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A MODIFIED MINIMUM ENERGY REGULATOR PROBLEM AND FEEDBACK STABIL--ETC(U)

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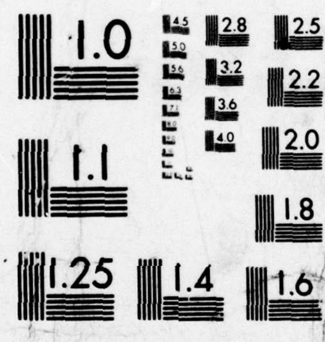

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A MODIFIED MINIMUM ENERGY REGULATOR PROBLEM AND FEEDBACK STABILIZATION OF A LINEAR SYSTEM*

by

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Abstract

This paper considers a feedback control law for linear time-varying and time invariant systems based on a modified minimum energy problem with fixed terminal constraints. The modified control laws are shown to be optimal for a certain cost function, asymptotically stable, and to result in a new method for stabilizing linear time-varying systems as well as extending some well known methods for stabilizing time invariant systems. In particular, the stabilizing gains of the feedback control laws are obtained from the solution of a Riccati equation over an arbitrary finite time interval, which is relatively easy to compute. Some stability results in [1], [2] and [10] will turn out to be special cases of our results.

I. *INTRODUCTION*

Consider a linear time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1.1}$$

$$y(t) = C(t)x(t) \tag{1.2}$$

where $A(t)$ and $B(t)$ are nxn and nxm piecewise continuous matrices. Consider also a cost function

$$J(u) = \int_{t_o}^{t_f} (y'(t)Q(t)y(t) + u'(t)R(t)u(t))dt, \tag{1.3}$$

where $Q(t)$ and $R(t)$ are piecewise continuous with $Q(t) \geq 0$ and $R(t) > 0$, and with the boundary conditions

$$x(t_o) = x_o \qquad x(t_f) = 0 . \tag{1.4}$$

The optimal solution is obtained by introducing the 2n-dimensional Hamiltonian system [4]

$$\begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & -B(t)R^{-1}(t)B'(t) \\ -C'(t)Q(t)C(t) & -A'(t) \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \tag{1.5}$$

Denote by $S(t,t_o)$ the 2n x 2n state transition matrix of the system (1.5) with four nxn partitions defined by

$$S(t,t_o) = \begin{bmatrix} \psi(t,t_o) & \Omega(t,t_o) \\ \chi(t,t_o) & \Lambda(t,t_o) \end{bmatrix} . \tag{1.6}$$

If the system (1.1) is completely controllable, then the open loop optimal control $u(t)$ is given by

$$u(t) = -R^{-1}(t)B'(t)[\chi(t,t_o) - \Lambda(t,t_o)\Omega^{-1}(t_f,t_o)\psi(t_f,t_o)]x(t_o). \tag{1.7}$$

The optimal closed-loop control is given by

$$u(t) = +R^{-1}(t)B'(t)\Omega^{-1}(t_f,t)\psi(t_f,t)x(t) . \tag{1.8}$$

The optimal control (1.8) can be represented in the following way:

$$u(t) = -R^{-1}(t)B'(t)P^{-1}(t,t_f)x(t) \tag{1.9}$$

where $P^{-1}(\tau,\sigma)$ is the inverse of a symmetric positive definite matrix $P(\tau,\sigma)$ satisfying

$$-\frac{\partial P(\tau,\sigma)}{\partial \sigma} = -A(\tau)P(\tau,\sigma) - P(\tau,\sigma)A'(\tau) - P(\tau,\sigma)C'(\tau)Q(\tau)C(\tau)P(\tau,\sigma) + B(\tau)R^{-1}(\tau)B'(\tau) , \quad \tau \leq \sigma \tag{1.10}$$

with the boundary condition

$$P(\sigma,\sigma) = 0 . \tag{1.11}$$

The representation (1.9)-(1.11) is not well-known but can be easily obtained from $P(t,t_f) = -\psi^{-1}(t_f,t)\Omega(t_f,t)$. The feedback control law (1.9) is singular at the terminal time, although $u(t_f)$ will be finite as a result of the terminal constraint (1.4).

We note that the Riccati equation (1.10)-(1.11) stems from the following free terminal regulator problem involving the adjoint system of (1.1)-(1.2):

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$$\dot{\hat{x}}(t) = -A'(t)\hat{x}(t) + C'(t)Q^{1/2}(t)\hat{u}(t) \quad (1.12)$$

$$\hat{y}(t) = B'(t)\hat{x}(t) \quad (1.13)$$

with the cost function

$$J(\hat{u}) = \int_{\tau}^{\sigma} [\hat{y}'(t)R^{-1}(t)\hat{y}(t) + \hat{u}'(t)\hat{u}(t)]dt \,. \quad (1.14)$$

The following definition is standard.

Definition: The pair $\{A(t),B(t)\}$ is uniformly completely controllable if for some $\delta_c > 0$ two of the following three conditions are satisfied:

(1) $\alpha_1 I \le W(t,t+\delta_c) \le \alpha_2 I$ for all t (1.15)

(2) $\alpha_3 I \le \Phi(t+\delta_c,t)W(t,t+\delta_c)\Phi'(t+\delta_c,t) \le \alpha_4 I$ for all t (1.16)

(3) $||\Phi(t,\tau)|| \le \alpha_5(|t-\tau|)$ for all t,τ (1.17)

where the controllability matrix $W(t,t+\sigma)$ is defined by

$$W(t_o,t_1) = \int_{t_o}^{t_1} \Phi(t_o,t)B(t)B'(t)\Phi'(t_o,t)dt \,, \quad (1.18)$$

$\Phi(t,t_o)$ is the state transition matrix for $A(t)$, α_1, α_2, α_3 and α_4 are positive constants, and $\alpha_5(\cdot)$ maps R into R and is bounded on bounded intervals.

Uniform complete observability of the pair $\{A(t),C(t)\}$ is defined similarly [1,6] with the observability matrix

$$M(t_o,t_1) = \int_{t_o}^{t_1} \Phi'(t,t_o)C'(t)C(t)\Phi(t,t_o)dt \quad (1.19)$$

and with an appropriate $\delta_o > 0$. In the remainder of the paper a fixed scalar $\delta > 0$ will be assumed to have been selected according to

$$\delta = \text{Max}(\delta_o,\delta_c) \,. \quad (1.20)$$

In this paper a modified control law stemming from the fixed terminal control (1.9) will be compared with the standard regulator problem without an end point constraint for the system (1.1)-(1.2) and cost function (1.3). In the latter case the optimal control law is given by

$$u(t) = -R^{-1}(t)B'(t)K(t,t_f)x(t) \quad (1.21)$$

where $K(t,t_f)$ is obtained from

$$-\frac{\partial K(\tau,\sigma)}{\partial \tau} = A'(\tau)K(\tau,\sigma) + K(\tau,\sigma)A(\tau) - K(\tau,\sigma)B(\tau)R^{-1}(\tau)B'(\tau)K(\tau,\sigma) + C'(\tau)Q(\tau)C(\tau) \quad (1.22)$$

with the boundary condition

$$K(\sigma,\sigma) = 0 \,.$$

It is well known that under uniform complete controllability and observability together with the conditions

$$\alpha_6 I \le Q(t) \le \alpha_7 I \quad \text{and} \quad \alpha_8 I \le R(t) \le \alpha_9 I \quad (1.23)$$

where α_6, α_7, α_8 and α_9 are positive constants, the steady state optimal control law (1.21) with $t_f = \infty$ is uniformly asymptotically stable. But practically speaking, it is very difficult to compute $K(t,\infty)$ due to the infinite integration interval. We will show that a modification of the control law (1.9) is uniformly asymptotically stable and that the gain matrix for this feedback control can be obtained by solving a Riccati equation on a finite time interval. Linear time varying systems are discussed in Section II and time invariant systems in Section III. The corresponding results for discrete time systems will be given in [9].

II. LINEAR TIME VARYING SYSTEMS

There exist very few methods for stabilizing the linear time varying system (1.1), one of which has been mentioned above. We suggest another feedback control which stabilizes (1.1) and is optimal in a certain sense. The following results are needed for the main theorem.

Lemma 1 The two-parameter matrix solution of (1.10)-(1.11) satisfies the following relation

$$P(\tau,\sigma_1) \le P(\tau,\sigma_2) \quad \text{for} \quad \tau \le \sigma_1 \le \sigma_2 \,.$$

That is,

$$\frac{\partial}{\partial \sigma} P(\tau,\sigma) \ge 0, \quad \text{for} \quad \tau \le \sigma \,. \quad (2.1)$$

Proof: From (1.12)-(1.14), we have

$$\hat{x}_o'P(\tau,\sigma_1)\hat{x}_o = \min_{\hat{u}} \int_{\tau}^{\sigma_1} [\hat{x}'(t)B(t)R^{-1}(t)B'(t)\hat{x}(t) + \hat{u}'(t)\hat{u}(t)]dt$$

$$\le \min_{\hat{u}} \int_{\tau}^{\sigma_2} [\hat{x}'(t)B(t)R^{-1}(t)B'(t)\hat{x}(t) + \hat{u}'(t)\hat{u}(t)]dt$$

$$= \hat{x}_o'P(\tau,\sigma_2)\hat{x}_o, \text{ for } \tau \le \sigma_1 \le \sigma_2 \,.$$

Lemma 2 (1) Assume $Q(t)$ and $R(t)$ satisfy (1.23). If $\{A(t),B(t)\}$ is uniformly completely controllable and $\{A(t),C(t)\}$ uniformly completely observable, then for each fixed T satisfying $\delta \le T \le \infty$, where δ is defined by (1.20), there exist positive constants α_{10} and α_{11} such that

$$\alpha_{10} I \le P(t,t+T) \le \alpha_{11} I \quad \text{for all } t.$$

(2) Assume $R(t)$ satisfies (1.23) and $Q(t)$ satisfies $0 \le Q(t) \le \alpha_7 I$. If $\{A(t),B(t)\}$ is uniformly completely controllable and $C(t)$ is bounded such that $||C(t)|| \le L$ for all t, then for all $\delta_c \le T < \infty$ there exist positive scalars α_{12} and α_{13} such that

$$\alpha_{12} I \le P(t,t+T) \le \alpha_{13} I \quad \text{for all } t.$$

Proof: The upper and lower bounds of Part (1) can be found in [1,5]; note that T may be infinite. The upper bound of Part (2) follows from the matrix variation of constant formula and (1.17); note that T is restricted to be finite. Since $P(t,t+T) \ge P(t,t+\delta_c)$, it suffices to check a lower bound of $P(t,t+\delta_c)$. It is known from [12] that

$$P(t,t+\delta_c) \ge \frac{1}{1+||G||} W(t,t+\delta_c)$$

where G is an operator on $[t,t+\delta_c]$ defined by

$$(Gu)(\sigma) = \int_t^\sigma R^{-\frac{1}{2}}(\sigma)B'(\sigma)\Phi'(\tau,\sigma)C'(\tau)Q^{\frac{1}{2}}(\tau)u(\tau)d\tau$$

$$t \le \sigma \le t+\delta_c .$$

But $||G||$ can be shown to satisfy $||G|| \le \alpha_8^{-1}\alpha_7 L^2 n\alpha_2\delta_c$, and is thus bounded independent of t. This completes the proof.

Now introduce the new control law (cf. (1.9))

$$u(t) = -R^{-1}(t)B'(t)P^{-1}(t,t+T)x(t), \quad T \ge \delta_c \tag{2.5}$$

where $P(t,t+T)$ is obtained by integrating (1.10) backward from $\tau = \sigma = t+T$ to $\tau = t$. Bounds for the cost function (1.3) using the control (2.5) are given in the following.

Theorem 1 For the system (1.1)-(1.2) with the control (2.5), the quadratic cost (1.3) has the following bound:

$$x'(t_o)K(t_o,t_1)x(t_o) \le \int_{t_o}^{t_1}[y'(t)Q(t)y(t) + u'(t)R(t)u(t)]dt$$

$$\le x'(t_o)[P^{-1}(t_o,t_o+T) - \Phi_p'(t_1,t_o)P^{-1}(t_1,t_1+T)\Phi_p(t_1,t_o)]x(t_o)$$

$$\le x'(t_o)P^{-1}(t_o,t_o+T)x(t_o) \tag{2.6}$$

where $\Phi_p(t,\tau)$ is the state transition matrix for the system

$$\dot{x}(t) = [A(t) - B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T)]x(t).$$

Proof: The lower bound is obvious. Let $F(t) \equiv A(t) - B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T)$. For the feedback control law (2.5), the quadratic cost is given by $x_o'N(t_o,t_1)x_o$ where $N(t_o,t_1)$ is the solution of the matrix Riccati equation

$$-\frac{d}{dt}N(t,t_1) = F'(t)N(t,t_1) + N(t,t_1)F(t) + P(t,t+T)B(t)R^{-1}(t)B'(t)P(t,t+T) + C'(t)Q(t)C(t) \tag{2.7}$$

with the boundary condition $N(t_1,t_1) = 0$. Also from (1.10) we have that

$$-\frac{d}{dt}P^{-1}(t,t+T) = F'(t)P^{-1}(t,t+T) + P^{-1}(t,t+T)F(t) + P^{-1}(t,t+T)B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T) + C'(t)Q(t)C(t) + P^{-1}(t,t+T)[\frac{\partial}{\partial\sigma}P(t,\sigma)\Big|_{\sigma = t+T}]P^{-1}(t,t+T) . \tag{2.8}$$

Let $E(t) \equiv N(t,t_1) - P^{-1}(t,t+T)$. Then we have

$$-\dot{E}(t) = F'(t)E(t) + E(t)F(t) - P^{-1}(t,t+T)[\frac{\partial}{\partial\sigma}P(t,\sigma)\Big|_{\sigma = t+T}]P'(t,t+T) \tag{2.9}$$

with the boundary condition $E(t_1) = -P^{-1}(t_1,t_1+T)$. From (2.9) we have

$$N(t_o,t_1) - P^{-1}(t_o,t_o+T) = E(t_o) = -\Phi_p'(t_1,t_o)P^{-1}(t_1,t_1+T)\Phi_p(t_1,t_o) - \int_{t_o}^{t_1}\Phi_p'(\tau,t_o)P^{-1}(\tau,\tau+T)[\frac{\partial}{\partial\sigma}P(\tau,\sigma)\Big|_{\sigma = \tau+T}]P^{-1}(\tau,\tau+T)\Phi_p(\tau,t_o)d\tau$$

$$\le -\Phi_p'(t_1,t_o)P^{-1}(t_1,t_1+T)\Phi_p(t_1 t_o). \tag{2.10}$$

The last inequality follows from Lemma 1. This completes the proof.

Note that Theorem 1 says

$$\int_{t_o}^{t_1}\{y'(t)Q(t)y(t) + u'(t)R(t)u(t)\}dt \le x'(t_o)P^{-1}(t_o,t_o+T)x(t_o) - x(t_1)P^{-1}(t_1,t_1+T)x(t_1) . \quad (2.11)$$

Implications of Theorem 1 will be discussed after we prove the stability property of (2.5) in Theorem 2.

Theorem 2 (1) Assume $Q(t)$ and $R(t)$ satisfy (1.23). If $\{A(t),B(t)\}$ and $\{A(t),C(t)\}$ are uniformly completely controllable and observable, respectively, then the system (1.1) with the feedback control law (2.5) is uniformly asymptotically stable for $\delta \le T \le \infty$.

(2) Assume $R(t)$ satisfies (1.23) and $Q(t)$ satisfies $0 \le Q(t) \le \alpha_7 I$. If $\{A(t),B(t)\}$ is uniformly completely controllable, and if $C(t)$ is bounded such that $||C(t)|| \le \alpha_{14}$ for all t for some finite constant α_{14}, then the system (1.1) with feedback control (2.5) and $\delta_c \le T < \infty$ is uniformly asymptotically stable. (Note: $Q(t)$ and $C(t)$ can be identically zero.)

Proof: Consider the adjoint system of (1.1) with the control law (2.5):

$$\dot{\hat{x}}(t) = -[A(t)-B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T)]'\hat{x}(t) \equiv -F'(t)\hat{x}(t), \quad (2.12)$$

and the associated scalar valued function

$$V(\hat{x},t) = \hat{x}'P(t,t+T)\hat{x} \quad (2.13)$$

From Lemma 2, $V(\hat{x},t)$ satisfies

$$\alpha_{10}|\hat{x}|^2 \le V(\hat{x},t) \le \alpha_{11}|\hat{x}|^2 \quad (2.14)$$

under the conditions of Part (1), or a similar inequality involving $(\alpha_{12},\alpha_{13})$ under Part (2), so that $V(\hat{x},t)$ is a positive definite function of $\hat{x}$ under either set of conditions. Since $\dot{x}(t) = F(t)x(t)$ can be shown to be uniformly exponentially decreasing if and only if the adjoint system (2.12) is uniformly exponentially increasing, it suffices to establish asymptotic exponential instability of (2.12) using (2.13). By direct calculation it is found that $\frac{dV(\hat{x}(t),t)}{dt}$ along solution curves of (2.12) is given by

$$\frac{dV(\hat{x}(t),t)}{dt} = \hat{x}'[B(t)R^{-1}(t)B'(t) + P(t,t+T)C'(t)Q(t)C(t)P(t,t+T) + \left.\frac{\partial P(t,\sigma)}{\partial\sigma}\right|_{\sigma=t+T}]\hat{x} \ge 0. \quad (2.15)$$

(The inequality follows from Lemma 1.) Hence, solutions of (2.12) are nondecreasing. From (2.15),

$$V(\hat{x}(t_1;\hat{x}_o,t_o),t_1) - V(\hat{x}_o,t_o) \ge \int_{t_o}^{t_1}\hat{x}'(t)B(t)R^{-1}(t)B'(t)\hat{x}(t)dt = \hat{x}_o'\int_{t_o}^{t_1}\phi_p(t_o,t)B(t)R^{-1}(t)B'(t)\phi_p'(t_o,t)dt\ \hat{x}_o . \quad (2.16)$$

Thus, if it can be shown that the pair $\{A(t) - B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T), B(t)\}$ is uniformly completely controllable, then there will exist a positive constant α_{15} such that

$$V(\hat{x}(t_1;\hat{x}_o,t_o),t_1) - V(\hat{x}_o,t_o) \ge \alpha_{15}|\hat{x}_o|^2 \quad \text{for } t_1 \ge t_o + \delta_c . \quad (2.17)$$

In turn, this will prove both parts of Theorem 2 since (2.17) implies that solutions of (2.12) are exponentially increasing and, hence, that solutions of the closed loop system $\dot{x}(t) = F(t)x(t)$ are exponentially decreasing. Using results in [6], the uniform complete controllability of the pair in question, i.e., the pair $\{A(t) - B(t)R^{-1}(t)B'(t)P^{-1}(t,t+T),B(t)\}$, is assured if it can be shown that

$$\int_t^{t+\delta_c}||R^{-1}(\tau)B'(\tau)P^{-1}(\tau,\tau+T)||^2d\tau \le \alpha_{16} \quad (2.18)$$

for some positive constant α_{16}. But

$$\int_t^{t+\delta_c}||R^{-1}(\tau)B'(\tau)P^{-1}(\tau,\tau+T)||^2d\tau \le \alpha_9^{-1}\alpha_{13}^{-1}\int_t^{t+\delta_c}||B'(\tau)||^2d\tau .$$

Furthermore, uniform complete controllability of the pair $\{A(t),B(t)\}$ implies

$$n\alpha_2 \ge \int_t^{t+\delta_c}tr[\phi(t,\tau)B(\tau)B'(\tau)\phi'(t,\tau)]d\tau = \int_t^{t+\delta_c}tr[B(\tau)B'(\tau)\phi'(t,\tau)\phi(t,\tau)]d\tau$$

$$\geq \int_t^{t+\delta_c} tr[B(\tau)B'(\tau)]\lambda_{min}(\Phi'(t,\tau)\Phi(t,\tau))d\tau$$

$$\geq \int_t^{t+\delta_c} tr[B(\tau)B'(\tau)]\lambda_{max}^{-1}(\Phi(\tau,t)\Phi'(\tau,t))d\tau$$

$$\geq \begin{bmatrix} \sup \alpha_5(\rho) \\ 0 \leq \rho \leq \delta_c \end{bmatrix}^{-2} \int_t^{t+\delta_c} ||B'(\tau)||^2 d\tau \qquad (2.19)$$

where the last inequality follows from (1.17). This finishes the proof.

The control law (2.5) has been obtained by a slight modification of the optimal control (1.9). Since $x(t_1) \to 0$ as $t_1 \to \infty$ under the control law (2.5), the cost function of Theorem 1 can be bounded for $t_1 \to \infty$ as follows:

$$x'(t_o)K(t_o,\infty)x(t_o) \leq \int_{t_o}^{\infty} [y'(t)Q(t)y(t) + u'(t)R(t)u(t)]dt \leq x'(t_o)P^{-1}(t_o,t_o+T)x(t_o). \qquad (2.20)$$

Also, under the conditions of Theorem 2(1) it can be seen that $\lim_{T\to\infty} \frac{\partial P(t,\sigma)}{\partial \sigma}\Big|_{\sigma = t+T} = 0$ since for fixed τ, $P(\tau,\sigma)$ is nondecreasing and bounded with respect to σ. Thus, $P^{-1}(t,\infty)$ and $K(t,\infty)$ satisfy the same matrix Riccati differential equation which implies that the control law (2.5) can be regarded as a suboptimal feedback control law to the infinite time regulator problem by choosing a sufficiently large T. However, it might be expected that (2.5) is optimal for some cost function since it is asymptotically stabilizing and is defined by an underlying Riccati equation. After careful inspection it can be deduced that (2.5) is the optimal control for the system (1.1)-(1.2) and the cost function

$$J(u) = \int_{t_o}^{\infty} [x'(t)H(t)x(t) + u'(t)R(t)u(t)]dt \qquad (2.21)$$

where

$$H(t) = C'(t)Q(t)C(t) + P^{-1}(t,t+T)\left[\frac{\partial P(t,\sigma)}{\partial \sigma}\Big|_{\sigma = t+T}\right] P^{-1}(t,t+T). \qquad (2.22)$$

Note that the weighting matrix $H(t)$ cannot be given a priori but rather a posteriori by a solution to (1.10). Of course, the control (2.5) is optimal for the system (1.1)-(1.2) with the moving cost function

$$J(u) = \int_t^{t+T} [y'(\tau)Q(\tau)y(\tau) + u'(\tau)P(\tau)u(\tau)]d\tau$$

and the moving terminal constraint $x(t+T) = 0$. This type of problem has been briefly discussed in [7].

III. LINEAR TIME INVARIANT SYSTEMS

In this section consider the fixed system

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = x_o \qquad (3.1)$$
$$y(t) = Cx(t)\ ,$$

where (A,B,C) are constant matrices, together with the cost function

$$J(u) = \int_o^{t_f} [y'(t)Qy(t) + u'(t)Ru(t)]dt \qquad (3.3)$$

where $Q \geq 0$ and $R > 0$ are constant weighting matrices. If the pair $\{A,B\}$ is completely controllable, the minimization of (3.3) subject to the end point constraint $x(t_f) = 0$ leads to the optimal feedback control law (cf. (1.9))

$$u(t) = -R^{-1}B'P^{-1}(t_f-t)x(t) \qquad (3.4)$$

where $P^{-1}(\tau)$ is the inverse of a symmetric positive matrix $P(\tau)$ satisfying

$$\frac{dP(\tau)}{d\tau} = -AP(\tau)-P(\tau)A'-P(\tau)C'QCP(\tau)+BR^{-1}B'$$
$$P(o) = 0. \qquad (3.5)$$

The result analogous to Theorem 2 for this case is contained in the following.

<u>Theorem 3</u> If the pair $\{A,B\}$ is completely controllable, then the system (3.1) with the fixed gain feedback control law

$$u(t) = -R^{-1}B'P^{-1}(T)x(t) \qquad (3.6)$$

is asymptotically stable, where $P(T)$ for any fixed $T > 0$ can be obtained by solving (3.5) corresponding to any chosen pair (Q,R) with $Q \geq 0$ and $R > 0$. With the additional condition that the pair $\{A,C\}$ is detectable and $Q > 0$, the result holds with $T = \infty$.

Proof: Although the proof is merely a specialization of Theorem 2 to the time invariant case, it is noted that a direct proof of asymptotic stability can be given in this case using the Lyapunov function $V(\hat{x}) = \hat{x}'P(T)\hat{x}$ and the system $\dot{\hat{x}}(t) = [A-BR^{-1}B'P^{-1}(T)]'\hat{x}(t)$.

The control law (3.6) is a generalization of a stabilizing feedback control given by Kleinman [2], involving the inverse of the controllability Gramian, in that the result of [2] is obtained by choosing $Q = 0$, i.e., if $Q = 0$, $P(T)$ is given by

$$P(T) = \int_0^T e^{-At} BR^{-1} B' e^{-A't} dt . \quad (3.7)$$

IV. CONCLUDING REMARKS

In the case of time invariant systems, the modified control law (3.6) can be interpreted as a practical way to avoid the singularity at the terminal time of the optimal control (3.4) when the argument $(t_f - t)$ in $P^{-1}(t_f - t)$ is frozen at some time $T = t_f - t > 0$. The important consideration is that such expediencies still render the resulting feedback control law asysmptotically stable. Similar considerations apply to the comparison between the control laws (1.9) and (2.5) which pertain to the time varying system (1.1)-(1.2). An advantage of the control law (2.5) for time varying systems is that the stabilizing feedback gains are obtained by integrating a Riccati equation backward in time over a finite interval, rather than an infinite time interval. The control law (3.6) for time invariant systems generalizes a well known method of feedback stabilization due to Kleinman, and can be interpreted as providing a means for weighting the state in the cost function by choosing $Q \neq 0$ in (3.3) and (3.5). Further results can be obtained, including extensions of results involving a nonlinear feedback control law in [3], stabilization with a prescribed degree of stability [11], and a dual problem in filtering theory.

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This paper considers a feedback control law for linear time-varying and time invariant systems based on a modified minimum energy problem with fixed terminal constraints. The modified control laws are shown to be optimal for a certain cost function, asmyptotically stable, and to result in a new method for stabilizing linear time-varying systems as well as extending some well known methods for stabilizing time invariant systems. In particular, the stabilizing gains of the feedback control laws are obtained from the solution of a Riccati equation over an arbitrary finite time interval, which is relatively easy to compute.

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