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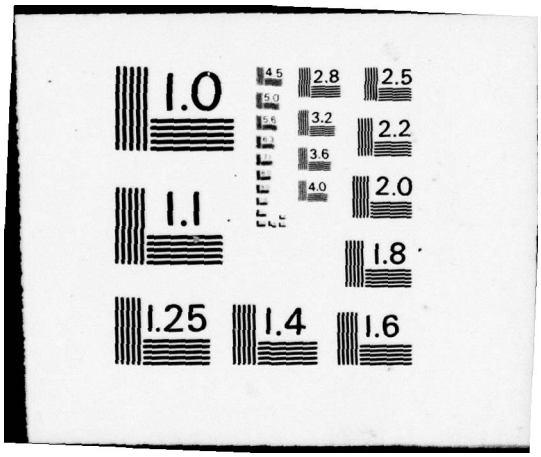
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REPORT ONR-CR215-234-2

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SEPARATED FLOWS AND THEIR REPRESENTATION BY BOUNDARY-LAYER EQUATIONS

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DEPARTMENT OF MECHANICAL ENGINEERING
CALIFORNIA STATE UNIVERSITY AT LONG BEACH
LONG BEACH, CALIFORNIA
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CONTRACT NO. N00014-75-C-0607
ONR TASK 215-234

SEPTEMBER 1976

PREPARED FOR THE

OFFICE OF NAVAL RESEARCH ● 800 N. QUINCY ST. ● ARLINGTON ● VA ● 22217

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This report presents an efficient numerical method for computing laminar and turbulent boundary layers with and without separation. It represents a major extension to the earlier work described by the author and makes use of Keller's two-point finite-difference method to solve the governing equations. The non-linear eigenvalue approach of the earlier work was found to be inappropriate to the separating boundary layers which are the main concern of this report and an alternative-approach, based on the solution procedures of Cebeci and next page

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cont.

Keller has been used. The solutions, which relate to attached and separating flows, were obtained with prescribed distributions of displacement thickness.

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PRINCIPAL NOTATION

A	Van Driest damping length parameter
c_f	local skin-friction coefficient
f	dimensionless stream function
f'	u/u_e
K	variable grid parameter
L	modified mixing length, or reference length
p	pressure
p^+	dimensionless pressure-gradient parameter
R_x, R_L	Reynolds number, $u_e x/\nu$ or $u_e L/\nu$
R_θ	Reynolds number, $u_e \theta/\nu$
u,v	velocity components in the x- and y-directions, respectively
u_o	reference velocity
u_τ	friction velocity (τ_w/ρ)
x,y	Cartesian coordinates
α	parameter in the outer eddy-viscosity formula, see (2.8)
δ	boundary-layer thickness
δ^*	displacement thickness, see (2.5)
ε_m	eddy viscosity
ε_m^+	dimensionless eddy viscosity
η	transformed y-coordinate
η_∞	transformed boundary-layer thickness
θ	momentum thickness, $\int_0^\infty u/u_e(1 - u/u_e)dv$
γ_{tr}	intermittency
κ	Von Karman's constant

μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress
ψ	stream function

Subscripts

e	outer edge of boundary layer
w	wall
∞	freestream conditions
$()'$	differentiation with respect to η

1. INTRODUCTION

Boundary-layer calculations are usually performed for prescribed boundary conditions which, for two-dimensional incompressible flows, may be provided in the form

$$u(x,0) = 0, \quad v(x,0) = v_w(x), \quad u(x,\delta) = u_e(x) \tag{1.1}$$

In some problems, however, the external velocity distribution is unknown and can be determined in order to satisfy an alternative boundary condition, such as prescribed displacement thickness, $\delta^*(x)$, or the wall shear, $\tau_w(x)$. These solutions are obtained with "inverse boundary-layer methods" and find application in many practical problems. For example, the airfoils discussed in a recent article by Liebeck (1976) are designed on the principle that, in certain regions of an airfoil flow, the deceleration results in values of wall shear stress close to zero. Other important applications of inverse boundary-layer procedures occur in the calculation of duct flows such as those discussed by Cebeci and Bradshaw (1977).

A particularly important application of inverse boundary-layer procedures is their potential to calculate the properties of separating and reattaching boundary-layer flows. It is well known that, for a prescribed external velocity distribution, the boundary-layer equations are singular at separation: they are not singular, however, when the displacement thickness is prescribed. This was demonstrated by Catherall and Mangler (1966) who solved the laminar boundary-layer equations in the usual way until separation was approached and then, by assuming a displacement-thickness distribution, calculated the external-velocity distribution and local flow properties through the recirculation region.

More recent investigations of the inverse boundary-layer method have been reported, for example, by Klineberg and Steger (1974), Horton (1974), Carter (1974,1975), Carter and Wornom (1975) and Williams (1975,1976), and have involved solutions to the laminar boundary-layer equations with prescribed displacement-thickness and shear-stress distributions in regions of negative shear stress.

The present report is concerned with incompressible turbulent boundary-layer flows with and without separation. It represents a major extension to

the work described by Cebeci (1975a), which was concerned with compressible, attached boundary-layer flows, and makes use of Keller's (1970) two-point finite-difference method to solve boundary-layer equations appropriate to turbulent flow. The nonlinear eigenvalue approach of the earlier work was found to be inappropriate to the separating boundary layers which are the main concern of this report and an alternative-approach, based on the solution procedures of Cebeci and Keller (1972), has been used. The solutions, which relate to attached and separating flows, were obtained with prescribed distributions of displacement thickness.

The equations and boundary conditions are presented in the following section together with the algebraic eddy-viscosity formulation. The succeeding sections describe the solution method, referred to as the Mechul*-function method, and these results and their implications. A brief statement of the more important conclusions forms the final section.

*lit. unknown (Turk.)

2. BASIC EQUATIONS

2.1 Boundary-Layer Equations

The conservation equations appropriate for steady two-dimensional incompressible laminar and turbulent boundary layers may be written as follows:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Momentum:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \tag{2.2}$$

$$\tau = \mu \frac{\partial u}{\partial y} - \rho \overline{u'v'} \tag{2.3}$$

Corresponding boundary conditions, for zero mass transfer, are:

$$\begin{aligned} y &= 0 && u, v = 0 \\ y &\to \infty && u \to u_e(x) \end{aligned} \tag{2.4}$$

With specified distributions $p(x)$ or $u_e(x)$, the equations and boundary conditions represent the usual statement of the two-dimensional boundary-layer problem for laminar and turbulent flows. For convenience, we shall call it the standard problem.

The inverse problem, which we consider here, is a flow with specified displacement thickness, $\delta^*(x)$, distribution. Here $\delta^*(x)$ is defined by

$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u}{u_e}\right) dy \tag{2.5}$$

2.2 Turbulence Model

The solution of the equations in section 2.1 requires a closure assumption for the Reynolds shear stress, $-\rho \overline{u'v'}$. In our study we use the eddy-viscosity concept

$$-\rho\overline{u'v'} = \rho\varepsilon_m \frac{\partial u}{\partial y} \tag{2.6}$$

and prescribe ε_m in the manner recommended by Cebeci and Smith (1974) and shown by them to represent various boundary-layer conditions such as those at high- and low-Reynolds numbers, transition flows, etc. According to this formulation, the turbulent boundary layer is regarded as a composite layer consisting of inner and outer regions with different eddy-viscosity formulas for each region. In the inner region the eddy viscosity is defined by

$$(\varepsilon_m)_i = L^2 \frac{\partial u}{\partial y} \gamma_{tr}$$

Here

$$L = 0.4y[1 - \exp(-y/A)]$$

$$A = 26 \frac{\nu}{N} u_\tau^{-1} \quad , \quad N = (1 - 11.8p^+)^{1/2} \quad , \quad p^+ = \frac{\nu u_e}{u_\tau^3} \frac{du_e}{dx} \tag{2.7}$$

$$u_\tau = \left(\frac{\tau_w}{\rho}\right)^{1/2}, \qquad \gamma_{tr} = 1 - \exp\left[-G(x - x_{tr}) \int_{x_{tr}}^{x} \frac{dx}{u_e}\right]$$

$$G = 8.35 \times 10^{-4}(u_e^3/\nu^2)\, R_{x_{tr}}^{-1.34}$$

In the outer region the eddy viscosity is given by

$$(\varepsilon_m)_o = \alpha \left| \int_0^\infty (u_e - u)dy \right| \gamma_{tr} \tag{2.8}$$

where

$$\alpha = 0.0168[1.55/(1 + \Pi)]$$

$$\Pi = 0.55[1 - \exp(-0.243z_1^{1/2} - 0.298z_1)] \tag{2.9}$$

$$z_1 = R_\theta/425 - 1$$

$$R_\theta = \frac{u_e \theta}{\nu}$$

2.3 Formulation of the Inverse Problems

In our approach to inverse boundary-layer problems, we use the stream function concept and write the continuity and momentum equations as a third-order system in dimensionless quantities defined by

$$\bar{y} = \frac{y}{L}\sqrt{R_L} \quad , \quad \bar{x} = \frac{x}{L} \quad , \quad \bar{p} = \frac{p}{\rho u_o^2} \quad , \quad \bar{u} = \frac{u}{u_o} \quad , \quad \bar{v} = \frac{v}{u_o}\sqrt{R_L} \quad , \quad R_L = \frac{u_o L}{\nu} \tag{2.10}$$

Then it can be shown that (2.1) and (2.2) can be written as

$$(b\psi'')' - \frac{d\bar{p}}{d\bar{x}} = \psi' \frac{\partial \psi'}{\partial \bar{x}} - \psi'' \frac{\partial \psi}{\partial \bar{x}} \tag{2.11}$$

Here primes denote differentiation with respect to $\bar{y}$ and b is given by:

$$b = 1 + \varepsilon_m^+ \quad , \qquad \varepsilon_m^+ = \varepsilon_m/\nu \tag{2.12}$$

The boundary conditions (2.5) become

$$\bar{y} = 0 \qquad \psi = \psi' = 0 \tag{2.13a}$$

$$\bar{y} = y_e \qquad \psi' = \bar{u}_e \quad , \quad \psi = \bar{u}_e[\bar{y}_e - \sqrt{R_L}\,\overline{\delta^*}(x)] \tag{2.13b}$$

where the subscript e denotes the boundary-layer edge and

$$\overline{\delta^*}(x) = \frac{\delta^*(x)}{L} \quad , \quad \bar{u}_e = \frac{u_e}{u_o} \tag{2.14}$$

The relation between $\bar{u}_e$ and $\bar{p}$ is given by Euler's equation which, in terms of dimensionless quantities, can be written as

$$\frac{d\bar{p}}{d\bar{x}} + \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} = 0 \tag{2.15}$$

Two approaches to the solution of the system (2.11), (2.13) and (2.15) were attempted. In the first approach, which we call the "nonlinear eigenvalue method," the pressure was treated, as an eigenvalue parameter. The second approach regarded the pressure as an unknown function; we shall refer to this method as the "Mechul-function method." The nonlinear eigenvalue

method is described in detail in Cebeci (1975a) and only a brief description is presented here. The Mechul-function method, which proved to be more appropriate to flows with separation, is described in detail in section 3.

For flows with negative wall shear, it is necessary to make approximations to the governing equations to continue the calculations past the separation point. Here we use the approximation first suggested by Reyhner and Flügge-Lotz (1968). This approximation referred to as FLARE by Williams (1975), neglects the $u(\partial u/\partial x)$ term in the region of negative u-velocity. All inverse boundary-layer procedures, including the ones described here, use this approximation for regions of separated flow. Once a solution has been obtained with this approximation, however, the assumption can be relaxed to allow solutions of the complete boundary-layer equations. This relaxation of the assumption is effected with a forward and backward difference procedure, as described in section 3.3.

2.3.1 Nonlinear Eigenvalue Method

Let us assume that at $x = x_0$ we are given the initial velocity profiles and we seek a solution of the system (2.11) subject to (2.13). In the nonlinear eigenvalue method, $\bar{u}_e$ is assumed and with (2.15) we then obtain a solution of (2.11) subject to (2.13a) and to the first relation in (2.13b), namely $\psi' = \bar{u}_e$ at $y = y_e$. Solutions were obtained by Cebeci (1975a) for compressible flows, in which the governing equations were solved in transformed variables rather than the physical variables considered here. For the solution we can compute $\bar{\delta}^*$ (which we shall denote by $\bar{\delta}^*_c$) from (2.5) which, in terms of stream function and dimensionless quantities, can be written as

$$\bar{\delta}_c(x) = \frac{1}{\sqrt{R_L}} (y_e - \psi_e/\bar{u}_e) \tag{2.16}$$

Recalling that the desired value for the displacement thickness is $\bar{\delta}^*(x)$, we form:

$$\phi[\bar{u}_e(x)] \equiv \bar{\delta}^*_c - \bar{\delta}^* \tag{2.17}$$

and seek $\bar{u}_e$ such that $\phi[\bar{u}_e(x)] \equiv 0$.

To solve $\phi[\bar{u}_e(x)] = 0$, we use Newton's method. With an estimate $\bar{u}_e^{(0)}(x)$ of the external velocity we define the sequence $\bar{u}_e^{(\nu)}(x)$ by setting

$$\bar{u}_e^{(\nu+1)}(x) = \bar{u}_e^{(\nu)} - \frac{\phi[u_e^{\nu}(x)]}{\partial\phi[u_e^{\nu}(x)] / \partial\bar{u}_e} \tag{2.18}$$

The derivative of ϕ with respect to $\bar{u}_e$ can be obtained from (2.17) by making use of the relation given by (2.16). This gives

$$\frac{\partial}{\partial\bar{u}_e}\phi[u_e^{\nu}(x)] = \frac{1}{\bar{u}_e\sqrt{R_L}} - \frac{\partial\psi_e}{\partial\bar{u}_e} + \frac{\psi_e}{\bar{u}_e} \tag{2.19}$$

One step of the iteration may be summarized as follows: $u_e^{(0)}(x)$ is assumed and a solution of the standard problem obtained. From this result and from the desired value of $\bar{\delta}^*(x)$, ϕ is found from (2.17). The next value of $\bar{u}_e$ can then be calculated from (2.18) provided that $\partial\phi/\partial\bar{u}_e$ is known. That is, obtained by solving a set of variational equations as described in Cebeci (1976). The iteration process is repeated until

$$\left|\bar{u}_e^{(\nu+1)}(x) - \bar{u}_e^{(\nu)}(x)\right| < \delta_1 \tag{2.20}$$

where δ_1 is a small error tolerance.

3. MECHUL FUNCTION METHOD

3.1 Numerical Formulation

In the Mechul function method, we solve the system (2.11), (2.13) with the numerical method of Keller (1970). This is an efficient accurate two-point finite-difference method that has been used extensively by Cebeci for two- and three-dimensional flows (see, for example, Cebeci (1974, 1975b)). According to this method we introduce new dependent variables $u(\bar{x},\bar{y})$, $v(\bar{x},\bar{y})$ so that (2.11) can be written as a first-order system. With the function $\bar{p}$ treated as unknown and with the help of the y-momentum equation $(-\partial\bar{p}/\partial\bar{y} = 0)$ we can write (2.11) as

$$\psi' = u \tag{3.1a}$$

$$u' = v \tag{3.1b}$$

$$p' = 0 \tag{3.1c}$$

$$(bv)' - \frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} - v\frac{\partial \psi}{\partial x} \tag{3.1d}$$

Similarly (2.13) becomes

$$y = 0 \qquad \psi = u = 0 \tag{3.2a}$$

$$y = y_e; \qquad u = u_e, \qquad \psi_e = u_e[y_e - \sqrt{R_L}\,\bar{\delta}^*(x)] \tag{3.2b}$$

Here, for convenience, we have dropped the bars on x, y, p and u_e.

On the rectangular net shown in figure 1, we denote the node points by

$$x_0 = 0, \qquad x_n = x_{n-1} + k_n \qquad n = 1, 2, \ldots, N \tag{3.3}$$

$$y_0 = 0, \qquad y_j = y_{j-1} + h_j \qquad j = 1, 2, \ldots, J : \; y_J = y_e$$

Nonuniform net spacings k_n and h_j are used. The quantities (ψ, u, v, p) at points (x_n, y_j) of the net are approximated by net functions denoted by $(\psi_j^n, u_j^n, v_j^n, p_j^n)$. We also employ the notation for points and quantities midway between net points and for <u>any</u> net function q_j^n:

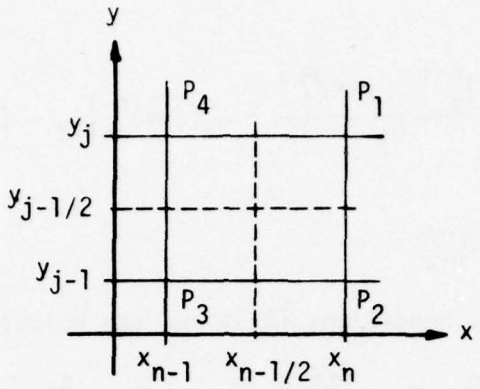


Figure 1. Net rectangle for difference approximations.

$$x_{n-1/2} \equiv \frac{1}{2}(x_n + x_{n-1}) \quad , \qquad \eta_{j-1/2} \equiv \frac{1}{2}(\eta_j + \eta_{j-1})$$
$$q_j^{n-1/2} \equiv \frac{1}{2}(q_j^n + q_j^{n-1}) \quad , \qquad q_{j-1/2}^n \equiv \frac{1}{2}(q_j^n + q_{j-1}^n) \tag{3.4}$$

The finite-difference forms of equations (3.1a,b,c), written for the midpoint $(x^n, y_{j-1/2})$ of the segment P_1P_2 shown in figure 1, are

$$\frac{\psi_j^n - \psi_{j-1}^n}{h_j} = u_{j-1/2}^n \tag{3.5a}$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n \tag{3.5b}$$

$$\frac{p_j^n - p_{j-1}^n}{h_j} = 0 \tag{3.5c}$$

Similarly (3.1d) is approximated by centering about the midpoint $(x_{n-1/2}, y_{j-1/2})$ of the rectangle $P_1P_2P_3P_4$ to obtain

$$\frac{b_j^n v_j^n - b_{j-1}^n v_{j-1}^n}{h_j} + \alpha_n[(\psi v)_{j-1/2}^n + v_{j-1/2}^{n-1}\psi_{j-1/2}^n - (u^2)_{j-1/2}^n - \psi_{j-1/2}^{n-1} v_{j-1/2}^n]$$
$$- 2\alpha_n[P_{j-1/2}^n - P_{j-1/2}^{n-1}] = R_{j-1/2}^{n-1} \tag{3.5d}$$

Here

$$R^{n-1}_{j-1/2} = -\frac{(bv)^{n-1}_j - (bv)^{n-1}_{j-1}}{h_j} + \alpha_n[(\psi v)^{n-1}_{j-1/2} - (u^2)^{n-1}_{j-1/2}] \tag{3.6a}$$

$$\alpha_n = \frac{1}{x_n - x_{n-1}} \tag{3.6b}$$

Similarly the boundary conditions (3.2) and the relation (2.15) become

$$\psi^n_0 = 0, \quad u^n_0 = 0, \quad u^n_J = u^n_e, \quad \psi^n_J = u^n_e[y_e - \bar{\delta}^*(x)] \tag{3.7}$$

$$p^n + \frac{(u^2_e)^n}{2} = p^{n-1} + \frac{(u^2_e)^{n-1}}{2} \tag{3.8}$$

3.2 Solution of the Difference Equations

If we assume $(\psi^{n-1}_j, u^{n-1}_j, v^{n-1}_j, p^{n-1}_j)$ to be known for $0 \le j \le J$, then (3.5) for $1 \le j \le J-1$ and the boundary conditions (3.7) yield a nonlinear algebraic system of 4J+4 equations with the same number of unknowns $(\psi^n_j, u^n_j, v^n_j, p^n_j)$. To solve the system we use Newton's method. We introduce iterates $[\psi^{(\nu)}_j, u^{(\nu)}_j, v^{(\nu)}_j, p^{(\nu)}_j]$ $\nu = 0, 1, 2, \ldots,$ with initial values

$$\begin{aligned}
&\psi^{(0)}_0 = 0, \quad u^{(0)}_0 = 0, \quad v^{(0)}_0 = v^{n-1}_0, \quad p^{(0)}_0 = p^{n-1}_0 && j = 0 \\
&\psi^{(0)}_j = \psi^{n-1}_j, \; u^{(0)}_j = u^{n-1}_j, \; v^{(0)}_j = v^{n-1}_j, \; p^{(0)}_j = p^{n-1}_j && 1 \le j \le J-1 \\
&\psi^{(0)}_J = \psi^{n-1}_J, \; u^{(0)}_J = u^{n-1}_e, \; v^{(0)}_J = v^{n-1}_J, \; p^{(0)}_J = p^{n-1}_J && j = J
\end{aligned} \tag{3.9}$$

For the higher-order iterates we set

$$\begin{aligned}
&\psi^{(\nu+1)}_j = \psi^{(\nu)}_j + \delta\psi^{(\nu)}_j, \quad u^{(\nu+1)}_j = u^{(\nu)}_j + \delta u^{(\nu)}_j, \quad v^{(\nu+1)}_j = v^{(\nu)}_j + \delta v^{(\nu)}_j \\
&p^{(\nu+1)}_j = p^{(\nu)}_j + \delta p^{(\nu)}_j
\end{aligned} \tag{3.10}$$

Then we insert these expressions in place of $[\psi_j, u_j, v_j, p_j]$ in (3.5) and drop the terms that are quadratic in $[\delta\psi^{(\nu)}_j, \delta u^{(\nu)}_j, \delta v^{(\nu)}_j, \delta p^{(\nu)}_j]$. This procedure yields the following linear system:

$$\delta\psi_j - \delta\psi_{j-1} - \frac{h_j}{2}(\delta u_j + \delta u_{j-1}) = (r_1)_j$$

$$\delta u_j - \delta u_{j-1} - \frac{h_j}{2}(\delta v_j + \delta v_{j-1}) = (r_3)_{j-1} \qquad (3.11)$$

$$\delta P_j - \delta P_{j-1} = (r_4)_{j-1}$$

$$(s_1)_j\delta v_j + (s_2)_j\delta v_{j-1} + (s_3)_j\delta\psi_j + (s_4)_j\delta\psi_{j-1} + (s_5)_j\delta u_j + (s_6)_j\delta u_{j-1}$$

$$+ (s_6)_j\delta u_{j-1} + (s_7)_j\delta P_j + (s_7)_j\delta P_{j-1} = (r_2)_j$$

for $j = 1, 2, \ldots, J$. Here for convenience we have dropped the superscripts ν and n, and have reordered the equations so that the A_0 matrix is not singular. (For details, see Cebeci and Bradshaw, 1977.) The coefficients $(r_k)_j$ $(k = 1$ to $4)$ are:

$$(r_1)_j = h_j u_{j-1/2} - \psi_j + \psi_{j-1}$$

$$(r_3)_{j-1} = h_j v_{j-1/2} - u_j + u_{j-1} \qquad (3.12)$$

$$(r_4)_{j-1} = 0$$

$$(r_2)_j = R^{n-1}_{j-1/2} - \frac{(bv)_j - (bv)_{j-1}}{h_j} + \alpha_n\left[(\psi v)_{j-1/2} + v^{n-1}_{j-1/2}\psi_{j-1/2} - \right.$$

$$\left. - (u^2)_{j-1/2} - \psi^{n-1}_{j-1/2}v_{j-1/2}\right] - 2\alpha_n(P_{j-1/2} - P^{n-1}_{j-1/2})$$

The coefficients $(s_k)_j$ $(k = 1$ to $7)$ of the differenced momentum equation are:

$$(s_1)_j = \frac{b_j}{h_j} + \frac{\alpha_n}{2}(\psi_j - \psi^{n-1}_{j-1/2})$$

$$(s_2)_j = -b_{j-1}/h_j + \frac{\alpha_n}{2}(\psi_{j-1} - \psi^{n-1}_{j-1/2})$$

$$(s_3)_j = \frac{\alpha_n}{2}(v_j + v^{n-1}_{j-1/2}) \qquad (3.13)$$

$$(s_4)_j = \frac{\alpha_n}{2}(v_{j-1} + v^{n-1}_{j-1/2})$$

$$(s_5)_j = -\alpha_n u_j$$

$$(s_6)_j = -\alpha_n u_{j-1}$$

$$(s_7)_j = -\alpha_n$$

The boundary conditions (3.7) become

$$\delta\psi_0 = (r_1)_0 = 0 \qquad \delta u_0 = (r_2)_0 = 0 \tag{3.14}$$

$$\delta\psi_J - [y_J - \sqrt{R_L}\,\bar{\delta}^*(x)]\delta u_J = (r_3)_J \quad -\psi_J + u_J(y_J - \sqrt{R_L}\,\bar{\delta}^*)$$

$$u_J\delta u_J + \delta P_J = (r_4)_J \equiv \frac{(u_J^2)^{n-1} - (u_J^2)^n}{2} + P_J^{n-1} - P_J^n \tag{3.14}$$

We use the block elimination method discussed by Isaacson and Keller (1966) to solve the linear system. According to this method, the linear system is written in the form

$$\mathbf{A}\,\underset{\sim}{\Delta}_j = \underset{\sim}{R}_j \tag{3.15}$$

Here

$$\mathbf{A} \equiv \begin{bmatrix} A_0 & C_0 & & & & \\ B_1 & A_1 & C_1 & & & \\ & B_j & A_j & C_j & & \\ & & B_{J-1} & A_{J-1} & C_{J-1} & \\ & & & B_J & A_J & \end{bmatrix}, \quad \underset{\sim}{\Delta}_j \equiv \begin{bmatrix} \underset{\sim}{\delta}_0 \\ \underset{\sim}{\delta}_1 \\ \vdots \\ \underset{\sim}{\delta}_j \\ \vdots \\ \underset{\sim}{\delta}_{J-1} \\ \underset{\sim}{\delta}_J \end{bmatrix}, \tag{3.16}$$

$$\underset{\sim}{R}_j = \begin{bmatrix} (\underset{\sim}{r})_0 \\ (\underset{\sim}{r})_1 \\ \vdots \\ (\underset{\sim}{r})_j \\ \vdots \\ (\underset{\sim}{r})_{J-1} \\ (r)_J \end{bmatrix} \qquad \underset{\sim}{\delta}_j \equiv \begin{bmatrix} \delta\psi_j \\ \delta u_j \\ \delta v_j \\ \delta P_j \end{bmatrix}$$

The matrices A_j, B_j, C_j are 4 x 4 matrices which are given by

$$A_o \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & -h_o/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad C_j \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -h_o/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad 0 \le j \le J-1$$

(3.17)

$$B_j = \begin{bmatrix} -1 & -h_j/2 & 0 & 0 \\ (s_4)_j & (s_6)_j & (s_2)_j & (s_7)_j \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad 1 \le j \le J; \quad A_j \equiv \begin{bmatrix} 1 & -h_j/2 & 0 & 0 \\ (s_3)_j & (s_5)_j & (s_1)_j & (s_7)_j \\ 0 & -1 & -h_j/2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad 1 \le j \le J$$

3.3 FLARE Approximation and DUIT Procedures

As previously mentioned, flows with negative wall shear require the use of the FLARE approximation in the region of reversed flow. According to this approximation, u_j^n and u_j^{n-1} are set equal to zero in the region of negative u_j. Once a solution is obtained with this approximation, more accurate calculations can be obtained without the FLARE approximation by using a forward and backward difference scheme. In the region of reverse flow ($u_j \le 0$), we write the difference approximations of (3.1d) for the midpoint $(x_{n+1/2}, y_{j-1/2})$ of the rectangle $P_1P_2P_3'P_4'$ (see figure 2) to get

$$\frac{(bv)_j^n - (bv)_{j-1}^n}{h_j} - \beta[(\psi v)_{j-1/2}^n + v_{j-1/2}^{n+1}\psi_{j-1/2}^n - (u^2)_{j-1/2}^n - \psi_{j-1/2}^{n+1}v_{j-1/2}^n]$$

$$- 2\beta[P_{j-1/2}^{n+1} - P_{j-1/2}^n] = R_{j-1/2}^{n+1} \qquad (3.18)$$

Here

$$R_{j-1/2}^{n+1} = -\frac{(bv)_j^{n+1} - (bv)_{j-1}^{n+1}}{h_j} + \beta[(\psi v)_{j-1/2}^{n+1} + (u^2)_{j-1/2}^{n+1}]$$

$$\beta = \frac{1}{x_{n+1} - x_n}$$

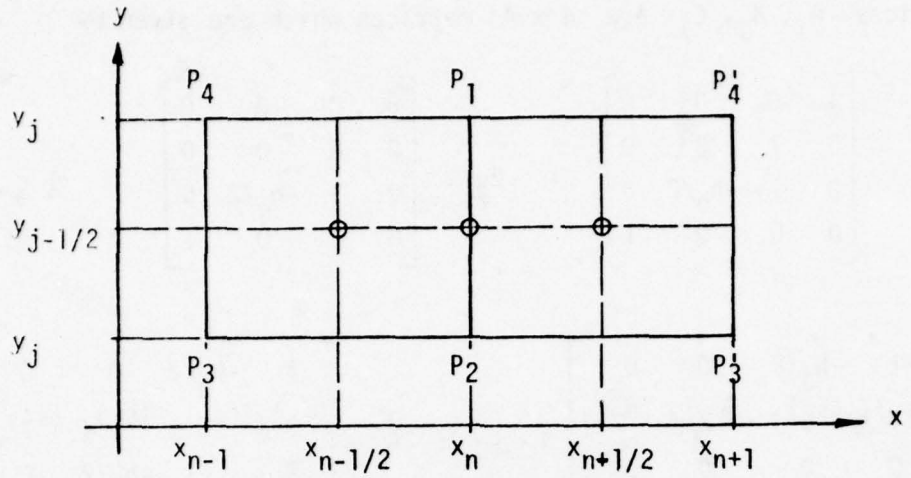


Figure 2. Net rectangle for difference approximations for the DUIT procedure.

With this procedure, the calculations are repeated, starting at the x-station where the calculations first indicated reverse flow and are continued to the final x-station, until the solutions do not indicate any changes. This procedure, referred to as DUIT (downstream upstream iteration) by Williams (1975), usually gives converged solutions with two or three DUITs.

3.4 Comments on the Solution Algorithm

The solution of the governing equations given by (2.1) to (2.4) or (2.11) and (2.13), requires initial conditions which can be obtained in the transformed variables,

$$\eta = \sqrt{u_e/\nu x}\, y \quad , \quad \psi = \sqrt{u_e \nu x}\, f(x,\eta) \tag{3.20}$$

Equations (2.1) to (2.4) become:

$$(bf'')' + \frac{m+1}{2} ff'' + m[1 - (f')^2] = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \tag{3.21}$$

where primes indicate differentiation with respect to η and

$$m = \frac{x}{u_e} \frac{du_e}{dx} \tag{3.22}$$

Equation (3.21) is subject to the following boundary conditions:

$$\eta = 0 \qquad f = f' = 0 \qquad \eta \to \eta_\infty \qquad f' \to 1 \tag{3.23}$$

Initial conditions were obtained by solving the governing equations for the standard problem to a location upstream of separation.

The present method has been developed to allow nonuniform net spacings in the streamwise direction and across the boundary layer. In the latter case, a geometric progression has been used with the property that the ratio of lengths of any two adjacent intervals is a constant; that is, $h_j = Kh_{j-1}$. The distance to the j-th line is given by the following formula:

$$\eta_j = h_1(K^j - 1)/(K - 1) \qquad K > 1 \tag{3.24}$$

There are two parameters: h_1, the length of the first $\Delta\eta$ step, and K, the ratio of two successive steps. The total number of points J can be calculated from the expression,

$$J = \frac{\ln[1 + (K - 1)(\eta_\infty/h_1)]}{\ln K} \tag{3.25}$$

For further details, see Cebeci and Bradshaw (1977).

4. RESULTS

To test the Mechul function method calculations were performed for laminar and turbulent boundary layers with and without separation, and compared with the results of previous numerical solutions for laminar flows and with experimental data for turbulent flows. The latter comparisons were made for a number of attached flows and only for one separated flow.

4.1 Results for Laminar Flows with Separation

For laminar flows with separation we have considered four separate flows which allow the displacement thickness distribution to be specified. These include the laminar separated flows of Williams (1976), Carter (1975) and Briley (1971). In the last case, the free-stream boundary condition corresponded to a linearly decreasing external velocity distribution followed by a constant velocity and resulted in separation and reattachment. Figure 3 allows a comparison of values of local skin-friction coefficient $c_f \sqrt{R_L}$ calculated with the present method with those obtained by Briley from the steady, two-dimensional form of the Navier-Stokes equations. Here c_f is defined by

$$c_f = \frac{\tau_w}{1/2\ \rho u_o^2} = \frac{2\psi''(0)}{\sqrt{R_L}} \tag{4.1}$$

and $c_f\sqrt{R_L}$ represents $2\psi''(0)$. These calculations were made for the displacement thickness distribution (see in figure 3) deduced from the Navier-Stokes solutions. In general, the agreement is very good and the small discrepancy may be associated with the difficulty of reading the input $\bar{\delta}^*(x)$ distribution from the graph presented by Briley. The Navier-Stokes solutions were obtained for $R_L = 2.08 \times 10^4$ and required 45 minutes on a UNIVAC 1108. The results obtained by the Mechul function approach required approximately 10 seconds on a CDC-6600 computer.

We next consider the two laminar flows with separation and reattachment computed by Carter (1975). These flows have displacement-thickness distributions $\delta^*(x)$ given by

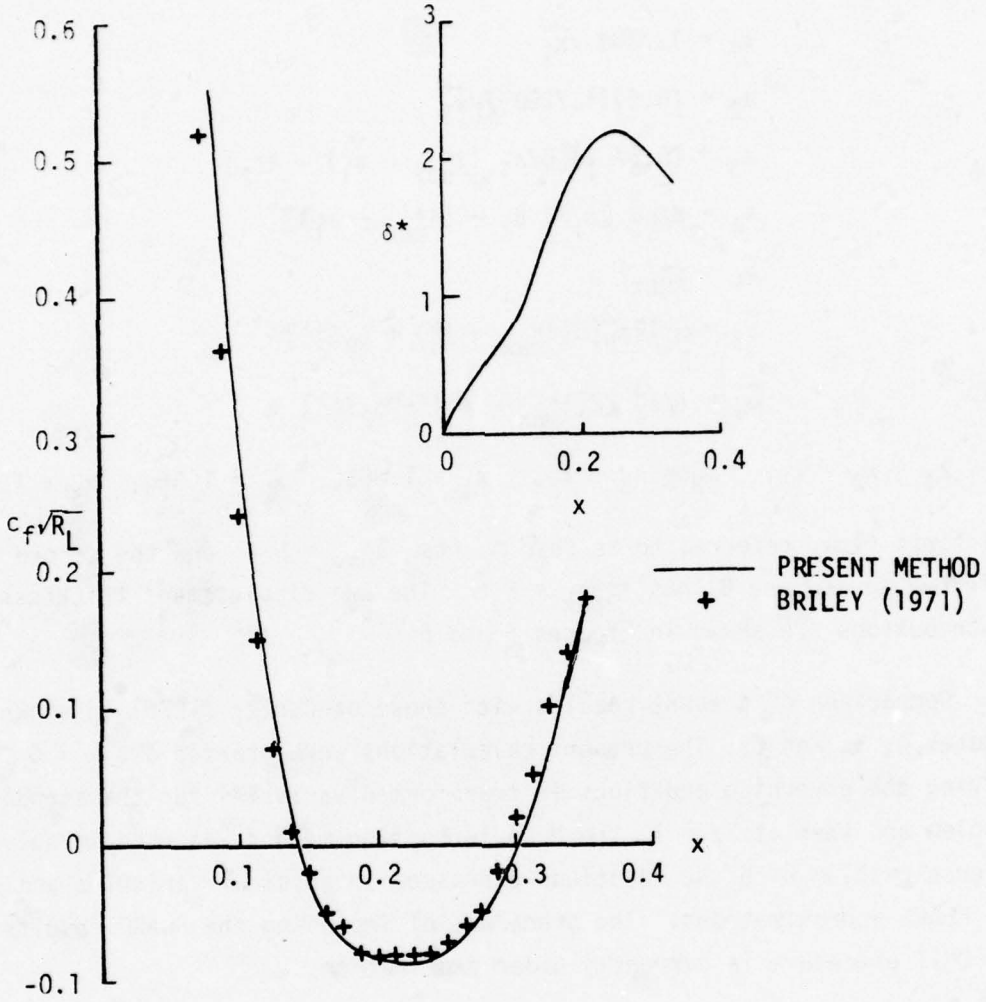


Figure 3. Calculated local skin-friction coefficient distribution for separating and reattaching flow computed by Briley (1971).

$$\bar{\delta}^*(x) \equiv \begin{cases} 1.7208\sqrt{x} & 1.0 \le x \le x_1 \\ a_1 + a_2(x - x_1) + a_3(x - x_1)^2 + a_4(x - x_1)^3 & x_1 \le x \le x_2 \\ \bar{a}_1 + \bar{a}_3(x - x_2)^2 + \bar{a}_4(x - x_2)^3 & x_2 \le x \le x_3 \end{cases} \tag{4.2}$$

with

$$
\begin{aligned}
a_1 &= 1.7208\ \sqrt{x_1} \\
a_2 &= (0.5)(1.7208)/\sqrt{x_1} \\
a_3 &= (0.5/\Delta_1)[6/\Delta_1\ (\delta^*_{max} - a_1) - 4a_2] \\
a_4 &= 2/\Delta_1^3\ [\Delta_1/2\ a_2 - (\delta^*_{max} - a_1)] \\
\bar{a}_1 &= \delta^*_{max} \\
\bar{a}_3 &= -1/\Delta_2^2[3(\delta^*_{max} - \delta^*_3) + \Delta_2\delta^{*\prime}_3] \\
\bar{a}_4 &= 1/\Delta_2^3\ [2(\delta^*_{max} - \delta^*_3) + \Delta_2\delta^{*\prime}_3]
\end{aligned}
\qquad (4.3)
$$

$$\Delta_1 = x_2 - x_1, \quad \Delta_2 = x_3 - x_2, \quad x_1 = 1.065, \quad x_2 = 1.35, \quad x_3 = 1.884$$

The first flow, referred to as Case A, has $\delta^*_{max} = 5.6$, and the second flow referred to as Case B, has $\delta^*_{max} = 8.6$. The two displacement thickness distributions are shown in Figures 5 and 6.

Comparison of present results with those of Carter (1975) is shown in Figures 4, 5, and 6. The present calculations were started at $x = 0$ by solving the governing equations in transformed variables for the standard problem and then at $x = 1$ the Mechul-function method was used to solve the inverse problem with the equations expressed in physical variables and with the FLARE approximations. The procedure of improving the FLARE results by the DUIT procedure is currently under examination.

As can be seen, the present results agree well with those of Carter which made use of the FLARE approximations and the DUIT procedure. Our skin-friction results are in better agreement with Carter's results obtained by the FLARE procedure. His FLARE approximation was slightly different from that considered here and may account for the disagreement with the present results. The computer time associated with the results was again of the order of 10s on a CDC 6600. At almost all x-stations the convergence rate was quadratic and required only two or three iterations, including regions of separated flow.

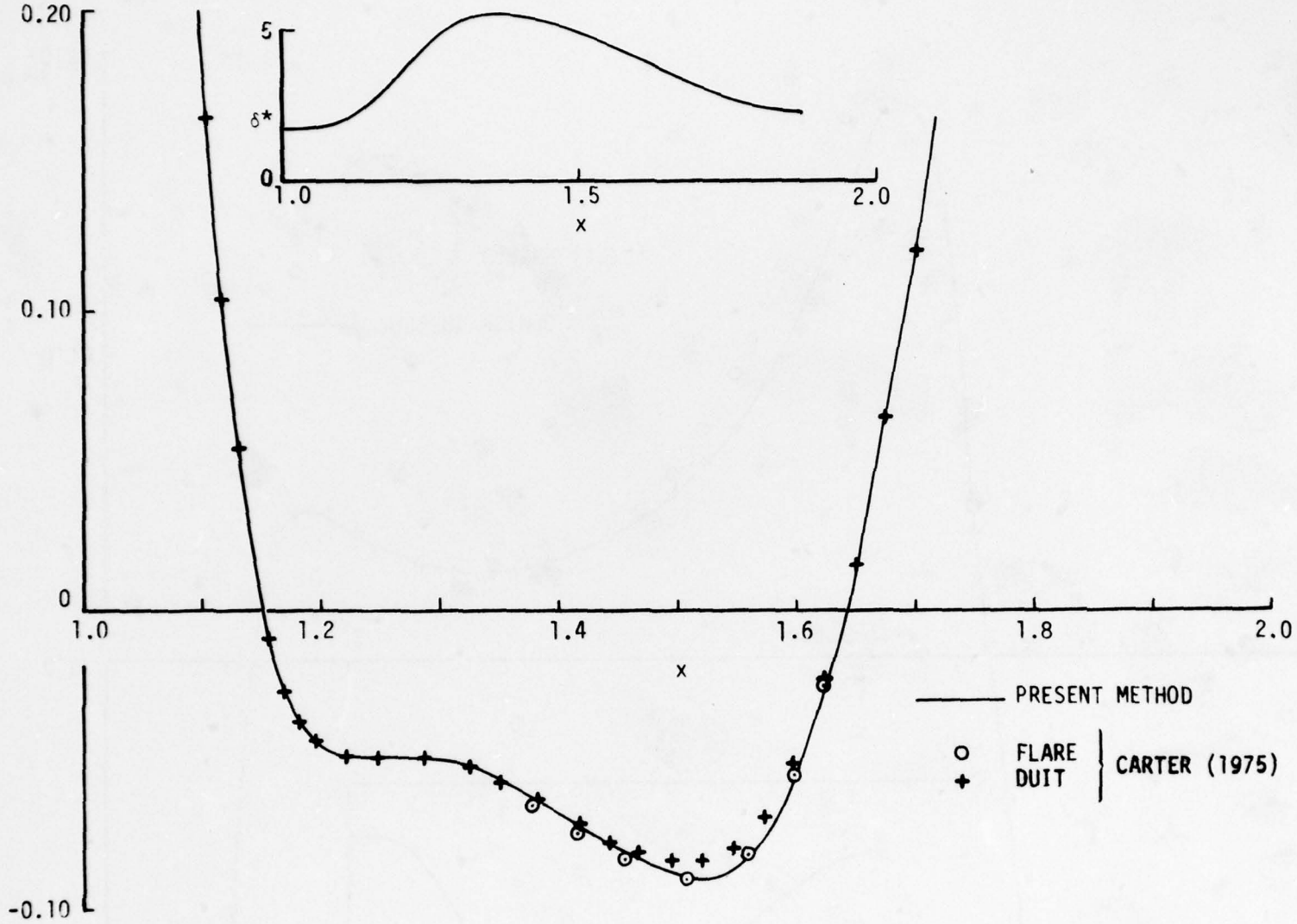


Figure 4. Local skin-friction distribution for Case A.

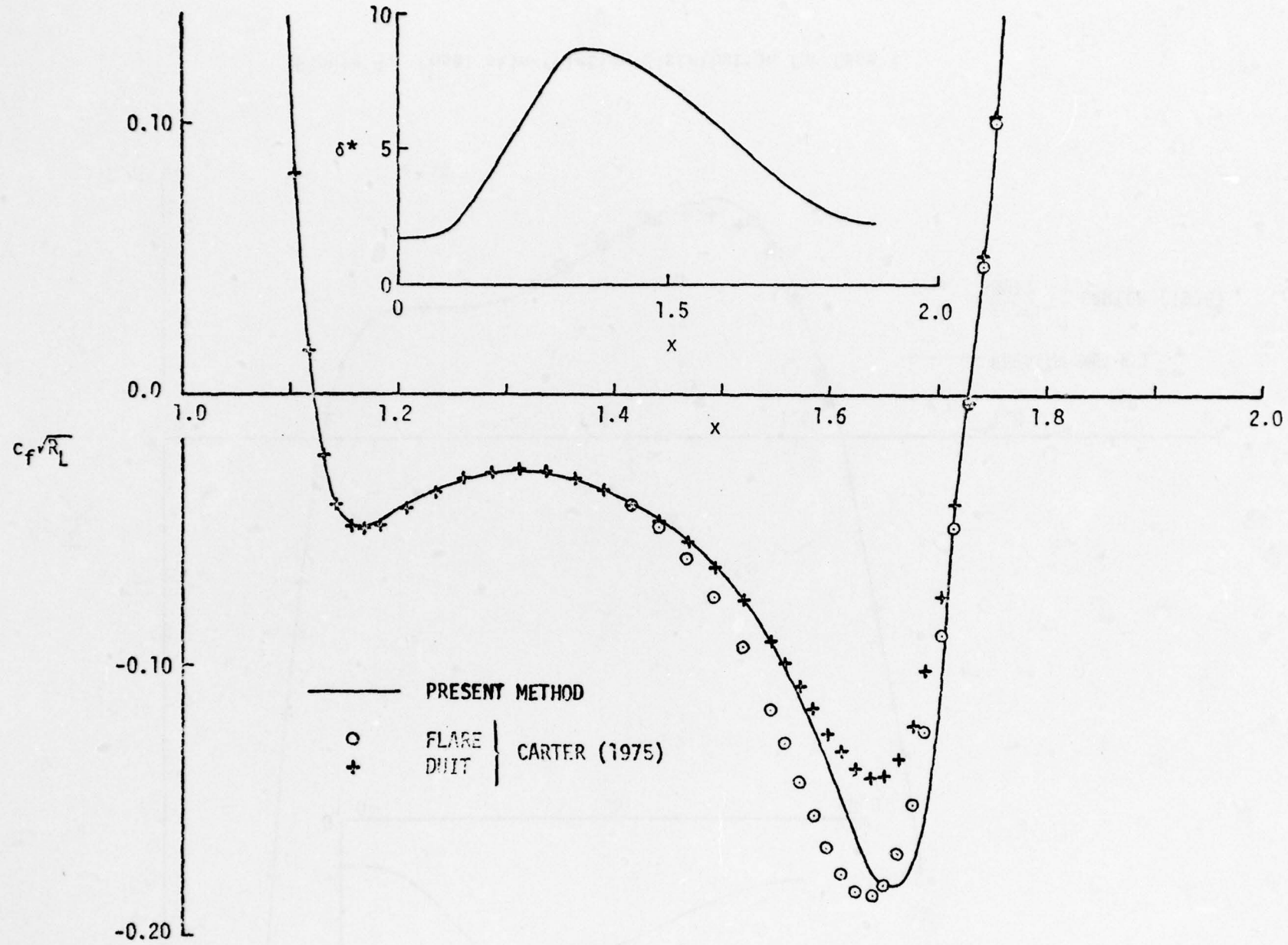


Figure 5. Local skin-friction distribution for Case B.

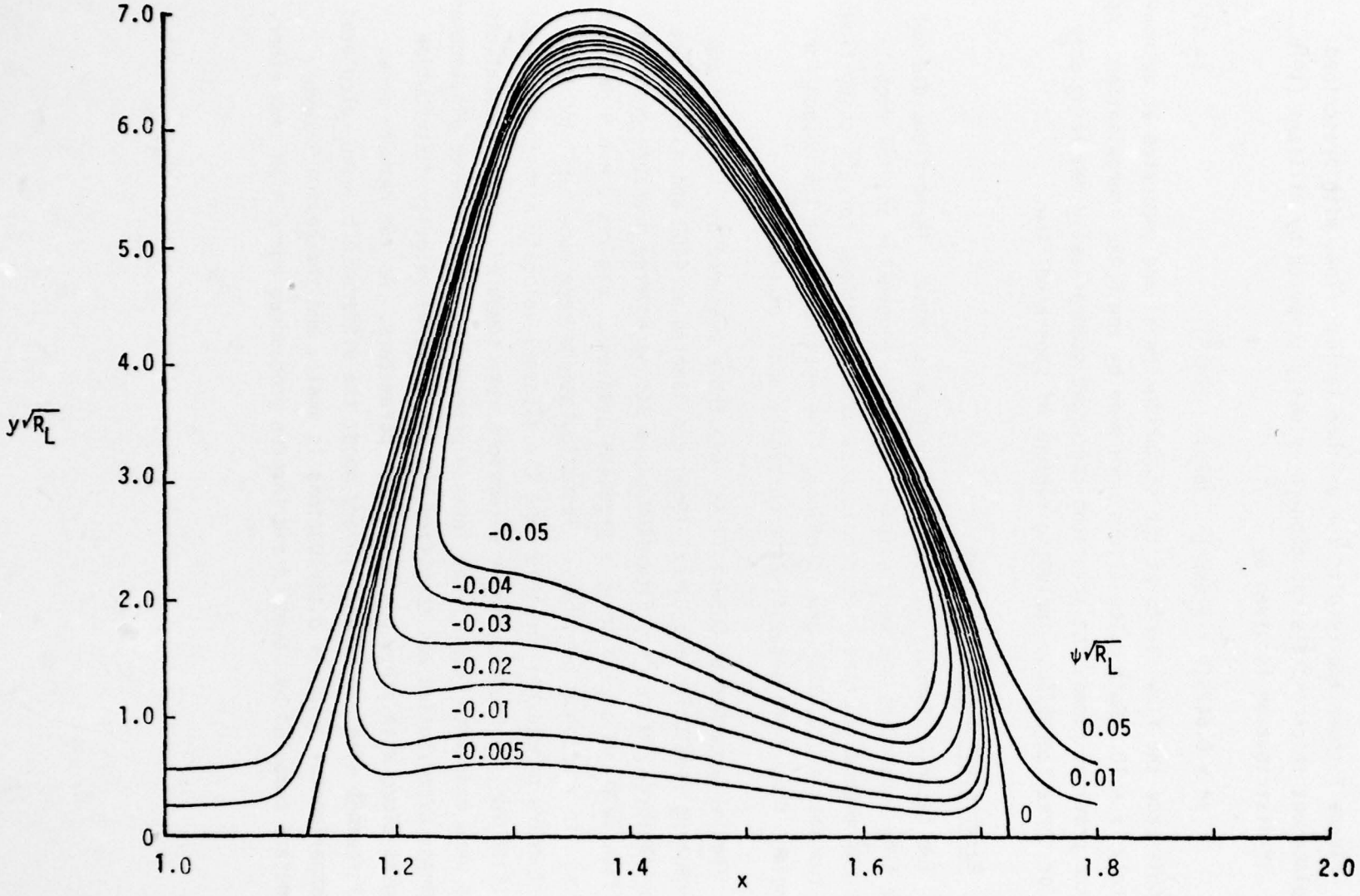


Figure 6. Streamline pattern in separation bubble for Case B.

Figure 7 shows the results for another laminar flow, with prescribed displacement thickness distribution, previously computed by Williams (1976). The δ^*-distribution is given by

$$\delta^* = 0.6479\{1 + 9 \exp [-0.0625(x - 14)^2]\} \tag{4.4}$$

In this case the flow starts at the stagnation-point and separates at approximately $x = 10$. Both calculations were made by the FLARE approximation. As in the previous cases, the solutions converged quadratically requiring only two or three iterations, including regions of separated flow.

4.2 Results for Turbulent Flows

Two separate turbulent flows have been considered. These flows did not have flow separation and were computed with the eigenvalue and the Mechul-function methods to check the predictions of both methods for attaching flows. The two sets of results were identical. The application of the method for turbulent flows with separation is currently under examination.

The two attached turbulent flows were those measured by Schubauer and Spangenberg (see Coles and Hirst, 1969) and labeled as 4400 and 4800 at the 1968 Stanford Conference. Flow 4400 has a strong adverse pressure gradient and flow 4800 has a mild adverse pressure gradient. Figures 8 and 9 present the results for these two flows. The calculations were made first by using the standard procedure: in this case the external velocity distribution was specified and the boundary-layer parameters were computed. Next, the calculations were made with both of the inverse procedures: the measured displacement thickness distribution was specified and the external velocity distribution computed along with other boundary-layer parameters. As the results show, the agreement with experiment is very good; the difference between calculated and experimental velocity distributions is small, and the boundary-layer parameters computed by standard and inverse procedures agree with each other.

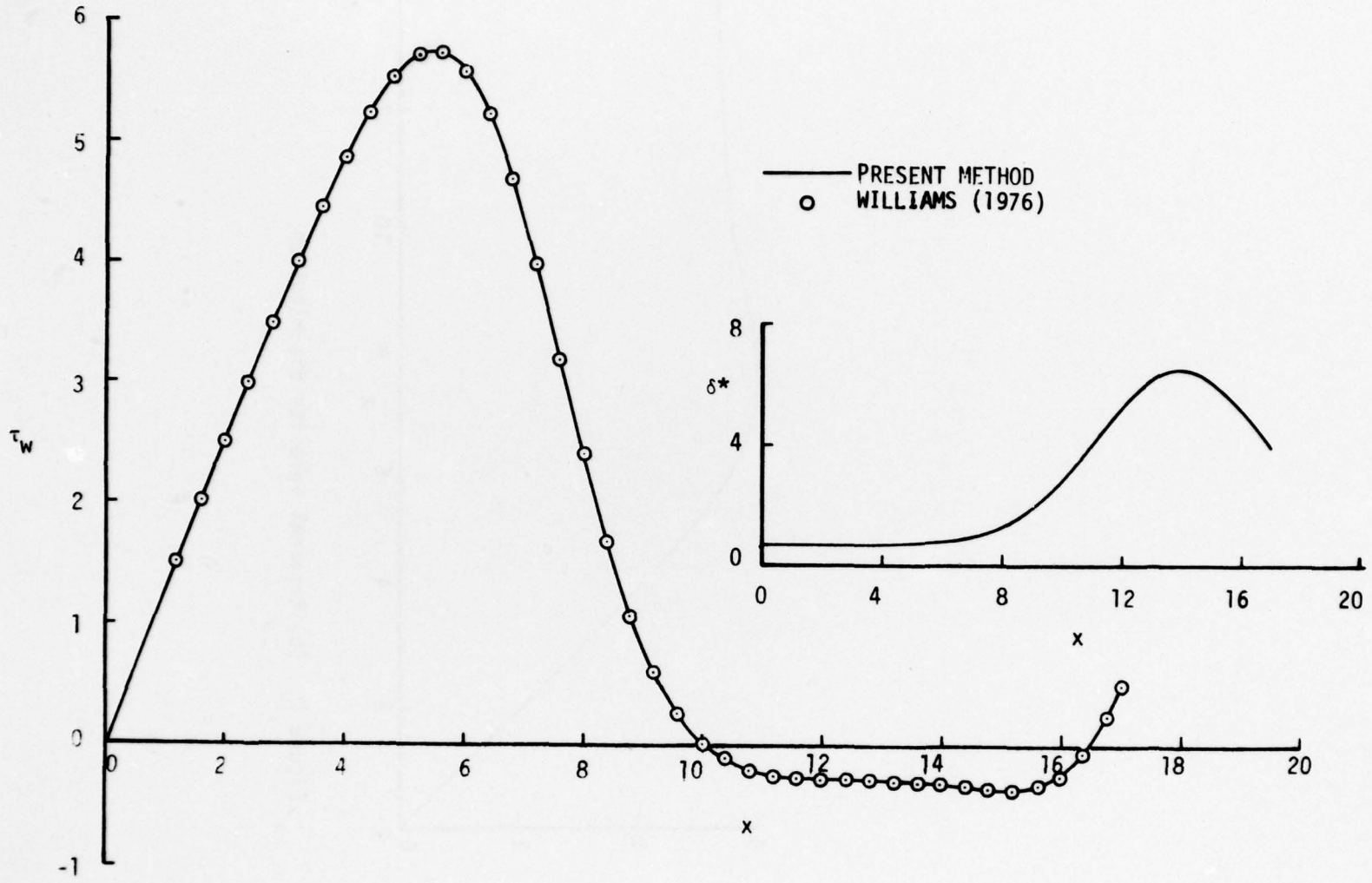


Figure 7. Comparison of calculated results for the separated flow for which displacement-thickness distribution is given by 4.4. (a) Wall shear distribution.

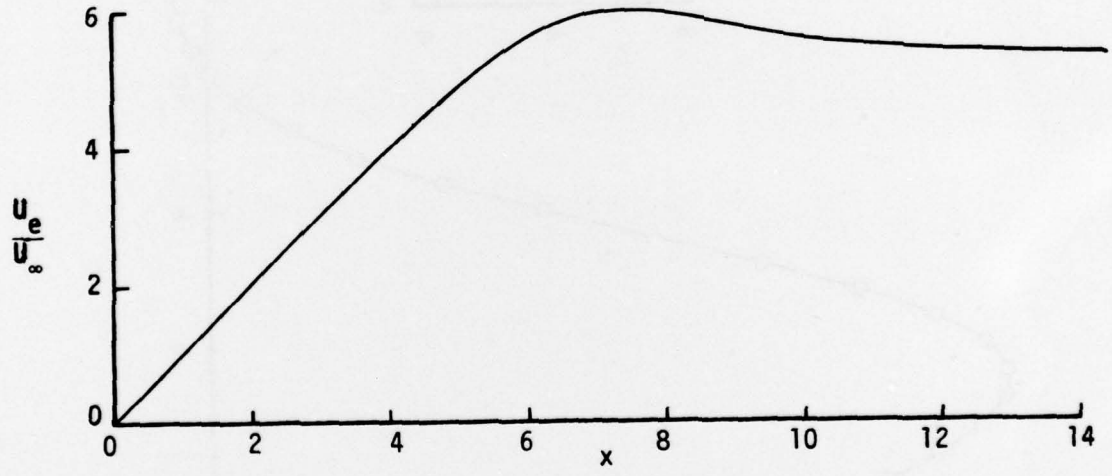


Figure 7. (b) External velocity distribution.

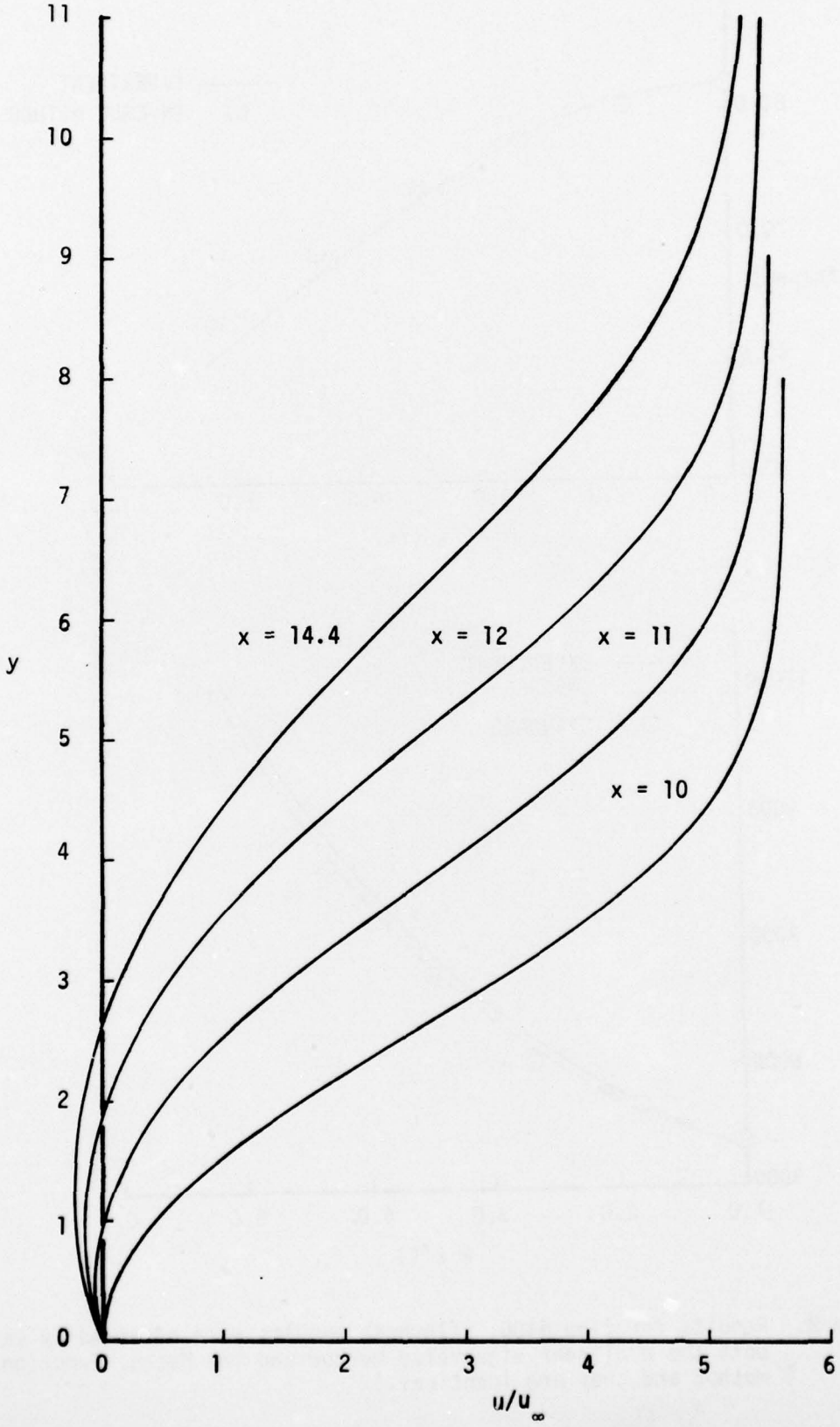
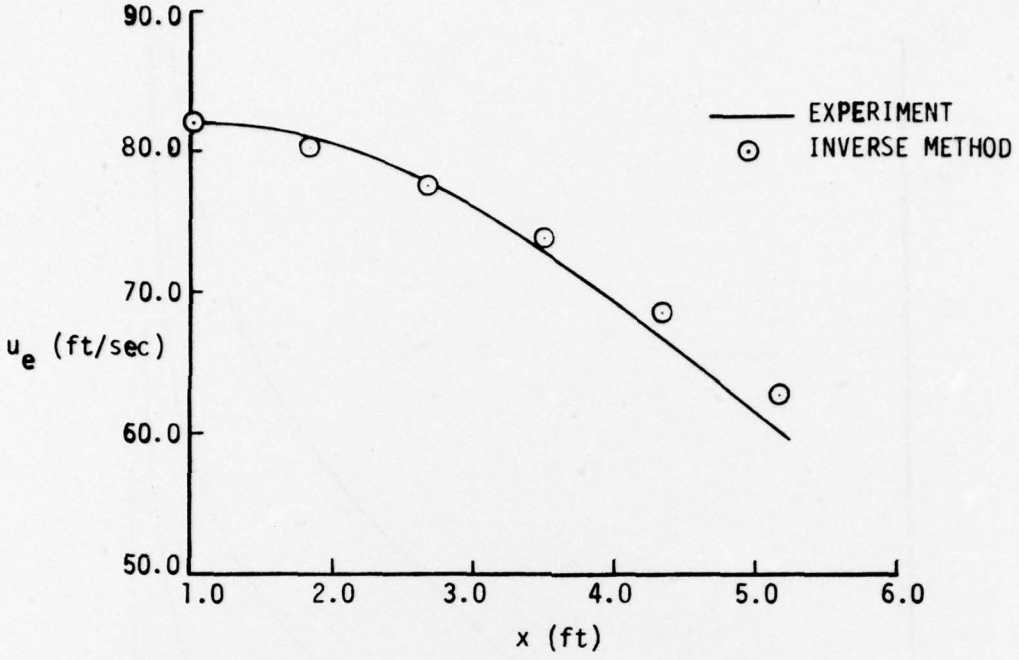


Figure 7. (c) Velocity profiles near and in the reverse flow region.



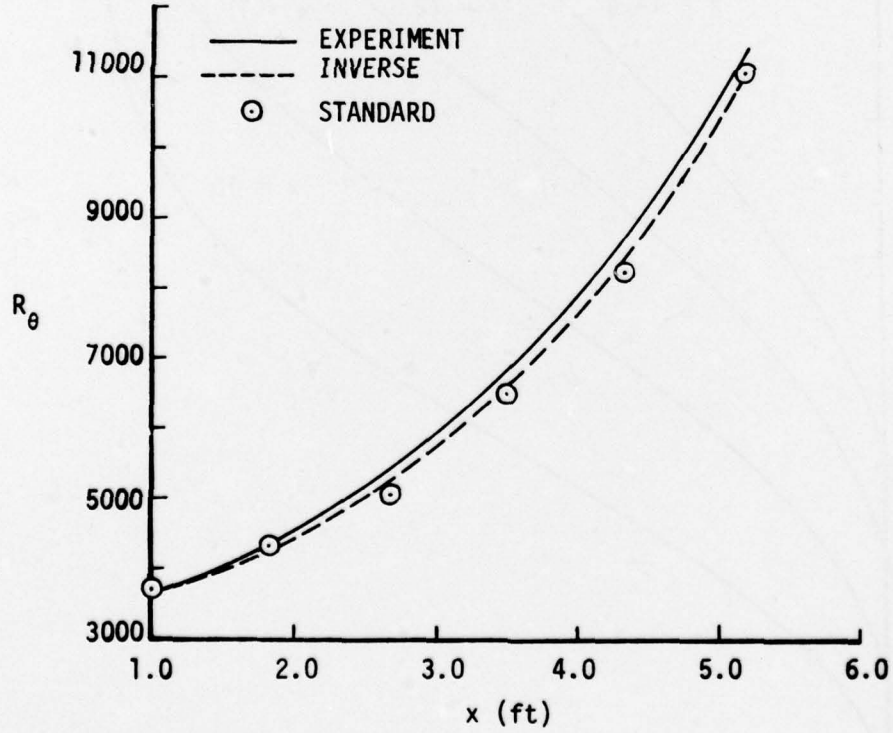
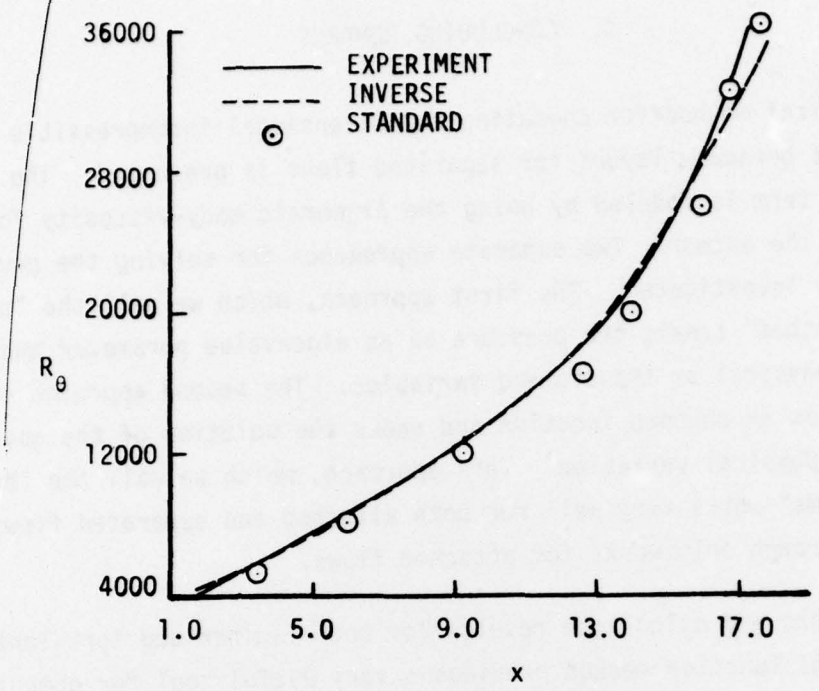


Figure 8. Results for flow 4400. (Inverse results were obtained by using both the nonlinear eigenvalue method and the Mechul-function method and they are identical.)



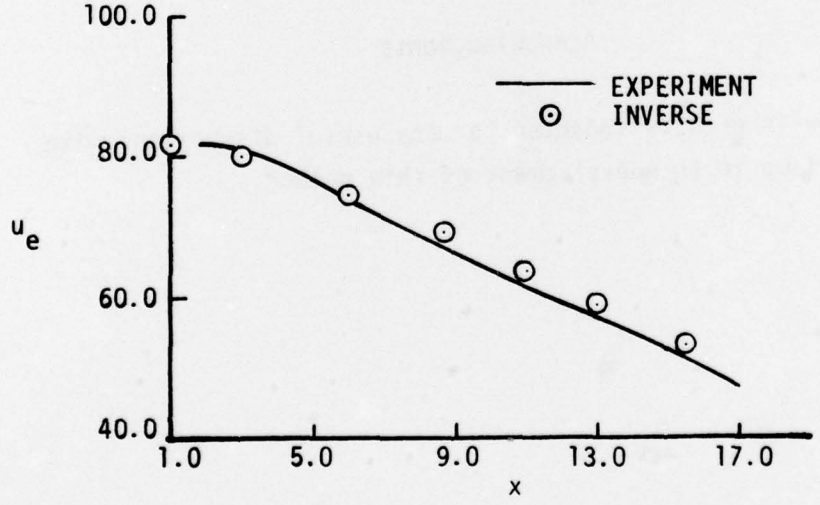


Figure 9. Results for flow 4800. (Inverse results were obtained by using both the nonlinear eigenvalue method and the Mechul-function method and they are identical.

5. CONCLUDING REMARKS

A numerical method for computing two-dimensional incompressible laminar and turbulent boundary layers for separated flows is presented. The Reynolds shear stress term is modeled by using the algebraic eddy-viscosity formulas developed by the author. Two separate approaches for solving the governing equations are investigated. The first approach, which we call the "nonlinear eigenvalue method" treats the pressure as an eigenvalue parameter and works with either physical or transformed variables. The second approach treats the pressure as an unknown function and seeks the solution of the governing equations in physical variables. This approach, which we call the "Mechul function scheme" works very well for both attached and separated flows while the first approach only works for attached flows.

Comparisons and calculated results for both laminar and turbulent flows show the Mechul function method provides a very useful tool for computing the separated flows with the FLARE approximation. The computed results agree well with the available numerical and experimental results and show no signs of numerical problems in regions of negative wall shear.

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