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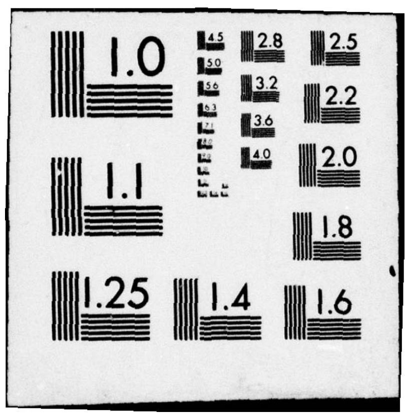
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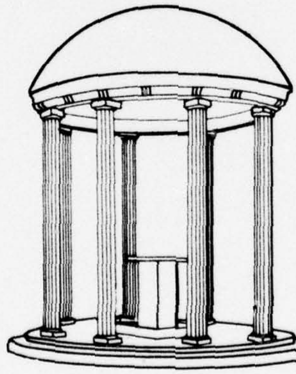
THE POWER APPROXIMATION

CONTROL OF MULTI-ITEM INVENTORY SYSTEMS WITH CONSTANT STANDARD-DEVIATION-TO-MEAN RATIO FOR DEMAND

YALE UNIVERSITY, NEW HAVEN, CONNECTICUT

NOVEMBER 1976

UNIVERSITY OF NORTH CAROLINA
AT CHAPEL HILL



SCHOOL OF BUSINESS ADMINISTRATION
AND
CURRICULUM IN
OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

THE POWER APPROXIMATION: CONTROL OF MULTI-ITEM INVENTORY SYSTEMS WITH CONSTANT STANDARD-DEVIATION-TO-MEAN RATIO FOR DEMAND

Technical Report #10

John G. Klincewicz*

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Decision Control Models in Operations Research

Work Performed under the Direction of

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this simulation study, we consider the performance of the Power Approximation Ehrhardt (1976) for a multi-item inventory system in which the underlying demand distributions are negative binomial with constant standard-deviation-to-mean ratio of 1.

The Power Approximation formulas Ehrhardt (1976) were obtained by least squares regression, using data from inventory items with Poisson demand distributions and negative binomial demand distributions with variance-to-mean ratios of 3 and 9. Previous experiments have all considered the effectiveness

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) of the Approximation for systems with constant variance-to-mean ratio.

We examine the constant standard-deviation-to-mean system for the situation of full information and for the situation in which the decision-maker's knowledge is limited to a sample of previously-realized demands. In addition, the research examines the accuracy of statistical forecasts that predict the future behavior of the operating characteristics. As a result, an inventory systems designer is apprised of both the costs of imperfect information and the extent of bias in the forecast estimates. In general, the behavior observed for the constant standard-deviation-to-mean system is comparable to the behavior observed for the constant standard-deviation-to-mean system is comparable to the behavior of the previously studied systems with constant variance-to-mean ratios of 9.

FOREWORD

As part of the on-going research program in "Decision and Control Models in Operations Research, Mr. John G. Klincewicz has investigated the behavior of multi-item inventory control systems in which the underlying demand distributions are estimated from a limited sample of historical observations. His work, which carries on from that of Mr. Richard Ehrhardt (Technical Report No. 7), utilizes demand distributions with constant standard deviation-to-mean ratio of unity. Ehrhardt's previous experiments with Power Approximation control considered systems with constant variance-to-mean ratios. To assist the reader in comprehending the results, Klincewicz presents his findings in tables that also exhibit the corresponding results of Ehrhardt; hence, this report is nearly self-contained.

Other related reports dealing with this research program are given below.

Harvey M. Wagner

Principal Investigator

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MacCormick, A. (1975), Statistical Problems in Inventory Control, ONR and ARO Technical Report, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 1), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, School of Organization and Management, Yale University, 58 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, School of Organization and Management, Yale University, 52 pp.

ABSTRACT

THE POWER APPROXIMATION: CONTROL OF MULTI-ITEM INVENTORY SYSTEMS WITH CONSTANT STANDARD-DEVIATION-TO-MEAN RATIO FOR DEMAND

John G. Klincewicz

In this simulation study, we consider the performance of the Power Approximation [Ehrhardt (1976)] for a multi-item inventory system in which the underlying demand distributions are negative binomial with constant standard-deviation-to-mean ratio of 1.

The Power Approximation formulas [Ehrhardt (1976)] were obtained by least squares regression, using data from inventory items with Poisson demand distributions and negative binomial demand distributions with variance-to-mean ratios of 3 and 9. Previous experiments have all considered the effectiveness of the Approximation for systems with constant variance-to-mean ratio.

We examine the constant standard-deviation-to-mean system for the situation of full information and for the situation in which the decision-maker's knowledge is limited to a sample of previously-realized demands. In addition, the research examines the accuracy of statistical forecasts that predict the future behavior of the operating characteristics. As a result, an inventory systems designer is apprised of both the costs of imperfect information and the extent of bias in the forecast estimates. In general, the behavior observed for the constant standard-deviation-to-mean system is comparable to the behavior of the previously studied systems with constant variance-to-mean ratios of 9.

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CONTENTS

1. DESIGN OF INVENTORY CONTROL POLICIES

1.1 The Model

Throughout this report, we deal with a single-item inventory model. We assume periodic review of an item's inventory level and employ a stationary, discrete-time stochastic process to generate the item's demand. The demand sequence $\xi_1, \xi_2, \ldots,$ consists of independent identically distributed negative binomial random variables taking non-negative integer values.

Demands are met as long as stock on hand is sufficient; when a stockout occurs, the unfilled demand is completely backlogged until a stock replenishment eventually arrives.

Items kept in inventory are assumed to be conserved, there being no losses by deterioration, obsolescence, or pilferage; disposal is not allowed. Inventory on hand at the end of a current period is the inventory from the previous period plus any replenishment that arrives, less demand in the current period. Negative inventory on hand represents the amount of backlogged demand. Replenishments are assumed to be delivered a fixed lead time λ periods after being ordered. The time sequence of events in any period is taken to be order, delivery, demand.

We assume no time discounting of costs and postulate an unbounded horizon over which the item is demanded and stocked. We seek to minimize expected total cost per period.

The cost of a replenishment quantity q is assumed linear with fixed ordering cost K and constant unit cost c

$$(1) \qquad c(q) = \begin{cases} K + cq & \text{for } q > 0 \\ 0 & \text{for } q = 0 \end{cases} .$$

Since items are not lost from inventory, demand is completely filled, and costs are not discounted, the constant unit cost c is not a factor in choosing a minimum cost policy, and is suppressed hereafter.

The inventory holding cost is proportional to any stock on hand at unit cost h

$$(2) \qquad h(i) = \begin{cases} hi & \text{for } i > 0 \\ 0 & \text{for } i \leq 0 \end{cases} ,$$

and the unit penalty cost π is applied to any quantity on backorder at the end of each period

$$(3) \qquad \pi(i) = \begin{cases} 0 & \text{for } i \geq 0 \\ -\pi i & \text{for } i < 0 \end{cases} .$$

The resulting total cost function, therefore, is linear in K, π, and h, and we may scale these parameters so that the value of the unit holding cost h is unity. Non-trivial changes in cost arise only with changes in the ratios K/h and π/h.

We postulate that control over replenishment is exercised by an (s,S) policy: whenever inventory x on hand and on order at the start of a period drops below the value "little s," an order is placed for a replenishment of size $S-x$.

Given our assumptions, when the demand distribution and the economic parameters are known, there is an optimal policy that has the (s,S) form

[Iglehart (1963a,b), Veinott & Wagner (1965)]. When the demand distribution is not known, even though this is the only assumption relaxed, an optimal policy may no longer be of the (s,S) form. Nevertheless, in this experiment we employ an (s,S) policy, since it is in popular use in the applied situation of incomplete information.

1.2 Experimental Design

1.2.1 The Power Approximation

The Power Approximation devised by Ehrhardt (1976) is an algorithm for computing approximately optimal values for (s,S) using only the mean μ and variance σ^2 of demand. The algorithm is executed as follows. Let

(1) $$D_p = (1.463)\mu^{.364}(K/h)^{.498}[(\lambda+1)\sigma^2]^{.0691}$$

and

(2)
$$s_1 = (\lambda+1)\,\mu + [(\lambda+1)\mu]^{.416}(\sigma^2/\mu)^{.603}U(z)$$

$$S_1 = s_1 + D_p \ ,$$

where $U(z)$ is given by

(3)
$$U(z) = .182/z + 1.142 - 3.466z \ ,$$

$$z = \left\{\frac{\mu^{.364}(K/h)^{.498}}{(1+\frac{\pi}{h})[(\lambda+1)\sigma^2]^{.431}}\right\}^{\frac{1}{2}} .$$

If D_p/μ is greater than 1.5, let $s = s_1$ and $S = S_1$. Otherwise, compute

(4) $$S_2 = (\lambda+1) + v[(\lambda+1)\sigma^2]^{\frac{1}{2}} \ ,$$

where v is the solution to

$$(5) \qquad \Phi(v) = \pi/(\pi+h),$$

and $\Phi(\cdot)$ is the cumulative distribution function of the unit normal distribution. The policy parameters are then given by

$$(6) \qquad \begin{aligned} s &= \text{minimum } \{s_1 , S_2\} \\ S &= \text{minimum } \{S_1 , S_2\} \ . \end{aligned}$$

If demands are integer valued, s_1 , D_p , and S_2 are rounded to the nearest integer. We analyze this algorithm's performance in Section 2.

1.2.2 The Statistical Power Approximation

Of course, in real applications, the mean and variance of demand are not always known. For the situation where only sample statistics of previous demands are available, the Statistical Power Approximation described below can be implemented. This decision rule implicitly assumes that demand is stationary. Such a rule derived for stationary conditions may be a reasonable approximation to an optimal rule when the demand process is mildly **nonstationary**, provided that the policy parameters s and S are revised periodically to meet the changing conditions.

We assume in this study that a demand history of fixed length is kept to make each revision, and equal weight is given to each observation. This is not optimal if the demand process is known to be stationary, for then the entire history should be accumulated to give progressively better knowledge and performance. Even when demand is known to be nonstationary, but varying in a regular manner, such as by a trend or periodic cycle, or both, an optimal decision rule would generally utilize the entire history.

The decision-maker usually is not in a position to know, however, that conditions observed, even over the entire past history, will continue to prevail. This provides justification for making frequent revisions, placing greater weight on observations from the immediate past and less on earlier history. For this study the admittedly arbitrary choice has been made to keep a history of fixed length and give equal weight to all observations in this history. Let T be the number of periods between policy revisions, which will be termed the revision interval; assume that a history of T periods' demands is kept for use at each revision.

The statistics required by our decision rules are the sample mean and variance of demand, $\bar{\xi}$ and $\bar{v}$, respectively. If t is a period at the beginning of which revision is made, then

$$\bar{\xi} = T^{-1} \sum_{\tau=1}^{T} \xi_{t-\tau}$$

(7)

$$\bar{v} = (T-1)^{-1} \sum_{\tau=1}^{T} (\xi_{t-\tau} - \bar{\xi})^2 .$$

When using the Statistical Power Approximation we periodically obtain values for (s,S) by substituting $\bar{\xi}$ and $\bar{v}$ for μ and σ^2 in equations (1) through (6). We investigate the properties of the Statistical Power Approximation in Sections 3 and 4 of this study.

1.2.3 Design Parameters

In the basic experiment, each run is made with a different combination of parameters. A negative binomial distribution is used to generate the demand sequences with standard-deviation-to-mean ratio fixed at 1. Runs are made with four mean values for the distributions, shown in Table 1.1.

Three values, 0, 2, and 4, are assigned to leadtime. Since the cost function is linear in the parameters K, h, and π, the value of the unit holding cost h is set at unity. Essential changes in cost arise only with changes in the ratios K/h and π/h. The stockout costs are $\pi = 4, 9,$ and 99 and the fixed cost values are $K = 32$ and 64. All combinations of input levels are included in the system corresponding to a full factorial design. Table 1.1 summarizes the parameter specifications of the system.

The resulting combinations of values for mean, leadtime, stockout cost, and fixed ordering cost total 72. Each set of input parameters is run with a revision interval and revision history length of 26 periods, using demand data accumulated over the same time interval. In each run of the experiment, 200 revisions and forecasts are made.

Table 1.1

System Parameters

Factor	Levels	Number of Levels
Demand distribution	Negative Binomial ($\sigma/\mu = 1$)	1
Mean demand	2, 4, 8, 16	4
Unit holding cost	1	1
Unit backlog penalty cost	4, 9, 99	3
Replenishment setup cost	32, 64	2
Replenishment leadtime	0, 2, 4	3

For a detailed discussion of demand generation, system initialization, and output analysis, see MacCormick [(1974, pp. 52-58)].

1.2.4 Item Operating Characteristics

Detailed operating characteristics of each item in the system are accumulated. These are the period-end inventory on hand, the period-end stockout quantity, the frequency of period-end stockouts, the replenishment quantity, the frequency of replenishment, and the total cost incurred. It is assumed that the decision-maker is interested only in the average performance between revisions, corresponding to the practice adopted by accountants of making reports to management at periodic intervals.

One objective of the experiment is to make inferences about the distributions of these operating characteristics for each of the three decision rules (optimal policies under full information, approximately optimal policies, and statistical approximation policies). Exact expected values are computed analytically for each operating characteristic under optimal control using full information and under approximately optimal control using full information. Under statistical control the simulation experiment yields a sequence of 200 sample values of each operating characteristic for which confidence intervals are constructed using the methods discussed in MacCormick (1974).

1.2.5 Forecasting

The forecasting method used here is the same as that described by MacCormick (1974), namely, retrospective simulation. This method takes the recent demand history for the items in the sample and estimates the performance of the chosen policy for this history. Thus, the method uses the same data twice, once to fix the policy parameters, and once to forecast the performance. As a result, the forecasts are biased, and this study investigates the extent of the bias.

At each revision, after the (s,S) parameters have been set to new values, a forecast is made of the properties of the system. A history of $T+\lambda$ demand values is kept to make the forecast. [We note that there are differences in information storage requirements of considerable importance in practice. To make a forecast of performance by retrospective simulation, the demands for the item in each of $T+\lambda$ periods must be stored, whereas the statistical decision rule may require only a handful of sufficient statistics to set (s,S). Also, for multi-item systems the cost of accumulating in storage the data needed to make forecasts by retrospective simulation is likely to be so high that histories will be kept for only a representative sample of items from the system.]

The forecasts are made by running the system, using the new values of s and S , and observing the operating characteristics as if these (s,S) values had been in force when the history occurred. Each time there is a policy revision, there is a forecast for each operating characteristic. The simulation initializes the stock on hand and on order at the actual value at the time of the forecast, that is, at the time of revision. As actual stock on hand is not recorded until the elapse of a leadtime after revision, the same interval is allowed to elapse before recording it for the forecast. The inventory on hand variable is therefore initialized at the initial value for inventory on hand and on order less the first λ demands in the history of $T+\lambda$ demands.

1.3 A Multi-Item System

Scientific techniques for inventory control are generally applied to systems of many items. This experiment combines the results from the 72 single-item simulations into a multi-item system. Since management generally assesses the performance of control techniques for a multi-item inventory system by observing indices that are aggregate operating characteristics [Wagner (1962)], certain aggregate characteristics have been computed.

The operating characteristics of the multi-item system under statistical control have been measured by aggregating the sample values of the corresponding characteristics for each item in the system. When the system is operated under perfect information, these characteristics are computed analytically. The aggregate of average total cost per period is computed as the arithmetic sum of the corresponding costs for each item. The components of total cost for inventory storage, backlog penalty, and for ordering replenishments are similarly computed. The aggregate backlog and replenishment frequencies are arithmetic averages of the corresponding frequencies observed for each item in the system. Since the unit inventory holding cost for all items is unity, the average number of units in inventory at period-end is identical to the aggregate average holding cost per period. Finally, a weighted proportion of demand backlogged is computed as the ratio of a weighted sum of the average quantity backlogged per period to a weighted sum of the (exact) mean values of demand. The weights used in both the numerator and denominator of the ratio are the unit cost of backlogging demand for the respective item.

1.4 Use of Constant Standard-Deviation-to-Mean Ratio

The Power Approximation formulas [Ehrhardt (1976)] were obtained by least squares regression, using values of s, S, and $D = S-s$ for 288 items as data points. Three types of demand distributions were included in these items: Poisson and negative binomial with variance-to-mean ratios of 3 and 9. Previous experiments have all considered the effectiveness of the approximation for systems with constant variance-to-mean ratio. In this study, we examine the performance of the Power Approximation algorithm for a negative binomial inventory system where the standard-deviation-to-mean ratio is constant and equal to 1.

The values of mean demand that appear in the 72 item system under study are $\mu = 2, 4, 8$ and 16. Table 1.2 compares the variance of the items in this system with the variance of items with the same mean in systems with constant variance-to-mean ratios of 3 and 9. Items with $\mu = 2$ have a lower variance, and items with $\mu = 16$ have a higher variance, for the standard deviation/mean = 1 system than for either the variance/mean = 3 or variance/mean = 9 systems. Variances for items with $\mu = 4$ and 8 in the constant standard deviation/mean system fall between those in the variance/mean = 3 and variance/mean = 9 systems.

Table 1.2

Variance of Demand for Items in

Multi-Item Negative Binomial Systems

	Variance σ^2		
Mean μ	$\sigma^2/\mu = 3$	$\sigma^2/\mu = 9$	$\sigma/\mu = 1$
2	6	18	4
4	12	36	16
8	24	72	64
16	48	144	256

Table 1.3 displays the standard deviation-to-mean ratio for items with mean $\mu = 2, 4, 8$ and 16 when the variance-to-mean ratio equals 3 and 9. The average standard-deviation-to-mean ratio for these items is 1.07.

Table 1.3

Standard-Deviation-to-Mean Ratios for Items in

Fixed Variance-to-Mean Systems

	Mean μ	Standard Deviation/Mean $= \sigma/\mu$
Variance/Mean = 9:	2	2.12
	4	1.50
	8	1.06
	16	.75
		Average = 1.36
Variance/Mean = 3:	2	1.22
	4	.87
	8	.61
	16	.43
		Average = .78

$$\text{Overall Average} = \frac{.78 + 1.36}{2} = 1.07$$

2. SYSTEM CONTROL WITH FULL INFORMATION ABOUT DEMAND

In this section we describe the inventory system with constant standard-deviation-to-mean ratio under optimal and Power Approximation control with full information about demand. We seek to establish benchmark values of the system operating characteristics for later comparison with a system controlled using only statistical information about demand.

2.1 Optimal Control with Full Information

Each item in the 72 item system described in Section 1 is controlled using an optimal (s,S) policy. Optimal values for the policy parameters are computed using the algorithm in Veinott and Wagner (1965). The expected values of the operating characteristics are calculated by first determining their conditional expectations for given values of inventory on hand plus on order after ordering. The distribution of inventory on hand plus on order after ordering is then used to form the unconditional expectations.

Table 2.1 lists the resulting expected values of average total cost per period and its components for the 72 item negative binomial system with standard-deviation-to-mean ratio of 1 and the 72 item systems with variance-to-mean ratios of 3 and 9. Each component's percent of total cost is shown in parentheses. For all quantities shown in the table, the value associated with the $\sigma/\mu = 1$ system is between the comparable values for the $\sigma^2/\mu = 9$ and $\sigma^2/\mu = 3$ systems. In general, however, the values are closer to those for the higher variance-to-mean ratio system.

Table 2.1

Average Costs per Period for a

Multi-Item System Optimally Controlled

Cost Component	Standard Deviation/Mean=1	Variance/Mean=9	Variance/Mean=3
Inventory	1897 (59.9)	1935 (59.7)	1273 (54.4)
Backlog	629 (19.9)	682 (21.0)	343 (14.6)
Replenishment	642 (20.3)	626 (19.3)	728 (31.1)
Total	3169 (100.0)	3243 (100.0)	2345 (100.0)

The apportionment of expected average costs for various classifications of the items in each multi-item system is shown in Table 2.2, as percentage distributions of the estimated expected average total cost per period. We note that items with mean = 16 contribute a much greater proportion of total costs in the $\sigma/\mu = 1$ system than in either the $\sigma^2/\mu = 9$ or the $\sigma^2/\mu = 3$ systems. This is due to the fact that items with $\mu = 16$ have a higher variance for the fixed standard-deviation-to-mean system than in either of the fixed variance-to-mean systems (see Section 1.4).

Aggregate values of the other operating characteristics for the various classifications are set out in Table 2.3. The classifications are one-way in the sense that items in the system are grouped according to the value taken by a single input parameter.

The sensitivity of expected costs per period and other operating characteristics is summarized in Table 2.4, which has been constructed by observing any monotonicity in the relation for the multi-item system (see Table 2.2). We note two differences between the patterns observed for the $\sigma/\mu = 1$ system and those for the $\sigma^2/\mu = 3$ and 9 systems. For the

Table 2.2

Apportionment of Aggregate Costs per Period for a 72-Item System Under Optimal Control with Full Information

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	59.9	11.3	16.2	32.5	28.7	31.2	14.3	20.6	24.9	5.1	8.8	16.0	30.0
BACKLOG	19.9	6.5	6.9	6.5	10.0	10.0	5.1	6.9	8.0	1.3	2.7	5.3	10.5
REPLENISHMENT	20.3	6.6	6.8	7.0	8.0	12.3	7.3	6.7	6.4	3.2	4.3	5.6	7.2
TOTAL	100.0	24.3	29.8	45.9	46.5	53.5	26.6	34.1	39.3	9.6	15.8	26.9	47.7
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	59.7	10.9	15.9	33.0	28.7	31.0	14.1	20.5	25.0	8.7	12.0	16.5	22.6
BACKLOG	21.0	6.6	7.1	7.2	10.4	10.6	5.2	7.3	8.5	3.7	4.6	5.6	7.1
REPLENISHMENT	19.3	6.3	6.4	6.6	7.6	11.7	7.0	6.3	6.0	2.5	3.6	5.3	7.8
TOTAL	100.0	23.8	29.4	46.8	46.7	53.3	26.4	34.2	39.4	15.0	20.2	27.4	37.4
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	54.3	11.7	15.6	27.0	25.1	29.2	14.3	18.5	21.6	7.8	10.8	15.0	20.7
BACKLOG	14.6	5.1	5.1	4.4	7.0	7.6	3.6	5.1	6.0	2.3	2.8	4.1	5.4
REPLENISHMENT	31.1	10.0	10.4	10.7	12.5	18.6	10.9	10.3	9.9	4.2	6.1	8.6	12.2
TOTAL	100.0	26.8	31.1	42.1	44.6	55.4	28.8	33.8	37.4	14.3	19.6	27.6	38.4

Table 2.3

Operating Characteristics of a 72-Item System Under Optimal Control with Full Information

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	1897	357	512	1028	909	988	453	653	789	161	279	507	950
BACKLOG FREQUENCY	.096	.186	.094	.009	.097	.096	.094	.097	.098	.088	.097	.099	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.031	.286	.134	.012	.031	.031	.024	.032	.038	.032	.031	.031	.031
REPLENISHMENT FREQUENCY	.194	.189	.194	.200	.219	.169	.209	.191	.183	.124	.165	.215	.274
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	1935	352	515	1070	931	1004	459	666	809	283	389	534	732
BACKLOG FREQUENCY	.098	.189	.095	.010	.098	.098	.096	.098	.100	.095	.098	.099	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.034	.299	.143	.013	.036	.034	.025	.035	.041	.091	.055	.034	.021
REPLENISHMENT FREQUENCY	.189	.186	.188	.193	.213	.165	.206	.186	.176	.097	.143	.210	.306
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	1273	274	366	633	588	685	335	434	506	183	253	352	485
BACKLOG FREQUENCY	.094	.182	.092	.009	.094	.095	.093	.095	.096	.087	.094	.097	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.017	.167	.074	.006	.016	.018	.013	.018	.021	.040	.025	.018	.012
REPLENISHMENT FREQUENCY	.222	.213	.222	.230	.254	.189	.235	.220	.210	.119	.173	.244	.350

$\sigma^2/\mu = 9$ system, proportion demand backlogged decreases monotonically with respect to setup cost, whereas for the $\sigma^2/\mu = 3$ system the relationship is monotonically increasing. For the $\sigma/\mu = 1$ system the relationship is essentially constant.

There is no discernible pattern in the relationship of expected backlog cost with respect to penalty cost for the constant standard-deviation-to-mean system. There is, however, a monotonically increasing relationship in the variance/mean = 9 system and a monotonically decreasing relationship for the variance/mean = 3 system.

The previous experiments of MacCormick (1974) and Estey and Kaufman (1975) for negative binomial systems, and of Kaufman and Klincewicz (1976) for a sporadic demand system, with constant variance-to mean ratio have indicated that the dependence of expected average total cost per period on the input parameters appears to be multiplicatively separable in form.

Table 2.4 shows the percentage distributions according to input parameter groupings of expected total cost per period for the 72 item system with $\sigma/\mu = 1$. There are three entries in each cell of Table 2.4. The first is the proportion of expected total cost per period. Values in the second entry are calculated as the product of the corresponding proportions observed for the one-way distributions that form the border to Table 2.5. For example, when **mean** = 2 and $\pi/h = 99$, the product is .096(.459) = .044 , which compares to the observed proportion of .040 in the two-way distribution. The third entry in the cell is the difference between the observed two-way proportion and the product of the observed one-way proportions.

Table 2.4

Sensitivity to Input Parameters of Operating Characteristics of an Optimally Controlled Inventory System with Negative Binomial Demands

	INPUT PARAMETERS			
	π/h	K/h	Leadtime	Mean Demand
STANDARD DEVIATION/MEAN = 1				
OPERATING CHARACTERISTIC				
E(COST)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(PERIOD-END INVENTORY)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(BACKLOG COST)	? ↑ ↓	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(BACKLOG FREQUENCY)	↓ ↓ ↓	↓ - ↑	↑ ↑ ↑	↑ ↑ ↑
PROPORTION DEMAND BACKLOGGED	↓ ↓ ↓	- ↓ ↑	↑ ↑ ↑	↓ ↓ ↓
E(REPLENISHMENT COST)	↑ ↑ ↑	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑
E(REPLENISHMENT FREQUENCY)	↑ ↑ ↑	↓ ↓ ↓	↓ ↓ ↓	↑ ↑ ↑
POLICY PARAMETERS				
D	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
s	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑
S	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑

The three symbols in each column refer to the following three sets of demand distributions: Negative Binomial (standard deviation/mean = 1), Negative Binomial (Variance/Mean = 9) and Negative Binomial (Variance/Mean = 3) respectively.

↑ indicates a monotonically increasing relationship
↓ indicates a monotonically decreasing relationship
- indicates an approximately constant relationship
? indicates that no simple relationship could be discerned

Table 2.5

System of 72 Items with Negative Binomial Demands with Standard Deviation/Mean = 1
Full Information, Optimal Control
Sources of Expected Total Cost

	PROPORTION	π/h 4	π/h 9	π/h 99	K/h 32	K/h 64	LEADTIME 0	LEADTIME 2	LEADTIME 4
PROPORTION		.243	.298	.459	.465	.535	.266	.341	.393
MEAN									
2	.096	.026	.030	.040	.043	.054	.028	.033	.035
		.023	.029	.044	.044	.051	.026	.033	.038
		.003	.001	-.004	-.001	.003	.002	.000	-.003
4	.158	.041	.048	.069	.072	.086	.044	.053	.060
		.038	.047	.073	.073	.084	.042	.053	.062
		.003	.001	-.004	-.001	.002	.002	.000	-.002
8	.269	.066	.080	.123	.125	.144	.072	.092	.105
		.065	.080	.123	.125	.144	.072	.092	.106
		.001	.000	.000	.000	.000	.000	.000	-.001
16	.477	.111	.140	.226	.226	.251	.122	.163	.172
		.112	.142	.219	.222	.255	.126	.163	.187
		-.001	-.002	.007	.004	-.004	-.004	.000	.005
LEADTIME									
0	.267	.066	.079	.121	.121	.146			
		.065	.080	.123	.124	.143			
		.001	-.001	-.002	-.003	.003			
2	.341	.083	.102	.157	.159	.182			
		.082	.102	.157	.159	.182			
		.001	.000	.000	.000	.000			
4	.392	.095	.117	.181	.185	.207			
		.095	.117	.180	.182	.210			
		.000	.000	.001	.003	-.003			
K/h									
32	.465	.110	.137	.218					
		.113	.139	.214					
		-.003	-.002	.004					
64	.534	.133	.160	.241					
		.130	.160	.246					
		.003	.000	.005					

The observed differences are all relatively small and give evidence of a multiplicative form for the dependence of expected total cost per period on input parameters for the constant σ/μ case as well. As in the fixed variance-to-mean ratio case, we observe trends in the differences with respect to changes in certain parameter settings. The trends that are observed across the rows for mean demand values, however, are all opposite to the trends observed for the constant variance-to-mean case. For instance, in Table 2.5 we note that for classifications with $\mu = 2$, the differences change by negative increments as we consider higher values of penalty cost π . For constant variance-to-mean ratios, these increments were positive. Further experiments may show if these observed trends extrapolate beyond the range of parameters considered here.

Finally, we explore an approximation to the backlog frequency that is well known to writers on inventory control. For an optimally controlled multi-period inventory model with linear costs, the ratio $1/(1+\pi)$ forms an upper bound to the probability of backlog. Table 2.6 shows the backlog frequencies are at or below the upper bounds for all three inventory systems we are considering.

Table 2.6

Backlog Frequencies for a Multi Item System

with Negative Binomial Demands

	π/h 4	9	99
$1/(1+\pi)$	.2	.1	.01
Standard Deviation/Mean = 1	.186	.094	.009
Variance/Mean = 9	.189	.095	.010
Variance/Mean = 3	.182	.092	.009

2.2 Power Approximation Control with Full Information

We obtain a Power Approximation policy for each item in our multi-item system using equations (1) through (6) in Section 1.2.3. Although the control is described as utilizing full information, only the mean and variance of the demand distribution need to be known in order to compute (s,S).

Table 2.7 lists the resulting expected value of average total cost per period and its components for the 72 item system with $\sigma/\mu = 1$, with $\sigma^2/\mu = 9$, and with $\sigma^2/\mu = 3$. Each component's percent of total cost is shown in parentheses.

Table 2.7

Average Costs per Period for a Multi-Item Negative Binomial System with Power Approximation Control

COST COMPONENT	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
INVENTORY	1939 (61.0)	1962 (60.3)	1262 (53.7)
BACKLOG	591 (18.6)	664 (20.4)	362 (15.4)
REPLENISHMENT	651 (20.5)	626 (19.3)	726 (30.9)
TOTAL	3181 (100.0)	3252 (100.0)	2350 (100.0)

The values for each component of total cost for the standard deviation/mean = 1 case fall between the values for the variance/mean = 9 and the variance/mean = 3 cases. As is the case under optimal control, however, the results are much closer to the variance/mean = 9 results, which show greater total cost than the variance/mean = 3 results.

In Table 2.8 we list absolute and percentage differences between optimal and Power Approximation control. We see that the fixed standard deviation/mean system shows only slightly worse degradation in total cost than the fixed variance/mean cases. The percentage increase in inventory holding costs and replenishment cost and the percentage decrease in backlog cost, compared to optimal control, are more pronounced for the $\sigma/\mu = 1$ case than for the $\sigma^2/\mu = 9$ case. For the $\sigma^2/\mu = 3$ case, it is the backlog costs which are above those for optimal control whereas the inventory and replenishment costs show percentage decreases.

Table 2.8

Average Costs Per Period for a Multi Item Negative Binomial System: Comparison of Optimal and Power Approximation Control

COST COMPONENT	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
INVENTORY	42 (2.2)	26 (1.4)	-13 (-1.0)
BACKLOG	-39 (-6.1)	-18 (-2.7)	17 (5.0)
REPLENISHMENT	9 (1.4)	1 (0.2)	-1 (-0.1)
TOTAL	12 (0.4)	9 (0.3)	3 (0.1)

Note: Table 2.8 shows absolute increase or decrease in the cost components of Table 2.7 over those in Table 2.1 and percentage changes in parentheses.

The apportionment of aggregate costs per period for various parameter classifications is shown in Table 2.9, which can be compared with Table 2.2. The differences in apportionment of aggregate costs between the constant standard deviation/mean system and the constant variance/mean systems, which can be seen in Table 2.9, reflect the differences in apportionment under optimal control as seen in Table 2.2. Detailed comparison of the three systems under optimal and approximately optimal control, expanding on Table 2.8, is in Table 2.10 showing percentage increases in each component

Table 2.9

Apportionment of Aggregate Costs per Period for a 72-Item System Under Power Approximation Control with Full Information

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	61.0	11.0	16.4	33.6	30.0	31.0	14.3	21.0	25.7	5.1	8.6	16.0	31.4
BACKLOG	18.6	6.5	6.5	5.6	8.6	10.0	5.1	6.4	7.1	1.4	2.8	5.3	9.1
REPLENISHMENT	20.5	6.8	6.8	6.8	8.1	12.4	7.2	6.7	6.6	3.2	4.3	5.7	7.3
TOTAL	100.0	24.3	29.7	46.0	46.6	53.4	26.6	34.1	39.4	9.6	15.7	26.8	47.8
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	60.3	11.0	16.0	33.3	29.0	31.3	14.3	20.8	25.2	8.8	12.1	16.6	22.8
BACKLOG	20.4	6.5	6.8	7.1	9.5	10.9	5.5	7.0	7.9	3.7	4.5	5.5	6.7
REPLENISHMENT	19.3	6.4	6.4	6.4	7.6	11.7	6.8	6.3	6.2	2.4	3.6	5.4	7.9
TOTAL	100.0	23.8	29.4	46.8	46.8	53.2	26.4	34.1	39.5	15.0	20.2	27.4	37.4
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	53.7	10.8	15.3	27.6	25.2	28.5	14.2	18.3	21.2	7.7	10.5	14.7	20.9
BACKLOG	15.4	5.7	5.4	4.3	7.1	8.3	3.8	5.2	6.4	2.5	3.3	4.3	5.3
REPLENISHMENT	30.9	10.3	10.3	10.3	12.4	18.5	10.8	10.2	9.9	4.1	6.0	8.6	12.1
TOTAL	100.0	26.8	31.0	42.2	44.7	55.3	28.8	33.8	37.4	14.3	19.8	27.5	38.4

Table 2.10

Percentage Excess of Costs Per Period of a 72-Item System Under Power Approximation Control Over the Costs of the Same System Under Optimal Control

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	2.2	-1.5	1.8	3.7	4.6	-0.1	0.1	2.0	3.5	-0.6	-2.2	0.3	5.0
BACKLOG	-6.1	-0.1	-5.2	-13.1	-12.6	0.3	1.5	-6.3	-10.8	4.2	6.1	-1.1	-13.1
REPLENISHMENT	1.4	4.2	1.5	-1.4	2.3	0.8	-0.6	1.4	3.6	-0.1	1.2	1.0	2.4
TOTAL	0.4	0.4	0.1	0.5	0.6	0.2	0.2	0.2	0.6	0.2	0.1	0.2	0.6
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	1.4	0.3	2.3	1.3	3.5	-0.6	0.4	1.3	2.0	2.0	1.1	0.3	2.0
BACKLOG	-2.7	-1.2	-4.9	-1.9	-8.7	3.2	4.9	-3.2	-7.0	-0.5	-1.2	-1.5	-5.7
REPLENISHMENT	0.2	1.8	0.7	-2.0	1.1	-0.5	-3.2	0.5	3.7	-3.1	-0.9	1.6	0.7
TOTAL	0.3	0.3	0.2	0.3	0.4	0.2	0.4	0.2	0.3	0.5	0.2	0.2	0.3
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	-1.0	-7.0	-2.0	2.2	0.3	-2.2	-0.8	-0.6	-1.5	-1.3	-3.0	-2.2	1.0
BACKLOG	5.0	9.9	7.3	-3.4	0.2	9.9	6.0	3.3	5.8	7.6	12.2	6.7	-1.4
REPLENISHMENT	-0.1	3.7	-0.2	-3.7	-0.1	-0.2	-0.6	-0.2	0.4	-1.3	-0.1	1.2	-0.7
TOTAL	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.1

of cost.

The percentage decrease in backlog cost and percentage increase in inventory cost from the optimal that were noted in Table 2.8 for the $\sigma/\mu = 1$ system are most noticeable for the subsystems with parameter values $\pi = 99$ and $\mu = 16$. The change in total cost from the optimal, however, is not greater than 0.6% for any subsystem. Overall, the Power Approximation with full information appears to perform close to optimal for the constant standard-deviation-to-mean system.

Table 2.11 shows the values of other operating characteristics for the systems under approximately optimal control. Percentage changes in the values of the operating characteristics in Table 2.11 from those in Table 2.3 are set out in Table 2.12. The percentages from optimality for proportion demand backlogged with parameter settings $\pi = 99$ and $\mu = 16$ reflect the pattern noted earlier for backlog cost. Overall, for the $\sigma/\mu = 1$ system, the period-end inventory and replenishment frequency increase and the proportion demand backlogged decreases from the optimal values as they do for the $\sigma^2/\mu = 9$ system; backlog frequency increases above optimal overall, as it does for the $\sigma^2/\mu = 3$ system.

Table 2.13 summarizes the sensitivity to the values of input parameters shown by the operating characteristics of a negative binomial system with standard-deviation-to-mean of 1 and with variance-to-mean of 9 and of 3 under Power Approximation control. No major discrepancies between the pattern for the constant standard deviation/mean case and the pattern for the constant variance/mean case are evident. We note, however, the sensitivity of the backlog characteristics with respect to leadtime and of the backlog cost with respect to penalty cost. For these relationships, no clear pattern is discernible for $\sigma^2/\mu = 9$; the patterns observed for

Table 2.11

Operating Characteristics of a 72-Item System Under Power Approximation Control with Full Information

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	1939	351	522	1066	952	987	456	667	818	161	273	508	996
BACKLOG FREQUENCY	.100	.200	.091	.008	.095	.105	.101	.099	.100	.101	.104	.101	.093
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.029	.286	.127	.010	.027	.032	.024	.030	.034	.033	.033	.031	.027
REPLENISHMENT FREQUENCY	.197	.197	.197	.197	.225	.170	.208	.194	.190	.124	.167	.218	.280
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	1962	353	526	1083	963	999	461	675	826	288	392	536	746
BACKLOG FREQUENCY	.096	.189	.088	.010	.089	.103	.097	.094	.096	.088	.095	.100	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.033	.295	.136	.013	.031	.035	.027	.034	.038	.090	.055	.033	.020
REPLENISHMENT FREQUENCY	.190	.190	.190	.190	.215	.164	.199	.188	.183	.095	.143	.214	.308
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	1262	255	360	649	592	670	333	430	498	180	246	345	491
BACKLOG FREQUENCY	.103	.204	.097	.009	.099	.108	.102	.102	.105	.102	.106	.105	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.018	.186	.078	.006	.017	.019	.013	.018	.022	.044	.029	.018	.012
REPLENISHMENT FREQUENCY	.221	.221	.221	.221	.253	.189	.232	.219	.212	.118	.173	.247	.346

Table 2.12

Percentage Excess of Operating Characteristics for a 72-Item System Under Power Approximation Control over the Same Characteristics for the System Under Optimal Control

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	2.2	-1.5	1.8	3.7	4.6	-0.1	0.1	2.0	3.5	-0.6	-2.2	0.3	5.0
BACKLOG FREQUENCY	3.6	7.5	-2.9	-8.0	-1.9	9.2	6.5	1.8	2.6	15.0	7.7	1.8	-8.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-6.1	-0.1	-5.2	-13.1	-12.6	0.3	1.5	-6.3	-10.8	4.2	6.1	-1.1	-13.1
REPLENISHMENT FREQUENCY	1.6	4.4	1.8	-1.2	2.3	0.8	-0.6	1.7	4.1	-0.1	1.7	1.5	2.5
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	1.4	0.3	2.3	1.3	3.5	-0.6	0.4	1.3	2.0	2.0	1.1	0.3	2.0
BACKLOG FREQUENCY	-2.4	-0.2	-7.4	2.3	-9.0	4.2	0.7	-4.3	-3.6	-7.7	-2.7	0.9	-0.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-2.7	-1.2	-4.9	-1.9	-8.7	3.2	4.9	-3.2	-7.0	-0.5	-1.2	-1.5	-5.7
REPLENISHMENT FREQUENCY	0.4	2.2	1.0	-1.8	1.1	-0.5	-3.2	1.0	4.1	-2.4	-0.2	1.9	0.6
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	-1.0	-7.0	-2.0	2.2	0.3	-2.2	-0.8	-0.6	-1.5	-1.3	-3.0	-2.2	1.0
BACKLOG FREQUENCY	9.4	11.6	6.3	-2.7	5.7	13.1	11.5	7.1	9.7	15.8	13.7	8.3	0.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	5.0	9.9	7.3	-3.4	-0.2	9.9	6.0	3.3	5.8	7.6	12.2	6.7	-1.4
REPLENISHMENT FREQUENCY	-0.1	3.8	-0.2	-3.8	-0.1	-0.2	-0.9	-0.4	1.0	-0.6	0.3	1.3	-1.2

Table 2.13

Operating Characteristics of a 72-Item Inventory System
Using the Power Approximation: Sensitivity to Input Parameters

	INPUT PARAMETERS			
	π/h	K/h	Leadtime	Mean Demand
OPERATING CHARACTERISTICS				
E(COST)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(PERIOD-END INVENTORY)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(BACKLOG COST)	↑ ? ↓	↑ ↑ ↑	↑ ? ↑	↑ ↑ ↑
E(BACKLOG FREQUENCY)	↓ ↓ ↓	↑ ↑ ↑	- ? -	? ? -
PROPORTION OF DEMAND BACKLOGGED	↓ ↓ ↓	↑ ↑ ↑	↑ ? ↑	↓ ↓ ↓
E(REPLENISHMENT COST)	- - -	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑
E(REPLENISHMENT FREQUENCY)	- - -	↓ ↓ ↓	↓ ↓ ↓	↑ ↑ ↑
POLICY PARAMETERS				
D	- - -	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
s	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑
S	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑

The three symbols in each column refer to the following three sets of demand distributions: (Standard Deviation/Mean = 1), Negative Binomial (Variance/Mean = 9), and Negative Binomial (Variance/Mean = 3), respectively, and

↑ indicates a monotonically increasing relationship;
↓ indicates a monotonically decreasing relationship;
- indicates an approximately constant relationship;
? indicates that no simple relationship could be discerned.

$\sigma/\mu = 1$ in these cases are identical to those observed for $\sigma^2/\mu = 3$.

Finally, we examine the ratio $1/(1+\pi)$ as an approximation to the probability of backlog in any period for each item. The entries in Table 2.14 have been extracted from Table 2.11. The values for the approximation are quite close to actual for the system under study, and quite comparable to the results for the constant variance-to-mean ratio.

Table 2.14

Backlog Frequencies for a Multi-Item System with Negative Binomial Demands

	Penalty cost = π/h		
	4	9	99
$1/(1+\pi)$	.2	.1	.01
Standard Deviation/Mean = 1	.200	.091	.008
Variance/Mean = 9	.189	.088	.010
Variance/Mean = 3	.204	.097	.009

3. SYSTEM CONTROL WITH STATISTICAL DEMAND INFORMATION

We suppose that a manager responsible for the control of the inventory system discussed in this section assumes that demand can be modeled sufficiently well by a sequence of independent demands, distributed stationarily, or approximately stationarily. But all that the manager actually knows about demand is the information provided by a sample from the realized demand history. The sample can be used to estimate central moments of the demand distribution; such estimates take the place of the true distribution moments in control policies derived to be approximately optimal for the case of perfect information. Details of the control policies were given previously in Section 1.

The operating characteristics describing the performance of inventory systems under control using only statistical information about demand have not been computed analytically. The results in this section are estimates of the performance of some selected inventory systems found by computer simulation. During each simulation run the control policy parameters are revised at regular intervals to allow for potential changes in the underlying demand distribution. Such changes do not, in fact, occur during the experiments, but in a real-life system periodic revision is essential to detect and adapt to changing demand structure.

We compare our simulation results for standard-deviation-to-mean ratio of 1 with those of Ehrhardt (1976) for the variance-to-mean ratios of 3 and 9. Estimated average total cost per period, and its components, are shown in Table 3.1 for each of these three systems.

Table 3.2 shows estimates of absolute and percentage incremental changes in expected costs per period when statistical control is compared

Table 3.1

Average Costs per Period for a 72-Item System:
Control with Statistical Information about Demand

Cost Component	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
Inventory	1990 (56.9)	1989 (55.0)	1297 (52.6)
Backlog	854 (24.4)	993 (27.5)	439 (17.8)
Replenishment	652 (18.7)	634 (17.5)	732 (29.6)
Total	3496 (100.0)	3616 (100.0)	2467 (100.0)

Table 3.2

Average Costs per Period for a Multi Item System:
Comparison of Statistical Control with Optimal Control

Cost Component	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
Inventory	91 (4.9)	54 (2.8)	22 (1.7)
Backlog	225 (35.8)	311 (45.7)	94 (27.4)
Replenishment	9 (1.5)	8 (1.3)	4 (0.6)
Total	327 (10.3)	373 (11.5)	120 (5.1)

Note: Table 3.2 shows the absolute increase in cost components of Table 3.1 over those of Table 2.1 **and** percentage increases shown in parentheses.

with optimal control given full information. The percentage excess in total cost for the standard-deviation-to-mean of 1 system (10.3%) is greater than that for the variance-to-mean of 3 system (5.1%) but slightly less than the percentage excess for the variance-to-mean of 9 system (11.5%).

When the items in each system are divided into subsystems according to the input parameter values, we obtain the apportionment of costs among the subsystems shown in Table 3.3. This table is similar in format to Table 2.2, which shows systems controlled with full information.

Comparison of Table 3.3 and Table 2.2 indicates that a higher proportion of expected total cost is formed by the backlog component when the $\sigma/\mu = 1$ system is controlled statistically. This same pattern was observed for the constant variance-to-mean systems as well.

Table 3.4 shows the estimated percentage incremental changes in the expected average total cost and its components. For the various subsystems this table elaborates on the changes summarized in Table 3.2. In comparison with the constant variance-to-mean case, the $\mu = 16$ subsystem shows a greater percentage excess in total cost for $\sigma/\mu = 1$. As noted in Section 1.4, the variance for $\mu = 16$ with $\sigma/\mu = 1$ is greater than the variance for $\mu = 16$ with either $\sigma^2/\mu = 3$ or $\sigma^2/\mu = 9$. Likewise, the $\mu = 2$ subsystem, which has a smaller variance when $\sigma/\mu = 1$, shows a smaller percentage excess in total cost when $\sigma/\mu = 1$ than when $\sigma^2/\mu = 3$ or 9.

We tabulate in Table 3.5 the physical operating characteristics for the three systems, and in Table 3.6 we set out the percentage increase of each over the corresponding values in Table 2.3. There is a general pattern of greater percentage increase in period-end inventory for the constant standard-deviation-to-mean system than for either of the constant variance-to-mean systems.

The largest percentage excesses for the $\sigma/\mu = 1$ case occur for backlog frequency (60.4%) and proportion demand backlogged (78.7%) in the $\pi = 99$ subsystem. These values are, however, not as large as for the

Table 3.3

Apportionment of Costs for a 72-Item System Controlled with Statistical Information about Demand
(Revision Interval 26 Periods, Revision History Length 26 Periods)

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	56.9	10.2	15.3	31.2	28.1	28.8	12.9	19.3	24.8	4.6	8.0	14.9	29.4
BACKLOG	24.4	6.6	7.3	10.5	11.5	12.9	5.7	8.4	10.3	1.6	3.4	6.8	12.6
REPLENISHMENT	18.7	6.2	6.2	6.2	7.4	11.3	6.6	6.2	5.9	2.9	3.9	5.2	6.7
TOTAL	100.0	23.2	28.9	47.9	47.0	53.0	25.2	33.8	41.0	9.1	15.3	26.8	48.7
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	55.0	9.7	14.6	30.6	26.9	28.1	12.5	18.7	23.8	7.8	10.8	15.1	21.3
BACKLOG	27.5	6.8	7.7	13.0	13.3	14.2	6.3	9.4	11.8	5.8	6.5	7.1	8.1
REPLENISHMENT	17.5	5.9	5.8	5.8	6.9	10.6	6.2	5.8	5.5	2.2	3.3	4.9	7.1
TOTAL	100.0	22.4	28.2	49.4	47.1	52.9	25.0	33.9	41.1	15.8	20.6	27.1	36.5
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	52.6	9.9	14.7	28.0	24.9	27.7	13.4	17.7	21.6	7.5	10.3	14.4	20.5
BACKLOG	17.8	6.5	5.9	5.4	8.3	9.5	4.0	6.2	7.5	3.1	3.9	4.8	6.0
REPLENISHMENT	29.6	9.9	9.9	9.8	11.8	17.8	10.4	9.7	9.4	4.0	5.8	8.3	11.6
TOTAL	100.0	26.3	30.5	43.2	45.1	54.9	27.8	33.7	38.5	14.5	19.9	27.4	38.2

Table 3.4

Percentage Excess of Costs of a 72-Item System under Statistical Control
Over the Costs of Controlling the Same System Optimally with Full Information
(Revision Interval 26 Periods, Revision History Length 26 Periods)

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	4.9	1.8	4.6	6.1	8.2	1.8	-0.6	3.2	9.5	0.0	0.1	2.8	8.2
BACKLOG	35.8	11.6	17.8	78.7	28.1	43.3	26.0	35.0	42.6	32.1	40.5	40.7	32.5
REPLENISHMENT	1.5	4.7	2.0	-1.9	2.2	1.0	-0.0	1.9	2.8	0.5	0.9	1.6	2.3
TOTAL	10.3	5.2	7.0	15.2	11.4	9.4	4.6	9.3	15.1	4.6	7.2	10.1	12.7
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	2.8	0.4	2.9	3.5	4.5	1.2	-1.4	1.6	6.2	-0.7	0.3	2.6	5.5
BACKLOG	45.7	13.4	20.0	100.7	42.7	48.6	32.9	44.6	54.5	71.5	58.0	40.9	27.9
REPLENISHMENT	1.3	3.8	2.2	-1.9	1.7	1.1	-0.9	2.0	3.3	0.3	1.5	1.9	1.1
TOTAL	11.5	4.9	6.9	17.8	12.5	10.6	5.5	10.8	16.1	17.6	13.6	10.3	8.9
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	1.7	-11.0	-1.2	8.9	4.2	-0.4	1.8	0.7	4.9	0.4	-0.4	0.6	4.0
BACKLOG	27.4	32.2	23.6	25.9	22.9	31.6	17.4	29.0	31.9	41.6	37.4	25.9	16.8
REPLENISHMENT	0.6	4.6	0.8	-3.4	0.1	0.9	0.6	0.1	1.0	-0.5	0.8	1.8	-0.1
TOTAL	5.1	3.2	3.5	7.6	6.0	4.4	1.5	4.7	8.2	6.8	5.7	4.7	4.5

Table 3.5

Operating Characteristics of a 72-Item System Controlled with Statistical Information About Demand
(Revision Interval 26 Periods, Revision History Length 26 Periods)

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	1990	364	535	1090	983	1007	452	674	865	161	281	519	1029
BACKLOG FREQUENCY	.111	.211	.106	.015	.106	.116	.101	.113	.118	.110	.115	.113	.105
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.042	.320	.158	.021	.040	.045	.030	.044	.054	.042	.044	.044	.041
REPLENISHMENT FREQUENCY	.198	.198	.198	.196	.224	.171	.209	.195	.189	.125	.167	.219	.280
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	1989	352	529	1108	973	1016	452	676	859	280	390	547	772
BACKLOG FREQUENCY	.110	.205	.107	.019	.106	.114	.097	.113	.120	.102	.111	.114	.114
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.049	.339	.172	.026	.048	.051	.034	.051	.063	.155	.087	.048	.027
REPLENISHMENT FREQUENCY	.192	.193	.192	.190	.216	.167	.204	.190	.182	.098	.146	.214	.309
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	1297	244	362	691	613	683	329	436	532	184	253	354	506
BACKLOG FREQUENCY	.115	.227	.106	.012	.110	.119	.107	.117	.120	.111	.115	.117	.115
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.022	.224	.090	.007	.020	.023	.015	.023	.028	.057	.036	.022	.014
REPLENISHMENT FREQUENCY	.222	.223	.223	.221	.254	.191	.234	.219	.213	.119	.174	.248	.348

Table 3.6

Percentage Excess of Operating Characteristics of a 72-Item System Under Statistical Control Over Those for System Under Optimal Control with Full Information (Revision Interval 26 Periods, Revision History Length 26 Periods)

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	4.9	1.8	4.6	6.1	8.2	1.8	-0.6	3.2	9.5	0.0	0.1	2.8	8.2
BACKLOG FREQUENCY	14.9	13.7	12.9	60.4	9.5	20.3	6.8	17.0	20.7	24.8	18.5	13.6	4.2
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	35.8	11.6	17.8	78.7	28.1	43.3	26.0	35.0	42.6	32.1	40.5	40.7	32.5
REPLENISHMENT FREQUENCY	1.7	4.8	2.2	-1.7	2.2	1.0	-0.1	2.3	3.2	0.9	1.3	1.8	2.3
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	2.8	0.4	2.9	3.5	4.5	1.2	-1.4	1.6	6.2	-0.7	0.3	2.6	5.5
BACKLOG FREQUENCY	12.4	8.1	12.0	102.2	8.7	16.0	1.3	15.0	20.4	7.5	13.7	15.2	13.0
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	45.7	13.4	20.0	100.7	42.4	48.6	32.9	44.6	54.5	71.5	58.0	40.9	27.9
REPLENISHMENT FREQUENCY	1.4	4.0	2.2	-1.8	1.7	1.1	-1.0	2.1	3.6	0.8	1.8	2.1	1.0
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	1.7	-11.0	-1.2	8.9	4.2	-0.4	-1.8	0.7	4.9	0.4	-0.4	0.6	4.0
BACKLOG FREQUENCY	21.5	24.0	16.0	25.3	17.9	25.0	15.9	22.7	25.5	25.7	23.5	21.5	15.7
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	27.4	32.2	23.6	25.9	22.9	31.6	17.4	29.0	31.9	41.6	37.4	25.9	16.8
REPLENISHMENT FREQUENCY	0.4	4.7	0.6	-3.7	0.1	0.9	0.2	-0.1	1.2	-0.0	0.9	1.7	-0.5

$\sigma^2/\mu = 9$ system where the percentage increments for these characteristics exceeded 100%.

In Table 3.7 we indicate the direction of sensitivity of the operating characteristics and policy variables to changes in input parameters for each of the three systems we have been comparing. The only discrepancy of note is the fact that, for the $\sigma/\mu = 1$ system, proportion demand backlogged shows no monotonic pattern with respect to mean demand whereas in both of the systems with σ^2/μ constant, a monotonic decreasing relationship is discernible.

As we did for the case of optimal control with full information, we consider the structural form of the dependence of expected total cost per period for the multi-item system on the levels of the input parameters. For optimal control in Section 2 (Table 2.5) it was observed that the distribution of expected total costs according to input parameter values closely conformed to a multiplicative relationship. Table 3.8 demonstrates the fit for the case of control with statistical information about demand, with a 26 period revision interval and utilizing a 26 period demand history. The differences in Table 3.8 are closely comparable to the differences noted in Table 2.3. The same trends can be observed by scanning the differences, and we caution against extrapolation. For the range of parameters tested, however, the use of constant standard-deviation-to-mean ratio has not impaired the fit of the multiplicative structure that was first observed for the case of constant variance-to-mean ratio.

We also consider the ratio $1/(1+\pi)$ for its value as an approximation to the frequency of backlog for a typical item in a multi-item system. As with the constant variance-to-mean systems, the actual value exceeds the estimate $1/(1+\pi)$ for all three values of π tested with standard-

Table 3.7

Operating Characteristics of a 72-Item Inventory System Using the Statistical Power Approximation: Sensitivity to Input Parameters (26-period Revision Interval, 26-period Revision History)

	INPUT PARAMETERS			
	π/h	K/h	Leadtime	Mean Demand
OPERATING CHARACTERISTICS				
E(COST)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(PERIOD-END INVENTORY)	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(BACKLOG COST)	↑ ↑ ?	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
E(BACKLOG FREQUENCY)	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑	? ? –
PROPORTION OF DEMAND BACKLOGGED	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑	? ↓ ↓
E(REPLENISHMENT COST)	– – –	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑
E(REPLENISHMENT FREQUENCY)	↓ ↓ ↓	↓ ↓ ↓	↓ ↓ ↓	↑ ↑ ↑
POLICY PARAMETERS				
D	– – –	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑
s	↑ ↑ ↑	↓ ↓ ↓	↑ ↑ ↑	↑ ↑ ↑
S	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑	↑ ↑ ↑

The three symbols in each column refer to Negative Binomial demand distributions with Standard Deviation/Mean = 1, Variance/Mean = 9, and Variance/Mean = 3, respectively.

↑ indicates a monotonically increasing relationship;
↓ indicates a monotonically decreasing relationship;
– indicates an approximately constant relationship;
? indicates that no simple relationship could be discerned.

Table 3.8

System of 72 Items with Negative Binomial Demand Distributions with Standard Deviation/Mean = 1
Statistical Information for 26 Period Demand History Used for Controls, with Revision Every 26 Periods
Sources of Average Total Cost

		π/h			K/h		LEADTIME		
	PROPORTION	4	9	99	32	64	0	2	4
PROPORTION		.232	.289	.479	.470	.530	.252	.338	.410
MEAN									
2	.091	.024	.028	.039	.041	.051	.026	.031	.035
		.021	.026	.044	.043	.048	.023	.031	.037
		.003	.002	-.005	-.002	.003	.003	.000	-.002
4	.153	.038	.046	.069	.070	.083	.041	.051	.061
		.035	.044	.073	.072	.081	.039	.052	.063
		.003	.002	-.004	-.002	.002	.002	-.001	-.002
8	.268	.062	.077	.129	.126	.143	.068	.091	.109
		.062	.077	.128	.126	.142	.068	.091	.110
		.000	.000	.001	.000	.001	.000	.000	-.001
16	.487	.107	.138	.242	.233	.254	.117	.165	.205
		.113	.141	.233	.229	.258	.123	.165	.200
		-.006	-.003	.009	.004	-.004	-.006	.000	.005
LEADTIME									
0	.252	.061	.073	.119	.115	.138			
		.058	.073	.121	.118	.134			
		.003	.000	-.002	-.003	.004			
2	.338	.078	.097	.162	.160	.178			
		.078	.098	.162	.159	.179			
		.000	-.001	.000	.001	-.001			
4	.410	.093	.118	.198	.196	.214			
		.095	.118	.196	.193	.217			
		-.002	.000	.002	.003	-.003			
C(FIX/C(IN)									
32	.470	.106	.134	.230					
		.109	.135	.225					
		-.003	-.001	.005					
64	.530	.126	.155	.250					
		.123	.153	.254					
		.003	.002	-.004					

deviation-to-mean ratio of 1. In fact, the values for the $\sigma/\mu = 1$ system are seen to be quite close to the observed values for the $\sigma^2/\mu = 9$ or $\sigma^2/\mu = 3$ systems.

Table 3.9

Backlog Frequencies for a Multi-Item Systems with Negative Binomial Demands

	π / h		
	4	9	99
1/(1+π)	.2	.1	.01
Standard Deviation/Mean = 1	.211	.106	.015
Variance/Mean = 9	.205	.107	.019
Variance/Mean = 3	.227	.106	.012

4. FORECASTING THE PERFORMANCE OF INVENTORY SYSTEMS UNDER STATISTICAL CONTROL

The manager of an inventory system operating with incomplete demand information will need to forecast system behavior, especially to justify the installation of scientific control. Forecasting may also be a routine requirement because of the periodic revision of the control parameters or even as part of a regular budgeting procedure.

Forecasting methods have been considered in MacCormick (1974), Estey and Kaufman (1975), Ehrhardt (1976), and Kaufman and Klincewicz (1976). We gather forecasting information using the same techniques; these involve double use of the demand history, for control and forecasting. Sample distributions of the forecasts are compared with those of the operating characteristics, which were obtained in Section 3.

The major issues to be resolved here are the extent of bias and the level of dispersion of the forecasts, together with the variation of these with system settings, including the demand process, unit costs, the replenishment leadtime, the revision interval, and the lengths of the histories used for revision and forecasting. Little consideration is given to an important sampling problem, namely, that since it is usually prohibitively expensive to gather a history of demands for every item in a large multi-item system, a sample of the items must be selected to make a forecast. The accuracy of such a forecast is likely to be critically dependent on the number of items in the sample and its composition. Forecasts made for the multi-item system in this simulation experiment have used every item in the system. Our work examines the case of a revision interval of 26 weeks and a revision history length of 26 weeks.

In the simulation experiments, point forecasts of the expected values of system operating characteristics are made each time the control parameters (s,S) are revised. The demand history used to make a forecast has been selected to contain demands for the same number of periods as the revision interval.

Average values of the forecasts of expected costs per period for the $\sigma/\mu = 1$, $\sigma^2/\mu = 9$, and $\sigma^2/\mu = 3$ multi-item systems are shown in Table 4.1, the sample standard deviations being shown in parentheses.

The sample statistics for forecasts shown in Table 4.1 can be compared with those in Table 3.1 for average costs per period. The comparison with average costs per period is explicit in Table 4.2, which contains the percentage differences between the forecasts and the estimated costs.

Table 4.1

Average Forecasted Values for Expected Costs per Period for a Multi Item Negative Binomial System Controlled with Statistical Power Approximation

Cost Components	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
Inventory	1959 (79) [65.5]	1966(68) [65.6]	1283(28) [56.2]
Backlog	384 (154) [12.8]	405(57) [13.5]	273(30) [12.0]
Replenishment	647 (14) [21.6)	626(15) [20.9]	727(11) [31.8]
Total	2991 (104) [100.0]	2996(97) [100.0]	2283(44) [100.0]

Note: Estimates of the standard deviations for the forecasts are shown in parentheses and each cost component's percent of the total is shown in brackets.

Table 4.2

Average Costs Per Period for Multi Item Negative Binomial System Controlled by Statistical Power Approximation: Comparison of Forecasts with Corresponding Estimated Expected Values

Cost Components	Standard Deviation/Mean = 1	Variance/Mean = 9	Variance/Mean = 3
Inventory	1.5	1.2	1.1
Backlog	55.0	59.3	37.7
Replenishment	0.8	1.2	0.6
Total	14.5	17.1	7.5

Note: Table 4.2 shows the percentage value of the underestimate by the forecast of the realized value.

On the average, the forecasts underestimate every component of expected cost for all systems, a result which is consistent with all previous analysis. The underestimation for the constant standard deviation/mean system is 14.5% which compares with 17.1% for the variance/mean = 9 system and 7.5% for the variance/mean = 3 system. Like these systems, the major source of underestimation is the backlog cost component which is **underestimated by 55.0%. We note, however, in Table 4.1, that for the standard deviation/mean = 1 system, the standard deviation of the backlog cost estimate is 154, which is much greater than the values for the variance/mean = 9 system (57) or the variance/mean = 3 system (30).**

Table 4.3 shows the percentage difference (bias) between the forecasted cost components and the observed simulation values. For the constant standard deviation/mean system, the bias in the forecast of expected average total cost and the backlog cost components tends to increase with increasing mean demand and the bias in the forecast of the replenishment cost component tends to increase with increasing leadtime. For the constant variance-to-mean systems these relationships were all monotonically decreasing. Other trends in the bias of cost component forecasts with respect to input parameters follow the same pattern in all three systems.

Table 4.4 demonstrates biases for forecasts of operating characteristics other than costs, expressed as percentages of the estimates of the actual expected values of the operating characteristics. We see that whereas, for the standard deviation/mean = 1 system, bias in the forecast of backlog frequency and proportion demand backlogged tends to increase with increasing mean demand and the bias in the forecasts of replenishment frequency tends to increase with leadtime, these same relationships are monotonically decreasing for the constant variance-to-mean ratio systems. The only characteristic where the bias in the forecast is consistently worse for the constant standard-deviation-to mean system is period-end inventory, but overall, the bias for this characteristic is only 1.5% below actual and should not indicate that use of the Power Approximation algorithm and retrospective simulation for a constant standard-deviation-to-mean system results in any significant problems which are not also present for constant variance-to-mean systems.

Table 4.3

Forecasting for a 72-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Average Costs per Period
(Revision Interval 26 Periods, Revision History 26 Periods, Forecasting History 26 Periods)

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
INVENTORY	1.5	3.8	2.6	0.2	1.8	1.3	0.5	0.9	2.5	0.7	1.3	1.5	1.7
BACKLOG	55.0	20.4	40.3	86.8	57.4	53.0	35.8	55.3	65.6	37.6	48.0	53.9	59.8
REPLENISHMENT	0.8	0.6	0.8	0.8	0.5	0.9	0.6	0.7	1.0	1.0	1.0	0.7	0.5
TOTAL	14.5	7.7	11.8	19.4	15.1	13.8	8.6	14.4	18.1	7.3	11.6	14.6	16.6
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
INVENTORY	1.2	2.8	1.6	0.5	1.0	1.3	0.4	1.0	1.7	1.3	1.2	1.4	0.9
BACKLOG	59.3	25.1	42.1	87.1	62.3	56.4	38.4	60.4	69.5	70.1	62.3	56.6	51.5
REPLENISHMENT	1.2	1.1	1.3	1.4	0.9	1.5	1.5	1.3	0.9	4.2	2.0	0.5	0.5
TOTAL	17.1	9.1	12.6	23.4	18.3	16.1	10.2	17.6	21.0	26.9	20.6	15.7	12.1
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
INVENTORY	1.1	2.8	1.7	0.2	1.1	1.1	0.1	0.5	2.1	1.2	1.3	0.9	1.0
BACKLOG	37.7	14.6	25.1	80.0	42.9	33.2	20.3	37.0	47.6	47.1	41.3	36.8	31.3
REPLENISHMENT	0.6	0.6	0.8	0.5	0.5	0.7	0.7	0.6	0.6	1.7	1.2	0.3	0.2
TOTAL	7.5	4.9	5.9	10.1	8.7	6.5	3.2	7.3	10.7	11.2	9.1	7.0	5.5

Table 4.4

Forecasting for a 72-Item System Under Statistical Control Estimated Percentage Difference between Forecasts and Expected Values of Operating Characteristics
(Revision Interval 26 Periods, Revision History 26 Periods, Forecasting History 26 Periods)

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS											
		π/h			K/h		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8	16
NEGATIVE BINOMIAL DEMANDS STANDARD DEVIATION/MEAN = 1													
PERIOD-END INVENTORY	1.5	3.8	2.6	0.2	1.8	1.3	0.5	0.9	2.5	0.7	1.3	1.5	1.7
BACKLOG FREQUENCY	12.3	5.3	18.0	71.8	14.4	10.3	3.7	12.3	19.6	9.8	10.7	12.6	16.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	55.0	20.4	40.3	86.8	57.4	53.0	35.8	55.3	65.6	37.6	48.0	53.9	59.8
REPLENISHMENT FREQUENCY	0.7	0.6	0.8	0.7	0.5	0.9	0.5	0.7	0.9	1.0	0.7	0.6	0.5
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 9													
PERIOD-END INVENTORY	1.2	2.8	1.6	0.5	1.0	1.3	0.4	1.0	1.7	1.3	1.2	1.4	0.9
BACKLOG FREQUENCY	16.3	7.9	22.0	73.9	18.7	14.1	4.6	17.1	25.1	18.6	16.7	15.0	15.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	59.3	25.1	42.1	87.1	62.3	56.4	38.4	60.4	69.5	70.1	62.3	56.6	51.5
REPLENISHMENT FREQUENCY	1.1	1.1	1.1	1.2	0.9	1.5	1.4	1.1	0.9	3.9	2.0	0.4	0.4
NEGATIVE BINOMIAL DEMANDS VARIANCE/MEAN = 3													
PERIOD-END INVENTORY	1.1	2.8	1.7	0.2	1.1	1.1	0.1	0.5	2.1	1.2	1.3	0.9	1.0
BACKLOG FREQUENCY	9.5	5.3	12.6	66.3	12.2	7.1	3.5	9.2	15.2	11.7	9.1	9.5	8.0
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	37.7	14.6	25.1	80.0	42.9	33.2	20.3	37.0	47.6	47.1	41.3	36.8	31.3
REPLENISHMENT FREQUENCY	0.6	0.6	0.7	0.5	0.5	0.7	0.7	0.6	0.6	1.6	1.2	0.3	0.2

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