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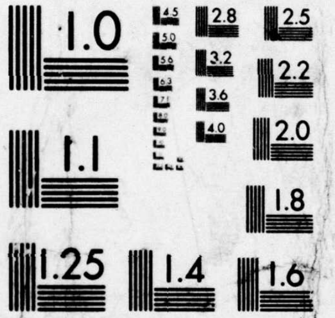
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This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In a previous report, the history of Complementary Pair Element Groups was discussed, and the application of this concept to the matching of electrically small antennas was shown. The measurements indicated a substantial improvement in efficiency as compared to matching with a simple load isolator absorbing the reflected power.

In this report, the power delivered to the external region is analyzed for both an individual monopole, and a pair of monopoles, by establishing the scattering matrices for both cases. The potential gain is calculated for the

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19. KEY WORDS (*Continued*)

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pair over the individual monopole, and it is shown that the pair can deliver up to twice the power to the exterior region if the mutual impedance term is optimized. Various special cases are discussed, varying from full complementarity to zero complementarity, and general conclusions are presented on the optimization of gain-bandwidth product.

Various types of decoys and ECM systems for aircraft and missiles are examples of potential applications of electrically small antennas and scatterers.

Large, hardened, ground-based phased arrays and wideband communication base station antennas are typical phased-array applications in fixed stations. Vehicular applications encompass wideband direction finders, aircraft conformal antennas for various functions, and high-performance spacecraft antennas for communications and radar systems.

CONTENTS

FIGURES

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I. INTRODUCTION

The complementary pair element group (CPEG) antenna has been described as a broadband antenna [1]. This configuration was originally conceived as a broadband matching system for elements in front of a reflector, or in a phased array environment. Recently an electrically small complementary pair (ESCP) was designed [2]. The antenna system was configured as two very fat identical monopoles above a ground plane. The input impedances were made complementary by using unequal cable lengths between each monopole and the hybrid. The monopole impedances were made to be exactly complementary with respect to the hybrid outputs at only one frequency, f_o, that is, when the differential cable length was exactly $\lambda/4$.

Measurements have shown that the feed (sum) port of the hybrid remains well matched, even at frequencies where the single monopole becomes very poorly matched (Fig. 1). How much of this decrease in reflected power has actually been converted to a gain in radiated power was investigated by observing the power dissipated in the hybrid difference port load, and in the hybrid itself [2]. These loss factors were then combined to form a "total matching efficiency". This total efficiency for the ESCP was shown to be substantially higher than for the isolated monopole for frequencies below f_o (Fig. 2). However, as was pointed out in [2] the actual radiation efficiency could not be determined exactly without a measurement of antenna gain, since it must also include I^2R losses in cables and elements.

It can be shown that if the monopoles were completely uncoupled, then the amount of power dissipated in the hybrid difference port is equal to the decrease in reflected power. This would produce no net increase in radiation efficiency. However, the experimental results of both [1] and [2] indicated that the Complementary Pair achieves maximum gain-bandwidth product when the monopoles are in proximity. In the present paper an attempt is made to analyze the coupled case and to find some theoretical bounds for radiation efficiency.

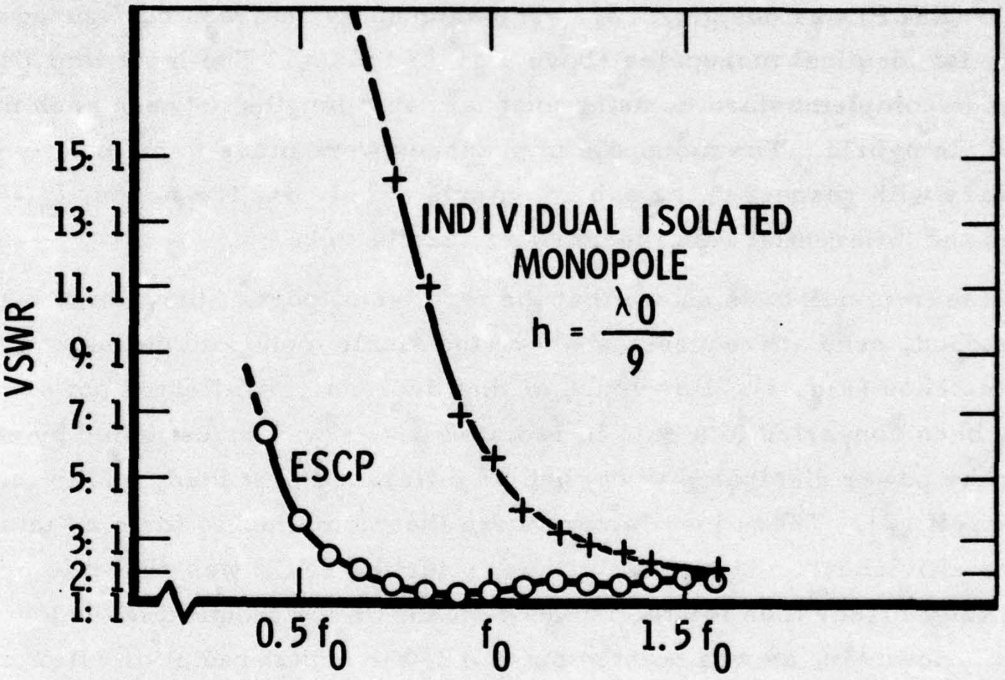

Fig. 1. ESCP Input Impedance Match

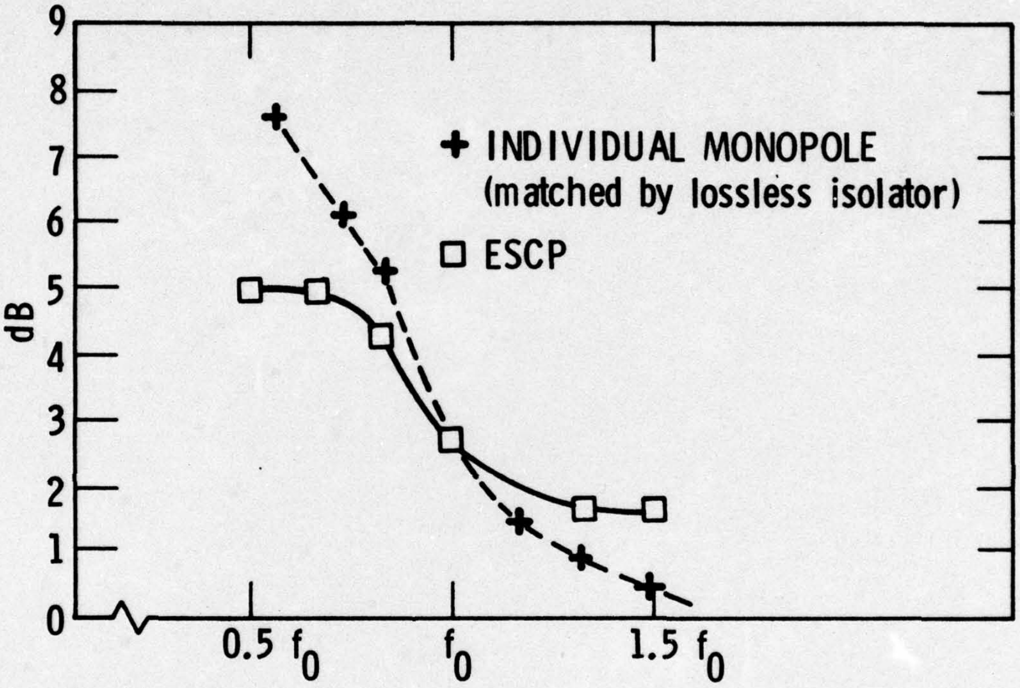

Fig. 2. ESCP Matching Loss

II. DESCRIPTION OF CONFIGURATION

A schematic drawing of the ESCP configuration is shown in Fig. 3. Energy is fed into port 2 (the sum port). Port 4 (the difference port) is terminated in a matched load. Ports 1 and 3 feed the two identical monopoles through a differential cable length ℓ. This configuration is to be compared with the configuration shown in Fig. 4, where one monopole is fed directly by the source and the other is terminated in a matched load. The advantage in comparing these two configurations is obvious. The exterior regions (i.e., the region above the ground plane) for the two configurations are identical. Therefore, we have reduced the problem to strictly a comparison of feeding techniques. However, the answers might also be applicable to the alternate problem of comparing the ESCP to an isolated monopole, since it has been found that for the electrically small case the measured VSWRs for the configuration shown in Fig. 4 and for a single isolated monopole are approximately equal.

In Fig. 3, the normalized amplitudes of waves entering and leaving the i^{th} hybrid port are a_i and b_i, respectively. The normalization is such that $\frac{1}{2} |a_2|^2$ is the incident power in watts. The incident power will be partially reflected, partially absorbed in the fourth port, and partially delivered to the monopoles. In Fig. 4, the incident power, $\frac{1}{2} |a_o|^2$, will be partially reflected, partially dissipated in the matched load, and partially delivered to the monopole. All of these quantities will be computed in the next section, and the two configurations will be compared according to the net power each can deliver to the exterior region (i.e., the power available for radiation).

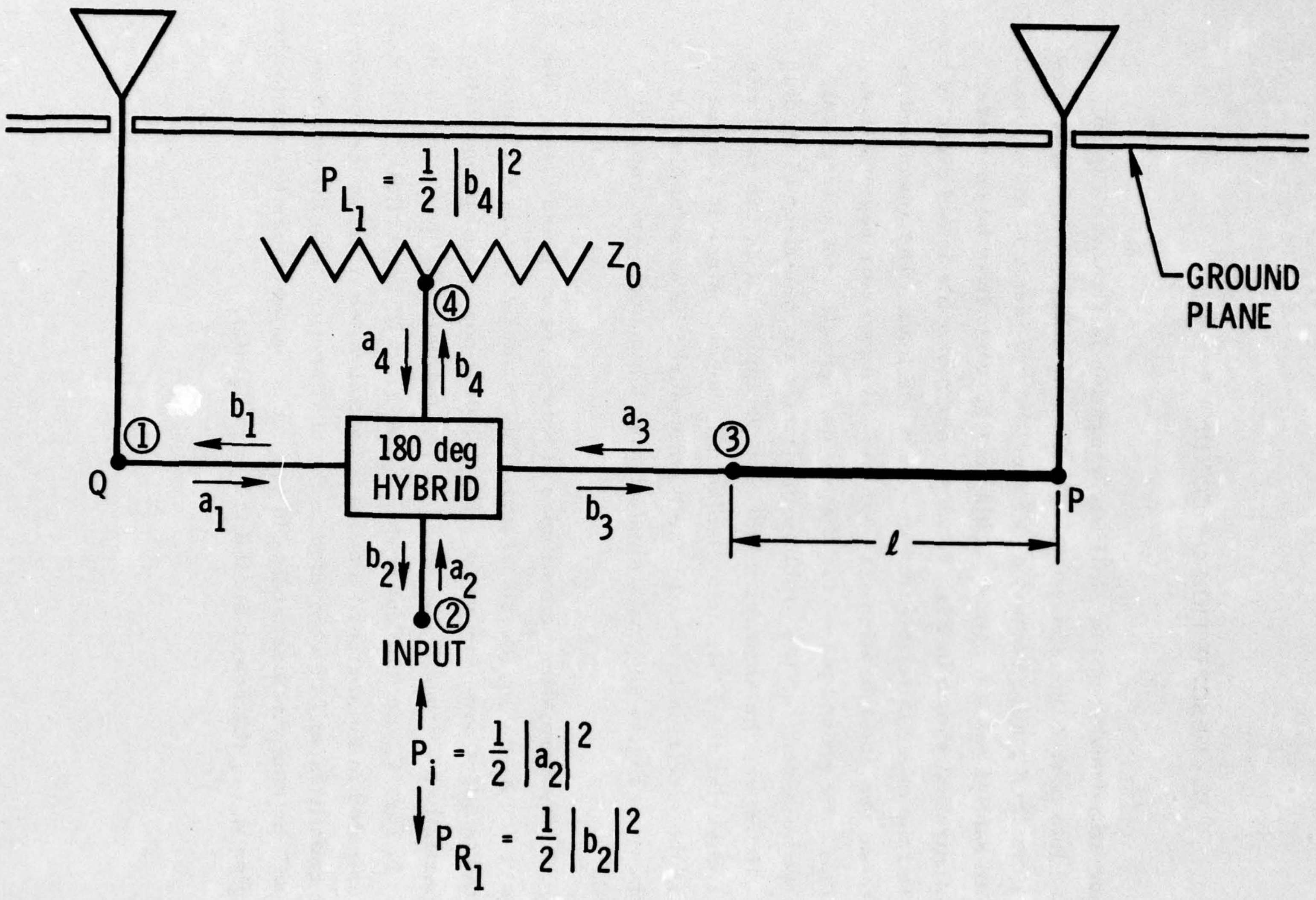

Fig. 3. ESCP Feed Network Schematic

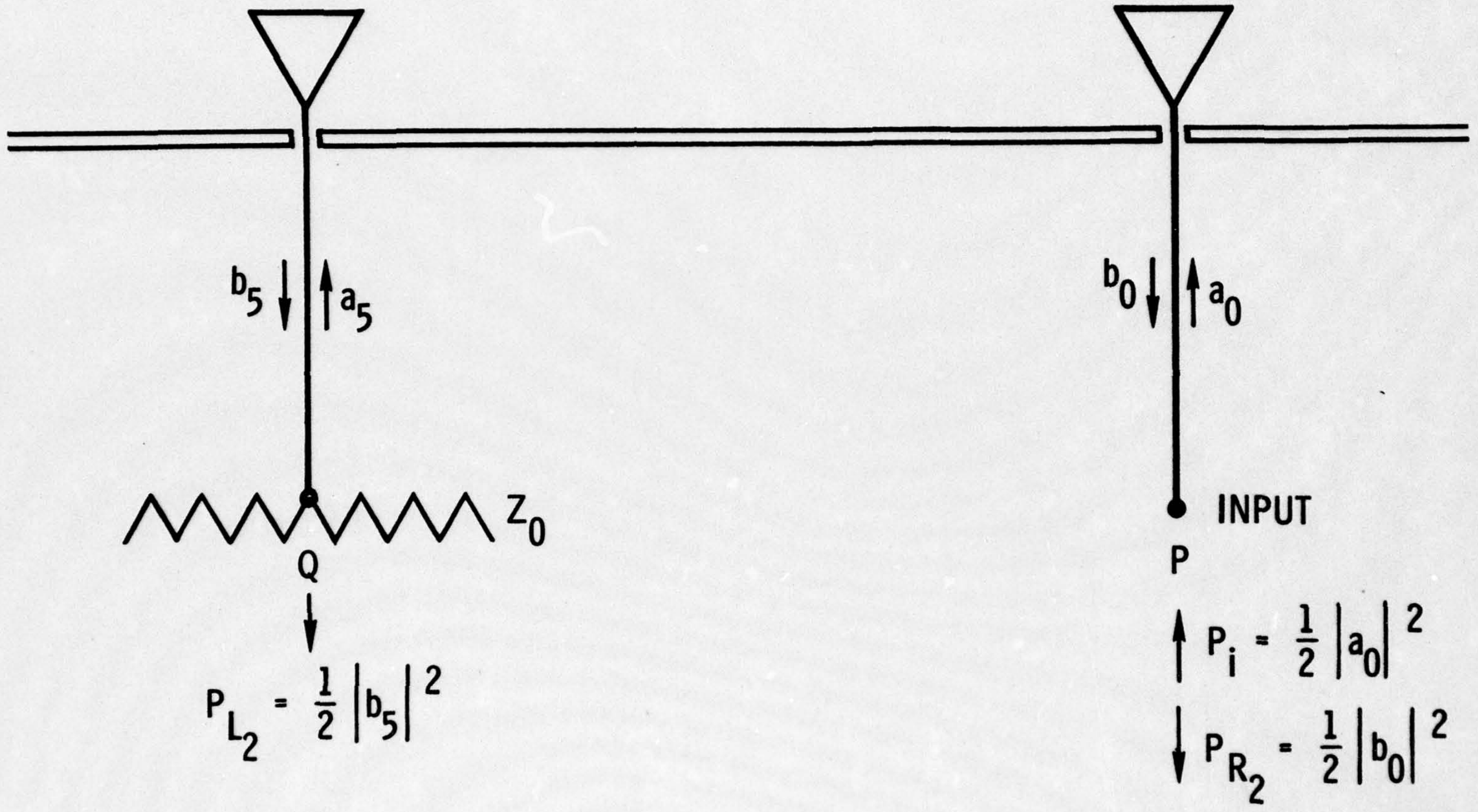

Fig. 4. Directly Fed Monopole

III. MATHEMATICAL DERIVATION

The ESCP will be considered first. It will be assumed that the hybrid, which often is referred to as a "magic T", is ideal and lossless, and that the fourth port is perfectly matched. Then it is possible to relate the incident and scattered wave amplitudes by a scattering matrix as follows [3]:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{j}{2} \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \end{bmatrix} \tag{1}$$

It is noted that a_4 has been set equal to zero, since the fourth port by assumption is perfectly matched. The incident and scattered wave amplitudes at ports 1 and 3 are related through the self and mutual impedances of the monopoles. As shown in Fig. 5 one can define a scattering matrix for the exterior region. That is, the exterior region can be represented by a two-port network as viewed from the reference terminals P and Q. Furthermore, only two quantities, S_{11} and S_{13}, are needed to describe this network, because of reciprocity and identical monopoles. In general S_{11} and S_{13} are quite difficult to calculate, but for the present we shall treat these quantities as if they were known. We therefore write

$$\begin{bmatrix} a_1 \\ a_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{13}e^{-j\beta \ell} \\ S_{13}e^{-j\beta \ell} & S_{11}e^{-2j\beta \ell} \end{bmatrix} \begin{bmatrix} b_1 \\ b_3 \end{bmatrix} \tag{2}$$

It should be obvious that the differential cable length ℓ has been incorporated into the scattering matrix. Also, $\beta = 2\pi/\lambda$ is the free space wave number.

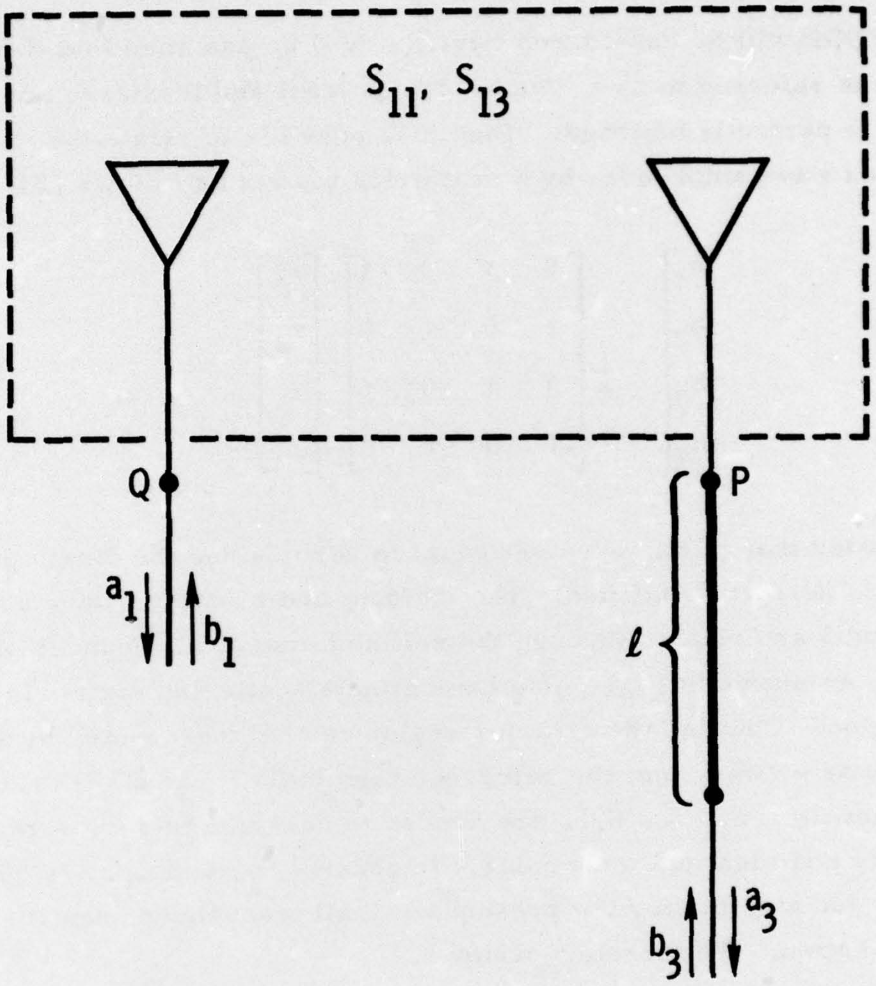

Fig. 5. Network Representation of Exterior Region

Equations (1) and (2) represent a set of six equations that may be solved for the six unknown wave amplitudes in terms of the incident wave at port 2. Of particular interest are b_2 and b_4, since these represent the reflected and fourth port waves respectively. The results are

$$b_2 = -a_2\, e^{-j\beta \ell} \left[S_{11} \cos \beta \ell + S_{13}\right] \tag{3}$$

$$b_4 = ja_2\, e^{-j\beta \ell}\, S_{11} \sin \beta \ell \tag{4}$$

Now consider the configuration of Fig. 4, which hereafter shall be referred to as the directly fed antenna. Since its exterior region is identical to the exterior region of the ESCP, it should satisfy the same scattering matrix. Therefore

$$\begin{bmatrix} b_0 \\ b_5 \end{bmatrix} \begin{bmatrix} S_{11} & S_{13} \\ S_{13} & S_{11} \end{bmatrix} \begin{bmatrix} a_0 \\ 0 \end{bmatrix} \tag{5}$$

Again a_5 has been set equal to zero because of the assumption that the second monopole is terminated in a matched load. Equation (5) by inspection gives

$$b_0 = S_{11} a_0 \tag{6}$$

$$b_5 = S_{13} a_0 \tag{7}$$

Now define the following quantities:

$P_i = \frac{1}{2} \mid a_2 \mid^2 = \frac{1}{2} \mid a_0 \mid^2$ = incident power for both ESCP and directly fed cases

$P_{R_1} = \frac{1}{2} | b_2 |^2$ = reflected power for ESCP

$P_{L_1} = \frac{1}{2} | b_4 |^2$ = power dissipated in port 4 for ESCP case

$P_{R_2} = \frac{1}{2} | b_0 |^2$ = reflected power for directly fed case

$P_{L_2} = \frac{1}{2} | b_5 |^2$ = power dissipated in second monopole for directly fed case

substituting Eqs. (3), (4), (6) and (7) into the above definitions gives

$$P_{R_1} = P_i \left[| S_{11} |^2 \cos^2 \beta \ell + | S_{13} |^2 + 2\mathrm{Re} \left\{ S_{11} S_{13}^* \right\} \cos \beta \ell \right] \tag{8}$$

$$P_{L_1} = P_i \, | S_{11} |^2 \sin^2 \beta \ell \tag{9}$$

$$P_{R_2} = P_i \, | S_{11} |^2 \tag{10}$$

$$P_{L_2} = P_i \, | S_{13} |^2 \tag{11}$$

It is noted that P_{L_1} is independent of coupling. Now let P_{A_1} and P_{A_2} be the power delivered to the exterior region for the ESCP and directly fed cases, respectively. Then from conservation of energy

$$P_{A_{1,2}} = P_i - P_{R_{1,2}} - P_{L_{1,2}} \tag{12}$$

Finally define the gain G to be the ratio of P_{A_1} to P_{A_2}. Substituting Eqs. (8) - (11) into (12) gives

$$G = \frac{1 - | S_{11} |^2 - | S_{13} |^2 - 2 \, \mathrm{Re} \left\{ S_{11} S_{13}^* \right\} \cos \beta \ell}{1 - | S_{11} |^2 - | S_{13} |^2} \tag{13}$$

It can be seen that G may be greater or less than unity depending on the sign of the last term in the numerator. The above equation will be examined in greater detail in the next section.

IV. SPECIAL AND GENERAL CASES

A. SPECIAL

1. EXACTLY COMPLEMENTARY PAIRS

For the impedances to be exactly complementary the differential cable length must be such that $\beta \ell = \pi/2$. Equations (8) and (9) then become

$$P_{R_1} = P_i \, |S_{13}|^2 \tag{14}$$

$$P_{L_1} = P_i \, |S_{11}|^2 \tag{15}$$

and Eq. (13) reduces to G = 1. Thus, for this special case the hybrid has simply redirected the power flow. The power that was originally reflected in the directly fed case is now dissipated in the matched load of the fourth hybrid port. The power that was originally dissipated in the matched load of the second monopole is now the reflected power. The net result is that both cases deliver the same amount of power to the exterior region.

2. NO COUPLING

When there is no coupling, $S_{13} = 0$, and Eqs. (8) and (9) reduce to

$$P_{R_1} = P_i \, |S_{11}|^2 \cos^2 \beta \ell \tag{16}$$

$$P_{L_1} = P_i \, |S_{11}|^2 \sin^2 \beta \ell \tag{17}$$

And Eq. (13) reduces to G = 1. Again there is merely a redistribution of power in the ESCP case such that there is no increase in net power delivered to the exterior region. If in addition $\beta \ell = \pi/2$, then it can be seen that all of the power reflected in the directly fed case can now be accounted for in the fourth hybrid port.

3. NO DIFFERENTIAL LINE LENGTH

When the differential line length is zero, Eqs. (8), (9) and (13) become

$$P_{R_1} = P_i \left| S_{11} + S_{13} \right|^2 \tag{18}$$

$$P_{L_1} = 0 \tag{19}$$

$$G = \frac{1 - \left| S_{11} + S_{13} \right|^2}{1 - \left| S_{11} \right|^2 - \left| S_{13} \right|^2} \tag{20}$$

Now since $\left| S_{11} + S_{13} \right|^2 \leq \left| S_{11} \right|^2 + \left| S_{13} \right|^2$, it is obvious that for this case $G \geq 1$. Thus we have found a case in which the ESCP appears to be superior to the directly fed antenna. Interestingly enough, this is a case in which the impedances at P and Q are not complementary, but identical.

B. GENERAL

For the general case it can be seen that G given by Eq. (13) is greater than unity whenever $\mathrm{Re}\left\{S_{11}S_{13}^{*}\right\} < 0$. It can be shown that $\mathrm{Re}\left\{S_{11}S_{13}^{*}\right\} < 0$ for parallel thin dipoles, spaced less than $\lambda/10$ apart. If this were also true for fat monopoles, then $G > 1$ whenever $\beta \ell < \pi/2$ and $G < 1$ whenever $\beta \ell > \pi/2$. This tends to agree with the findings of [1] and [2] where the differential line lengths were designed to be $\lambda/4$ at the center frequency f_o leading to improved matching efficiency at frequencies below f_o.

Of course, G cannot be computed unless S_{11} and S_{13} are known. However, there are restrictions on scattering matrices in general which up to now have not been considered. One is the condition of physical realizability, which states that the scattering matrix of any network that dissipates real power must satisfy the relationship [4]

$$\mathrm{Det}\left\{I - S^{*}S\right\} \geq 0 \tag{21}$$

where I is the identity matrix. Now, taking S to be the matrix defined by Eq. (2), and substituting into Eq. (21) gives

$$[1-|S_{11}|^2-|S_{13}|^2]^2 - 4[\mathrm{Re}\,\{S_{11}S_{13}^*\}]^2 \geq 0$$

or

$$2\,|\mathrm{Re}\,\{S_{11}S_{13}^*\}| \leq |1-|S_{11}|^2-|S_{13}|^2| \tag{22}$$

Thus there is an upper bound on $|\mathrm{Re}\,\{S_{11}S_{13}^*\}|$ given by Eq. (22). Furthermore, since the maximum value of $\cos \beta \ell$ is unity, Eq. (22) imposes an upper bound on G given by

$$G \leq \frac{1-|S_{11}|^2-|S_{13}|^2+|1-|S_{11}|^2-|S_{13}|^2|}{1-|S_{11}|^2-|S_{13}|^2}$$

or

$$G \leq 2 \tag{23}$$

We have therefore determined that the CPEG gives at most a 3-dB improvement in the amount of power delivered to the exterior region, due to mutual coupling.

V. CONCLUSIONS

The following conclusions have been drawn from the analysis presented:

1. The ESCP may be very effective in reducing the reflection loss from an electrically short monopole. It may also be effective in converting a portion of this loss into radiated power. However, the improvement in net power delivered to the elements can be no more than 3 dB.
2. Without mutual coupling there can be no improvement in net power delivered to the antenna elements. In this case the hybrid simply acts as an isolator, with varying degrees of mismatch improvement depending on the degree of complementarity. A definite advantage over a load isolator, of course, is the fact that the hybrid is bidirectional.
3. The ESCP may be most effective in reducing reflections when the impedances are exactly complementary, but for this case there is also no improvement in net power delivered to the antenna. However, for the case of an externally complementarized endfire pair, such as described here, using differential cable lengths, the element spacing can be chosen so as to maximize the array factor directive gain, and the degree of complementarity can be optimized.
4. The best configuration from the standpoint of maximum gain without any directivity is to have the element impedances equal with respect to the hybrid output ports, but for this case the power reflected may remain large.

The overall conclusion is that in a broadband ESCP configuration the same conditions apply as already found for the (large) CPEG in [1], i.e., at the lowest operating frequency the two impedances seen by the hybrid are most nearly equal, and hence maximum improvement in matching efficiency occurs, whereas at the center frequency (where near-perfect complementarity exists) this improvement is not required, because the directive gain of the pair may approach 3 dB. The optimum compromise is thus one where just enough complementarity is provided at the lowest operating frequency to achieve an acceptable input impedance match.

Very small length-to-diameter ratios (high "fatnesses"), of the two elements are necessary to assure the coupling required for this improvement, with L/D ratios of less than unity being typical. Such an optimized configuration can yield substantial improvements in gain-bandwidth product, but at the expense of having a directional pattern over much of this bandwidth.

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