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FINAL REPORT  
ON GRANT DAH C04-74-G-0030

Strain Rate and History Effects  
on the Deformation of Metals

by

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Dec. 28, 1976

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## ABSTRACT

This report is a summary of the research work accomplished during the last three years under grant DAH-C04-74-G-0030, entitled "Strain Rate and History Effects on the Deformation of Metals". The work falls into two main categories: (i) The Endochronic Constitutive Theory and (ii) The Endochronic Theory of Fracture. Both these theories constitute significant innovations over what was the state of the art prior to their development. They are based on the concept of intrinsic time put forward by Valanis.

Of far greater significance, however, is the fact that both theories have been demonstrated to be valid mathematical representations of material behavior as observed in the laboratory, and they unify, possibly for the first time constitutive and fracture behavior of metals.

On the conceptual level they establish two conclusions: (a) The yield surface is not necessary for the description of plastic behavior (b) History yet rate independent behavior can be formulated within the framework of the theory of thermodynamics of internal variables as formulated and developed by the principal investigator.

Recommendation. Both theories are in need of further investigation and development. The results obtained are so fruitful that they should be examined further in depth as well as breadth. They both hold promise of interconnecting at the mesoscopic level atomic and phenomenological view points and of providing simple, yet effective, mathematical representation of the mechanical response of metals to triaxial deformation histories.

REVIEW OF RESEARCH ACCOMPLISHED  
UNDER GRANT DAH C04-74-G-0030

INTRODUCTION

The research work during the last three years under Grant DAH C04-74-G-0030 falls into two main categories:

(i) The Endochronic Constitutive Theory which is the underlying basis for the mathematical representation of the mechanical behavior of metals.

(ii) The Endochronic Theory of Failure which is the basis for the analytical prediction of fatigue and fracture of metals.

We believe that the project has been eminently successful. Twelve papers have been published or accepted in leading journals or proceedings of national conferences. In addition four Ph.D. theses were completed under the grant and ten Divisional Reports were printed.

The technical aspects of what was achieved are discussed at some length in the body of the Report.

ENDOCHRONIC CONSTITUTIVE THEORY

The endochronic theory of plasticity was formulated in 1971 by the principal investigator. This was accomplished by stipulating a material memory with respect to the length of a deformation path  $\zeta$  in a Riemannian nine dimensional strain space with a metric  $\underline{P}$ , where  $\underline{P}$  is a material property. The timelike "measure"  $d\zeta$  is evidently intrinsic to the material in question.

As a consequence the adjective "endochronic" was coined. The purposes of formulating the theory were fourfold:

- (i) to place the mathematical representation of rate independent yet history dependent material behavior on a sound thermodynamic foundation;
- (ii) to eliminate constitutive ambiguities associated with a non-unique definition of the yield surface;
- (iii) to demonstrate that such a theory gives rise to results that are in accord with observed plastic behavior of metals;
- (iv) to unify the theories of plasticity and viscoplasticity.

As a result of (iv) it was shown that a viscoplasticity theory follows as a natural generalization of the plasticity theory by defining an intrinsic time measure, as a distance between two adjacent points, in a Riemann space of strain-time.

During the last three years the principal investigator and his co-investigator have worked mainly toward tasks (iii) and (iv) above, while broadening, at the same time, the conceptual foundations of the theory to accommodate, with greater quantitative accuracy, observed behavior of a wider variety of metals subject to a greater diversity of mechanical histories.

The following two theses were completed in the course of the above investigation:

"Thermodynamics of Internal Variables in the Context of the Absolute Rate Theory and the Notion of Intrinsic Time", by S. Lalwani, 1976.

"A Probabilistic Theory of the Time to Fracture . . ." by U. Yilmazer, 1976.

ENDOCHRONIC CONSTITUTIVE EQUATIONS

Following the inception of the endochronic theory we explored the applicability of the theory by first deriving the simplest three-dimensional explicit forms possible and by limiting our investigation (a) to complexities of dimensionality in the presence of essentially monotonic histories and (b) to one dimension while exploring the effects of complexities of the deformation history.

With the above in mind we developed the "linear functional form of the theory" as it has been presented in Ref.'s (1) and (2) i.e., in the notation of those references

$$g = \int_0^z \lambda(z-z') \frac{\partial \epsilon_{kk}}{\partial z'} dz' + 2 \int_0^z \mu(z-z') \frac{\partial \xi}{\partial z'} dz' \quad (1)$$

where  $z$  is an intrinsic time scale such that

$$dz = \frac{d\zeta}{f(\zeta)} \quad (2)$$

and  $d\zeta$  is an intrinsic time measure where

$$d\zeta = P_{ijkl} d\epsilon_{ij} d\epsilon_{kl}, \quad (3)$$

$P$  is a material fourth order tensor and  $f$  is a non-negative function of  $z$ .

Equation (1) was generalized somewhat in a later reference (3) to

account for situations where the material is not in an annealed state at the onset of deformation. The early investigations proved to be extremely fruitful and promising, a fact to which References (1) - (3) will easily attest. Deformation induced hardening is an effect which the theory could predict naturally to a remarkable quantitative degree. Cyclic hardening and softening as a result of cyclic loading in one dimension were also depicted, by the theory, in References (2), (3) and (4).

Mechanical behavior at the onset and during unloading was also the subject of extensive study during the last three years, particularly in one dimension. One of the attractive features of the endochronic theory lay in the fact that it could predict the phenomenon of constitutive discontinuity, naturally, without a need for additional assumptions which in effect postulate one constitutive equation that applies during loading and another during unloading, thus creating a dichotomy in the mathematical representation of material response. However, certain problems arose which are explained below:

With specific reference to uniaxial tests the "linear" endochronic theory predicts, at the onset of unloading, a slope which is equal to  $2E_0 - E_t$  where  $E_0$  is the initial modulus and  $E_t$  the tangent modulus at the point of unloading. This is practically twice the observed slope in the "flat" portion of the stress-strain curve in the case of aluminum and steel. The high slope of the unloading stress-strain curve persists during unloading giving rise to permanent strains which are overestimated. In any event, a "straight" unloading curve, as observed in the case of steels could not be obtained by the "linear" theory.

To predict linear unloading a non-linear form of the endochronic theory was developed in Ref. (5). What was even more remarkable was that a linear unloading curve was predicted exactly by a non-linear constitutive equation proposed therein. It was thus demonstrated that linear unloading behavior need not be assumed separately but can be obtained as a specific response to a specific history from one and the same constitutive equation.

There remained the question of repetitive loading and unloading with particular reference to high tensile aluminum and steel. Particularly in the case of steels and in certain strain ranges unloading and subsequent loading follow essentially identical linear paths until the initial unloading point is reached beyond which the loading response follows the virgin curve. This kind of behavior is illustrated in Fig. 1.

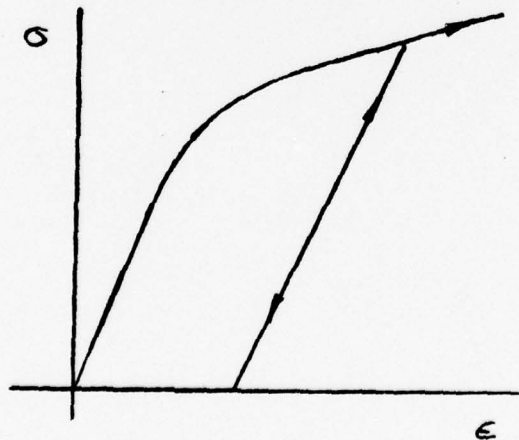


Fig. 1.

Repetitive loading unloading response.

At this point we are not aware of a non-dichotomized constitutive equation - with the exception of the one to be discussed below - which can accommodate exactly such behavior. It was found, in particular, that the endochronic theory (linear or non-linear) could not depict this kind of response. Though the non-linear equation of Ref. (5) could predict linear unloading, it cannot exactly predict linear reloading.

#### The Autochronic Theory

The search for an "exact" theory in the above context lead to a broadening of the conceptual framework of the endochronic theory (6, 7). First the idea of an intrinsic time spectrum was introduced by assigning to each internal variable a particular and distinct time scale. This lead further to the idea of the autochronic time scale whereby the autochronic time measure was defined strictly in terms of the internal variable to which it applies. More precisely if  $d\tau_r$  is the autochronic time measure appropriate to the internal variable  $q_r$  then

$$d\tau_r^2 = Q_{ijkl}^{(r)} dq_{ij}^{(r)} dq_{kl}^{(r)} \quad (4)$$

where  $Q^r$  is a fourth order material tensor.

Further the concept of internal barriers, which has existed in the field of physics for many years, was introduced, for what is believed to be the first time, in the internal variable theory. In effect, it was stated that to each internal variable  $q_r$  there exists an internal barrier  $\mu_r$  such that  $q_r$  is not activated unless the value of the norm  $\left\| \frac{\partial \psi}{\partial q_r} \right\|$

of the "internal force"  $\frac{\partial \psi}{\partial q_r}$  equals or exceeds  $\mu_r$ , i.e.,

$$\left\| \frac{\partial \psi}{\partial q_r} \right\| = \left( \frac{\partial \psi}{\partial q_r} \cdot \underset{\sim}{\phi}^r \cdot \frac{\partial \psi}{\partial q_r} \right)^{\frac{1}{2}} \geq \mu_r \quad (5)$$

where  $\underset{\sim}{\phi}^r$  is a fourth order material tensor. A more precise definition of this condition is given in Ref. (6).

The autochronic theory is dealt with in detail in Ref. (6). Specifically, an application was made to the one dimensional response of a "stable" metal i.e., one that does not manifest significant hardening in tension following unloading and subsequent compression in the plastic range. The behavior of such a material is illustrated in Fig. 2 below, within a certain strain range as discussed in Ref. (6):

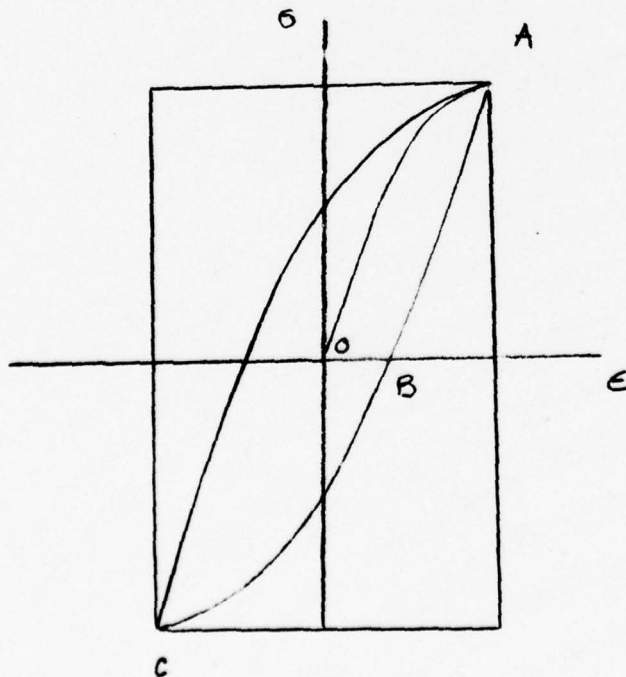


Fig. 2.

It is interesting that the autochronic theory, when applied to material such as normalized mild steel with stable cyclic behavior (possibly after a period of cyclic hardening or softening), predicts a cyclic behavior which is consistent with the Massing hypothesis (7). This hypothesis asserts that if OA in Fig. 2 is the curve of the function  $f(\epsilon)$  then the unloading curve OBC is the curve of the function  $g(\epsilon)$  where  $g(\epsilon)$  is obtained from  $f(\epsilon)$  by the transformation,

$$g(\epsilon) = 2f\left(\frac{\epsilon}{2}\right) \quad . \quad (6)$$

Whereas Massing postulated the consequences of the above equation on the basis of observation, the autochronic theory gives equation (6) as a derived result. The application of the theory to multi-dimensional response requires further detailed investigation.

#### Plastic Fluids

The foregoing discussion limited itself to small deformation situations. As we weave the thread of research at Iowa, always in the context of the endochronic theory and irreversible thermodynamics, we come upon our large deformation investigations (8). In the reference just cited we faced a number of issues. One concerned the mathematical representation of the behavior of simple fluids i.e. those materials whose behavior is in general given by the constitutive equation

$$\underline{\sigma} = \underline{F} \left( \overset{0}{\underset{-\infty}{\int}} \underline{C} (s); \rho \right) \quad (7)$$

where  $\underset{\sim}{C}_t$  is the relative Finger tensor,  $s$  is the past time and  $\rho$  the current density. Linear versions of the above constitutive equation for incompressible media have been given by Lodge, Bogue and Carreau and others (9-11). One such version is

$$\underline{\sigma} = -p\underline{\delta} + \int_0^t \mu(t-\tau) \underset{\sim}{C}_t(\tau) d\tau \quad (8)$$

where

$$C_{tij}(\tau) = \frac{\partial y_i(t)}{\partial y_k(\tau)} \frac{\partial y_j(t)}{\partial y_k(\tau)} \quad (9)$$

Note that such materials are not cognizant of a specific reference configuration; as such they are classified as fluids. Hitherto, the internal variable theory has failed to give rise to such an equation as (8). We felt that this was a curiosity as well as an omission. The difficulty was resolved by ascribing to the tensorial internal variables a covariant, a contravariant or mixed character, and ensuring that covariance (say) was preserved in the equation of evolution of the internal variables. Typically, such an equation has the following appropriate form:

$$\frac{\partial \psi}{\partial q_{\alpha\beta}}(\underline{r}) + b q_{\alpha\beta}(\underline{r}) = 0 \quad (10)$$

where the balance of indices is worth noting. It transpired that equations such as equation (10) are appropriate to simple fluids as shown in Ref. (8).

The natural question that ensues, is whether in fact one could

derive equations for plastic fluids i.e. rate independent but history dependent materials which have no cognizance of a specific reference configuration. The answer is in the affirmative. The task at hand is accomplished by defining an intrinsic time by the relation

$$\left(\frac{dz}{dt}\right)^2 = P_{ijkl} d_{ij} d_{kl} \quad (11)$$

where  $\dot{d}$  is the deformation rate tensor and  $\overset{P}{\underset{\dot{d}}{P}}$  is positive semidefinite and symmetric and may at most depend on  $\dot{d}$  in a fashion that renders  $\overset{P}{\underset{\dot{d}}{P}}$  independent of the strain rate. Note that  $z$  defined above is independent of the reference configuration as well as the newtonian time scale. We then showed in Ref. (8), that one may derive with little effort the constitutive equation:

$$\overset{\sigma}{\underset{\dot{d}}{\sigma}} = -P\overset{\dot{d}}{\underset{\dot{d}}{\dot{d}}} + \int_0^z \mu(z-z') \overset{C}{\underset{\dot{d}}{C}}(z') dz' \quad (12)$$

where

$$C_{zij} = \frac{\partial y_i(z)}{\partial y_k(z')} \frac{\partial y_j(z)}{\partial y_k(z')} \quad (13)$$

Equation (12) does in fact pertain to an incompressible plastic fluid in the above sense. Thus we have laid the foundation for the application of the endochronic theory to finite deformation.

#### Viscoplasticity

The uncertainty associated with the representation of plastic response of a material is a major factor causing difficulty in dynamic

plasticity. This difficulty, of course, has its roots in the theory of plasticity and can fortunately be overcome by the endochronic theory (12). In that investigation, it was first demonstrated that it is possible to reproduce the experimentally determined constant strain-rate stress-strain curves for annealed aluminum using the endochronic theory (12,13). In Ref. (13) in particular the strain-rate effect is assumed to exist and the intrinsic time  $\zeta$  is assumed to depend on the strain rate in the fashion given by equation (14):

$$d\zeta = \{k_a - k_b \log(\dot{\epsilon}/\dot{\epsilon}_0)\} |d\epsilon| \quad (14)$$

where  $k_a$  and  $k_b$  are material parameters and  $\dot{\epsilon}_0$  is a reference strain rate "under quasi-static conditions".

An endochronic theory of viscoplastic wave propagation under conditions of variable strain-rate was then developed. Emphasis was placed on the plastic wave propagation under combined loads and also on the strain-rate and the strain-rate history effects. In the investigation of combined loading, the strain-rate effect was suppressed. On the other hand, the strain-rate and strain-rate history effects were investigated only in the one dimensional problem.

The problem of interest in the case of combined loading is that of a long slender thin-walled cylindrical tube subject to a suddenly applied combined longitudinal and torsional motion (or stress) at one end. This problem has acquired the attention of researchers only rather recently. Most of the published work is, however, to our knowledge, based on the

flow theory of plasticity, and both qualitative and quantitative discrepancies have been found upon comparison of theory and experiment. The most important discrepancy of all is a constant strain region between the fast and slow waves, predicted theoretically yet not experimentally observed. Upon application of the endochronic theory to this problem (Ref. (14, 15)), it is shown theoretically that the endochronic theory does not lead to such a constant strain region and thereby agrees with experiment. The experimentally observed final strain levels are also predicted better by the endochronic theory.

The problem of longitudinal wave propagation in thin rods was investigated (13, 16) to show the influence of strain-rate and strain-rate history on the dynamic material response. The theoretical strain-time profiles are in quantitative agreement with the experimental results. It was shown theoretically that the strain-rate effect in the endochronic theory lowers the final constant strain states in the strain-time profiles. The dynamic stress-strain curves for the impacts considered were also investigated. The strain-rate is not constant along these curves. The final state for each impact is represented by a point on the stress-strain diagram. It is interesting to note that the endochronic theory predicts a final state which lies above the quasi-static stress-strain curve. This effect is attributed to the fact that the strain-rate history plays a role in this calculation, and its effects cannot be predicted by an "overstress" type theory.

Further development of the endochronic theory of plastic wave propagation has been made using the stress defined intrinsic time which was

introduced in Ref. (5). In Ref. (17), it was shown that the stress defined intrinsic time can be used conveniently in the description of mechanical behavior of strain-hardening materials. This investigation is an application of the endochronic theory in its Gibbs free energy form.

In Ref. (18), a nonlinear form of the endochronic theory has been derived based on the Helmholtz free energy form. Again, the stress defined intrinsic time was employed in the investigation. The nonlinear constitutive equation was then applied to investigate effects which have not been so well predicted by the linear form of the theory. Plastic wave propagation was a very important part of this investigation.

We would like to mention the following two Ph.D theses, written in the course of this investigation:

"Plastic Wave Propagation in the Endochronic Theory of Viscoplasticity",  
by H. C. Lin, 1974

"A Nonlinear Form of the Endochronic Theory and Its Application  
to Dynamic Plasticity", by Albert R. F. Yao, 1976

#### Internal Variable Theory in a Deformation Kinetics Context

In the many instances where there was occasion to discuss the internal variable theory, a question was almost always asked of the speaker on the physical meaning of the internal variables. Though it has often been conjectured that the internal variables are "in some sense, averages of atomic motions," the quantitative correlation between the former and the latter has remained elusive. In Ref. (19) we addressed this problem in

an effort to answer the above question. The results have been extremely rewarding.

Our point of departure was Eyring's absolute rate theory (20). Eyring was able to develop strain rate types of constitutive equations by asserting that the potential barrier to a process - chemical or mechanical - is distorted by the application of a driving external field. He assumed that in the case of deformation the driving field is in fact the stress.

Eyring's constitutive rate equations gave the right qualitative trends but contained a certain degree of rigidity that made their generalization difficult. Very recently Eyring and his associates have strived to overcome the difficulty by introducing mechanical spring and dashpot models into the theory.

We followed instead another path in which we were guided by the concepts of the internal variable theory. We stipulated that an internal variable  $q_r$  is in fact the average displacement of atoms which are impeded by a barrier of a specific "height"  $\epsilon_0^r$ . To the extent the  $q_r$  are representative of internal motion it is physically consistent to expect that the barrier  $\epsilon_0^r$  will be distorted not by the applied stress but by the "internal force"  $\frac{\partial \psi}{\partial q_r}$ , where in standard notation  $\psi$  is the Helmholtz free energy.

It was shown in Ref. (19) that the resulting equation of evolution of the internal variables in the presence of initially equal forward and backward barriers is of the form

$$\frac{\partial \psi}{\partial q_r} + \frac{1}{k_r} \sinh^{-1} \left( \frac{\dot{q}_r}{k_l} \right) = 0 \quad (15)$$

In equation (15),  $k_1^r$  is the rate constant appropriate to the internal variable  $q_r$ . Specifically  $k_1^r$  is given by equation (16)

$$k_1^r = \frac{2\lambda^r}{\tau^r} \frac{\sum_{\substack{r \\ \epsilon_i^r > \epsilon_0^r}} e^{-\frac{\epsilon_i^r}{kT}}}{\sum_{\epsilon_i^r} e^{-\frac{\epsilon_i^r}{kT}}} \quad (16)$$

where  $\lambda^r$  is an interatomic distance,  $\tau^r$  a time constant of the process and  $\epsilon_i^r$  the energy states of the group of atoms appropriate to  $q_r$ . In addition  $\epsilon_0^r$  is the potential barrier,  $k$  is the Boltzmann constant and  $T$  the absolute temperature. The constant  $k_2^r$  is proportional to the barrier distortion per unit value of  $\frac{\partial \psi}{\partial q_r}$ . See page 19, Ref. (19).

It is significant that equation (15) represents an important case of successful intermarriage of continuum and particle mechanics.

A linear form of equation (15) was proposed much earlier by the principal investigator (21). However, a generalization of this linear form was not at all obvious without recourse to the fundamentals of statistical mechanics of rate processes. Equation (15) was the basis for the study of nonlinear relaxation and creep phenomena in metals, as discussed at length in Ref. (19), where adequate agreement between theory and experiment was demonstrated.

ENDOCHRONIC THEORY OF FAILURE

In this section we describe our work in fatigue and fracture using the endochronic theory of failure initially proposed by the principal investigator (35).

This theory is a combination of the theory of intrinsic time to failure and the endochronic theory of viscoplasticity. Both theories were originally formulated by Valanis at Iowa. The endochronic theory of plasticity has gained its acceptance in the field of plasticity through the years since its initiation in 1971. Considerable amount of work has since been done at Iowa aiming at the application of the endochronic theory in predicting experimentally observed effects as discussed in the previous section.

Progress has also been made (36, 37) in the theory of intrinsic time to failure since its birth in 1974. The application of the theory has now been extended to include plastic materials under monotonic loadings. The reports on the contributions to the development of this theory are listed for information. A brief account of the theory of intrinsic time to failure is given below:

The theory accounts for the three causes which have long been recognized as contributing strongly to the phenomenon of fracture. These are: the role of the strain energy density, the stochastic nature of fracture, and the phenomenon of cumulative damage which led to "the time to fracture" hypothesis by Zhurkov (38). In addition, the theory utilizes the notion of intrinsic time which was discussed in the previous section.

According to the endochronic theory of failure, which is an energy-probability theory, fracture occurs on an intrinsic time scale  $\zeta$  which in the

case of plasticity is given by

$$d\zeta^2 = P_{ijkl} dE_{ij} dE_{kl} \quad (17)$$

where  $P_{ijkl}$  is a positive semi-definite symmetric material tensor which may depend on the Green tensor  $E_{ij}$ . Fracture of a microelement will have occurred if the intrinsic time  $\zeta$  of the microelement has reached a critical value  $\zeta_c$ , which is given by the equation:

$$\int_0^{\zeta_c} \{1 - \exp(-\frac{\gamma}{kT} \langle \psi - \psi_0 \rangle)\} d\zeta = 1 \quad (18)$$

where  $\gamma$  is a material parameter;  $k$  is the Boltzmann constant;  $T$  is the absolute temperature;  $\psi$  is the change in energy of the microelement relative to its unstressed state and  $\psi_0$  a fracture activation energy.

Applying equation (18), it was possible to obtain results for a number of interesting cases in fracture which, otherwise, appear to bear no apparent relation to one another. The results obtained include:

- (i) the derivation of analytical expressions for "S-N" curve of a specimen in a uniaxial cyclic stress field, for asymptotically small and asymptotically large stress;
- (ii) a criterion for the brittle extension of a crack which includes the Griffith crack propagation criterion as a special case;
- (iii) a rational explanation for the disparity in the strengths of a glass beam and rod which have the same geometry and are made of the same material;
- (iv) the derivation of a polar form of the fracture surface in

strain (stress) space under proportional straining (loading) for linearly elastic isotropic materials.

The polar form of the fracture surface has been subsequently studied for plastic materials. In this study, monotonic proportional straining was assumed. The endochronic constitutive equations were utilized to obtain an expression for the free energy function. Furthermore, it has been shown that the free energy of a plastic material under proportional loading and at "large" strain is a quadratic function of the stress.

The following fracture criterion was then derived:

$$f(\cos\phi) \left\{ S_c - \frac{\bar{n}}{2 \sqrt{\frac{\gamma \mu_0}{\pi k T} \left( 1 + \frac{3\nu}{1-2\nu} \cos^2 \phi \right)}} \cdot \operatorname{erf} \left( \sqrt{\frac{\gamma \mu_0}{k T} \left( 1 + \frac{3\nu}{1-2\nu} \cos^2 \phi \right)} \right) \cdot \left( \frac{1 + \beta f S_c}{\beta f \bar{n}} \right) \right\} = 1 \quad (19)$$

This criterion is applicable to materials that undergo plastic deformation prior to failure under proportional straining. In equation (19),  $f$  is defined by  $f^2 = P_{ij} l_i l_j$ , where  $l_i$  are constant direction cosines of a radial strain path in the principal strain space;  $\phi$  is the angle between the strain vector and the hydrostatic strain direction;  $S_c$  is the critical length of the strain path; and  $\mu_0$ ,  $\nu$ ,  $\bar{n}$  and  $\beta$  are material constants.

Symmetries of the fracture surface have also been discussed at length in this study. Specifically, it has been shown that the fracture locus in the deviatoric plane ( $\phi = \frac{\pi}{2}$ ) must possess a six-fold symmetry for isotropic materials, so that only a  $30^\circ$  sector of the fracture locus need

to be found for the entire locus to be completely determined.

The polar form of the fracture criterion was then applied to the description of the experimental data obtained by Coffin (39) on gray cast iron. In doing so the function  $f(\phi)$  for gray cast iron has been determined. The results also show that the fracture surface for gray cast iron in principal strain space is indeed rotationally symmetric with respect to the hydrostatic strain vector.

Gray cast iron is a material which is considered to be 'brittle' but without either a linear or an elastic stress-strain curve. A moderate amount of plastic deformation is known to occur prior to fracture even under uniaxial tension. Hence, the plastic deformation plays a role in the prediction of failure of this material.

In this investigation, the following conclusions were also obtained:

(i) In the event that the constitutive equation of a material is known, then biaxial stress data suffice to determine the polar form of the fracture surface in the fully three dimensional strain space and thereby make possible the prediction of failure in three dimensional strain fields.

(ii) In terms of strain space coordinates, the form of the fracture surface becomes  $i_2 = g(i_1)$ , where  $i_1$  and  $i_2$  are the first and second invariants of the strain respectively. Thus, the fracture criterion is indeed a strain criterion which was previously discussed by Wu (40, 41) in connection with brittle materials.

(iii) Under proportional straining (loading) the nature of fracture is controlled by the magnitude of the hydrostatic strain (stress) at fracture.

On the basis of the foregoing discussion the following points are apparent:

(a) If during fracture experiments with proportional loading, strain measurements are made, then the strain failure surface may be determined either in polar form, or in strain space, without knowledge of the constitutive properties of a material.

(b) If stresses are measured, then a stress failure surface can be constructed in stress space, again without knowledge of the constitutive properties of the material.

However, when the above conditions apply, it is not possible to map a strain failure surface into a stress failure surface. In other words it is not possible to obtain the failure surface in stress space from its counterpart in strain space (or vice-versa) without knowledge of the constitutive properties of the material.

On the other hand, if both stresses and strains are measured during fracture experiments with proportional loading, much richer results can be obtained. The fracture surfaces in both spaces can be determined, but in addition constitutive properties can also be deduced.

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Of far greater significance, however, is the fact that both theories have been demonstrated to be valid mathematical representations of material behavior as observed in the laboratory, and they unify, possibly for the first time constitutive and fracture behavior of metals.

On the conceptual level they establish two conclusions: (a) The yield surface is not necessary for the description of plastic behavior (b) History yet rate independent behavior can be formulated within the framework of the theory of thermodynamics of internal variables as formulated and developed by the principal investigator.

Recommendation. Both theories are in need of further investigation and development. The results obtained are so fruitful that they should be examined further in depth as well as breadth. They both hold promise of interconnecting, at the mesoscopic level, atomic and phenomenological view points and of providing simple, yet effective, mathematical representation of the mechanical response of metals to triaxial deformation histories.