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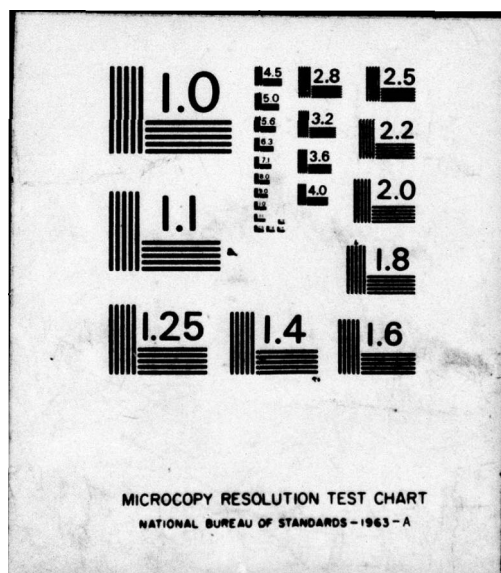
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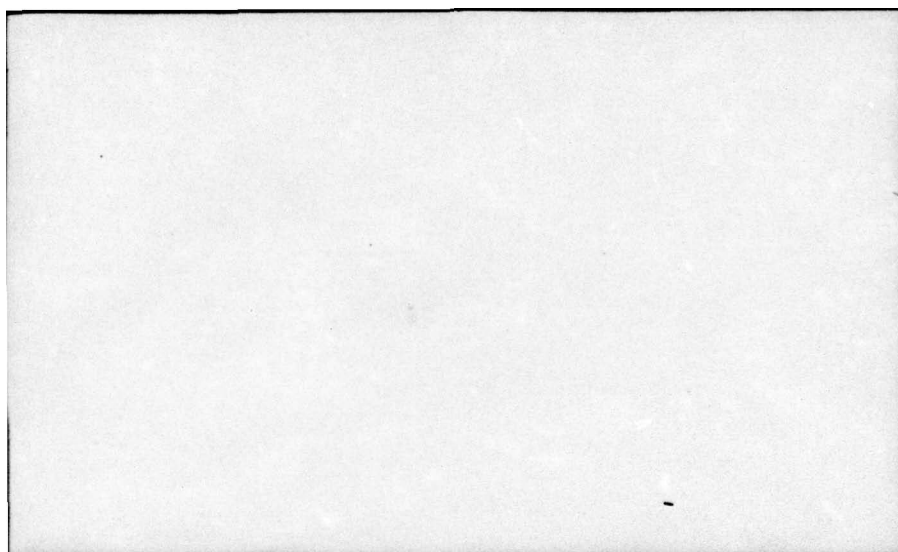


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FENCHEL'S DUALITY THEOREM IN
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by

10 Elmor L. Peterson

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Fenchel's Duality Theorem in
Generalized Geometric Programming

by

Elmor L. Peterson*

Abstract. Fenchel's duality theorem is extended to generalized geometric programming with explicit constraints -- an extension that also generalizes and strengthens Slater's version of the Kuhn-Tucker theorem.

Key words: Fenchel's duality theorem, generalized geometric programming, convex programming, ordinary programming, Slater's constraint qualification, Kuhn-Tucker theorem.

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1. Introduction. Although many implications of this extension have already been discussed in the author's recent survey paper [1], a proof of it is given here for the first time.

This proof utilizes the unconstrained version that has already been established by independent and somewhat different arguments in [2] and [3]. In doing so, it exploits the main result from [4] and also requires some of the convexity theory in [3]--especially the theory having to do with the "relative interior" ($\text{ri } S$) of an arbitrary convex set $S \subseteq E_N$ (N -dimensional Euclidean space).

2. The unconstrained case. We begin with the following notation and hypotheses:

$\mathcal{X}$ is a nonempty closed convex cone in E_n ,

g is a (proper) closed convex function with a nonempty (effective) domain $C \subseteq E_n$.

Now, given $\mathcal{X}$ and g , consider the resulting "geometric programming problem" $\mathcal{Q}$.

PROBLEM $\mathcal{Q}$. Using the feasible solution set

$$\mathcal{J} \triangleq \mathcal{X} \cap C,$$

calculate both the problem infimum

$$\varphi \triangleq \inf_{x \in \mathcal{J}} g(x)$$

and the optimal solution set

$$\mathcal{J}^* \triangleq \{x \in \mathcal{J} \mid g(x) = \varphi\}.$$

Geometric duality is defined in terms of both the "dual cone"

$$\mathcal{V}^{\Delta} = \{y \in E_n \mid 0 \leq \langle x, y \rangle \text{ for each } x \in \mathcal{X}\}$$

and the "conjugate transform function" h whose (effective) domain

$$\mathcal{D}^{\Delta} = \{y \in E_n \mid \sup_{x \in \mathcal{C}} [\langle y, x \rangle - g(x)] \text{ is finite}\}$$

and whose functional value

$$h(y) \stackrel{\Delta}{=} \sup_{x \in \mathcal{C}} [\langle y, x \rangle - g(x)].$$

In particular, given the geometric programming problem $\mathcal{P}$, consider the resulting "geometric dual problem" $\mathcal{B}$.

PROBLEM $\mathcal{B}$. Using the feasible solution set

$$\mathcal{J}^{\Delta} = \mathcal{V} \cap \mathcal{D}^{\Delta},$$

calculate both the problem infimum

$$\psi \stackrel{\Delta}{=} \inf_{y \in \mathcal{J}^{\Delta}} h(y)$$

and the optimal solution set

$$\mathcal{J}^{\Delta*} = \{y \in \mathcal{J}^{\Delta} \mid h(y) = \psi\}.$$

Fenchel's duality theorem in the context of dual problems $\mathcal{P}$ and $\mathcal{B}$ is one of the most important theorems in geometric programming. It can be stated in the following way.

Theorem 1. If problem $\mathcal{P}$ has both a feasible solution $y^0 \in (ri \mathcal{V}) \cap (ri \mathcal{B})$ and a finite infimum ψ , then

(I) problem $\mathcal{Q}$ has both a nonempty feasible solution set $\mathcal{S}$ and a finite infimum φ , and

$$0 = \varphi + \psi,$$

(II) problem $\mathcal{Q}$ has a nonempty optimal solution set $\mathcal{S}^*$.

This theorem is established as Theorem 31.4 on page 335 of [3].

The implications of Theorem 1 are given on page 26 of [1]. An important extension of it is established in the next section.

3. The constrained case. To incorporate explicit constraints into generalized geometric programming, we introduce the following notation and hypotheses:

I and J are two nonintersecting (possibly empty) positive-integer index sets with finite cardinality $o(I)$ and $o(J)$ respectively;

x^k and y^k are independent vector variables in E_{n_k} for $k \in \{0\} \cup I \cup J$, and x^I and y^I denote the respective Cartesian products of the vector variables x^i , $i \in I$, and y^i , $i \in I$ while x^J and y^J denote the respective Cartesian products of the vector variables x^j , $j \in J$, and y^j , $j \in J$; so the Cartesian products $(x^0, x^I, x^J) \stackrel{\Delta}{=} x$ and $(y^0, y^I, y^J) \stackrel{\Delta}{=} y$ are independent vector variables in E_n , where

$$n \stackrel{\Delta}{=} n_0 + \sum_I n_i + \sum_J n_j;$$

α and λ are independent vector variables with respective components α_i and λ_i for $i \in I$, and β and κ are independent vector variables with

respective components β_j and κ_j for $j \in J$;

X and Y are nonempty closed convex dual cones in E_n , and g_k and h_k are (proper) closed convex conjugate functions with respective (effective) domains $C_k \subseteq E_{n_k}$ and $D_k \subseteq E_{n_k}$ for $k \in \{0\} \cup I \cup J$.

Now, let

$$Z \triangleq \{(x^0, x^I, \alpha, x^J, \kappa) \in E_n \mid (x^0, x^I, x^J) \in X; \alpha = 0; \kappa \in E_{o(J)}\},$$

where $n + o(I) + o(J) = n$. In addition, let

$$C \triangleq \{(x^0, x^I, \alpha, x^J, \kappa) \in E_n \mid x^0 \in C_0; x^i \in C_i, \alpha_i \in E_1, \text{ and}$$

$$g_1(x^1) + \alpha_1 \leq 0, i \in I; (x^j, \kappa_j) \in C_j^+, j \in J\},$$

and let

$$\varphi(x^0, x^I, \alpha, x^J, \kappa) \triangleq g_0(x^0) + \sum_j g_j^+(x^j, \kappa_j),$$

where the (closed convex) function g_j^+ has a domain

$$C_j^+ \triangleq \{(x^j, \kappa_j) \mid \text{either } \kappa_j = 0 \text{ and } \sup_{d^j \in D_j} \langle x^j, d^j \rangle < +\infty, \text{ or } \kappa_j > 0 \text{ and } x^j \in \kappa_j C_j\}$$

and functional values

$$g_j^+(x^j, \kappa_j) \triangleq \begin{cases} \sup_{d^j \in D_j} \langle x^j, d^j \rangle & \text{if } \kappa_j = 0 \text{ and } \sup_{d^j \in D_j} \langle x^j, d^j \rangle < +\infty \\ \kappa_j g_j(x^j / \kappa_j) & \text{if } \kappa_j > 0 \text{ and } x^j \in \kappa_j C_j. \end{cases}$$

The resulting problem $\mathcal{Q}$ can clearly be stated in the following way.

PROBLEM A. Consider the objective function G whose domain

$$C \stackrel{\Delta}{=} \{(x, \kappa) \mid x^k \in C_k, k \in \{0\} \cup I, \text{ and } (x^j, \kappa_j) \in C_j^+, j \in J\}$$

and whose functional value

$$G(x, \kappa) \stackrel{\Delta}{=} g_0(x^0) + \sum_j g_j^+(x^j, \kappa_j).$$

Using the feasible solution set

$$S \stackrel{\Delta}{=} \{(x, \kappa) \in C \mid x \in X, \text{ and } g_i(x^i) \leq 0, i \in I\},$$

calculate both the problem infimum

$$\varphi \stackrel{\Delta}{=} \inf_{(x, \kappa) \in S} G(x, \kappa)$$

and the optimal solution set

$$S^* \stackrel{\Delta}{=} \{(x, \kappa) \in S \mid G(x, \kappa) = \varphi\}.$$

Now, section 3 of [4] shows that

$$\mathcal{Y} = \{(y^0, y^I, \lambda, y^J, \beta) \in E_n \mid (y^0, y^I, y^J) \in Y; \beta = 0, \lambda \in E_0(I)\}.$$

Section 3 of [4] also shows that

$$\mathcal{D} = \{(y^0, y^I, \lambda, y^J, \beta) \in E_n \mid y^0 \in D_0; (y^i, \lambda_i) \in D_i^+, i \in I; y^j \in D_j,$$

$$\beta_j \in E_1, \text{ and } h_j(y^j) + \beta_j \leq 0, j \in J\},$$

and that

$$h(y^0, y^I, \lambda, y^J, \beta) = h_0(y^0) + \sum_I h_i^+(y^i, \lambda_i),$$

where the (closed convex) function h_1^+ has a domain

$$D_1^+ = \{(y^1, \lambda_1) \mid \text{either } \lambda_1 = 0 \text{ and } \sup_{c^1 \in C_1} \langle y^1, c^1 \rangle < +\infty, \text{ or } \lambda_1 > 0 \text{ and } y^1 \in \lambda_1 D_1\}$$

and functional values

$$h_1^+(y^1, \lambda_1) \triangleq \begin{cases} \sup_{c^1 \in C_1} \langle y^1, c^1 \rangle & \text{if } \lambda_1 = 0 \text{ and } \sup_{c^1 \in C_1} \langle y^1, c^1 \rangle < +\infty \\ \lambda_1 h_1(y^1/\lambda_1) & \text{if } \lambda_1 > 0 \text{ and } y^1 \in \lambda_1 D_1. \end{cases}$$

The resulting problem $\mathcal{B}$ can clearly be stated in the following way.

PROBLEM B. Consider the objective function H whose domain

$$D \triangleq \{(y, \lambda) \mid y^k \in D_k, k \in [0] \cup J, \text{ and } (y^1, \lambda_1) \in D_1^+, i \in I\}$$

and whose functional value

$$H(y, \lambda) \triangleq h_0(y^0) + \sum_I h_1^+(y^1, \lambda_1).$$

Using the feasible solution set

$$T \triangleq \{(y, \lambda) \in D \mid y \in Y, \text{ and } h_j(y^j) \leq 0, j \in J\},$$

calculate both the problem infimum

$$\psi \triangleq \inf_{(y, \lambda) \in T} H(y, \lambda)$$

and the optimal solution set

$$T^* \triangleq \{(y, \lambda) \in T \mid H(y, \lambda) = \psi\}.$$

It is worth noting that dual problems A and B provide the only completely symmetric duality that is presently known for general (closed) convex programming with explicit constraints. Moreover, [1] and some of the references cited therein show that all other duality in convex programming can be viewed as a special case. For the fundamental relations between geometric duality and ordinary Lagrangian duality see [5].

Fenchel's duality theorem in the context of dual problems A and B is one of the most important theorems, as well as one of the deepest theorems, in geometric programming. It can be stated in the following way.

Theorem 2. If

(i) problem B has a feasible solution (y', λ') such that

$$h_j(y'^j) < 0 \quad j \in J,$$

(ii) problem B has a finite infimum ψ ,

(iii) there exists a vector (y^+, λ^+) such that

$$y^+ \in (ri Y),$$

$$y^{+k} \in (ri D_k) \quad k \in \{0\} \cup J,$$

$$(y^{+1}, \lambda_1^+) \in (ri D_1^+) \quad i \in I,$$

then

(I) problem A has both a nonempty feasible solution set S and a finite infimum φ , and

$$0 = \varphi + \psi,$$

(II) problem A has a nonempty optimal solution set S^* .

Proof. We obviously need only show that the Fenchel hypothesis in Theorem 1 (i.e. the hypothesis that there exists a vector $y^0 \in (ri \mathcal{Y}) \cap (ri \mathcal{B})$) is equivalent to hypotheses (i) and (iii) in Theorem 2.

Toward that end, we first use the formulas for $\mathcal{Y}$ and $\mathcal{B}$ to derive comparable formulas for $(ri \mathcal{Y})$ and $(ri \mathcal{B})$ -- two derivations that make crucial use of the following basic facts:

(A) $(ri U) = U$ when U is a vector space,

(B) $(ri V) = \bigcap_{k=1}^n (ri V_k)$ when $V = \bigcap_{k=1}^n V_k$ and the sets V_k are convex,

and

(C) $(ri W) = (int W)$, the "interior" of W , when W is a convex set with the same "dimension" as the space in which it is embedded.

Fact (A) is established on page 44 of [3]; fact (B) can be obtained inductively from the formula at the top of page 49 of [3]; and fact (C) is explained on page 44 of [3].

Now, the formula for $\mathcal{Y}$ along with facts (A) and (B) implies that

$$(ri \mathcal{Y}) = \{(y^0, y^I, \lambda, y^J, \beta) \in E_n \mid (y^0, y^I, y^J) \in (ri Y); \lambda \in E_{O(I)}; \beta = 0\}.$$

Moreover, the formula for $\mathcal{B}$ along with facts (A) and (B) implies that

$$(ri \mathcal{B}) = \{(y^0, y^I, \lambda, y^J, \beta) \in E_n \mid y^0 \in (ri D_0); \lambda_i > 0 \text{ and } y^I \in \lambda_i (ri D_i),$$

$$i \in I; y^J \in (ri D_j), \beta_j \in E_1, \text{ and } h_j(y^J) + \beta_j < 0, j \in J\},$$

by virtue of both the equation

$$(ri D_1^+) = \{(y^1, \lambda_1) \mid \lambda_1 > 0 \text{ and } y^1 \in \lambda_1 (ri D_1)\}$$

and the equation

$$\begin{aligned} (ri \{(y^j, \beta_j) \mid y^j \in D_j \text{ and } h_j(y^j) + \beta_j \leq 0\}) = \\ \{(y^j, \beta_j) \mid \beta_j \in E_1, y^j \in (ri D_j), \text{ and } h_j(y^j) + \beta_j < 0\}. \end{aligned}$$

To derive the latter equation, simply use Theorem 6.8 on page 49 of [3] along with fact (C). To derive the former equation, first consider the point-to-set mapping $Y_1^+ : \Lambda_1^+$ where

$$Y_1^+[\lambda_1] \stackrel{\Delta}{=} \{y^1 \mid (y^1, \lambda_1) \in D_1^+\}$$

and

$$\Lambda_1^{+\Delta} = \{\lambda_1 \mid Y_1^+[\lambda_1] \text{ is not empty}\}.$$

Now, Corollary 6.8.1 on page 50 of [3] implies that

$$(ri D_1^+) = \{(y^1, \lambda_1) \mid \lambda_1 \in (ri \Lambda_1^+) \text{ and } y^1 \in (ri Y_1^+[\lambda_1])\}.$$

Moreover, the definition of D_1^+ clearly shows that $\Lambda_1^+ = \{\lambda_1 \geq 0\}$, which means of course that

$$(ri \Lambda_1^+) = \{\lambda_1 > 0\}.$$

Furthermore, for $\lambda_1 > 0$ the definition of D_1^+ clearly shows that

$$Y_1^+[\lambda_1] = \lambda_1 D_1, \text{ which means that}$$

$$(ri Y_1^+[\lambda_1]) = \lambda_1 (ri D_1) \text{ for } \lambda_1 \in (ri \Lambda_1^+),$$

by virtue of Corollary 6.6.1 on page 48 of [3]. Consequently, our derivation of the preceding formula for $(ri\beta)$ is complete.

In particular then, the Fenchel hypothesis in Theorem 1 simply asserts that

there exists a vector $(y^0, y^I, \lambda, y^J, 0) = y^0$

such that $(y^0, y^I, y^J) \in (ri Y)$; $y^0 \in (ri D_0)$;

$\lambda_i > 0$ and $y^i \in \lambda_i (ri D_i)$, $i \in I$; $y^j \in (ri D_j)$

and $h_j(y^j) < 0$, $j \in J$.

To complete our proof, we now show that this hypothesis is in fact equivalent to the hypothesis

there exists a vector $(y'^0, y'^I, \lambda', y'^J)$

such that $(y'^0, y'^I, y'^J) \in Y$; $y'^0 \in D_0$;

$(y'^i, \lambda'_i) \in D_i^+$, $i \in I$; $y'^j \in D_j$ and $h_j(y'^j) < 0$, $j \in J$

--- and there exists a vector

$(y^{+0}, y^{+I}, \lambda^+, y^{+J})$ such that

$(y^{+0}, y^{+I}, y^{+J}) \in (ri Y)$; $y^{+0} \in (ri D_0)$; $\lambda_i^+ > 0$

and $y^{+i} \in \lambda_i^+ (ri D_i)$, $i \in I$; $y^{+j} \in (ri D_j)$, $j \in J$.

Obviously, a vector (y^0, y^I, λ, y^J) that satisfies the former hypothesis satisfies both parts of the latter hypothesis. On the other hand,

Theorem 6.1 on page 45 of [3] and Theorem 7.1 on page 51 of [3] imply that a convex combination $\alpha(y'^0, y'^I, \lambda', y'^J) + \beta(y^{+0}, y^{+I}, \lambda^+, y^{+J})$ of vectors $(y'^0, y'^I, \lambda', y'^J)$ and $(y^{+0}, y^{+I}, \lambda^+, y^{+J})$ that satisfy the latter hypothesis will satisfy the former hypothesis for sufficiently small $\beta > 0$. q.e.d.

Although the condition $h_j(y'^J) < 0$, $j \in J$ in hypothesis (i) of Theorem 2 resembles the well-known "Slater constraint qualification", it is of course to be deleted when J is empty -- which is the situation in most applications. However, the analogous condition $g_i(x'^I) < 0$, $i \in I$ in hypothesis (i) of the (unstated) dual of Theorem 2 (obtained from Theorem 2 by interchanging the symbols A and B , the symbols x and y , the symbols κ and λ , the symbols g and h , the symbols i and j , the symbols I and J , the symbols ϕ and ψ , the symbols X and Y , the symbols C and D , the symbols S and T , and the symbols S^* and T^*) is essentially the Slater constraint qualification. In fact, we shall now see that the "ordinary programming" case of the dual of Theorem 2 actually strengthens Slater's version of the "Kuhn-Tucker theorem".

The ordinary programming case occurs when

$$J = \emptyset,$$

$$n_k = m \text{ and } C_k^\Delta = C_0 \text{ for some set } C_0 \subseteq E_m \quad k \in \{0\} \cup I,$$

and

$$X^\Delta = \text{column space of } \begin{bmatrix} U \\ U \\ \vdots \\ U \end{bmatrix} \text{ where there is a total of } 1 + o(I) \text{ identity matrices } U \text{ that are } m \times m.$$

In particular, an explicit elimination of the vector space condition $x \in X$ by the linear transformation

$$\begin{pmatrix} x^0 \\ x \\ x^I \end{pmatrix} = \begin{bmatrix} U \\ U \\ \vdots \\ U \end{bmatrix} z$$

shows that the resulting problem A is equivalent to the very general ordinary programming problem

Minimize $g_0(z)$ subject to

$$g_i(z) \leq 0 \quad i \in I$$

$$z \in C_0.$$

Now, the Slater constraint qualification for the preceding problem simply requires the existence of a feasible solution z' such that $g_i(z') < 0$, $i \in I$. Moreover, Slater's version of the Kuhn-Tucker theorem asserts that the existence of such a "Slater solution" z' and the existence of a finite infimum φ are sufficient to guarantee the existence of a Kuhn-Tucker (Lagrange) multiplier vector λ^* .

To strengthen the preceding theorem with the aid of the dual of Theorem 2, first note that the image $x' = (z', z', \dots, z')$ of a Slater solution z' under the given linear transformation satisfies hypothesis (i) of the dual of Theorem 2. Then, note that the existence of a finite infimum φ is simply hypothesis (ii) of the dual of Theorem 2. Now, the convexity of C_0 implies the existence of a vector $z^+ \in (\text{ri } C_0)$, by virtue of Theorem 6.2 on page 45 of [3]. Moreover, its image $x^+ = (z^+, z^+, \dots, z^+)$ under the given linear transformation clearly satisfies hypothesis (iii)

of the dual of Theorem 2 -- because $(ri\ X) = X$ by virtue of fact (A), and because $J = \emptyset$. Consequently, the dual of Theorem 2 implies that both T and T^* are nonempty and that $0 = \varphi + \psi$. In view of Corollary 7A of [6], we conclude from the nonemptiness of T^* that a Kuhn-Tucker (Lagrange) vector λ^* exists. Finally, note that we have also shown the existence of another vector y^* ; so the Slater version of the Kuhn-Tucker theorem has actually been strengthened.

More significant implications of Theorem 2 are given on page 47 of [1].

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