

1708

File Cy:

BRL CR 323

BRL

AD A035171

CONTRACT REPORT NO. 323

THE RESPONSE OF THIN TARGETS TO PROJECTILE IMPACT

Prepared by

Systems, Science and Software
P. O. Box 1620
La Jolla, California 92037

December 1976

Approved for public release; distribution unlimited.

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

Destroy this report when it is no longer needed.
Do not return it to the originator.

Secondary distribution of this report by originating
or sponsoring activity is prohibited.

Additional copies of this report may be obtained
from the National Technical Information Service,
U.S. Department of Commerce, Springfield, Virginia
22151.

The findings in this report are not to be construed as
an official Department of the Army position, unless
so designated by other authorized documents.

*The use of trade names or manufacturers' names in this report
does not constitute indorsement of any commercial product.*

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CONTRACT REPORT NO. 323	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE RESPONSE OF THIN TARGETS TO PROJECTILE IMPACT		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER SSS-R-75-2630
7. AUTHOR(s) J. K. Dienes		8. CONTRACT OR GRANT NUMBER(s) DAAD05-75-C-0724
9. PERFORMING ORGANIZATION NAME AND ADDRESS Systems, Science and Software P. O. Box 1620 La Jolla, California 92037		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS USA Ballistic Research Laboratory Aberdeen Proving Ground, MD 21005		12. REPORT DATE DECEMBER 1976
		13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) US Army Materiel Development & Readiness Command 5001 Eisenhower Avenue Alexandria, VA 22333		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Helmet	Personnel Armor	Woven Roving
Helmet Penetration	Membrane Theory	XP Plastic
Helmet Perforation	Nylon	KEVLAR
Residual Velocity	Hadfield Steel	Titanium
Ballistic Limit	Glass Fabric	
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
The response of thin targets to projectile impact is investigated under the assumption that membrane theory is sufficient to describe the interaction theoretically derived. Residual velocities of penetrating projectiles for a number of materials are compared with experimental data. The method allows data for a specific target thickness, projectile size and projectile velocity to be used in determining the ballistic properties under other impact conditions. This requires that a value for the ballistic figure of merit, W, be obtained for each material, which can then be used to rank a number of		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. materials in a quantitative fashion, even though the tests performed were quite dissimilar.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

TABLE OF CONTENTS

	Page
I. INTRODUCTION AND SUMMARY	1
II. ONE DIMENSIONAL IMPACT	3
III. AXISYMMETRIC IMPACT	7
IV. COMPARISON OF MEMBRANE THEORY WITH EXPERIMENT	15
V. CORRELATION OF DATA FOR DIFFERENT VELOCITIES	19
VI. RANKING OF MATERIALS	21
VII. CONCLUSIONS AND RECOMMENDATIONS	23
REFERENCES	25
APPENDIX I	27
APPENDIX II	31
DISTRIBUTION LIST.	51

I. INTRODUCTION AND SUMMARY

The response of thin targets to projectile impact is investigated under the assumption that membrane theory is sufficient to describe the interaction. Though it is possible with existing computer codes to compute such problems in greater detail as described, for example, by Dienes et al⁽¹⁾, the membrane theory is amenable to analytic treatment since the equation of motion is of only second order. This makes it possible to gain an overview of the problem which exhibits the main dynamic phenomena. It also makes it possible to analyze approximately the response of materials such as cloth and skin which are not represented by bending theory either in analytic or in computer solutions.

The organization of this report is strongly influenced by a companion document by Dienes and Miles⁽²⁾ which describes the membrane theory in detail in a form suitable for publication in the open literature. In this report the main conclusions are summarized, and applied to a variety of ballistic problems. In Section II the one dimensional plastic response of a wire to impact is analyzed. The conclusions are similar to those for axisymmetric impact, but the mathematics is considerably simpler because of the elementary nature of solutions to the one dimensional wave equation. The results of the axisymmetric impact problem are summarized in Section III. One of the main conclusions is that the residual velocity does not go to zero as the impact velocity decreases to the ballistic limit. Projectiles that penetrate will always do so with a substantial velocity. Residual velocities of penetrating projectiles are compared with experimental data in Section IV. The method allows data for a specific target thickness, projectile size and projectile velocity to be used in determining the ballistic properties under other impact conditions. This requires that a value for the ballistic figure of merit, w , be obtained for each material. Since the resistance of materials to impact is determined by this parameter, it can be used to rank materials in a quantitative fashion, even when the tests performed are quite dissimilar. It is shown in Section V that this approach works successfully for a nylon for which low speed data is available. At high speeds, however, penetration at the ballistic limit may involve cratering, melting, spallation and other phenomena which the current theory does not allow for. In Section VI, the figure of merit, w , is given for a number of materials. This allows them to be ranked in order of their ballistic figure of merit. Finally, conclusions and recommendations are given in Section VII.

In Appendix I it is shown that membrane theory correlates well with experimental data on the response of circular plates. A listing of the computer program used to evaluate the ballistic limit and residual velocity is given in Appendix II.

II. ONE DIMENSIONAL IMPACT

To illustrate the physical character of the membrane approach to target deformation, consider the response of a rigid, perfectly plastic wire to impact. Since the tension in the wire is constant, its motion is governed by the equation

$$\rho A \ddot{y} = \sigma A y'' \quad (2.1)$$

where A denotes the cross-sectional area of the wire; $y(x,t)$, its deflection; ρ , the density and σ the stress. In view of the assumption of ideal plasticity

$$\sigma = \zeta Y \quad (2.2)$$

where Y is the flow stress and ζ is ± 1 , the stress being positive when the strain rate is positive and negative otherwise. Consequently, the equation of motion reduces to the classical wave equation,

$$\ddot{y} = c^2 y'' \quad (2.3)$$

where

$$c = \sqrt{Y/\rho}$$

during the initial phase of the impact, with the quantity, c , being interpreted as the speed of plastic waves.

The solution can be written in the form

$$y = f(x - ct) + g(x + ct). \quad (2.4)$$

We consider only the right going waves in the region $x > 0$ of Fig. 1, so that the solution reduces to the form

$$y = f(x - ct). \quad (2.5)$$

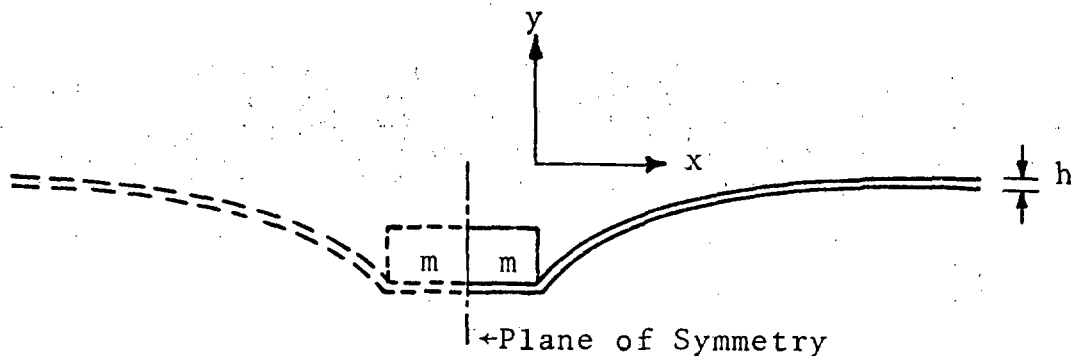


Fig. 1 Sketch of geometry for the problem of an infinite, ductile wire struck by a mass, m .

Only the region to the right of the projectile need be considered, since the geometry is symmetric with respect to the center of the impacting mass. The boundary condition to be imposed at the origin is

$$m\ddot{z} = hYy' \quad (2.6)$$

where m denotes the mass per unit width and

$$z = y(0, t). \quad (2.7)$$

denotes the displacement of the mass. In view of Eq. 1.5 the boundary condition can be written

$$\alpha y' = y'' \quad (2.8)$$

where

$$\alpha = h\sigma/c^2m = h\rho/m. \quad (2.9)$$

The argument of y in Eq. 1.8 is

$$\xi = - ct \quad (2.10)$$

since the equation is valid only at $x = 0$. It has the solution

$$f(\xi) = y_0 (e^{\alpha \xi} - 1) \quad (2.11)$$

under the constraint that the deflection must initially vanish. It is straightforward to show that the overall string response is, then, given by

$$y = y_0 (e^{\alpha(x - ct)} - 1) \quad (2.12)$$

The initial condition for this problem, based on conservation of momentum, is

$$(m + \rho h a) \dot{y} = mv \quad (2.13)$$

where v is the initial projectile velocity. Solving for the unknown coefficient, one finds

$$y = \frac{m}{m + \rho h a} \frac{m}{h \rho} \frac{v}{c} \left(1 - e^{\alpha(x - ct)} \right), \quad x < ct \quad (2.14a)$$

$$= 0, \quad x > ct \quad (2.14b)$$

At late times the deflection approaches the value

$$y_{\infty} = \frac{m}{m + h \rho a} \frac{m}{h \rho c} v. \quad (2.15)$$

Using the small strain approximation

$$\epsilon = \frac{1}{2} y'^2 \quad (2.16)$$

we find the maximum strain equal to

$$\epsilon_{\max} = \frac{1}{2} \left(\frac{m}{m + \rho h a} \frac{v}{c} \right)^2. \quad (2.17)$$

If it is assumed that the material is ductile and fails when the strain reaches a critical value, ϵ_f , (sometimes termed the breaking index) then failure occurs when the impact velocity exceeds the critical value

$$v_c = \frac{m + \rho h a}{m} \sqrt{2\epsilon_f Y / \rho}. \quad (2.18)$$

The quantity

$$w = \sqrt{2\epsilon_f Y / \rho} \quad (2.19)$$

is characteristic of the material and can be determined either from static measurements or from a measurement of the critical velocity, v_c . For a rigid-plastic material $w^2/2$ is equal to the amount of energy per unit mass that the material can absorb without failure.

III. AXISYMMETRIC IMPACT

The response of an infinite target impacted by a cylindrical projectile, under the assumption that the flow stress is constant in the target material, is discussed in detail in the companion report by Dienes and Miles.⁽²⁾ In this section the main results are summarized with an emphasis on their physical interpretation.

The equation of motion for the deflection, y , of a membrane under the assumption of axial symmetry is

$$\rho y_{tt} = Y(ry_r)_r \quad (3.1)$$

where Y is the flow stress; ρ , the density and r , the radial coordinate. On the contact circle the membrane stresses cause a deceleration of the projectile which leads to the boundary condition

$$2\pi a Y h y_r = (m + \pi a^2 \rho h) y_{tt} \quad (3.2)$$

where m denotes the projectile mass; a , its radius and h is the membrane thickness. It proves convenient to define the dimensionless variables

$$\eta = r/a \quad (3.3)$$

$$\tau = ct/a \quad (3.4)$$

and

$$\zeta = y/(1 - \mu)\beta a \quad (3.5)$$

where

$$\mu = \frac{\pi a^2 \rho h}{m + \pi a^2 \rho h} \quad (3.6)$$

and

$$\beta = v/c. \quad (3.7)$$

In terms of these dimensionless variables the boundary value problem can be cast into the form

$$(\eta \zeta_{\eta})_{\eta} = \eta \zeta_{\tau\tau} \quad (3.8)$$

with the boundary condition at $\eta = 1$

$$2\mu \zeta_{\eta} = \zeta_{\tau\tau} \quad (3.9)$$

and the initial conditions

$$\zeta = \dot{\zeta} = 0 \quad (\eta > 1, \tau = 0) \quad (3.10)$$

and

$$\zeta = 0, \dot{\zeta} = 1 \quad (\tau = 0) \quad (3.11)$$

By the method of Laplace transforms it can be shown that the solution can be expressed as

$$\zeta = 4\mu \operatorname{Re} [A e^{s_0 \tau}] + 2\mu J(\tau) \quad (3.12)$$

where Re denotes the real part of the expression in brackets. Here

$$A = \frac{1}{s_0^3 + 4\mu(1 - \mu)s_0} \quad (3.13)$$

and

$$J(\tau) = \int_0^{\infty} \frac{e^{-x\tau} dx}{[x^2 K_0(x) + 2\mu K_1(x)]^2 + \pi^2 [x^2 I_0(x) - 2\mu I_1(x)]^2} \quad (3.14)$$

where K_0 , K_1 , I_0 and I_1 are modified Bessel functions. In the expression for A , s_0 denotes the root of the transcendental equation

$$sK_0(s) + 2\mu K_1(s) = 0, \quad (3.15)$$

The term, A , arises from the contribution of a pair of poles in the s plane. The maximum dimensionless deflection $\zeta_{\max}$, was obtained numerically and is shown in Fig. 2.

The strain in the membrane is given, as in the previous section, by

$$\epsilon = \frac{1}{2} y_r^2 \quad (3.16)$$

for small deformations. Denoting by a the deceleration of the projectile, it can be readily shown that the strain is given by

$$\epsilon = \frac{1}{2} (1 - \mu)^2 (\beta a)^2. \quad (3.17)$$

Under the assumption of a critical strain failure criterion, it follows that penetration occurs for impact velocities above

$$v_c = \frac{w}{(1 - \mu) a_m} \quad (3.18)$$

where a_m denotes the maximum value of a . As in the previous section, the quantity w can be taken as a figure of merit and its value is given by

$$w = \sqrt{2\epsilon_f Y/\rho} \quad (3.19)$$

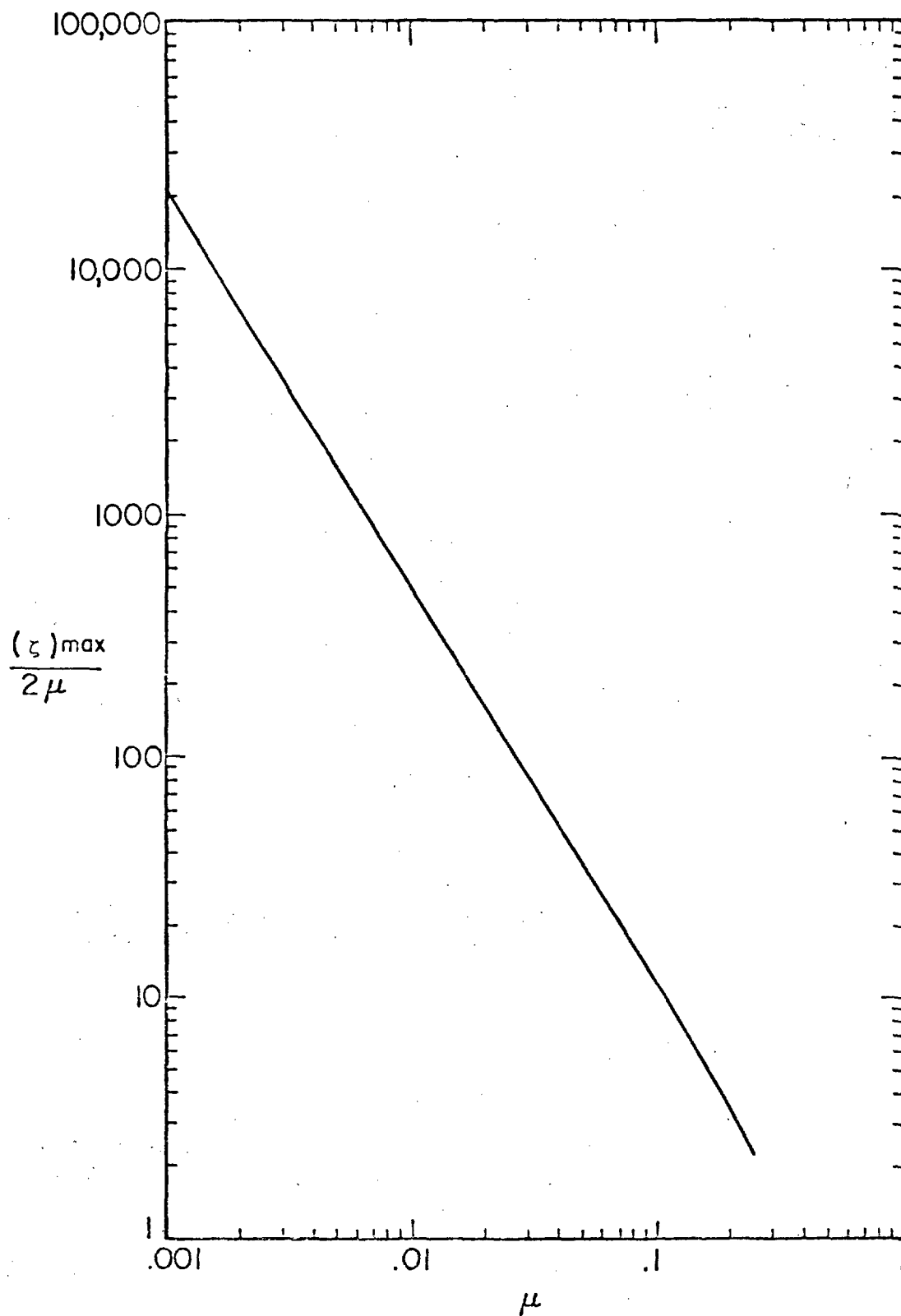


Fig. 2. The maximum value of the dimensionless deflection. The actual peak deflection is given by $y_{\max} = (1-\mu)\beta a \zeta_{\max}$.

The quantity a_m is plotted in Fig. 3 as a function of μ . Its value always exceeds unity. Thus, the ballistic limit for a cylinder of radius a is always less than that for a rectangular prism of width a . The ballistic limit decreases as μ becomes small which, from inspection of (3.6), occurs when the projectile length increases and the radius decreases with a fixed projectile mass. This is, of course, in agreement with one's expectations. The dimensionless time at which peak deflection occurs, τ_d , is shown in Fig. 4, which also includes the dimensionless time to maximum strain, τ_m . The actual times, based on Eq. 3.4, are

$$t_d = a\tau_d/c \quad (3.21)$$

for maximum deflection, and

$$t_m = a\tau_m/c \quad (3.22)$$

for maximum strain. Maximum strain always occurs prior to maximum deflection. This implies that when failure occurs, based on a maximum strain criterion, that the projectile is still in motion and, hence, that it will penetrate at a finite velocity. It is shown in Ref. 2 that the residual velocity is

$$v_r = F(\mu)v,$$

with $F(\mu)$ being plotted in Fig. 5.

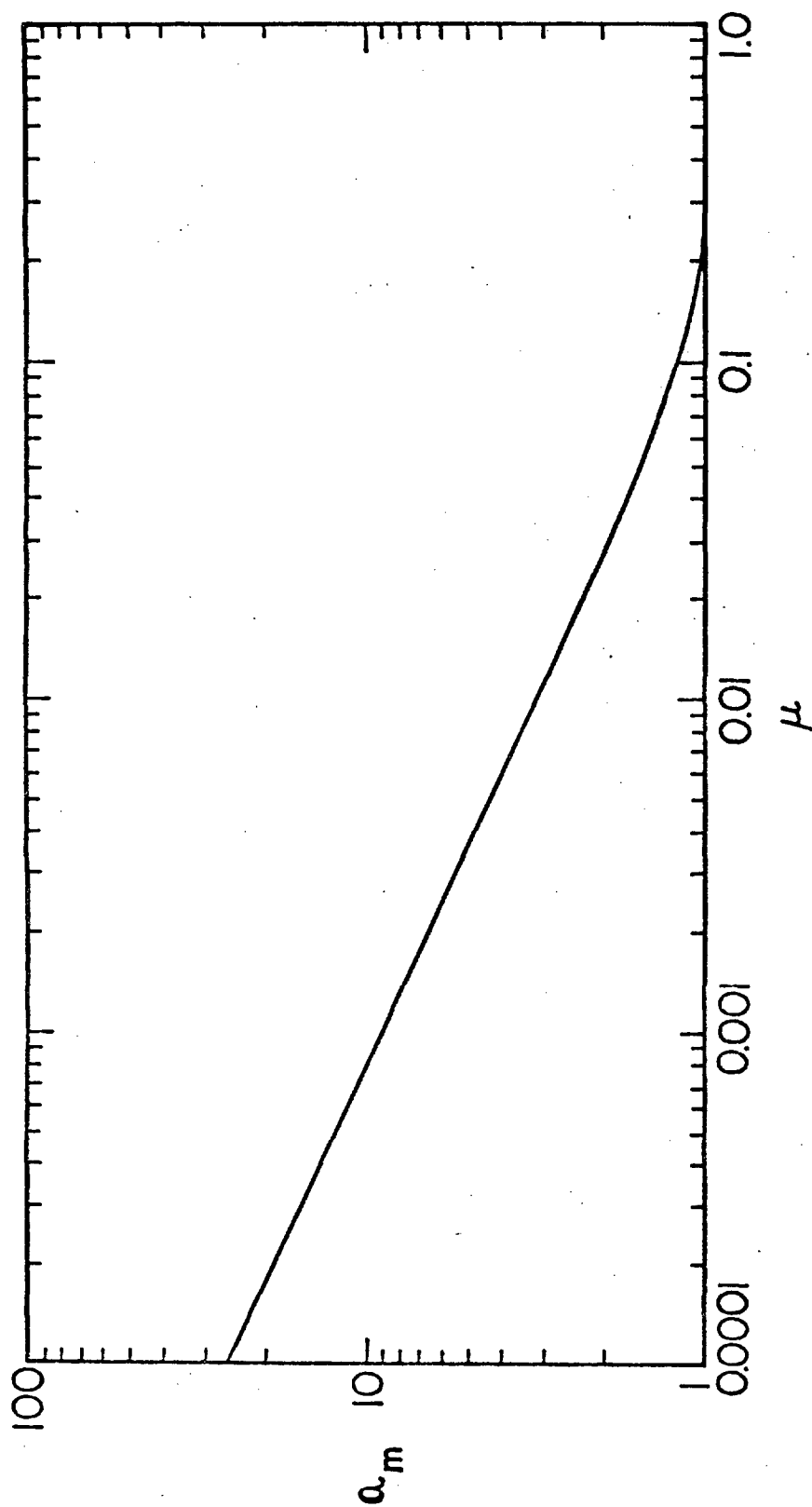


Fig. 3. The maximum value of the normalized deceleration, $a_m(\mu) = a(\tau_m)$; τ_m is plotted in Fig. 4.

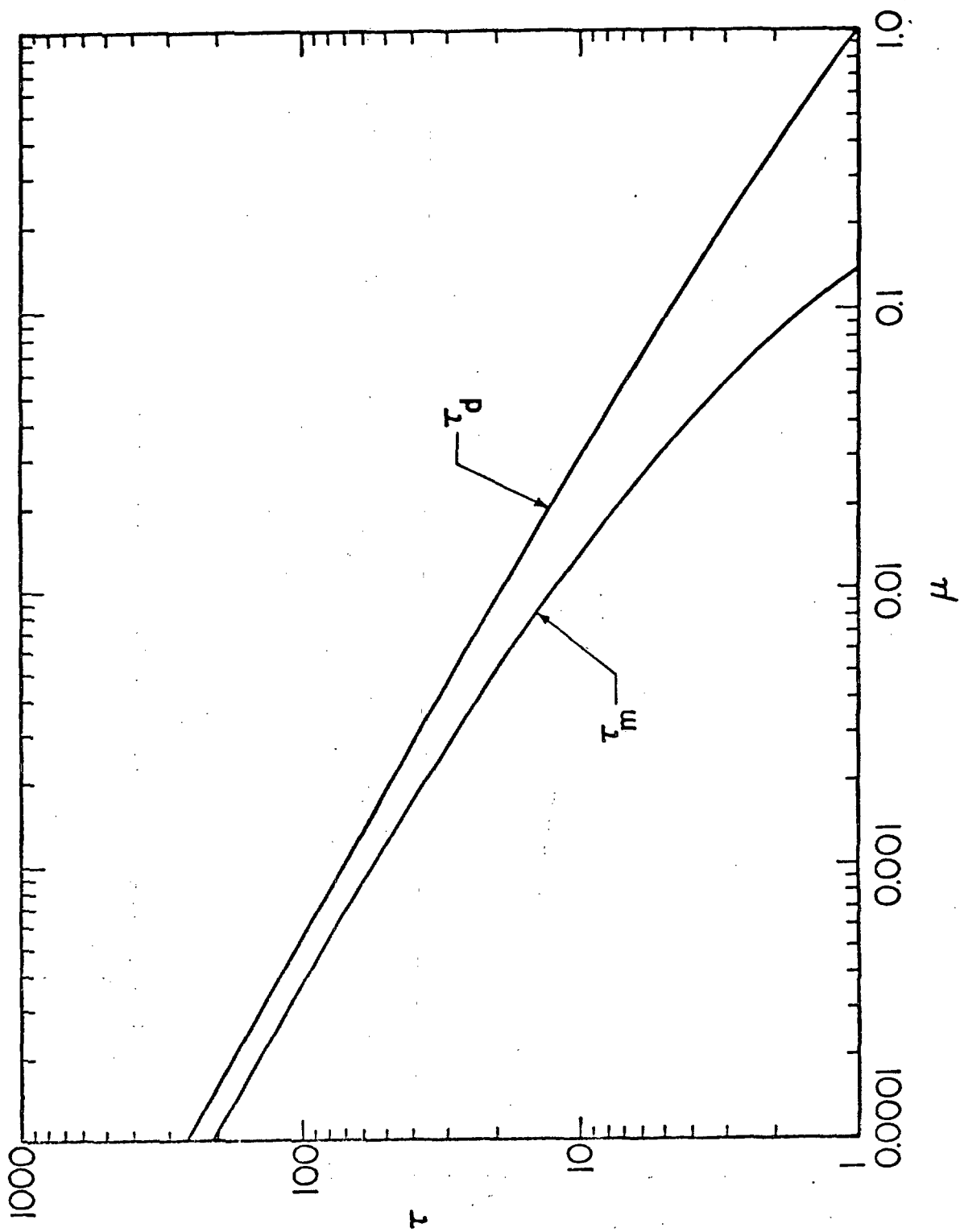


Fig. 4. The time τ_d at which the deflection is a maximum and the time τ_m at which the strain is a maximum. Note that maximum strain always precedes maximum deflection.

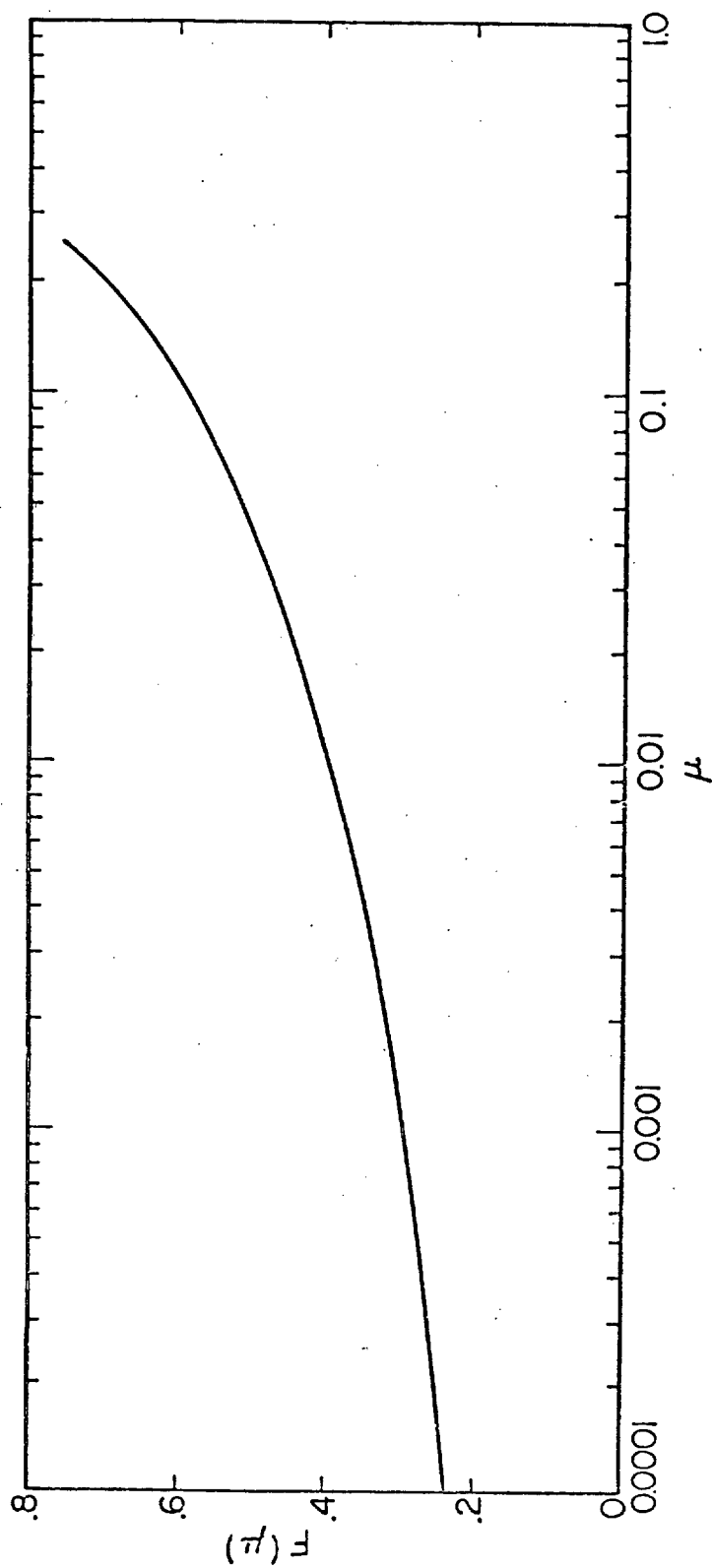


Fig. 5. The ratio of the residual velocity to the initial velocity of the projectile.

IV. COMPARISON OF MEMBRANE THEORY WITH EXPERIMENT

It is known that for small deflections, particularly when the materials lie in the elastic regime, the deformation of plates is governed by bending theory. With increasing deflections, stretching of the middle surface becomes important and the membrane stresses begin to dominate. Ultimately, for very large deflections, the resistance is governed virtually in its entirety by the membrane stresses, which may be considered uniform through the thickness and independent of radius. A comparison between calculations based on this approach and measured deflections of impulsively loaded circular metal plates is given in Appendix I. The comparison shows that membrane theory describes the deformation well when the deflection exceeds 15% of the radius.

To indicate the potential accuracy of the method for determining the ballistic limit, we consider here the resistance of a sheet of the titanium alloy Ti5Al-2.5Sn. The residual velocities measured in a series of tests described by Bruchey⁽³⁾ in which steel cylinders were fired into sheets 0.1295 cm in thickness are shown in Fig. 6 for a cylinder of radius .284 cm and mass 1.037 gm. Based on a titanium density of 4.5 g/cm³ we find $\mu = 0.125$ and $F(\mu) = 1.11$. If we take the ballistic limit from Fig. 6 as the lowest striking velocity at which residual velocities are clustered, about 340 m/s, then $w = 330$ m/s. Now, the flow stress for a similar alloy, Ti6Al-4V, is given by Lindholm, Yeakley and Bessey⁽⁴⁾ as 188ksi (13.0×10^9 d/cm²) and the cited strain to failure based on post-test measurements is 18 percent. (The similarity in properties of the alloys can be verified by comparing stress-strain curves given by Wolf⁽⁵⁾). Using these data we find $w = 324$ m/s, which is only 2 percent less than the value obtained from ballistic data. The strain-to-failure of 18% based on post-test measurements is significantly larger than is indicated by their stress-strain curve, but such data is not commonly available in handbooks, and for this reason we have had to refer to the more specialized data for Ti6Al-4V.

The minimum residual velocity can be obtained using the value of μ cited above and Fig. 7. For a projectile velocity of 340 m/s we find $v_r = 210$ m/s, in good agreement with the lowest values observed by Bruchey. This result, that the residual velocity does not go to zero as the impact velocity decreases to the ballistic limit, is our most striking conclusion.

To determine the residual velocity above the ballistic limit, we note that at $r = a$ the strain is given by Eq. 3.16.

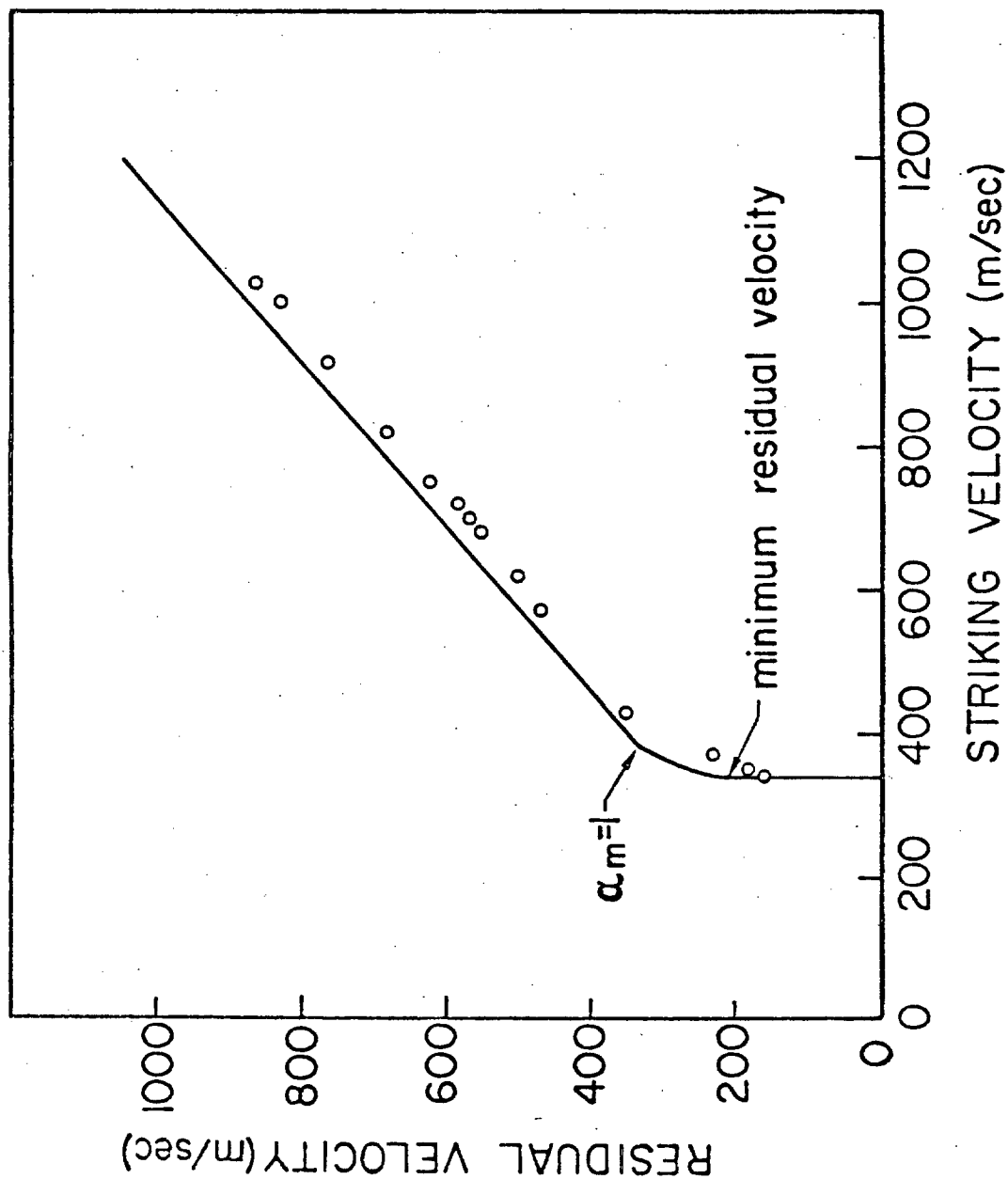


Fig. 6. A comparison of theoretical and measured residual velocities for a 1.04gm steel cylinder fired into an 0.13 cm (thick) titanium plate (Bruchey 1973). Failure occurs without membrane deflection for velocities above that at which $\alpha_m(\mu) = 1$.

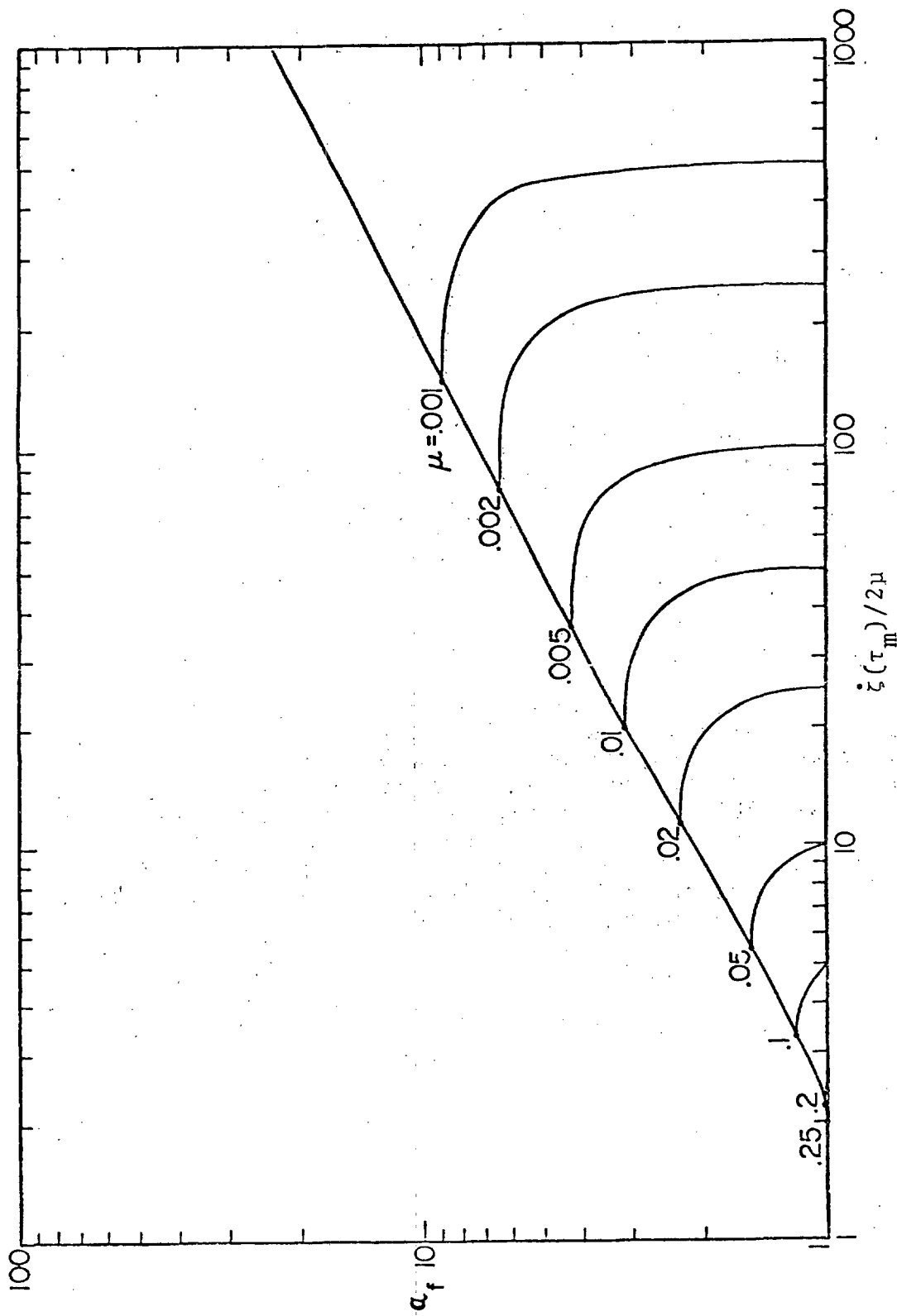


Fig. 7. The relation between $\dot{\zeta}_m = \dot{\zeta}(\tau_m)$ and $a_m \equiv a(\tau_m)$ for various values of the parameter μ ; see (3.17), (4.2) and (4.3). The upper boundary is the locus of $a_f = a_m$ and determines the residual velocity at the ballistic limit. For $a_f = 1$ plug failure occurs without membrane deformation.

The value of y_r at the time of maximum strain may be obtained by solving

$$\sqrt{2\epsilon_f} = - (1 - \mu)\beta\ddot{\zeta}(\tau_m) \quad (4.1)$$

for the time of maximum strain, τ_m , at a specific impact velocity (recalling that $\dot{\zeta} < 0$). Then the residual velocity is given by

$$v_r = 2\mu(1 - \mu)v\dot{\zeta}(\tau_m) \quad (4.2)$$

In Fig. 7 we have plotted $a_f = -\ddot{\zeta}(\tau_m)/2$ versus $\zeta(2\mu)/2\mu$ for various values of μ . Thus, $\dot{\zeta}(\tau_m)$ can be obtained from (4.1) without explicitly determining τ_m by means of the graphs. In practise, it is convenient to determine a_f from the relation

$$a_f = \frac{w/v}{2\mu(1 - \mu)} \quad (4.3)$$

and the residual velocity is given by 4.2. The residual velocities for $a_f \geq 1$ shown in Fig. 6 were obtained in this way, and though the experimental data are too scattered to confirm the theory in detail in this range, the overall trend is reasonable. As the impact velocity increases, the time to maximum strain decreases until at $\tau_m = 0$, $\zeta = 1$. The corresponding value of v , which we denote by $\bar{v}$, is the upper bound of the impact velocities that result in membrane deflection followed by membrane failure.

At impact velocities above $\bar{v}$ the membrane fails instantly (in the current theoretical model) and the residual velocity is reduced only by transfer of momentum to the ruptured plug that is removed from the membrane. Then

$$v_r = \frac{m}{m + \pi a^2 \rho h} v \quad (4.3a)$$

$$= (1 - \mu)v \quad (4.3b)$$

This trend agrees well with the measured residual velocities shown in Fig. 6.

V. CORRELATION OF DATA FOR DIFFERENT VELOCITIES

The theory outlined in the preceding sections is based on a number of strong assumptions concerning projectile shape and rigidity and the characteristics of the target. In this section we consider two experimental determinations of the ballistic limit and examine the correlation provided by membrane theory.

Laible, Figucia and Ferguson⁽⁶⁾ have examined the resistance to penetration of a number of high-modulus fibers, primarily nylons and other organic fibers. In one pair of experiments the ballistic limit (v_{50}) was determined for an organic fiber described as X-500, type I. Its tenacity is reported as 12-14 grams per denier; the modulus as 500 grams per denier; elongation to failure as 2-4%; and density as 1.47 g/cm^3 . The relevance of these properties to the properties of the fabric is somewhat obscure, however, since the mechanical behavior of a fabric may be very complex even when its constituent fibers behave in a linear fashion. The tests involved impact by a 17 grain, 0.22 caliber missile. Its velocity was varied until the value at which 50% of the projectiles were defeated by the target was determined, and this value was defined as the ballistic limit, v_{50} . The results are summarized in Table I. It is remarkable that the values of the ballistic figure of merit, w , agree within 2 percent between the two tests in view of the many respects in which the details of the encounter differ from the assumptions made in the analysis.

Bruchey⁽³⁾ has determined the ballistic limit for two steel projectiles fired into titanium targets of the same thickness. The results for the heavier projectile, with a mass of 1.037 g, were described in the previous section. The projectile radius was 0.284 cm, and the areal density of the target was $.583 \text{ g/cm}^2$. As previously described, this leads to a value of 330 m/sec for the ballistic figure of merit, w . A cylinder having dimensions which were approximately half those of the low speed projectile ($a = .131 \text{ cm}$) was also tested, and the ballistic limit was determined to be 585 m/s. It follows that the value of μ is .1945, $F(\mu) = 1.02$ and $w = 481 \text{ m/s}$. This figure is significantly higher (46%) than that determined from the low velocity projectile, indicating that at higher speeds the target material behaves more efficiently. This can probably be attributed to the greater dissipation of energy per unit mass involved with high speed projectiles, with a substantial fraction of the energy going into the cratering process. It would be of interest to examine the interaction in careful experiments or numerical calculations to determine in greater detail why the target is more efficient in resisting high speed impacts.

TABLE I
COMPARISON OF THE BALLISTIC RESISTANCE OF TWO THICKNESSES OF X-500

Areal density oz/ft ²	Areal density, g/cm ²	m_1 $\pi a^2 \rho h$ g	μ	$F(\mu)$	v_{50} m/s	$(1-\mu)F(\mu)v_{50}$ m/s
18.4	.560	.1369	.1107	1.18	326	342
11.1	.337	.0824	.0697	1.37	263	335

VI. RANKING OF MATERIALS

The impact resistance of a number of materials for which ballistic data are available is shown in Table II. Based on this data, the Hadfield steel which has been traditionally used for infantry helmets and many kinds of armor plate ranks lowest among the materials considered with $w = 260$ m/s. Kevlar, recently developed by Dupont, ranks highest among the materials analyzed, with $w = 872$ m/s. These data are taken from widely differing sources, however, and it would be desirable to conduct a systematic series of tests to provide a more direct verification of this approach to ranking materials. It is not entirely clear from the raw data what properties constitute a good ballistic material. It may be that woven materials are better than plate materials, and it may be that local melting plays a role in some cases. Though the approach proposed here may serve as a rough guide to the relative efficiency of different materials, the choice of materials for specific protective missions should involve careful consideration of the specific penetration mechanisms.

TABLE II

COMPARISON OF THE BALLISTIC RESISTANCE OF VARIOUS
MATERIALS RANKED BY THE PARAMETER w

Material	Reference	Areal Density, ρ gm/cm ²	Projectile Mass, m grams	Projectile Radius, a cm	Mass Ratio, μ	Ballistic Limit, v_{50} meters/sec	w meters/sec
Hadfield Steel	McManus ⁷	.87	1.10	.279	.162	290	260
Glass Fabric	Laible ⁶	.55	1.10	.279	.109	282	296
Titanium Woven	Bruchey ³	.58	1.04	.284	.125	340	330
Roving X-500	Bruchey ³	.64	1.04	.284	.135	360	342
Type I Nylon	Laible ⁶	.56	1.10	.279	.111	326	342
Tire Yarn Aluminized	Laible ⁶	.57	1.10	.279	.112	373	391
Nylon	Flaherty ⁸	.014	.13	.159	.0091	132	429
Nylon 728	Bruchey ³	.41	1.104	.284	.090	485	543
XP Plastic	Bruchey ³	.40	1.037	.284	.089	510	576
Kevlar	Kennel ⁹	.098	10.2	.483	.0074	244	872

VII. CONCLUSIONS AND RECOMMENDATIONS

It has proved possible to determine the motion of a membrane impacted by a cylindrical mass in closed form with the assumption that the flow stress in the membrane material is uniform. A comparison of the computed deflection with measurements published by Florence in 1966 has indicated that the assumption that the flow stress is constant and equal to the yield strength of the material, for circular plates, leads to reasonably accurate results when the deflection exceeds 15% of the radius.

Of the various possible failure theories, a critical strain criterion is the simplest to implement in the current theoretical approach. It appears to lead to good correlation when compared with the low speed titanium results obtained by Bruchey, both with regard to the ballistic limit and the residual velocity. It is found that the residual velocity is finite, even when the target is penetrated at the ballistic limit. This is due to the fact that maximum strain occurs before maximum deflection and, consequently that failure occurs before the motion has stopped. In the example of the low speed titanium impact experiments carried out by Bruchey, the theoretical residual velocity was 210 m/s at the ballistic limit of 340 m/s.

Configurations for which the stress and strain are uniform through the thickness, as assumed in membrane theory, are efficient in comparison with structures that undergo bending stresses, for when loads are resisted by bending moments, the middle surface is strained less than adjacent surfaces, and consequently the material is not being used in an optimum fashion. This may serve as an explanation of a rather surprising result obtained by Laible et al.⁽⁶⁾ They find that the addition of resin to fabric laminates lowers the ballistic limit. In one example, the addition of 8.5 oz/ft² of resin to 10.7 oz/ft² of fabric lowered the ballistic limit from 830 to 623 ft/sec. This suggests that the filled laminate is subject to bending stress which are not present in the unfilled laminate, and that failure occurs when the bending stress exceeds the allowable value. This occurs at a lower velocity than when the laminates are unbonded, and they are all stressed to the same level. This line of thinking suggests that fabrics are more efficient than plates of the same material. In fact, some tests were run at S³ in which woven wire (340 stainless) was compared with plates of the same material. The woven fabric turned out to have a relatively low ballistic limit. This was subsequently attributed to the fact that the wire had an elongation of only 15%, where the plate had an elongation

of 40%. We believe that if a fabric of wire which was not strain-hardened were fabricated, then it would indeed prove more efficient than plate.

Few materials exhibit a stress-strain relation which closely resembles the rigid-plastic behavior assumed in the current theory. It would be desirable to find a good method for approximating real material behavior by a solution to the linear wave equation. That is, how can one select an equivalent strength, Y_e , which in some sense causes the solutions to

$$\rho \ddot{y} = Y(y') \nabla^2 y$$

to be closely approximated by

$$\rho \ddot{y} = Y_e \nabla^2 y$$

Procedures for this kind of analysis, termed the method of equivalent linearization, have been developed for ordinary differential equations, as discussed, for example, by Booten⁽¹¹⁾ and Dienes⁽¹²⁾, but they do not appear to have been developed adequately for partial differential equations.

The problem of oblique impact has not been solved, even in the simple case of a linear membrane. The separation of the target into regions of tension and compression seems to present a major difficulty. Perhaps it would be best to use a numerical approach to some of these problems, retaining the assumptions of membrane theory, in order to gain a better insight into the mechanics of oblique impact. The results might suggest some approximations that would be useful in the oblique problem.

REFERENCES

1. Dienes, J. K., M. M. Baligh, L. Morino and J. W. Miles, "Investigation of Helmet Dynamics," Systems, Science and Software Report SSS-R-73-1985, Dec., 1973.
2. Dienes, J. K. and J. W. Miles, "A Membrane Model for High Speed Impact," Systems, Science and Software Report SSS-R-2626, May, 1975.
3. Bruchey, W. J., Jr., "Penetration of Lightweight Personnel Armor Materials by Munition Fragments," BRL interim memorandum report no. 97, April, 1973.
4. Lindholm, U. S., L. M. Yeakley and R. L. Bessey, "An Investigation of the Behavior of Materials under High Rates of Deformation," Air Force Materials Laboratory Report AFML-TR-68-194, July, 1968.
5. Wolf, J. (Coordinating Editor), Aerospace Structural Metals Handbook, Belfour Stulen, Traverse City, Michigan, 1970.
6. Laible, R. C., F. Figucia and W. J. Ferguson, "The Application of High-Modulus Fibers to Ballistic Protection," J. Macromol. Sci.-Chem., A7(1), pp. 295-322, (1973).
7. McManus, L. R., "Protective Helmets of NATO and Other Countries," United States Army Natick Laboratories Report 73-29-CE, Jan., 1973.
8. Flaherty, R. E., "A Study of Low-Velocity Impacts into Thin-Sheet Aluminum and Nylon Cloth," National Aeronautics and Space Administration Report TN D-6324, May, 1971.
9. Kennel, R., Private Communication, Aerospace Corporation, Nov., 1973.
10. Florence, A. L., "Circular Plate under a Uniformly Distributed Impulse," Int. J. Solids and Structures, Vol. 2, pp. 37, 1966.
11. Booten, R. C., Jr., "The Analysis of Non-Linear Control Systems with Random Input," Proceedings, Symposium on Nonlinear Circuit Analysis, Polytechnic Institute, Brooklyn, N. Y., Vol. 2, 1953.
12. Dienes, J. K., "Some Applications of the Theory of Continuous Markoff Processes to Random Oscillation Problems," Ph.D. thesis, California Institute of Technology, 1961.

APPENDIX I. RESPONSE OF CIRCULAR PLATE TO UNIFORM IMPULSE

Florence(10) has determined the response of both aluminum (6061-T6) and steel (CR-1018) circular plates to sheet explosive, which provides an approximately uniform impulse, and compared the maximum deflection with that calculated from plate (bending) theory. His results, which are shown in Fig. 8, suggest that bending theory is inadequate for deflections that exceed 10% of the plate radius, and he remarked that membrane stresses might account for the discrepancies. We proceed to apply the membrane model to his configuration.

The notation remains as in Section III, except that a is now the radius of the circular membrane. The motion is governed by the axisymmetric wave equation (3.1), the boundary condition (3.2) is replaced by

$$y = 0 \quad (r = a, t > 0) , \quad (A.1)$$

and the initial conditions are

$$y = 0, y_t = v_0 \equiv I/\rho h \quad (r < a, t = 0) , \quad (A.2a,b)$$

where I is the impulse per unit area. [In reality, v_0 is only approximately uniform and falls sharply to zero as $r \uparrow a$ in consequence of the restraint (A.1) at $r = a$; however, this deficiency of the model is relatively unimportant for the prediction of y in $r < a$].

The solution of (3.1), (A.1) and (A.2) is given by

$$y = \sum_{i=1}^{\infty} A_i J_0(\lambda_i \eta) \sin(\lambda_i \tau) , \quad (A.3)$$

where λ_i is the i 'th root of the Bessel function J_0 ,

$$J_0(\lambda_i) = 0 \quad (0 < \lambda_1 < \lambda_2 < \dots \infty) , \quad (A.4)$$

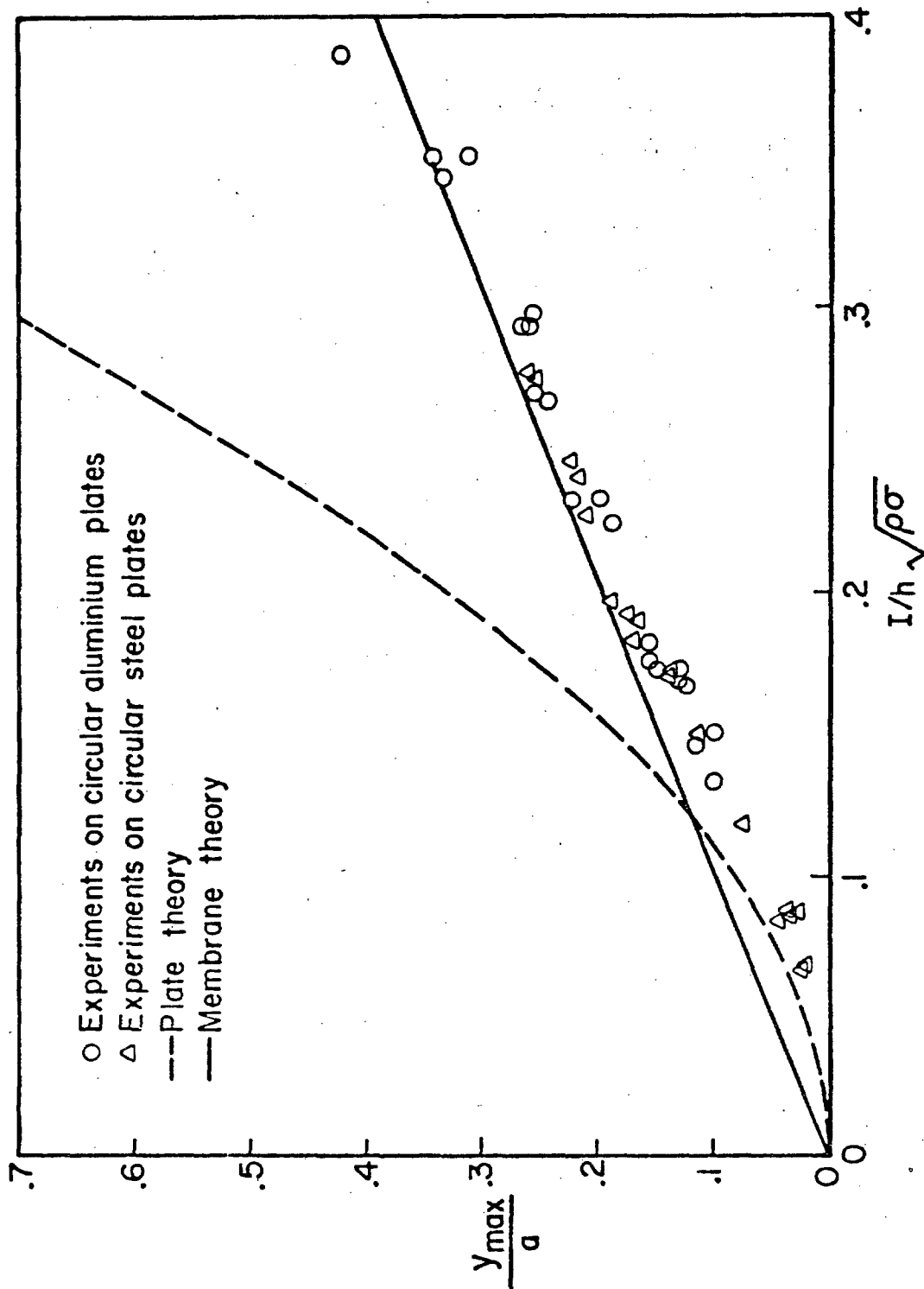


Fig. 8. The maximum deflection of a circular plate, subjected to a uniform impulse I , as determined by membrane theory (—), bending theory (---), and Florence's (1966) measurements.

η and τ are defined by (3.3) and (3.4), and

$$A_i = \frac{a}{c\lambda_i} \frac{\int_0^1 v_0 J_0(\lambda_i \eta) \eta d\eta}{\int_0^1 J_0^2(\lambda_i \eta) \eta d\eta} = \frac{2v_0 a}{c\lambda_i^2 J_1(\lambda_i)} . \quad (A.5)$$

The deflection at the center of the plate, $r = 0$, is given by

$$y_0/a = (v_0/c)f(\tau) , \quad (A.6)$$

$$\text{where } f(\tau) = 2 \sum_{i=1}^{\infty} [\lambda_i^2 J_1(\lambda_i)]^{-1} \sin(\lambda_i \tau) . \quad (A.7)$$

Its first maximum (determined numerically) of 1.0 is the only one that has physical significance, since the formulation is valid only for increasing deflection.

The theoretical prediction $y_{\max}/a = 1.0(v_0/c)$ is compared with Florence's results in Fig. 8. It is evident that the membrane model provides a significantly better correlation with the measured results than does the bending theory if $v_0/c > 0.15$ and that the correlation increases with v_0/c within the range of the measurements.

APPENDIX II

LISTING OF COMPUTER PROGRAM

3FOR+IS INT
FOR SEP-03/24/75-13:22:42 (00)

MAIN PROGRAM

STORAGE USED: C03E(1) C00362: DATA(1) C0010C: BLANK COMMON(2) C0000C

COMMON 4BLOCKS:

C003 EPSDEF C00072

EXTERNAL REFERENCES (BLOCK NAME)

C004 CRF
C005 XFORM
C006 EXIT
C007 NINTRS
C008 NPRTS
C009 NIO2S
C010 CEXP
C011 COVS
C012 NSTOPS

STORAGE ASSIGNMENT (BLOCK TYPE, RELATIVE LOCATION, NAME)

C001 C00004 1053 C001 C00105 1316 C00C C00031 51CF 0000 C00026 80CF
C000 R 703312 DX C000 R C00011 E C003 C 000000 EPS 0000 C 000002 FF 0000 R 000016 FO
C000 R C00017 F1 C000 R C00020 F2 C00C R C00021 F3 C00C I 000010 IE 0000 I 000014 II
C001 I 000013 K C000 C 000006 S C000 C 000010 SSIRT 0000 R 000015 T
C005 R 000000 XFORM C000 R C00022 Z0 C000 R C00023 Z1 C00C R 000024 Z2 0000 R C00025 Z3

C0101 1* COMPLEX EPS+SSIRT*FF+S*0 000000
C0103 2* COMMON/EPSDEF/EPS 000004
C0104 3* DO 20 IE=1,1 000004
C0107 4* E=25*0. 000004
C0110 5* PRINT 51C+E 000006
C0113 6* DX=.001 000013
C0114 7* EPS=CMPLX(E*0.) 000015
C0115 8* IF(IE.E3.1) SSIRT=(-1.8E-1*1.4E-1) 000020
C0117 9* IF(IE.NE.1) SSIRT=S 000025
C0121 10* CALL CRF(SSIRT*DX*FF+S*K) 000032
C0122 11* PRINT 80C+S*FF*K 000041
C0127 12* 80J FORMAT (1P4E16.8,18) 000105
C0130 13* DO 10 IT=1,501 000105
C0133 14* I=0020*FLOAT(IT-1) 000105
C0134 15* FC=XFORM(I,E*0.) 000114
C0135 16* F1=XFORM(I,E*1.) 000122
C0136 17* F2=XFORM(I,E*2.) 000130
C0137 18* F3=XFORM(I,E*3.) 000136

00140	19*	Q=CXHF(S,T)/(S**3+2**E*S-E**2*S)	000144
00141	21*	ZC=F0+2**REAL(H)	000246
00142	21*	Z1=-F1+2**REAL(S*Q)	000251
00143	22*	Z2=F2+2**REAL(S**2*Q)	000272
00144	23*	Z3=-F3+2**REAL(S**3*Q)	000313
00145	24*	PRINT 520,I,F0,F1,F2,F3,Z0,Z1,Z2,Z3	000334
00160	25*	10 CONTINUE	000355
00162	25*	20 CONTINUE	000355
00164	27*	CALL EXIT	000355
00165	28*	510 FORMAT(4H1E =1PE11.5//,6X,1HT,12X,4HF(0),9X,4HF(1),	000361
00165	29*	1 9X,4HF(2),9X,4HF(3),9X,4HZ(0),9X,4HZ(1),9X,4HZ(2),9X,4HZ(3))	000361
00165	31*	520 FORMAT(1X,1P9E13.4)	000361
00167	31*	END	000361

END OF COMPILATION: NO DIAGNOSTICS.

3FOR1S CICIP,CICIP

FOR SE2C-03/24/75-13:22:45 (1C)

SUBROUTINE CICIP ENTRY POINT=00C202

STORAGE USED: CODE(1) 00C226; DATA(7) 00C033; BLANK COMMON(2) 00G000

EXTERNAL REFERENCES (BLOCK NAME)

00J3 CDVS
00C8 NERR3S

STORAGE ASSIGNMENT (BLOCK TYPE RELATIVE LOCATION NAME)

00J1 00J32 1136 0000 C 000305 DENC1 0000 I 000037 I 0000 000020 INJPS
00C0 I 00C004 NI 0000 C 000000 Y 0000 C 000002 YI

00101	1*	SUBROUTINE CICIP(Z,C1,CIP)	000000
00103	2*	IMPLICIT COMPLEX(A-H,O-Z)	000000
00104	3*	Y=0.25*Z**2	000000
00105	4*	YI=1	000017
00106	5*	NI=10	000021
00107	6*	CI=1.7	000023
00110	7*	CIP=1.0	000024
00111	8*	DENC1=1.0	000025
00112	9*	DO 11 I=1,NI	000032
00115	11*	YI=Y*YI	000032
00116	11*	DENC1=DENC1*I**2	000051
00117	12*	DENCIP=DENC1*(I+1)	000075
00120	13*	CI=CI+YI/DENC1	000121
00121	14*	CIP=CIP+YI/DENCIP	000132
00122	15*	11 CONTINUE	000145
00124	16*	CIP=CIP*0.5*Z	000145
00125	17*	RETURN	000167
00126	18*	END	000225

END OF COMPILATION: NO DIAGNOSTICS.

3FOR+IS CKZK1+CKZK1
FOR SE2C-03/24/75-13:22:48 (•C)

SUBROUTINE CKZK1 ENTRY POINT C00357

STORAGE USED: CODE(1) C00415; DATA(1) C0006C; BLANK COMMON(2) C0000C

EXTERNAL REFERENCES (BLOCK+NAME)

C003 CICIP
C004 CLOG
C005 C0VS
C006 NERR3S

STORAGE ASSIGNMENT (BLOCK+TYPE+RELATIVE LOCATION+NAME)

C001 C00132 1236 C000 C 000011 CI C000 C 000017 CKZ2 C000 C 000015 CKZP
C000 C 000021 CL C000 C 000023 DEN C000 C 000000 GAMMA C000 I C00025 I C000 000041 INJPS
C000 I 000002 NI C000 C 000007 WS1 C000 C 000033 Y C000 C 000005 Y1

36
C0101 1* SUBROUTINE CKZK1(2+CKZ+CK1) 000000
C0103 2* IMPLICIT COMPLEX (A-H,O-Z) 000000
C0104 3* GAMMA=1.577215655 000000
C0105 4* NI=10 000001
C0106 5* Y=1.25*Z**2 000003
C0107 6* YI=1 000023
C0110 7* WS1=CLDB(0.5*Z)+GAMMA 000025
C0111 8* CALL CICIP(2+CI+CIPI) 000043
C0112 9* CKZP=-WS1*CIPI/2 000050
C0113 10* CKZ=-CI*WS1 000100
C0114 11* CKZP=1. 000120
C0115 12* CL=1. 000122
C0116 13* DEN=1. 000123
C0117 14* DO 11 I=1,NI 000132
C0122 15* YI=YI*Y 000132
C0123 16* CL=CL+1./I 000151
C0124 17* DEN=DEN*I**2 000163
C0125 18* CKZ=CKZ+YI/DEN*CL 000207
C0126 19* CKZP=CKZP+YI/Y/DEN*I*CL 000235
C0127 20* 11 CONTINUE 000313
C0131 21* CK1=-CKZP-CKZP*0.5*Z 000313
C0132 22* RETURN 000342
C0133 23* END 000414

END OF COMPILATION: NO DIAGNOSTICS.

3FOR,IS F,F
FOR SE2C-33/2u/75-13:22:54 (.0)

FUNCTION F ENTRY POINT 00105

STORAGE USED: CODE(1) 000120; DATA(1) 00012; BLANK COMMON(2) 000000

COMMON BLOCKS:

0003 EPSDEF 000002

EXTERNAL REFERENCES (BLOCK, NAME)

0004 CKZK1
0005 NERR3S

STORAGE ASSIGNMENT -- (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 C 000002 CKZ 0000 C 000004 CK1 0003 C 000000 EPS 0000 C 000000 F 0000 C 000006 INJPS

	1*	COMPLEX FUNCTION F(Z)	
00101	1*	IMPLICIT COMPLEX(A-H,O-Z)	000000
00103	2*	COMMON/EPSDEF/EPS	000000
00104	3*	CALL CKZK1(/,CKZ,CK1)	000000
00105	4*	F=Z**2*CKZ+EPS*Z*CK1	000004
00106	5*	RETURN	000074
00107	6*	END	000117
00110	7*		

END OF COMPILATION: NO DIAGNOSTICS.

ENTRY POINT 000267

EXTERNAL REFERENCES (BLOCK NAME)

0003	F
0004	NPR15
0005	NI025
0006	NST005
0007	CDV5
0010	CAB5
0011	VERR35

STORAGE ASSIGNMENT	(BLOCK, TYPE, RELATIVE LOCATION, NAME)
1	1000, 1, 1, 1000
2	1000, 1, 1, 1000
3	1000, 1, 1, 1000
4	1000, 1, 1, 1000
5	1000, 1, 1, 1000
6	1000, 1, 1, 1000
7	1000, 1, 1, 1000
8	1000, 1, 1, 1000
9	1000, 1, 1, 1000
10	1000, 1, 1, 1000
11	1000, 1, 1, 1000
12	1000, 1, 1, 1000
13	1000, 1, 1, 1000
14	1000, 1, 1, 1000
15	1000, 1, 1, 1000
16	1000, 1, 1, 1000
17	1000, 1, 1, 1000
18	1000, 1, 1, 1000
19	1000, 1, 1, 1000
20	1000, 1, 1, 1000
21	1000, 1, 1, 1000
22	1000, 1, 1, 1000
23	1000, 1, 1, 1000
24	1000, 1, 1, 1000
25	1000, 1, 1, 1000
26	1000, 1, 1, 1000
27	1000, 1, 1, 1000
28	1000, 1, 1, 1000
29	1000, 1, 1, 1000
30	1000, 1, 1, 1000
31	1000, 1, 1, 1000
32	1000, 1, 1, 1000
33	1000, 1, 1, 1000
34	1000, 1, 1, 1000
35	1000, 1, 1, 1000
36	1000, 1, 1, 1000
37	1000, 1, 1, 1000
38	1000, 1, 1, 1000
39	1000, 1, 1, 1000
40	1000, 1, 1, 1000
41	1000, 1, 1, 1000
42	1000, 1, 1, 1000
43	1000, 1, 1, 1000
44	1000, 1, 1, 1000
45	1000, 1, 1, 1000
46	1000, 1, 1, 1000
47	1000, 1, 1, 1000
48	1000, 1, 1, 1000
49	1000, 1, 1, 1000
50	1000, 1, 1, 1000
51	1000, 1, 1, 1000
52	1000, 1, 1, 1000
53	1000, 1, 1, 1000
54	1000, 1, 1, 1000
55	1000, 1, 1, 1000
56	1000, 1, 1, 1000
57	1000, 1, 1, 1000
58	1000, 1, 1, 1000
59	1000, 1, 1, 1000
60	1000, 1, 1, 1000
61	1000, 1, 1, 1000
62	1000, 1, 1, 1000
63	1000, 1, 1, 1000
64	1000, 1, 1, 1000
65	1000, 1, 1, 1000
66	1000, 1, 1, 1000
67	1000, 1, 1, 1000
68	1000, 1, 1, 1000
69	1000, 1, 1, 1000
70	1000, 1, 1, 1000
71	1000, 1, 1, 1000
72	1000, 1, 1, 1000
73	1000, 1, 1, 1000
74	1000, 1, 1, 1000
75	1000, 1, 1, 1000
76	1000, 1, 1, 1000
77	1000, 1, 1, 1000
78	1000, 1, 1, 1000
79	1000, 1, 1, 1000
80	1000, 1, 1, 1000
81	1000, 1, 1, 1000
82	1000, 1, 1, 1000
83	1000, 1, 1, 1000
84	1000, 1, 1, 1000
85	1000, 1, 1, 1000
86	1000, 1, 1, 1000
87	1000, 1, 1, 1000
88	1000, 1, 1, 1000
89	1000, 1, 1, 1000
90	1000, 1, 1, 1000
91	1000, 1, 1, 1000
92	1000, 1, 1, 1000
93	1000, 1, 1, 1000
94	1000, 1, 1, 1000
95	1000, 1, 1, 1000
96	1000, 1, 1, 1000
97	1000, 1, 1, 1000
98	1000, 1, 1, 1000
99	1000, 1, 1, 1000
100	1000, 1, 1, 1000

0000	000022 10F	0001	000011 21L	0000	000022 80JF	0003	C 000030 F	0000	C 000016 FIM
0000	C 000014 51P	0000	C 000010 FRM	0000	C 000006 FRP	0000	000031 INJP	0000	C 000002 X
0000	000031 XM	0000	R 000000 XR	0000	C 000034 XS	0000	C 000012 XT		

```

1* SUBROUTINE CRF(XSTRT,DX,FF,XX,K)
2* IMPLICIT COMPLEX(A-H,O-Z)
3* REAL XR,XM,UX
4* X=XSTRT
5* PRINT 1,X,XSTRT
6* 10 FORMAT(1F4E16.8)
7* K=7
8* 21 K=K+1
9* IF(K.GT.10)STOP K*MX
10* XR=REAL(X)
11* XM=AIMAG(X)
12* XS=X
13* FF=F(X)
14* X=CMPLX(XR+DX,XM)
15* FRP=F(X)
16* X=CMPLX(XR-DX,XM)
17* FRM=F(X)
18* XS=FF+2.*DX/(FRP-FR)
19* XR=REAL(X)
20* XM=AIMAG(X)
21* FF=F(X)
22* XI=X
23* X=CMPLX(XR,XM+UX)
24* FIP=F(X)

```

00136	25*	X=CMPLX(XR,XI-DX)	000142
00137	26*	FIM=F(X)	000147
00140	27*	X=XI-FF*2+CMPLX(0+DX)/(FID-FIM)	000153
00141	28*	PRINT 800,X,XI,XS,FF,FRP,FRM,FID,FIM	000220
00153	29*	800 FORMAT(1P8E14.7)	000234
00154	30*	IF(CABS(X-XS).GT.1.E-8)GO TO 21	000234
00155	31*	FF=F(X)	000250
00157	32*	XX=X	000254
00160	33*	RETURN	000256
00161	34*	END	000313

END OF COMPILATION: NO DIAGNOSTICS.

GFOR,IS BSSLIK

FOR U11A-10/29/73-12:38:19 (.9)

SUBROUTINE BSSLIK ENTRY POINT 000245

STORAGE USED: CODE(1) 000336; DATA(0) 000124; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 NERR65
0004 SQRT
0005 EXP
0006 ALOG
0007 NERR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	00033	1436	0001	000071	1616	0001	000005	20L	0001	000143	2326	0001	000176	2176
0001	000015	30L	0001	000056	50L	0001	000132	70L	0001	000163	90L	0000	R	000007
0000	R	000027	B11	0000	R	000047	BK0	0000	R	000065	BK1	0000	R	000104
0000	R	000105	EPXLNT	0000	000115	INHPS	0000	I	000075	N	0000	R	000000	S10
0000	R	000040	SK0	0000	R	000056	SK1	0000	R	000076	SQRTX	0000	R	000074
0000	R	000101	X10	0000	R	000102	X11	0000	R	000077	XX	0000	R	000100

1* SUBROUTINE BSSLIK(X,XEMX10,XEMX11,XEPXK0,XEPXK1)
2* DIMENSION S10(7),B10(9),S11(7),B11(9),SK0(7),BK0(7),SK1(7),BK1(7)
3* DATA S10/1.,3.5156229,3.0899424,1.2067492,.2659732,.0360768,
4* .0045813/
5* DATA B10/.39894228,.01328592,.0025319, -.00157565,.00916281,
6* -.02057706,.02635537, -.01647633,.00392377/
7* DATA S11/.5,.87890594,.51498869,.15084934,.02658733,
8* .00301532,.00032411/
9* DATA B11/.39894228, -.03988024, -.00362018,.00163801, -.01031555,
10* .02282967, -.02895312,.01787654, -.00420059/
11* DATA SK0/-.57721566,.4278420,.23069756,.03488590,.00262698,
12* .00010750,.00000740/
13* DATA BK0/1.25331414, -.07832358,.02189568, -.01062446,.00587872,
14* -.00251540,.00353208/
15* DATA SK1/1.,.15443144, -.67278579, -.18156897, -.01919402,
16* -.00110494, -.00054686/
17* DATA BK1/1.25331414,.23498619, -.03655620,.01504268, -.00780353,
18* .00325614, -.00068245/
19* IF(X)10,20,30
20* 10 RETURN 0
21* 20 XEMX10=0.
22* XEMX11=0.
23* XEPXK0=Q.
XEPXK0=Q.

```

00133 24* XEPXKI=1.
00134 25* RETURN
00135 26* 30 IF(X-1T-3.75)GO TO 50
00137 27* T=3.75/X
00140 28* XEMXIC=B10(9)
00141 29* XEMXII=B11(9)
00142 30* DO 40 N=6,1,-1
00145 31* XEMXIC=XEMXIC+810(N)
00146 32* XEMXII=XEMXII+811(N)
00150 33* SQRTX=SQR(X)
00151 34* XEMXIC=SQR(X)*XEMXIC
00152 35* XEMXII=SQR(X)*XEMXII
00153 36* GO TO 70
00154 37* 50 XX=X*X
00155 38* TT=XX/14.0625
00156 39* XIO=SIC(7)
00157 40* XII=SII(7)
00160 41* DO 60 N=6,1,-1
00163 42* XIG=XIC+TT+SIC(N)
00164 43* XII=XII+TT+SII(N)
00166 44* XIG=X*XIG
00167 45* XII=X*XII
00170 46* EMX=EXP(1-X)
00171 47* XEMXIC=EMX*XIG
00172 48* XEMXII=EMX*XII
00173 49* IF(X-1T-2)GO TO 90
00175 50* SQR(X)=SQR(X)
00176 51* 70 T=2/X
00177 52* XEPXK2=BK0(7)
00200 53* XEPXKI=BK1(7)
00201 54* DO 80 N=6,1,-1
00204 55* XEPXK2=XEPXK2+T*BK0(N)
00205 56* XEPXKI=XEPXKI+T*BK1(N)
00207 57* XEPXK2=SQR(X)*XEPXK2
00210 58* XEPXKI=SQR(X)*XEPXKI
00211 59* RETURN
00212 60* 90 T=X/2.
00213 61* TT=T*T
00214 62* XEPXK2=SK0(7)
00215 63* XEPXKI=SK1(7)
00216 64* DO 100 N=6,1,-1
00221 65* XEPXK2=XEPXK2+TT*SK0(N)
00222 66* XEPXKI=XEPXKI+TT*SK1(N)
00224 67* EPX=EXP(X)
00225 68* EPXLNT=EPX*ALOG(T)
00226 69* XEPXK2=X*EPX*XEPXK2-EPXLNT*XIG
00227 70* XEPXKI=X*EPX*XEPXKI-EPXLNT*XII
00230 71* RETURN
00231 72* END

```

DIENES

DATE 102973

PAGE 6

FOR:IS DIENES
FOR U11A-10/29/73-12:38:27 (10)

FUNCTION DIENES ENTRY POINT 000073

STORAGE USED: CODE(1) 000104; DATA(0) 000022; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 BSSLIK
0004 EXP
0005 NERR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 R 000000 DIENES 0000 R 000002 E 0000 000012 INJPS 0000 R 000001 T 0000 R 000003 X10
0000 R 000004 X11 0000 R 000005 XKO 0000 R 000006 XK1

00101 1* FUNCTION DIENES(X,A)
00103 2* DIMENSION A(1)
00104 3* T=A(1)
00105 4* E=A(2)
00106 5* CALL BSSLIK(X,X10,X11,XKO,XK1)
00107 6* DIENES=1./(EXP(X*(T-2.))+(X*XKO+E*XK1))*2
00107 7* 1 +9.8696044*EXP(X*(T-2.))+(X*X10-E*XK1))*2
00110 8* RETURN
00111 9* END

END OF COMPILATION: NO DIAGNOSTICS.

0006 DISMOD

DNMOD

FOR IS DNMOD
FOR U11A-10/29/73-12:38:29 (10)

FUNCTION DNMOD ENTRY POINT 000021

STORAGE USED: CODE(1) 000032; DATA(0) 000005; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 DIENES
0004 NCRR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0003 R 000000 DIENES 0000 R 000000 DNMOD 0000 000001 INJPS

00101 1* FUNCTION DNMOD(X,A)
00103 2* DIMENSION A(1)
00104 3* DNMOD=X*3*DIENES(X,A)
00105 4* RETURN
00106 5* END

END OF COMPILATION: NO DIAGNOSTICS.

ENDG CADRE

CADRE

GPOR,IS CADRE
FOR U1A-10/29/73-12:38:31 (10)

FUNCTION CADRE ENTRY POINT 002426

STORAGE USED: CODE(1) 002477: DATA(0) 005021: BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 XPII
0004 NVDUS
0005 NI02\$
0006 NI01\$
0007 ALOG10
0010 XPRI
0011 NERR3\$

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	000207	10L	0001	002226	10126	0001	002241	10206	0001	002352	10616	0001	002361	10666
0001	000305	12L	0001	000465	19L	0001	000570	18L	0001	000617	20L	0001	000660	21L
0001	000662	22L	0001	000253	221G	0001	000341	235G	0001	000432	255G	0001	000504	272G
0001	000746	30L	0001	001015	31L	0001	001106	33L	0001	001166	333L	0001	001227	34L
0001	001247	35L	0001	001261	36L	0001	001265	37L	0001	001370	40L	0001	001072	426G
0001	001134	42G	0001	001147	450G	0001	001162	456G	0001	000122	5L	0001	001462	50L
0001	001275	517G	0001	001360	544G	0001	00123	6L	0001	001535	60L	0000	004534	609F
0000	004541	619F	0000	004545	630F	0001	001562	624G	0000	004552	629F	0000	004556	630F
0000	004562	631F	0000	004565	632F	0000	004567	633F	0000	004606	638F	0000	004613	641F
0000	004623	649F	0000	004627	650F	0000	004644	660F	0000	004651	667F	0000	004665	670F
0000	004671	671F	0000	004700	682F	0000	004716	6900F	0000	004713	692F	0000	004731	6959F
0001	001624	70L	0001	001770	700G	0001	001777	705G	0001	001657	71L	0001	001676	72L
0001	002057	85L	0001	002140	782G	0001	001736	80L	0001	002010	82L	0001	002021	83L
0001	002151	93L	0001	002165	94L	0001	002104	90L	0001	002306	900L	0001	002106	91L
0001	002401	999L	0000	004512	A051	0000	002205	95L	0001	002325	950L	0001	002372	959L
0000	004447	A1TTL	0000	004453	ALG402	0000	R 000157	A1T	0000	L 004403	A1TKEN	0000	R 004445	A1TLOW
0000	004246	BEGIN	0000	R 000000	CADRE	0000	R 004524	ALPHA	0000	R 004473	A5TEP	0000	R 004464	BEG
0000	004467	END	0000	R 004516	ENG0AL	0000	R 004461	CUREST	0000	R 000171	D1F	0000	R 004521	DIFF
0000	004342	EST	0000	R 004465	FDEG	0000	R 004455	ERRA	0000	R 004520	ERRR	0000	R 004454	ERRR
0000	004517	FEXTRP	0000	R 004304	FINIS	0000	R 004531	FBE62	0000	R 004470	FEND	0000	R 004523	FEXTM1
0000	004402	H2CONV	0000	R 004525	H2NEXT	0000	R 004501	FN	0000	R 004462	FNSIZE	0000	R 004504	HOVN
0000	004466	IBEG	0000	R 004210	10E6TS	0000	R 004527	H2TFEX	0000	R 004446	H2TOL	0000	R 004506	I
0000	004767	1NJP5	0000	R 004460	15TAGE	0000	R 004471	1END	0000	R 004503	I1	0000	R 004505	I11
0000	004532	J	0000	R 004431	JUMPTL	0000	R 004502	1STEP	0000	R 004507	1STEP2	0000	R 004513	IT
0000	004451	MAXTBL	0000	R 004450	MAXTS	0000	R 004475	L	0000	R 004400	LENGTH	0000	R 004477	LMI
0000	004500	N2	0000	R 004463	PREVER	0000	R 000145	R	0000	R 004476	N	0000	R 004533	NNLEFT
0000	004404	RIGHT	0000	R 000203	RN	0000	R 000452	SING	0000	R 004405	REGLAR	0000	R 004406	REGLSV
0000	004457	STAGE	0000	R 004472	STEP	0000	R 004522	SING	0000	R 004526	SINGNX	0000	R 004530	SLOPE
0000	000001	T	0000	R 004474	TABS	0000	R 004456	STEPMN	0000	R 004510	SUM	0000	R 004511	SUMABS
0000	004514	VINT	0000	R 004474	TABS	0000	R 004515	TABTLM	0000	R 004444	TOLHCH	0000	R 000207	TS

```

00101 1* FUNCTION CADRE(F,ARG,A,B,AERR,RERR,LEVEL,ERROR,IFLAG)
00103 2* PARAMETER Q10=10, Q30=30, Q2049=2*(Q10+1)+1
00104 3* DIMENSION T(Q10,Q10),R(Q10),AIT(Q10),DIF(Q10),RN(4),
00104 4* > TS(Q2049),IBEGS(Q30),BEGIN(Q30),FINIS(Q30),EST(Q30),ARG(I)
00105 5* REAL LENGTH, JUMPTL
00106 6* LOGICAL H2CONV,AITKEN,RIGHT,REGLAR,REGLSV,Q30)
00107 7* DATA TOLMCH,AITLOW,H2TOL,AITTOL,JUMPTL,MAXTS,MAXTBL,MXSTGE
00107 8* / 1.E-07, 1.1, .15, .1, .01, Q2049, Q10, Q30/
00120 9* DATA RN/.7142CG53,.34662815,.843751,.12633046/
00122 10* DATA ALG402 /.301029996/
00124 11* CADRE = 0.
00125 12* ERROR = 0.
00126 13* IFLAG = 1
00127 14* LENGTH = ABS(B-A)
00130 *DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00130 15* IF (LENGTH .EQ. 0.) RETURN
00132 16* ERR = AMIN(.1,AMAX1(ABS(RERR), 10.*TOLMCH))
00133 17* ERRA = ABS(AERR)
00134 18* STEPMN = AMAX1(LENGTH/FLOAT(2.*MXSTGE),
00134 19* AMAX1(LENGTH,ABS(A),ABS(B))*TOLMCH)
00135 20* STAGE = .5
00136 21* ISTAGE = 1
00137 22* CUREST = 0.
00140 23* FNSIZE = 0.
00141 24* PREVER = 0.
00142 25* REGLAR = .FALSE.
00143 26* BEG = A
00144 27* FBEG = F(BEG,ARG)/2.
00145 28* TS(1) = FBEG
00146 29* IBEG = 1
00147 30* END = B
00150 31* FEND = F(END,ARG)/2.
00151 32* TS(2) = FEND
00152 33* IEND = 2
00153 34* 5 RIGHT = .FALSE.
00154 35* 6 STEP = END - BEG
00155 36* ASTEP = ABS(STEP)
00156 37* IF (ASTEP .LT. STEPMN) GO TO 950
00160 38* IF (LEVEL .GE. 3) WRITE(6,609) BEG,STEP,ISTAGE
00166 39* 609 FORMAT(1DH BEG,STEP,2E16.8,15)
00167 40* T(1,1) = FBEG + FEND
00170 41* TABS = ABS(FBEG) + ABS(FEND)
00171 42* L = 1
00172 43* N = 1
00173 44* H2CONV = .FALSE.
00174 45* AITKEN = .FALSE.
00175 46*
00176 47* 9 IF (LEVEL .GE. 4) WRITE(6,692) L,T(1,LM1)
00203 48* 10 LM1 = L
00204 49* L = L + 1
00205 50* N2 = N*2
00206 51* FN = N2
00207 52* ISTEP = (IEND - IBEG)/N

```

CADRE

```

00210 53* IF (ISTEP .GT. 1) GO TO 12
00212 54* II = IEND
00213 55* IEND = IEND + N
00214 56* IF (IEND .GT. MAXIS) GO TO 900
00216 57* HOVN = STEP/FN
00217 58* III = IEND
00220 59* DO 11 I=1,N2,2
00223 60* TS(III) = TS(II)
00224 61* TS(III-1) = F(END - FLOAT(I)*HOVN, ARG)
00225 62* III = III-2
00226 63* 11 II = II-1
00230 64* ISTEP = 2
00231 65* 12 ISTEP2 = IBEG + ISTEP/2
00232 66* SUM = 0.
00233 67* SUMABS = 0.
00234 68* DO 13 I=ISTEP2,IEND,ISTEP
00237 69* SUM = SUM + TS(I)
00240 70* 13 SUMABS = SUMABS + ABS(TS(I))
00242 71* T(L,I) = T(L-1,I)/2. + SUM/FN
00243 72* TABS = TABS/2. + SUMABS/FN
00244 73* ABSI = ASTEP*TABS
00245 74* N = N2
00246 75* IT = 1
00247 76* VINT = STEP*T(L,I)
00250 77* TABTLM = TABS*TOLMCH
00251 78* FNSIZE = AMAXI(FNSIZE,ABS(T(L,I)))
00252 79* ERGOAL = AMAXI(FASTEP*TOLMCH*FNSIZE,
00252 80* STAGE*AMAXI(ERRRERRR*ABS(CUREST+VINT)))
00252 81* COMPLETE ROW L AND COLUMN L OF *O* ARRAY.
00252 82* FEXTRP = 1.
00254 83* DO 14 I=1,LMI
00257 84* FEXTRP = FEXTRP*4.
00260 85* T(I,I) = T(L,I) - T(L-1,I)
00261 86* 14 T(L,I+1) = T(L,I) + T(I,L)/(FEXTRP-1.)
00263 87* ERROR = ASTEP*ABS(T(I,L))
00264 88* IF (L .GT. 2) GO TO 15
00266 89* IF (ABS(T(I,2)) .LE. TABTLM) GO TO 60
00270 90* GO TO 10
00271 92* CALCULATE NEXT RATIOS FOR COLUMNS 1,...,L-2 OF T-TABLE
00274 93* 15 DO 16 I=2,LMI
00275 94* DIFF = 0.
00277 95* IF (ABS(T(I-1,I)) .GT. TABTLM) DIFF = T(I-1,LMI)/T(I-1,L)
00301 96* 16 T(I-1,LMI) = DIFF
00303 97* IF (ABS(4.-T(I,LMI)) .LE. H2TOL) GO TO 20
00305 98* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00307 99* IF (T(I,LMI) .EQ. 0.) GO TO 18
00311 100* IF (ARS(2,-ABS(T(I,LMI))) .LT. JUMPTL) GO TO 50
00312 101* IF (L .EQ. 3) GO TO 9
00314 102* H2CONV = .FALSE.
00316 103* IF (ABS(T(I,LMI)-T(I,L-2)) .LE. ATITOL*ABS(T(I,LMI)))
00320 105* 17 IF (REGLAR) GO TO 30
00314 103* IF (L .EQ. 4) GO TO 18
00316 104* 18 IF (ERRR .LE. ERGOAL) GO TO 70

```

```

00322 106* IF (LEVEL .GE. 4) WRITE (6,692) L,T(1,LM1)
00327 107* GO TO 91
00327 108* CAUTIOUS ROMBERG EXTRAPOLATION
00330 109* 20 IF (LEVEL .GE. 4) WRITE (6,619) L,T(1,LM1)
00335 110* 619 FORMAT(15,E16.8,5X6HH2CONV)
00336 111* IF (H2CONV) GO TO 21
00340 112* AITKEN = .FALSE.
00341 113* H2CONV = .TRUE.
00342 114* IF (LEVEL .GE. 3) WRITE (6,620) L
00346 115* 620 FORMAT(22H H2 CONVERGENCE AT ROW,13)
00347 116* 21 FEXTRP = 4.
00350 117* 22 IT = IT + 1
00351 118* VINT = STEP*IT(L,IT)
00352 119* ERROR = ABS(STEP/(FEXTRP-1.))*T(IT-1,L))
00353 120* IF (ERROR .LE. ERGOAL) GO TO 80
00355 121* IF (IT .EQ. LM1) GO TO 40
00357 122* *DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00361 123* IF (T(IT,LM1) .EQ. 0.) GO TO 22
00363 124* IF (T(IT,LM1) .LE. FEXTRP) GO TO 40
00363 125* IF (ABS(T(IT,LM1)/4.-FEXTRP) .LT. FEXTRP*AITTOL)
00365 126* FEXTRP = FEXTRP*4.
00366 127* GO TO 22
00373 128* 30 IF (LEVEL .GE. 4) WRITE (6,629) L,T(1,LM1)
00374 129* 629 FORMAT(15,E16.8,5X6HAIKEN)
00376 130* IF (T(1,LM1) .LT. AITLOW) GO TO 91
00400 131* IF (AITKEN) GO TO 31
00401 132* H2CONV = .FALSE.
00402 133* AITKEN = .TRUE.
00406 134* IF (LEVEL .GE. 3) WRITE (6,630) L
00407 135* 630 FORMAT(14H AITKEN AT ROW,13)
00410 136* 31 FEXTRP = T(L-2,LM1)
00412 137* IF (FEXTRP .GT. 4.5) GO TO 21
00414 138* IF (FEXTRP .LT. AITLOW) GO TO 91
00414 139* IF (ABS(FEXTRP-T(L-3,LM1)) .GT. T(1,LM1)*H2TOL)
00416 140* GO TO 91
00422 141* IF (LEVEL .GE. 3) WRITE (6,631) FEXTRP
00423 142* 631 FORMAT(6H RATIO,F12.8)
00425 143* SING = FEXTRP
00425 144* FEXTM1 = FEXTRP - 1.
00430 145* DO 32 J=2,L
00431 146* AIT(1) = T(1,1) + (T(1,1)-T(J-1,1))/FEXTM1
00432 147* R(1) = T(1,1-1)
00434 148* 32 DIF(1) = AIT(1) - AIT(1-1)
00435 149* IT = 2
00436 150* 33 VINT = STEP*AIT(L)
00440 151* IF (LEVEL .LT. 5) GO TO 333
00446 152* WRITE (6,632) (R(I+1),I=IT,LM1)
00454 153* WRITE (6,632) (AIT(I),I=IT,L)
00462 154* WRITE (6,632) (DIF(I+1),I=IT,LM1)
00463 155* 632 FORMAT(1X,E15.8)
00464 156* 333 ERROR = ERROR/FEXTM1
00466 157* IF (ERROR .GT. ERGOAL) GO TO 34
00467 158* ALPHA = ALOGIC(SING)/ALC402 - 1.
IF (LEVEL .GE. 2) WRITE (6,633) ALPHA,BEG,END

```

```

00475 159* 633 FORMAT(11X42HINTEGRAND SHOWS SINGULAR BEHAVIOR OF TYPE
00476 160* 4HX*(F4.2,9H) BETWEEN I5.8,4H ANDE I5.8)
00477 161* IFLAG = MAXG(IFLAG,2)
00478 162*
00500 163* 34 IT = IT + 1
00501 164* IF (IT.EQ. LM1)
00502 165* IF (IT.GT.3)
00503 166* H2NEXT = 4.
00504 167* SINGNX = SING+SING
00505 168* IF (H2NEXT.LT. SINGNX)
00506 169* FEXTRP = SINGNX
00507 170* SINGNX = SINGNX+SINGNX
00513 171*
00514 172* 36 FEXTRP = H2NEXT
00515 173* H2NEXT = 4.*H2NEXT
00516 174* 37 DO 38 I=1T,LM1
00521 175* R(I+1) = 0.
00522 176* 38 IF (ABS(DIF(I+1)) .GT. TAB1LM) R(I+1) = DIF(I)/DIF(I+1)
00525 177* IF (LEVEL .GE. 4) WRITE (6,638) FEXTRP,R(LM1),R(L)
00533 178* 638 FORMAT(16H FEXTRP + RAT I05,3E15.8)
00534 179* H2FEX = -H2TOL*FEXTRP
00535 180* IF (R(L) .LT. H2FEX+FEXTRP)
00537 181* IF (R(L-1) .LT. H2FEX+FEXTRP)
00541 182* ERROR = ASTEP*ABS(DIF(L))
00542 183* FEXTM1 = FEXTRP - 1.
00543 184* DO 39 I=1T,L
00546 185* AIT(I) = AIT(I) + DIF(I)/FEXTM1
00547 186* 39 DIF(I) = AIT(I) - AIT(I-1)
00551 187*
00552 188* 40 FEXTRP = AMAX1(PREVER/ERROR, AITLOW)
00553 189* PREVER = ERROR
00554 190* IF (L.LT. 5)
00556 191* IF (LEVEL .GE. 3) WRITE (6,641) ERROR,ERGOAL,FEXTRP,IT
00565 192* 641 FORMAT(23H ERROR,ERGOAL,FEXTRP,IT,2E15.8,F14.5,I3)
00566 193* IF (L-IT .GT. 2 .AND. 1STAGE .LT. MXSTGE) GO TO 90
00570 194* IF (ERROR .LT. ERGOAL*FEXTRP*(MAXTBL-L)) GO TO 10
00572 195*
00573 196* 50 IF (LEVEL .GE. 4) WRITE (6,649) L,IT(L,LM1)
00600 197* 649 FORMAT(15,E16.8,5X4HJUMP)
00601 198* IF (ERROR .GT. ERGOAL)
00603 199* DIFF = ABS(T(I,L))*(FN + FN)
00604 200* IF (LEVEL .GE. 2) WRITE (6,650) DIFF,BEG,END
00612 201* 650 FORMAT(13X30HINTEGRAND SEEMS TO HAVE JUMP OF SIZEE13.6,
00613 202* 8H BETWEEN I5.8,4H ANDE I5.8)
00614 203*
00614 204* 60 IF (LEVEL .GE. 4) WRITE (6,660) L
00620 205* 660 FORMAT(15,21X,13HSTRAIGHT LINE)
00621 206* SLOPE = (FEND-FBEG) + (FEND-FBEG)
00622 207* FBEG2 = FBEG+FBEG
00623 208* DO 61 I=1,4
00626 209* DIFF = ABS((F(BEG+RN(I))*STEP + ARG) - FBEG2-RN(I)*SLOPE)
00627 210* IF (DIFF .GT. TAB1L)
00631 211* 61 CONTINUE
00633 212* IF (LEVEL .GE. 3) WRITE (6,667) BEG, END

```

CADRE

DATE 102973

PAGE 13

```

00640 213* 667 FORMAT(27X43HINTEGRAND SEEMS TO BE STRAIGHT LINE BETWEEN
00640 214*      E15.8,4H AND C15.8)
00641 215*
00642 216*      GO TO 80
00647 217* 70 IF (LEVEL .GE. 4) WRITE (6,670) L,T(1,LM1)
00650 218* 670 FORMAT(15,E16.8,5XSHNOISE)
00651 219*      SLOPE = (FEND-FBEG) / (FEND-FBEG)
00652 220*      FBEG2 = FBEG*FBEG
00653 221*      I = 1
00654 222* 71 DIFF = ABS(FBEG2-RN(1)*STEP, ARG) - FBEG2-RN(1)*SLOPE)
00655 223* 72 ERROR = AMAX1(ERROR,ASTEP*DIFF)
00657 224*      IF (EPRGR .GT. ERGOAL)      GO TO 91
00660 225*      I = I+1
00662 226*      IF (I .LE. 4)      GO TO 71
00667 227*      IF (LEVEL .GE. 3) WRITE (6,671) BEG,END
00670 228* 671 FORMAT(15H NOISE BETWEEN ,E15.8,4H AND,E15.8)
00671 229*      IFLAG = 3
00672 230* 80 CADRE = CADRE + VINT
00673 231*      ERROR = ERROR + ERROR
00675 232*      IF (LEVEL .LT. 3)      GO TO 83
00677 233*      IF (LEVEL .LT. 5)      GO TO 82
00712 235*      DO 81 I=1,L
00720 236* 81 WRITE (6,692) I,(T(1,J),J=1,L)
00721 237* 82 WRITE (6,682) VINT,ERROR,L,IT
00722 238* 682 FORMAT(12H INTEGRAL 15,E16.8,7H, ERROR,E15.8,9H FROM T(,
00723 239*      I,1H,11.1H))
00724 240*      IF (RIGHT)      GO TO 85
00726 241*      ISTAGE = ISTAGE - 1
00727 242*      IF (ISTAGE .EQ. 0)      RETURN
00730 243*      REGLAR = REGLSV(ISTAGE)
00731 244*      BEG = BEGIN(ISTAGE)
00732 245*      END = FINIS(ISTAGE)
00733 246*      CUREST = CUREST - EST(ISTAGE+1) + VINT
00734 247*      JEND = IBEG - 1
00735 248*      FEND = TS(IEND)
00736 249*      IBEG = IBEGS(ISTAGE)
00737 250*      GO TO 94
00740 251* 85 CUREST = CUREST + VINT
00741 252*      STAGE = STAGE+STAGE
00742 253*      JEND = IBEG
00743 254*      IBEG = IBEGS(ISTAGE)
00744 255*      END = BEG
00745 256*      BEG = BEGIN(ISTAGE)
00746 257*      FEND = FBEG
00747 258*      FBEG = TS(IBEG)
00750 259*      GO TO 5
00752 260* 90 REGLAR = .TRUE.
00753 261* 91 IF (ISTAGE .EQ. MXSTAGE)      GO TO 950
00754 262*      IF (LEVEL .LT. 5)      GO TO 93
00757 263*      DO 92 I=1,L
00767 264* 92 WRITE (6,692) I,(T(1,J),J=1,L)
00770 265* 692 FORMAT(15,7E16.8/3E16.8)
00772 266* 93 IF (RIGHT)      GO TO 95
00773 267*      REGLSV(ISTAGE+1) = REGLAR
00774 268*      BEGIN(ISTAGE) = BEG

```

```

00774 267* IBEGS(ISTAGE) = IBEG
00775 268* STAGE = STAGE/2.
00776 269* 94 RIGHT = .TRUE.
00777 270* REG = (BEG+END)/2.
01000 271* IBEG = (IBEG+IEND)/2
01001 272* TS(IREG) = TS(IREG)/2.
01002 273* FBEG = TS(IREG)
01003 274*
01004 275* GO TO 6
01005 276* 95 NNLEFT = IBEG - IBEGS(ISTAGE)
01007 277* IF (IEND-NNLEFT .GE. MAXTS) GO TO 900
01010 278* III = IBEGS(ISTAGE)
01011 279* II = IEND
01014 280* DO 96 I=III,IREG
01015 281* II = II + 1
01017 282* 96 TS(II) = TS(II)
01022 283* DO 97 I=IBEG,II
01023 284* TS(III) = TS(II)
01025 285* 97 III = III + 1
01026 286* IEND = IEND + 1
01027 287* IBEG = IEND - NNLEFT
01030 288* FBEG = TS(IREG)
01031 289* FINIS(ISTAGE) = END
01032 290* END = BEG
01033 291* BEG = BEGIN(ISTAGE)
01034 292* BEGIN(ISTAGE) = END
01035 293* REGLSV(ISTAGE) = REGLAR
01036 294* ISTAGE = ISTAGE + 1
01037 295* REGLAR = REGLSV(ISTAGE)
01040 296* EST(ISTAGE) = VINT
01041 297* CUREST = CUREST + EST(ISTAGE)
01042 298* GO TO 5
01043 299* 900 IF (LEVEL .GE. 2) WRITE (6,6900) BEG, END
01050 300* 6900 FORMAT(37H TOO MANY FUNCTION EVALUATIONS AROUND/
01050 301* 10X,E15.8,4H AND,E15.8)
01051 302* IFLAG = 4
01052 303* GO TO 999
01053 304* 950 IFLAG = 5
01054 305* IF (LEVEL .LT. 2)
01056 306* IF (LEVEL .LT. 5)
01060 307* DO 958 I=1,L
01063 308* 958 WRITE (6,692) I,(T(I),J),J=1,L)
01073 309* 959 WRITE (6,6959) BEG, END
01077 310* 6959 FORMAT(12X38HINTEGRAND SHOWS SINGULAR BEHAVIOUR OF
01077 311* 20HUNKNOWN TYPE BETWEEN E15.8,4H AND E15.8)
01100 312* 999 CADRE = CUREST + VINT
01101 313* RETURN
01102 314* END

```

END OF COMPILATION: 3 DIAGNOSTICS.

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
12	Commander Defense Documentation Center ATTN: DDC-TCA Cameron Station Alexandria, VA 22314	1	Commander US Army Electronics Command ATTN: DRSEL-RD Fort Monmouth, NJ 07703
1	Director of Defense Research & Engineering (OSD) Washington, DC 20301	1	Director Research and Development Technical Support Agency ATTN: DRSEL-GG-E, Mr. Goldy US Army Electronics Command Fort Monmouth, NJ 07703
1	Director Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, VA 22209	1	Commander US Army Tank Automotive Development Command ATTN: DRDTA-RWL Warren, MI 48090
1	Director Weapons Systems Evaluation Group Washington, DC 20305	1	Commander US Army Missile Command ATTN: DRSMI-R Redstone Arsenal, AL 35809
1	Chairman Defense Science Board Washington, DC 20301	2	Commander US Army Mobility Equipment Research & Development Command ATTN: Tech Docu Cen, Bldg. 315 DRSME-RZT Fort Belvoir, VA 22060
1	Commander US Army Materiel Development and Readiness Command ATTN: DRCDMA-ST 5001 Eisenhower Avenue Alexandria, VA 22333	1	Commander US Army Armament Command Rock Island, IL 61202
1	Commander US Army Aviation Systems Command ATTN: DRSAV-E 12th and Spruce Streets St. Louis, MO 61366	1	Commander US Army Picatinny Arsenal ATTN: SARPA-DW-6 Dover, NJ 07801
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035	2	Commander US Army Frankford Arsenal ATTN: SARFA-J7400 SARFA-J7600 Philadelphia, PA 19137

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
1	Commander US Army Harry Diamond Labs ATTN: DRXDO-TI 2800 Powder Mill Road Adelphi, MD 20783	1	Commander US Army Aviation School ATTN: Aviation Agency Fort Rucker, AL 36362
4	Commander US Army Materials and Mechanics Research Center ATTN: Dr. R. Frost Mr. A. Alesi Mr. K. Abbott Mr. E. DeLuca Watertown, MA 02172	1	Commander US Army Field Artillery School ATTN: Field Artillery Agency Fort Sill, OK 73405
6	Commander US Army Natick Research and Development Center ATTN: Mr. T. Bailey Mr. T. Keville Mr. W. Wright Mr. F. Figucia Mr. M. Denomee MAJ W. R. Belcher Natick, MA 01760	1	The Surgeon General Department of the Army Washington, DC 20315
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SA White Sands Missile Range NM 88002	1	Chief of Naval Operations ATTN: OP-03EG Department of the Navy Washington, DC 20350
1	Commander US Army Training and Doctrine Command ATTN: MAJ R. Young Fort Monroe, VA 23651	1	Commander US Naval Air Systems Command ATTN: AIR-604 Washington, DC 20360
1	Commander US Army Armor School ATTN: Armor Agency Fort Knox, KY 40121	2	Commander US Naval Weapons Center ATTN: Code 753 Code 12 China Lake, CA 93555
		1	Commander US Naval Medical Field Research Laboratories Camp LeJuene, NC 28542
		1	Commander US Naval Research Laboratories ATTN: Code 8434, W. Ferguson Washington, DC 20375
		1	Commandant US Marine Corps Washington, DC 20380

DISTRIBUTION LIST

<u>No. of Copies</u>	<u>Organization</u>
1	Director, MCDED ATTN: MAJ D. Crowe Quantico, VA 22134
1	AFATL (DLRW) Eglin AFB, FL 32542
1	US Department of Commerce National Bureau of Standards Law Enforcement Standards Lab Washington, DC 20236

Aberdeen Proving Ground

Cdr, USAEA
ATTN: SAREA-BL-B
Dir, USAMSAA
Dir, USAHEL
ATTN: Mr. B. Corona
Marine Corps Ln Ofc