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A NUMERICAL STUDY OF AN IDEALIZED EMP PROBLEM.(U)

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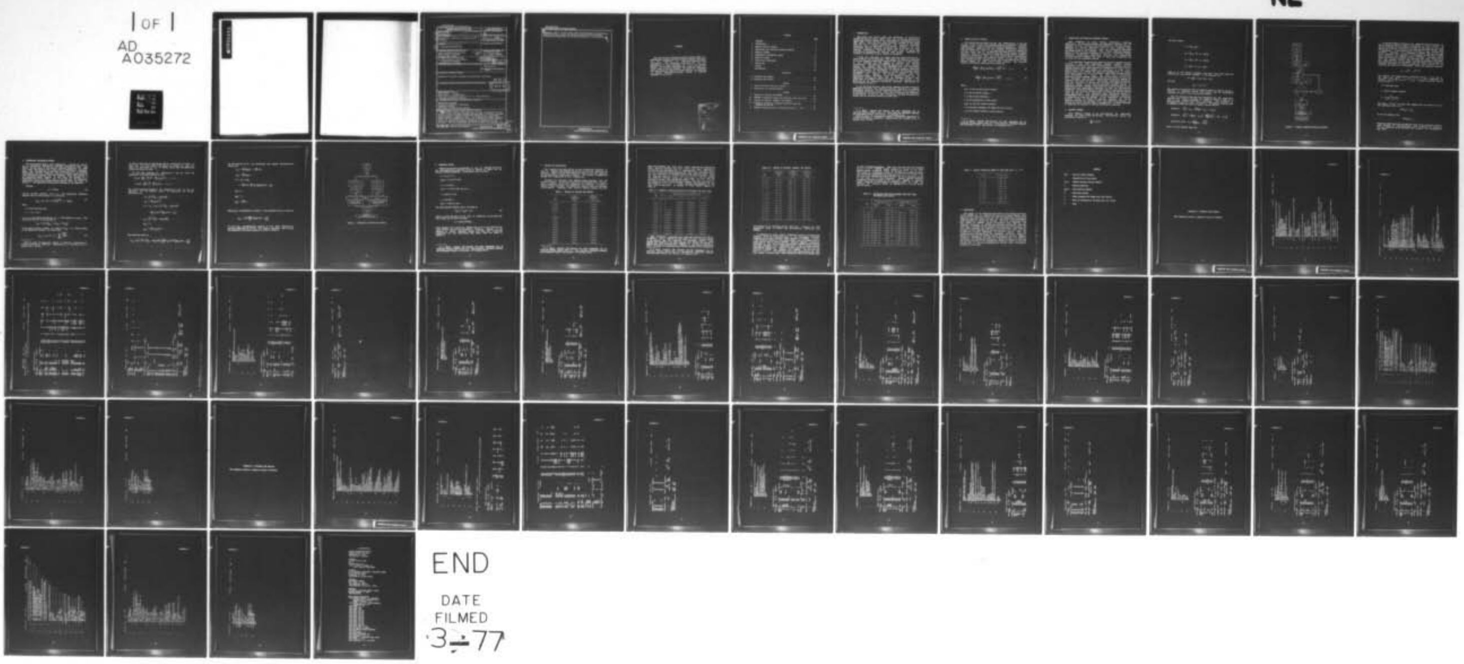

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Accurate results were obtained for the time history of the electric-field strength between two parallel, infinite, aluminum plates caused by Compton electrons generated by a transient, high-intensity, gamma-ray flux. Two numerical methods, Hamming's method and an exponential differencing technique, were used to solve the resulting ordinary differential equations of the problem. The two techniques and their results are examined and → over

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compared, and it is set forth that the exponential differencing method provides a more efficient solution to the ordinary differential equations of the type involved in this problem.

FOREWORD

The Army Multiple Systems Evaluation Program (MSEP) is a comprehensive program developing general analytic techniques for the prediction of high-electromagnetic-pulse vulnerability and hardening technology and for the application of these techniques to a list of critical systems. The analytic techniques have been verified for a large class of tactical systems. The hardening techniques have been applied to specific systems and are now resulting in product improvement programs leading to hardened equipment in the field.

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1. INTRODUCTION

This effort was written under the sponsorship of the Multiple Systems Evaluation Program (MSEP) which has as its main objective to harden Army tactical systems to the exoatmospheric electromagnetic-pulse (EMP) threat. Along with this major objective, the MSEP is also tasked with the aim to develop experimental and analytical evaluation techniques that are applicable to all systems' problems. This work illuminates several numerical techniques that can be used to solve ordinary differential equations that might arise in making EMP vulnerability assessments for tactical Army systems. Therefore, the objective of providing analytic methods is satisfied.

A study was undertaken to expand upon the results obtained in a paper by Wyatt[1] and to investigate and compare two numerical techniques in solving ordinary differential equations that occur in an EMP problem. It was conducted with the purpose of refining the previous calculations.[1] The numerical techniques employed are fairly standard methods, but whereas Wyatt was concerned only with approximate solutions, this work deals mainly with the specific numerical techniques used in the solution of the problem. An analysis of truncation errors was employed to deal with the accuracy of the solutions that better solidifies the results. The methods utilized were the predictor-corrector technique known as Hamming's method and an exponential differencing method that was developed by Pope.[2] This effort does not attempt to explain the physics of the problem or the derivation of the ordinary differential equations. This material is fully detailed by Wyatt.[1]

In the following sections, some general comments are made concerning Runge-Kutta solutions and predictor-corrector solutions of ordinary differential equations. The two techniques employed in the solution of the problem are described fully, along with a discussion and comparison of the results. At the end, some conclusions are made concerning the results and the numerical methods that were used.

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

[2] *David A. Pope, An Exponential Method of Numerical Integration of Ordinary Differential Equations, Communications of the ACM 6, No. 8 (August 1963), 491-493.*

2. GENERAL PHYSICAL PROBLEM

The problem that was dealt with was the determination of the time history of the electric-field strength between two parallel, infinite, aluminum plates caused by Compton electrons generated by a transient, high-intensity, gamma-ray flux. The ordinary differential equations that are solved are a result of the application of Poisson's equation to the free charge that is the spatially distributed current of Compton electrons. Since the conductivity of the air is significant, Poisson's equation is modified by Ohm's law, which relates the air conductivity, electric-field strength, and conduction current. In the case examined here, methane was used instead of air, and it was found by Wyatt[1] that methane reduced the peak electric-field strength. The resulting ordinary differential equations to be solved are

$$\frac{dE(t)}{dt} + \frac{q\mu}{\varepsilon} N_e(0) e^{rt} E(t) = \frac{J_c(t)}{\varepsilon} e^{rt} \,, \quad t < 0 \qquad (1)$$

$$\frac{dE(t)}{dt} + \frac{q\mu}{\varepsilon} N_e(t) E(t) = \frac{J_c(t)}{\varepsilon} \,, \quad t \geq 0 \,, \qquad (2)$$

where

$E(t)$ is the electric-field strength,

q is the electronic charge,

μ is the electron mobility,

ε is the permittivity of free space,

$N_e(t)$ is the free electron density,

r is the model parameter for gamma-flux rate history,

J_c is the Compton electron current density.

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

3. RUNGE-KUTTA AND PREDICTOR-CORRECTOR METHODS

When attempting to solve ordinary differential equations numerically, two well-known methods, the Runge-Kutta technique and the predictor-corrector technique, are often employed to find the solution. Depending on various constraints such as time, computing funds available, accuracy, and stability, there are accompanying advantages and disadvantages to both methods. Thus, before examining Hamming's method in the solution of equations (1) and (2), a few general remarks concerning fourth-order Runge-Kutta solutions and fourth-order predictor-corrector methods are made to give some background and rationale for the technique utilized.

It is generally recognized that predictor-corrector methods are more difficult to code than Runge-Kutta methods. However, although Runge-Kutta techniques are more straightforward, predictor-corrector methods provide a much easier analysis and examination of errors and are generally much faster. For example, the evaluation of $f(x,y)$ (i.e., $dy/dx = f(x,y)$) is usually the most time-consuming part of solving differential equations, and fourth-order Runge-Kutta methods require four evaluations of $f(x,y)$ per step, while fourth-order predictor-corrector methods require only two evaluations of $f(x,y)$ per step. This means that the fourth-order predictor-corrector methods are generally nearly twice as fast as fourth-order Runge-Kutta techniques. Thus, the evaluation of errors and the speed of computation are two compelling reasons to use predictor-corrector methods instead of Runge-Kutta methods. However, since predictor-corrector methods are not self-starting and Runge-Kutta techniques do have the self-starting capability, Runge-Kutta methods are quite useful in generating starting values for the solution and may be used to change the interval between steps when desired. This usefulness makes Runge-Kutta methods an indispensable tool in using predictor-corrector techniques. Therefore, a combination of these two methods, (1) the Runge-Kutta to determine starting values and change the per-step interval and (2) the predictor-corrector to actually solve the differential equation and analyze the errors involved, gives a technique that is highly desirable in computing the solution of ordinary differential equations.

4. HAMMING'S METHOD

Since Hamming's method is not self-starting, the Runge-Kutta technique was used to start the solution. This involves writing the differential equation as

$$\frac{dy}{dx} = f(x,y)$$

and then finding

$$k_1 = f\left(x_i, y_i\right)h \ ,$$

$$k_2 = f\left(x_i + \tfrac{1}{2}h,\ y_i + \tfrac{1}{2}k_1\right)h \ ,$$

$$k_3 = f\left(x_i + \tfrac{1}{2}h,\ y_i + \tfrac{1}{2}k_2\right)h \ ,$$

$$k_4 = f\left(x_i + h,\ y_i + k_3\right)h \ ,$$

where h is the desired increment step size. Once these values are computed for a particular increment, Δy is calculated by

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \ ,$$

and then

$$y_{i+1} = y_i + \Delta y \ .$$

This process is continued until the desired number of starting values is obtained. In Hamming's method, three values from the Runge-Kutta technique are used to start the solution process.

Once the starting values have been determined from the Runge-Kutta technique, the general procedure for Hamming's method is used, as is done by most predictor-corrector techniques. This procedure is illustrated by the flow chart in figure 1. The specific formulas used in Hamming's method and shown in figure 1 are as follows:

Predictor: $$y_{i+1}^{(0)} = y_{i-3} + \frac{4h}{3}\left(2y'_i - y'_{i-1} + 2y'_{i-2}\right),$$

Corrector: $$y_{i+1}^{(j+1)} = \frac{1}{8}\left(9y_i - y_{i-2}\right) + \frac{3h}{8}\left(\left[y_{i+1}^{(j)}\right]' + 2y'_i - y'_{i-1}\right),$$

Truncation error: $$T_i \simeq \frac{9}{121}\left[y_{i+1} - y_{i+1}^{(0)}\right] \ ,$$

where h is the interval step size.

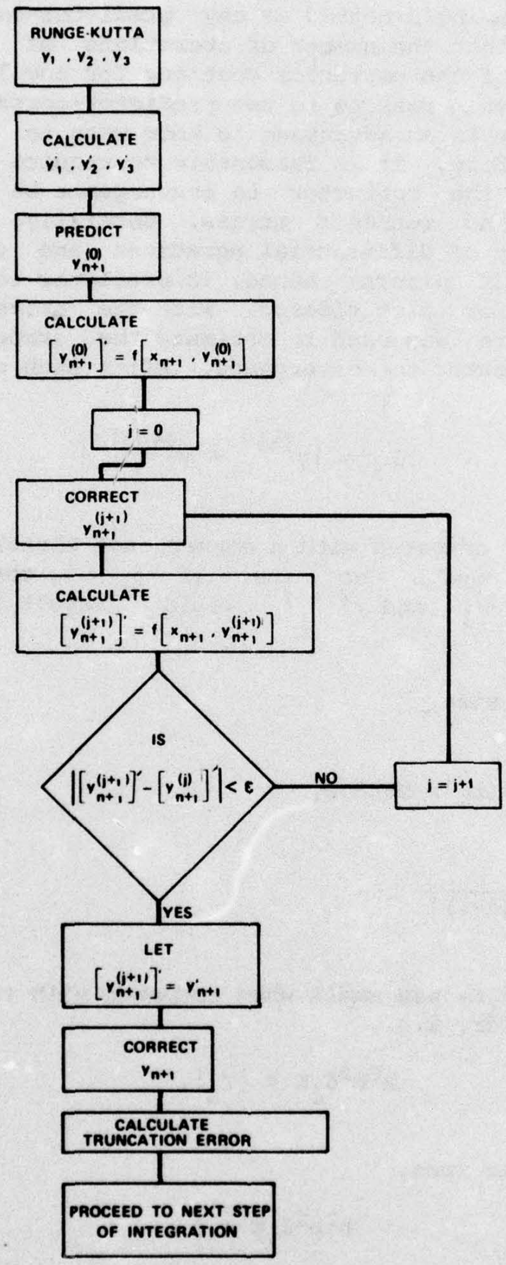

Figure 1. General Predictor-Corrector Method.

In applying Hamming's method or any predictor-corrector method, one must be cautious about the number of iterations of the corrector. To let the iteration of the corrector continue for any length of time would defeat one of the main reasons to use predictor-corrector methods, speed of computation. It is an advantage to know when to stop iterating the corrector. Therefore, it is reasonable to require that any error made by not iterating the corrector to convergence be small when compared with truncation and roundoff errors. Generally, truncation errors affect the solution of differential equations and cause instabilities more than roundoff errors; hence, in predictor-corrector techniques, only truncation error is considered. With the above point in mind, the following procedure was used to estimate the error incurred by not iterating the corrector to convergence. After each corrector iteration,

$$\delta_i = |y^{(i)'} - y^{(i-1)'}|$$

was computed and compared with a convergence factor, ε (ε in fig. 1). The value ε was chosen so that if $\delta_i < \varepsilon$, then terminating the iteration with $\left[y^{(i)}\right]'$ and $y^{(i+1)}$ would result in an error value of $h^2 b^2 \delta_i K$, where

h = time step size,

$b = \frac{3}{8}$ for Hamming's method,

$$K = \frac{\delta_i}{|y^{(i)} - y^{(i-1)}|} .$$

This value, $h^2 b^2 \delta_i K$, was small when compared with the absolute value of the truncation error, i.e.,

$$h^2 b^2 \delta_i K < |T_n| .$$

In all the computer runs,

$$h^2 b^2 \delta_i K = 0 ,$$

which was indeed less than the absolute value of the truncation error in every instance of the calculations. Also, utilizing this procedure, only two iterations of the corrector were required for convergence in most cases.

5. EXPONENTIAL DIFFERENCING METHOD

The second method used to solve differential equations (1) and (2) was an exponential differencing technique developed by Pope.[2] This method has been shown to have superior stability properties for large step sizes when dealing with a large class of differential equations. Hence, this technique may be used with a large step size to decrease significantly the total computing time for a solution, particularly in those engineering problems, like EMP problems, where high accuracy is not necessarily needed. However, in this work, the accuracy is a significant part of the effort, and, as it turns out, the exponential differencing method does provide precise results, which are shown in section 7. The exponential differencing technique is now described.

Consider

$$y' = f(x,y) \tag{3}$$

with the initial condition $y(x_o) = y_o$. The exponential difference equation used to solve equation (3) is given by

$$y_{n+1} = y_n + hf + y''f_y^{-2}\left(e^{hf_y} - 1 - hf_y\right) , \tag{4}$$

where

h is the time step size,

$y'' = f_x + f \cdot f_y$,

f_x, f_y are the partial derivatives of f with respect to x and y. The truncation error for this algorithm is

$$T_{n+1} \approx \frac{1}{6} h^3 \left(f_{xx} + 2f \cdot f_{xy} + f^2 \cdot f_{yy}\right) .$$

If the value of $|hf_y|$ is small, at least if $|hf_y| < 0.1$, then in place of the exponential formula, the series form

$$y_{n+1} = y_n + hf + y'' \sum_{k=2}^{\infty} \frac{h^k f_y^{k-2}}{k!} \tag{5}$$

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[2] *David A. Pope, An Exponential Method of Numerical Integration of Ordinary Differential Equations, Communications of the ACM, 6, No. 8 (August 1963), 491-493.*

is used to avoid loss of significance due to cancellation of terms. In this case, only a few terms of the series are needed; we used only three. If the value of hf_y is fairly large, then the exponential subroutines should be used.

To find the solutions for equations (1) and (2) using the exponential differencing method, we let

$$f(t,E) = \frac{dE}{dt} = \frac{J_c}{\varepsilon} e^{rt} - \frac{q\mu}{\varepsilon} N_e(0) e^{rt} E \ , \quad t < 0 \ ,$$

$$g(t,E) = \frac{dE}{dt} = \frac{J_c}{\varepsilon} - \frac{q\mu}{\varepsilon} N_e(t) E \ , \quad t > 0 \ .$$

The various equations needed to use equations (4) and (5) are now generated. For the function f(t,E), its derivatives and partial derivatives are

$$f_t = \frac{r}{\varepsilon} e^{rt} \left[J_c - q\mu N_e(0) E \right] ,$$

$$f_E = - \frac{q\mu}{\varepsilon} N_e(0) e^{rt} ,$$

$$E'' = f_t + f \cdot f_E = \frac{r}{\varepsilon} e^{rt} \left[J_c - q\mu N_e(0) E \right]$$

$$+ \frac{q\mu}{\varepsilon^2} N_e(0) e^{2rt} \left[q\mu N_e(0) E - J_c \right] ,$$

$$f_{tt} = \frac{r^2}{\varepsilon} e^{rt} \left[J_c - q\mu N_e(0) E \right] ,$$

$$f_{EE} = 0 ,$$

$$f_{tE} = - \frac{q\mu}{\varepsilon} N_e(0) r e^{rt} .$$

The truncation error is

$$T_{n+1} \approx \frac{1}{6} h^3 \frac{r^2}{\varepsilon} e^{rt} \left[J_c - q\mu N_e(0) E \right] + \left\{ \frac{2q\mu}{\varepsilon^2} N_e(0) e^{2rt} \left[q\mu N_e(0) E - J_c \right] \right\} .$$

For the function g(t,E), its derivatives and partial derivatives are found to be

$$g_t = -\frac{q\mu}{\varepsilon} E N_e'(t) = -\frac{q\mu}{\varepsilon} S \cdot E ,$$

$$g_E = -\frac{q\mu}{\varepsilon} N_e(t) ,$$

$$E'' = g_t + g \cdot g_E$$

$$= -\frac{q\mu}{\varepsilon} S \cdot E + \frac{q\mu}{\varepsilon} N_e(t) \left[q\mu N_e(t) E - J_c \right] ,$$

$$g_{tt} = 0 ,$$

$$g_{EE} = 0 ,$$

$$g_{tE} = -\frac{q\mu}{\varepsilon} S ,$$

where $N_e(t)$ is described in section 6. The truncation error is given by

$$T_{n+1} \approx \frac{1}{6} h^3 \left\{ \frac{2q\mu}{\varepsilon^2} \left[q\mu N_e(t) E - J_c \right] \right\} .$$

It is an easy, straightforward process to code these quantities to calculate equations (4) and (5). The general procedure used to solve the exponential difference equation is given in figure 2.

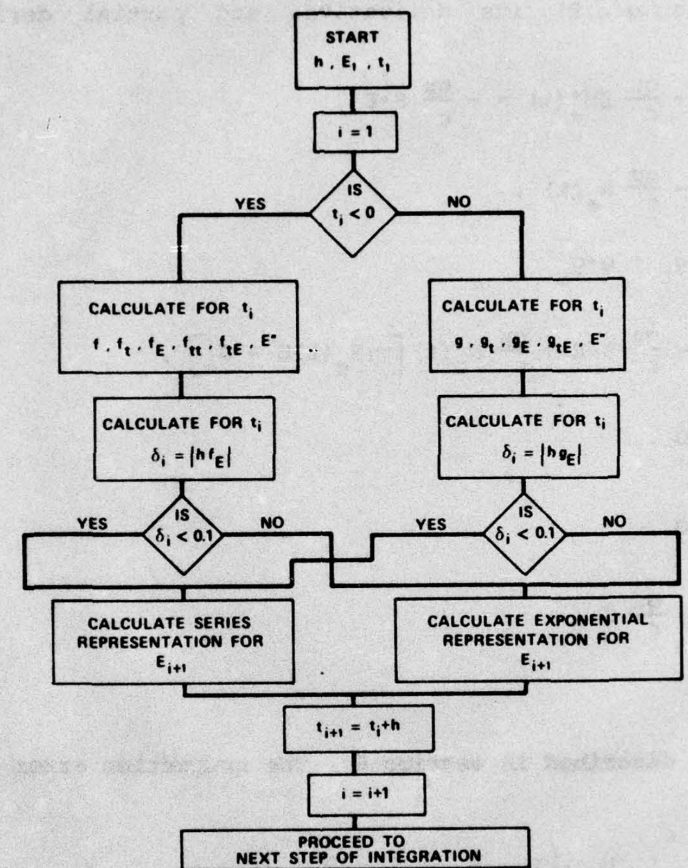

Figure 2. Exponential Differencing Method.

6. NUMERICAL VALUES

Finding the solution of equations (1) and (2) required the use of the same hypothetical numerical values as applied by Wyatt[1] for the various constant parameters. These values are as follows:

$q = 1.6 \cdot 10^{-19}$ C,

$N_e(0) = 1.133 \cdot 10^{11}/cm^3$,

$\mu = 2.0\ m^2/V/s$,

$J_c(t) = 1.34 \cdot 10^3\ A/m^2$ for all t ,

$\varepsilon = 8.85 \cdot 10^{-2}$ F/m ,

$r = 5.0 \cdot 10^8/s$,

$E(0) = 3.697 \cdot 10^4$ V/m .

The free-electron density, $N_e(t)$, is given by

$$N_e(t) = N_e(0) + St , \tag{6}$$

where t is the time and S is the rate of production of ion pairs per unit volume and is given the value

$$S = 5.664 \cdot 10^{25}/m^3/s .$$

Since equation (6) is valid for methane only to 9 ns, any solutions for times beyond this were not possible to calculate. However, it was possible to obtain excellent results and make some interesting comparisons for this restrictive time frame, which are shown in section 7.

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[1]*W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

7. RESULTS AND COMPARISONS

Two computer codes were written to solve differential equations (1) and (2). Computer code HAMMING was written to solve the equations by Hamming's method, and the code EXPDIFF was used to solve the equations by the exponential differencing methods. The listings of HAMMING and EXPDIFF are in appendices A and B, respectively.

Although the solutions from HAMMING compare favorably with the results of Wyatt,[1] the solutions obtained are considered more accurate, since truncation errors are calculated and examined. In table I, specific values from Wyatt[1] and HAMMING are listed to show the agreement of the results. Regarding these results, numerous computer runs were

TABLE I. RESULTS OF ORIGINAL AND HAMMING

Time (s)	Original Electric field strength (V/m)	HAMMING Electric field strength ($\Delta t = 1.0 \cdot 10^{-12}$) (V/m)
$-4.0.10^{-9}$	$3.80 \cdot 10^4$	$3.6970 \cdot 10^4$
$-3.0 \cdot 10^{-9}$	$3.80 \cdot 10^4$	$3.6965 \cdot 10^4$
$-2.0 \cdot 10^{-9}$	$3.80 \cdot 10^4$	$3.6961 \cdot 10^4$
$-1.0 \cdot 10^{-9}$	$3.80 \cdot 10^4$	$3.6960 \cdot 10^4$
0	$3.80 \cdot 10^4$	$3.6959 \cdot 10^4$
$5.0 \cdot 10^{-10}$	$3.00 \cdot 10^4$	$3.2129 \cdot 10^4$
$1.0 \cdot 10^{-9}$	$2.48 \cdot 10^4$	$2.6259 \cdot 10^4$
$1.5 \cdot 10^{-9}$	$2.10 \cdot 10^4$	$2.2092 \cdot 10^4$
$2.0 \cdot 10^{-9}$	$1.85 \cdot 10^4$	$1.9107 \cdot 10^4$
$3.0 \cdot 10^{-9}$	$1.47 \cdot 10^4$	$1.5093 \cdot 10^4$
$4.0 \cdot 10^{-9}$	$1.20 \cdot 10^4$	$1.2496 \cdot 10^4$
$5.0 \cdot 10^{-9}$	$1.00 \cdot 10^4$	$1.0670 \cdot 10^4$
$6.0 \cdot 10^{-9}$	$8.80 \cdot 10^3$	$9.3132 \cdot 10^3$
$7.0 \cdot 10^{-9}$	$8.00 \cdot 10^3$	$8.2648 \cdot 10^3$
$8.0 \cdot 10^{-9}$	$7.40 \cdot 10^3$	$7.4296 \cdot 10^3$
$9.0 \cdot 10^{-9}$	$7.00 \cdot 10^3$	$6.7483 \cdot 10^3$

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

made with different time step sizes, which revealed some interesting facts. When the time step size $\Delta t = 1.0 \cdot 10^{-10}$ was used, stability of the solution was obtained from 0 to approximately 6.5 ns, while from 6 to 9 ns, the solution became dominated by very large truncation errors and, thus, resulted in an unstable solution over this time frame. Using the time step sizes $\Delta t = 1.0 \cdot 10^{-11}$ and $\Delta t = 1.0 \cdot 10^{-12}$ did not produce any stability problems during the time frame 0 to 9 ns, and the truncation errors were considerably smaller than for $\Delta t = 1.0 \cdot 10^{-10}$. The truncation errors did not decrease, however, when the time step size was lowered to $\Delta t = 1.0 \cdot 10^{-13}$, as they remained approximately on the same order of magnitude. An example of these results appears in table II.

TABLE II. HAMMING'S TRUNCATION ERRORS FOR DIFFERENT TIME STEP SIZES

Time (s)	Truncation errors for time step sizes (Δt)			
	$\Delta t = 1.0 \cdot 10^{-10}$	$\Delta t = 1.0 \cdot 10^{-11}$	$\Delta t = 1.0 \cdot 10^{-12}$	$\Delta t = 1.0 \cdot 10^{-13}$
0	0	0	0	0
$1.0 \cdot 10^{-10}$	0	$3.4909 \cdot 10^{-6}$	$-8.6590 \cdot 10^{-11}$	$-1.2123 \cdot 10^{-10}$
$2.0 \cdot 10^{-10}$	0	$4.1106 \cdot 10^{-6}$	$-1.7318 \cdot 10^{-11}$	$-6.9272 \cdot 10^{-11}$
$3.0 \cdot 10^{-10}$	0	$4.0913 \cdot 10^{-6}$	$-5.1954 \cdot 10^{-11}$	$-1.2123 \cdot 10^{-10}$
$4.0 \cdot 10^{-10}$	$1.9735 \cdot 10^{-1}$	$3.6824 \cdot 10^{-6}$	$-6.9272 \cdot 10^{-11}$	$-1.3854 \cdot 10^{-10}$
$1.0 \cdot 10^{-9}$	$8.5146 \cdot 10^{-2}$	$5.5282 \cdot 10^{-7}$	$-6.9272 \cdot 10^{-11}$	$-6.0613 \cdot 10^{-11}$
$2.0 \cdot 10^{-9}$	$-1.1649 \cdot 10^{-3}$	$-1.5846 \cdot 10^{-8}$	$-5.1954 \cdot 10^{-11}$	$-4.3295\ 10^{-11}$
$3.0 \cdot 10^{-9}$	$-1.7558 \cdot 10^{-4}$	$-3.2038 \cdot 10^{-9}$	$-6.4942 \cdot 10^{-11}$	$-5.1954 \cdot 10^{-11}$
$4.0 \cdot 10^{-9}$	$-1.0871 \cdot 10^{-4}$	$-8.9188 \cdot 10^{-10}$	$-3.8965 \cdot 10^{-11}$	$-2.5977 \cdot 10^{-11}$
$5.0 \cdot 10^{-9}$	$-8.4018 \cdot 10^{-3}$	$-3.4203 \cdot 10^{-10}$	$-2.1647 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$6.0 \cdot 10^{-9}$	$-1.2692 \cdot 10^{1}$	$-1.6019 \cdot 10^{-10}$	$-3.0306 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$7.0 \cdot 10^{-9}$	$-2.3989 \cdot 10^{5}$	$-8.6590 \cdot 10^{-11}$	$-3.4636 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$8.0 \cdot 10^{-9}$	$-5.0072 \cdot 10^{10}$	$-6.4942 \cdot 10^{-11}$	$-1.9483 \cdot 10^{-11}$	$-2.1647 \cdot 10^{-11}$
$9.0 \cdot 10^{-9}$	$-9.8303 \cdot 10^{16}$	$-3.2471 \cdot 10^{-11}$	$-1.2988 \cdot 10^{-11}$	$-1.0824 \cdot 10^{-11}$

Some interesting occurrences were noted when analyzing the results of EXPDIFF. First, EXPDIFF was very simple to code. In fact, because of the relative ease and simplicity in programming EXPDIFF, valid results were obtained on the first computer run. Second, the results of EXPDIFF were quite favorable when compared to the results obtained by Wyatt[1] and again are considered more accurate because of the calculation

[1]*W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

TABLE III. RESULTS OF ORIGINAL, HAMMING, AND EXPDIFF

Time (s)	Original Electric field strength (V/m)	HAMMING Electric field strength $\Delta t = 1.0 \cdot 10^{-12}$ (V/m)	EXPDIFF Electric field strength $\Delta t = 1.0\ 10^{-12}$ (V/m)
$-4.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6970 \cdot 10^{4}$	$3.6970 \cdot 10^{4}$
$-3.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6965 \cdot 10^{4}$	$3.6965 \cdot 10^{4}$
$-2.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6961 \cdot 10^{4}$	$3.6961 \cdot 10^{4}$
$-1.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6960 \cdot 10^{4}$	$3.6960 \cdot 10^{4}$
0	$3.80 \cdot 10^{4}$	$3.6959 \cdot 10^{4}$	$3.6959 \cdot 10^{4}$
$5.0 \cdot 10^{-10}$	$3.00 \cdot 10^{4}$	$3.2129 \cdot 10^{4}$	$3.2135 \cdot 10^{4}$
$1.0 \cdot 10^{-9}$	$2.48 \cdot 10^{4}$	$2.6259 \cdot 10^{4}$	$2.6264 \cdot 10^{4}$
$1.5 \cdot 10^{-9}$	$2.10 \cdot 10^{4}$	$2.2092 \cdot 10^{4}$	$2.2095 \cdot 10^{4}$
$2.0 \cdot 10^{-9}$	$1.85 \cdot 10^{4}$	$1.9107 \cdot 10^{4}$	$1.9110 \cdot 10^{4}$
$3.0 \cdot 10^{-9}$	$1.47 \cdot 10^{4}$	$1.5093 \cdot 10^{4}$	$1.5094 \cdot 10^{4}$
$4.0 \cdot 10^{-9}$	$1.20 \cdot 10^{4}$	$1.2496 \cdot 10^{4}$	$1.2497 \cdot 10^{4}$
$5.0 \cdot 10^{-9}$	$1.00 \cdot 10^{4}$	$1.0670 \cdot 10^{4}$	$1.0670 \cdot 10^{4}$
$6.0 \cdot 10^{-9}$	$8.80 \cdot 10^{3}$	$9.3132 \cdot 10^{3}$	$9.3138 \cdot 10^{3}$
$7.0 \cdot 10^{-9}$	$8.00 \cdot 10^{3}$	$8.2648 \cdot 10^{3}$	$8.2652 \cdot 10^{3}$
$8.0 \cdot 10^{-9}$	$7.40 \cdot 10^{3}$	$7.4296 \cdot 10^{3}$	$7.4299 \cdot 10^{3}$
$9.0 \cdot 10^{-9}$	$7.00 \cdot 10^{3}$	$6.7483 \cdot 10^{3}$	$6.7486 \cdot 10^{3}$

and analysis of the truncation errors (table III). Probably the most significant aspect of EXPDIFF was the results of comparisons with HAMMING.

In comparing the codes, several significant points of interest were revealed. There was a noticeable ease in the programming of EXPDIFF compared with the more difficult effort required for HAMMING. In fact, a very generous analysis of this coding effort was that HAMMING took at least twice as long to code as EXPDIFF. Although HAMMING is generally considered to give more accurate calculations, EXPDIFF yielded results that were extremely close (sometimes exact) to the solutions obtained from HAMMING (table III). However, the most significant result was the stability of EXPDIFF when compared with HAMMING. For the time step size $\Delta t = 1.0 \cdot 10^{-11}$, the truncation errors were approximately the same size, except in several cases where truncation errors from HAMMING were an order of magnitude smaller. When the time step size was increased to $\Delta t = 1.0 \cdot 10^{-10}$, HAMMING became unstable at about 6.5 ns, while EXPDIFF yielded truncation errors on the order of 10^{-6} for 0 to 9 ns. This timing is illustrated in table IV. An even more meaningful result was noticed when considering the large step size of $\Delta t = 1.0 \cdot 10^{-9}$. For this step size, EXPDIFF was stable through 9 ns, since truncation errors on the order of 10^{-2} were tolerated. Whereas EXPDIFF exhibited reasonable truncation errors for this step size, HAMMING, as expected, became dominated by the buildup of truncation errors and, thus, was unstable. The truncation errors of EXPDIFF for $\Delta t = 1.0 \cdot 10^{-9}$ can be seen in table V.

TABLE IV. TRUNCATION ERRORS FOR DIFFERENT TIME STEP SIZES FOR HAMMING AND EXPDIFF

Time (s)	Truncation errors at time step size $1.0 \cdot 10^{-10}$		Truncation errors at time step size $1.0 \cdot 10^{-11}$	
	HAMMING	EXPDIFF	HAMMING	EXPDIFF
0	0	$7.4679 \cdot 10^{-5}$	0	$5.0168 \cdot 10^{-8}$
$1.0 \cdot 10^{-10}$	C	$5.9718 \cdot 10^{-11}$	$3.4909 \cdot 10^{-6}$	$3.9421 \cdot 10^{-9}$
$2.0 \cdot 10^{-10}$	0	$5.1673 \cdot 10^{-6}$	$4.1106 \cdot 10^{-6}$	$6.740 \cdot 10^{-9}$
$3.0 \cdot 10^{-10}$	0	$8.4422 \cdot 10^{-6}$	$4.0913 \cdot 10^{-6}$	$8.3.3 \cdot 10^{-9}$
$4.0 \cdot 10^{-10}$	$1.9735 \cdot 10^{-1}$	$1.0322 \cdot 10^{-5}$	$3.6824 \cdot 10^{-6}$	$9.0334 \cdot 10^{-9}$
$1.0 \cdot 10^{-9}$	$8.5146 \cdot 10^{-2}$	$9.7018 \cdot 10^{-6}$	$5.5282 \cdot 10^{-7}$	$7.0506 \cdot 10^{-9}$
$2.0 \cdot 10^{-9}$	$-1.1649 \cdot 10^{-3}$	$5.3903 \cdot 10^{-6}$	$-1.5846 \cdot 10^{-8}$	$3.6655 \cdot 10^{-9}$
$3.0 \cdot 10^{-9}$	$-1.7558 \cdot 10^{-4}$	$3.5640 \cdot 10^{-6}$	$-3.2038 \cdot 10^{-9}$	$2.2718 \cdot 10^{-9}$
$4.0 \cdot 10^{-9}$	$-1.0871 \cdot 10^{-4}$	$2.6160 \cdot 10^{-6}$	$-8.9188 \cdot 10^{-10}$	$1.5602 \cdot 10^{-9}$
$5.0 \cdot 10^{-9}$	$-8.4018 \cdot 10^{-3}$	$2.0477 \cdot 10^{-6}$	$-3.4203 \cdot 10^{-10}$	$1.1436 \cdot 10^{-9}$
$6.0 \cdot 10^{-9}$	$-1.2692 \cdot 10^{1}$	$1.6749 \cdot 10^{-6}$	$-1.6019 \cdot 10^{-10}$	$8.7743 \cdot 10^{-10}$
$7.0 \cdot 10^{-9}$	$-2.3989 \cdot 10^{5}$	$1.4142 \cdot 10^{-6}$	$-8.6590 \cdot 10^{-11}$	$6.9648 \cdot 10^{-10}$
$8.0 \cdot 10^{-9}$	$-5.0072 \cdot 10^{10}$	$1.2230 \cdot 10^{-6}$	$-6.4942 \cdot 10^{-11}$	$5.6760 \cdot 10^{-10}$
$9.0 \cdot 10^{-9}$	$-9.8303 \cdot 10^{16}$	$1.0775 \cdot 10^{-6}$	$-3.2471 \cdot 10^{-11}$	$4.7242 \cdot 10^{-10}$

TABLE V. EXPDIFF TRUNCATION ERRORS AT TIME STEP SIZE $1.0 \cdot 10^{-9}$

Time (s)	Truncation errors
0	$8.7131 \cdot 10^{-1}$
$1.0 \cdot 10^{-9}$	$2.1944 \cdot 10^{-7}$
$2.0 \cdot 10^{-9}$	$5.1673 \cdot 10^{-2}$
$3.0 \cdot 10^{-9}$	$3.4598 \cdot 10^{-2}$
$4.0 \cdot 10^{-9}$	$2.5851 \cdot 10^{-2}$
$5.0 \cdot 10^{-9}$	$2.0672 \cdot 10^{-2}$
$6.0 \cdot 10^{-9}$	$1.7226 \cdot 10^{-2}$
$7.0 \cdot 10^{-9}$	$1.4766 \cdot 10^{-2}$
$8.0 \cdot 10^{-9}$	$1.2920 \cdot 10^{-2}$
$9.0 \cdot 10^{-9}$	$1.1484 \cdot 10^{-2}$

8. CONCLUSIONS

With regard to the comparisons made between the predictor-corrector routine HAMMING and the exponential differencing method EXPDIFF, there seem to be two compelling factors that make the exponential differencing technique superior. First, the ease and simplicity of the programming effort required for EXPDIFF far outweigh the more complicated coding work needed for HAMMING. Second and probably more important, the stability properties of EXPDIFF are excellent, whereas the solutions calculated by HAMMING became dominated by truncation errors during the time frame examined for particular time step sizes. This stability property is best exemplified by the large step size ($1.0 \cdot 10^{-9}$) that can be used with relative assuredness of accurate results. Thus, this stability factor represented by the reasonable truncation errors of EXPDIFF far outweigh the somewhat smaller truncation errors of HAMMING. Therefore, considering the ease of programming and the stability properties, the exponential differencing method provides an efficient and reliable solution to the differential equations involved in this EMP problem and is strongly recommended for solving other differential equations of this type.

SYMBOLS

$E(t)$	Electric-field strength
ϵ	Permittivity of free space
$J_c(t)$	Compton electron current density
μ	Electron mobility
$N_e(t)$	Free electron density
q	Electronic charge
r	Model parameter for gamma flux rate history
s	Rate of production of ion pairs per unit volume
t	Time

APPENDIX A.--COMPUTER CODE HAMMING

This appendix contains a complete listing of HAMMING.

```
PROGRAM HAMMING    74/74   OPT=1                    FTN 4.5+414       06/04/76  13.30.02       PAGE     1

      PROGRAM HAMMING(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      INTEGER FLAG
      REAL NORD(10),NABS(10)
      COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
      COMMON/B/VAL(5000),CF(5000)
      COMMON/C/RNE(5000),SNE(5000)
      DIMENSION K(5000),TITLE(20)
      DIMENSION ERROR(5000)
      DIMENSION TERR(5000)
      DIMENSION ECOREC(60),ECORECP(60)
   10 L=1
      IT=0
      READ(5,20)DEL,TMAX,EPSLON,START,ESTART,M
   20 FORMAT(5E10.3,I5)
      CF(1)=EPSLON
C
C  CALCULATE STARTING VALUES Y1,Y2,Y3,USING RUNGE-KUTTA TECHNIQUE
C
      CALL RUNGE
      DO 30 J=1,4
      CF(J)=EPSLON
   30 EP(J)=FET(H(J),E(J),J)
      I=4
C
C  PREDICT Y(N+1) USING MILNE"S PREDICTOR
C
   40 CALL PREDICT(I,EPDIC)
      CF(I+1)=EPSLON
      H(I+1)=H(I)+DEL
      EPDICP=FET(H(I+1),EPDIC,0)
      ECOREC(L)=EPDIC
      ECORECP(L)=EPDICP
C
C  CORRECT Y(N+1) USING HAMMING"S CORRECTOR
C
   50 CALL CORRECT(I,ECORECP(L),ECOREC(L+1))
      ECORECP(L+1)=FET(H(I+1),ECOREC(L),0)
      IF(L.EQ.1)  GO TO 55
C
C  CHECK TO SEE IF ERROR RESULTS FROM NOT ITERATING TO CONVERGENCE
C
      CALL EROR(I,L,EPDIC,ECOREC(L),ECOREC(L+1),ECORECP(L),
     1ECORECP(L+1),FLAG,IT)
      IF(FLAG) 53,60,60
   53 ERROR(I+1)=ABS(ECORECP(L+1)-ECORECP(L))
      IF(L.GT.50)  GO TO 65
   55 L=L+1
      GO TO 50
   60 K(I+1)=L-1
C
C  CALCULATE CORRECT VALUE OF Y(N+1) USING HAMMING"S CORRECTOR
C
      CALL CORRECT(I,ECORECP(L+1),ECOR)
      E(I+1)=ECOR
      EP(I+1)=FET(H(I+1),E(I+1),I+1)
C
C  CALCULATE TRUNCATION ERROR
```

```
PROGRAM HAMMING    74/74   OPT=1                    FTN 4.5+414        06/04/76  13.30.02        PAGE     2

      C
            TERR(I+1)=(9./121.)*(E(I+1)-EPDIC)
            N=I+1
            I=I+1
            L=1
            IF(H(I).LE.TMAX)  GO TO 40
         65 WRITE(6,70)
         70 FORMAT(1H1)
            WRITE(6,80) DEL,TMAX,M,EPSLON,START
         80 FORMAT(1X,17HDELTA TIME STEP: ,1PE10.3,5X,
           120HMAXIMUM TIME TO BE PLOTTED: ,1PE10.3,5X,
           220HINCREMENT OF OUTPUT VALUES: ,I5,/,
           344X,23HCONVERGENCE TOLERANCE: ,1PE10.3,
           45X,15HSTARTING TIME: ,1PE10.3,/,/,/,/)
            IF(L.GT.50.AND.FLAG.EQ.-1)  GO TO 140
            WRITE(6,90)
         90 FORMAT(/,/,/,53X,9HNUMBER OF,10X,8HERROR BY)
            WRITE(6,100)
        100 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,3X,
           118HCONVERGENCE CYCLES,3X,13HNOT ITERATING,3X,
           218HCONVERGENCE FACTOR,5X,6HOLD NE,8X,6HNEW NE)
            WRITE(6,110)
        110 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,3X,
           118H------------------,3X,13H-------------,3X,
           218H------------------,5X,6H------,8X,6H------)
            DO 130 J=1,N,M
            WRITE(6,120) H(J),E(J),TERR(J),K(J),VAL(J),CF(J),RNE(J),SNE(J)
        120 FORMAT(1X,3(1PE11.4,5X),7X,I3,11X,1PE11.4,8X,1PE11.4,7X,1PE11.4,
           13X,1PE11.4)
        130 CONTINUE
      C
      C
            TITLE(1)=10HEMP TIME D
            TITLE(2)=10HEPENDENCY
            TITLE(3)=5HSTUDY
            TITLE(4)=1H
            TITLE(5)=1H
            NORD(1)=10HELECTRIC F
            NORD(2)=10HIELD STREN
            NORD(3)=10HGTH  V/M
            NABS(1)=4HTIME
            NABS(2)=1H
            NABS(3)=1H
      C
      C
            CALL PLOT(N,MAX)
            CALL PRTPLT(H,E,MAX,TITLE,NABS,NORD,0.,0.,100)
            GO TO 155
        140 KL=I+1
            WRITE(6,150) KL,ERROR(KL)
        150 FORMAT(/,/,/,/,1X44HDID NOT CONVERGE IN 50 ITERATIONS WHILE CALC,
           110HULATING E(,I5,25H)  -  CONVERGENCE ERROR: ,1PE11.4)
        155 READ(5,160) IK
        160 FORMAT(9X,I1)
            GO TO (10,170),IK
        170 CONTINUE
            END
```

```
           PROGRAM HAMMING    74/74   OPT=1                         FTN 4.5+414        06/04/76  13.30.02        PAGE     3
CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM
   112     I                  AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.


         SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS       DEF LINE     REFERENCES
 4111  HAMMING          1

VARIABLES     SN  TYPE              RELOCATION
11610  CF         REAL      ARRAY      B          REFS          5         84   DEFINED        15        21        28
35230  DEL        REAL                 A          REFS          4         29        66   DEFINED        13
    0  E          REAL      ARRAY      A          REFS          4         22        55        59        84       104
                                               DEFINED         54
 4645  ECOR       REAL                            REFS         53         54
42152  ECOREC     REAL      ARRAY                 REFS         10         36        37      2*42   DEFINED        31
42246  ECORECP    REAL      ARRAY                 REFS         10         36      2*42      2*45        53
                                               DEFINED         32         37
23420  EP         REAL      ARRAY      A          REFS          4    DEFINED        22        55
 4643  EPDIC      REAL                            REFS         27         30        31        42        59
 4644  EPDICP     REAL                            REFS         32    DEFINED        30
 4637  EPSLON     REAL                            REFS         15         21        28        66   DEFINED        13
16532  ERROR      REAL      ARRAY                 REFS          8        107   DEFINED        45
35232  ESTART     REAL                 A          REFS          4    DEFINED        13
 4633  FLAG       INTEGER                         REFS          2         42        44        72
11610  H          REAL      ARRAY      A          REFS          4         22        29        30        37        55        63
                                                    84        104    DEFINED        29
 4642  I          INTEGER                         REFS         27         28      2*29        30        36        37        42
                                                    45         49         53        54      4*55      2*59        60        61
                                                    63        106    DEFINED        23        61
 4651  IK         INTEGER                         REFS        112    DEFINED       110
 4635  IT         INTEGER                         REFS         42    DEFINED        12
 4641  J          INTEGER                         REFS         21       4*22      8*84   DEFINED        20        83
 4676  K          INTEGER   ARRAY                 REFS          7         84   DEFINED        49
 4650  KL         INTEGER                         REFS      2*107    DEFINED       106
 4634  L          INTEGER                         REFS         31         32      2*36      2*37        38      5*42      2*45
                                                    46         47         49        53        72   DEFINED        11        47
                                                    62
 4640  M          INTEGER                         REFS         66         83   DEFINED        13
 4647  MAX        INTEGER                         REFS        103        104
 4646  N          INTEGER                         REFS         83        103   DEFINED        60
 4664  NABS       REAL      ARRAY                 REFS          3        104   DEFINED        98        99       100
 4652  NORD       REAL      ARRAY                 REFS          3        104   DEFINED        95        96        57
    0  RME        REAL      ARRAY      C          REFS          6         84
11610  SME        REAL      ARRAY      C          REFS          6         84
35231  START      REAL                 A          REFS          4         66   DEFINED        13
30342  TERR       REAL      ARRAY                 REFS          9         84   DEFINED        59
16506  TITLE      REAL      ARRAY                 REFS          7        104   DEFINED        90        91        92        93
                                                    94
 4636  TMAX       REAL                            REFS         63         66   DEFINED        13
    0  VAL        REAL      ARRAY      B          REFS          5         84
```

APPENDIX A

```
        PROGRAM HAMMING    74/74    OPT=1                    FTN 4.5+414       06/04/76  13.30.02       PAGE     4

FILE NAMES         MODE
     0  INPUT
  2041  OUTPUT
     0  TAPE5      FMT                    READS        13       110
  2041  TAPE6      FMT                    WRITES       64        66        73        75        79        84       107

EXTERNALS          TYPE    ARGS        REFERENCES
        CORRECT             3              36          53
        EROR                9              42
        FET        REAL     3              22          30        37        55
        PLOT                2             103
        PREDICT             2              27
        PRTPLT              9             104
        RUNGE               0              19

INLINE FUNCTIONS   TYPE    ARGS          DEF LINE   REFERENCES
        ABS        REAL     1  INTRIN                  45

STATEMENT LABELS             DEF LINE    REFERENCES
  4312  10                      11          112
  4425  20      FMT             14           13
     0  30                      22           20
  4134  40                      27           63
  4150  50                      36           48
     0  53         INACTIVE     45           44
  4200  55                      47           38
  4202  60                      49         2*44
  4226  65                      64           46
  4433  70      FMT             65           64
  4445  80      FMT             67           66
  4473  90      FMT             74           73
  4503  100     FMT             76           75
  4524  110     FMT             80           79
  4555  120     FMT             85           84
     0  130                     87           83
  4310  140                    106           72
  4572  150     FMT            108          107
  4316  155                    110          105
  4613  160     FMT            111          110
  4327  170                    113          112

LOOPS   LABEL    INDEX      FROM-TO     LENGTH     PROPERTIES
  4122  30      * J          20 22        11B                 EXT REFS
  4244  130     * J          83 87        23B                 EXT REFS

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
        A         15003             0 E      (5000)            5000 H      (5000)           10000 EP     (5000)
                                15000 DEL    (1)              15001 START  (1)              15002 ESTART (1)
        B         10000             0 VAL    (5000)            5000 CF     (5000)
        C         10000             0 PNE    (5000)            5000 SNE    (5000)

STATISTICS
  PROGRAM LENGTH                362378    15519
  BUFFER LENGTH                  41038     2115
  CM LABELED COMMON LENGTH     1042738    35003
```

APPENDIX A

```
SUBROUTINE RUNGE      74/74   OPT=1                    FTN 4.5+414        06/04/76  13.30.02       PAGE      1

      1                  SUBROUTINE RUNGE
                         COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                         H(1)=START
                         E(1)=ESTART
      5                  I=1
                      10 R1=FET(H(I),E(I),0)*DEL
                         T2=H(I)+.5*DEL
                         E2=E(I)+.5*R1
                         R2=FET(T2,E2,0)*DEL
     10                  E3=E(I)+.5*R2
                         R3=FET(T2,E3,0)*DEL
                         T4=H(I)+DEL
                         E4=E(I)+R3
                         R4=FET(T4,E4,0)*DEL
     15                  DELE=(R1+2.*R2+2.*R3+R4)/6.
                         E(I+1)=E(I)+DELE
                         I=I+1
                         H(I)=H(I-1)+DEL
                         IF(I.GT.3)  RETURN
     20                  GO TO 10
                         END


          SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    1  RUNGE          1            19

VARIABLES     SN  TYPE              RELOCATION
35230  DEL        REAL                  A           REFS        2         6         7         9        11        12        14
                                                     18
  113  DELE       REAL                              REFS       16    DEFINED       15
    0  E          REAL        ARRAY     A           REFS        2         6         8        10        13        16
                                                 DEFINED        4        16
23420  EP         REAL        ARRAY     A           REFS        2
35232  ESTART     REAL                  A           REFS        2         4
  104  E2         REAL                              REFS        9    DEFINED        8
  106  E3         REAL                              REFS       11    DEFINED       10
  111  E4         REAL                              REFS       14    DEFINED       13
11610  H          REAL        ARRAY     A           REFS        2         6         7        12        18
                                                 DEFINED        3        18
  101  I          INTEGER                           REFS      2*6         7         8        10        12        13      2*16
                                                     17      2*18        19   DEFINED         5        17
  102  R1         REAL                              REFS        8        15   DEFINED         6
  105  R2         REAL                              REFS       10        15   DEFINED         9
  107  R3         REAL                              REFS       13        15   DEFINED        11
  112  R4         REAL                              REFS       15    DEFINED       14
35231  START      REAL                  A           REFS        2         3
  103  T2         REAL                              REFS        9        11   DEFINED         7
  110  T4         REAL                              REFS       14    DEFINED       12

EXTERNALS          TYPE     ARGS         REFERENCES
       FET         REAL       3              6         9        11        14
```

```
      SUBROUTINE RUNGE      74/74   OPT=1                      FTN 4.5+414       06/04/76  13.30.02       PAGE     2

STATEMENT LABELS             DEF LINE    REFERENCES
    6  10                        6           20

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
      A          15003                   0 E      (5000)           5000 H      (5000)          10000 EP     (5000)
                                    15000 DEL     (1)             15001 START  (1)             15002 ESTART (1)

STATISTICS
   PROGRAM LENGTH                    114B       76
   CM LABELED COMMON LENGTH       35233B     15003
```

```
SUBROUTINE PREDICT     74/74    OPT=1                          FTN 4.5+414        06/04/76  13.30.02        PAGE     1

1                 SUBROUTINE PREDICT(I,EPDIC)
                  COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                  V=2.*EP(I)-EP(I-1)+2.*EP(I-2)
                  EPDIC=E(I-3)+((4.*DEL)/3.)*V
5                 RETURN
                  END


          SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS       DEF LINE      REFERENCES
    3 PREDICT          1              5

VARIABLES      SN  TYPE                RELOCATION
35230 DEL          REAL                  A               REFS          2          4
    0 E            REAL         ARRAY    A               REFS          2          4
23420 EP           REAL         ARRAY    A               REFS          2          3*3
    0 EPDIC        REAL                      F.P.    DEFINED           1          4
35232 ESTART       REAL                  A               REFS          2
11610 H            REAL         ARRAY    A               REFS          2
    0 I            INTEGER                   F.P.        REFS          3*3        4     DEFINED          1
35231 START        REAL                  A               REFS          2
   22 V            REAL                                  REFS          4     DEFINED          3

COMMON BLOCKS    LENGTH       MEMBERS - BIAS NAME(LENGTH)
    A             15003                0 E        (5000)               5000 H      (5000)              10000 EP     (5000)
                                   15000 DEL      (1)                 15001 START  (1)                 15002 ESTART (1)

STATISTICS
  PROGRAM LENGTH                   23B      19
  CM LABELED COMMON LENGTH      35233B   15003
```

APPENDIX A

```
SUBROUTINE CORRECT     74/74   OPT=1                         FTN 4.5+414       06/04/76  13.30.02        PAGE     1

1                  SUBROUTINE CORRECT(I,EDICP,ECREC)
                   COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                   V1=(9.*E(I)-E(I-2))/8.
                   V2=EDICP+2.*EP(I)-EP(I-1)
5                  ECREC=V1+((3.*DEL)/8.)*V2
                   RETURN
                   END


     SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    3  CORRECT        1           6

VARIABLES      SN  TYPE            RELOCATION
35230  DEL         REAL                 A              REFS       2        5
    0  E           REAL        ARRAY    A              REFS       2       2*3
    0  ECREC       REAL                   F.P.      DEFINED       1        5
    0  EDICP       REAL                   F.P.         REFS       4     DEFINED        1
23420  EP          REAL        ARRAY    A              REFS       2       2*4
35232  ESTART      REAL                 A              REFS       2
11610  H           REAL        ARRAY    A              REFS       2
    0  I           INTEGER                F.P.         REFS      2*3      2*4    DEFINED        1
35231  START       REAL                 A              REFS       2
   26  V1          REAL                                REFS       5     DEFINED        3
   27  V2          REAL                                REFS       5     DEFINED        4

COMMON BLOCKS   LENGTH     MEMBERS - BIAS NAME(LENGTH)
    A           15003             0 E       (5000)          5000 H      (5000)            10000 EP     (5000)
                              15000 DEL     (1)            15001 START  (1)               15002 ESTART (1)

STATISTICS
  PROGRAM LENGTH                  30B        24
  CM LABELED COMMON LENGTH     35233B     15003
```

```
SUBROUTINE ERDR       74/74   OPT=1                          FTN 4.5+414      06/04/76  13.30.02       PAGE     1

                SUBROUTINE ERDR(I,L,EPDIC,P,C,PP,CP,FLAG,IT)
                COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                COMMON/B/VAL(5000),CF(5000)
                INTEGER FLAG
                B=3./8.
                DELTA=ABS(CP-PP)
             10 IF(DELTA.LE.CF(I+1))  GO TO 20
                IF(L.LE.10)  GO TO 15
                CF(I+1)=DELTA*CF(I+1)
             15 FLAG=-1
                RETURN
             20 ARG=C-P
                IF(ARG.EQ.0.)  ARG=1.0E-5
                RK=DELTA/ABS(ARG)
                VAL(I+1)=(DEL**2)*(B**2)*DELTA*RK
                CALL CORRECT(I,PP,C)
                EC=C
                ERR=(9./121.)*(EC-EPDIC)
                IF(VAL(I+1).GT.ABS(ERR))  GO TO 40
                FLAG=0
                RETURN
             40 IP1=I+1
                IF(IT.GT.0)  GO TO 80
                WRITE(6,60)
             60 FORMAT(1H1)
                WRITE(6,70)
             70 FORMAT(/,/,/,1X,40HTHE ERROR INCURRED BY NOT ITERATING THE ,
               161HCORRECTOR TO CONVERGENCE IS GREATER THAN THE TRUNCATION ERROR,
               2/,16X,12HERROR BY NOT,/,16X,12HITERATING TO,5X,10HTRUNCATION,
               3/,6X,4HE(I),6X,11HCONVERGENCE,6X,5HERROR)
                IT=1
             80 WRITE(6,90) IP1,VAL(I+1),ERR
             90 FORMAT(5X,2HE(,I4,1H),5X,1PE11.4,5X,1PE11.4)
                FLAG=1
                RETURN
                END

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
   3 ERDR             1             11        21        35

VARIABLES       SN  TYPE            RELOCATION
   162 ARG          REAL                            REFS     13       14   DEFINED      12       13
   160 B            REAL                            REFS     15  DEFINED         5
     0 C            REAL                   F.P.     REFS     12       16        17  DEFINED         1
 11610 CF           REAL        ARRAY     B         REFS      3        7         9  DEFINED         9
     0 CP           REAL                   F.P.     REFS      6  DEFINED         1
 35230 DEL          REAL                  A         REFS      2       15
   161 DELTA        REAL                            REFS      7        9        14       15  DEFINED        6
     0 E            REAL        ARRAY     A         REFS      2
   164 EC           REAL                            REFS     18  DEFINED        17
 23420 EP           REAL        ARRAY     A         REFS      2
```

```
      SUBROUTINE EROR        74/74    OPT=1                                       FTN 4.5+414         06/04/76  13.30.02        PAGE      2

VARIABLES      SN  TYPE              RELOCATION
    0  EPDIC       REAL                      F.P.          REFS          18    DEFINED          1
  165  ERR         REAL                                    REFS          19         32   DEFINED          18
35232  ESTART      REAL                   A                REFS           2
    0  FLAG        INTEGER                   F.P.          REFS           4    DEFINED          1          10          20          34
11610  M           REAL          ARRAY    A                REFS           2
    0  I           INTEGER                   F.P.          REFS           7         29         15          16          19          22          32
                                                       DEFINED            1
  166  IP1         INTEGER                                 REFS          32    DEFINED         22
    0  IT          INTEGER                   F.P.          REFS          23    DEFINED          1          31
    0  L           INTEGER                   F.P.          REFS           8    DEFINED          1
    0  P           REAL                      F.P.          REFS          12    DEFINED          1
    0  PP          REAL                      F.P.          REFS           6         16   DEFINED           1
  163  RK          REAL                                    REFS          15    DEFINED         14
35231  START       REAL                   A                REFS           2
    0  VAL         REAL          ARRAY    B                REFS           3         19         32   DEFINED          15

FILE NAMES         MODE
       TAPE6       FMT                          WRITES          24         26         32

EXTERNALS               TYPE   ARGS         REFERENCES
       CORRECT                    3             16

INLINE FUNCTIONS    TYPE   ARGS          DEF LINE   REFERENCES
       ABS          REAL      1  INTRIN                 6          14         19

STATEMENT LABELS               DEF LINE   REFERENCES
    0  10          INACTIVE          7
   17  15                           10          8
   21  20                           12          7
   52  40                           22         19
  101  60         FMT               25         24
  106  70         FMT               27         26
   63  80                           32         23
  142  90         FMT               33         32

COMMON BLOCKS     LENGTH      MEMBERS - BIAS NAME(LENGTH)
       A          15003                  0 E      (5000)              5000 M       (5000)           10000 EP      (5000)
                                     15000 DEL    (1)                15001 START   (1)              15002 ESTART  (1)
       B          10000                  0 VAL    (5000)              5000 CF      (5000)

STATISTICS
   PROGRAM LENGTH                   167B        119
   CM LABELED COMMON LENGTH      60653B      25003
```

```
SUBROUTINE PLOT       74/74   OPT=1                        FTN 4.5+414      06/04/76  13.30.02      PAGE    1

                      SUBROUTINE PLOT(N,MAX)
                      COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                      RT=N/100.
                      IDEL=INT(RT)
                      IF(IDEL.EQ.0)  IDEL=1
                      L=1
                      DO 10 I=1,N,IDEL
                      H(L)=H(I)
                      E(L)=E(I)
                      L=L+1
                   10 CONTINUE
                      MAX=L
                      RETURN
                      END


    SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    3  PLOT          1           13

VARIABLES     SN  TYPE            RELOCATION
35230  DEL        REAL                A         REFS      2
    0  E          REAL       ARRAY    A         REFS      2        9   DEFINED     9
23420  EP         REAL       ARRAY    A         REFS      2
35232  ESTART     REAL                A         REFS      2
11610  H          REAL       ARRAY    A         REFS      2        8   DEFINED     8
   34  I          INTEGER                       REFS      8        9   DEFINED     7
   32  IDEL       INTEGER                       REFS      5        7   DEFINED     4       5
   33  L          INTEGER                       REFS      8        9       10     12   DEFINED      6      10
    0  MAX        INTEGER             F.P.    DEFINED     1       12
    0  N          INTEGER             F.P.      REFS      3        7   DEFINED     1
   31  RT         REAL                          REFS      4   DEFINED       3
35231  START      REAL                A         REFS      2

INLINE FUNCTIONS    TYPE     ARGS        DEF LINE   REFERENCES
    INT            INTEGER     1  INTRIN                 4

STATEMENT LABELS              DEF LINE    REFERENCES
    0  10                        11           7

LOOPS  LABEL    INDEX    FROM-TO    LENGTH    PROPERTIES
  17   10         I        7 11       4B       INSTACK

COMMON BLOCKS   LENGTH     MEMBERS - BIAS NAME(LENGTH)
    A           15003             0 E       (5000)          5000 H      (5000)          10000 EP      (5000)
                              15000 DEL     (1)            15001 START  (1)            15002 ESTART  (1)

STATISTICS
  PROGRAM LENGTH                 35B       29
  CM LABELED COMMON LENGTH    35233B    15003
```

```
      FUNCTION FET        74/74    OPT=1                              FTN 4.5+414        06/04/76  13.30.02        PAGE     1

  1                 FUNCTION FET(TI,ET,K)
                    REAL JC,MUP,NE
                    R=5.0E+8
                    EPS=8.85E-12
  5                 Q=1.6E-19
                    MUP=2.0
                    IF(TI.GT.0.)  GO TO 10
            C  NE(T) IS MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                    FET=(JC(TI)-1.E+6*NE(TI,K)*Q*MUP*ET*EXP(R*TI))/EPS
 10                 RETURN
            C  NE(T) IS MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
               10   FET=(JC(TI)-1.E+6*NE(TI,K)*Q*MUP*ET)/EPS
                    RETURN
                    END


      SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS       DEF LINE      REFERENCES
     4  FET              1          10        13

 VARIABLES     SN  TYPE            RELOCATION
    73  EPS        REAL                          REFS         9        12     DEFINED        4
     0  ET         REAL              F.P.        REFS         9        12     DEFINED        1
    70  FET        REAL                       DEFINED         9        12
     0  K          INTEGER           F.P.        REFS         9        12     DEFINED        1
    71  MUP        REAL                          REFS         2         9        12     DEFINED        6
    74  Q          REAL                          REFS         9        12     DEFINED        5
    72  R          REAL                          REFS         9    DEFINED        3
     0  TI         REAL              F.P.        REFS         7       3*9      2*12     DEFINED        1

 EXTERNALS          TYPE   ARGS       REFERENCES
        EXP         REAL     1 LIBRARY       9
        JC          REAL     1               2         9        12
        NE          REAL     2               2         9        12

 STATEMENT LABELS             DEF LINE    REFERENCES
    36  10                       12           7

 STATISTICS
   PROGRAM LENGTH              75B     61
```

```
      FUNCTION NE         74/74    OPT=1                         FTN 4.5+414        06/04/76  13.30.02        PAGE     1

   1                  REAL FUNCTION NE(T,K)
                      COMMON/C/RNE(5000),SNE(5000)
                      REAL NEO
                      NEO=1.133E+11
   5                  S=5.664E+19
                      IF(T.GE.0.)  GO TO 10
                      NE=1.133E+11
                      RNE(K)=0.
                      SNE(K)=0.
  10                  RETURN
               10     NE=NEO+S*T
                      IF(K.EQ.0)  RETURN
                      SNE(K)=NE
                      RNEO=1.133E+11
  15                  RS=1.022E+20
                      DELTA=1.0008E+8
                      A=4.E-7
                      ARG=A*RS*(T**2)
                      X=.5*ARG
  20                  Y=DELTA*T
                      Z=-X-Y
                      F1=RS/(2.*DELTA)
                      F2=1.+EXP(X)
                      F3=EXP(Y)-1.
  25                  F4=EXP(Z)
                      RNE(K)=(F1*F2*F3+RNEO)*F4
                      RETURN
                      END


     SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS     DEF LINE      REFERENCES
     4  NE            1            10         12         27

 VARIABLES     SN  TYPE             RELOCATION
    70  A          REAL                               REFS        18    DEFINED      17
    71  ARG        REAL                               REFS        19    DEFINED      18
    67  DELTA      REAL                               REFS        20         22   DEFINED      16
    75  F1         REAL                               REFS        26    DEFINED      22
    76  F2         REAL                               REFS        26    DEFINED      23
    77  F3         REAL                               REFS        26    DEFINED      24
   100  F4         REAL                               REFS        26    DEFINED      25
     0  K          INTEGER              F.P.          REFS         8          9      12      13       26
                                                   DEFINED         1
    62  NE         REAL                               REFS        13    DEFINED       7      11
    63  NEO        REAL                               REFS         3         11   DEFINED       4
     0  RNE        REAL        ARRAY    C             REFS         2    DEFINED       8      26
    65  RNEO       REAL                               REFS        26    DEFINED      14
    66  RS         REAL                               REFS        18         22   DEFINED      15
    64  S          REAL                               REFS        11    DEFINED       5
 11610  SNE        REAL        ARRAY    C             REFS         2    DEFINED       9      13
     0  T          REAL                 F.P.          REFS         6         11      18      20   DEFINED       1
    72  X          REAL                               REFS        21         23   DEFINED      19
```

```
      FUNCTION NE          74/74   OPT=1                                FTN 4.5+414        06/04/76  13.30.02         PAGE      2

VARIABLES      SN  TYPE            RELOCATION
   73  Y           REAL                              REFS        21         24   DEFINED        20
   74  Z           REAL                              REFS        25    DEFINED         21

EXTERNALS          TYPE    ARGS          REFERENCES
       EXP         REAL      1 LIBRARY       23          24          25

STATEMENT LABELS               DEF LINE    REFERENCES
   15  10                         11           6

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
       C          10000                 0 RNE     (5000)           5000 SNE    (5000)

STATISTICS
  PROGRAM LENGTH                     101B        65
  CM LABELED COMMON LENGTH         23420B     10000
```

```
FUNCTION JC         74/74   OPT=1                   FTN 4.5+414     06/04/76  13.30.02      PAGE    1

               REAL FUNCTION JC(T)
               R=5.E+8
               IF(T.GT.0.)  GO TO 10
               JC=1.34E+3*EXP(R*T)
               RETURN
            10 JC=1.34E+3
               RETURN
               END


     SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    4  JC            1            5        7

VARIABLES      SN  TYPE           RELOCATION
   20  JC          REAL                         DEFINED      4        6
   21  R           REAL                           REFS       4    DEFINED      2
    0  T           REAL                 F.P.      REFS       3        4    DEFINED      1

EXTERNALS          TYPE   ARGS        REFERENCES
       EXP         REAL      1 LIBRARY      4

STATEMENT LABELS              DEF LINE    REFERENCES
   14  10                         6           3

STATISTICS
  PROGRAM LENGTH                  22B         18
```

APPENDIX A

```
SUBROUTINE PRTPLT      74/74    OPT=1                        FTN 4.6+420          10/27/76  09.31.03          PAGE      1

          SUBROUTINE PRTPLT (HORZ,VERT,NPTS,HEAD,HLAB,VLAB,HREF,VREF,J)
C
C         THIS FORTRAN SUBROUTINE PRODUCES A FULL PAGE X-Y PRINTER PLOT.
C
C         HORZ - REAL ARRAY TO BE PLOTTED ALONG HORIZONTAL AXIS.
C         VERT - REAL ARRAY TO BE PLOTTED ALONG VERTICAL AXIS.
C         NPTS - NUMBER OF VALUES IN HORZ,VERT (NTE 101)
C         HEAD - PLOT HEADING (MUST BE 5 WORDS)
C         HLAB - LABEL FOR HORIZONTAL AXIS (MUST BE 3 WORDS)
C         VLAB - LABEL FOR VERTICAL AXIS (MUST BE 3 WORDS)
C         HREF - REFERENCE VALUE FOR HORIZONTAL AXIS.
C         VREF - REFERENCE VALUE FOR VERTICAL AXIS.
C         J  THIS DETERMINES THE SIZE OF THE ARRAYS HORZ AND VERT--NTE 100
C
          DIMENSION HORZ(J),VERT(J),HEAD(5),HLAB(3),VLAB(3),LINE(101),
         C IVER(101),JHOR(101),HORIZ(101),VERTIC(101),YAXIS(30),DOWNT(21)
          INTEGER BLANK,STAR,DOT,EYE
C
C
          DATA BLANK/1H /,STAR/1H*/,DOT/1H./,MINUS/1H-/,EYE/1HI/
          CALL DATE(DAT)
          CALL TIME(TIM)
          HEAD(4)=DAT
          HEAD(5)=TIM
          HO = 1
          NUM = 1
          N = NPTS
          N = MINO(N,101)
          DO 10 I = 1,N
          HORIZ(I) = HORZ(I)
          VERTIC(I) = VERT(I)
       10 CONTINUE
C
          DECODE (30,1000,VLAB(1)) (YAXIS(I),I=1,30)
     1000 FORMAT(30A1)
       20 NA = N - 1
          DO 30 I = 1,NA
          IF (VERTIC(I).GE.VERTIC(I+1)) GO TO 30
          HOLD = VERTIC(I)
          VERTIC(I) = VERTIC(I+1)
          VERTIC(I+1) = HOLD
          SAVE = HORIZ(I)
          HORIZ(I) = HORIZ(I+1)
          HORIZ(I+1) = SAVE
       30 CONTINUE
          DO 40 I = 1,NA
          IF (VERTIC(I).LT.VERTIC(I+1)) GO TO 20
       40 CONTINUE
          VMAX = VERTIC(1)
          VMIN = VERTIC(N)
          HMAX = HORIZ(1)
          HMIN = HORIZ(1)
          DO 50 I = 2,N
```

```
SUBROUTINE PRTPLT      74/74    OPT=1                    FTN 4.6+420         10/27/76  09.31.03         PAGE      2

          HZ = HORIZ(I)
          HMAX = AMAX1(HMAX,HZ)
          HMIN = AMIN1(HMIN,HZ)
     50 CONTINUE
          DO 60 I = 1,N
          JVER = 50.*(VERTIC(I)-VMIN)/(VMAX-VMIN) + 1.5
          IVER(I) = 52 - JVER
     60 JHOR(I) = 100.*(HORIZ(I)-HMIN)/(HMAX-HMIN) + 1.5
          KHREF = 100.*(HREF-HMIN)/(HMAX-HMIN) + 1.5
          IF (KHREF.LT.1) KHREF = 150
          IF (KHREF.GT.101) KHREF = 150
          KVREF = 50.*(VREF-VMIN)/(VMAX-VMIN) + 1.5
          KVREF = 52 - KVREF
          IF (KVREF.LT.1) KVREF = 1
          IF (KVREF.GT.51) KVREF = 1
          UP = (VMAX-VMIN)/50.
          PRINT 2000 , HEAD
   2000 FORMAT(1H1,35X,5A10//)
          DO 200 K = 1,51
          LINE(1) = EYE
          LINE(101) = EYE
          DO 70 I = 2,100
     70 LINE(I) = BLANK
          IF (K.EQ.1) GO TO 90
          IF (K.EQ.51) GO TO 90
          IF (K.NE.KVREF) GO TO 120
          DO 80 I = 2,100
     80 LINE(I) = DOT
          GO TO 120
     90 DO 100 I = 2,100
    100 LINE(I) = MINUS
          DO 110 I = 6,100,5
    110 LINE(I) = EYE
          GO TO 130
    120 IF (KHREF.EQ.150) GO TO 130
          IF (KHREF.EQ.1) GO TO 130
          LINE(KHREF) = DOT
    130 IF (K.EQ.IVER(NUM)) GO TO 140
          IF (K.LT.IVER(NUM)) GO TO 160
          NUM = NUM + 1
          GO TO 130
    140 IF (JHOR(NUM).GT.101) GO TO 150
          IF (JHOR(NUM).LT.1) GO TO 150
          JH = JHOR(NUM)
          LINE(JH) = STAR
    150 NUMBER = NUM + 1
          IF (IVER(NUM).NE.IVER(NUMBER)) GO TO 160
          NUM = NUMBER
          GO TO 140
    160 T = K - 1
          UPT = VMAX - T*UP
          IF (K.LT.11) GO TO 170
          IF (K.GT.41) GO TO 170
```

```
SUBROUTINE PRTPLT      74/74    OPT=1                    FTN 4.6+420        10/27/76  09.31.03        PAGE      3

                  TAXIS = YAXIS(MQ)
                  MQ = MQ + 1
                  PRINT 3000 , TAXIS,UPT,LINE
             3000 FORMAT(4X,A1,4X,1PE9.2,2X,101A1)
                  GO TO 200
              170 PRINT 4000 , UPT,LINE
             4000 FORMAT(9X,1PE9.2,2X,101A1)
              200 CONTINUE
                  DOWN = (HMAX-HMIN)/20.
                  T = HMIN
                  DO 210 I = 1,21
                  DOWNT(I) = T
                  T = T + DOWN
              210 CONTINUE
                  PRINT 5000 , (DOWNT(I),I=1,21,2)
             5000 FORMAT(/15X,1P11E10.2)
                  PRINT 6000 , (DOWNT(I),I=2,20,2)
             6000 FORMAT(20X,1P10E10.2)
                  PRINT 7000 , HLAB
             7000 FORMAT(/50X,3A10)
                  RETURN
                  END
```

APPENDIX B.--COMPUTER CODE EXPDIFF

This appendix contains a complete listing of EXPDIFF.

```
PROGRAM EXPDIFF    74/74   OPT=1                    FTN 4.5+414        06/04/76  13.30.36        PAGE     1

          PROGRAM EXPDIFF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
          REAL JC,MU,NE,NORD(3),NABS(3)
          COMMON/A/R,EPS,Q,MU,S
          COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
          DIMENSION TITLE(5)
          S=5.664E+19
          R=5.0E+8
          EPS=8.85E-12
          Q=1.6E-19
          MU=2.0
   10     CONTINUE
          I=1
          READ(5,20) DEL,TSTART,TMAX,ESTART,N,J
   20     FORMAT(4E10.3,I5,I5)
          T(1)=TSTART
          E(1)=ESTART
          TERR(1)=0.
          IF(J.NE.1)  GO TO 30
          T(1)=T(N)
          E(1)=E(N)
          TSTART=T(1)
          ESTART=E(N)
   30     CONTINUE
          IF(T(I).GT.TMAX)  GO TO 70
          IF(T(I).GE.0.)  GO TO 50
          CALL FCALC(I,F,FT,FE,FTT,FTE)
          EDP=FT+F*FE
          DELTA(I)=ABS(DEL*FE)
          IF(DELTA(I).LT..1)  GO TO 40
          X=EXP(DEL*FE)-1.-DEL*FE
          E(I+1)=E(I)+DEL*F+EDP*(FE**(-2))*X
          TERR(I+1)=((DEL**3)*(FTT+2.*F*FTE))/6.
          T(I+1)=T(I)+DEL
          I=I+1
          GO TO 30
   40     S1=(DEL**2)/2.
          S2=((DEL**3)*FE)/6.
          S3=((DEL**4)*(FE**2))/24.
          E(I+1)=E(I)+DEL*F+EDP*(S1+S2+S3)
          TERR(I+1)=(DEL**3)*(FTT+2.*F*FTE)/6.
          T(I+1)=T(I)+DEL
          I=I+1
          GO TO 30
   50     CALL GCALC(I,G,GT,GE,GTE)
          EDP=GT+G*GE
          DELTA(I)=ABS(DEL*GE)
          IF(DELTA(I).LT..1)  GO TO 60
          X=EXP(DEL*GE)-1.-DEL*GE
          E(I+1)=E(I)+DEL*G+EDP*(GE**(-2))*X
          TERR(I+1)=(DEL**3)*(2.*G*GTE)/6.
          T(I+1)=T(I)+DEL
          I=I+1
          GO TO 30
   60     S1=(DEL**2)/2.
          S2=((DEL**3)*GE)/6.
          S3=((DEL**4)*(GE**2))/24.
          E(I+1)=E(I)+DEL*G+EDP*(S1+S2+S3)
```

```
                TERR(I+1)=(DEL**3)*(2.*G*GTE)/6.
                T(I+1)=T(I)+DEL
                I=I+1
                GO TO 30
             70 CONTINUE
                N=I-1
                CALL OTWRIT(N,M,DEL,TSTART,TMAX)
          C
          C
                TITLE(1)=10HEMP TIME D
                TITLE(2)=10HEPENDENCY
                TITLE(3)=5HSTUDY
                TITLE(4)=1H
                TITLE(5)=1H
                NORD(1)=10HELECTRIC F
                NORD(2)=10HIELD STREN
                NORD(3)=10HGTH  V/M
                NABS(1)=4HTIME
                NABS(2)=1H
                NABS(3)=1H
          C
          C
                CALL PLOT(N,MAX)
                CALL PRTPLT(T,E,MAX,TITLE,NABS,NORD,0.,0.,100)
                READ(5,80) IK
             80 FORMAT(9X,I1)
                GO TO (10,90),IK
             90 STOP
                END
```

```
CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM

     84    I                 AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.
```

```
         SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS      DEF LINE      REFERENCES
  4111  EXPDIFF        1

 VARIABLES      SN  TYPE            RELOCATION
  4455  DEL         REAL                          REFS       28      2*30       31        32        33        36        37
                                                    30       39        40       41        46      2*48        49        50
                                                    51       54        55       56        57        58        59        64
                                                DEFINED      13
 23420  DELTA       REAL       ARRAY    B         REFS        4        29       47   DEFINED        28        46
 11610  E           REAL       ARRAY    B         REFS        4        20       22        31        39        49        57
                                                    81  DEFINED        16       20        31        39        49        57
  4471  EDP         REAL                          REFS       31        39       49        57   DEFINED        27        45
     1  EPS         REAL                A         REFS        3   DEFINED        8
  4460  ESTART      REAL                          REFS       16   DEFINED       13        22
```

```
      PROGRAM EXPDIFF     74/74   OPT=1                          FTN 4.5+414      06/04/76  13.30.36       PAGE     3

VARIABLES       SN  TYPE            RELOCATION
 4464  F             REAL                          REFS        26        27        31        32        39        40
 4466  FE            REAL                          REFS        26        27        28      2*30        31        37        38
 4465  FT            REAL                          REFS        26        27
 4470  FTE           REAL                          REFS        26        32        40
 4467  FTT           REAL                          REFS        26        32        40
 4476  G             REAL                          REFS        44        45        49        50        57        58
 4500  GE            REAL                          REFS        44        45        46      2*48        49        55        56
 4477  GT            REAL                          REFS        44        45
 4501  GTE           REAL                          REFS        44        50        58
 4454  I             INTEGER                       REFS        24        25        26        28        29      2*31        32
                                                 2*33        34      2*39        40      2*41        42        44        46
                                                   47      2*49        50      2*51        52      2*57        58      2*59
                                                   60        63   DEFINED        12        34        42        52        60
 4503  IK            INTEGER                       REFS        84   DEFINED        82
 4462  J             INTEGER                       REFS        18   DEFINED        13
 4452  JC         *  REAL          *UNDEF          REFS         2
 4461  M             INTEGER                       REFS        64   DEFINED        13
 4502  MAX           INTEGER                       REFS        80        81
    3  NU            REAL                   A      REFS         2         3   DEFINED        10
 4463  N             INTEGER                       REFS        19        20        22        64        80
                                                DEFINED        63
 4507  NABS          REAL          ARRAY           REFS         2        81   DEFINED        75        76        77
 4453  NE         *  REAL          *UNDEF          REFS         2
 4504  NORD          REAL          ARRAY           REFS         2        81   DEFINED        72        73        74
    2  Q             REAL                   A      REFS         3   DEFINED         9
    0  R             REAL                   A      REFS         3   DEFINED         7
    4  S             REAL                   A      REFS         3   DEFINED         6
 4473  S1            REAL                          REFS        39        57   DEFINED        36        54
 4474  S2            REAL                          REFS        39        57   DEFINED        37        55
 4475  S3            REAL                          REFS        39        57   DEFINED        38        56
    0  T             REAL          ARRAY    B      REFS         4        19        21        24        25        33        41
                                                   51        59        81   DEFINED        15        19        33        41
                                                   51        59
35230  TERR          REAL          ARRAY    B      REFS         4   DEFINED        17        32        40        50        58
 4512  TITLE         REAL          ARRAY           REFS         5        81   DEFINED        67        68        69        70
                                                   71
 4457  TMAX          REAL                          REFS        24        64   DEFINED        13
 4456  TSTART        REAL                          REFS        15        64   DEFINED        13        21
 4472  X             REAL                          REFS        31        49   DEFINED        30        48

FILE NAMES          MODE
    0  INPUT
 2041  OUTPUT
    0  TAPE5        FMT                   READS        13        82
 2041  TAPE6

EXTERNALS            TYPE     ARGS         REFERENCES
       EXP           REAL       1  LIBRARY      30        48
       FCALC                    6               26
       GCALC                    5               44
       DTWRIT                   5               64
       PLOT                     2               80
       PRTPLT                   9               81

INLINE FUNCTIONS     TYPE     ARGS         DEF LINE  REFERENCES
       ABS           REAL       1  INTRIN               28        46
```

APPENDIX B

APPENDIX B

```
        PROGRAM EXPDIFF     74/74   OPT=1                        FTN 4.5+414       06/04/76  13.30.36       PAGE     4

STATEMENT LABELS                DEF LINE    REFERENCES
 4121  10                           11          04
 4412  20         FMT               14          13
 4136  30                           23          18          35          43          53          61
 4175  40                           36          29
 4223  50                           44          25
 4256  60                           54          47
 4302  70                           62          24
 4422  80         FMT               83          82
 4337  90                           85          84

COMMON BLOCKS   LENGTH      MEMBERS   BIAS NAME(LENGTH)
       A             5                  0 R        (1)                  1 EPS    (1)                2 Q       (1)
                                        3 MU       (1)                  4 S      (1)
       B         20000                  0 T        (5000)            5000 E      (5000)         10000 DELTA   (5000)
                                    15000 TERR     (5000)

STATISTICS
  PROGRAM LENGTH                 414B      268
  BUFFER LENGTH                41030B     2115
  CM LABELED COMMON LENGTH    47045B    20005
```

```
SUBROUTINE FCALC        74/74    OPT=1                         FTN 4.5+414        06/04/76  13.30.36        PAGE     1

  1                SUBROUTINE FCALC(I,FDFTE,FSUBT,FSUBE,FSUBTT,FSUBTE)
                   REAL JC,MU,NE
                   COMMON/A/R,EPS,Q,MU,S
                   COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
  5                RT=R*T(I)
                   FDFTE=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))/EPS
                   FSUBTE=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))*(R/EPS)
                   FSUBE=-(Q*MU*NE(I)*EXP(RT))/EPS
                   FSUBTT=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))*(R**2)/EPS
 10                FSUBTE=-(Q*MU*NE(I)*R*EXP(RT))/EPS
                   RETURN
                   END


        SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    3  FCALC          1            11

VARIABLES        SN  TYPE             RELOCATION
23420  DELTA         REAL      ARRAY    B          REFS       4
11610  E             REAL      ARRAY    B          REFS       4      6      7      9
    1  EPS           REAL               A          REFS       3      6      7      8      9     10
    0  FDFTE         REAL                  F.P.    DEFINED    1      6
    0  FSUBE         REAL                  F.P.    DEFINED    1      8
    0  FSUBT         REAL      *UNUSED     F.P.    DEFINED    1
    0  FSUBTE        REAL                  F.P.    DEFINED    1      7     10
    0  FSUBTT        REAL                  F.P.    DEFINED    1      9
    0  I             INTEGER               F.P.    REFS       5    3*6    3*7      8    3*9     10
                                                   DEFINED    1
    3  MU            REAL               A          REFS       2      3      6      7      8      9     10
    2  Q             REAL               A          REFS       3      6      7      8      9     10
    0  R             REAL               A          REFS       3      5      7      9     10
  113  RT            REAL                          REFS       6      7      8      9     10
                                                   DEFINED    5
    4  S             REAL               A          REFS       3
    0  T             REAL      ARRAY    B          REFS       4      5
35230  TERR          REAL      ARRAY    B          REFS       4

EXTERNALS          TYPE   ARGS        REFERENCES
       EXP         REAL     2 LIBRARY      6      7      8      9     10
       JC          REAL     1              2      6      7      9
       NE          REAL     1              2      6      7      8      9     10

COMMON BLOCKS    LENGTH     MEMBERS - BIAS NAME(LENGTH)
      A             5             0 R      (1)              1 EPS    (1)              2 Q      (1)
                                  3 MU     (1)              4 S      (1)
      B         20000             0 T      (5000)        5000 E      (5000)       10000 DELTA  (5000)
                              15000 TERR   (5000)

STATISTICS
  PROGRAM LENGTH                1140      76
  CM LABELED COMMON LENGTH    470450   20005
```

APPENDIX B

```
     SUBROUTINE GCALC       74/74    OPT=1                           FTN 4.5+414        06/04/76  13.30.36        PAGE     1

       1                   SUBROUTINE GCALC(I,GDFTE,GSUBT,GSUBE,GSUBTE)
                           REAL JC,MU,NE
                           COMMON/A/R,EPS,Q,MU,S
                           COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
       5                   RT=R*T(I)
                           GDFTE=(JC(I)-Q*MU*NE(I)*E(I))/EPS
                           GSUBT=-(Q*MU*E(I)*S)/EPS
                           GSUBE=-(Q*MU*NE(I))/EPS
                           GSUBTE=-(Q*MU*S)/EPS
      10                   RETURN
                           END


     SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS      DEF LINE      REFERENCES
     3  GCALC          1            10

 VARIABLES      SN  TYPE            RELOCATION
 23420  DELTA       REAL       ARRAY   B              REFS        4
 11610  E           REAL       ARRAY   B              REFS        4        6        7
     1  EPS         REAL               A              REFS        3        6        7        8        9
     0  GDFTE       REAL                   F.P.    DEFINED        1        6
     0  GSUBE       REAL                   F.P.    DEFINED        1        8
     0  GSUBT       REAL                   F.P.    DEFINED        1        7
     0  GSUBTE      REAL                   F.P.    DEFINED        1        9
     0  I           INTEGER                F.P.       REFS        5      3*6        7        8   DEFINED        1
     3  MU          REAL               A              REFS        2        3        6        7        8        9
     2  Q           REAL               A              REFS        3        6        7        8        9
     0  R           REAL               A              REFS        3        5
    40  RT        * REAL                           DEFINED        5
     4  S           REAL               A              REFS        3        7        9
     0  T           REAL       ARRAY   B              REFS        4        5
 35230  TERR        REAL       ARRAY   B              REFS        4

 EXTERNALS          TYPE    ARGS       REFERENCES
        JC          REAL      1            2        6
        NE          REAL      1            2        6        8

 COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
        A            5                  0 R       (1)             1 EPS     (1)               2 Q       (1)
                                        3 MU      (1)             4 S       (1)
        B        20000                  0 T       (5000)       5000 E       (5000)        10000 DELTA   (5000)
                                    15000 TERR    (5000)

 STATISTICS
   PROGRAM LENGTH                   41B      33
   CM LABELED COMMON LENGTH     47045B    20005
```

```
SUBROUTINE OTWRIT      74/74    OPT=1                          FTN 4.5+414        06/04/76  13.30.36        PAGE      1

1                  SUBROUTINE OTWRIT(N,M,DEL,TSTART,TMAX)
                   COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                   IT=0
                   K=1
5                  WRITE(6,10)
                10 FORMAT(1H1)
                   WRITE(6,20)DEL,TSTART,TMAX,M
                20 FORMAT(1X,17HDELTA TIME STEP: ,1PE10.3,5X,15HSTARTING TIME: ,
                  11PE10.3,5X,28HMAXIMUM TIME TO BE PLOTTED: ,1PE10.3,/,34X,
10                228HINCREMENT OF OUTPUT VALUES: ,15,/,/,/,/)
                   WRITE(6,30)
                30 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,8X,5HDELTA)
                   WRITE(6,40)
                40 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,8X,5H-----)
15                 DO 110 I=1,N,M
                   WRITE(6,50)T(I),E(I),TERR(I),DELTA(I)
                50 FORMAT(1X,4(1PE11.4,5X))
                   IF(IT.EQ.1)  GO TO 60
                   IF(K.EQ.50)  GO TO 70
20              60 IF(K.EQ.60)  GO TO 70
                   K=K+1
                   GO TO 110
                70 CONTINUE
                   WRITE(6,80)
25              80 FORMAT(1H1)
                   WRITE(6,90)
                90 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,8X,5HDELTA)
                   WRITE(6,100)
               100 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,8X,5H-----)
30                 IT=1
                   K=1
               110 CONTINUE
                   RETURN
                   END

    SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
   3 OTWRIT           1             33

VARIABLES      SN  TYPE            RELOCATION
     0 DEL         REAL                 F.P.       REFS        7   DEFINED       1
 23420 DELTA       REAL       ARRAY    B           REFS        2        16
 11610 E           REAL       ARRAY    B           REFS        2        16
   206 I           INTEGER                         REFS     4*16   DEFINED      15
   204 IT          INTEGER                         REFS       18   DEFINED       3        30
   205 K           INTEGER                         REFS       19        20      21   DEFINED       4       21       31
     0 M           INTEGER              F.P.       REFS        7        15  DEFINED        1
     0 N           INTEGER              F.P.       REFS       15   DEFINED       1
     0 T           REAL       ARRAY    B           REFS        2        16
 35230 TERR        REAL       ARRAY    B           REFS        2        16
     0 TMAX        REAL                 F.P.       REFS        7   DEFINED       1
     0 TSTART      REAL                 F.P.       REFS        7   DEFINED       1
```

```
        SUBROUTINE OTWRIT     74/74   OPT=1                          FTN 4.5+414        06/04/76  13.30.36       PAGE     2

    FILE NAMES          MODE
          TAPE6         FMT                   WRITES        5         7         11        13        16        24        26        28

    STATEMENT LABELS              DEF LINE    REFERENCES
        70  10        FMT              6           5
       101  20        FMT              8           7
       124  30        FMT             12          11
       135  40        FMT             14          13
       152  50        FMT             17          16
        64  60                        20          18
        50  70                        23          19        20
       160  80        FMT             25          24
       165  90        FMT             27          26
       176  100       FMT             29          28
        60  110                       32          15        22

    LOOPS  LABEL     INDEX       FROM-TO     LENGTH     PROPERTIES
       30  110     * I            15 32       33B                   EXT REFS

    COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
         B            20000                  0 T       (5000)              5000 E      (5000)            10000 DELTA  (5000)
                                         15000 TERR    (5000)

    STATISTICS
      PROGRAM LENGTH                  221B      145
      CM LABELED COMMON LENGTH      47040B    20000
```

```
SUBROUTINE PLOT        74/74   OPT=1                          FTN 4.5+414       06/04/76  13.30.36       PAGE      1

     1                SUBROUTINE PLOT(N,MAX)
                      COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                      RT=N/100.
                      IDEL=INT(RT)
     5                IF(IDEL.EQ.0)  IDEL=1
                      L=1
                      DO 10 I=1,N,IDEL
                      T(L)=T(I)
                      E(L)=E(I)
    10                L=L+1
                   10 CONTINUE
                      T(L)=T(N)
                      E(L)=E(N)
                      MAX=L
    15                RETURN
                      END


    SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    3 PLOT           1           15

VARIABLES     SN  TYPE          RELOCATION
23420  DELTA      REAL       ARRAY   B          REFS       2
11610  E          REAL       ARRAY   B          REFS       2        9        13     DEFINED      9       13
   37  I          INTEGER                       REFS       8        9     DEFINED     7
   35  IDEL       INTEGER                       REFS       5        7     DEFINED     4          5
   36  L          INTEGER                       REFS       8        9        10        12        13       14
                                             DEFINED       6       10
    0  MAX        INTEGER              F.P.  DEFINED       1       14
    0  N          INTEGER              F.P.     REFS       3        7        12        13     DEFINED      1
   34  RT         REAL                          REFS       4    DEFINED       3
    0  T          REAL       ARRAY   B          REFS       2        8        12     DEFINED      8       12
35230  TERR       REAL       ARRAY   B          REFS       2

INLINE FUNCTIONS     TYPE    ARGS         DEF LINE  REFERENCES
     INT          INTEGER     1  INTRIN                  4

STATEMENT LABELS             DEF LINE   REFERENCES
    0  10                       11          7

LOOPS  LABEL     INDEX     FROM-TO     LENGTH     PROPERTIES
   17  10          I          7 11        4B        INSTACK

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
      B          20000                  0 T       (5000)              5000 E       (5000)           10000 DELTA  (5000)
                                    15000 TERR    (5000)

STATISTICS
  PROGRAM LENGTH                  40B      32
  CM LABELED COMMON LENGTH     47040B   20000
```

APPENDIX B

APPENDIX B

```
      FUNCTION NE          74/74    OPT=1                        FTN 4.5+414        06/04/76  13.30.36        PAGE    1

    1                 REAL FUNCTION NE(I)
                      REAL NEO
                      COMMON/A/R,EPS,Q,NU,S
                      COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
    5                 NEO=1.133E+11
                      IF(T(I).GE.0.)  GO TO 10
                C  NE MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                      NE=NEO*1.E+6
                      RETURN
   10           C  NE MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                   10 NE=(NEO+S*T(I))*1.E+6
                      RETURN
                      END


      SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS      DEF LINE      REFERENCES
     4  NE              1            9         12

 VARIABLES      SN  TYPE              RELOCATION
 23420  DELTA       REAL        ARRAY    B             REFS        4
 11610  E           REAL        ARRAY    B             REFS        4
     1  EPS         REAL                 A             REFS        3
     0  I           INTEGER                F.P.        REFS        6        11    DEFINED        1
     3  NU          INTEGER              A             REFS        3
    22  NE          REAL                             DEFINED       8        11
    23  NEO         REAL                               REFS        2         8          11    DEFINED        5
     2  Q           REAL                 A             REFS        3
     0  R           REAL                 A             REFS        3
     4  S           REAL                 A             REFS        3        11
     0  T           REAL        ARRAY    B             REFS        4         6          11
 35230  TERR        REAL        ARRAY    B             REFS        4

 STATEMENT LABELS              DEF LINE    REFERENCES
    13  10                        11           6

 COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
       A              5                 0 R      (1)                 1 EPS    (1)                 2 Q       (1)
                                        3 NU     (1)                 4 S      (1)
       B          20000                 0 T      (5000)           5000 E      (5000)          10000 DELTA   (5000)
                                    15000 TERR   (5000)

 STATISTICS
   PROGRAM LENGTH                248       20
   CM LABELED COMMON LENGTH   47045B    20005
```

```
FUNCTION JC         74/74    OPT=1                                FTN 4.5+414        06/04/76  13.30.36       PAGE     1

                      REAL FUNCTION JC(I)
                      COMMON/A/R,EPS,Q,MU,S
                      COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                      IF(T(I).GE.0.)  GO TO 10
                      JC=1.34E+3*EXP(R*T(I))
                      RETURN
                10    JC=1.34E+3
                      RETURN
                      END


     SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    4  JC              1           6        8

VARIABLES       SN  TYPE           RELOCATION
23420  DELTA        REAL       ARRAY   B              REFS        3
11610  E            REAL       ARRAY   B              REFS        3
    1  EPS          REAL               A              REFS        2
    0  I            INTEGER               F.P.        REFS        4        5    DEFINED       1
   17  JC           REAL                           DEFINED        5        7
    3  MU           INTEGER            A              REFS        2
    2  Q            REAL               A              REFS        2
    0  R            REAL               A              REFS        2        5
    4  S            REAL               A              REFS        2
    0  T            REAL       ARRAY   B              REFS        3        4        5
35230  TERR         REAL       ARRAY   B              REFS        3

EXTERNALS           TYPE   ARGS         REFERENCES
       EXP          REAL      1 LIBRARY        5

STATEMENT LABELS                DEF LINE    REFERENCES
   14  10                           7           4

COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
     A              5                 0 R      (1)                   1 EPS    (1)                  2 Q       (1)
                                      3 MU     (1)                   4 S      (1)
     B          20000                 0 T      (5000)             5000 E      (5000)           10000 DELTA   (5000)
                                  15000 TERR   (5000)

STATISTICS
  PROGRAM LENGTH                 20B      16
  CM LABELED COMMON LENGTH    47045B   20005
```

APPENDIX B


```
      SUBROUTINE PRTPLT (HORZ,VERT,NPTS,HEAD,HLAB,VLAB,HREF,VREF,J)
C
C     THIS FORTRAN SUBROUTINE PRODUCES A FULL PAGE X-Y PRINTER PLOT.
C
C     HORZ - REAL ARRAY TO BE PLOTTED ALONG HORIZONTAL AXIS.
C     VERT - REAL ARRAY TO BE PLOTTED ALONG VERTICAL AXIS.
C     NPTS - NUMBER OF VALUES IN HORZ,VERT (NTE 101)
C     HEAD - PLOT HEADING (MUST BE 5 WORDS)
C     HLAB - LABEL FOR HORIZONTAL AXIS (MUST BE 3 WORDS)
C     VLAB - LABEL FOR VERTICAL AXIS (MUST BE 3 WORDS)
C     HREF - REFERENCE VALUE FOR HORIZONTAL AXIS.
C     VREF - REFERENCE VALUE FOR VERTICAL AXIS.
C     J  THIS DETERMINES THE SIZE OF THE ARRAYS HORZ AND VERT--NTE 100
C
      DIMENSION HORZ(J),VERT(J),HEAD(5),HLAB(3),VLAB(3),LINE(101),
     C IVER(101),JHOR(101),HORIZ(101),VERTIC(101),YAXIS(30),DOWNT(21)
      INTEGER BLANK,STAR,DOT,EYE
C
C
      DATA BLANK/1H /,STAR/1H*/,DOT/1H./,MINUS/1H-/,EYE/1HI/
      CALL DATE(DAT)
      CALL TIME(TIM)
      HEAD(4)=DAT
      HEAD(5)=TIM
      HO = 1
      NUM = 1
      N = NPTS
      N = MINO(N,101)
      DO 10 I = 1,N
      HORIZ(I) = HORZ(I)
      VERTIC(I) = VERT(I)
   10 CONTINUE
C
      DECODE (30,1000,VLAB(1)) (YAXIS(I),I=1,30)
 1000 FORMAT(30A1)
   20 NA = N - 1
      DO 30 I = 1,NA
      IF (VERTIC(I).GE.VERTIC(I+1)) GO TO 30
      HOLD = VERTIC(I)
      VERTIC(I) = VERTIC(I+1)
      VERTIC(I+1) = HOLD
      SAVE = HORIZ(I)
      HORIZ(I) = HORIZ(I+1)
      HORIZ(I+1) = SAVE
   30 CONTINUE
      DO 40 I = 1,NA
      IF (VERTIC(I).LT.VERTIC(I+1)) GO TO 20
   40 CONTINUE
      VMAX = VERTIC(1)
      VMIN = VERTIC(N)
      HMAX = HORIZ(1)
      HMIN = HORIZ(1)
      DO 50 I = 2,N
```

APPENDIX B

```
SUBROUTINE PRTPLT     74/74   OPT=1                    FTN 4.6+420        10/27/76  09.31.03       PAGE     2

          HZ = HORIZ(I)
          HMAX = AMAX1(HMAX,HZ)
          HMIN = AMIN1(HMIN,HZ)
   50 CONTINUE
      DO 60 I = 1,K
          JVER = 50.*(VERTIC(I)-VMIN)/(VMAX-VMIN) + 1.5
          IVER(I) = 52 - JVER
   60 JHOR(I) = 100.*(HORIZ(I)-HMIN)/(HMAX-HMIN) + 1.5
          KHREF = 100.*(HREF-HMIN)/(HMAX-HMIN) + 1.5
          IF (KHREF.LT.1) KHREF = 150
          IF (KHREF.GT.101) KHREF = 150
          KVREF = 50.*(VREF-VMIN)/(VMAX-VMIN) + 1.5
          KVREF = 52 - KVREF
          IF (KVREF.LT.1) KVREF = 1
          IF (KVREF.GT.51) KVREF = 1
          UP = (VMAX-VMIN)/50.
          PRINT 2000 , HEAD
 2000 FORMAT(1H1,35X,5A10//)
          DO 200 K = 1,51
          LINE(1) = EYE
          LINE(101) = EYE
          DO 70 I = 2,100
   70 LINE(I) = BLANK
          IF (K.EQ.1) GO TO 90
          IF (K.EQ.51) GO TO 90
          IF (K.NE.KVREF) GO TO 120
          DO 80 I = 2,100
   80 LINE(I) = DOT
          GO TO 120
   90 DO 100 I = 2,100
  100 LINE(I) = MINUS
          DO 110 I = 6,100,5
  110 LINE(I) = EYE
          GO TO 120
  120 IF (KHREF.EQ.150) GO TO 130
          IF (KHREF.EQ.1) GO TO 130
          LINE(KHREF) = DOT
  130 IF (K.EQ.IVER(NUM)) GO TO 140
          IF (K.LT.IVER(NUM)) GO TO 160
          NUM = NUM + 1
          GO TO 130
  140 IF (JHOR(NUM).GT.101) GO TO 150
          IF (JHOR(NUM).LT.1) GO TO 150
          JH = JHOR(NUM)
          LINE(JH) = STAR
  150 NUMBER = NUM + 1
          IF (IVER(NUM).NE.IVER(NUMBER)) GO TO 160
          NUM = NUMBER
          GO TO 140
  160 T = K - 1
          UPT = VMAX - T*UP
          IF (K.LT.11) GO TO 170
          IF (K.GT.41) GO TO 170
```

APPENDIX B

```
SUBROUTINE PRTPLT     74/74    OPT=1                          FTN 4.6+420       10/27/76  09.31.03       PAGE     3

              TAXIS = VAXIS(NO)
              NO = NQ + 1
              PRINT 3000 , TAXIS,UPT,LINE
        3000 FORMAT(4X,A1,4X,1PE9.2,2X,101A1)
              GO TO 200
         170 PRINT 4000 , UPT,LINE
        4000 FORMAT(9X,1PE9.2,2X,101A1)
         200 CONTINUE
              DOWN = (HMAX-HMIN)/20.
              T = HMIN
              DO 210 I = 1,21
              DOWNT(I) = T
              T = T + DOWN
         210 CONTINUE
              PRINT 5000 , (DOWNT(I),I=1,21,2)
        5000 FORMAT(/15X,1P11E10.2)
              PRINT 6000 , (DOWNT(I),I=2,20,2)
        6000 FORMAT(20X,1P10E10.2)
              PRINT 7000 , HLAB
        7000 FORMAT(/50X,3A10)
              RETURN
              END
```

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