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MAGNETOHYDRODYNAMICS, PLASMAS, AND NONLINEAR OSCILLATIONS

Final Report

US-AFOSR Project 71-2122

September 1, 1971 - August 31, 1976

Prepared by

Lamberto Cesari

Department of Mathematics

The University of Michigan

Ann Arbor, Michigan

October 1976

I. Personnel

Project Director:

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II. List of Reports Issued in the Period
September 1, 1971 - April 30, 1976
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Reports which have appeared in print are denoted by an asterisk(*) and the quotation is given.

*1. L. Cesari, An existence theorem in optimal control without convexity conditions. SIAM Journal Control 12, 1974, 319-331.

2. L. Cesari, Closure theorems and weak convergence. (Included in Report 11).

3. L. Cesari, Controllability of time dependent linear, and nonlinear systems.

4. L. Cesari, Problems of approximation in optimal control theory.

*5. J. A. Smoller and M. E. Taylor, Wave front sets and the viscosity methods. Bull. Amer. Math. Soc. 79, 1973, 98-103.

6. L. Cesari, Optimization problems, maxima and minima in functional analysis.

*7. D. E. Cowles, Problems of optimal control monitored by integral equations. SIAM J. Control 11, 1973, 595-606.

*8. M. B. Suryanarayana, Existence theorems for optimization problems concerning hyperbolic partial differential equations. J. Optimization Theory and Appl. 15, 1975, 361-392.

*9. M. B. Suryanarayana, Existence theorems for optimization problems concerning linear hyperbolic partial differential equations without convexity conditions. J. Optimization Theory and Appl. 19, 1976, 47-61.

*10. L. Cesari, Convexity and seminormality in the calculus of variations. Atti e Memorie Accad. Scienze Lettere Arti Modena (6) 14, 1972, 119-133.

*11. L. Cesari, Closure theorems for orientor fields. Bull. Amer. Math. Soc. 79, 1973, 684-689.

*12. L. Cesari, Closure theorems for orientor fields and weak convergence. Archive Rational Mechanics and Analysis 55, 1974, 332-356.

*13. L. Cesari and M. B. Suryanarayana, Closure theorems without seminormality conditions. J. Optimization Theory and Appl. 15, 1975, 441-465.

*14. L. Cesari and M. B. Suryanarayana, Lipschitz condition implies property (Q). Appeared under the title "Convexity and property (Q) in optimal control theory", SIAM J. Control, 1974, 705-720.

*15. T. Nishida and J. Smoller, Solutions in the large for some nonlinear hyperbolic conservation laws, Communication, pure applied Math. 26, 1973, 183-200.

*16. L. Cesari, Nonlinear analysis. Lectures Centro Internazionale Matematico Estivo, Bressanone 1972. Cremonese, Roma, 1973, 1-90.

*17. L. Cesari, Convexity of the range of certain integrals. SIAM J. Control 13, 1975, 666-676.

*18. L. Cesari, Lower semicontinuity and lower closure theorems without seminormality conditions. Annali Matematica pura e appl. 98, 1974, 381-397.

19. L. Cesari, Periodic solutions of an equation of nonlinear wave theory. Included in Report 47.

20. L. Cesari, Summary of research on nonlinear oscillations in plasmas, nonlinear optics, and radio waves.

*21. L. Cesari and R. Kannan, Functional analysis and nonlinear differential equations. Bull. Amer. Math. Soc. 79, 1973, 1216-1219.

22. R. Kannan, Quelques resultats d'existence pour une classe d'equations differentielles non-lineaires abstraits. Included in Report 35.

*23. J.A. Smoller and C.C. Conley, Shock waves as limits of progressive wave solutions of higher order equations, Communications on pure and applied Math. 24, 1971, 459-472.

*24. R. Kannan, On periodically perturbed conservative systems. J. Differential Equations, 16, 1974, 506-514.

25. R. Kannan, Existence of periodic solutions for a Liénard system. Included in Report 52.

*26. R. Kannan, Existence of periodic solutions for nonlinear differential equations. Trans. Amer. Math. Soc. 217, 1976, 225-236.

27. L. Cesari, Boundary value problems for hyperbolic quasilinear systems of first order partial differential equations. Included in Report 32.

*28. P. J. Kaiser, Existence theorems in the calculus of variations. Appeared as a joint paper: L. Cesari and P. J. Kaiser, "Closed operators and existence theorems in multidimensional problems of the calculus of variations", Bull. Amer. Math. Soc. 80, 1974, 473-478.

29. L. Cesari, The Cauchy problem for quasilinear hyperbolic systems in canonic form. Included in Report 37.

30. L. Cesari, The Cauchy problem for quasilinear hyperbolic systems. Included in Report 38.

31. L. Cesari, A boundary value problem for quasilinear hyperbolic systems. Included in Report 39.

32. L. Cesari, Boundary value problems for hyperbolic quasilinear systems of first order partial differential equations. Included in Report 40.

33. L. Cesari, Periodic solutions of an equation of nonlinear wave theory. Included in Report 47.

*34. L. Cesari, Geometric and analytical views in existence theorems for optimal control in Banach spaces. I. Distributed parameters. J. Optimization Theory and Appl. 14, 1974, 505-520.

35. R. Kannan, Existence of solutions of semilinear operator equations. To appear in Proc. Amer. Math. Soc.

*36. R. Kannan, Existence of solutions of a nonlinear problem in potential theory. Michigam Math. J. 21, 1974, 257-263.

*37. L. Cesari, Boundary value problems for hyperbolic quasilinear systems of first order partial differential equations. Rend. Accad. Naz. Lincei (8) 56, 1974, 1-4.

*38. L. Cesari, Boundary value problems for hyperbolic quasilinear systems in Schauder's canonic form. Rend. Accad. Naz. Lincei (8) 57, 1974, 303-307.

39. L. Cesari, A boundary value problem for quasilinear hyperbolic systems. To appear in Rivista di Matematica dell'Universita di Parma.

*40. L. Cesari, A boundary value problem for quasilinear hyperbolic systems in Schauder's canonic form. Annali Scuola Normale Superiore di Pisa (4) 1, 1974, 311-358.

*41. L. Cesari, Alternative methods in nonlinear analysis. Invited lecture. International Conference on Differential Equations, Los Angeles, (September 3-7, 1974). Academic Press 1975, 95-148.

*42. L. Cesari, Geometric and analytic views in existence theorems for optimal control. II. Distributed and boundary controls. J. Optimization Theory and Appl. 15, 1975, 467-497.

*43. M. B. Suryanarayana, Remarks on lower semicontinuity and lower closure. J. Optimization Theory and Appl. 19, 1976, 125-140.

*44. L. Cesari and M. B. Suryanarayana, Nemitsky's operators and lower closure theorems. J. Optimization Theory and Appl. 19, 1976, 165-184.

*45. C. C. Conley and J. A. Smoller, The existence of heteroclinic orbits, and applications. Dynamical Systems, Theory and Applications, Springer Lecture Notes Physics (ed. J. Moser), 1975, 511-524.

46. L. Cesari, Nonlinear oscillations in the frame of alternative methods. Invited lecture. International Symposium on Dynamical Systems, Providence, R.I. (1974). Academic Press, 1976, vol.1, 29-50.

47. L. Cesari, Nonlinear oscillations under hyperbolic systems. International Symposium on Dynamical Systems, Providence, R.I. (1974). Academic Press, 1976, vol.1, 251-261.

48. C. C. Conley and J. A. Smoller, The structure of magnetohydrodynamic shock waves. II. To appear in J. de Mathematiques Pures et Appliquees.

49. R. Kannan, Periodically perturbed conservative systems. To appear as an appendix to Report 26.

50. P. J. Kaiser, Seminormality properties of convex sets. To appear in Rendiconti Circolo Matematico Palermo.

*51. P. J. Kaiser and M. B. Suryanarayana, Orientor field equations in Banach spaces. J. Optimization Theory and Appl. 19, 1976, 141-164.

*52. L. Cesari and R. Kannan, Periodic solutions in the large of nonlinear ordinary differential systems. Rendiconti di Matematica Universita Roma (2) 8, 1975, 633-654.

*53. L. Cesari, Geometric and analytic views in existence theorems for optimal control. III. Weak solutions. J. Optimization Theory and Appl. 19, 1976, 185-214.

*54. L. Cesari, Nonlinear analysis and alternative methods. Accademia Nazionale dei Lincei. Centro Interdisciplinare. Lecture Notes, Roma 1974. pp. 1-103.

55. J. Smoller and T. Nishida, Mixed problems for nonlinear conservation laws. Journal Differential Equations, to appear.

56. J. Smoller and J. Rauch, Qualitative theory of the Fitzhugh-Nagumo equation. Advances in Mathematics, to appear.

57. P. Bassanini, On the boundary value problem for a class of quasilinear hyperbolic systems in two independent variables. To appear in Bollettino Unione Matematica Italiana.

58. L. Cesari and P. Bassanini, A nonlinear hyperbolic problem arising from a question of nonlinear optics.

59. L. Cesari and R. Kannan, Solutions in the large of Lienard systems with forcing terms. To appear in Annali di Matematica pura e applicata.

*60. L. Cesari and M. B. Suryanarayana, Existence theorems for Pareto optimization in Banach spaces, Bull. Amer. Math. Soc. 82, 1976, 306-308.

61. L. Cesari, Qualitative conditions for nonlinear oscillations. Invited address. Seventh International Conference in Nonlinear Oscillations at Humboldt University, Berlin (September 8-13, 1975). To appear in the Acta of the Conference.

62. L. Cesari, The alternative method in nonlinear oscillations. To appear in a volume of the Math. Assoc. of America "Studies in Mathematics" (ed. J. K. Hale).

63. L. Cesari, Existence theorems across a point of resonance. To appear in the Bull. Amer. Math. Soc.

64. L. Cesari and R. Kannan, An abstract existence theorem at resonance. To appear in Proc. Amer. Math. Soc.

65. L. Cesari, An abstract existence theorem across the point of resonance. Invited address. International Conference on Dynamical Systems, Gainesville, Florida, March 24-26, 1976. To appear in the Acta of the Conference.

66. L. Cesari, Nonlinear oscillations across a point of resonance for nonselfadjacent systems. To appear in Journal Differential Equations.

67. L. Cesari, Nonlinear problems across a point of resonance for nonselfadjoint systems. To appear in the volume Nonlinear Analysis, Academic Press.

*68. L. Cesari and M.B. Suryanarayana, Existence theorems for Pareto problems of opimization. Invited lecture delivered by Cesari at the Conference on Calculus of Variations and Optimal Control Theory, Madison, Wisconsin. Calculus of variations and Control Theory (ed. D. Russel). Academic Press 1976, 139-154.

69. L. Cesari and M.B. Suryanarayana, Existence theorems for Pareto optimization. Multivalued and Banach space valued functionals. To appear in Trans. Amer. Math. Soc.

70. L. Cesari and T.T. Bowman, Some error estimates by the alternative method. To appear in Quart. Applied Math.

71. P.J. McKenna, Nonselfadjoint semilinear equations at simple resonance in the alternative method. International Conference on Dynamical Systems, Gainesville, Florida, March 24-26, 1976. To appear in the Acta of the Conference.

72. R. Kannan and P.J. McKenna, An existence theorem by alternative methods for semilinear abstract equations at resonance. Bolletins Unione Matematica Italiana. To appear.

73. Richard DeVries, Periodic solutions of differential systems of Liénard and Rayleigh type. International Conference on Dynamical Systems, Gainesville, Florida, March 24-26, 1976. To appear in the Acta of the Conference.

74. P.J. McKenna, Nonselfadjoint semilinear problems in the alternative method. A Ph.D. thesis at the University of Michigan.

75. K. Nagle, Boundary value problems for nonlinear ordinary differential equations, A Ph.D. thesis at the University of Michigan.

76. H.C. Shaw, Nonlinear elliptic boundary value problems at resonance. A Ph.D. thesis at the University of Michigan.

III. Ph.D. Theses at the University of Michigan

The following theses have been completed in the frame of the project:

K. Nagle, Boundary value problems for nonlinear ordinary differential equations. June 1975.

H.C. Shaw, Nonlinear elliptic boundary value problems at resonance. July 1975.

P.J. McKenna, Nonselfadjoint semilinear problems in the alternative method. July 1976.

IV. Lectures and Other Activities

Cesari was on leave from the University of Michigan for the Winter Term 1975, and was at the University of Florida, Gainesville, for study and research. At the University of Florida, Cesari gave a series of lectures on problems of nonlinear oscillations, together with Den Hartog from M.I.T.. The interplay between the engineering viewpoint presented by Den Hartog and the mathematical one presented by Cesari was the main point of the series of lectures.

In March 1975 Cesari attended the St. Louis, Missouri, meeting of the American Mathematical Society. Cesari gave a paper on the existence in the large of periodic solutions of Lienard systems.

In June 1975 Cesari was the principal lecturer at the Michigan State University Conference on Functional Analysis and Nonlinear Differential Equations, East Lansing, Michigan June 9-12, 1975. Cesari gave six one-hour lectures. The written version of these lectures (200 pages), together with the papers of the other participants to the Conference, are in process of publication as a book: Functional Analysis and Differential Equations (Cesari, Kannan, Schuur eds.). M. Dekker, New York 1976.

In September 1975 Cesari gave the inaugural lecture at the International Conference on Nonlinear Oscillations, Humboldt University, Berlin, September 8-13, 1975.

Also in September 1975 Cesari gave a one-hour lecture at the Conference on Calculus of Variations at the University of Wisconsin, Madison, Wisconsin, September 22-24, 1975.

Cesari was the Editor for the two special issues Vol.15, No.4, April 1975, and Vol.19, No.1, May 1976, of the "Journal of Optimization Theory and Applications" on the topic of existence theorems for problems of optimization.

Cesari has been awarded the R.L. Wilder Chair of Mathematics at the University of Michigan, September 1975.

Cesari was a member of the program committee of the Conference on Dynamic Systems which has taken place at the University of Florida, Gainesville, Florida, March 24-26, 1976. Cesari gave a one-hour lecture on existence of solutions at resonance. McKenna gave a half-hour lecture, and DeVries and Ku gave contributed papers.

Cesari was awarded the Russel lecturship at the University of Michigan for 1976. Cesari read his lecture March 23, 1976 on "Mathematics in the Mediterranean. Today's view."

Cesari gave a one-hour lecture at the Conference on Nonlinear Systems and Applications at the University of Texas at Arlington, and also a short course on the alternative method, July 19-28, 1976.

Cesari was awarded a Doctor Degree honoris causa at the University of Perugia, Italy, October 5, 1976.

Smoller gave a half-hour lecture at the Symposium on applications of functional analysis to problems in mechanics, Marseille, France, August 1975.

Smoller gave a half-hour lecture at the American Mathematical Society meeting in Washington, D.C., January 1975.

Smoller gave a series of lectures at the Atomic Energy Commission Laboratory, Los Alamos, New Mexico, April 1975, and April 1976.

Smoller gave a half-hour lecture at the American Mathematical Society Regional Conference on Topological Methods in Dynamical Systems, Boulder, Colorado, May 1976.

Smoller gave a half-hour lecture at the American Mathematical Society Regional Conference on Reaction Diffusion Equations, Houston, Texas, June 1975.

V. Summaries of Reports

(a) Entrainement of frequency "in the large". Oscillations of physical systems "at resonance", and "across a point of resonance".

Reports Nos. 52, 59, 61, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76.

This research touches an important point of physical and mechanical systems. If we know that such a system possesses "free oscillations" of a given frequency ω, then, under the action of periodic external forces of the same frequency ω (resonance), the system -- in a purely linear theory -- should give rise to phenomena of instability, as, for instance, oscillations of the same frequency whose amplitude increases with time beyond control. This behavior does not correspond to reality, because physical and mechanical systems are in general nonlinear. In reality, we expect the system to give rise to steady oscillations of the same frequency ω (syntony). A good agreement between theory and reality has been shown long ago by mathematical models in which the nonlinearity in the equations appears as a "small perturbation", or nonlinear systems of the perturbation type, and the theory is often referred to as bifurcation theory. In reports 59, 60, 64 such an agreement is sought for mathematical models, not of the perturbation type, but "with large nonlinearities" as they often occurred in applications.

In reports 52, 59 Cesari and Kannan consider Liénard systems of high order, thus representing, for instance, complex electrical networks, say n circuits inductively connected, with large nonlinearities (as diodes, triodes, solid state elements, impedances). Under suitable assumptions on the mathematical model, it is shown that external periodic electromagnetic forces of given frequency ω give rise to currents of the same frequency ω, though ω is the frequency of free oscillations of the underlying linear system.

In report 73 DeVries uses the same method of Cesari and Kannan to prove some further existence theorems for periodic solutions of periodic Lienard systems, so as to include some recent results of Knoller and to guarantee invariance properties of the systems under consideration.

In report 64 Cesari and Kannan show the existence of solutions "at resonance" in an extremely general context, namely for nonlinear operational equations in Hilbert spaces, in particular for general selfadjoint boundary value problems for partial differential equations with large nonlinearities.

In report 72 McKenna and Kannan give a new proof of the result in report 64.

In report 63 Cesari starts from the remark that exact "resonance" is an abstraction, since the coincidence between frequency of the external forces and frequency of the free oscillations of the underlying linear system cannot be verified exactly in any circumstance. Actually, this abstraction is not needed. Cesari, in reports 63, 65 proves that the same hypotheses which have been used to guarantee the existence of solutions at resonance in the same mathematical models, actually guarantee the existence of steady smooth solutions of the same frequency ω' of the external forces, even if ω' is only close to a frequency ω of the linear system, and that no phenomenon of instability seems to appear, as ω' varies in a small interval $(\omega - \delta, \omega + \delta)$ around a frequency ω of the free oscillations of the underlying linear system. This phenomenon, which in most applications is called "entrainement of frequency" and is well known in models of the perturbation type, is being proved here in models containing large nonlinearities.

Actually, in report 65, Cesari studies the problem in the context of nonlinear selfadjoint operational equations in Hilbert spaces, and in this generality, the phenomenon is formulated as the existence of steady smooth solutions across a point of resonance. In reports 66, 67 Cesari extends the results to operational equations in Banach spaces, (nonselfadjoint problems), and then considers a number of models with scalar equations.

Let $E : \mathcal{D}(E) \to Y$, $\mathcal{D}(E) \subset X$, and $N : X \to Y$ be operators, E linear, N nonlinear, X, Y Banach spaces. Let λ be an eigenvalue of E, and W the space of all solutions x of $Ex + \lambda x = 0$. For $1 \le \dim W < \infty$ and suitable hypotheses on E, N, X, Y, W, we prove in report 66 that there are numbers $\alpha > 0$, $C > 0$ such that, for every $|\alpha| < \alpha$ the equation $Ex + (\lambda + {}^{o}\alpha)x = Nx$ has solutions $x \in X$ with $\|x\|^{o} \le C$ (existence across the point of resonance $\alpha = 0$). The conditions on E, N, X, Y, W are stated in abstract form and are shown to extend the specific conditions proposed by Landesman, Lazer, Leach, Williams, DeFigueiredo for existence at resonance only of the problems taken into consideration by these authors. For scalar equations with the nonlinearity of the form $Nx = f(t) + h(x)$, h nonlinear, Cesari shows that the well known condition $xh(x) \ge 0$ [or $xh(x) \le 0$], in a suitable context, is relevant in proving the existence of steady smooth solutions $x \in X$ to $Ex + (\lambda + \alpha)x = f(t) + h(x)$ for $|\alpha| \le \alpha_o$ λ eigenvalue of E, f a known function.

In report 75 Nagle examines questions of existence and approximation for the solutions of perturbation problems in ordinary differential systems of the form

$$x' - \dot{A} = \varepsilon f(t, x, x'), \quad a \le t \le b,$$

with boundary conditions

$$B_1 x(a) + B_2 x(b) = 0,$$

where A is an $n \times n$ matrix, B_1, B_2, are constant $m \times n$ matrices, $0 \le m \le n$, $x = (x_1, \ldots, x_n)$, $x' = dx/dt$, and ε a small parameter. The novel treatment is based on the alternative method, and is relevant because of the rather general boundary conditions under consideration, because of the fact that derivatives appear in the nonlinear terms $f(t, x, x')$, and because the coefficients are only measurable and bounded, and the terms $f(t, x, x')$ are measurable in t , Lipschipzian in x, x' , and bounded for x, x' bounded. The early results of Cesari, Hale, Gambill (1952-60) concerning periodic solutions are thus extended to arbitrary homogeneous boundary conditions, and under the mentioned general requirements.

In report 76 Shaw considers nonselfadjoint problems at resonance for nonselfadjoint elliptic partial differential equations of any order. Shaw gives direct extensions of the Landersman and Lazer theorem for selfadjoint problems only. Particularly relevant and new is the treatment of the bifurcation equation in terms of point set topology and of algebraic topology.

In reports 71 and 74 McKenna develops a new treatement of nonselfadjoint problems in terms of alternative and bifurcation methods, which improves on previous work of Osborn and Sadler. Great many applications and examples show the power of the new treatment, which could be used for the determination of approximate solutions to the problems under investigation.

(b) Pareto Optimization

Reports Nos. 60, 68, 69.

Pareto in 1896 proposed a concept of optimization, or state of equilibrium in problems of economics, which has been shown to be relevant, and has been extensively studied. Recently many authors have studied necessary conditions for Pareto optima (Smale, Simon, Yu), in connection with optimal control theory, game theory, Lagrange and Pontryagin multipliers. In the reports 60, 68, 69, Cesari and Suryanarayana frame the problem of Pareto optimization in terms of optimal control theory with vector valued functionals, and prove existence theorems for Pareto optima.

These theorems are proved in great generality, namely for functionals with values in a real reflexive Banach space, and the plant expressed in terms of an operational relation also in Banach spaces. Examples and applications are given for the case of n functionals with the plant expressed in terms of ordinary differential equations in Euclidean spaces.

(c) Optical resonance

Reports Nos. 37, 38, 39, 40, 57, 58.

In the reports 37, 38, 39, 40 Cesari proved theorems of existence of smooth solutions for systems of first order partial differential equations and an extremely general class of boundary value problems in a thin infinite slab $S = [0 \leq x \leq a, \ y \in E^2]$, of thickness $a > 0$. In particular, these theorems hold for the Cauchy problem (all data on the hyperplane $x = 0$). These theorems contain estimates by defect of the maximum thickness a above which the solutions cease to be smooth and phenomena of discontinuity, or shock, appear. Applications of these theorems to problems of optical resonance with laser beams (Franken, Ward, Bloemberger) have been described in report numbers 20, 32, 33, 47, and in summaries of the same reports and of report numbers 37, 38, 39, 40.

In report numbers 57, 58 the authors determine sharper estimates by defect of a, and compute some of these estimates for the case of a laser beam through a thin quartz crystal.

(d) Evolution equations and related problems in shock wave theory, semiconductors, diffusion, combustion kinetics, ecological systems.

Smoller and collaborators have been working on systems of partial differential equations, of the Fitzhugh-Nagumo and of the Hodgkin-Huxley types, which describe the transmission of nerve impulse signals along an axon (see 1, 2, 3 below). They have proved the existence of global (in time) solutions to these problems, and have also obtained qualitative properties of the solutions.

Smoller recently has been working on some mathematical problems in ecology. In 4, Smoller and Conway have proved the existence of global solutions to the system of partial differential equations which describe ecological processes and effects of a very general predator-prey problem. They have also analyzed the solutions proving relevant qualitative properties.

Smoller and Conley, in a rather general paper on systems of nonlinear diffusion equations (see 1 below), have devised methods which enabled them to obtain global existence theorems for systems of nonlinear diffusion equations. They then apply these results to various systems arising in physics, chemistry and biology. In particular, they take into consideration (a) a system of equations which arises in the theory of semiconductors (see 5 below), (b) the equation of combustion kinetics (flame propagation, see 6 below); and (c) systems of equations describing certain general chemical and biological processes (see 7, 8 below).

Smoller has been working on weak solutions of systems of nonlinear hyperbolic partial differential equations (see10, 11 below). These include the equations of gas dynamics, chemical chromotography, nonlinear elasticity, nonlinear optics, and magnetohydrodynamics. In particular, Smoller has studied the solutions of the so called piston and double piston problems of gas dynamics (isothermal flow).

1. C. C. Conley and J. A. Smoller, Positively invariant regions for systems of nonlinear diffusion equations. To appear in Indiana Math. J.

2. C. C. Conley and J. A. Smoller, Remarks on traveling wave solutions to nonlinear diffusion equations. To appear.

3. J. B. Rauch and J. A. Smoller, Qualitative theory of the Fitzhugh-Nagumo equation. To appear in Advances in Mathematics.

4. E. D. Conway and J. A. Smoller, A predatory-prey problem. To appear.

5. Abraham and Tsuneto, Time variation of the Ginzberg-Landau order parameter. Phys. Rev. 152, 1966, 416-422.

6. Gavalas, Nonlinear diffusion equations of chemically reacting systems. Springer, New York, 1968.

7. Gelfand, Some problems in the theory of quasilinear equations, Uspehi. Mat. Nauk 14, 1959, 87-158.(in Russian).

8. Amundson, Nonlinear problems in chemical reactor theory. In Mathematical Aspects of Chemical and Biochemical Problems in Quantum Chemistry, SIAM-AMS Proc. Vol. 8, Amer. Math. Soc., Providence, 59-84.

9. T. Nishida and J. A. Smoller, Mixed problems for nonlinear conservation laws. To appear in J. Diff. Equations.

10. C. C. Conley and J. A. Smoller, Topological methods in the theory of shock waves. Proc. of Symposia in Pure Mathematics, Vol. 23, Partial Differential Equations, 1973, 293-302.

11. C. C. Conley and J. A. Smoller, On the structure of magnetohydrodynamic shock waves. Comm. Pure Appl. Math. 28, 1974, 367-375.

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During this period the nine scientists associated with the project produced 76 papers of which 35 have appeared in journals at this time. Many invited lectures, editorships and a chair in mathematics attest to the ability of the principal investigators. In the area of entrainment of frequency in the large involving oscillations of physical systems at resonance and across point of resonance 14 papers were produced. Pareto optimization theorems in optimal control theory account for three papers. Six papers on optical resonance of laser beams in crystals and 11 papers on evolution equations and related problems in shock

next page

20 Abstract

cont.

wave theory, semiconductors, diffusion, combustion kinetics and ecological systems give an idea of the diversity of areas addressed by the functional analytic techniques of Professors Ceasari, Smoller, Suryanarayana, Kannan and associates.