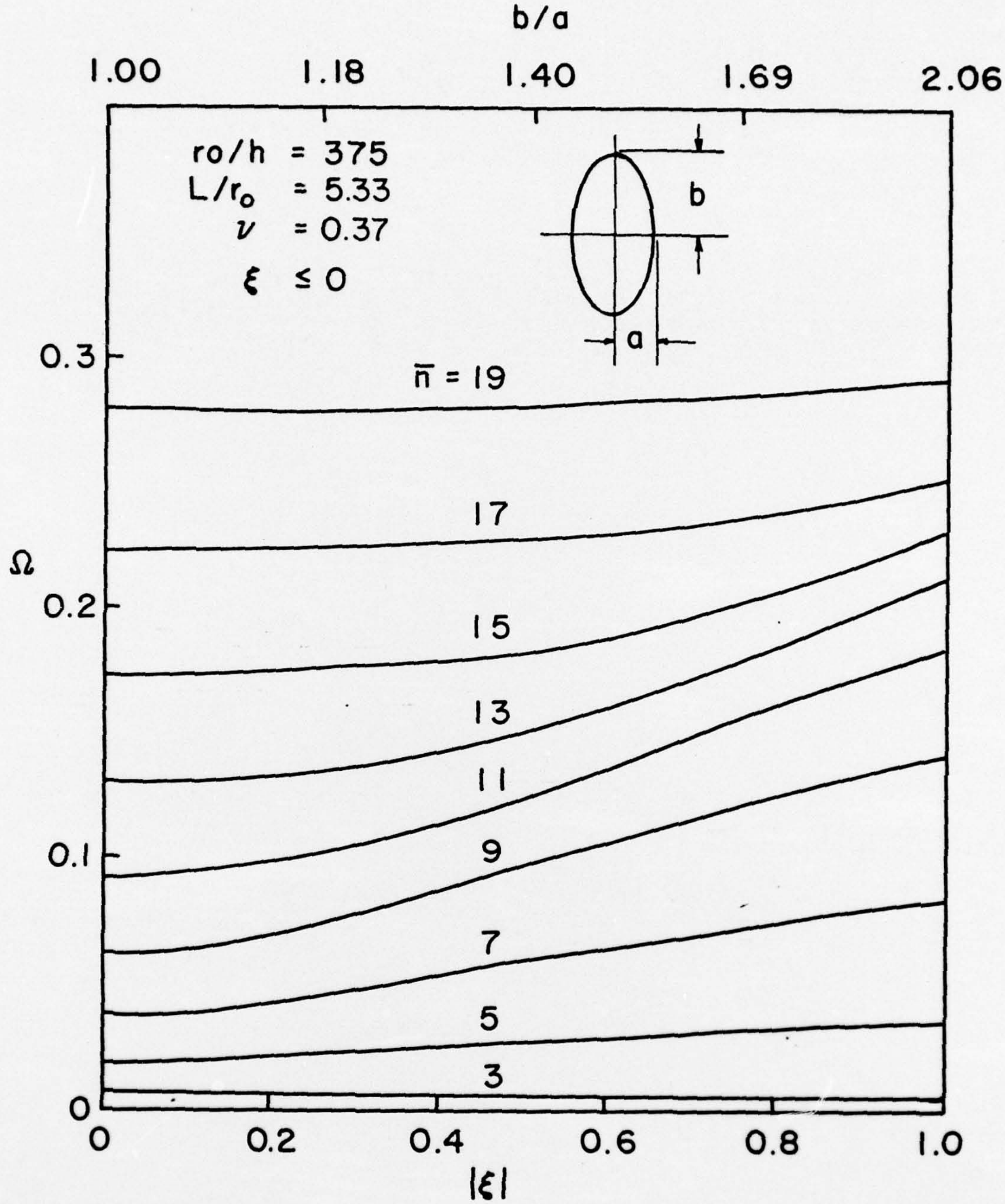

FIG. 6 VARIATION OF NONDIMENSIONAL FREQUENCY VS. ECCENTRICITY PARAMETER OF FREE-FREE OVAL SHELLS, FIRST LONGITUDINALLY SYMMETRIC (RAYLEIGH) MODES (m = 1).

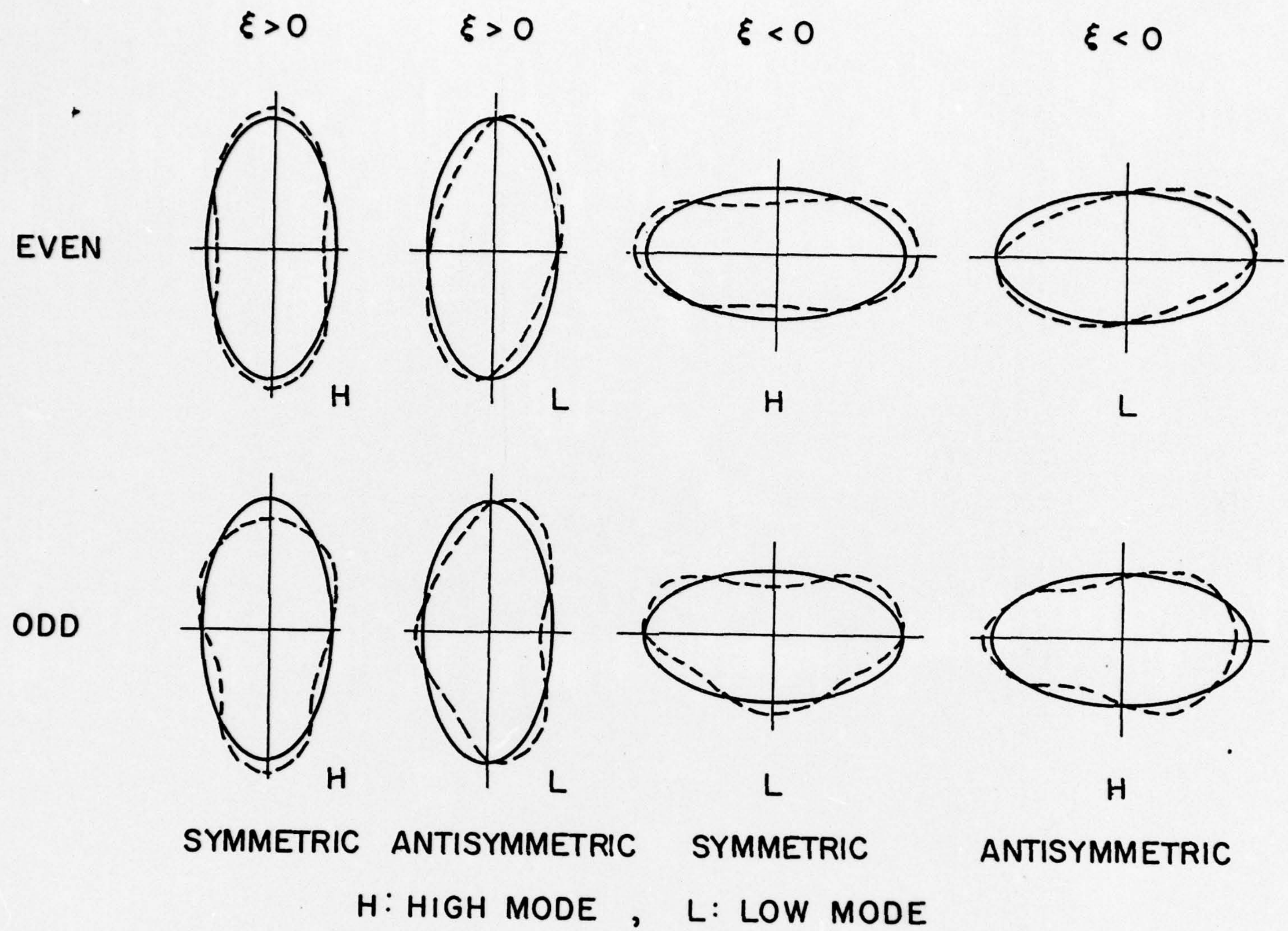

FIG. 7 HIGH MODES AND LOW MODES OF DEFORMATION (ILLUSTRATED BY $\bar{m} = 2$ AND $\bar{m} = 3$).