

AD-A035 312

NAVAL WEAPONS ENGINEERING SUPPORT ACTIVITY WASHINGTON D C F/G 11/6  
MECHANICAL RELIABILITY DESIGN EVALUATION GUIDE STATUS REPORT.(U)  
DEC 76 F M HALL, C ELLIS, W SKEWIS

UNCLASSIFIED

NAVWESA-37-76

NL

1 OF 1  
AD  
A035312



END  
DATE  
FILMED  
3-77

ADA035312

SYSTEMS ENGINEERING DEPARTMENT

12

MECHANICAL RELIABILITY DESIGN EVALUATION GUIDE

STATUS REPORT

1 DECEMBER 1976

REPORT NUMBER 37-76

NAVAL WEAPONS ENGINEERING SUPPORT ACTIVITY  
WASHINGTON NAVY YARD  
WASHINGTON, DC 20374

97 DDC  
RECEIVED  
FEB 8 1977  
D

**DISTRIBUTION STATEMENT A**

Approved for public release;  
Distribution Unlimited

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE   |  | READ INSTRUCTIONS<br>BEFORE COMPLETING FORM                                     |
|---|--|---|
| 1. REPORT NUMBER<br>37-76   | 2. GOVT ACCESSION NO.<br>(14) NAVWE SA-37-76 | 3. RECIPIENT'S CATALOG NUMBER   |
| 4. TITLE (and Subtitle)<br>(6) Mechanical Reliability Design Evaluation Guide Status Report.  |  | 5. TYPE OF REPORT & PERIOD COVERED<br>Research Report, July 1976-December 1976  |
| 7. AUTHOR(s)<br>(10) F. M. Hall, C. Ellis<br>W. Skewis  |  | 6. PERFORMING ORG. REPORT NUMBER<br>N/A   |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Naval Weapons Engineering Support Activity (ESA-116)<br>Wash. Navy Yard, Washington, DC 20374  |  | 8. CONTRACT OR GRANT NUMBER(s)<br>N/A   |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>Same as 9.   | (12) 56p.                                    | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS<br>6270N/53-532/403 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br>Same as 9.   | (11)   | 12. REPORT DATE<br>1 Dec 1976   |
|   |  | 13. NUMBER OF PAGES<br>48   |
|   |  | 15. SECURITY CLASS. (of this report)<br>Unclassified                            |
|   |  | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE                                      |
| 16. DISTRIBUTION STATEMENT (of this Report)<br>Approved for public release; distribution unlimited.<br>(9) Research rept. Jul-Dec 76,   |  |   |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)<br>Approved for public release; distribution unlimited.  |  |   |
| 18. SUPPLEMENTARY NOTES   |  |   |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Mechanical Reliability<br>Failure Modes and Effects Analysis<br>Dimensional Analysis  |  |   |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>This interim report reviews the progress at the Naval Weapons Engineering Support Activity in development of a mechanical reliability design guide. Design evaluation techniques under consideration are included, with emphasis on the preparation of a Failure Mode and Effects Analysis (FMEA) Military Standard. Existing FMEA procedures are examined and a FMEA standard procedure is recommended as a basis for developing improved reliability design evaluation techniques. |  |   |

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-LF-014-6601

Unclassified 407 935  
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)



ABSTRACT

This interim report reviews the progress at the Naval Weapons Engineering Support Activity in development of a mechanical reliability design guide. Design evaluation techniques under consideration are included, with emphasis on the preparation of a Failure Mode and Effects Analysis (FMEA) Military Standard. Existing FMEA procedures are examined and a FMEA standard procedure is recommended as a basis for developing improved reliability design evaluation techniques.

|                                 |               |                                     |
|---------------------------------|---------------|-------------------------------------|
| ACCESSION FOR                   |               |                                     |
| NTIS                            | White Section | <input checked="" type="checkbox"/> |
| ONS                             | Buff Section  | <input type="checkbox"/>            |
| UNPRODUCED                      |               | <input type="checkbox"/>            |
| IDENTIFICATION                  |               |                                     |
| BY                              |               |                                     |
| DISTRIBUTION/AVAILABILITY CODES |               |                                     |
| DECL.                           | AVAIL.        | and/or SPECIAL                      |
| A                               |               |                                     |

DDC  
RECEIVED  
FEB 8 1977  
D

## TABLE OF CONTENTS

1. INTRODUCTION
2. RELIABILITY PREDICTION
3. FAILURE MODE AND EFFECTS ANALYSIS (FMEA)
4. FMEA PROCEDURAL CONSIDERATIONS

### Appendix

"Application of Dimensional Analysis to the Prediction of Mechanical Reliability"-Louisiana Tech University

## 1. Introduction

NAVWESA (ESA-11) has been tasked by NAVAIR (Air-340C) under Air Task A3400000/184B/4F53-532-403 to analyze mechanical reliability prediction techniques and to develop design evaluation procedures for advanced-development stage non-electronic equipment. The ultimate objective of this research is publication of a mechanical reliability design guide which will assist mechanical equipment designers in evaluating and verifying equipment reliability on the drawing board.

To establish a basis for this development, ESA-116 initiated a comparative analysis of mechanical reliability determination techniques in FY76 to identify present design evaluation capabilities. Investigations were conducted in the following areas:

- a. Reliability estimating and prediction techniques
  - (1) Accelerated testing
  - (2) Functional Reliability Evaluation Techniques (FRET)
  - (3) Electric circuit analogy
  - (4) Summation of failure rates
  - (5) Physics of failure
  - (6) Regression analysis
- b. Failure Mode and Effects Analysis (FMEA)
- c. Equipment standardization approach for problem identification
- d. Mechanical system modeling
- e. Test/validation methods

The FMEA surfaced as the most effective design evaluation technique for early identification of design problems and the most promising base from which to develop further design evaluation techniques. In Section 3, the value of the FMEA as a design evaluation tool is addressed. The major benefit of performing on FMEA early in the equipment design process is to provide an accurate means of predicting design-related failures to allow low-cost design modification and establish a realistic developmental test program.

## 2. Reliability Prediction

The key to achieving cost effective mechanical system reliability is found in advanced development where, because of the early point in time of the life cycle, engineering changes can be controlled and can be incorporated relatively inexpensively.

One of the tools used by reliability engineers in evaluating design is mathematical modeling of proposed systems based on summation of predictable component failure rates. Quantitative estimates of reliability permit the comparison of alternate designs, the effects of design changes on equipment reliability, and the evaluation of equipment reliability in relation to its system allocation. Reliability estimation is fairly accurate for electronic equipment because confidence levels for electronic component failure rates published in MIL-HDBK-217B are relatively high.

For mechanical components, prediction of reliability is much more involved because component failure rates are dependent to a large degree upon the design configuration, equipment age, operational environment and application. Mechanical parts are usually not standardized because of the need for special designs that often perform more than one function. Therefore, developing valid part failure rates for mechanical components is an extremely difficult task. Those failure rates for mechanical equipment which have been published are usually based upon the exponential distribution and assume a constant failure rate. Their value for design evaluation purposes is questionable.

Meaningful failure rates for mechanical components will not be available unless components are more standardized, failure modes and failure causes more thoroughly defined, and a data base established. There is little doubt about the feasibility of accurately evaluating reliability of non-electronic systems, but a valid data base does not yet exist. Therefore, current efforts for improvement and documentation of non-electronic reliability evaluation techniques are not based on summation of component failure rates but are intended to assist the designer in design evaluation and correction of potential reliability problems very early in the development phase.

### 3. Failure Mode and Effects Analysis

In the advanced development stage, a thorough FMEA provides valuable assistance for the design team. Its results identify potential problems, further testing requirements, and required design modifications. The FMEA may evaluate a system, subsystem or component, and the interrelationships of elements to determine the effects of potential failures on the system and the consequences of the failure modes on equipment performance.

The value of the FMEA as a design evaluation tool lies in its potential to:

- a. Simplify a design
- b. Improve materials selection
- c. Identify potential hazards
- d. Identify test program requirements
- e. Identify critical items for design review, inspection and process control
- f. Pinpoint unreliable areas for logistics consideration

FMEA procedures have been prepared by various agencies and industrial groups and each one guides the user through an evaluation of a design with different requirements. In order to locate the most appropriate procedure or combination thereof, the Naval Weapons Engineering Support Activity reviewed the existing FMEA bulletins, standards and advisory procedures. Results of the investigation provided a better understanding of the guidance and doctrine within private industry and the Department of Defense. The following procedures were selected as germane.

- a. SAE Aerospace Recommended Practice, ARP 926, of 15 SEP 1967. "Design Analysis Procedure for Failure Mode, Effects and Criticality Analysis (FMECA)". Society of Automotive Engineers, Inc., 485 Lexington Ave., New York, N.Y. 10017.
- b. Military Standard, MIL-STD-1629 (SHIPS), of 1 NOV 1974. "Procedure for Performing a Failure Mode and Effect Analysis for Shipboard Equipment."

Commander, Naval Ship Engineering Center (COMNAVSEC), Department of the Navy, Center Bldg, Prince George's Center, Hyattsville, MD 20782.

c. EIA Reliability Bulletin No. 9 of NOV 1971. "Failure Mode and Effect Analyses". Electronic Industries Association, G-41 Committee on Reliability, Engineering Department 2001 Eye St., N.W., Washington, D.C. 20006.

d. NASA FMEA Document No. AHB 5326-1 of MAR 1973. AMES Research Center-National Aeronautics and Space Administration, Moffett Field, CA 94035.

e. NAVAIR Preliminary Drafts, FMECA MIL-STD or Specification. Sourced from AIR-340C (OCT 1976), AIR-5205 (AUG 1976) and NWESA-1183 (OCT 1976).

A review of the above references suggests a need to maintain currency and to standardize FMEA procedures for intra-DOD agencies and for Defense Department contractor use. ARP 926 is now under revision and NASA is originating a new FMEA procedure.

In Section 4 of this report, it is emphasized that there are now in wide use at least two different basic procedural approaches to the FMEA. For example MIL-STD-1629 uses a "Top-down" methodology in its deductive analysis of shipboard equipment design, whereas ARP 926 prescribes an inductive analysis, "from the "Bottom-up". A third approach combines the desirable aspects of the first two. "Criticality Analysis", for instance, is approached in depth in ARP 926, but with little emphasis in the forthcoming NASA document. It is due to the above lack of industry-wide standardization that authority has been requested of the Defense Material Specifications and Standards Office to generate a military-wide FMEA standard procedure publication. Efforts will be coordinated with all appropriate sources in order to generate an effective FMEA Military Standard.

#### 4. FMEA Procedural Considerations

Preparing a standard FMEA document requires that each element of the procedure be coordinated with interested activities to ensure completeness and compatibility. The procedural steps requiring examination and definition include:

- a. Define the system and its requirements
- b. Establish a reliability model

- c. Establish the assumptions and rules for performing the analysis
- d. Identify failure modes and their effects
- e. Identify failure detection and isolation methods
- f. Evaluate criticality of the failure modes
- g. Document the analysis

FMEA methodologies from various activities form a collective basis for a universal military procedure. The following paragraphs describe these procedural steps and their variations.

- a. Define the system and its requirements

The first step in performing an FMEA is defining the system being analyzed. A complete definition of the product or system includes its use, performance expected, system restraints and failure definitions. Defining the system and its requirements is accomplished by breaking the system down into its major functional blocks and describing the function of each block and the interfaces between the blocks.

Any system is designed for one or more operational modes and may be active during various times of the day or phases of a mission. Defining the system and its requirements consists of functional narratives of the system's operation for each operational mode or mission phase. It also includes statements of primary and secondary objectives.

The amount of detail for this step will vary to a considerable extent with system complexity, the number of indenture levels<sup>1/</sup> within the system and the uniqueness of the system function performed. The narrative should contain as a minimum:

- (1) System and element description of operation in each operational mode or phase.
- (2) Expected mission times and equipment utilization.
- (3) Operational and environmental stresses the system is expected to encounter as well as stress limits.
- (4) Functions and outputs of each element including the performance parameters and allowable limits for each specified output.
- (5) Conditions which constitute system and element failure.

<sup>1/</sup>"Indenture level" in an FMEA refers to a group of components within a system which have the same level of relative complexity.

A proper definition is important in order to establish a meaningful purpose for conducting the FMEA. This step is particularly valuable during exploratory and advanced development since it enhances a thorough understanding of the system and its anticipated environment prior to beginning the actual analysis. Existing FMEA procedures exhibit some weakness in the requirements for this step of the procedure and could be strengthened by incorporating some minimum requirements similar to those mentioned above.

b. Establish reliability model

This step graphically displays the system elements to be analyzed; the series and redundant relationships among the elements; and functional interdependencies between the system components so that the effects of a functional failure can be tracked through the system. The model provides for analyzing the loss effects of any output and the failure relationships among the system elements. A reliability block diagram must be prepared for each alternative mode of operation. While each of the FMEA procedures specifies a reliability model, obtaining desired results can be a problem and specific details of generating a reliability model are needed in the FMEA procedure. The reliability block diagram should contain as a minimum:

- (1) A breakdown of the system or subsystem into its major blocks including functional relationships.
- (2) All inputs and outputs of each block and the inputs and outputs of the system clearly labeled.
- (3) System redundancies or other design means for preventing component malfunctions from affecting overall system performance.
- (4) Separate logic diagrams as required for each operational phase or mode.

c. Establish assumptions and rules for performing the FMEA

Depending upon the type equipment, its complexity, utilization, or progress in development, several different approaches are recommended by users of FMEA procedures. The basic concept of each of the approaches nevertheless remains the same: To determine what items in an equipment can fail, the modes of failure that are possible for each of these items, and the effect of each mode of failure on the complete equipment or lower indenture level.

Two primary approaches to analysis methodology exist within the basic concept of an FMEA: "Top-down" (sometimes referred to as "System Technique") and "Bottom-up" (because it starts at the "Parts level").

(1) The Top-down approach proceeds from the top indenture level (total system or equipment) to successively lower levels. The Top-down analysis usually identifies non-desirable system functions as in a Fault-tree analysis, and the equipment is analyzed at the next lower indenture level to identify any failure and associated failure mode which could result in the originally identified failure effect. For each of these second-level failures the process is repeated by tracing back through the functional paths and relationships to the next lower level. This process is repeated until the desired end level is reached. The Top-down approach is generally preferred when detailed definitions are not yet available during conceptual phases of design or when the system complexity is such that it is easier to list system malfunctions as a starting point rather than listing components. The Top-down method of analysis is event oriented and can easily include external influences in the analysis such as the environment and human error. Thus it is a good technique for evaluating multiple failures.

(2) The parts-level approach is a Bottom-up technique identifying failure modes at the part level, and for each of the part failure modes the corresponding effect on performance at the next higher indenture level is determined. The resulting failure effect becomes the failure mode at the next higher indenture level, and the failure effects of each failure mode are analyzed at this level. Successive iterations result in the eventual identification of the failure effects at all functional levels up to the system or highest level. Parts level-up analysis is a hardware oriented, more rigorous technique for identifying all single failures. Many different failure modes will relate to each ultimate end failure effect and this relationship permits the grouping of various part failure modes. The parts-level approach is favored when all the components can be identified and the equipment is not so complex that the analysis becomes cumbersome.

ARP 926 commences the FMEA from the bottom-up. MIL-STD-1629 prescribes a Top-down analysis. Electronic Industries Association (EIA) Bulletin No. 9,

on the other hand, encourages either approach be used, dependent upon the circumstances of the analysis. For abbreviated reference; the following comparison emphasizes the contrast in the two techniques:

"Top-Down"

Event oriented. Identifies higher level events or events subsequent to failure.

Certain failure effects are easily analyzed.

Failures with no effects are not considered.

Optimum technique for multiple failures.

Considers external influences such as human error.

Deductive in nature.

"Bottom-Up"

Rigorous technique for identifying all single failures (component failure modes).

Hardware oriented. Identifies functional failures of components, sub-systems or systems.

Inductive in nature.

Neither approach can be considered the most appropriate technique to be used. The Top-down approach is much easier to manipulate than a parts level analysis, but there is a certain probability that failure modes of individual components can be overlooked, thereby raising serious questions as to the value of the analysis. The FMEA procedures in existence provide one or the other analytical techniques but individual equipment types and situations dictate which approach should be used. In some applications a combination of the two approaches or a modification to one of the approaches should be considered. The FMEA procedure should present guidelines for deciding on a Top-down or Parts level-up approach.

In addition to determining which analytical technique should be used, other ground rules must be established prior to performing the actual FMEA:

- (1) The indenture level at which the analysis should begin and at which level it should end.

- (2) The environmental and operational stress limits to be considered in the analysis.
- (3) The worksheet format and the coding system to be used which will permit the tracking of failure modes and causes throughout the analysis.
- (4) A definition of failure for system hardware or software elements.

d. Identify Failure Modes and Their Effects

The procedures for identifying failure modes and related failure effects can be effectively employed only after adequate preliminary preparation has been conducted.

For each system component each potential failure mode and its effect on the system is analyzed. Where system definition has not reached down to the piece part level, the system component will be an assembly. Where system hardware definition has not reached the stage of identifying system functions with specific hardware that will perform these functions, the FMEA should be based upon failure of the system functions. In this case, the general type of hardware envisioned to be the basis for system design must be stated. Whatever the indenture level being considered, the failure modes for the particular system component are listed. Four modes of component or functional failure typically considered include:

- (1) Premature operation
- (2) Failure to operate at a prescribed time
- (3) Failure to cease operation at a prescribed time
- (4) Failure during equipment operation

Other failure modes must be considered dependent upon the particular component involved, the system application, mission time and the anticipated environment.

The FMEA assumes that only the failure under consideration has occurred. When safety or back-up devices exist this assumption should be broadened to include the failure conditions which resulted in the need for the back-up function.

Where the procedure is initiated as a bottom-up technique, the failure effects in the direct functional chain to higher levels are determined. It

is also important to identify what happens to related elements in the same functional level and the interactions that could result at lower level within any of these related elements. Similarly where the initial procedure is Top-down, it is important to consider all of the higher level effects that could result from lower level failure modes initially identified with a particular higher level failure effect. This is especially pertinent where an intermediate function feeds or couples directly to more than one higher level function.

In some of the top-down FMEA's reviewed, several of the undesirable system level events were tracked down to an intermediate indenture level in such a way as to leave voids in continuity. In completed analyses that utilized a parts level-up approach, many of the component failure modes were not considered.

The FMEA procedure needs to provide guidance for identifying the indenture levels at which the analysis should start and stop. Also, a list of standard failure modes for components which should be considered by the analyst would be extremely beneficial. Ultimately, the value of the analysis remains dependent upon the analyst. FMEA procedures must be tailored to specific needs and a cookbook approach is not recommended. General guidelines with minimum acceptable requirements in the procedure would help prevent an FMEA being performed as an end to itself where form becomes more important than content.

e. Failure Detection and Isolation

To a large extent the ability of the equipment operator to detect a malfunction, the ability of the maintenance technician to repair an item without introducing secondary failures, and the reaction time required to institute corrective action determine the effects of a given failure mode on the next higher indenture level. To be effective an analysis must identify those critical items which give the operator identical indications and those critical items which present similar malfunction symptoms to the maintenance technician.

Minimum requirements for considering failure detection and repair need to be derived for a DOD wide procedure. Generally these requirements should include:

(1) The indication(s) of failure presented to the operator(s) or maintenance personnel during use or checkout.

(2) Controls or features of the equipment which eliminate or reduce failure mode probability and/or the seriousness of the occurrence. In the FMEA procedures to date, very little emphasis is given to failure detection and isolation.

Because of the dependency of the FMEA upon maintenance philosophy, a close relationship must exist between the maintenance analysis and the FMEA.

f. Evaluate Criticality of the Failure Modes

After failure modes and their effects are identified, probabilities of occurrence must be considered. The most serious failure mode is not generally significant if its probability of occurrence is insignificant. For this reason the criticality of a failure mode is usually considered to reflect both the seriousness of the malfunction and the probability that the malfunction or undesired event will occur. This relationship permits a predetermined criticality threshold in order to identify "critical" situations.

The design team conducting the analysis is in the best position to evaluate and rank the seriousness of the malfunction. The analyst has described the system and developed the reliability models. If the equipment is electronic, MIL-HDBK-217B can be used to determine the probability of a malfunction or undesirable event occurring. However, if the equipment is mechanical, quantitative predictions may not be possible since valid failure rates for mechanical components are usually not available. In this case a relative approximation may have to be assigned by the analyst.

It is in the Criticality Analysis (CA) that further differences exist in methodology among the publications referenced in this report. Each addresses probabilities of failure mode occurrence; how they are derived and used will show different approaches. ARP 926 uses generic failure rates specified by the procuring activity. EIA Bulletin No. 9, being electronics industry-oriented, quantifies mode frequency through failure rates derived from MIL-HDBK-217B, Rome Air Development Center (RADC) Reliability Notebook or

Government-Industry Data Exchange Program (GIDEP) data. MIL-STD-1629 largely uses failure probability levels based on contractor estimated quantitative failure mode probabilities.

After establishing probabilities of occurrence, the degree of criticality is addressed. EIA Bulletin No. 9 establishes the extent of degradation of mission performance ("critical", "major", "minor") along with desired corrective action. MIL-STD-1629 establishes relative values for "Level of Severity" and "Failure Probability" for each mode. Based upon these, a criticality matrix is used to provide relative criticality tabulation. ARP 926 establishes a separate procedure for Criticality Analysis wherein the critical failure modes are identified first. Then a "Criticality Number" is calculated for the system components having critical failure modes. The Criticality Number ranks the relative impact of all part and assembly failure modes according to the magnitude of the number.

While it is desirable to have a quantitative ranking of criticality, the cost of generating such numbers may be greater than the benefits derived if the equipment is in the early design stages of development. A qualitative treatment which simply identifies individual part failure modes in relation to a specific system failure effect is sufficient where the objective is to eliminate the occurrence of the particular failure effect.

g. Document the Analysis

For the FMEA to contribute meaningfully to the development process, conclusions which strongly lead to design improvements or further testing must be clearly documented. Calculated probabilities of a criticality analysis merely help to quantify and rank failure effects for further analysis, and the documentation effort is not complete until critical components are identified and the design is "debugged". The important contribution of the documented analysis to the development program is the increased awareness of the design group of the interaction between factors that contribute to malfunctions and normal operation. The group can then more effectively correct any design deficiencies.

Minimum requirements for documenting the analysis need to be devised. Generally these requirements should include:

- (1) System definition
- (2) Reliability block diagram
- (3) Assumptions and rules established for performing the FMEA and CA
- (4) FMEA and CA worksheets
- (5) List of critical items
- (6) Conclusions and recommendations

Section 4 has addressed procedural considerations in conducting an FMEA. It is intended to develop a service-wide military standard on the subject which would provide the base for development of a mechanical reliability design evaluation guide. Contents of the guide will be structured to include the FMEA as a design analysis tool and to optimize results of the FMEA as a design input.

The formulation of mathematical modeling techniques will require development effort. Louisiana Tech University is presently determining the feasibility of using dimensional analysis techniques to aid the designer in determining critical factors in reliability. The preliminary report is included as Appendix A. The realistic challenge of a design guide will require additional supportive analytical research, as well as cooperation from agencies, associations and individuals interested in concrete methods of improving mechanical reliability through guidelines for design analysis.

APPENDIX  
APPLICATION OF DIMENSIONAL  
ANALYSIS TO THE PREDICTION  
OF MECHANICAL RELIABILITY

September, 1976

Prepared for  
Naval Weapons Engineering Support Activity  
Building 200, Washington Navy Yard  
Washington, D.C. 20374

Under Contract N68335 - 76 - C - 2288

by

Dr. Jack Canterbury  
Dr. James D. Lowther

Division of Engineering Research  
Department of Mechanical Engineering  
Louisiana Tech University  
Ruston, Louisiana 71270

## ABSTRACT

This report documents a study to determine the feasibility of applying dimensional analysis to the prediction of mechanical reliability. The study concludes that dimensional analysis is an essential tool for the development of empirical models for mechanical reliability. As an example, dimensional analysis is applied to the prediction of mean-time-to failure for involute splines. Advantages of dimensional analysis are enumerated.

## ACKNOWLEDGMENT

The authors acknowledge with appreciation the assistance of Dr. J. B. Keats, Acting Head of the Department of Industrial Engineering and Computer Science, Louisiana Tech University. Dr. Keats performed the regression analysis for the spline example.

## CONTENTS

|   | Page |
|---|------|
| I. INTRODUCTION . . . . .   | 1    |
| II. DIMENSIONAL ANALYSIS . . . . .  | 2    |
| III. APPLICATION OF DIMENSIONAL<br>ANALYSIS TO A TYPICAL CASE:<br>SPLINE RELIABILITY PREDICTION . . . . . | 9    |
| IV. CONCLUSIONS . . . . .   | 20   |
| V. RECOMMENDATIONS FOR FUTURE WORK . . . . .  | 22   |
| VI. LIST OF REFERENCES . . . . .  | 23   |
| VII. APPENDICES . . . . .   | 24   |

## INTRODUCTION

In conjunction with research in "Analysis of Mechanical Reliability Prediction Techniques" being performed by the Naval Weapons Engineering Support-Activity (ESA-11), the ultimate goal of which is the publication of a mechanical reliability design guide, Louisiana Tech University has performed studies aimed at improving mechanical reliability estimation and prediction techniques.

The objective of the research on which this report is based was to demonstrate that dimensional analysis is an essential tool for the development of empirical mathematical models for mechanical reliability. While dimensional analysis by itself cannot produce a mathematical model, dimensional analysis reduces the quantity of data required and makes the resulting model simpler and more general. Dimensional analysis also simplifies the application of regression analysis in the development of mathematical models for mechanical reliability prediction. Furthermore, the empirical approach using dimensional analysis does not require a theoretical understanding of the inner workings of the phenomenon being modeled.

## DIMENSIONAL ANALYSIS

Dimensional analysis has been used extensively in the fields of convection heat transfer and fluid mechanics to reduce the quantity of experimental data and theoretical knowledge required for the development of empirical mathematical models. Applications of dimensional analysis may be found in almost every text on these subjects. References (1)\* and (2) are examples of such texts. The application of dimensionless groupings of variables produced by application of dimensional analysis is used universally in developing empirical mathematical models (mathematical models based directly on experimental data rather than theory) in these fields. Dimensionless groupings of variables such as the Reynolds Number, Prandtl Number, friction factor, Mach Number and Nusselt Number are familiar to all who work in these areas.

Dimensional analysis has also been applied to electrical circuits (3, 4, 5, 6), electromagnetics (4, 7), electronics (4), mechanics (3, 4, 7, 8), stress analysis (3, 4, 5, 7), celestial mechanics (8), geology (4), thermodynamics (3, 4, 7), radiation heat transfer (4, 5), and conduction heat transfer (3, 7).

Although the reduction in experimental data required and the elimination of the need for detailed theoretical knowledge of the inner mechanisms of the phenomenon are exactly what is needed for mechanical reliability, workers in the field of mechanical reliability seem to be

---

\*Numbers in parentheses refer to an entry in the List of References.

unaware of the tremendous potential of dimensional analysis. The only application of dimensional analysis to mechanical reliability that has been found thus far is incomplete and was not used to exploit the advantages of the method at all (9). Reference (10) cites as a guideline for parameter selection in using regression analysis to predict reliability of mechanical equipments, "meaningful parameter combinations will be made so that the number of parameters can be reduced with little or no information loss." No significant actual use of "parameter combinations" is reported in reference (10), however.

#### Principles of Dimensional Analysis

Dimensional analysis is a tool that can be used along with data to develop an empirical mathematical model of a physical phenomenon. The use of dimensional analysis is based on ". . . the single premise that the phenomenon can be described by a dimensionally correct equation among certain variables." (7) It is important to note that we do not have to be able to write the equation at the outset in order to apply dimensional analysis. Indeed, if we were able to write the equation (mathematical model), we would not need to employ dimensional analysis. We merely need the assurance that the physical phenomenon could be expressed by an equation. One of the main advantages of dimensional analysis is the fact that dimensional analysis does not require that we have a knowledge of the inner mechanisms of a physical phenomenon.

In order to apply dimensional analysis, we do need to know enough about the physical phenomenon to identify all the variables that affect the phenomenon. If a variable that affects the phenomenon is omitted,

the resulting dimensionless variables obviously cannot be used to accurately model the phenomenon. Once the pertinent variables are selected, dimensional analysis is used to group the variables into dimensionless combinations of these variables. Thus, dimensional analysis reduces the number of variables that describes a physical phenomenon.

#### Dimensions and Units

Before proceeding further, the difference between units and dimensions must be made clear. Depending on which system of dimensions is used, the dimension of a physical variable may be expressed in terms of mass [M]\*, length [L], time [T] and temperature [θ] (mass system); force [F], length [L], time [T] and temperature [θ] (force system), etc. It can be shown that the different systems are equivalent. In this report, the mass system (sometimes called the MLTθ system) will be used exclusively. A physical variable has but one set of dimensions; however, its magnitudes may be expressed in a number of different units. For example, the diameter of a cylinder has the dimension length [L], but the numerical value of the diameter may be expressed in a number of different units such as millimeters, centimeters, inches, feet, etc. Likewise, shear stress has the dimension  $[ML^{-1}T^{-2}]$  but its numerical values may be expressed in pounds per square inch, pounds per square foot, newtons per square millimeter, etc.

Dimensional analysis (a) predicts the number of independent dimensionless groups that can be formed from a group of physical variables and (b) provides a systematic means of forming the groups. Dimensional

---

\*The symbols for basic dimensions are enclosed in brackets.

analysis does not provide the functional relationship between the dimensionless variables (mathematical model)--this relationship must be determined by some other means, usually from experimental data. If experimental or other numerical data is available, regression analysis can be employed to determine an equation relating the dimensionless variables.

#### Advantages of Dimensional Analysis

The important advantages of dimensional analysis are summarized as follows:

1. The number of variables required to describe a physical phenomenon is reduced. The number of dimensionless variables is given by Van Driest (11) as:

The number of dimensionless products in a complete set is equal to the total number of variables minus the maximum number of these variables that will not form a dimensionless product.

A rule that is equivalent and easier to apply is given by Langhaar (7):

The number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix.

In most instances, the "maximum number of these variables that will not form a dimensionless product" or the "rank of their dimensional matrix" is just the number of primary dimensions required to express the dimensional formulas of the variables involved. For example, if a physical phenomenon is described by 7 variables and the dimensional formulas of

these variables can be expressed using the 3 primary dimensions [M], [L], and [T], then the physical phenomenon will be described by  $7-3 = 4$  dimensionless variables.

Corollary advantages resulting from the reduction in the number of variables are:

- a. Data requirements are greatly reduced.
- b. The fitting of equations to data is facilitated.
- c. The resulting mathematical models are easier to use and to present.

To illustrate these advantages, consider again a physical phenomenon that is described by 7 variables. We wish to establish a relationship (model) that predicts the dependent variable in terms of the 6 independent variables. If we use experimental data to determine the relationship by obtaining experimentally a value for the dependent variable for 5 values of each of the independent variables, then  $5^6 = 15,625$  data points are required. If the results are presented in graphical form, with each chart representing the dependent variable as a function of 2 independent variables, the remaining 4 independent variables being constant for that particular chart, then an unwieldy  $5^4 = 625$  charts are required. If the relationship is expressed in equation form, then an equation containing 6 independent variables must be fitted to the data. The latter is a formidable task since the relations are most often non-linear and many different variable transformations must be tried for each variable in order to obtain an equation that fits the data well.

If dimensional analysis is used to reduce the number of variables from 7 to 4 and experimental data is taken for 5 values of each of the

3 independent variables, then only  $5^3 = 125$  data points are required. In this case data requirements are reduced by over 99 percent through the use of dimensional analysis! Additionally, the results can be presented graphically by a set of 5 charts rather than 625. If an equation is to be fitted to the data, the equation would contain only 3 independent variables; consequently, the equation would be easier to obtain and simpler to use than an equation containing 6 independent variables.

2. A knowledge of the inner mechanism of the phenomenon is not required.

3. The model that results from the utilization of dimensionless groupings is very general in nature. For example, a particular value of a dimensionless grouping of 4 variables may result from any one of an infinite number of combinations of values of the 4 variables. This characteristic of dimensional analysis will be discussed in a subsequent example.

4. The value of a dimensionless grouping of variables can be varied in an experiment by varying the value of only one of the variables. The experiment can therefore be planned so that the value of the dimensionless grouping is controlled by varying the variable that is most easily and economically controlled.

5. Dimensionless groupings can be interpreted physically, thus adding to the understanding of the phenomenon.

## Characteristics of Dimensional Analysis

Some important characteristics of dimensional analysis may be summarized as follows:

1. In order to successfully apply dimensional analysis, the phenomenon must be understood well enough so that all the variables that affect the phenomenon are identified.
2. All variables must be quantifiable.
3. Dimensional analysis by itself does not provide a model of the phenomenon. Information concerning the relationship between the variables, most often in the form of experimental data, is required.
4. Dimensional analysis does not explain the inner mechanism of a phenomenon.
5. The more we know about the phenomenon, the easier the dimensional analysis and curve fitting procedures.

APPLICATION OF DIMENSIONAL ANALYSIS TO A TYPICAL  
CASE: SPLINE RELIABILITY PREDICTION

The use of dimensionless ratios as a tool in mechanical reliability has potential for a wide range of components and systems. For illustration purposes a single component with simplified assumptions will be used in the following paragraphs. The class of mechanical equipment which fails mainly due to wear in constant rotation has a large number of variables suitable for dimensionless grouping and a method suitable for one component could be expected to be similar to a method suitable for another component. Components in this class include gears, cams, splines, sprockets, bearings, seals, impellers, rollers, casters and tires.

Since splines are of special interest in mechanical reliability studies by the Navy at the present time they were chosen as an example to illustrate the method of dimensionless ratios. Splines may be classified as fixed or working, with tooth types of different geometries (such as full depth involute, half depth involute, circular, etc.) and possible failure modes include fatigue, shear failure, and fretting corrosion as well as the primary failure mode--wear.

Forming the Dimensionless Groups

A listing of the more common independent factors affecting the wear life of splines with a specified type of teeth includes: length, diameter, torque, speed, tooth hardness, and misalignment. Other factors

such as contact area, bearing stress, and horsepower transmitted are dependent upon the above factors and are not included as variables.

In order to be expressible in equation form, a variable must be quantifiable. The present state of knowledge for splines does not permit the effect of lubrication on reliability to be quantified. Similarly, shock loading effects and tooth type have not been quantified. It should be noted that shock loading effects can be quantified; however, much experimental work would be required and a better solution would be to have a separate mathematical model for each application (general type of shock effect). In addition, the best approach is a separate mathematical model for each type of non-quantifiable parameter, i.e., a different model for each type of lubrication and tooth type.

Adding a dependent variable, mean-time-to failure, to the list of six independent variables gives seven variables. A letter symbol will be used for each as follows:

t -- Torque, inch-pounds

N -- Speed, revolutions per minute

ℓ -- Spline length, inches

D -- Spline diameter, inches

B -- Tooth hardness (Brinell hardness, with units of kilograms per square millimeter)

α -- Misalignment angle, radians

F -- Mean time to failure, hours

Using the pi theorem (See Appendix A) the seven variables are put into four dimensionless groups as follows:

$$FN; \frac{BD^3}{t}; \frac{\ell}{D}; \alpha$$

Brinell hardness is used because it has a specific set of units. If other values, such as Rockwell C are used, they must be converted to Brinell.

To simplify the frequent use of these four parameters throughout the example to follow, two new symbols are to be used, namely  $\theta$  and  $\phi$ .

$\theta = 60FN$  and may be considered as the life of the spline in total revolutions.

$\phi = \frac{1422BD^3}{t}$  and may be considered as a "strength-to-load" ratio; with higher numbers indicating longer life for a given  $\ell/D$  ratio and misalignment,  $\alpha$ .

The  $\ell/D$  ratio is large for long slender splines and small for short stubby splines. In this case the diameter was taken to be the pitch diameter.

Since  $F$  is in hours and  $N$  is in revolutions per minute, the constant 60 minutes per hour is included to make  $\theta$  dimensionless. Similarly 1422 is included in  $\phi$  to make the units of Brinell hardness compatible with the units of  $D$  and  $t$ .

The procedure now is to find the relationship that exists among the four variables from some existing theory or data. Although any one of the four variables may be used as the explicit variable, we will choose  $\theta$  as the most desirable. That is,

$$\theta = f(\phi, \ell/D, \alpha). \quad (1)$$

#### Data

In order to find the unknown function in the above equation some spline test data was sought that included all the parameters. Some test data was available, such as from Southwest Research Institute (12),

but the values of  $N$ ,  $\ell$ ,  $D$ , and  $t$  were the same for all the runs. An involute spline design procedure outlined by Raymond J. Drago (13) includes all the needed variables in chart form to establish the above functional relationship. It was decided to use this procedure since results would be more complete. A copy of this procedure from Machine Design, February 12, 1976, is included as Appendix B.

Table 1 in Appendix C was constructed by choosing several different values of  $L$  while holding all other variables constant to give different  $\ell/D$  values. The second half of the table is constructed to give different values of  $\phi$  while holding all other variables constant.

Table 2 in Appendix C is a more complete set of data for full depth involute splines taken from the Drago charts.

The quantities in Tables 1 and 2 such as  $J$  (geometry factor),  $T_F$  (torque factor),  $B_s$  (bearing stress, and  $K_L$  (life factor) are explained in Appendix B. Also the numerical values of  $\alpha$  in both tables were obtained by taking the mid-range value from those given in Appendix B for light, moderate, and heavy misalignment.

#### Establishing the Relationship Among the Dimensionless Variables

Graphical. The curves shown in Figures 1 and 2 show how  $\theta$  varies with  $\phi$  and  $\ell/D$ . When the two curves are compared it is noted that they are similar if not identical. This implies that the two variables,  $\ell/D$  and  $\phi$ , may be combined into a single dimensionless variable to reduce the number of variables to 3 if so desired. This provides an extra measure of flexibility in using the dimensionless variables. Reduction to 3 variables reduces still further the amount of test data needed to establish a mathematical life model for a new type of spline; but keeping

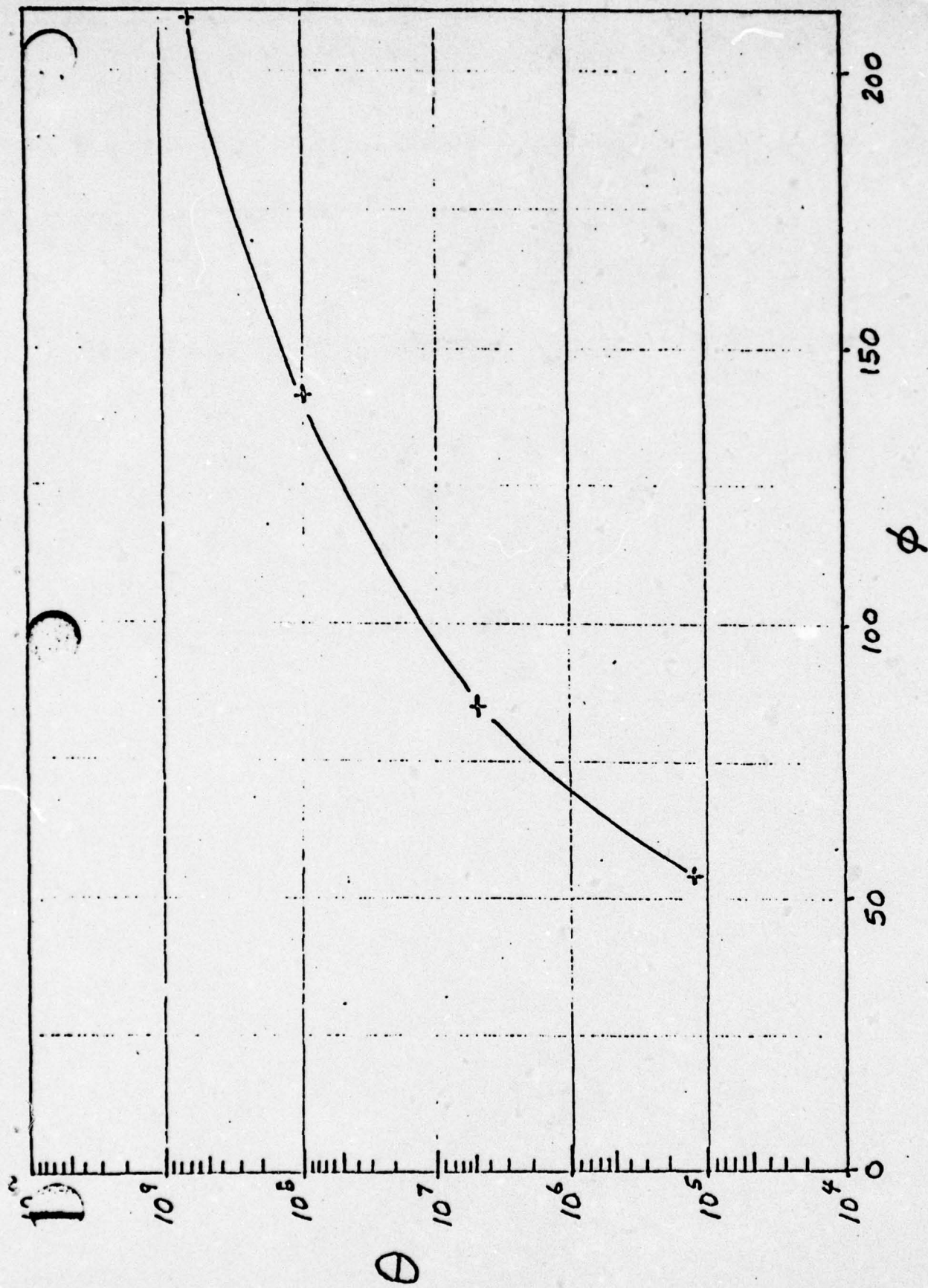


Figure 1.  $\theta$  vs.  $\phi$ .

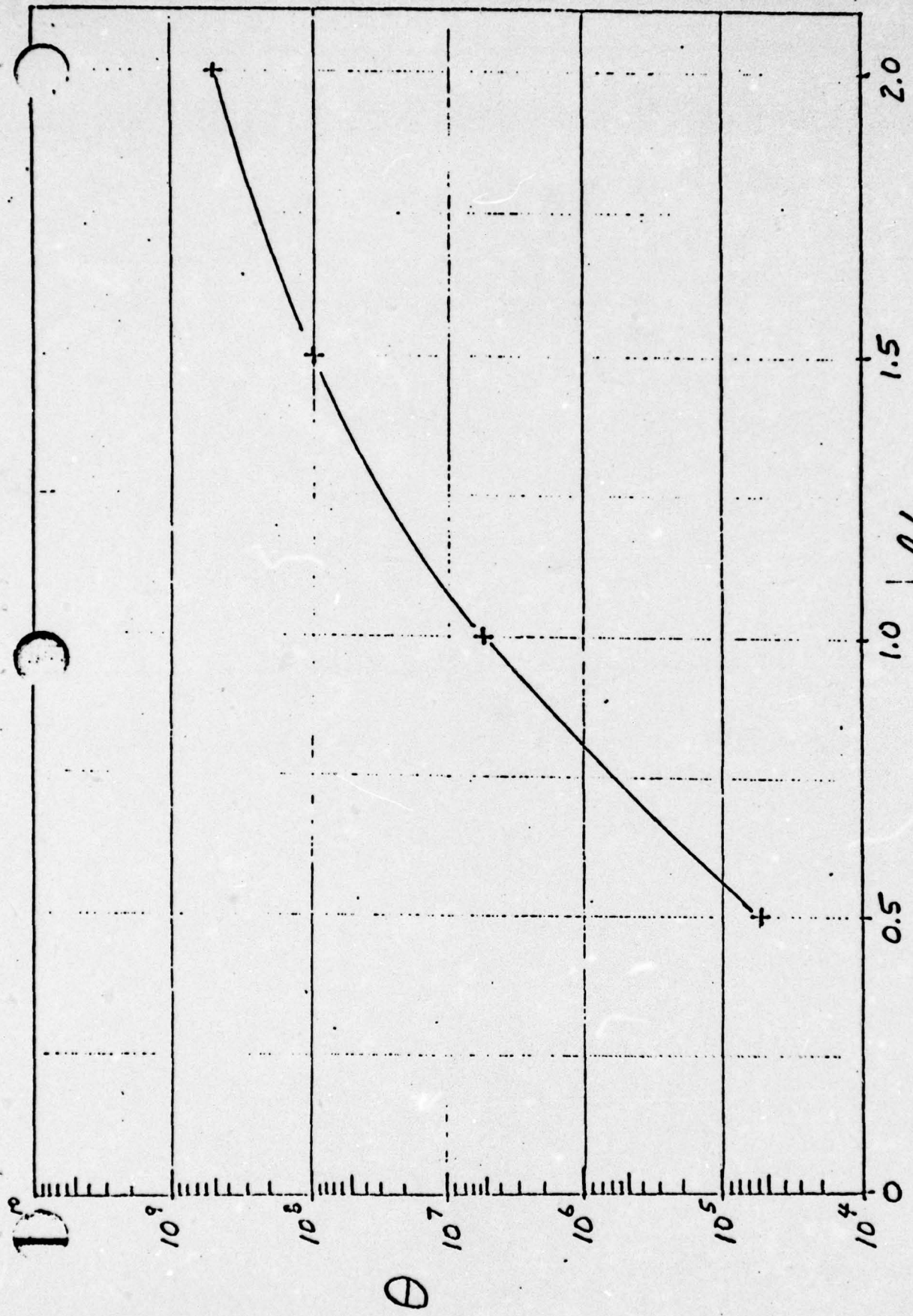


Figure 2.  $\theta$  vs.  $l/D$ .

$l/D$  as a separate variable gives the designer a special flexibility factor for selecting spline geometry for maximum life.

Figure 3 is a set of curves of  $\log \theta$  vs.  $\log \left(\frac{\phi l}{D}\right)$  for different values of  $\alpha$ . This represents the relationship among the variables in graphical form. The physical significance of  $\frac{\phi l}{D}$  is that it represents a ratio of tooth hardness to bearing pressure for the particular spline and its application.

Regression Analysis. Regression analysis was used to determine the exact functional relationship for  $\theta$  as a function of  $l/D$ ,  $\phi$ , and  $\alpha$  using the 33 "data" points previously discussed. Several linear regression models were investigated.

The first approach used a stepwise regression technique to predict  $\theta$  with the variables  $l/D$ ,  $\phi$ ,  $\alpha$ , their squares, and their cross products. In stepwise regression, variables are added to the model one at a time until no further improvement can be found. The variable with the highest correlation with the dependent variable is added first. Other variables are chosen on the basis of their ability to produce a significant F statistic. The F statistic is indicative of the predictive power of the model being tested. It also reflects the goodness of the fit. The higher the F value, the better the model describes the data. The term significant means that the F value being examined has a very small probability ( $<.05$ ) of occurring by chance.

The stepwise regression procedure yielded a model of the form  $\theta = a_0 + a_1 \alpha + a_2 \alpha^2$ . For this model  $R^2 = .26$ .  $R^2$  indicates the proportion of the variability in  $\theta$  explained (or accounted for) by the

current regression model. Since 74% of the variance in  $\theta$  was unexplained by the above model, it was deemed unsatisfactory and another approach was initiated.

Figure 3, a plot of  $\log \theta$  vs.  $\log (\ell/D \cdot \phi)$  for each of the three values of  $\alpha$  indicated a strong linear relationship. A regression model of the form  $\log \phi = a_0 + a_1 \log (\ell/D \cdot \phi)$  offered support for this notion as  $R^2 = .81$  for the case  $\alpha = .0005$ ,  $R^2 = .82$  for  $\alpha = .0060$  and  $R^2 = .93$  for  $\alpha = .0025$ .

A single model including the contribution of  $\alpha$  was then investigated. This model was

$$\log \theta = -9.15 + 4.56 \log (\ell/D \cdot \phi) - 2.36 \log \alpha.$$

The resulting  $R^2$  value was .89. Several other models were investigated, but each of the other models had an  $R^2$  less than 89%. The above model was chosen as the best predictor of  $\log \theta$  since it explains 89% of the variance in  $\log \theta$ .

Taking the antilog of both sides of this model, we have

$$\theta = 7.08 \times 10^{-10} (\ell/D \cdot \phi)^{4.56} \alpha^{-2.36}. \quad (2)$$

This equation was used to generate 33 predicted values of  $\theta$  and comparisons were made with the actual data points. Figure 4 shows a comparison of the values of  $\theta$  predicted by equation (2) vs. the actual data point values of  $\theta$  for the 33 data points. Figure 4 along with the value of  $R^2 = 89\%$  demonstrates that MTF for this spline case can be predicted by developing a mathematical model using dimensionless groupings.

It should be pointed out that some of the scatter in the "data" is due to the fact that each value of  $\theta$  is a result of reading values from 4 charts in sequence so that there can be considerable accumulation of

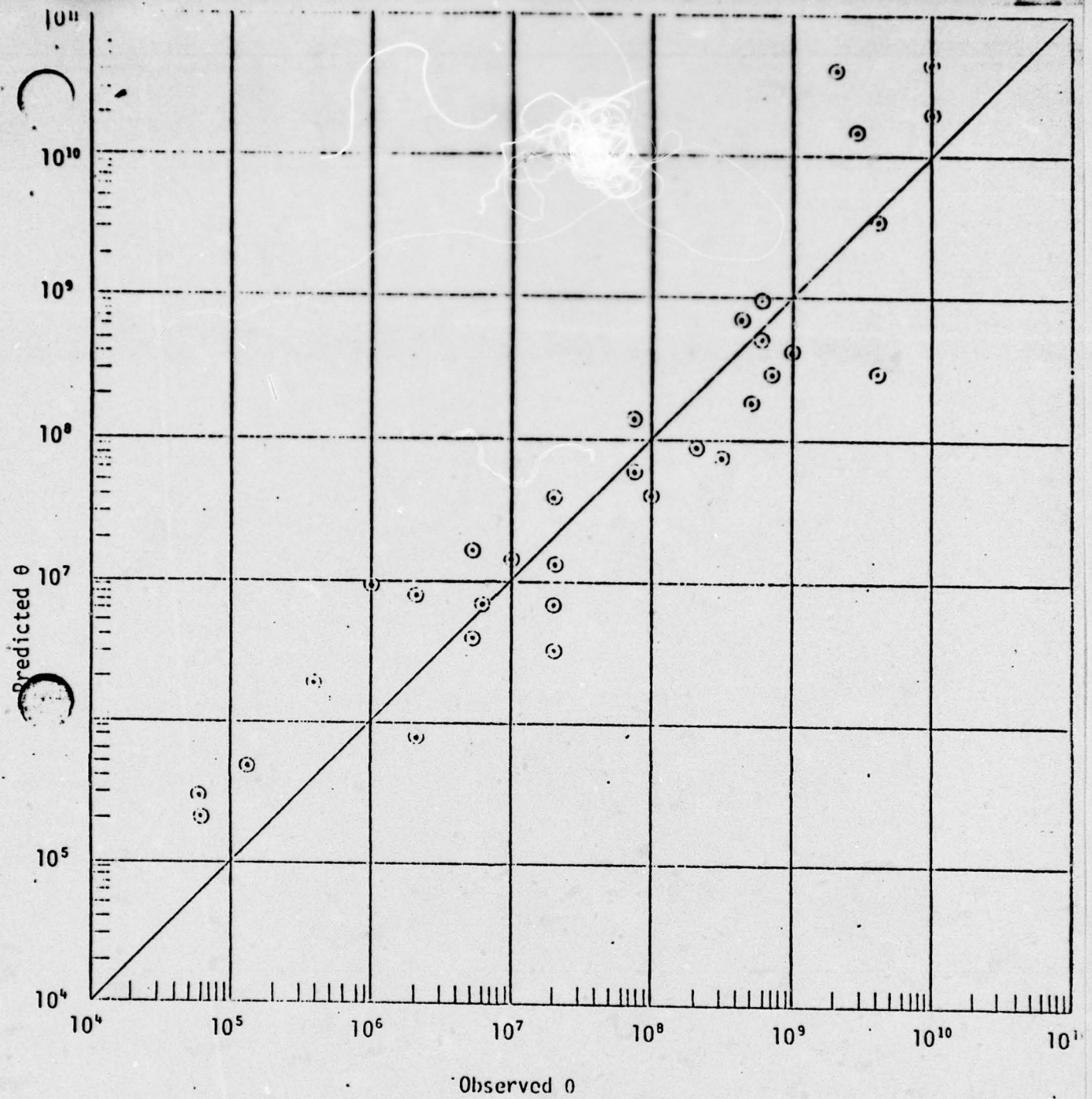


Figure 4. Comparison of  $\theta$  predicted by equation (2) to actual data point values.

error due to reading of the charts. This scatter is not considered detrimental to the present study since any actual experimental data would exhibit scatter due to experimental error, variation in material properties, etc. Another point of note is the fact that the Drago method represents a very elementary mathematical model and predicted values may differ considerably from actual experimental values; therefore, the mathematical model obtained using dimensional analysis and regression analysis will predict MTTF with an accuracy that is no better than the accuracy of the data on which it is based, (i.e., the Drago method).

## CONCLUSIONS

The preceding example demonstrates that dimensional analysis is an important tool for use in the prediction of mechanical reliability and should be implemented in all future test programs. Through the use of dimensionless groupings of variables, a predicting equation for mean time to failure as a function of 7 independent variables was fitted to 33 "data" points. The primary advantages of the use of dimensional analysis are:

1. The number of variables in the problem is reduced, thus reducing data requirements and simplifying the application of regression analysis.

2. Dimensional analysis can be very useful in planning experimental work to get maximum information from a minimum amount of raw data since only the magnitude of each dimensionless group needs to have a range of values and every individual variable does not need to be changed. Also, the most important or most influential variables may be isolated easier when the variables are in dimensionless groups. By performing a dimensional analysis before an experiment is planned, the testing can be done more economically. Since one dimensionless group can be varied by varying only one of the variables in the group, the experimenter can choose to control or vary the dimensionless group by varying or controlling the variables that are most easily varied or controlled.

3. A mathematical model written in the form of dimensionless groups of variables for a specific design or type of item can be more readily adapted to a new design of the item. For example, it is likely that a correlation for involute splines could be revised for circular splines in an easier manner if the correlation is in the form of dimensionless groups of variables.

In addition, the use of dimensional analysis allows prediction of reliability for certain operating conditions for which data is unavailable. For example, suppose in the process of designing a spline that is to operate at a speed of 4000 rpm, we find that the data is available only for a speed of 2000 rpm. If a predicting equation in dimensionless form is fitted to the 2000 rpm data, then, in the absence of 4000 rpm data, the predicting equation can be used to predict reliability of splines at 4000 rpm.

## RECOMMENDATIONS FOR FUTURE WORK

1. There are strong indications that dimensional analysis could be used in developing improved methods of accelerated testing. Further work should be done to determine the feasibility of using dimensionless variables to relate the results of accelerated testing to actual use conditions.

2. The applicability of dimensional analysis to the prediction of the reliability of variety of mechanical components should be investigated. Such an investigation would include further work on the prediction of reliability for splines as well as devices such as gears, pumps, etc. It is expected that such an investigation would provide input for planning and analysis of current and future mechanical reliability test programs.

## LIST OF REFERENCES

1. Kreith, Frank, Principles of Heat Transfer, Second Edition, International Textbook Company, Scranton, Pa., 1965.
2. Vennard, John K. and Robert L. Street, Elementary Fluid Mechanics, Fifth Edition, John Wiley and Sons, Inc., New York, 1975.
3. Bridgeman, P. W., Dimensional Analysis, Revised Edition, 1931, Yale University Press, New Haven.
4. Focken, C. M., Dimensional Methods and Their Applications, 1953, Edward Arnold and Company, London.
5. Ipsen, D. C., Units, Dimensions, and Dimensionless Numbers, McGraw-Hill Book Company, Inc., New York, 1951.
6. Sutherland, Robert L., Engineering Systems Analysis, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1958.
7. Langhaar, H. L., Dimensional Analysis and Theory of Models, McGraw-Hill Book Company, Inc., New York, 1951.
8. Sedov, L. I., Similarity and Dimensional Methods in Mechanics, Translated by Morris Friedman from the 4th Russian Edition, 1959, Academic Press, New York.
9. Lockheed-Georgia Company, Investigation of Reliability of Mechanical Systems, Prepared for the Bureau of Naval Weapons (AD 475-977), Marietta, Georgia, 1965.
10. ARINC Research Corporation, A Technique for Design Prediction of Reliability and Maintainability of Mechanical Equipment (Task 4), Publication No. 594-01-4-962, Annapolis, Maryland, 1969.
11. Van Driest, E. R., "On Dimensional Analysis and Presentation of Data in Fluid Flow Problems," J. Appl. Mech., Vol. 13, (1940), p. A-34.
12. Ku, P. M. and M. L. Valtierra, "Spline Wear-Effects of Design and Lubrication," Transactions of the ASME, November, 1975, pp. 1257-1265.
13. Drago, Raymond J., "Rating the Load Capacity of Involute Splines," Machine Design, February 12, 1976, pp. 104-109.

APPENDICES

## APPENDIX A

### Determination of Dimensionless Variables for Spline Example

We wish to form a complete set of dimensionless variables for the case of full-depth involute splines as discussed in the section Forming the Dimensionless Groups. The method outlined in Reference (7) will be used. The pertinent variables along with their dimensions are:

| <u>Variable</u>      | <u>Symbol</u> | <u>Dimension</u>                    |
|----------------------|---------------|-------------------------------------|
| Mean Time to Failure | F             | [T]                                 |
| Speed                | N             | [T <sup>-1</sup> ]                  |
| Torque               | t             | [ML <sup>2</sup> T <sup>-2</sup> ]  |
| Length               | ℓ             | [L]                                 |
| Diameter             | D             | [L]                                 |
| Hardness             | B             | [ML <sup>-1</sup> T <sup>-2</sup> ] |
| Angular Misalignment | α             | Dimensionless                       |

Since the misalignment is already dimensionless, we will deal with the first 6 variables only. Any dimensionless grouping of these 6 variables will be of the form:

$$\Pi = F^a N^b t^c \ell^d D^e B^f$$

where  $\Pi$  represents a dimensionless group (sometimes called a "pi-term"). The values of the exponents a, b, c, etc., will be determined such that we have a complete set of dimensionless variables formed from the 6 variables. The dimension equation for  $\Pi$  is

$$[M]^0 [L]^0 [T]^0 = [T]^a [T^{-1}]^b [ML T^{-2}]^c [L]^d [L]^e [ML^{-1} T^{-2}]^f.$$

The above equation may also be written as

$$[M]^0 [L]^0 [T]^0 = [M]^{c+f} [L]^{2c+d+e-f} [T]^{a-b-2c-2f};$$

therefore, we can write the following 3 equations:

$$c+f = 0$$

$$2c+d+e-f = 0$$

$$a-b-2c-2f = 0$$

The matrix of the coefficients of the 6 unknown in the above equation is:

$$\begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 & -1 \\ 1 & -1 & -2 & 0 & 0 & -2 \end{array}$$

The rank of the above matrix is 3; therefore, the number of pi-terms to be formed from the 6 variables is  $6 - 3 = 3$ .

Noting that we have 3 equations and 6 unknowns, we must choose values for 3 of the unknowns and solve for the remaining 3 in determining each pi-term. For the first pi-term, we choose  $a = 1$  (This insures that F will appear in  $\Pi_1$ ),  $c = d = 0$ . The 3 simultaneous equations now become:

$$f = 0$$

$$e-f = 0$$

$$1-b-2f = 0$$

The solution of this set of equations yields  $f = 0$ ,  $e = 0$ ,  $b = 1$  so that

$$\Pi_1 = F^1 N^1 t^0 \ell^0 D^0 B^0$$

$$\Pi_1 = FN$$

We want the dependent variable,  $F$ , to appear in one pi-term only; therefore, in determining  $\Pi_2$ , we let  $a = 0$ . We also let  $c = 0$ ,  $d = 1$ .

The set of equations becomes:

$$f = 0$$

$$1 + e - f = 0$$

$$b - 2f = 0,$$

the solution of which is  $f = 0$ ,  $b = 0$ ,  $e = -1$ . Therefore,

$$\Pi_2 = F^0 N^0 t^0 \ell^1 D^{-1} B^0$$

$$\Pi_2 = \ell/D$$

Finally, let  $a = d = 0$ ,  $f = 1$  so that the set of equations becomes

$$c + 1 = 0$$

$$2c + e - 1 = 0$$

$$-b - 2c - 2 = 0$$

The above set of equations has the solution  $c = -1$ ,  $e = 3$ ,  $b = 0$ , yielding

$$\Pi_3 = F^0 N^0 t^{-1} \ell^0 D^3 B^1$$

$$\Pi_3 = \frac{BD^3}{t}$$

These 3 pi-terms along with  $\alpha$  which is already dimensionless are a complete set of 4 dimensionless variables formed from the variables pertinent to the spline case:

$$\Pi_1 = FN$$

$$\Pi_2 = \lambda/D$$

$$\Pi_3 = \frac{BD^3}{t}$$

$$\Pi_4 = \alpha$$

The above dimensionless variables form a complete set because each dimensionless variable in the set is independent of the others in the sense that no dimensionless variable in the set is a product of powers of the other dimensionless variable. Although innumerable other dimensionless variables can be formed, any other dimensionless variable is a product of powers of the dimensionless variables in the set.

Obviously, there is an infinite number of complete sets of dimensionless variables other than the set developed above. Selection of the best complete set depends on the way in which the model is to be used and the experience of the engineer. Any knowledge available concerning the phenomenon enhances the ability of the engineer to select the best complete set.

# Rating the Load Capacity of

Much work has been done to standardize spline geometries, but little has been done to standardize load-capacity ratings. Thus, spline performance is difficult to predict with conventional design procedures. Now, with a new set of charts, service life can be determined quickly.

**RAYMOND J. DRAGO**  
Senior Engineer  
Boeing Vertol Co.  
Philadelphia, Pa.

**INVOLUTE SPLINES** connect rotating parts in applications as diverse as helicopter rotors and machine-tool gearboxes. But too often, each application uses its own special design. There is very little literature on how to determine the interchangeability of a spline among different operating conditions.

As a result, a new spline application must be analyzed in depth to determine how long a particular design will last or, conversely, which design best fits the functional requirements. Now, with a newly developed set of design charts, a spline can be checked quickly to determine its life under specific operating conditions. Or, the performance requirements can be used to find a spline design that will last for the desired life.

The charts are based on equations common to the industries that use involute splines. They represent "average" operating conditions, and they apply to almost any spline application.

The prime factors to consider in spline design are the compressive (surface bearing) stress  $S_c$  and the shear (tooth) stress  $S_s$ . The equations for these stresses are

$$S_c = (2T/D) (1/NLI) (K_m) \\ \leq S_{c,0} K_r / K_t K_f$$

$$S_s = (2T/D) (1/DNL) (K_m) \\ \leq S_{s,0} K_r / K_t$$

These equations (with  $N$ ,  $H$ , and  $t$  converted to functions of diametral pitch) are the basis for the design charts presented here. The only approximation involved in the charts is in the use of "nominal" tooth thickness in the shear stress equation.

For most designs, this approximation introduces a maximum error of less than 5%.

Before getting into a discussion of how to use the charts, it is important to understand how splines work, how they fail, and how various processes help prolong life. This understanding makes the load factors ( $K_m$  and  $K_n$ ) easier to estimate.

## Types of Spline Joints

Generally, spline joints fall into two categories: fixed and working. In a working joint, the mating members are free to move relative to each other. But in a fixed joint, both ends of the spline are clamped and piloted to prevent relative motion. In addition, many intermediate (semiworking or semi-fixed) spline joints are possible. For example, a spline with loose pilots at each end or with O-rings to retain a grease pack, while not fully fixed, cannot be considered fully working either because relative motion is restricted to some degree.

A working spline generally joins two independently supported shafts that may also be slightly offset axially (from tolerance buildups). This type of spline accommodates the offset easily without excessive loss in basic load capacity. Fixed splines may also join independently supported shafts, but the shafts must be aligned more accurately. Most frequently, however, fixed splines mount precision gears to shafts, particularly in high-load, high-speed, lightweight applications.

Usually, the surface load capacity of a fixed spline is considerably higher than that of a working spline. In a working spline, relative motion between mating surfaces induces a high wear rate, even at relatively low bearing stresses. But in a fixed joint, restricted motion results in a low wear rate, making surface fatigue the primary failure mode.

## Spline Failure Modes

Spline joints "fail" in any number of different ways, but many failure modes (such as fretting corrosion) cannot be analyzed directly. Fortunately, physical failures can be analyzed, and they are generally divided into four classes. Two classes, shaft shear (torsional shear failure of the shaft section under the spline teeth) and bursting of an internal spline (from radial and centrifugal loads),

# Involute Splines

are primarily shaft related and are not treated here because they do not occur often.

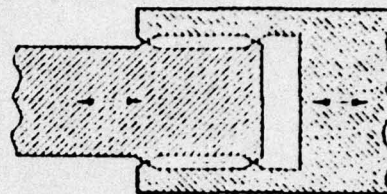
The other two tooth-related failure modes are much more common. These modes are tooth shear (actual shearing off of a tooth projection from the shaft) and surface distress of the teeth (from wear or surface fatigue). In the shear mode, a spline immediately ceases to transmit power, while in the wear mode it degrades gradually before complete failure. Wear failure can only be detected during periodic inspection.

Surface distress is by far the most common failure mode encountered in spline joints. This failure may take the form of wear (material removed from mating surfaces) or spalling of mating surfaces from fatigue. Frequently, over a long period of time, these two types of failure combine, with fatigue resulting from a worn surface. On the other hand, depending on spline configuration, the joint may "wear-in" to remove a potential failure zone. Here, highly loaded local areas caused by high spots, slight misalignment, or tooth index errors may wear to relieve the load and distribute it over a larger area, thus prolonging life.

One of the most significant factors causing wear in a spline joint is misalignment, which causes high bearing stresses at the contact points. Normally, with a well aligned shaft, wear can be prevented by lengthening a spline to reduce bearing stress, thus lowering wear rate. However, if wear is caused by excessive misalignment, lengthening the spline is not likely to reduce wear rate. Better solutions are to reduce length and increase diameter, or perhaps crown the teeth of the external member. Of course, the best solution is to eliminate misalignment altogether; however, this fix may not be practical from a design standpoint (the misalignment may be the very reason why the spline is used). Generally, though, if substantial misalignment is inherent in the application, a crowned spline, a large-diameter short-length spline, or a more complicated coupling should be custom designed for the application.

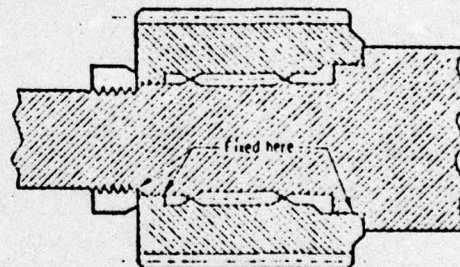
One way to overcome slight misalignment (particularly in high speed, high-load, precision gearboxes) uses a "quill" shaft between the driving and driven shafts. Basically, a quill shaft is an extra shaft, splined at both ends, that floats between the other two shafts. This free-floating capability allows the quill shaft to self-align to some degree

Working Spline



In a working spline joint, the mating parts are free to move relative to each other. This type of spline usually connects two independently supported shafts that also may be slightly offset axially. It accommodates the offset with little loss in load bearing capacity. Wear is the predominant failure mode.

Fixed Spline



A fixed spline joint restricts relative motion between mating parts. This type of spline usually mounts precision gears to shafts in high-load, high-speed, lightweight applications. Wear combined with surface fatigue is the primary failure mode.

to accommodate the mismatch between the driving and driven members. But quill shafts are difficult to lubricate and position. And, if misalignment is large, a significant bending moment may be applied to the quill shaft, so it behaves somewhat like a rotating fatigue-test specimen. Crowned teeth add to the misalignment capability of a quill shaft, especially in high-speed, high-load applications in aircraft.

Shear failure, as a primary mode, is relatively

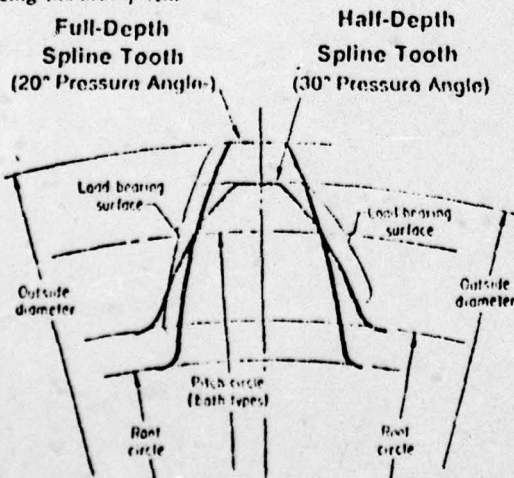
rare in most spline applications. This failure usually occurs as a secondary mode after the teeth have worn excessively thin or as a result of severe overload or impact loading. However, shear can be a primary mode in a spline joint subjected to severe impact loading throughout its life. In this case, shear analysis should be emphasized because the spline may fail in shear before it has a chance to wear in. Therefore, even though shear may not be a determining factor in the design of a spline joint, shear stresses should always be checked as a matter of course.

### Full-Depth vs. Half-Depth Teeth

Both full and half depth splines can be analyzed with these charts. Most splines in use are half-depth because they have a number of advantages over full-depth splines. First, because tooth height generally determines the amount of backup material required behind the teeth, the half-depth spline provides a more compact, and often lighter, overall design. Second, the relatively high teeth on a full-depth spline are somewhat more sensitive to misalignment. In addition to these operational considerations, several manufacturing methods, such as broaching or roll forming, are easier to perform on the smaller half-depth spline teeth.

In spite of these considerations, full-depth splines have about double the surface (bearing) load capacity of equivalent half-depth splines. In applications where this increased capacity is needed, it is worth the effort to overcome the inherent disadvantages of the full-depth design. A typical application for full-depth splines is on the rotor shafts of some large helicopters; joints that are generally fixed so that other variables can be controlled. Another application is in high-speed, "gear" couplings that use full-depth (often crowned) teeth to connect rotating shafts.

The standard pressure angle for half-depth splines is  $30^\circ$ . The pressure angle of full-depth splines commonly is  $20^\circ$ , however, it sometimes must be varied according to the particular design to avoid pointed teeth. The drawing shows the differences in tooth size and bearing area between full and half-depth splines. The addendum of a full-depth spline equals  $1/P_d$ , while that of a half-depth spline equals  $1/2P_s$ , where  $P_d$  = diametral pitch (ratio of number of teeth to pitch diameter) and  $P_s$  = "stub" pitch ( $2P_d$ ). The pitch of a spline commonly is given as a ratio (16/32, 20/40, and so on) with the numerator being the diametral pitch and the denominator being the stub pitch.



### Coatings and Surface Treatment

Wear is often caused by poor lubrication. Sometimes, despite the most elaborate lubrication system, the only way to prevent wear is to apply a coating or surface treatment.

The most common coatings are soft ones like black oxide, phosphate, nylon, silver, and copper, and various hard coatings such as flame or plasma-sprayed materials. The soft coatings aid initial wear-in and also lubricate the surface with a pliant film during operation. The hard coatings approach the wear problem from the opposite end of the spectrum; that is, they produce a surface that resists wear by virtue of extreme hardness.

If the surface bearing stresses are low (less than 3,000 psi), a nylon coating is often the best way to prevent wear and damp the spline joint. The coating, a few thousandths of an inch thick, is applied to the entire spline surface (generally on the external member). Usually, to allow room for the thickness of the nylon and still provide proper fit, the cut size of the spline must be reduced. The other coatings are thin enough (on the order of a few thousandths of an inch) so that no modification is required.

Other alternatives to improve spline load capacity include case hardening processes such as carburizing and nitriding. These processes generally produce a very hard ( $R_c$  58 to 64) wear resistant surface. Unfortunately, they also increase the cost of the parts because special operations are needed to control or correct heat treatment distortions.

### EXAMPLES

**Problem 1:** Consider a fixed, half-depth spline transmitting torque  $T = 1,000$  lb-in. and operating under light shock and misalignment (less than 0.001 in./in.). The spline is made of AISI 9310 case carburized steel with a surface hardness of  $R_c$  60 to 64. Dimensions are  $L = 0.30$  in. and  $D = 0.75$  in. Find the bearing and shear stresses and the projected life.

1. On Chart 1, at  $L = 0.30$  in. and  $D = 0.75$  in., find  $J = 9.5$ .
2. On Chart 2 (half-depth bearing stress plots, light misalignment), at  $J = 9.5$ , find  $K = 0.5$  for bearing stress.
3. On Chart 2 (shear stress plots, light misalignment), at  $J = 9.5$ , find  $K = 6.0$  for shear stress.
4. Calculate  $S_{t4} = (1000)(9.5) = 9,500$  psi.
5. Calculate  $S_{s4} = (1000)(6.0) = 6,000$  psi.
6. On Chart 3A, for  $S_{t4} = 9,500$  psi,  $K_s = 1.0$  (light shock), and  $R_c$  60 carburized steel, find bearing stress life factor  $K_L = 1.30$ .
7. On Chart 3B, for  $S_{t4} = 6,000$  psi,  $K_s = 1.0$ , and  $R_c$  60 carburized steel, find shear stress life factor  $K_L = 0.125$ .
8. On Chart 4, at  $K_L = 1.30$  for a fixed spline, find bearing stress life  $= 7.4 \times 10^9$  revolutions.
9. On Chart 4 again, at  $K_L = 0.125$ , find shear stress life  $= \infty$ .
10. Probable life is  $7.4 \times 10^9$  revolutions.

Although surface-hardened parts have a high wear life, they may perform poorly if not made accurately. These parts do not "wear-in" very well and may destroy themselves if each tooth does not carry its share of the load. On smaller splines, lapping is an inexpensive and often effective way to finish hardened surfaces. This process improves the finish and corrects minor errors at considerably lower cost than grinding.

### Lubrication

Because splines are often outside an enclosure, a pressurized oil system may not be available to lubricate the joint. Therefore, grease is a common spline lubricant. Generally, the spline joint is packed full of grease and a retention method devised so that the grease does not centrifuge out of the joint.

One drawback to grease lubrication is that wear particles are trapped in the grease. After a time, the mixture becomes abrasive and may accelerate wear. For this reason, grease lubricated working splines should be regreased periodically, and the old grease (and wear particles) purged rather than merely replenished. One exception to this rule is the case of a fully locked or fixed spline. If relative motion is severely restricted, the initial grease pack may last for hundreds or even thousands of hours.

If the spline is internal and the gearbox is oil lubricated, the joint may be lubricated by oil splash or direct oil jet. In either case, the oil flow both

lubricates and removes wear particles, thus reducing wear.

Another method of lubrication that has been used with some success directs oil down the center of a hollow shaft to radial holes in the spline. The oil flows by centrifugal force through the spline and out through external metering holes. The holes should be large enough to prevent clogging yet small enough to "flood" the tooth area. This method is an effective means of cooling and lubricating the joint and flushing out sludge.

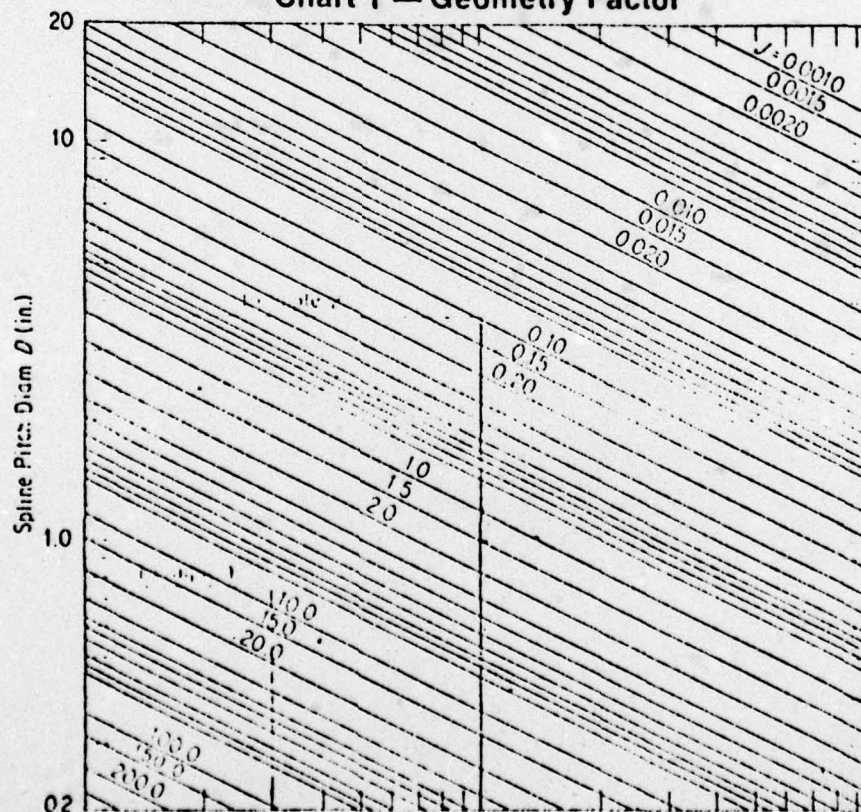
Coatings and lubricants generally work in concert. For example, black oxide enhances the oil retention or oiliness of spline teeth, and silver plate acts like an extreme pressure film during break-in and operation. In some cases, however, lubrication is superfluous, as with some forms of nylon coatings (these coatings operate best when unlubricated).

### Rating Spline Load and Life

With this basic understanding of the factors that affect spline life, the use of the design charts for rating load and life will be more clear. The charts can be used in either of two cases. First, if spline material, geometry, and operating conditions are known, the load capacity and potential life of the joint can be determined. Second, if the desired life, loading conditions, and material are known, the geometry can be found.

Two factors must be determined before the

Chart 1 — Geometry Factor



analysis can begin: the load distribution factor  $K_m$  and the overload factor  $K_o$ .  $K_m$  is a function of the misalignment present in the joint. Since misalignment can have a significant effect on the load capacity, care should be taken to choose as realistic a value as possible for  $K_m$ . Typically, for light misalignment (0.000 to 0.001 in./in.),  $K_m = 1.0$ ; for moderate misalignment (0.001 to 0.004 in./in.),  $K_m = 2.0$ ; and for heavy misalignment (0.004 to 0.008 in./in.),  $K_m = 3.0$ .

The actual peak load applied to a spline may be considerably higher than the steady-state design load. Factor  $K_s$  adjusts the load rating to account for any shock loads on the driven and driving members. Typical values for  $K_s$  are listed in the table near Chart 3.

The procedure when load and life must be determined from material and geometry is:

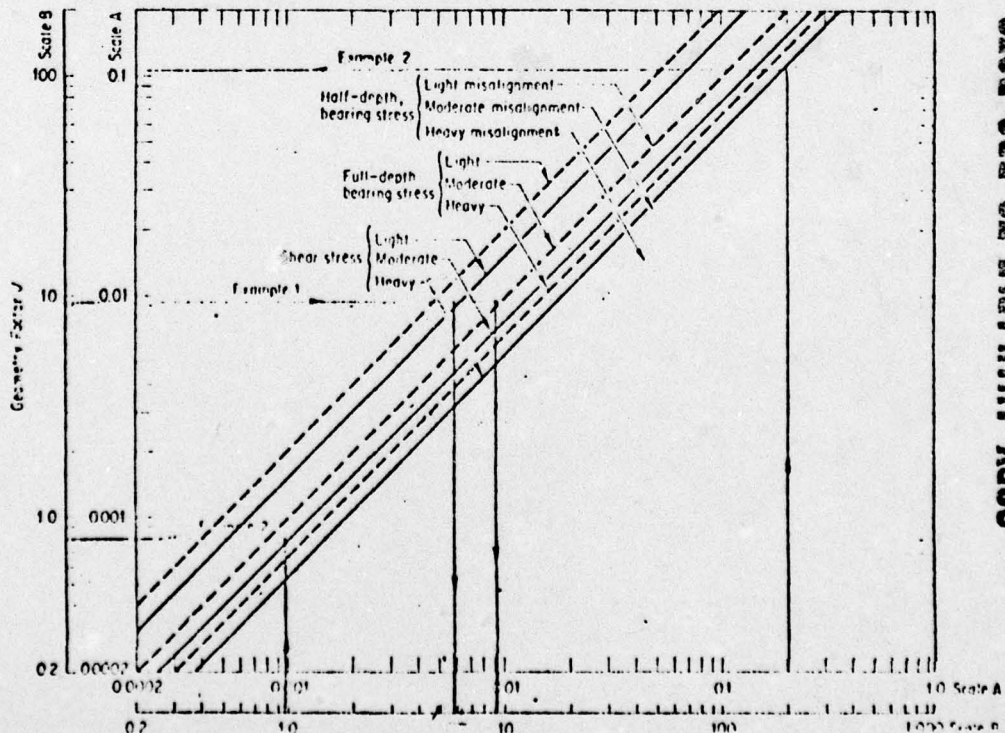
1. Chart 1: Find geometry factor  $J$  at known pitch diameter  $D$  and length  $L$ .
2. Chart 2: On the proper curve (relating misalignment, stress, and spline design), find torque factor  $K$  at known  $J$  and known (or estimated)  $K_m$ .
3. Calculate allowable bearing stress  $S_{cA}$  and allowable shear stress  $S_{sA}$  from  $S = TK$ .
4. Charts 3A and 3B: Find two life factors  $K_L$  at known values of  $S_{cA}$  and  $S_{sA}$  for the given material.
5. Chart 4: At known values of  $K_L$ , find the projected spline life. Use lower life as projection.

**Problem 2:** Consider a spline made of 300 Bhn steel that must transmit a torque  $T = 8,000$  lb-in. under conditions of moderate shock and misalignment. Required life is  $10^7$  revolutions. Determine the spline dimensions for this application.

1. On Chart 4, at life =  $10^7$  revolutions, find  $K_L = 0.3$  for shear stress and  $K_L = 0.9$  for bearing stress.
2. On chart 3A, at  $K_L = 0.9$ , 300 Bhn steel, and  $K_s = 1.5$  (moderate shock), find  $S_{cA} = 1,700$  psi.
3. On Chart 3B, at  $K_L = 0.3$ , 300 Bhn steel, and  $K_s = 1.5$ , find  $S_{sA} = 8,000$  psi.
4. Calculate bearing stress torque factor  $K = 1700/8000 = 0.21$ .
5. Calculate shear stress torque factor  $K = 8000/8000 = 1.00$ .

6. On chart 2 (half-depth bearing stress plots, medium misalignment), at  $K = 0.21$ , find  $J = 0.108$  for bearing stress.
7. On Chart 2 (shear stress plots, medium misalignment, at  $K = 1.00$ , find  $J = 0.80$  for shear stress.
8. Since  $J$  for bearing stress is lower than  $J$  for shear stress, bearing stress is the more important factor. Therefore, any combination of  $L$  and  $D$  that meets along the  $J = 0.108$  line on Chart 1 will satisfy the stress and life requirements. The choice of  $L$  and  $D$  should be based on the available space, keeping Ratio  $L/D$  as small as practicable. For example, a 3.6 in. diameter spline, about 1.0 in. long will do the job here.

Chart 2 — Torque Factor



COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LIABLE PRODUCTION

For determining geometry from desired life and material, the procedure is:

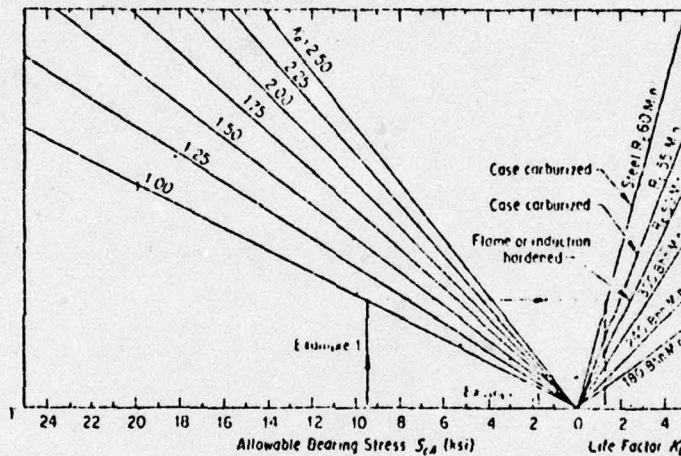
1. Chart 4: At desired life, find life factor  $K_L$ .
2. Charts 3A and 3B: At known  $K_L$ , find allowable bearing and shear stresses  $S_{cA}$  and  $S_{sA}$ .
3. Calculate a torque factor  $K$  for both bearing and shear stress from  $K = S/T$ .
4. Chart 2: On the relevant curves, at known  $K$  and known (or estimated)  $K_m$ , find a geometry factor  $J$  for both bearing and shear stress.
5. Chart 1: Use the smaller  $J$  from Step 4 to find length  $L$  and pitch diameter  $D$ .

Both procedures determine the bearing stress and the shear stress. Generally, bearing stress limits long-term life while shear stress limits the impact or shock load capacity. □

Nomenclature

- $D$  = Spline pitch diam, in.
- $H$  = Tooth height, in.
- $K_L$  = Life factor
- $K_m$  = Load distribution factor
- $K_o$  = Overload factor
- $K_r$  = Spline type factor
- $L$  = Spline length, in.
- $N$  = Number of teeth
- $S_c$  = Compressive (surface bearing) stress, psi
- $S_{cA}$  = Allowable bearing stress, psi
- $S_s$  = Shear (tooth) stress, psi
- $S_{sA}$  = Allowable shear stress, psi
- $T$  = Torque, lb-in.
- $t$  = Tooth thickness, in.

Chart 3A — Bearing Stress Life Factor



How Overload Factor Reflects Shock Loads

| Shock on Driving Member | Overload Factor $K_o$ | —Shock on Driven Member— |          |       |
|-------------------------|-----------------------|--------------------------|----------|-------|
|                         |                       | Light                    | Moderate | Heavy |
| Light                   | 1.00                  | 1.00                     | 1.25     | 1.75  |
| Moderate                | 1.25                  | 1.25                     | 1.50     | 2.25  |
| Heavy                   | 1.50                  | 1.50                     | 1.75     | 2.50  |

Chart 3B — Shear Stress Life Factor

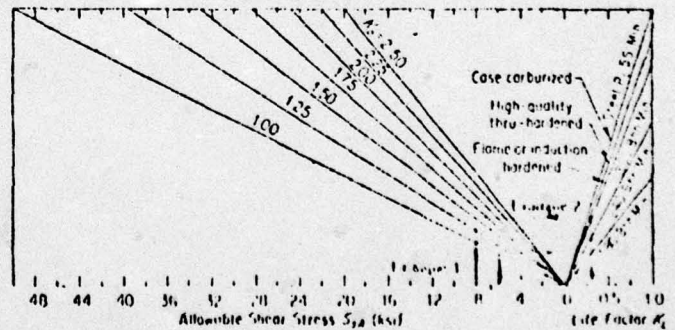
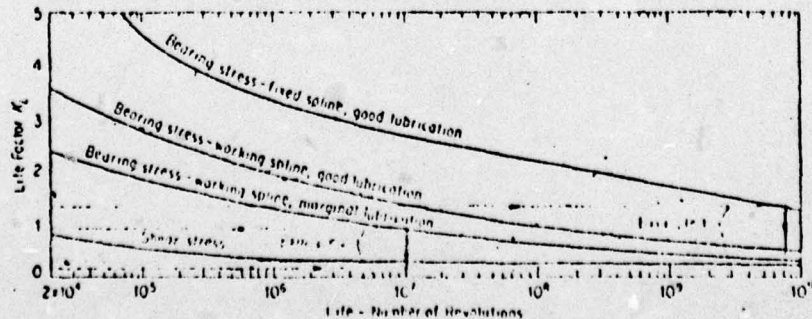


Chart 4 — Probable Life



APPENDIX C. TABLE 1

| L   | D   | B   | t    | $\alpha$ | J    | $T_F$ | $B_S$ | $K_L$ | $\theta$          | L/D | $\phi$ | $\phi L/D$ |
|-----|-----|-----|------|----------|------|-------|-------|-------|-------------------|-----|--------|------------|
| 0.5 | 1.0 | 300 | 3000 | .0025    | 3.0  | 3.0   | 9000  | 3.0   | $5.6 \times 10^4$ | 0.5 | 142    | 71         |
| 1.0 | 1.0 | 300 | 3000 | .0025    | 1.5  | 1.5   | 4500  | 1.5   | $6.0 \times 10^6$ | 1.0 | 142    | 142        |
| 1.5 | 1.0 | 300 | 3000 | .0025    | 1.0  | 1.0   | 3000  | 1.0   | $1.0 \times 10^8$ | 1.5 | 142    | 213        |
| 2.0 | 1.0 | 300 | 3000 | .0025    | 0.75 | 0.75  | 2250  | 0.75  | $5.0 \times 10^8$ | 2.0 | 142    | 284        |
| 1.5 | 1.0 | 300 | 2000 | .0025    | 1.0  | 1.0   | 2000  | 0.67  | $7.0 \times 10^8$ | 1.5 | 213    | 320        |
| 1.5 | 1.0 | 300 | 3000 | .0025    | 1.0  | 1.0   | 3000  | 1.0   | $1.0 \times 10^8$ | 1.5 | 142    | 213        |
| 1.5 | 1.0 | 300 | 5000 | .0025    | 1.0  | 1.0   | 5000  | 1.67  | $5.0 \times 10^6$ | 1.5 | 85     | 128        |
| 1.5 | 1.0 | 300 | 8000 | .0025    | 1.0  | 1.0   | 8000  | 2.67  | $1.2 \times 10^5$ | 1.5 | 53     | 80         |

APPENDIX C. TABLE 2

| L   | D   | B   | t      | $\alpha$ | J     | T <sub>F</sub> | B <sub>S</sub> | K <sub>L</sub> | $\theta$               | L/D   | $\phi$ | $\phi$ /L/D |
|-----|-----|-----|--------|----------|-------|----------------|----------------|----------------|------------------------|-------|--------|-------------|
| 1.0 | 0.5 | 300 | 1,000  | .0025    | 6.00  | 6.20           | 6,200          | 1.50           | 5.0 X 10 <sup>6</sup>  | 2.0   | 53     | 106         |
| 1.0 | 2.0 | 180 | 6,000  | .0025    | 0.38  | 0.39           | 2,340          | 1.40           | 1.0 X 10 <sup>7</sup>  | 0.5   | 341    | 171         |
| 2.0 | 0.5 | 600 | 3,000  | .0025    | 3.00  | 3.0            | 9,000          | 1.20           | 2.0 X 10 <sup>7</sup>  | 4.0   | 35.5   | 142         |
| 2.0 | 3.0 | 500 | 30,000 | .0025    | .0833 | .0833          | 2,500          | 0.70           | 6.0 X 10 <sup>8</sup>  | 0.667 | 640    | 427         |
| 1.0 | 4.0 | 240 | 15,000 | .0025    | .094  | .094           | 1,410          | 0.70           | 6.0 X 10 <sup>8</sup>  | 0.25  | 1,456  | 364         |
| 1.0 | 3.0 | 300 | 15,000 | .0025    | .167  | .167           | 2,500          | 0.85           | 2.0 X 10 <sup>8</sup>  | 0.33  | 768    | 256         |
| 1.0 | 0.5 | 180 | 1,000  | .0005    | 6.00  | 2.8            | 2,800          | 1.8            | 2.0 X 10 <sup>6</sup>  | 2.00  | 32     | 64          |
| 0.5 | 1.0 | 300 | 3,000  | .0005    | 3.0   | 1.4            | 4,200          | 1.2            | 2.0 X 10 <sup>7</sup>  | 0.5   | 142    | 71          |
| 1.0 | 0.5 | 300 | 1,000  | .0005    | 6.0   | 3.0            | 3,000          | 0.8            | 3.0 X 10 <sup>8</sup>  | 2.0   | 53     | 106         |
| 1.0 | 2.0 | 180 | 6,000  | .0005    | 0.38  | 0.2            | 1,200          | 0.75           | 4.0 X 10 <sup>8</sup>  | 0.5   | 341    | 171         |
| 2.0 | 0.5 | 600 | 3,000  | .0005    | 3.0   | 1.4            | 4,200          | 0.5            | 4.0 X 10 <sup>9</sup>  | 4.0   | 35.5   | 142         |
| 2.0 | 3.0 | 500 | 30,000 | .0005    | .0833 | .0417          | 1,250          | 0.35           | 1.0 X 10 <sup>10</sup> | 0.67  | 640    | 427         |
| 1.0 | 4.0 | 240 | 15,000 | .0005    | .094  | .047           | 705            | 0.35           | 1.0 X 10 <sup>10</sup> | 0.25  | 1,456  | 364         |
| 1.0 | 3.0 | 300 | 15,000 | .0005    | .167  | .083           | 1,250          | 0.43           | 2.0 X 10 <sup>9</sup>  | 0.33  | 768    | 256         |
| 1.5 | 2.5 | 500 | 20,000 | .0005    | .160  | .080           | 1,600          | 0.40           | 3.0 X 10 <sup>9</sup>  | 0.60  | 555    | 333         |
| 2.5 | 1.5 | 300 | 10,000 | .0005    | .270  | .135           | 1,350          | 0.50           | 4.0 X 10 <sup>9</sup>  | 1.67  | 144    | 240         |
| 1.0 | 0.5 | 300 | 1,000  | .006     | 6.0   | 9.0            | 9,000          | 3.0            | 6.0 X 10 <sup>4</sup>  | 2.0   | 53     | 106         |
| 1.0 | 2.0 | 180 | 6,000  | .006     | 0.38  | 0.58           | 3,480          | 2.2            | 3.6 X 10 <sup>5</sup>  | 0.5   | 341    | 171         |
| 2.0 | 0.5 | 600 | 3,000  | .006     | 3.0   | 4.4            | 13,200         | 1.8            | 2.0 X 10 <sup>6</sup>  | 4.0   | 35.5   | 142         |
| 2.0 | 2.0 | 500 | 10,000 | .006     | 0.19  | 0.3            | 3,000          | 0.6            | 1.0 X 10 <sup>9</sup>  | 1.0   | 569    | 569         |
| 2.0 | 3.0 | 500 | 30,000 | .006     | .0833 | .125           | 3,750          | 1.05           | 7.0 X 10 <sup>7</sup>  | 0.67  | 640    | 427         |
| 1.0 | 4.0 | 240 | 15,000 | .006     | .094  | .141           | 2,115          | 1.05           | 7.0 X 10 <sup>7</sup>  | 0.25  | 1,456  | 364         |
| 1.0 | 3.0 | 300 | 15,000 | .006     | .167  | .250           | 3,750          | 1.28           | 2.0 X 10 <sup>7</sup>  | 0.25  | 768    | 256         |
| 1.5 | 2.5 | 500 | 20,000 | .006     | .160  | .240           | 4,800          | 1.20           | 2.0 X 10 <sup>7</sup>  | 0.60  | 555    | 333         |
| 2.5 | 1.5 | 300 | 10,000 | .006     | .270  | .405           | 4,050          | 2.0            | 1.0 X 10 <sup>6</sup>  | 1.67  | 144    | 240         |