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Analytical expressions are developed which relate the translational and angular accelerations, velocities, and displacements of a body with the outputs of six correctly placed internal motion-sensing transducers. These expressions are in a form that is easily converted into computer programs for analytis of data taken by motion-sensing transducers in any sort of free vehicle. Thus, they are appropriate for use in experimental studies in which the motion of torpedoes, missiles, etc. is to be measured.

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NOTATION

$\vec{A}$	Kinematic acceleration of the origin of a moving coordinate system
$\vec{A}_B$	Kinematic acceleration of point B
A_{Bx}, A_{By}, A_{Bz}	Components of $\vec{A}_B$ along the (x,y,z) axes
a	Acceleration
a_i	Output of accelerometer "number i"
a_g	Gravitational component of an accelerometer's output
a_k	Kinematic component of an accelerometer's output
a_x, a_y, a_z	Components of $\vec{A}$ along the (x,y,z) axes
B	A point in space
$\vec{b}$	Position vector of B in fixed coordinate system
$\vec{b}'$	Position vector of B in moving coordinate system
c	Spring constant
d	Displacement of a spring from its rest position
$\hat{e}_x, \hat{e}_y, \hat{e}_z$	Unit vectors along the (x,y,z) axes
$\hat{e}_{axis}$	Unit vector along a given axis
g	Gravitational constant
$\vec{g}$	Gravitational acceleration vector
$\vec{g}_a$	Gravitational effect on an accelerometer
g_{ax}, g_{ay}, g_{az}	Components of $\vec{g}_a$ along the (x,y,z) axes
g_x, g_y, g_z	Components of $\vec{g}$ along the (x,y,z) axes
g_{xo}, g_{yo}, g_{zo}	Components of $\vec{g}$ along the (x_o, y_o, z_o) axes
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors along the fixed (x_o, y_o, z_o) axes

ℓ_i	Longitudinal distance from body CG to accelerometer a_i
$\vec{m}$	Position vector of the origin of the moving coordinate system
O	Origin of the fixed coordinate system (x_o, y_o, z_o)
O'	Origin of the moving coordinate system (x, y, z)
p, q, r	Components of angular velocity, $\vec{\omega}$, along the (x, y, z) axes
[R]	Transformation from coordinates (x_o, y_o, z_o) to (x, y, z)
$[R^{-1}]$	Transformation from coordinates (x, y, z) to (x_o, y_o, z_o)
$[R_g]$	Transformation from coordinates (x_o, y_o, z_o) to (x_g, y_g, z_g)
$[R_o]$	Transformation [R] at $t = 0$
$[R_\phi], [R_\theta], [R_\psi]$	Transformations causing rotation of angles ϕ, θ, and ψ about the x,y,z axes, respectively
t	Time
$\vec{U}$	Translational velocity of O'
u, v, w	Components of $\vec{U}$ along the (x, y, z) axes
x, y, z	Moving coordinates
x_o, y_o, z_o	Fixed coordinates
x_1, y_1, z_1	Coordinate system rotated by an angle ψ about the z_o-axis from (x_o, y_o, z_o)
x_2, y_2, z_2	Coordinate system rotated by an angle θ about the y_1-axis from (x_1, y_1, z_1)
x_B, y_B, z_B	Components of $\vec{b}'$ along the (x, y, z) axes
x_g, y_g, z_g	Coordinate system fixed such that the z_g-axis is parallel to $\vec{g}$
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle

ϕ_o	Roll angle at $t = 0$
θ_o	Pitch angle at $t = 0$
ψ_o	Yaw angle at $t = 0$
ϕ_g, θ_g, ψ_g	Euler angles between coordinate systems (x_g, y_g, z_g) and (x_o, y_o, z_o)
$\vec{\omega}$	Angular velocity vector

ABSTRACT

Analytical expressions are developed which relate the translational and angular accelerations, velocities, and displacements of a body with the outputs of six correctly placed internal motion-sensing transducers. These expressions are in a form that is easily converted into computer programs for analysis of data taken by motion-sensing transducers in any sort of free vehicle. Thus they are appropriate for use in experimental studies in which the motion of torpedoes, missiles, etc. is to be measured.

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INTRODUCTION

Over the past several years, the David W. Taylor Naval Ship Research and Development Center (DTNSRDC) has been requested to perform model launching studies on a variety of torpedoes and missiles. Performance of these experiments has required the development of techniques for measuring the motion of rigid bodies in up to six degrees of freedom. Two main approaches to this problem have been tried.

One approach is to make the measurements photographically. Films taken by a stereoscopic pair of synchronized, high-speed motion picture cameras may, in principle, be analyzed photogrammetrically to yield a complete time history of the motion of a body. This technique has the advantage of requiring no attachments of any kind to the body itself. The main disadvantage of this technique is that the analysis involved is quite laborious and time consuming. Also, experience has shown that extreme care must be used in setting up and calibrating the required systems. Photographic measurement techniques have been used at DTNSRDC with some success and are still being developed.

The alternative to photographic measurement is to equip the body with internal motion-sensing transducers. This technique is viable if the model is large enough to contain the transducers and if the physical situation allows the use of a trailing unbilical cable for telemetering the transducer signals. This report is concerned with this second technique.

Complete determination of the motion of the body, i.e., all six components of motion, requires at least six transducers. The most common type of motion-sensing transducer is the accelerometer. Physical size limitations and other considerations, however, often preclude the use of accelerometers for the measurement of all components of motion. Other types of motion sensors include angular rate gyros and angular accelerometers. However, the use of gyros or other moving mass devices is often undesirable in model testing because a moving mass may significantly alter the desired mass distribution or impart undesired stability to the model. A new type of electrofluidic angular rate transducer has recently become available commercially.[1] Since this device does not utilize a moving mass, it does not exhibit the undesirable characteristics (for this type of work) of rate gyros. These devices, as well as accelerometers, are available in sizes small enough to be practical for use in models.

Several combinations of accelerometers and angular rate sensors can be used to determine the motion of a body. The particular combination chosen would depend on the availability of the various types of transducers. The useful combinations are (1) three accelerometers and three angular rate sensors and (2) five accelerometers and one angular rate sensor.

Data analysis procedures are needed for converting the outputs of motion-sensing transducers into quantities which describe the motion of a body in a reasonable and straightforward way. Thus, the equations of motion must be put into a form which makes obvious the dependence of those parameters normally used to describe motion on those quantities which are measurable with transducers. Furthermore, the types of mathematical relationships developed for this purpose should be easily converted into software for use on high-speed digital computers. Since no readily available references were found containing relationships of this type, this report develops the equations of motion from this point of view and presents analytical relationships appropriate for reduction of data acquired by two useful transducer combinations.

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[1] Humphrey, Inc., "A Solid State Angular Rate Sensor," Brochure BR0064-272 (undated).

GENERAL EQUATIONS OF MOTION

Although the equations of motion for a rigid body are well known, it is useful to derive them from basic principles in order to gain physical and mathematical insight into the problem. While going through the derivation, it is important to keep in mind that the purpose is to develop equations for converting measurable quantities into useful information. Thus, we should be on the lookout for directly measurable quantities in the equations.

The problem involves two coordinate systems. One system, with axes x_0, y_0, and z_0 and unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, respectively, is fixed in space. The second coordinate system, which shall be called prime ('), is moving with respect to the fixed system. The prime coordinate system has axes x, y, and z with unit vectors $\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$, respectively. Time derivatives taken in the prime coordinate system will be denoted by d'/dt. Derivatives taken in the fixed system will be denoted in the usual way.

In the fixed coordinate system, let $\vec{m}$ be the position vector of the origin 0' of the moving coordinate system. See Figure 1. The translational velocity of 0' is given by

$$\frac{d\vec{m}}{dt} = \vec{U} = u\hat{e}_x + v\hat{e}_y + w\hat{e}_z \qquad [1]$$

The translational acceleration of 0' is given by

$$\frac{d^2\vec{m}}{dt^2} = \vec{A} = a_x\hat{e}_x + a_y\hat{e}_y + a_z\hat{e}_z \qquad [2]$$

The rotational motion of the prime system can be described by the angular velocity vector $\vec{\omega}$ where

$$\vec{\omega} = p\hat{e}_x + q\hat{e}_y + r\hat{e}_z \qquad [3]$$

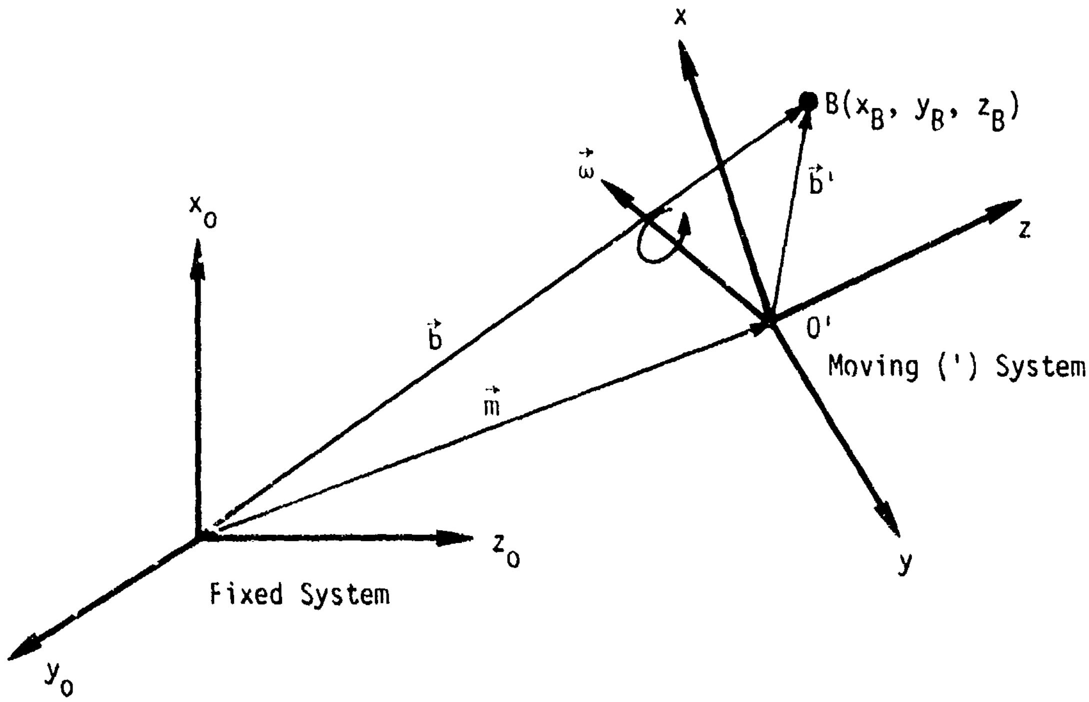

Figure 1 - Relation between Fixed and Moving Coordinate Systems

Consider a point B with coordinates x_B, y_B, z_B in the prime coordinate system. Its position vector in the prime system is

$$\vec{b}' = x_B\hat{e}_x + y_B\hat{e}_y + z_B\hat{e}_z \qquad [4]$$

The position vector of B in the fixed system is $\vec{b}$. It is easily seen in Figure 1 that

$$\vec{b} = \vec{b}' + \vec{m},$$

and, therefore, that

$$\frac{d^2\vec{b}}{dt^2} = \frac{d^2}{dt^2}[\vec{b}' + \vec{m}] = \frac{d^2\vec{b}'}{dt^2} + \frac{d^2\vec{m}}{dt^2} \qquad [5]$$

If the first term of [5] is examined more closely, it is seen that

i.e.,
$$\frac{d^2\vec{b}'}{dt^2} = \frac{d}{dt}\left[\frac{d'\vec{b}'}{dt} + \vec{\omega} \times \vec{b}'\right]$$

$$\frac{d^2\vec{b}'}{dt^2} = \frac{d'^2\vec{b}'}{dt^2} + \vec{\omega} \times (\vec{\omega} \times \vec{b}') + 2\vec{\omega} \times \frac{d'\vec{b}'}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{b}' \qquad [6]$$

Substituting [6] into [5] yields

$$\frac{d^2\vec{b}}{dt^2} = \frac{d'^2\vec{b}'}{dt^2} + \vec{\omega} \times (\vec{\omega} \times \vec{b}') + 2\vec{\omega} \times \frac{d'\vec{b}'}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{b}' + \frac{d^2\vec{m}}{dt^2} \qquad [7]$$

Equation [7] is the vector equation of motion for a point on a rigid body and is often called the Coriolis theorem. The term $\vec{\omega} \times (\vec{\omega} \times \vec{b}')$ is the "centripetal acceleration", and the term $2\vec{\omega} \times d'\vec{b}'/dt$ is the "Coriolis acceleration."

If the point B is assumed fixed with respect to the prime coordinate system, then

$$\frac{d'^2\vec{b}'}{dt^2} = 0$$

and

$$2\vec{\omega} \times \frac{d'\vec{b}'}{dt} = 0$$

Therefore, Equation [7] reduces to

$$\frac{d^2\vec{b}}{dt^2} = \vec{\omega} \times (\vec{\omega} \times \vec{b}') + \frac{d\vec{\omega}}{dt} \times \vec{b}' + \frac{d^2\vec{m}}{dt^2} \qquad [8]$$

If the first term of [8] is evaluated in terms of its components, then

$$\begin{aligned}\vec{\omega} \times (\vec{\omega} \times \vec{b}') = {} & [y_B(pq) - x_B(q^2 + r^2) + z_B(pr)]\ \hat{e}_x \\ & + [z_B(qr) - y_B(r^2 + p^2) + x_B(pq)]\ \hat{e}_y \\ & + [x_B(pr) - z_B(p^2 + q^2) + y_B(qr)]\ \hat{e}_z \end{aligned} \qquad [9]$$

To evaluate the second term of [8], note that

$$\frac{d\vec{\omega}}{dt} = \dot{p}\ \hat{e}_x + \dot{q}\ \hat{e}_y + \dot{r}\ \hat{e}_z$$

Then in terms of components

$$\frac{d\vec{\omega}}{dt} \times \vec{b}' = (\dot{q}z_B - \dot{r}y_B)\ \hat{e}_x + (\dot{r}x_B - \dot{p}z_B)\ \hat{e}_y + (\dot{p}y_B - \dot{q}x_B)\ \hat{e}_z \qquad [10]$$

Substituting [9] and [10] into [8] yields

$$\begin{aligned} \frac{d^2\vec{b}}{dt^2} = & [y_B(pq - \dot{r}) + z_B(pr + \dot{q}) - x_B(q^2 + r^2)]\ \hat{e}_x \\ & + [z_B(qr - \dot{p}) + x_B(pq + \dot{r}) - y_B(r^2 + p^2)]\ \hat{e}_y \\ & + [x_B(pr - \dot{q}) + y_B(qr + \dot{p}) - z_B(p^2 + q^2)]\ \hat{e}_z \\ & + \frac{d^2\vec{m}}{dt^2} \end{aligned} \qquad [11]$$

If Equation [2] is substituted into [11], then

$$\begin{aligned} \frac{d^2\vec{b}}{dt^2} = & [a_x + y_B(pq - \dot{r}) + z_B(rp + \dot{q}) - x_B(q^2 + r^2)]\ \hat{e}_x \\ & + [a_y + z_B(rq - \dot{p}) + x_B(pq + \dot{r}) - y_B(p^2 + r^2)]\ \hat{e}_y \\ & + [a_z + x_B(pr - \dot{q}) + y_B(qr + \dot{p}) - z_B(p^2 + q^2)]\ \hat{e}_z \end{aligned} \qquad [12]$$

Now consider the left side of Equation [12]. The term $d^2\vec{b}/dt^2$ is the acceleration of point B relative to the origin of the fixed coordinate system. The term $d^2\vec{b}/dt^2$ is expressed as follows:

$$\frac{d^2\vec{b}}{dt^2} = \vec{A}_B = A_{Bx}\hat{e}_x + A_{By}\hat{e}_y + A_{Bz}\hat{e}_z \qquad [13]$$

where A_{Bx}, A_{By}, and A_{Bz} are the components of A_B along the axes of the prime coordinate system.

Substituting [13] into [12] yields the following system of equations:

$$A_{Bx} = a_x + y_B(pq - \dot{r}) + z_B(rp + \dot{q}) - x_B(q^2 + r^2)$$

$$A_{By} = a_y + z_B(rq - \dot{p}) + x_B(pq + \dot{r}) - y_B(p^2 + r^2) \qquad [14]$$

$$A_{Bz} = a_z + x_B(pr - \dot{q}) + y_B(qr + \dot{p}) - z_B(p^2 + q^2)$$

These expressions relate A_{Bx}, A_{By}, and A_{Bz}, the prime system components of the total kinematic acceleration of point B with respect to the fixed coordinate system, with the acceleration of the moving origin O', and the components of the angular velocity vector. The quantities A_{Bx}, A_{By}, A_{Bz} are measurable with accelerometers. The components of angular velocity - p, q, and r - can be measured with angular rate sensors. Thus, Equations [14] are a useful formulation of this problem for the purpose of measuring motion.

If desired, the derivation can be carried one step beyond [14]. If Equation [1] is recalled, then

$$\frac{d\vec{m}}{dt} = \vec{U} = u\hat{e}_x + v\hat{e}_y + w\hat{e}_z$$

Then

$$\frac{d^2\vec{m}}{dt^2} = \frac{d\vec{U}}{dt} = \dot{u}\hat{e}_x + u(\vec{\omega} \times \hat{e}_x) + \dot{v}\hat{e}_y + v(\vec{\omega} \times \hat{e}_y)$$

$$+ \dot{w}\hat{e}_z + w(\vec{\omega} \times \hat{e}_z)$$

$$= \dot{u}\hat{e}_x + \dot{v}\hat{e}_y + \dot{w}\hat{e}_z + u(r\hat{e}_y - q\hat{e}_z)$$

$$+ v(p\hat{e}_z - r\hat{e}_x) + w(q\hat{e}_x - p\hat{e}_y)$$

$$\frac{d^2\vec{m}}{dt^2} = [\dot{u} - vr + qw]\,\hat{e}_x + [\dot{v} + ur - wp]\,\hat{e}_y + [\dot{w} - uq + vp]\,\hat{e}_z \qquad [15]$$

The components of $d^2\vec{m}/dt^2$ in Equation [15] are equal to a_x, a_y, and a_z, respectively. Substituting into Equation [14] results in

$$\begin{aligned} A_{Bx} &= \dot{u} - vr + qw + y_B(pq - \dot{r}) + z_B(rp + \dot{q}) - x_B(q^2 + r^2) \\ A_{By} &= \dot{v} + ur - wp + z_B(rq - \dot{p}) + x_B(pq + \dot{r}) - y_B(p^2 + r^2) \\ A_{Bz} &= \dot{w} - uq + vp + x_B(pr - \dot{q}) + y_B(qr + \dot{p}) - z_B(p^2 + q^2) \end{aligned} \qquad [16]$$

Equations [16] are a standard formulation of this problem. For measurement purposes, however, Equations [14] are a more direct and useful formulation.

EULERIAN ANGLES

ANGULAR RELATIONSHIP OF THE TWO COORDINATE SYSTEMS

In the above discussion, all vectors were defined in the moving coordinate system. It is both useful and necessary to express vectors in the fixed coordinate system. In order to do this, the angular relationship between the two coordinate systems must be known. Several systems of angles, known as Eulerian angles, exist which describe this relationship. The system which will be used here follows the conventions prescribed by the Society of Naval Architects and Marine Engineers.[2]

The Eulerian angles are defined by three successive rotations. The order in which the rotations is done is significant and is part of the definition of any set of Eulerian angles.

Consider a fixed coordinate system x_0, y_0, z_0. A rotation by an angle ψ about the z_0-axis will result in a new set of axes x_1, y_1, z_1. This rotation can be accomplished mathematically by a transformation $[R_\psi]$

---

[2]Society of Naval Architects and Marine Engineers, "Nomenclature for Treating the Motion of a Submerged Body through a Fluid," SNAME Technical and Research Bulletin 1-5 (Apr 1950).

whose components are the direction cosines relating the axes:

$$[R_\psi] = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [17]$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = [R_\psi] \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

Note that the transformation $[R_\psi]$ does not affect the z_o-axis. Thus,

$$z_1 = z_0$$

Now, consider a rotation $[R_\theta]$ of an angle θ about the y_1-axis, resulting in a new set of axes x_2, y_2, z_2.

$$[R_\theta] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \qquad [18]$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = [R_\theta] \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

Note that $[R_\theta]$ does not affect the y_1-axis. Thus,

$$y_2 = y_1$$

Finally, consider a rotation $[R_\phi]$ of an angle ϕ about the x_2-axis resulting in a set of axes x, y, z.

$$[R_\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \qquad [19]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R_\phi] \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

Note that $[R_\phi]$ does not affect the x_2-axis. Thus,

$$x = x_2$$

These three rotations define the Eulerian angles ϕ, θ, and ψ which are called the angles of roll, pitch, and yaw, respectively. The total transformation [R] from the fixed axes to the rotated axes is the result of successively performing $[R_\psi]$, $[R_\theta]$, and $[R_\phi]$. That is,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R] \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad [20]$$

where

$$[R] = [R_\phi]\,[R_\theta]\,[R_\psi]$$

Performing the indicated matrix multiplications results in

$$[R] = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ (-\sin\psi\cos\phi + \cos\psi\sin\phi\sin\theta) & (\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) & \cos\theta\sin\phi \\ (\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta) & (-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi) & \cos\theta\cos\phi \end{bmatrix} \quad [21]$$

Since $[R_\psi]$, $[R_\theta]$, and $[R_\phi]$ are direction cosine matrices, they and hence $[R]$ are orthogonal. Therefore, $[R^{-1}]$, the inverse of $[R]$ is equal to the transpose of $[R]$.

$$[R^{-1}] = \begin{bmatrix} \cos\theta\cos\psi & (-\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi) & (\sin\psi\sin\phi + \sin\theta\cos\psi\cos\phi) \\ \cos\theta\sin\psi & (\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) & (-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi) \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad [22]$$

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = [R^{-1}] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

RELATION WITH ANGULAR VELOCITY

Since the Eulerian angles are needed for some calculations, they must be determined from some measurable quantity, such as the angular velocity. Thus, the relation is needed between ϕ, θ, and ψ (and their derivatives) and the components of $\vec{\omega}$, namely p, q, and r.

Consider $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$, the first time derivatives of the Eulerian angles. These may be expressed as vectors with directions along the respective axes of rotation:

$$\vec{\dot{\phi}} = \dot{\phi}\hat{e}_{x_2}$$
$$\vec{\dot{\theta}} = \dot{\theta}\hat{e}_{y_1}$$
$$\vec{\dot{\psi}} = \dot{\psi}\hat{e}_{z_0}$$

where $\hat{e}_{axis}$ indicates the unit vector along the appropriate axis. It can be seen from [17], [18], and [19] that

$$\hat{e}_{x_2} = \hat{e}_x$$

$$\hat{e}_{y_1} = \hat{e}_{y_2}$$

$$\hat{e}_{z_0} = \hat{k} = \hat{e}_{z_1}$$

$$\vec{\dot{\phi}} = \dot{\phi}\hat{e}_{x_2}$$

$$\vec{\dot{\theta}} = \dot{\theta}\hat{e}_{y_2} \qquad [23]$$

$$\vec{\dot{\psi}} = \dot{\psi}\hat{e}_{z_1}$$

The vectors $\vec{\dot{\phi}}$, $\vec{\dot{\theta}}$, and $\vec{\dot{\psi}}$ are a set of components of $\vec{\omega}$. However, they are generally not mutually orthogonal and cannot be transformed to the prime coordinate system (x, y, z) by one of the orthogonal transformations. However, notice that $\vec{\dot{\phi}}$ and $\vec{\dot{\theta}}$ are directed along the x_2 and y_2 axes, respectively. Therefore, if $\vec{\dot{\psi}}$ can be resolved along the axes of the coordinate system (x_2, y_2, z_2), an orthogonal vector triad exists as components of $\vec{\omega}$ in the (x_2, y_2, z_2) system. In the coordinate system (x_1, y_1, z_1), the vector $\vec{\dot{\psi}}$ can be expressed

$$\vec{\dot{\psi}} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

If the transformation $[R_\theta]$ (Equation [18]) is used to transform $\vec{\dot{\psi}}$ to the system (x_2, y_2, z_2), then

$$[R_\theta] \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = -\dot{\psi}\sin\theta\ \hat{e}_{x_2} + \dot{\psi}\cos\theta\ \hat{e}_{z_2} \qquad [24]$$

thus

$$\vec{\omega} = (\dot{\phi} - \dot{\psi}\sin\theta)\hat{e}_{x_2} + \dot{\theta}\hat{e}_{y_2} + \dot{\psi}\cos\theta\hat{e}_{z_2} \qquad [25]$$

Equation [25] expresses $\vec{\omega}$ in terms of its orthogonal components in the (x_2, y_2, z_2) coordinate system. If the transformation $[R_\phi]$, Equation [19], is applied to Equation [25], the result is

$$\begin{aligned}\vec{\omega} &= [R_\phi] \begin{bmatrix} \dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\theta} \\ \dot{\psi}\cos\theta \end{bmatrix} \\ &= (\dot{\phi} - \dot{\psi}\sin\theta)\hat{e}_x \\ &+ (\dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi)\hat{e}_y \\ &+ (\dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi)\hat{e}_z\end{aligned}$$

Thus

$$\begin{aligned} p &= \dot{\phi} - \dot{\psi}\sin\theta \\ q &= \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ r &= \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{aligned} \qquad [26]$$

Equations [26] can be manipulated algebraically into the form

$$\begin{aligned} \dot{\phi} &= p + (q\sin\phi + r\cos\phi)\tan\theta \\ \dot{\theta} &= q\cos\phi + r\sin\phi \\ \dot{\psi} &= (q\sin\phi + r\cos\phi)/\cos\theta \end{aligned} \qquad [27]$$

These equations are simultaneous differential equations relating the time derivatives of the Eulerian angles with the components of the angular velocity vector. They may be solved by numerical techniques to yield values for the Eulerian angles.

APPLICATION OF THE GENERAL EQUATIONS

Now that the general equations of motion have been developed for a point fixed on a rigid body, let us consider how the general equations can be used to determine, from measurements, a complete time history of the motion of the body.

DEFINITION OF COORDINATE SYSTEMS

Define two coordinate systems: one attached to and moving with the body and one fixed with respect to the reference frame, the earth.

Most bodies which are likely to be subject to an investigation of this type - projectiles, airplanes, missiles, torpedoes, etc. - have a definite preferred orientation. That is, there is a definite "fore and aft," "port and starboard," etc. associated with the body shape. And even if a preferred orientation were not obvious, one could arbitrarily define the directions forward, aft, port, and starboard on the body. Once this orientation is established, the coordinate system (x, y, z) - called the body axes - can be defined.

Hydrodynamic conventions for submerged bodies,[2] which will be followed here, call for the origin of the body axes to be located at the body center of gravity (CG). The x-axis is directed forward, the y-axis to starboard, and the z-axis transverse to the x- and y-axes such that the coordinate system is right-handed. Thus, for an airplane flying parallel to the ground, the z-axis is directed downward.

The coordinate system (x_o, y_o, z_o) - called the ground axes - is fixed with respect to the earth; it can be defined in any of several ways depending on the problem. It will often be convenient to define the fixed coordinate system such that the body axes and the ground axes are initially coincident. In general, determination of the relation between the fixed and moving coordinate systems can be based on the initial position in the ground axis coordinate system of the body CG and on initial values - ϕ_o, θ_o, and ψ_o - of the Eulerian angles of roll, pitch, and yaw.

ALIGNMENT OF TRANSDUCERS

Now consider the body and its internal transducers. Two desirable combinations of six transducers have previously been stated as (1) three accelerometers and three angular rate transducers and (2) five accelerometers and one angular rate sensor. The analyses for these two configurations will naturally differ slightly, but the general approach is the same for either case.

Combination (1): Three Accelerometers and Three Rate Sensors

If Combination (1) is used, the components of angular velocity along the body axes, p, q, and r can be measured directly. In order to accomplish this, the angular rate sensors must be aligned such that the sensitive axis of one rate sensor is parallel to each axis of the body coordinate system (x, y, z). Similarly, the accelerometers should be aligned such that the sensitive direction of one accelerometer is parallel to each of the body axes. Referring to Equations [14] and recognizing that each accelerometer measures the total acceleration of the point at which it is located in its sensitive direction, it can be seen that it would be most desirable to locate all of the accelerometers at the CG of the body. Although this is a physical impossibility, it is usually possible to locate the accelerometers such that their points of action are all on the longitudinal axis. This situation will eliminate two terms from each of Equations [14].

If three accelerometers are placed in the body as shown in Figure 2, an accelerometer a_1 will be directed in the x-direction at coordinates $(\ell_1, 0, 0)$ of the body system. Accelerometer a_2 is directed in the y-direction at coordinates $(\ell_2, 0, 0)$ and accelerometer a_3 is directed in the z-direction at coordinates $(-\ell_3, 0, 0)$. In this situation Equation [14] can be reduced to

$$
\begin{aligned}
a_1 &= a_x - \ell_1(q^2 + r^2) + g_x \\
a_2 &= a_y + \ell_2(pq + \dot{r}) + g_y \\
a_3 &= a_z - \ell_3(pr - \dot{q}) + g_z
\end{aligned}
\qquad [28]
$$

where g_x, g_y, and g_z are the components of gravitational acceleration felt by the accelerometers along the x-, y-, and z- axes, respectively.

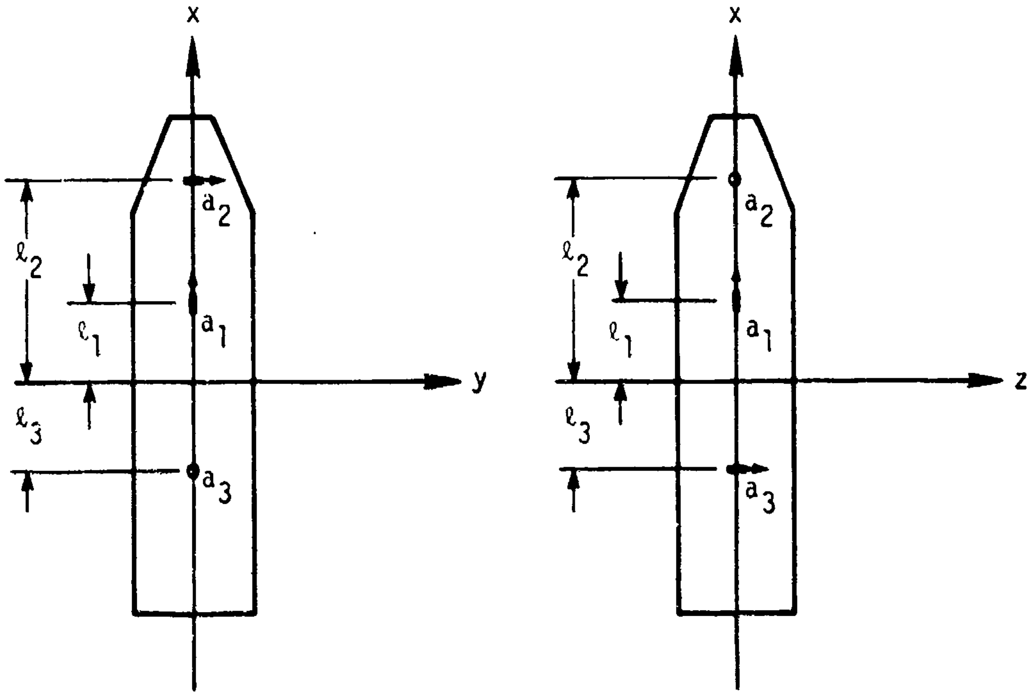

Figure 2 - Alignment of Three Accelerometers Inside a Model

Since p, q, and r are known at all times from direct measurement, $\dot{p}$, $\dot{q}$, and $\dot{r}$ can be determined numerically. Thus, if it is assumed that somehow g_x, g_y, and g_z can be calculated, Equations [28] can be converted into expressions for the components a_x, a_y, and a_z of the translational acceleration of the CG in terms of known quantities:

$$a_x = (a_1 - g_x) + \ell_1(q^2 + r^2)$$

$$a_y = (a_2 - g_y) - \ell_2(pq + \dot{r}) \qquad [29]$$

$$a_z = (a_3 - g_z) + \ell_3(pr - \dot{q})$$

Combination (2): Five Accelerometers and One Rate Sensor

If the model is instrumented with Combination (2), the rate sensor would be used to measure p, the component of the angular velocity in the x-direction. The other two angular velocity components, q and r, must be determined from the outputs of the accelerometers. If the accelerometers are located inside the model as shown in Figure 3, a_1 is directed along the x-axis at coordinates $(\ell_1, 0, 0)$ of the body system. Accelerometers a_2 and a_4 are directed parallel to the y-axis at coordinates $(\ell_2, 0, 0)$ and $(-\ell_4, 0, 0)$, respectively; a_3 and a_5 are directed parallel to the z-axis at coordinates $(\ell_3, 0, 0)$ and $(-\ell_5, 0, 0)$, respectively. For this configuration, Equations [14] become

$$
\begin{aligned}
&(a) \quad a_1 = a_x - \ell_1(q^2 + r^2) + g_x \\
&(b) \quad a_2 = a_y + \ell_2(pq + \dot{r}) + g_y \\
&(c) \quad a_3 = a_z + \ell_3(pr - \dot{q}) + g_z \\
&(d) \quad a_4 = a_y - \ell_4(pq + \dot{r}) + g_y \\
&(e) \quad a_5 = a_z - \ell_5(pr - \dot{q}) + g_z
\end{aligned}
\qquad [30]
$$

where g_x, g_y, and g_z are the components of gravitational acceleration felt by the accelerometers along the x-, y-, and z-axes, respectively. Subtracting [30d] from [30b] results in

$$a_2 - a_4 = (\ell_2 + \ell_4)(pq + \dot{r})$$

or

$$\dot{r} = (a_2 - a_4)/(\ell_2 + \ell_4) - pq \qquad [31]$$

Subtracting [30e] from [30c] results in

$$a_3 - a_5 = (\ell_3 + \ell_5)(pr - \dot{q})$$

or [32]

$$\dot{q} = (a_5 - a_3)/(\ell_3 + \ell_5) + pr$$

Since p is known from direct measurement, Equations [31] and [32] are a pair of simultaneous differential equations in two unknowns, q and r. These equations can be solved for q and r by a numerical method such as the Runge-Kutta technique.[3]

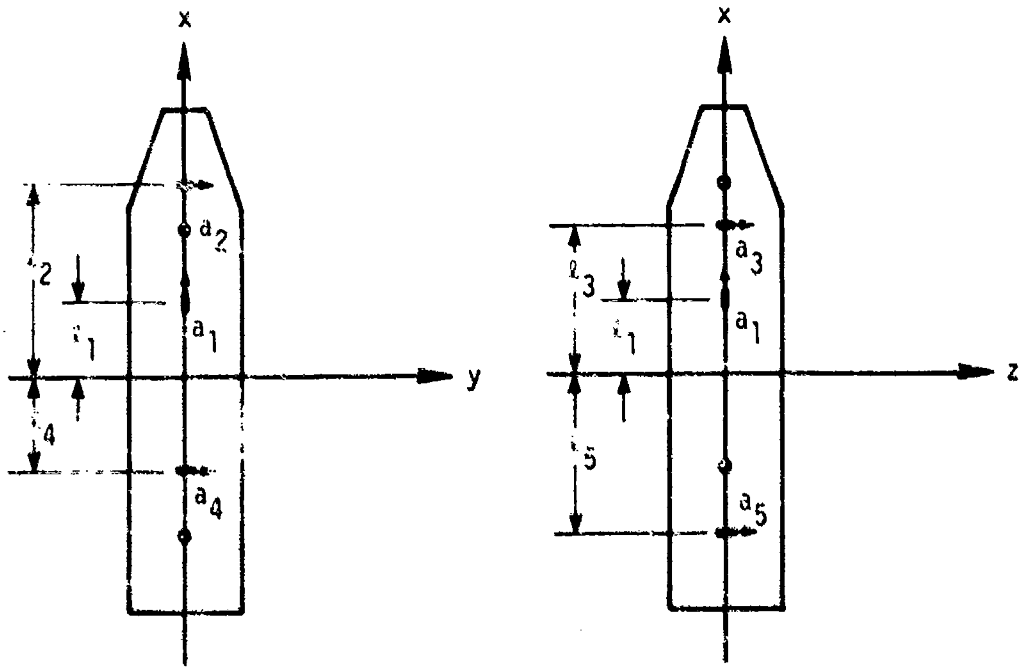

Figure 3 - Arrangement of Five Accelerometers Inside a Model

[3] Mikhlin, S. G. and K. L. Smolitskiy, "Approximate Methods for Solution of Differential and Integral Equations," American Elsevier Publishing Company, Inc., New York (1967) Chapter 1.

If Equation [30b] is multiplied by ℓ_4, Equation [30d] is multiplied by ℓ_2, and the resulting equations are added, the result is

$$\ell_4(a_2 - g_y) + \ell_2(a_4 - g_y) = (\ell_2 + \ell_4)\, a_y$$

and [33]

$$a_y = [\ell_4(a_2 - g_y) + \ell_2(a_4 - g_y)]/(\ell_2 + \ell_4)$$

Similarly, if [30c] is multiplied by ℓ_5 [30e] is multiplied by ℓ_3, and the resulting equations are added, then

$$\ell_5(a_3 - g_z) + \ell_3(a_5 - g_z) = (\ell_3 + \ell_5)a_z$$

and [34]

$$a_z = [\ell_5(a_3 - g_z) + \ell_3(a_5 - g_z)]/(\ell_3 + \ell_5)$$

Rearranging Equation [30a] yields

$$a_x = (a_1 - g_x) + \ell_1(q^2 + r^2) \qquad [35]$$

Thus if the gravitational components g_x, g_y, and g_z can be calculated, expressions can be derived for the components a_x, a_y and a_z in the body coordinate system of the translational acceleration of the CG.

CALCULATING THE EULERIAN ANGLES

Depending on the instrumentation configuration, either the components p, q, and r of the angular velocity are measured directly or p is measured directly and q and r are found from Equations [31] and [32]. These equations relate q and r with measured quantities and can be solved simultaneously by a numerical method. Before other desired quantities can be calculated, the values of the Eulerian angles ϕ, θ, ψ, must be calculated first. Since the actual values of p, q, and r are known, Equations [27] can be solved simultaneously by a numerical method for the values of ϕ, θ, and ψ. Thus, the members of the transformations [R] and $[R^{-1}]$ can now be calculated.

CORRECTING FOR GRAVITATIONAL EFFECTS

Expressions were derived above for the components of the translational acceleration of the body CG. These expressions contain gravitational terms which must be evaluated if the translational acceleration is actually to be calculated.

The gravitational acceleration vector $\vec{g}$ has magnitude approximately taken as $g = 9.8\ m/s^2$ at the surface of the earth and is generally thought of as being directed downward. In order to determine the effect of gravitational acceleration on the internal motion sensors of the body, it will ultimately be necessary to evaluate the components of $\vec{g}$ along the body axes. The Eulerian angles may be calculated independent of gravitational effects (see Equations [27], [31], and [32]) for either of the instrumentation configurations. Thus, the transformation [R], Equation [21], which transforms vectors from the ground coordinate system to the body system may be evaluated, and the problem becomes one of finding the components of $\vec{g}$ along the ground axes.

The definition of the ground coordinate system (x_o, y_o, z_o) did not specify any particular orientation of the axes. It will often be convenient to define the ground coordinate system such that one of its axes is directed parallel to $\vec{g}$. In such a case, of course, the components of $\vec{g}$ along the ground axes are obvious. In general, however, the ground axis components of $\vec{g}$ may be found by the following method.

Consider a coordinate system (x_g, y_g, z_g) in which the z_g-axis is directed downward, i.e., parallel to $\vec{g}$. The vector $\vec{g}$ may be expressed in terms of its components in this coordinate system as

$$\vec{g} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

The coordinate system (x_g, y_g, z_g) can always be expressed in terms of a rotation of the ground axes (x_0, y_0, z_0) by Eulerian angles ψ_g, θ_g, ϕ_g. The angles may be found by measurement. The rotation from the ground axes to the (x_g, y_g, z_g) coordinate system may be performed analytically by the transformation $[R_g]$ which is just the transformation [R] from Equation [21] with the Eulerian angles taken as $\psi = \psi_g$, $\theta = \theta_g$, and $\phi = \phi_g$.

Conversely, the transformation $[R_g]^{-1}$ transforms vectors in the (x_g, y_g, z_g) system to the ground system. Thus, if g_{x_0}, g_{y_0}, and g_{z_0} are the components of $\vec{g}$ along the ground axes.

$$\begin{bmatrix} g_{x_0} \\ g_{y_0} \\ g_{z_0} \end{bmatrix} = [R_g]^{-1} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \qquad [36]$$

In order to be sure of the sense of the effect of gravity on an accelerometer, consider a simple model consisting of a disk of mass M free to slide without friction inside a tube and attached to one end of the tube by a linear spring of spring constant c. (See Figure 4)

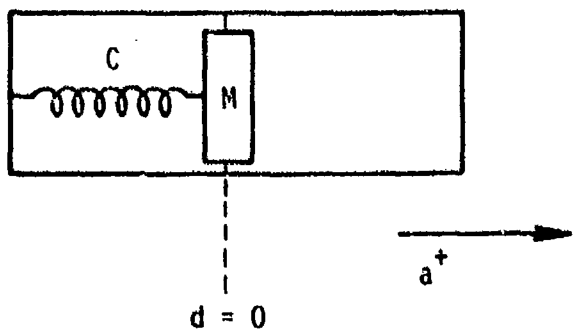

Figure 4 - A Simple Model of an Accelerometer

The displacement d of the disk from its rest position is defined to be positive in the sense of compression of the spring. Thus, the restoring force on the disk is given by

$$F = cd$$

If an acceleration a, as shown in Figure 4, results in a positive output of the accelerometer, then

$$F = Ma = cd$$

or

$$a = \frac{cd}{M}$$

Now, if the accelerometer is directed with its positive sense vertically upwards, the acceleration due to gravity will pull the disk downward, resulting in compression of the spring and a positive output from the accelerometer. Thus the gravitational acceleration causes an output from an accelerometer $\vec{g}_a$ equal in magnitude but opposite in sign to the gravitational acceleration itself. Thus

$$\vec{g}_a = -\vec{g}$$

The effect of gravitational acceleration as seen by accelerometers in the body axes is then determined by using the transformation [R] as

$$\begin{bmatrix} g_{a_x} \\ g_{a_y} \\ g_{a_z} \end{bmatrix} = [R] \begin{bmatrix} -g_{x_o} \\ -g_{y_o} \\ -g_{z_o} \end{bmatrix}$$

Before actually performing the transformation, one should consider a further aspect of the gravitational correction. Normal testing procedure often calls for the transducers to be "zeroed" before making a run. In the case of accelerometers initially directed horizontally, this practice yields the correct initial value. In the case of accelerometers not initially horizontal, this practice effectively subtracts the initial value of the component of gravitational acceleration along the accelerometer axis from the accelerometer output. If the output of an accelerometer is given by a, This is then

$$a(t) = a_k(t) + a_g(t) - a_g(0)$$

where a_k denotes the kinematic component of acceleration and a_g denotes the gravitational component of acceleration.

If each of the accelerometers is aligned parallel to one of the body axes, as stated above, then the components of gravitational acceleration along the body axes must be determined. Thus if the accelerometers are zeroed prior to a run, the components of the gravitational acceleration seen by the accelerometers are

$$\begin{aligned} g_x(t) &= g_{a_x}(t) - g_{a_x}(0) \\ g_y(t) &= g_{a_y}(t) - g_{a_y}(0) \\ g_z(t) &= g_{a_z}(t) - g_{a_z}(0) \end{aligned} \qquad [37]$$

where $g_{a_x}(0)$, $g_{a_y}(0)$, and $g_{a_z}(0)$ are given by

$$\begin{bmatrix} g_{a_x}(0) \\ g_{a_y}(0) \\ g_{a_z}(0) \end{bmatrix} = [R_o] \begin{bmatrix} -g_{x_o} \\ -g_{y_o} \\ -g_{z_o} \end{bmatrix} \qquad [38]$$

where $[R_o]$ denotes the transformation [R], Equation [21], with the values of the Eulerian angles taken as ϕ_o, θ_o, and ψ_o, respectively.

As an example of the calculation of the gravitational correction, consider the case where the ground axes are defined such that the x_o-axis is directed vertically upward and the body axes are initially coincident with the ground axes. In this case, the angle θ_g = -90 degrees, $\phi_g = 0$, and $\psi_g = 0$. Equations [36] yield for this case,

$$g_{x_o} = -g$$

$$g_{y_o} = 0$$

$$g_{z_o} = 0$$

The angles ϕ_o, θ_o, and ψ_o are all zero and so

$$g_{a_x}(0) = g$$

$$g_{a_y}(0) = g_{a_z}(0) = 0$$

Thus

$$\begin{bmatrix} g_{a_x} \\ g_{a_y} \\ g_{a_z} \end{bmatrix} = [R] \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix}$$

and

$$g_{a_x} = g \cos\theta \cos\psi$$

$$g_{a_y} = g(\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi)$$

$$g_{a_z} = g(\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi)$$

Thus

$$g_x = g_{a_x} - g_{a_x}(0) = g \cos\theta \cos\psi - g$$

$$g_x = g(\cos\theta \cos\psi - 1)$$

$$g_y = g_{a_y} - 0$$

$$g_z = g_{a_z} - 0$$

Thus for the case where the x_o-axis is directed vertically upward and the body axes are initially coincident with the ground axes, the components of gravitational effect on accelerometers along the body axes are given by

$$\begin{aligned} g_x &= g(\cos\theta \cos\psi - 1) \\ g_y &= g(\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi) \\ g_z &= g(\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi) \end{aligned} \qquad [39]$$

CALCULATING THE TRAJECTORY

The final step in determining the motion of the body is to find the time history of the position of the CG. Equations [36], [37], and [38] are used to calculate the components g_x, g_y, and g_z of the gravitational effect on the accelerometers. These values are inserted into Equations [29] if the body is equipped with three accelerometers and three rate sensors, or into Equations [33], [34], and [35] if the body is equipped with five accelerometers and one angular rate sensor. These equations can now be solved in the body coordinates for a_x, a_y, and a_z, the components of the kinematic acceleration of the CG. That is, one can calculate

$$\vec{A} = a_x\hat{e}_x + a_y\hat{e}_y + a_z\hat{e}_z$$

However, it is known that, in the ground axis coordinate system, $\vec{A}$ can be expressed as

$$\vec{A} = \ddot{x}_o(t)\hat{i} + \ddot{y}_o(t)\hat{j} + \ddot{z}_o(t)\hat{k}$$

where $x_o(t)$, $y_o(t)$, and $z_o(t)$ represent the components of the displacement of the body CG from the origin of the ground axes. So by using the transformation $[R^{-1}]$ from Equation [22],

$$\begin{bmatrix} \ddot{x}_o \\ \ddot{y}_o \\ \ddot{z}_o \end{bmatrix} = [R^{-1}] \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \qquad 40]$$

These expressions can be integrated by using Simpson's rule or some other numerical method to obtain

$$\dot{x}_o(t) = \int_o^t \ddot{x}_o(t)\,dt$$

$$\dot{y}_o(t) = \int_o^t \ddot{y}_o(t)\,dt \qquad [41]$$

$$\dot{z}_o(t) = \int_o^t \ddot{z}_o(t)\,dt$$

Equations [41] can be further integrated to yield

$$x_o(t) = \int_o^t \dot{x}_o(t)\, dt$$

$$y_o(t) = \int_o^t \dot{y}_o(t)\, dt \qquad [42]$$

$$z_o(t) = \int_o^t \dot{z}_o(t)\, dt$$

Equations [42] represent the trajectory of the body CG. The values of $x_o(t)$, $y_o(t)$, and $z_o(t)$, together with the values of $\phi(t)$, $\theta(t)$, and $\psi(t)$, represent a complete time history of the motion of the body.

DISCUSSION

The analytical methods described in this report permit a determination of the motion of a body in six degrees of freedom on the basis of outputs of internal motion sensors. It should be noted, however, that the results of any such analysis can certainly be no better than the accuracy of the particular transducers utilized. Thus it is extremely important that transducers used in this type of experiment be chosen for accuracy and that the experiment be very carefully set up.

The choice of configuration may often depend on the availability of transducers or on physical considerations such as the size and shape of the body. However, if a choice is available, the configuration with three accelerometers and three angular rate sensors will generally be preferable to that with five accelerometers and one angular rate sensor.

For the preferred configuration, all of the components of angular velocity are measured directly and, therefore, some calculations detrimental to the accuracy of the results are avoided.

Since calculations generally have a detrimental effect on the accuracy of the results, one would expect that those quantities directly measured or nearly directly measured to have the smallest uncertainty. In this type of analysis, then, one would have the greatest confidence in the translational accelerations and the angular velocities and/or accelerations. In the alternative photographic method, the most nearly directly measured quantities are linear and angular displacement. Therefore, if the accuracy of measurement is approximately the same for the two methods, one would have greater confidence in the photographic results for displacement and in the internal transducer results for acceleration and angular velocity. It should be pointed out, however, that a three-dimensional photographic analysis, especially for a body in water, requires a rather involved camera calibration and an extremely careful setup. So far as is known to the author, a detailed error analysis has not been performed for either of these techniques. Nevertheless it is felt that there is some justification for the suspicion that the difficulty in obtaining accurate basic measurements is greater for the three-dimensional photographic technique than for the internal transducer method. In any case, it would certainly be desirable to perform an experiment wherein both techniques were utilized to enable a direct comparison of the results.

One problem is that translational velocity is not measured directly by either of these techniques and therefore it must be calculated. Thus for the internal transducer method, the velocity must be obtained by numerically integrating the acceleration, and for the photographic method, it must be obtained by numerically differentiating the displacement. The results will reflect the properties of these mathematical operations. In general, numerical integration is a "smoothing process" whereas the opposite is ture for numerical differentiation. In general, then, a velocity versus time curve obtained by integrating a measured acceleration will be "better looking," i.e., smoother, than that obtained by differentiating a measured displacement. Several launching tests performed at DTNSRDC have indicated that the velocity at the tube exit obtained from the internal transducer method was in good agreement with that obtained by measuring the frequency of light pulses reflected from alternate red and black stripes painted on the model.

The internal-transducer technique has two significant operational advantages over photographic techniques. First, measurements can be made throughout the entire domain of the body motion whereas there are often parts of the domain in which the body is not visible to cameras.

Second, if appropriate data-processing equipment is available, the internal-instrumentation technique can yield immediate onsite results whereas the photographic technique requires a multistage process including film processing, film reading, and the running of a rather elaborate computer program.

The main operational disadvantage of the internal transducer technique is the need for a trailing umbilical cable.

SUMMARY

1. Analytical methods have been presented for determining the motion of a rigid body in six degrees of freedom on the basis of the outputs of internal motion sensors.

2. When the physical situation is appropriate, the internal-transducer measurement technique is a useful alternative to photographic techniques for measuring the motion of a body in space.

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