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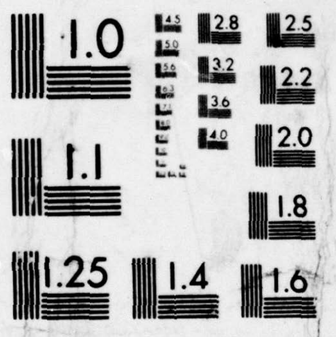

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ABSTRACT

The statistical testing of the sonar return data, consisting of individual stave outputs from several hydrophone arrays, has been delayed due to the inavailability of the data gathering equipment, notably the computer link between the CDC 3200 and the Honeywell DDP-516. This link has been in heavy use during the past quarter by the Signal Physics Division which is involved in the automatic classifier program. This equipment will become available during the next quarter and the stave data will be obtained in digital form.

The analysis and interpretation of the reverberation data will require attention to the envelope properties of the narrowband process. This report contains preliminary work required for such a study. The work was performed and reported at this time due to the delay in data collection.

THE COVARIANCE FUNCTION AND INTENSITY SPECTRUM OF THE ENVELOPE OF A NARROWBAND REVERBERATION PROCESS

by

T. D. Plemons

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I. INTRODUCTION

To detect and/or classify target echoes, the receiving systems of active sonars frequently extract and process, in some manner, the envelope of the received signal, which may be reverberation or reverberation plus the echo from a target. The analysis of the performance of these processors requires a minimum amount of information about the statistical descriptions of the reverberation envelope. To a considerable extent, the properties of the envelope are determined by the characteristics of the transmitted waveform. Thus, the covariance of the reverberation process is very similar to the autocorrelation function of the transmitted signal. The signals most commonly transmitted are the pulsed sinusoid (cw) and the linearly frequency modulated (FM) pulse. Determining the properties of the envelope of the reverberation processes generated by these two signal types is the goal of this study. In particular, we wish to know the covariances and intensity spectra of the fluctuating component of the envelope, since knowing these two second-order statistics is necessary for the analysis of a receiving system that makes decisions based on the extracted envelope.

This portion of the study is organized as follows. In Chapter II the narrowband reverberation process is discussed, with emphasis placed on the covariance and intensity spectrum. The material presented here is not new, but it is needed as an introduction and will be used, in Chapters III and IV, in the computation of the covariance and intensity spectrum of the fluctuating component of the envelope of the reverberation. In Chapter V, the results obtained thus far are summarized and future research in this direction will be discussed.

II. THE NARROWBAND REVERBERATION PROCESS

We begin by assuming that the reverberation process X(t), defined on the time (range) interval $I = (T_1, T_2)$, is narrowband (since the transmitted signals that generate X(t) are assummed to be narrowband) and Gaussianly distributed with zero means. Thus,

$$\langle X(t) \rangle = 0 \quad , \tag{2.1}$$

where <·> denotes the infinite ensemble average. Furthermore, we assume that X(t) is a *stationary* process over I, an assumption generally not true in practical situations. However, for reasons of analytical expedience, this assumption is made.

The narrowband process X(t) can be written as

$$X(t) = E(t)\cos\left[2\pi f_o t + \phi(t)\right] \quad , \tag{2.2}$$

where f_o, $\phi(t)$, and E(t) are, respectively, the center frequency, the phase, and the envelope of X(t). Since X(t) is narrowband, E(t) and $\cos\phi(t)$ are slowly varying functions when compared to $\cos 2\pi f_o t$.

By expanding Eq. (2.2), X(t) can be expressed as

$$X(t) = x_c(t)\cos 2\pi f_o t + x_s(t)\sin 2\pi f_o t \quad , \tag{2.3}$$

where

$$x_c(t) \equiv E(t)\cos\phi(t) \quad , \tag{2.4.a}$$

and

$$x_s(t) \equiv -E(t)\sin\phi(t) \tag{2.4.b}$$

are the cosine and sine quadrature components of X(t).

The covariance of X(t) is

$$K_X(t_1,t_2) \equiv \left\langle X(t_1)X(t_2)\right\rangle \quad . \tag{2.5}$$

When X(t) is stationary, then $K_X(t_1,t_2) = K_X(\tau)$, with $\tau = t_2 - t_1$. The mean intensity (variance) of X(t) is

$$\Psi \equiv K_X(t_1,t_1) = \left\langle X^2(t_1)\right\rangle = K_X(0) \quad , \tag{2.6}$$

which is independent of t under the assumption of stationarity.

If the quadrature representation of X(t) (Eqs. (2.4.a) and (2.4.b)) is used in Eq. (2.5), then the covariance can be written as (Ref. 1, Sec.9)

$$K_X(\tau) = \Psi\left[\rho_o(\tau)\cos 2\pi f_o\tau + \lambda_o(\tau)\sin 2\pi f_o\tau\right] \quad , \tag{2.7}$$

where

$$\Psi\rho_o(\tau) = \frac{1}{2}\left\langle x_c(t_1)x_c(t_2)\right\rangle + \frac{1}{2}\left\langle x_s(t_1)x_s(t_2)\right\rangle \quad , \tag{2.8.a}$$

and

$$\Psi\lambda_o(\tau) = -\frac{1}{2}\left\langle x_c(t_2)x_s(t_1)\right\rangle + \frac{1}{2}\left\langle x_c(t_1)x_s(t_2)\right\rangle \quad . \tag{2.8.b}$$

The covariance can also be written as

$$K_X(\tau) = K_o(\tau)\cos\left[2\pi f_o\tau + \Phi_o(\tau)\right] \quad , \tag{2.9}$$

where

$$K_o(\tau) = \Psi\left[\rho_o^{\,2}(\tau) + \lambda_o^{\,2}(\tau)\right]^{1/2} \quad , \tag{2.10}$$

and

$$\Phi_o(\tau) = \tan^{-1}\left[\lambda_o(\tau)/\rho_o(\tau)\right] \quad , \tag{2.11}$$

are the envelope and phase, respectively, of $K_X(\tau)$. From Eqs. (2.8.b) and (2.11) we see that $\Phi(0) = 0$, and, therefore, using Eq. (2.9) and Eq. (2.6),

$$K_X(0) = K_o(0) = \Psi \quad . \tag{2.12}$$

The normalized envelope of $K_X(\tau)$ is

$$k_o(\tau) \equiv \left[\rho_o^{\,2}(\tau) + \lambda_o^{\,2}(\tau)\right]^{1/2} \quad . \tag{2.13}$$

The maximum of $k_o(\tau)$ occurs at $\tau = 0$,

$$k_o(0) = 1 \quad . \tag{2.14}$$

III. THE FIRST AND SECOND MOMENTS OF THE ENVELOPE OF THE REVERBERATION

This study is primarily interested in the statistical description of the envelope E(t) (see Eq. (2.2)) of the reverberation process, X(t), whose partial description was outlined in Chapter II. The first and second moments of E(t) are (Ref. 2, Sec. 9.1)

$$\langle E(t) \rangle = \left(\frac{\pi}{2} \Psi \right)^{1/2} , \qquad (3.1)$$

and

$$\langle E(t)^2 \rangle = 2\Psi , \qquad (3.2)$$

where $\Psi = \langle X(t)^2 \rangle$ is the variance of X(t). Of interest here is the *fluctuating component* of E(t), which is the variation of E(t) about its mean, $\langle E(t) \rangle$, defined by

$$y(t) \equiv E(t) - \langle E(t) \rangle , \qquad (3.3)$$

with $\langle y(t) \rangle = 0$. The variance of y(t) is, using Eqs. (3.1) and (3.2),

$$\begin{aligned} \sigma_y^2 &= \langle y(t)^2 \rangle \\ &= \langle E^2(t) \rangle - \langle E(t) \rangle^2 \\ &= \left(2 - \frac{\pi}{2} \right) \Psi . \end{aligned} \qquad (3.4)$$

The covariance of $y(t)$ is

$$K_y(t_1,t_2) \equiv \left\langle y(t_1)y(t_2)\right\rangle \quad , \tag{3.5}$$

which, using Eq. (3.3), is

$$\begin{aligned} K_y(t_1,t_2) &= \left\langle E(t_1)E(t_2)\right\rangle - \left\langle E(t_1)\right\rangle \left\langle E(t_2)\right\rangle \\ &= K_E(t_1,t_2) - \frac{\pi}{2}\Psi \quad , \end{aligned} \tag{3.6}$$

where

$$K_E(t_1,t_2) \equiv \left\langle E(t_1)E(t_2)\right\rangle$$

is the covariance of $E(t)$. If we assume that $X(t)$ is a normal random process, then we can use a relationship previously obtained for $K_E(t_1,t_2)$ (Ref. 2, Sec. 9.1),

$$K_E(t_1,t_2) = \frac{\Psi\pi}{2}\, {}_2F_1(-1/2,-1/2;1,k_o^{\,2}) \quad , \tag{3.7}$$

where ${}_2F_1$ is a hypergeometric function with the argument $k_o(t_1,t_2) = k_o(\tau)$ (see Eq. (2.13)).* A general series form of ${}_2F_1$ is

$${}_2F_1(\alpha,\beta;\gamma;x) = 1 + \frac{\alpha\beta}{\gamma}\frac{x}{1!} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)2!}x^2 + \ldots \quad . \tag{3.8}$$

Assuming that the first two terms of this series accurately represent ${}_2F_1$ we find, with Eqs. (3.6), (3.7), (3.8) that

$$K_y(t_1,t_2) = K_y(\tau) = \frac{\pi}{8}\Psi k_o^{\,2}(\tau) \ , \ \tau = t_2 - t_1 \quad ; \tag{3.9}$$

*A key assumption used in the derivation of Eq. (3.7) is that the envelope $E(t)$ has a Rayleigh first-order probability density function; we also make this assumption.

that is, the covariance function of the fluctuating component of the envelope of the reverberation is proportional to the square of the envelope of the covariance function of the reverberation process X(t). This result, with comparisons to experimental data, is given by Ol'shevskii in Ref. 3, Ch. IV.

The expression for $K_y(\tau)$, Eq. (3.9), is approximate since only the first two terms of the series of Eq. (3.8) were retained. To determine the amount of energy lost in this approximation we note that the time variance of y(t) is given by Eq. (3.4),

$$\sigma_y^2 = (2 - \frac{\pi}{2})\Psi \simeq 0.43\ \Psi \quad ,$$

whereas our approximation of σ_y^2 is (Eq. (3.9))

$$K_y(0)\Big|_{approx} = \frac{\pi}{8}\Psi = 0.39\Psi = 0.91\ \sigma_y^2 \quad .$$

Therefore, about 10% of the energy of y(t) is contained in the terms following the second of the series of Eq. (3.8).

IV. THE INTENSITY SPECTRUM OF THE FLUCTUATING COMPONENT OF THE ENVELOPE

Applying the Wiener-Khintchine (W-K) theorem (Ref. 2, p. 144), the intensity spectrum of the fluctuating component (see Eq. (3.3)) can be obtained from the covariance, $K_y(\tau)$, of $y(t)$:

$$W_y(f) = 4\int_0^\infty K_y(\tau)\cos \omega\tau d\tau, \quad \omega = 2\pi f \quad . \tag{4.1}$$

Using the approximation Eq. (3.9) for $K_y(\tau)$, the intensity spectrum of $y(t)$ is

$$W_y(f) = \frac{\Psi\pi}{2}\int_0^\infty k_o^2(\tau)\cos \omega\tau d\tau \quad . \tag{4.2}$$

The explicit form of $k_o(\tau)$ is determined mainly by the characteristics of the waveforms of the transmitted signals that generate the reverberation process $X(t)$. The covariance $K_X(t_1,t_2)$ is approximately equivalent to the autocorrelation function of the transmitted signal (Ref. 4, Ch. V)

$$C(\tau) \equiv \int_{-\infty}^\infty S(t)S(t+\tau)dt \quad , \tag{4.3}$$

where $S(t)$ is the narrowband, transmitted signal. Under certain conditions, such as no scatterer Doppler or platform motion, local stationarity, etc., we can write (within a scale factor)

$$K_X(\tau) = C(\tau) \quad . \tag{4.4}$$

When $S(t)$ is narrowband, with center frequency f_o, then the autocorrelation function can be written as (Ref. 4, Sec. V)

$$C(\tau) = C_o(\tau)\cos\left[2\pi f_o\tau + \xi_o(\tau)\right] \quad , \qquad (4.5)$$

where $C_o(\tau)$ and $\xi_o(\tau)$ are the envelope and phase, respectively, of $C(\tau)$. Comparing Eqs. (2.4), (2.5), and (1.7), we have

$$k_o(\tau) = C_o(\tau) \quad , \qquad (4.6)$$

$$W_y(f) = \frac{\Psi\pi}{2}\int_o^\infty C_o^2(\tau)\cos \omega\tau d\tau \quad . \qquad (4.7)$$

Before proceeding, the transmitted signal must be specified. Currently the two types of signals that are most often transmitted by an active sonar are the cw pulse and the linearly frequency modulated (FM) waveform. Each of these will be considered in the next progress report.

V. CONCLUDING REMARKS

With the necessary first- and second-order statistics of the reverberation envelope we can go further and determine the properties of the reverberation envelope that has been passed through various types of linear systems. One problem to be solved is that of determining the statistical properties of the envelope that has been subjected to a *smoothing* process, i.e., averaged with a fixed-length, lagged time window. Of considerable interest here is the comparison of the FM and cw envelope processes that have been subjected to the same linear system. Since the input spectra of these two processes are different, it can be assumed that the corresponding output spectra will be different. In the case of smoothing, a variable parameter is the integration time which, when increased, results in a decrease in the spectral widths of the envelope processes, and hence a decrease in the background reverberation level in the sonar receiver.

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