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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS.

PART 33.

ON THE VALUE OF MARKET INFORMATION.

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Martin/Schubik

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 33

ON THE VALUE OF MARKET INFORMATION\*

by

Martin Shubik\*\*

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1. INTRODUCTION

→ In this paper a model is presented of trade where individuals have differing sets of information and are able to trade in information prior to trading in commodities or contingent commodities. This paper concentrates upon modelling and the conceptual problems which must be overcome in constructing models which reflect adequately the process of trade in information. General proofs of the existence of equilibria and the study of their properties will be given in a subsequent joint paper with P. Dubey.

Although this paper is written to be self-contained, it is related with a series of models presented in previous papers by Shubik,<sup>1</sup> Shapley,<sup>2</sup>

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page 1

Shapley and Shubik,<sup>3</sup> Dubey and Shubik<sup>4</sup> and several others.<sup>5,6,7</sup> The study of the functioning of markets using money and/or other financial instruments poses many subtly interrelated problems. The approach adopted in this series of papers has been to isolate them as much as possible, to study them separately and to produce models which combine more than one phenomenon only when needed to understand a new phenomenon which depends upon the interactions of several phenomena.

In the model presented below a closed trading economy (the usual Walrasian model) is modelled as an n-person game which can be studied for the existence of noncooperative equilibria. The following additional features however are added. "Nature" randomizes to select which state the economy is in. This type of modification has been considered by Arrow<sup>8</sup> and Debreu<sup>9</sup> in their treatment of trade in contingent commodities. That treatment however was limited to models in which all traders have the same level of uncertainty. In more recent work, and using different approaches Radner<sup>10</sup> and Dubey and Shubik<sup>11,12</sup> have considered trade where the traders have different information concerning Nature's move.

The work of Dubey and Shubik was based upon a specific model of the trading process which will be described below. Several other models could have been used. The specific problem posed in modelling an exchange economy as a single simultaneous move game of strategy has been discussed in detail by Shubik.<sup>13</sup> He has suggested that there are essentially four basically different models, although many slight variations can be considered. Shubik<sup>1,14</sup> Shapley and Shubik<sup>3</sup> and Dubey and Shubik<sup>4</sup> have considered three of these four models. Two of them will be utilized here.

Both of the models of the trading process used here have the traders make their payments in a commodity money. A more realistic model would

call for the use of a fiat money or credit. This feature can be added to the work described, however in order to do so bankruptcy conditions must be introduced in order to account for the possibility that in an economy with credit an individual may not be in a position to cover his debts. This feature is not critical to the understanding of the market for information hence we do not add it here. Discussion of models with bankruptcy is given elsewhere.<sup>6, 15</sup>

The key additional feature to the model presented here is the introduction of specific markets for information and a mechanism which prices the information.

## 2. MODELLING TRADE WITH NONSYMMETRIC INFORMATION

### 2.1. Trade with Nonsymmetric Information Conditions

Suppose that there are  $n$  traders,  $g$  states and  $m+1$  commodities. The  $m+1^{\text{st}}$  commodity is used as a means of payment in the manner described below. The preferences of each trader  $i$  can be represented by a utility function of the form  $\sum_{k=1}^g \rho^k \phi_1(x_1^{ik}, x_2^{ik}, \dots, x_{m+1}^{ik})$  which is concave and specified up to a linear transformation. Each trader  $i$  has an initial bundle of goods of  $(A_1^{ik}, A_2^{ik}, \dots, A_{m+1}^{ik})$  in state  $k$  where  $A_j^{ik} \geq 0$  for  $i = 1, \dots, n$ ,  $j = 1, \dots, m$ ,  $k = 1, \dots, g$  and  $A_j^k > 0$  for all  $j$ ,  $k$  ( $A_j^k = \sum_i A_j^{ik}$ ). Furthermore there are at least two traders who desire any commodity  $j$  in any state  $k$ .

There are  $g$  states of Nature where the  $k^{\text{th}}$  state occurs with probability  $\rho^k$ .

There are  $gm$  trading posts, where each post sells one contingent commodity such as "oranges if state 2 occurs." A move by a trader is an

assignment of the  $m+1^{\text{st}}$  commodity to the  $g_m$  trading posts. It is possible that the individual may have to bid without knowing how much commodity money he has. In which case we must specify a rule to convert his bid into one that is feasible. The easiest way to do this is by scaling back his bid in all markets in proportion.

At this point the reader may feel that the model is grossly unrealistic. First people do not bid by splitting money among a group of "target goods" without knowing how much they are going to buy in advance. Furthermore even if this were the case, the rationing convention which calls for a scaling back of all bids is not necessarily reasonable. The author is well aware of the validity of these observations. Yet the model is defended on two grounds. First, it is one of the simplest completely defined process models of trade. Second, that there are other more complicated mechanisms which will lead to substantially the same results.

We now complete the definition of the model. All contingent goods  $A_j^k$  are offered for sale at the trading post which deals only in good  $j$  under state  $k$ . We may regard the post as trading in "options." If you buy an option and state  $k$  occurs you will obtain an amount of good  $j$ . If state  $k$  does not occur you will have still paid for the option but will obtain nothing.

If you happen to be informed before you buy that state  $k$  has or has not occurred you will know in advance whether a purchase of the option will be worthwhile. It is as though each contingent good options trading post were a black box where individuals are paying for a share of the contents. Those who are informed in advance are able to "see into the box" to determine whether it is empty or full before they bid. The others find out only after they have bid.

As the notation for the general case is rather complex and the full mathematical treatment of this model is given in separate papers<sup>11, 12</sup> it is easier to proceed by presenting a specific example in detail.

Let us consider an economy with two types of traders. There are  $n_1$  traders of type 1 and  $n_2$  of type 2. There is one commodity and one commodity money and two states of Nature. Traders of type 1 are informed about the state of Nature before trade; traders of type 2 are not informed.

A move by a trader of type 1 or 2 is a triad of numbers, the percentage of his income to be spent on the option market for good 1 in state 1, for good 1 in state 2 and the percentage of commodity money to go unspent.

A strategy for a trader of type 2 is the same as a move as he has no information prior to making his choice. A strategy for a trader of type 1 will consist of six numbers, one triad of bids for state 1 and another triad for state 2.

An initial endowment for a trader  $i$  of type  $s$  is  $(s_{A_1}^{i1}, s_{A_2}^{i1}, s_{A_1}^{i2}, s_{A_2}^{i2})$  where the subscript indicates the type of good (here there are only two) and the superscripts indicate the player type ( $s$ ) the player's "name" ( $i$ ) and the state of the system.

A strategy of a trader  $i$  of type 1 is given by  $(b_1^{i1}, b_2^{i1}, b_3^{i1}; b_1^{i2}, b_2^{i2}, b_3^{i2})$ . Because a trader  $i$  of type 2 cannot distinguish among states his strategy is  $(b_1^i, b_2^i, b_3^i)$ .

The actual registered bid of a trader will be scaled up or down to convert proportions to actual amounts. Let the scaling factor be  $r^{ik}$  for trader  $i$  of type 1 if state  $k$  holds. Then

$$\sum_{j=1}^3 l_{r_j}^{1, ik} l_{b_j}^{1, ik} = l_{A_2}^{1, ik} .$$

For a trader of type 2 this is

$$\sum_{j=1}^3 l_{r_j}^{2, ik} l_{b_j}^{2, i} = l_{A_2}^{2, ik} .$$

The difference between these two conditions is that for traders of type 1, bids change with the state, while for traders of type 2 they do not.

If state k exists a trader i of type 1 obtains the following amount of commodity 1

$$l_{x_1}^{1, i} = \frac{l_{r_1}^{1, ik} l_{b_1}^{1, ik, k} A_1}{\sum_{i=1}^{n_1} l_{r_1}^{1, ik} l_{b_1}^{1, ik} + \sum_{i=1}^{n_2} l_{r_1}^{2, ik} l_{b_1}^{2, i}} .$$

The extensive form of the bidding game representing this market can be represented as follows:

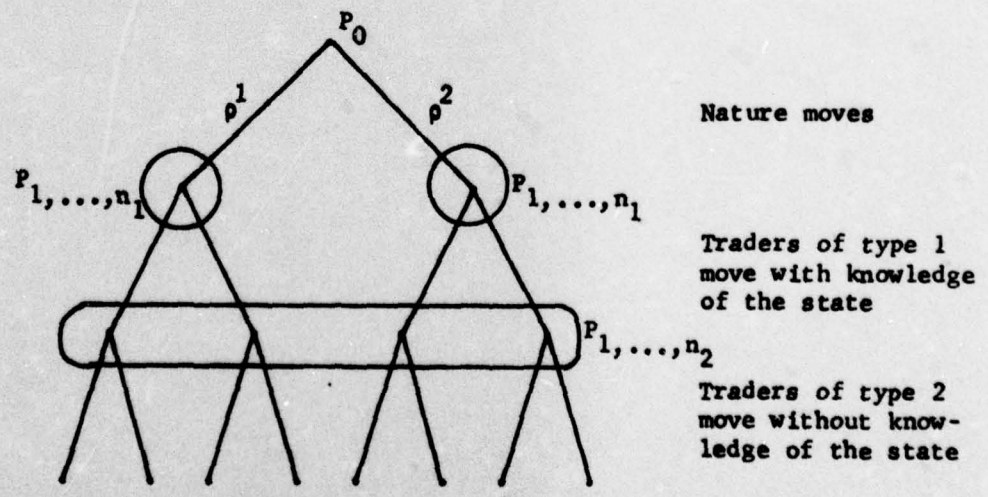


FIGURE 1

## 2.2. An Explicit Example

We consider a trading economy with one good, one commodity money and two states. Traders of type 1 have endowments of 10 if state 1, 2 if state 2 and traders of type 2 have endowments of 2 if state 1, 10 if state 2.

There are  $n_1$  traders of type 1 and  $n_2$  of type 2. All have a large amount of money  $M$  in either state. Each traders of either type has a utility function of the form:

$$(1) \quad U^i = \log x_1^i x_2^i + y^i$$

where  $(x_1^i, x_2^i, y^i)$  are final endowments of the contingent goods 1 and 2 and money. There exist separate markets for  $x_1$  and  $x_2$ .

Traders of type 1 move with knowledge of the state and traders of type 2 without this knowledge (see Figure 1). Nature randomizes with probabilities  $\rho_1 = \rho_2 = 1/2$ .

Let the symbol  $b_s^{ijk}$  stand for the bid of trader  $i$  of type  $j$  at market  $k$  under state  $s$ .  $b^{ijk}$  stands for a bid which is state independent. The omission of a superscript indicates summation, e.g.  $b^{jk} = \sum_i b^{ijk}$ .

A trader  $i$  of type 1 attempts to maximize

$$\begin{aligned}
 (2) \quad \Pi_1^i = & \frac{1}{2} \left\{ \log(10n_1 + 2n_2) \frac{b_1^{i11}}{b_1^{11} + b^{21}} - b_1^{i11} + \{b_1^{11} + b^{21}\} \left( \frac{10}{10n_1 + 2n_2} \right) \right. \\
 & + 0 - b_2^{i11} + \{b_2^{11} + b^{21}\} \left( \frac{10}{10n_1 + 2n_2} \right) \left. + \frac{1}{2} \left\{ 0 - b_1^{i12} \right. \right. \\
 & + \{b_1^{12} + b^{22}\} \left( \frac{2}{2n_1 + 10n_2} \right) + \log(2n_1 + 10n_2) \frac{b_2^{i12}}{b_2^{12} + b^{22}} - b^{i12} \\
 & \left. \left. + \{b_2^{12} + b^{22}\} \left( \frac{2}{2n_1 + 10n_2} \right) \right\} \right\}.
 \end{aligned}$$

A trader  $j$  of type 2 attempts to maximize

$$\begin{aligned}
 (3) \quad \Pi_2^j = & \frac{1}{2} \log(10n_1 + 2n_2) \frac{b^{j21}}{b^{21} + b_1^{11}} + \frac{1}{2} \log(2n_1 + 10n_2) \frac{b^{j22}}{b^{22} + b_2^{12}} - b^{j21} - b^{j22} \\
 & + \frac{1}{2} (2b^{21} + b_1^{11}) \left\{ \frac{2}{10n_1 + 2n_2} \right\} + \frac{1}{2} (2b^{22} + b_2^{12}) \left\{ \frac{10}{2n_1 + 10n_2} \right\}.
 \end{aligned}$$

In (3) a simplification has already been made which can also be made for (2). Specifically because a trader of type 1 knows that "box 1" is empty under state 2 and "box 2" is empty under state 1 we may set  $b_2^{i11} = 0 = b_1^{i12}$ . A trader of type 2 does not have this advantage.

Further, first order conditions for maximization give us:

$$(4) \quad \frac{1}{b_1^{i11}} - \frac{1}{b_1^{11} + b^{21}} - 1 + \frac{10}{10n_1 + 2n_2} = 0$$

$$(5) \quad \frac{1}{b_2^{i12}} - \frac{1}{b_2^{12} + b^{22}} - 1 + \frac{2}{2n_1 + 10n_2} = 0$$

$$(6) \quad \frac{1}{2} \left[ \frac{1}{b^{j21}} - \frac{1}{b^{21} + b_1^{11}} \right] - 1 + \frac{2}{10n_1 + 2n_2} = 0$$

$$(7) \quad \frac{1}{2} \left[ \frac{1}{b^{j22}} - \frac{1}{b^{22} + b_1^{12}} \right] - 1 + \frac{10}{2n_1 + 10n_2} = 0 .$$

We limit ourselves to the case where  $n_1 = n_2 = n$  and consider a type symmetric noncooperative equilibrium (T.S.N.E.). We may replace  $b_s^{jk}$  by  $nb_s^{ijk}$  and for ease we drop the superscript for the individual trader thereby reinterpreting  $b_s^{jk}$  as the bid of a typical trader of type  $j$ . Rewriting the equations (4)-(7) we obtain

$$(8) \quad \frac{1}{b_1^{11}} - \frac{1}{n(b_1^{11} + b^{21})} = 1 - \frac{5}{6n}$$

$$(9) \quad \frac{1}{b_2^{12}} - \frac{1}{n(b_2^{12} + b^{22})} = 1 - \frac{1}{6n}$$

$$(10) \quad \frac{1}{b^{21}} - \frac{1}{n(b^{21} + b_1^{11})} = \left(1 - \frac{1}{6n}\right) \frac{1}{2}$$

$$(11) \quad \frac{1}{b^{22}} - \frac{1}{n(b^{22} + b_1^{12})} = \left(1 - \frac{1}{6n}\right) \frac{1}{2} .$$

For  $n \rightarrow \infty$  we obtain  $b_1^{11} = b_2^{12} = 1$ ,  $b^{21} = b^{22} = 1/2$ .

If we assumed that traders of type 2 also had full information we would obtain as a solution

$$b_1^{11} = b_2^{12} = 1 = b_1^{21} = b_2^{22} .$$

When traders of type 2 have incomplete information, the payoffs will be

$$(12) \quad \Pi_1 = \frac{1}{2} \left\{ \log_{12} \left( \frac{2}{3} \right) - 1 + \frac{3}{2} \left( \frac{10}{12} \right) + \frac{1}{2} \left( \frac{10}{12} \right) \right\} + \frac{1}{2} \left\{ \log_{12} \left( \frac{2}{3} \right) - 1 + \frac{1}{2} \left( \frac{2}{12} \right) + \frac{3}{2} \left( \frac{2}{12} \right) \right\}$$

$$= \log 8$$

$$(13) \quad \Pi_2 = \frac{1}{2} \log_{12} \left( \frac{1}{3} \right) + \frac{1}{2} \log_{12} \left( \frac{1}{3} \right) - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} (2) \left( \frac{2}{12} \right) + \frac{1}{2} (2) \left( \frac{10}{12} \right)$$

$$= \log 4 .$$

When all have complete information, the payoffs are:

$$(14) \quad \Pi_1 = \Pi_2 = \log 6 .$$

The payoffs for finite  $n$  can be calculated from the general solution to equations (8)-(11). This is a straightforward and not particularly edifying computational problem and as it is not necessary to the main thread of the argument in this paper, the calculations are not given.

### 3. MODELLING TRADE IN INFORMATION

#### 3.1. Preliminary Observations

In the context of a communication linkage such as a telephone wire it has been possible to define a measure of the amount of information which can be transmitted. The work of Shannon<sup>16</sup> on information theory answers many basic questions on the measurement of the amount of information, but is not concerned with the value of the message to the recipients.

Here the stress is upon the value of the different messages which can be sent to the various participants. The total amount of information in the system can be measured in Shannon's sense it is  $\log_2 g$  where  $g$  is the number of recognizable states. We are concerned however with the worth of those messages. If there are  $g$  possible states of Nature,

an individual may be in any one of  $2^g$  information states. Each information state is represented by a partition of the  $g$  states of Nature into information sets. These information sets describe the ability of an individual to distinguish among states of Nature. Any states which belong to the same information set of an individual cannot be distinguished by that individual. A better name for an information set might be a "lack-of-information" set.

If two individuals have different information about the states of Nature they might wish to trade in information. The formal theory of games treats the partition of choice points into information sets as a datum of the description of a game. Here we are going to allow for trading in messages as part of the structure of the moves of the game. The result of these trades will be that individuals who buy information may refine their information sets before they bid in a market for options or contingent commodities.

### 3.2. Message Trading Posts

A trading economy will now be described in three stages. Nature randomizes among  $g$  states. Then simultaneously all individuals go to (at most)  $2^g$  message trading posts where they offer to sell or buy information. The specific mechanism is described below. After trade in information has been completed each individual now simultaneously bids in a market of the type described in Section 2 where his information sets are given by the refinement of his original information sets with those he has purchased.

The information purchased at any message trading post is that the state of the system either is or is not a member of the information set

at that message trading post. A simple example will help to make this clear. Consider an economy with 3 states of Nature and 5 traders. Their information sets are as follows:

Trader 1 distinguishes 1, 2 or 3.

Trader 2 distinguishes 1 from 2 or 3.

Trader 3 distinguishes 2 from 1 or 3.

Trader 4 distinguishes 3 from 1 or 2.

Trader 5 cannot distinguish among 1, 2 or 3.

There are formally  $2^3$  message trading posts where one can purchase the answer to the question "Does the current state of the system belong to your information set?" The null set and the set of all states can be eliminated.

In the example above there will be 6 message trading posts which might be active.

At Post 1 the question is: "Is the system in State 1?"

At Post 2 the question is: "Is the system in State 2?"

At Post 3 the question is: "Is the system in State 3?"

At Post 4 the question is: "Is the system in States 2 or 3?"

At Post 5 the question is: "Is the system in States 1 or 3?"

At Post 6 the question is: "Is the system in States 1 or 2?"

At Post 1, traders 1 or 2 can sell and 3, 4 and 5 might wish to buy.

At Post 2, traders 1 or 3 can sell and 2, 4 and 5 might wish to buy.

At Post 3, traders 1 or 4 can sell and 2, 3 and 5 might wish to buy.

At Post 4, traders 1 or 2 can sell and 3, 4 and 5 might wish to buy.

At Post 5, traders 1 or 3 can sell and 2, 4 and 5 might wish to buy.

At Post 6, traders 1 or 4 can sell and 2, 3 and 5 might wish to buy.

If the system were actually in State 2 the messages would be:

At Post 1    no  
At Post 2    yes  
At Post 3    no  
At Post 4    yes  
At Post 5    no  
At Post 6    yes.

The specific functioning of a message trading post is as follows. Any individual who can answer the question to be answered may offer his information for sale at that post at a price he specifies. He may only sell his information at the message trading posts. He cannot offer for sale information which he does not possess.

Any buyer who wishes to purchase information specifies the price he is willing to pay for it at the message trading post. All actions by sellers and buyers are simultaneous. No buyer has the opportunity to resell the information he has bought prior to bidding in the final market for goods.

We have several alternative methods of modelling the fixing of the price of information and the clearing of the message markets. We explore several of them.

For simplicity we assume that there are no transactions costs involved in the purchase and sale of information.

It is natural to assume that the same answer can be supplied from a single individual to many questioners. Hence we may regard each supplier as offering an unlimited supply of information. This is shown graphically in Figure 2a. An alternative, less reasonable, but well defined mechanism is where each supplier is credited with the ability to sell

his information only once. This is shown in Figure 2b (drawn for 5 offerers and 5 potential buyers).

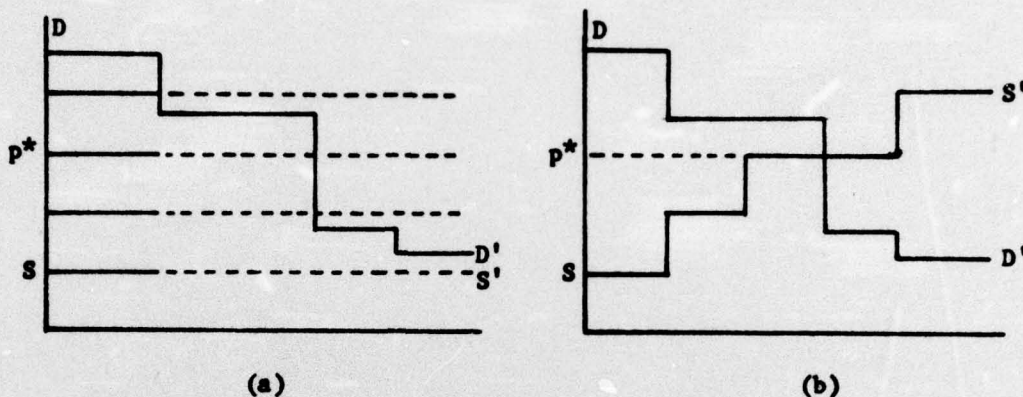


FIGURE 2

In Figure 2a we see that as each offeror can supply all demanders we need a convention for deciding who will be picked. In a manner somewhat reminiscent of the Bertrand price duopoly model the convention adopted is that supplier with the lowest price is chosen. If his price is higher than any buyer will pay, no information will be sold. If it is less than or equal to the price offered by some buyers the seller with the lowest price, say  $p_1$  will supply all buyers who are willing to pay  $p_1$  or more. Suppose that  $s$  buyers are willing to pay  $p_1$  or more, then the revenue of  $sp_1$  will be paid in toto to the seller of the information. If there is a tie for the low price, then the sellers share the revenue. Thus if there were  $r$  sellers with the lowest price each would obtain  $sp_1/r$ .

If we adopt the other convention which credits each seller with the ability to sell only one unit of information than a more conventional supply diagram as is shown by  $SS'$  in Figure 2b can be drawn. We may use as the market clearing convention the price determined by the inter-

section of  $SS'$  and  $DD'$ . Under this convention if this price is say,  $p^*$  (as shown in Figure 2b) then 3 buyers will buy information at the price  $p^*$  and the first two sellers each obtain  $p^*$  while the remaining income must be split between the two marginal sellers. A simple convention would be to randomize or another reasonable convention would be to share the income.

A third model that we might wish to consider is as follows. If there are  $k$  states of nature  $2^k$  parameters  $c_j \geq 0$  where  $j = 1, \dots, 2^k$ ; are introduced. These are prices assigned to the messagetrading posts. Any buyer who wishes to buy must pay  $c_j$  however only sellers who have registered their willingness to sell obtain any payments. Furthermore if  $r$  wish to sell and  $s$  wish to buy each seller obtains  $sc_j/r$ .

### 3.3. Selling Information and the Extensive Form

In order to bring the sale of information into a formal market structure we have had to impose several severe restrictions on the communication and information system. In particular it is important to note the following:

- (1) The uncertainty is only about the outcome of an exogenous random variable with given probabilities.
- (2) There are no markets for "spy information," i.e. information concerning strategies or moves of others.
- (3) Individuals either know about or form beliefs about the occurrence of certain events based upon their knowledge of the probabilities used by Nature and upon their information sets. In this model there are no other factors which make one individual more expert or believable than another.

- (4) There is no communication or dealing outside of the model, hence informal arrangements for the pooling or quick resale of information are ruled out.
- (5) Because there is no communication net beyond "the rules of the game" and because there are organized message trading posts it is possible to guard the trade in information and to prevent the appropriation of information without payment.

All of the conditions noted above are undoubtedly stringent but are needed in order, at least, to be able to capture one recognizable property of message sending and information evaluation which can be treated in a fully formal game theoretic manner.

Before the game tree can be fully defined one more feature must be specified. We assume that after all traders have purchased or sold information they are informed of these sales and purchases prior to bidding in the markets for contingent commodities.

Figure 3 provides the extensive form for the example provided in 2.2 for the case  $n_1 = n_2 = 1$ . Nature moves first randomizing with  $p_1 = p_2 = 1/2$ . Trader 1 decides either to offer or not to offer his information for sale. For simplicity in this example we choose the third convention for pricing information, i.e. a price is set as an outside parameter hence the information purchase or sale decision is actually binary.

After the traders have bid for or offered information they are told who has bid or offered thus even if trader 2 decided not to buy he is told if trader 1 had or had not made the offer to sell. This is reflected in the information sets shown for trader 2's move to bid for goods.

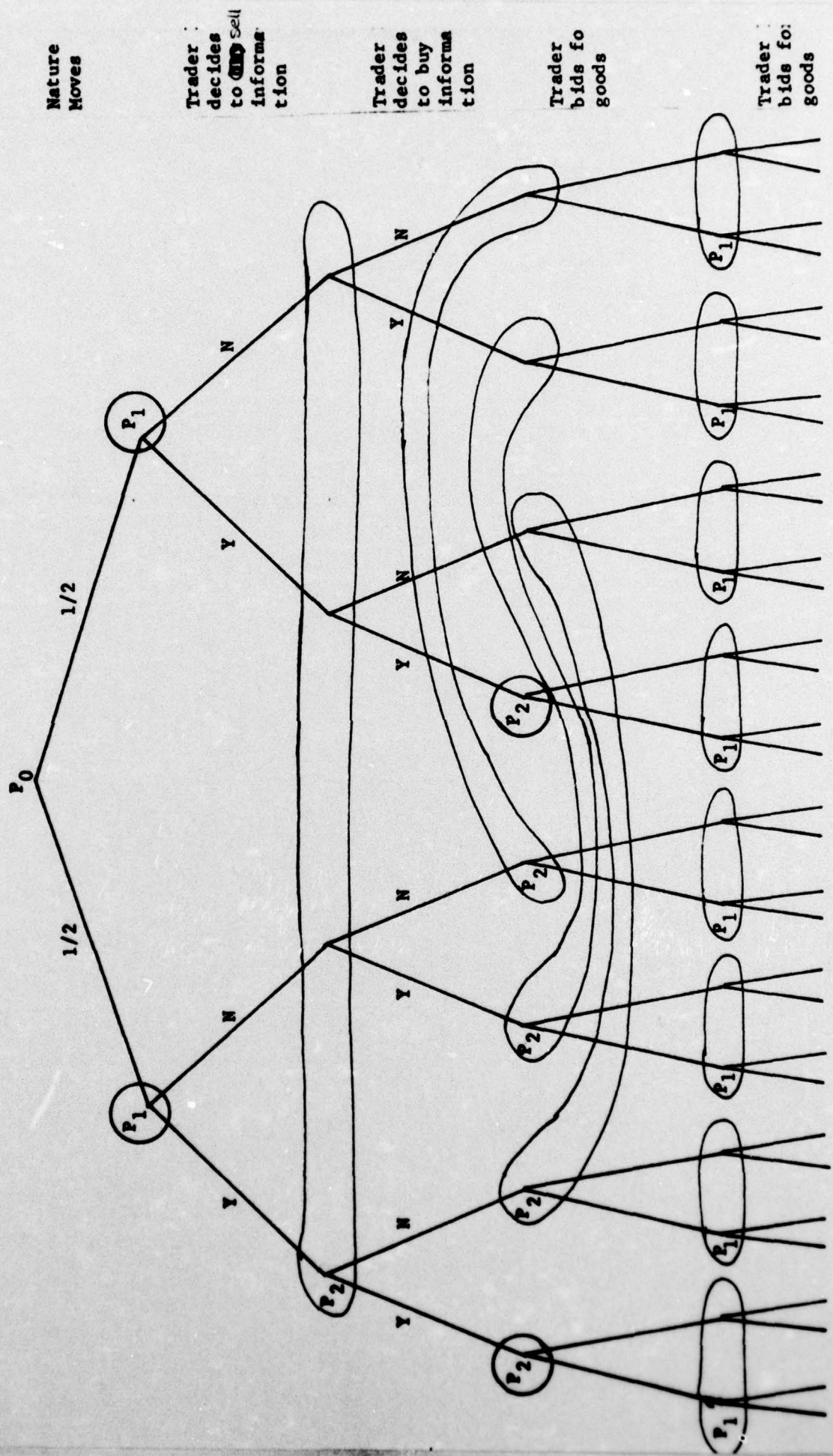


FIGURE 3

#### 4. NONCOOPERATIVE EQUILIBRIA AND TRADE IN INFORMATION

##### 4.1. Two Difficulties

There are two conceptual difficulties which pertain to all two stage games which are not solved adequately here. However, it is conjectured that, at least for large markets, neither difficulty is fatal. The first concerns our ability to define a state dependent noncooperative equilibrium and the second is to show that there will be a unique equilibrium.

It is argued elsewhere<sup>6,7</sup> that the introduction of fiat money and a bankruptcy law of the appropriate type may help uniqueness. It might be conjectured that we could usefully regard these games as consisting of two stages. In the first stage information is traded. Then in the second stage the traders check to see what trading game with nonsymmetric information they are in and bid accordingly. An example will show that a state strategy may not always be reasonable.

The example selected in 2.2 was picked to have a unique noncooperative equilibrium for purposes of illustration.

##### 4.2. Solutions to the Example

We now consider the example presented in 2.2 to serve as the second stage for a game with information trading as described in Section 3. All three conventions are considered. They are the two illustrated in Figures 2a and 2b and the setting of prices for messages parametrically.

(a) The Bertrand Case

If an individual is allowed to collect revenue from the multiple sale of the same message this market can be regarded as precisely the equivalent of a Bertrand price oligopoly with each seller with a supply of a costless good sufficient to saturate the market. The only noncooperative equilibrium, if we consider state strategies, has all sellers providing information at zero price. However suppose that there were only two sellers of information. One might use a threat strategy amounting to "I will not offer information for sale; if you do not, I will play the second stage as a one stage game; if you offer information I will bid in such a way as to cause you a loss relative to what you would have obtained with no sale of information."

When there are many individuals in a market it might be argued that these types of threat cannot really be communicated or carried out and state strategies are more reasonable. Even this is not necessarily always true.

(b) Restricted Supply

If the message market is as shown in Figure 2b then it has been argued elsewhere<sup>14</sup> that the only equilibrium for this type of market requires that all sellers charge the same price. If there are more sellers than buyers at a price of zero this becomes the equivalent of the Bertrand case. If there are fewer sellers than buyers at a price of zero then a positive price can be supported.

The restriction on message sales might be reasonable for copyrighted material but for "forecasting" knowledge or "trade secrets" it does not appear to be warranted.

(c) The Price Parameter

A way in which the reproducibility of information can be accommodated without introducing extreme competition among the sellers is to have a market agent fix prices to the highest levels consistent with all purchasing full information.

Reconsidering the limit case as  $n \rightarrow \infty$  for the example in 2.2 we observe that information can command a positive price. Suppose that individual 1 were to consider his position given that all  $2n-1$  others had obtained information. In effect he would know that the price of a future for state 1 or state 2 is  $1/6$ . Thus his payoffs can be given by

$$(15) \quad \Pi = \frac{1}{2} \log 6x + \frac{1}{2} \log 6y - x - y + 1 + M$$

where  $x$  and  $y$  are his expenditures on futures and  $M$  is the amount of commodity money held by him. We see that even though all others are informed and even though he may be informed of their strategies this still does not give him the information about Nature that he needs. All he knows is that every one else will use strategies of the form:

"If state 1 then (1,0); if state 2 then (0,1)," but as he cannot distinguish state 1 from 2 he still has to split his bid even though he can calculate that the price of either future will be approximately  $1/6$ . We can observe that  $x = y = 1/2$  and the expected payoff to the uninformed trader is

$$\log 3 + M ;$$

by buying information he can convert this to:

$$\log 6 - c + M$$

where  $c$  is the cost of the information; thus  $0 \leq c \leq \log 2$  are prices which will support the purchase of information.

In testing for equilibria we may note that as others are more informed the value of information to the remainder may change. Suppose that all traders of type 2 have not purchased information. By considering the solution to the example in 2.2 a single trader without information obtains  $\log 4 + M$ . If he buys information he obtains  $\log 8 + M - c$ . He will purchase information if  $0 \leq c \leq \log 2$ . In this example the value has remained the same. This equality appears to be an artifact of the particular example.

#### 4.3. Further Problems

The models presented above are offered in an attempt to isolate some specific simple aspects of trade in economic information. In particular the appropriation, reproducibility and transmission of information are isolated by the formal rules of the games governing the message market centers. The demand side of the market is then studied in the third model version by attaching a parametric price to the message at each message market.

This approach of isolating properties of the exchange of information is clearly so limited that there are several further problems which would require more elaborate modelling to investigate them. In particular we do not deal with subjective probability. Furthermore the analysis presented here is not dynamic.

A key question that has been asked in the economics of finance and information\* is "does the market price of a stock reflect the full

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\* See for example,<sup>17</sup>

discounting of insider information?" In terms of the analysis presented here this is not a completely well defined question until the following specifications are made (a) is the model envisioned to be in the context of a closed economic model or Walrasian system? (b) does it have subjective probabilities? (c) can information be traded? (d) are prices given to the traders or do they evolve from the trading?

In the context of this analysis if we assume a Walrasian model with no subjective probabilities, and prices which evolve from trade the answers suggested are: (1) If no information is traded then the price system does reflect the differences in information and the price system will not be that given by the competitive equilibrium. (2) If information is traded, depending upon the mechanisms of trade and the control on reproduction, transmission and appropriation, the message markets will go active before the commodity markets and prices in the commodity markets will be influenced by the prior exchange of information.

The complex activities in preparing crop estimates, making economic or stock market forecasts, assessing the success of new technology, finding "deals" mergers, acquisitions or other new transformations and recombinations of assets are all examples of trading in information. Forecasters are paid, finders fees are often paid and insiders exchange information. The markets are there, but given difficulties in appropriation and communication they are hard to formulate. The models presented in Section 3 are a first attempt to formalize some of the elements of information markets.

#### 4.4. Institutions and a Price for Amnesia

The remarks in this section may be regarded by some as somewhat cryptic and are based heavily upon the elaboration of a previous model.<sup>18</sup> They are noted as a warning against making the assumption that because the value of more information to some may support a positive price, then more information is necessarily optimal for the system as a whole. The reverse may be true, as is shown in an example noted in the paper referred to here.<sup>18</sup> If this is the case then we could conceive of a "message amnesia" market, i.e. a market in which certain individuals are paid or "bribed" so that they will "forget" information. This possibility can be modelled in a manner as suggested in Section 3, but a working market system for "amnesia" has not yet been constructed. It appears that the solution to this problem lies in the construction of rules of the game which can be interpreted as certain rudimentary institutions.<sup>18</sup>

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