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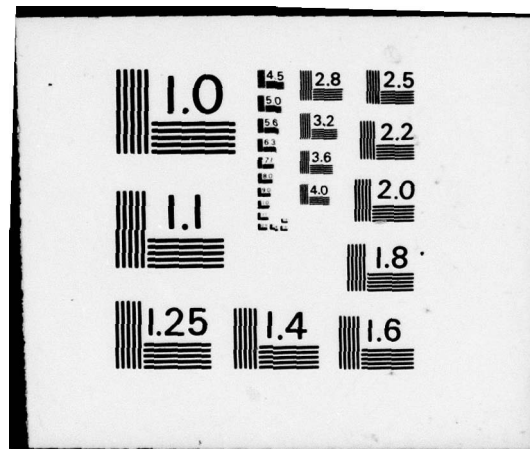
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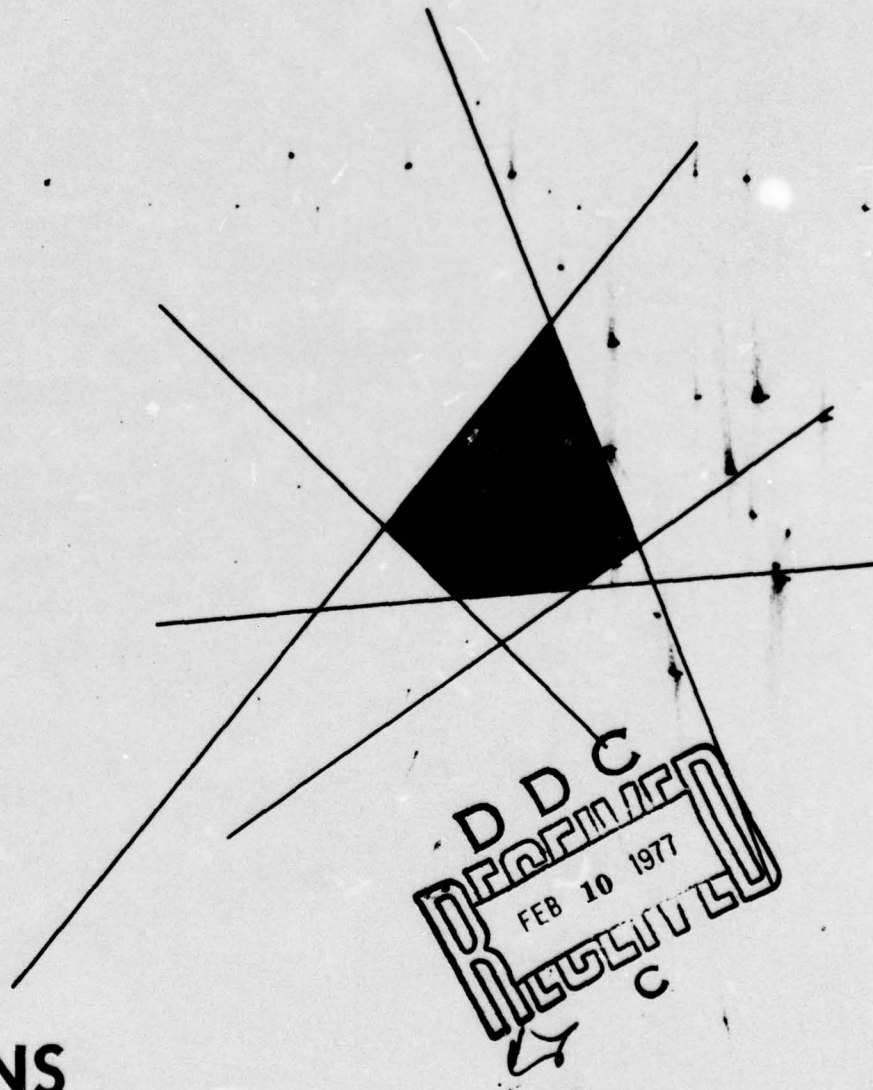
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RICHARD C. GRINOLD

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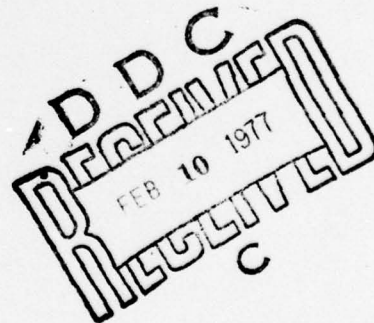
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OPTIMAL DESIGN OF A MANPOWER SYSTEM

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ABSTRACT

An equilibrium model of a manpower system is developed based on the notion of a career flow. Institutional constraints and measures of system performance are linear functions of the career flow. A typical optimal design problem is formulated and a solution procedure is developed. The optimization problem is a generalized linear program in which columns are generated by solving a shortest path problem. Upper and lower bounds on the optimal value function can be developed at each stage of the calculations.

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OPTIMAL DESIGN OF A MANPOWER SYSTEM

0. INTRODUCTION

This paper presents a model for the design of a manpower system and develops a procedure for its solution. The manpower system is assumed to be in temporal equilibrium and to be a strict hierarchy; i.e., a person in rank i either stays in rank i , moves to rank $i + 1$, or leaves the system. The equilibrium assumption is critical to the model. The assumption of strict hierarchy is made for simplicity; a more complicated system could be considered in the same way with more complicated results.

The approach to modeling the manpower system was suggested by the equilibrium chain-flow model of Oliver and Hopkins [7]. Their idea of chain flow is exactly the notion of career flow used in this paper. Oliver and Hopkins stipulate a set of possible student and faculty flow paths through a university and then calculate the path flows that are consistent with meeting institutional constraints at minimum cost. This paper presents these ideas in a more general setting and shows how the problem of having a very large number of possible career paths can be handled.

The motivation for the procedure developed in this paper can be found in Schmidt, et al. [8], [9]. The problem posed in [8] and [9] is to design a manpower system in order to maximize the effectiveness to cost ratio subject to several institutional constraints. A rather complicated nonlinear optimization approach that deals with an aggregate version of the problem is presented in [8]. In contrast the approach described here is flexible, general, deals with the entire problem, and is easily solved.

The problem of system design should be contrasted with the problem of system control. In [4], [5], procedures for controlling graded manpower systems were developed. These procedures assumed that certain ratios among

manpower stocks and flows were known, stationary, and governed the manpower flow process. In contrast, the approach developed in this paper actually designs the flow process and yields the ratios as policy variables.

Section I describes the manpower system and the equilibrium flow equations. The notion of a career is introduced in Section II and several important characteristics of the manpower system are expressed in terms of career flows. In Section III linear programming optimization model is presented and Section IV describes a column generation technique for its solution; the columns are generated by solving a shortest path problem. In Section V a termination criterion is developed. The special structure of the problem allows one to determine if the current solution is within a specified percentage error of the optimal solution. A subsequent paper will deal with applications of the techniques developed here to the enlisted force of the U.S. Navy.

I. THE SYSTEM

The manpower system is an N -rank hierarchy that is observed at equally spaced points in time $t = 0, 1, 2, \dots$. The time interval $(t - 1, t]$, after time $t - 1$ and up to and including time t , is called period t . At observation time t each individual is given a classification (i, u) , according to their rank, i , and period of service, u . A period of service equal to u at time t indicates the individual entered the system in period $t + 1 - u$: i.e. in the time interval $(t - u, t + 1 - u]$. An individual's length of service is the number of *completed* periods of service; thus an individual in class (i, u) has length of service $u - 1$. The maximum length of service is $M - 1$; thus an individual in class (i, M) must leave the system.

Figure 1 shows the possible movements from class (i, u) . The individual in (i, u) moves either out of the system (to class 0), stays in rank i , or, if $i < N$, is promoted. There are no demotions or double promotions, and each person enters in class $(1, 1)$.

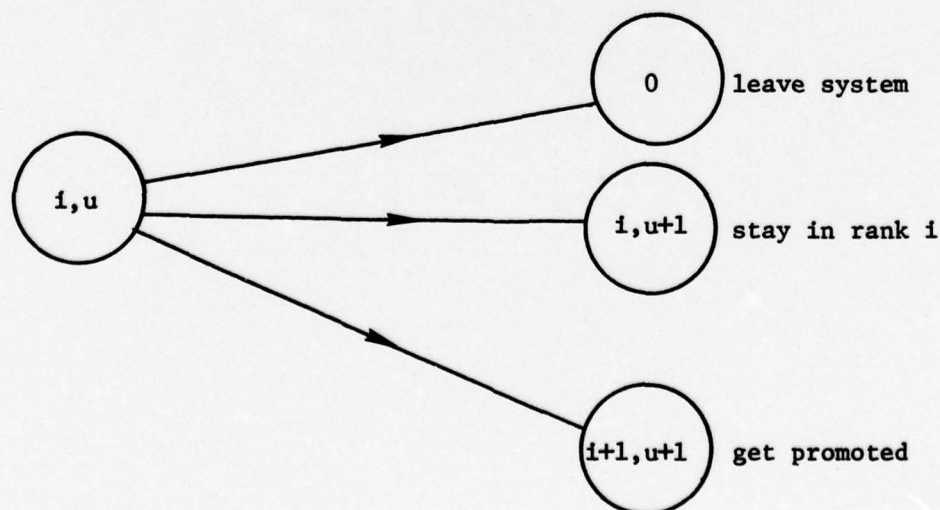


FIGURE 1: Possible Transitions from Class (i, u) .

We assume the system is in equilibrium. Let $s(i,u)$ be the number of people in class (i,u) , and let $f(i,u,j)$ be the number of people per period who move from class (i,u) to class $(j,u+1)$. Notice the assumption on possible movements implies $f(i,u,j) = 0$ if j is not equal to 0 , i , or $i+1$. Of course, $f(N,u,N+1) = 0$, since there are only N ranks, and $f(i,M,i) = f(i,M,i+1) = 0$ since one must leave the system during the M^{th} period of service.

The equilibrium flow equations are

- (2) (i) for $u = 1, i = 1$

$$s(1,1) = f(1,1,0) + f(1,1,1) + f(1,1,2) .$$
- (ii) for $1 < u < M, i = 1$

$$f(1,u-1,1) = s(1,u) = f(1,u,0) + f(1,u,1) + f(1,u,2) .$$
- (iii) for $1 < u < M, 1 < i < N$

$$f(i-1,u-1,i) + f(i,u-1,i) = s(i,u) = f(i,u,0) + f(i,u,i) + f(i,u,u+1) .$$
- (iv) for $1 < u < M, i = N$

$$f(N-1,u-1,N) + f(N,u-1,N) = s(N,u) = f(N,u,0) + f(N,u,N) .$$
- (v) for $u = M, i = 1$

$$f(1,M-1,1) = s(1,M) = f(1,M,0) .$$
- (vi) for $u = M, 1 < i$

$$f(i-1,M-1,i) + f(i,M-1,i) = s(i,M) = f(i,M,0) .$$

The equilibrium flow for $N = 3, M = 5$ is shown in Figure 2, retirement flows $(i,u,0)$ are not shown.

The variable $s(i,u)$ and $f(i,u,j)$ allow for a measure of the effectiveness and cost of different equilibrium flow patterns. Let $e(i,u)$ be the effectiveness of an individual in class (i,u) , $c(i,u)$ the cost, $p(i,u)$ the cost associated with a promotion from (i,u) to $(i+1,u+1)$,

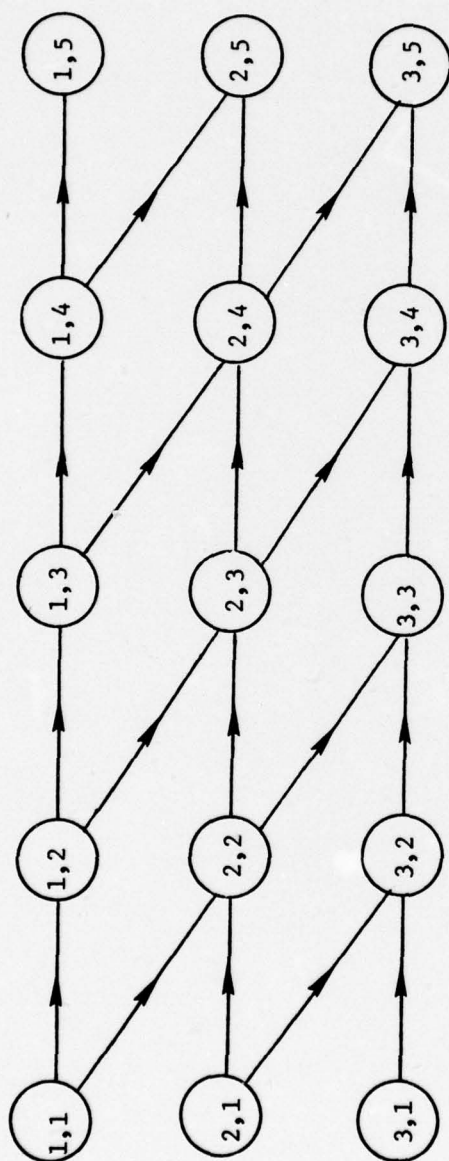


FIGURE 2: Internal Flows for $N = 3$, $M = 5$.

and $r(i,u)$ the cost of a retirement from (i,u) . The cost and effectiveness are respectively:

$$(3) \quad (i) \quad \sum_i \sum_u c(i,u)s(i,u) + p(i,u)f(i,u,i+1) + r(i,u)f(i,u,0) .$$

$$(ii) \quad \sum_i \sum_u e(i,u)s(i,u) .$$

Although Equations (2) and (3) describe the equilibrium conditions and possible criteria they are not in a suitable form to describe the structure of allowable flow patterns. That structure will be discussed in the next section.

II. CAREERS

A career is a path through the manpower system that commences at rank 1 with $u = 1$. Let k index the career. Each career has several important characteristics. Some of these are:

- (4) (i) $n(i,k) \sim$ The number of times counted in class i .
- (ii) $m(i,k) \sim$ Attainment of class i : this variable is one if career k includes class i (i.e. if $n(i,k) > 0$), zero if career k does not include class i : i.e. $n(i,k) = 0$.
- (iii) $a(i,k) \sim$ Length of service upon entry to class i .
- (iv) $d(v,k) \sim$ The number of periods with length of service greater than v : i.e. time spent in the professional force.
- (v) $e(k) \sim$ The total effectiveness of career k .
- (vi) $c(k) \sim$ The total cost of career k .
- (vii) $n(k) \sim$ Total length of career k , $n(k) = \sum_i n(i,k)$.

A career may be characterized by transitions between nodes. Let

$$f^k(i,u,j) = \begin{cases} 1 & \text{if career } k \text{ moves from } (i,u) \text{ to } (j,u+1) \\ 0 & \text{otherwise.} \end{cases}$$

The characteristics described in (4) can be expressed in terms of the variables f^k .

- (5) (i) $n(i,k) = \sum_u \sum_j f^k(i,u,j)$.
- (ii) $m(i,k) = \sum_u f^k(i-1,u,i)$ for $i \geq 2$.
- (iii) $a(i,k) = \sum_u u f^k(i-1,u,i)$ for $i \geq 2$.
- (iv) $d(v,k) = \sum_{u \geq v} \sum_i \sum_j f^k(i,u,j)$.

$$\begin{aligned}
 \text{(v)} \quad c(k) &= \sum_u \sum_i \sum_j c(i,u,j) f^k(i,u,j) . \\
 \text{(vi)} \quad e(k) &= \sum_u \sum_i e(i,u) \sum_j f^k(i,u,j) . \\
 \text{(vii)} \quad n(k) &= \sum_i \sum_u \sum_j f^k(i,u,j) .
 \end{aligned}$$

The costs $c(i,j,j)$ are calculated from basic costs described in Section I.

$$\begin{aligned}
 \text{(6)} \quad \text{(i)} \quad c(i,u,0) &= c(i,u) + r(i,u) . \\
 \text{(ii)} \quad c(i,u,i) &= c(i,u) . \\
 \text{(iii)} \quad c(i,u,i+1) &= c(i,u) + p(i,u) .
 \end{aligned}$$

The special structure of the organization implies that $f^k(i,u,i+1) = 1$ for *at most* one value of u . Thus at most one term in the summations (5:ii) and (5:iii) is equal to one.

Let $g(k)$ be the number of people starting career k each period. Under the steady-state assumptions there will be $g(k)$ individuals in each phase of the career. It follows that:

$$\begin{aligned}
 \text{(7)} \quad \text{(i)} \quad \sum_k n(i,k)g(k) &\text{ is the total number of people in class } i . \\
 \text{(ii)} \quad \sum_k m(i,k)g(k) &\text{ is the number of entrants that ever reach class } i . \\
 \text{(iii)} \quad \sum_k a(i,k)g(k) &\text{ is the total length of service of individuals} \\
 &\text{entering class } i . \\
 \text{(iv)} \quad \sum_k d(v,k)g(k) &\text{ is the number of people with length of service} \\
 &\text{greater than } v . \\
 \text{(v)} \quad \sum_k c(k)g(k) &\text{ is the total cost incurred.} \\
 \text{(vi)} \quad \sum_k e(k)g(k) &\text{ is the total effectiveness.} \\
 \text{(vii)} \quad \sum_k n(e)g(k) &\text{ is the total number of people in the system.}
 \end{aligned}$$

This description of the system is based on the notion of a career. The next section demonstrates how the system can be designed in an optimal manner.

III. THE OPTIMIZATION PROBLEM

The model can be used with several objectives. It is possible to maximize effectiveness subject to a cost constraint. An alternative is to minimize cost subject to an effectiveness constraint. A third objective is to maximize an effectiveness/cost ratio. These objectives are discussed at length in an appendix. The choice of an objective does not change the optimization procedure. The example presented below is to minimize cost subject to effectiveness and system constraints.

Let $q(i,k) = a(i,k) - a(i)m(i,k)$ be excess experience upon entry to rank i . The system constraints are

- (8) (i) $\sum_k n(i,k)g(k) \geq r(i)$, at least $r(i)$ people in rank i :
- $i = 1, 2, \dots, N$.
- (ii) $\sum_k q(i,k)g(k) \geq 0$, the average length of service upon entry to class i at least $a(i)$ for $i = 2, \dots, N$.
- (iii) $\sum_k d(v,k)g(k) \geq d$, a minimum size of the professional staff.
- (iv) $\sum_k e(k)g(k) \geq e$ a minimum level of effectiveness.
- (v) $\sum_k n(k)g(k) = r$; fixed total strength.
- (vi) $\sum_k c(k)g(k)$ the cost.

The linear program is to minimize (8:vi) subject to the constraints (8:i-v). This linear program has $2(1 + N)$ constraints and a nonnegative variable, $g(k)$ for each career. The next section describes a method for solving this problem.

Equation (8) describes several examples of constraints. There are other possibilities: three additional examples are presented below. The first is

$$\sum_k \{m(i+1, k) - \alpha(i+1, i)m(i, k)\}g(k) \geq 0.$$

The fraction of those who reach rank $i+1$ given they have reached rank i exceeds $\alpha(i+1, i)$. A second example is

$$\sum_k \{m(i+1, k) - \delta(i+1, i)n(i, k)\}g(k) \geq 0.$$

The fraction of those in rank i that are promoted to rank $i+1$ per year is at least $\delta(i+1, i)$. Finally,

$$\sum_k \{d(h, k) - \zeta d(\ell, k)\} \geq 0.$$

The fraction of those with length of service greater than k who remain to have length of service greater than $h > \ell$ is at least ζ .

IV. SOLUTION PROCEDURE

The solution procedure is outlined in this section. It is a column generation algorithm: see [1], [2], and [3] for the origins and applications of this idea. Column generation is a variant of the revised simplex method. The columns of the activity matrix are not stored explicitly. They are generated by solving a sub-problem.

The section describes one iteration of the procedure. Assume a feasible basis K is available. Let k index the career in the bases. There are simplex multipliers associated with this basis; each multiplier is associated with a constraint.

- (9) (i) $\Psi(i) \sim$ stock constraints $i = 1, \dots, N$.
- (ii) $\lambda(i) \sim$ average length of service constraints $i = 2, \dots, n$.
- (iii) $\mu \sim$ professional staff constraint.
- (iv) $\gamma \sim$ effectiveness constraint.
- (v) $\alpha \sim$ total size constraint.

These simplex multipliers satisfy the following linear equality for careers k in the current bases.

$$(10) \quad \sum_i \Psi(i)n(i,k) + \sum_{i \geq 2} \lambda(i)\{q(i,k)\} + d(v,k) + \gamma e(k) + \alpha n(k) + c(k) = 0.$$

The current basis is optimal if $(\psi, \lambda, \mu, \gamma)$ are nonpositive and for all careers k ,

$$(11) \quad \sum_i \Psi(i)n(i,k) + \sum_{i \geq 2} \lambda(i)q(i,k) + d(v,k) + \gamma e(k) + \alpha n(k) + c(k) \geq 0.$$

This condition can be checked by solving the sub-problem

$$(12) \quad \text{Min}_k \left[c(k) + \sum_i \Psi(i)n(i,k) + \sum_{i \geq 2} \lambda(i)q(i,k) + d(v,k) + \gamma e(k) + \alpha n(k) \right].$$

From (10) note that the optimal value of the subproblem (12) is less than or equal to zero. If the optimal value of the subproblem is zero, then (11) is satisfied and the current basis is optimal. If the optimal value of the subproblem is negative, then a new career can be found that is eligible to enter the current basis. The procedure for solving (12) and generating the new career is presented below.

Substitute the Equations (5), from Section 2, into (12) and combine terms. This yields

$$(13) \quad \text{Min}_k \left[\sum_u \sum_j \sum_i l(i,u,j) f^k(i,u,j) \right].$$

The correct values of $l(i,u,j)$ are given below where $d(u) = 1$ if $u \geq v$, otherwise.

$$(14) \quad (i) \quad l(i,u,i) = c(i,u) + \psi(i) + \mu d(u) + \gamma e(i,u) + \alpha.$$

$$(ii) \quad l(i,u,0) = l(i,u,i) + r(i,u).$$

$$(iii) \quad l(i,u,i+1) = l(i,u,i) + p(i,u) + \lambda(i+1)(u - a, i+1)$$

$$\text{for } i \leq N-1.$$

Problem (13) is a shortest path problem. The network was described in Figure 1, Section 1. The $l(i,u,j)$ are the lengths. Let $V(i,u) =$ length of the shortest path starting from node (i,u) out of the system.

$$v(i,M) = l(i,M,0).$$

For $u < M$

$$(15) \quad (i) \quad v(i,u) = \text{Min} \begin{cases} l(i,u,0) \\ l(i,u,i) + v(i,u+1) \\ l(i,u,i+1) + v(i+1,u+1) \end{cases} \quad \text{if } i < N$$

$$(ii) \quad v(N,u) = \text{Min} \begin{cases} l(N,u,0) \\ l(N,u,N) + v(N,u+1) \end{cases}.$$

Finally $v^* = \min_i [v(i,1)]$ is the length of the shortest path. If $v^* < 0$, the shortest path defines a career that is eligible to enter the bases. If $v^* = 0$, the current solution is optimal.

V. BOUNDS ON THE OBJECTIVE

The column generation scheme described in Section IV can be modified slightly to give a series of upper and *lower* bounds on the optimal value of the objective. Once a feasible solution has been obtained the cost of the current feasible solution is an upper bound on the optimal value. Let U be this upper bound.

$$(16) \quad U = - \sum_{i=1}^N \psi(i)r(i) + \mu p + \gamma e + \alpha r .$$

Recall that the shortest path calculation solves the problem

$$(17) \quad \text{Min}_k \left[c(k) + \sum_i \psi(i)n(i,k) + \sum_{i \geq 2} \lambda(i)q(i,k) + \mu p(k) + \gamma e(k) + \alpha n(k) \right] .$$

Without loss of generality, $\psi, \lambda, \mu, \gamma$ are nonpositive. If they were positive a slack variable would have been admitted to the basis. Let $\phi = (\psi, \lambda, \mu, \gamma, \alpha)$, and call the minimand in (17), $x(k, \phi)$. The lower bound is developed by studying a fractional program (18). Notice that

$$(18) \quad \text{Min}_k \left[\frac{x(k, \phi)}{n(k)} \right]$$

the fractional optimization problem (18), will have an optimal value of the same sign as (17). Let $g^*(k)$ be the number of people following career k in an optimal solution of the overall problem and let V be the optimal value of the overall problem.

$$(19) \quad V = \sum_k c(k)g^*(k) \leq U .$$

Now define $z(k) = \{n(k)g^*(k)\}/r$; z is a nonnegative vector that sums to one. Suppose career h solves (18), with value W , then for all k

$$(20) \quad W = \frac{x(h, \phi)}{n(h)} \leq \frac{x(k, \phi)}{n(k)} .$$

Multiply (20) by $z(k)$ and sum; this yields

$$(21) \quad W \leq \frac{1}{r} \left[\sum c(k)g^*(k) + \sum_i \psi_i \sum_k n(i, k)g^*(k) + \sum_{i \geq 2} \lambda(i) \sum q(i, k)g^*(h) + \right. \\ \left. \mu \sum_k p(k)g^*(k) + \gamma \sum_k e(k)g^*(k) + \alpha \sum_k n(k)g^*(n) \right] .$$

From the feasibility of g^* and the nonpositivity of $(\psi, \lambda, \mu, \gamma)$, this becomes

$$(22) \quad W \leq \frac{1}{r} [V - U] \quad \text{or} \quad U + rW \leq V .$$

If ϵ is a desired percentage error in the calculation of V , then the stopping rule is to terminate if

$$(23) \quad \left(\frac{-rW}{U} \right) \leq \epsilon .$$

Notice that calculation of this lower bound involves the solution of (18) rather than (17). However, (17) is a rather easy to solve shortest path problem and (18) is a more difficult to solve fractional program. It is possible to approximate the value of W using (17). Let $Y(k, \phi, \omega)$ and $L(\omega)$ be defined as

$$(24) \quad Y(k, \phi, \omega) = x(k, \phi) - \omega n(k) \\ L(\omega) = \min_k [Y(k, \phi, \omega)] .$$

Notice that evaluating $L(\omega)$ is equivalent to solving problem (17) with α replaced by $(\alpha - \omega)$. The optimal solution of (18) is found by finding the ω^* that satisfies $L(\omega^*) = 0$, in fact $\omega^* = W$. The reader can show that $L(\omega)$ is a decreasing, piecewise linear, concave function of ω . Furthermore, suppose h is the career that solves (17). If there is no unique optimal career, then h is selected to minimize $n(h)$ among all shortest paths. Problem (17) evaluates $L(0)$, and the left derivative of L at $\omega = 0$, see [3], is $-n(h)$. Therefore $\omega_1 = L(0)/n(h)$ is an upper bound for ω^* . Define ω_2 by $\omega_2 = -\epsilon U/r$; notice from (23) and the monotonicity of L that cutoff occurs if and only if $\omega_2 \leq \omega^*$.

The three possible cases that can occur are shown in Figure 3. First, in 3.a, $\omega^* \leq \omega_1 < \omega_2$ and no cutoff is possible. If $\omega_2 \leq \omega_1$, then $L(\omega_2)$ is evaluated by solving (17) with α replaced by $\alpha - \omega_2$. In Figure 3b., $L(\omega_2) < 0$, therefore $\omega^* < \omega_2$ and cutoff does not occur. In Figure 3c. $L(\omega_2) \geq 0$, so $\omega_2 \leq \omega^*$ and calculations are terminated with a percentage error less than ϵ .

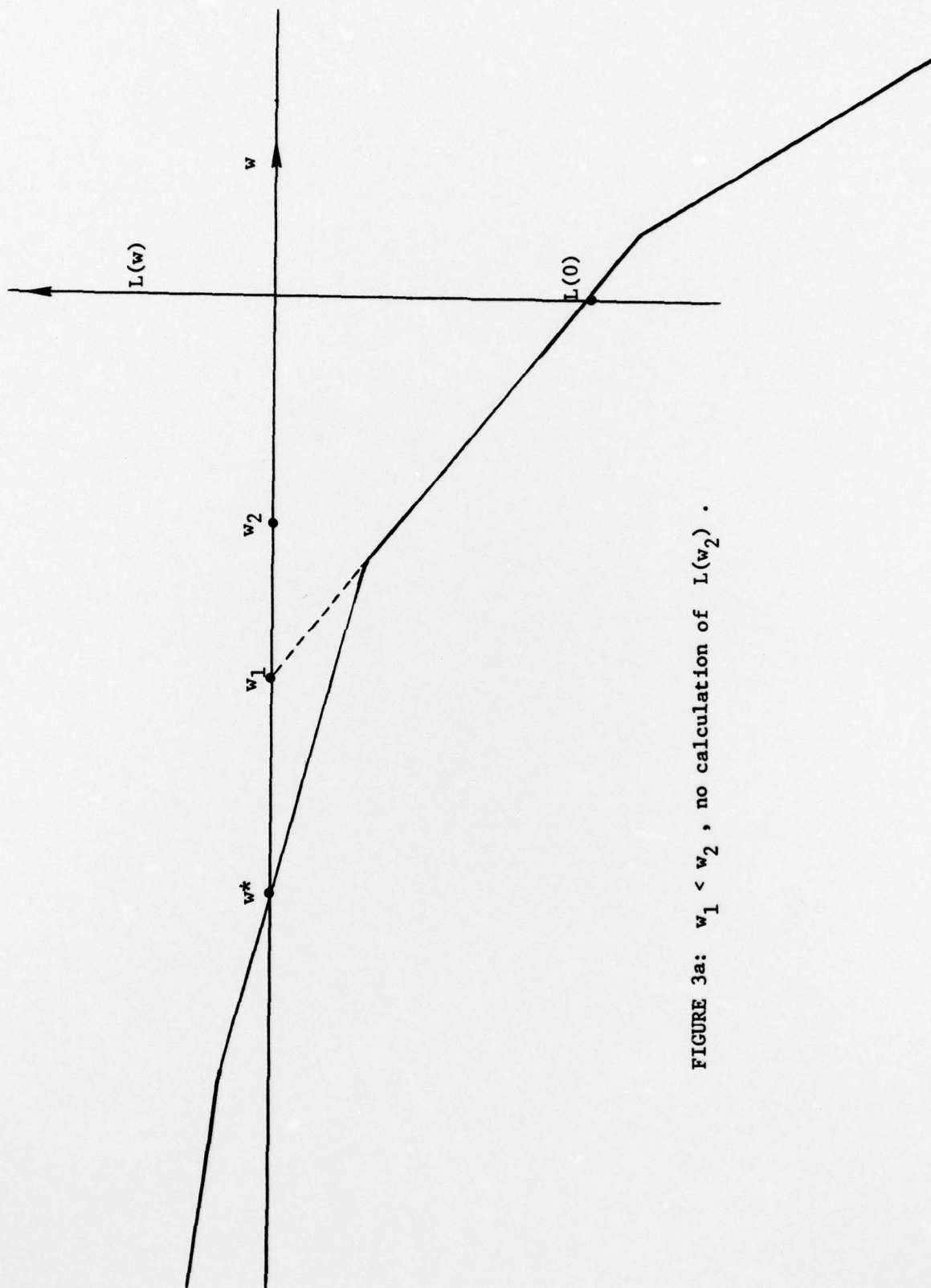


FIGURE 3a: $w_1 < w_2$, no calculation of $L(w_2)$.

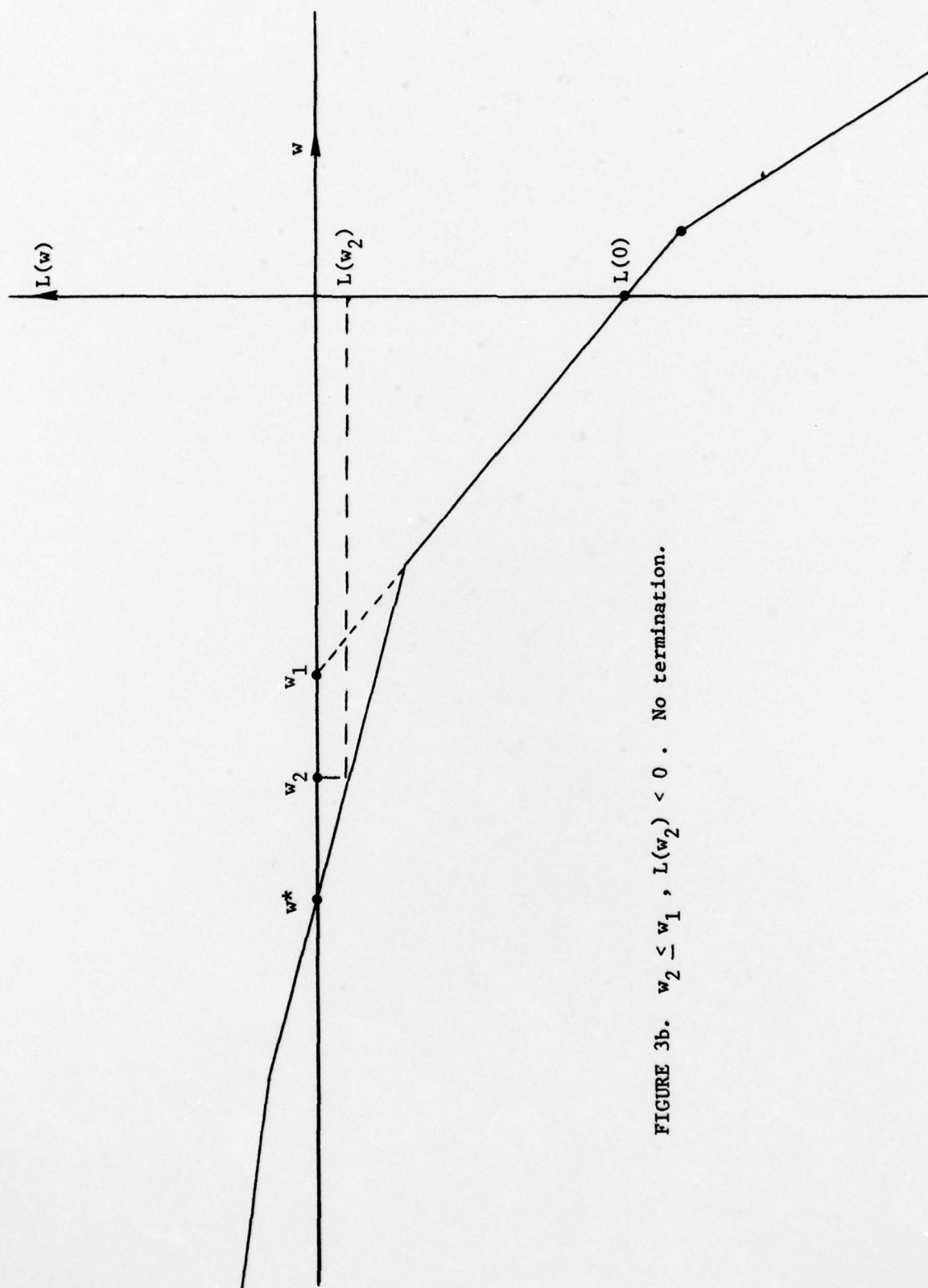


FIGURE 3b. $w_2 \leq w_1$, $L(w_2) < 0$. No termination.

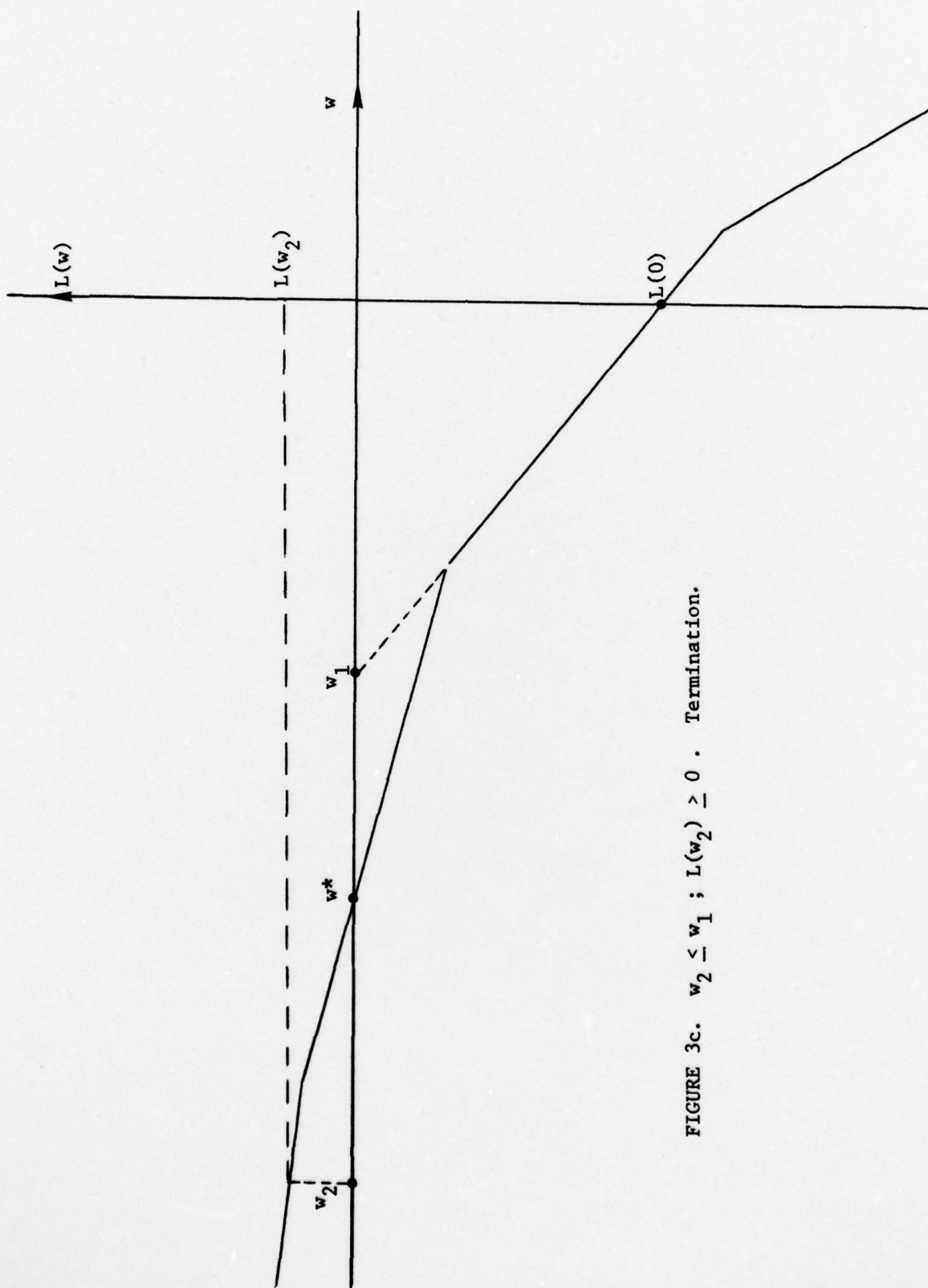


FIGURE 3c. $w_2 \leq w_1$; $L(w_2) \geq 0$. Termination.

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