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This report discusses a method of using an adaptive array with AM signals. A technique is suggested in which phase modulation is added to a conventional AM signal to make it possible to separate the desired signal from interference. The new signal will be transmitted in the same bandwidth as the conventional AM signal and hence can be intermixed with standard signals in a communication net. Digital simulations of such a system have been done and a few of the results are shown in this report. These results indicate that the array will provide suitable interference protection with such signals.		

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I. INTRODUCTION

The LMS (least mean square) adaptive array[1,2] has tremendous potential as a technique for protecting communication systems from interference. However, to date this potential has been realized only for certain spread spectrum[3] and time division multiple access systems[4]. An important research challenge is to find methods for using these adaptive arrays in conventional communication systems, such as AM and FM. Spread spectrum systems, although more complicated than conventional systems, are more easily combined with adaptive arrays, because the signals contain code modulation that can be used to distinguish the desired signal from other signals. In conventional modulation systems, the interfering signals often have virtually the same characteristics as the desired signal, so it is difficult for the array to separate them. On the other hand, the interference rejecting capability of adaptive arrays would be extremely useful in conventional modulation systems because of the large number of such systems in operation and because of the heavy interference these systems are subject to (particularly in the HF band).

This report discusses a method of using an adaptive array with AM signals. A technique is suggested in which phase modulation is added to a conventional AM signal to make it possible to separate the desired signal from interference. This phase modulation will cause some envelope distortion, but it is believed that if the modulation parameters are properly chosen, the distortion will be minor.[†] The new signal will be transmitted in the same bandwidth as the conventional AM signal and hence can be intermixed with standard signals in a communication net.

Section II of this report gives an overview of the proposed adaptive array system and the modified AM signal. Section III discusses the problem of coupling between the phase and envelope modulation of the signal. Section IV discusses the control loop bandwidth of the adaptive array and the concept of signal correlation. In Section V, design relations are developed for choosing the system parameters properly in the adaptive array. Section VI discusses the relationship between the analog array system and its digital simulation. Section VII shows some preliminary simulation results on the interference rejecting capability of the proposed system, and Section VIII presents a summary and the conclusions of the study.

[†]The type of phase modulation that can be added to an AM signal without increasing its bandwidth or changing its envelope is severely limited. See the discussion on phase envelope coupling later in this report, and also the excellent papers by Voelcker[5,6].

II. THE PROPOSED AM SYSTEM

The basic structure of an LMS adaptive array is shown in Fig. 1 [1,2]. The signals $x_i(t)$ from the array elements are adjusted in magnitude and phase by the complex weights W_i and then added to produce the array output $s(t)$. The magnitude and phase adjustments w_i are under the control of a feedback system that attempts to minimize the squared value of the error signal $\epsilon(t)$. (The details of the feedback have been described elsewhere[1,2].) $\epsilon(t)$ is the difference between the array output $s(t)$ and the reference signal $R(t)$. This reference signal $R(t)$ is a locally generated signal which ideally should be a perfect replica of the desired signal, i.e., the signal we wish to receive with the array. If $R(t)$ is identical to the desired signal, the error signal $\epsilon(t)$ consists of thermal noise and any other unwanted signals, so minimizing the error signal corresponds to eliminating these signals from the array output.

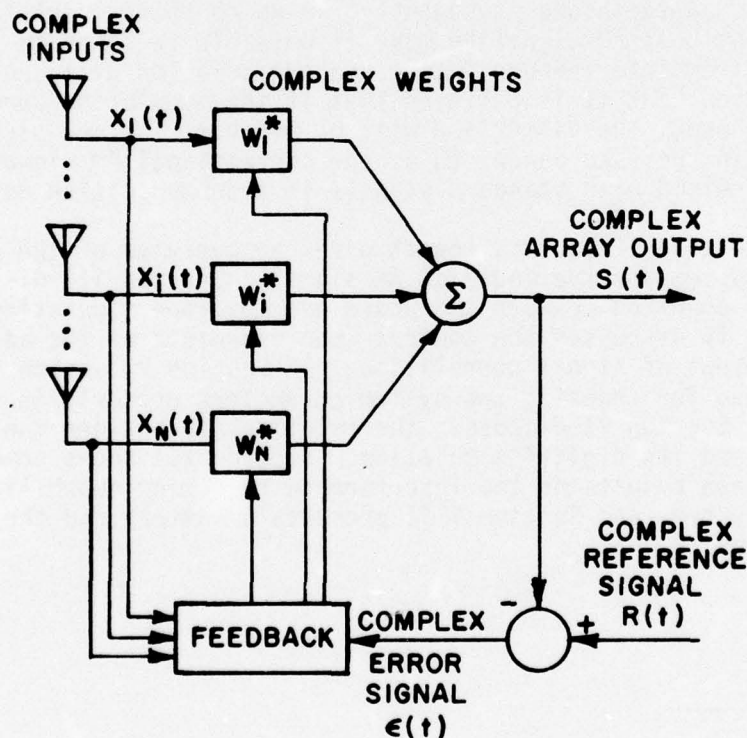


Fig. 1. The LMS adaptive array.

In practice, of course, one cannot obtain a perfect replica of the desired signal to use for the reference signal. (It is fundamental that the desired signal must be unknown in some respect if it is to convey useful information to the receiver!) But it turns out that $R(t)$ does not need to be a perfect replica of the desired signal; any signal reasonably well correlated with the desired signal and uncorrelated with the interference signals will do.[†] An LMS array is essentially a correlation system in which the array performance depends on the amount of correlation between the reference, desired and the interference signals. When a signal received by the array is highly correlated with the reference signal, the feedback retains that signal in the array output. A signal uncorrelated with the reference signal is nulled by the array. Hence the array can be used to protect a communication signal from interference if the reference signal can be made strongly correlated with the desired signal but uncorrelated with the interference. From this viewpoint it appears less difficult to obtain a suitable reference signal, because one may be able to derive a reference signal from the array output. I.e., if the desired signal and interference signal waveforms are sufficiently different, it may be possible to construct a signal processing loop that generates a highly correlated estimate of the desired signal but alters (decorrelates) the interference waveform. This technique has been successfully used in spread spectrum systems[7].

We consider here the application of adaptive arrays to an ordinary AM communication system. A conventional AM signal has the form

$$(1)^{++} \quad y(t) = A[1 + mf(t)] \cos [\omega_c t + \theta]$$

[†] The subject of what is meant by "correlation" with regard to adaptive arrays will be discussed in Section IV.

⁺⁺The signal in Eq. (1) is expressed in real form. For convenience in the analysis that follows, we shall also often express signals in their analytic signal form[8,9]. The analytic form of $y(t)$ is

$$(2) \quad Y(t) = y(t) + j\hat{y}(t) = A[1 + mf(t)] e^{j(\omega_c t + \theta)}$$

where

$$\hat{y}(t) = \text{Hilbert transform of } y(t) \text{ [8].}$$

Throughout this report both real and analytic forms of various signals will be used as appropriate. In general, lower-case letters will be used to denote real signals, while upper-case letters will be used to denote their analytic forms.

where

A = an amplitude constant,

m = a constant controlling the percentage modulation,

$f(t)$ = a narrow band audio waveform to be transmitted by the signal.

ω_c = carrier frequency,

θ = carrier phase angle.

Since we are interested in the case where the adaptive array will receive interfering signals that are similar to the desired signal, it will be necessary to modify the desired signal in some manner so the array can distinguish between it and interference. In this report we consider the inclusion of an extra phase modulation in the desired signal in addition to the conventional envelope modulation. We assume the following type of desired signal is generated:[†]

$$(3) \quad d(t) = A[1 + mf(t)] \cos[\omega_c t + \theta + \phi(t)]$$

where the extra phase modulation $\phi(t)$ is a binary waveform switching between 0 and π , with bit duration T_p , as shown in Fig. 2. It is assumed here that $\phi(t)$ is a pseudonoise code derived from a feedback shift register[10]. However, for some applications (such as friendly interference), it may be possible to use a square-wave for $\phi(t)$.

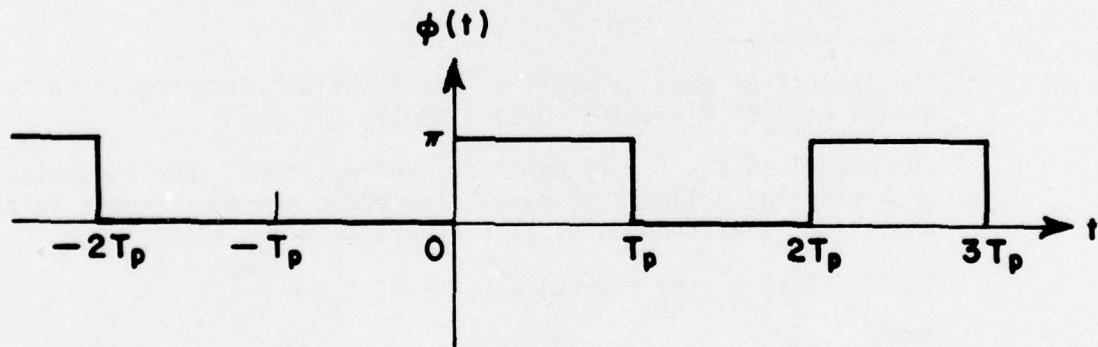


Fig. 2. The phase modulation $\phi(t)$.

[†]The transmitted signal in the proposed system will be a bandlimited version of $d(t)$.

The inclusion of $\phi(t)$ in $d(t)$ means that the reference signal $r(t)$ must contain similar phase modulation. We assume the reference signal has the form

$$(4) \quad r(t) = C \cos[\omega_c t + \hat{\phi}(t)]$$

where C is an amplitude constant and $\hat{\phi}(t)$ denotes an estimate of $\phi(t)$. Ideally, $\hat{\phi}(t)$ should be identical to $\phi(t)$, but in a real system the timing of $\hat{\phi}(t)$ must be derived from the received signal. (The waveform of $\phi(t)$ is known at the receiving site, but not its timing.) Including the modulation $\hat{\phi}(t)$ in the reference signal makes the reference signal uncorrelated with interference signals, which is the purpose of adding the phase modulation.

The reference signal in Eq. (4) does not contain the envelope modulation in $d(t)$. For a signal with the form of $d(t)$ (i.e., having a d -term in the envelope " $1 + mf(t)$ "), it is not necessary to include the envelope modulation in $r(t)$ in order to have good correlation between $d(t)$ and $r(t)$.[†] Not having to estimate the envelope of $d(t)$ simplifies the generation of $r(t)$.

$\hat{\phi}(t)$ may be derived from the received desired signal by passing the array output signal through a limiter to strip off the envelope modulation and then applying the result to a delay-lock loop. The loop timing may be synchronized by slewing the local code relative to the received signal code and applying the sum channel output to a threshold. The difference channel output is then used to track the timing of $\phi(t)$. With this procedure, the lockup time is not affected by interference, because the array nulls interference and keeps it off the delay-lock loop during the slewing process. (Interference nulling by the array does not depend on proper code timing, so the array nulls interference before the PN code is synchronized, i.e., while the delay-lock loop is slewing.) This procedure has been applied successfully in a spread spectrum adaptive array system[3].

In the next section, we discuss the effect of $\phi(t)$ on the desired signal envelope modulation.

[†]The authors are grateful to Dr. L.E. Brennan of Technology Service Corporation for pointing this out.

III. PHASE-ENVELOPE COUPLING

The signal $d(t)$ in Eq. (3) has envelope $A[1 + mf(t)]$, the same as the envelope of the conventional AM signal $y(t)$ in Eq. (1). However, the bandwidth of $d(t)$ is infinite. If $d(t)$ is passed through a filter to limit its bandwidth, envelope fluctuations will result due to $\phi(t)$. This effect is easily illustrated by noting that $d(t)$ may be written as

$$(5) \quad d(t) = A p(t)[1 + mf(t)] \cos \omega_c t$$

where $p(t)$ has values ± 1 , corresponding to $\phi(t)=0$ and π , as shown in Fig. 3.

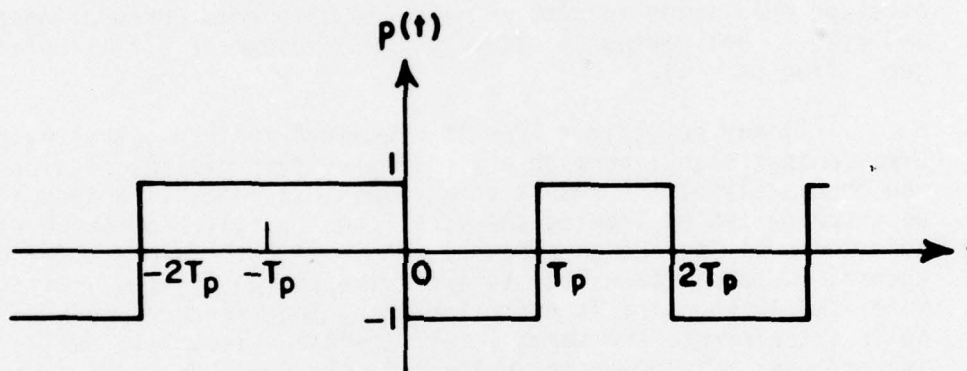


Fig. 3. The amplitude modulation $p(t)$.

If we momentarily assume, to simplify the discussion, that $p(t)$ is a square wave[†] with period $2T_p$ as shown in Fig. 4, then $p(t)$ may be written as

$$(6) \quad p(t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \sin n\omega_p t$$

[†]A square wave is a non-maximal length pseudonoise code.

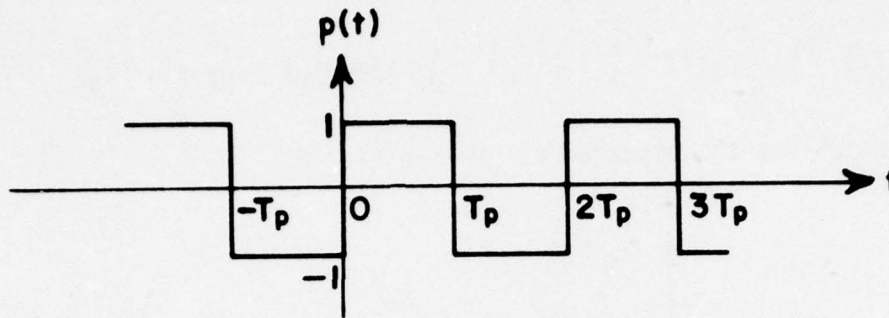


Fig. 4. Square wave $p(t)$.

where

$$\omega_p = \frac{\pi}{T_p} = \text{switching frequency of the square wave.}$$

Since $p(t)$ possesses half-wave symmetry, only odd harmonics exist. The Fourier Transform $P(\omega)^\dagger$ of $p(t)$ is shown in Fig. 5.

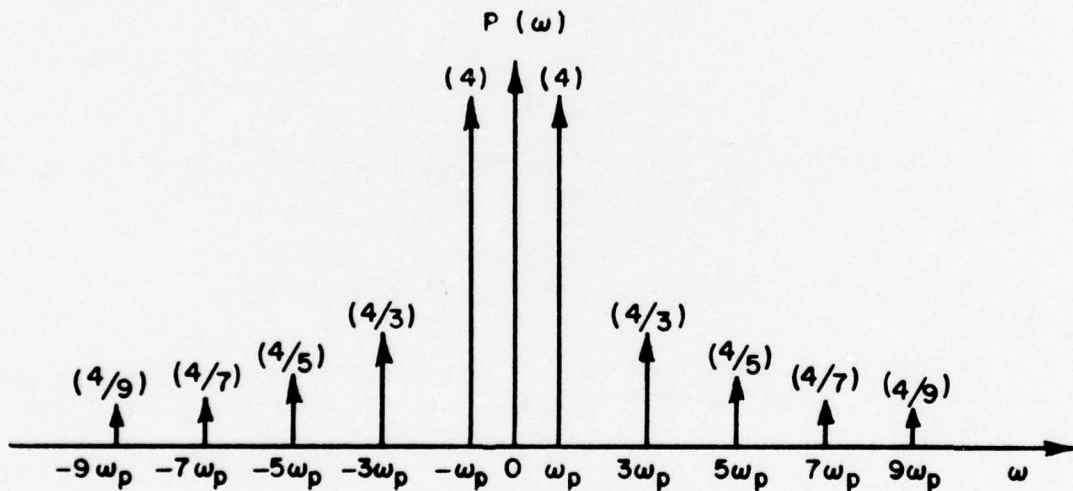


Fig. 5. Fourier Transform of the Square wave $p(t)$.

$$^\dagger \quad P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p(t) e^{-j\omega t} dt$$

If $p(t)$ is limited in bandwidth to, say, the first three frequency components at ω_p , $3\omega_p$ and $5\omega_p$, the resulting waveform, $p_a(t)$ is

$$(7) \quad p_a(t) = \frac{4}{\pi} \sin \omega_p t + \frac{4}{3\pi} \sin 3\omega_p t + \frac{4}{5\pi} \sin 5\omega_p t$$

which has the appearance shown in Fig. 6.

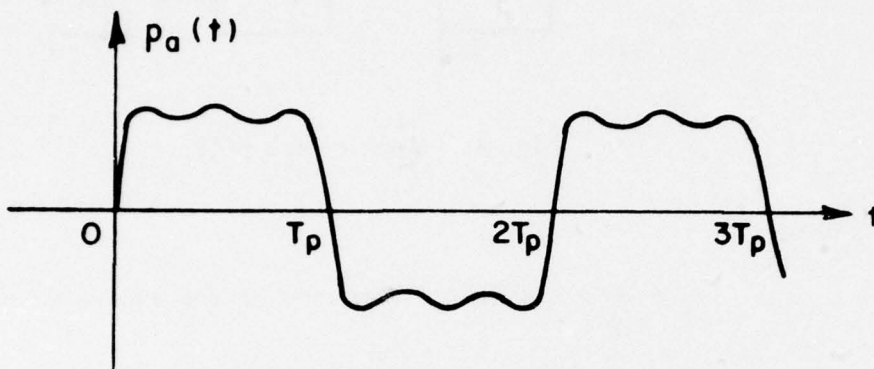


Fig. 6. A bandlimited square wave.

Let us further assume, to simplify the discussion, the $f(t)=0$ in $d(t)$ (see Eqs. (3) and (5)). Then

$$(8) \quad d(t) = A \cos[\omega_c t + \phi(t)] = A p(t) \cos \omega_c t$$

If this signal is passed through a bandpass filter which passes only the first three sidebands of $d(t)$ on either side of ω_c , the filter output will be

$$(9) \quad d_a(t) = A P_a(t) \cos \omega_c t$$

which has the appearance shown in Fig. 7. We find that $d_a(t)$ has AM modulation on it due to bandlimiting $d(t)$.

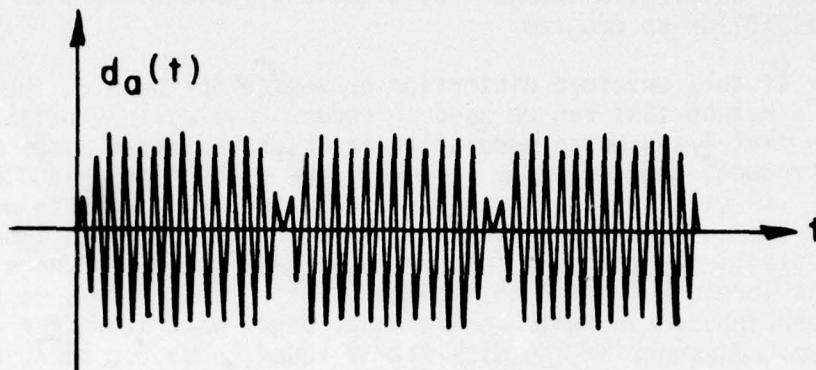


Fig. 7. A bandlimited version of $d(t)$.

$f(t)$ will not usually be zero, of course, but similar phase-envelope coupling will occur with a nonzero $f(t)$. For a given envelope modulation, the spectrum of $d(t)$ can be calculated and the waveform of its bandlimited version obtained. However, the above example is adequate for our purposes to illustrate how bandlimiting converts $\phi(t)$ to envelope modulation.

It is not surprising that the introduction of $\phi(t)$ in $d(t)$ produces envelope modulation when bandlimited, because there is a close coupling between the amplitude and phase modulation of a signal. This subject has been carefully discussed in two excellent papers by Voelcker[5,6], and the reader is referred there for details. Voelcker has shown that once the bandwidth and (periodic) envelope of a signal are specified, there are only certain (finitely many) phase modulations that the signal may have. The signals having these precisely defined phase modulations are the members of a so-called common envelope set. All the waveforms of the common envelope set may be obtained from any one of the set by a process of zero conjugation in the complex time plane. Any phase modulation other than one in the common envelope set (such as the $\phi(t)$ we have introduced!) must produce either an infinite bandwidth or a different envelope modulation. The waveform discussed above clearly illustrates this principle. $\phi(t)$ can be added to the AM signal without changing its envelope, but the resulting signal has an infinite bandwidth. If the bandwidth is limited, envelope distortion results.

The effect of the envelope distortion illustrated in Fig. 5 on the fidelity of a voice signal has not been determined at this writing. It is believed, however, that this effect will not be objectionable if the switching rate of $\phi(t)$ is below the lowest audio frequency transmitted by the AM system and if the envelope rollover at each phase transition of $\phi(t)$ occurs at a higher rate than the highest audio frequency transmitted.

The tolerance of the human listener to such envelope distortion is a subjective matter that is difficult to evaluate from theoretical considerations. We plan to examine the effects of this distortion in the experimental follow-up program.

If this envelope distortion proves objectionable, there is an alternative method that can be used to reduce envelope distortion. It is well known that 4-ary phase modulation in which the phase jumps never exceed $\pi/2$ produces substantially less envelope distortion than biphase modulation. In particular, 4-ary phase modulation with constrained jumps does not produce the envelope rollover seen in Fig. 5. Such 4-phase modulation has essentially the same interference rejection performance as biphase, but is more complicated to implement. For this reason, we have initially chosen biphase modulation and plan to begin with it in the experimental system. However, if the fidelity is found to be too poor, constrained 4-phase modulation will be used.

IV. FEEDBACK-LOOP BANDWIDTH AND THE CONCEPT OF SIGNAL CORRELATION

The concept of signal correlation in an LMS array is closely related to the concept of the feedback loop bandwidth for the array weights. In an LMS array, the feedback loop equation for the complex weights is[11],

$$(10) \quad \frac{dW(t)}{dt} = -K_A \nabla |\epsilon(t)|^2$$

where

$W(t)$ = complex weight vector of order N , where N is the number of elements in the array,

K_A = analog loop gain constant,

$$\nabla |\epsilon(t)|^{2\dagger} = -2X(t)\epsilon^*(t) \quad .$$

$$(11) \quad \epsilon(t) = R(t) - S(t)$$

[†] $\nabla |\epsilon(t)|^2$ is derived from the gradient of the real squared error signal, $\epsilon^2(t)$, with respect to the real weights.

$$(12) \quad S(t) = W^\dagger(t)X(t)$$

\dagger = adjoint (transpose conjugate) operation,
 $X(t)$ = complex input signal vector

and where the signals $\epsilon(t)$, $R(t)$ and $S(t)$ are expressed in analytic form.

With Eqs. (11) and (12), Eq. (10) can be written as

$$(13) \quad \frac{dW(t)}{dt} = 2K_A X(t)\epsilon^*(t)$$

or

$$(14) \quad \frac{dW(t)}{dt} + [2K_A R_X(t)]W(t) = 2K_A X(t)R^*(t)$$

where the asterisk denotes the complex conjugate and

$$R_X(t) = X(t)X^\dagger(t) = \text{the complex correlation matrix of the input signals.}$$

Equation (14), which is a first-order inhomogeneous coupled system of differential equations with time varying coefficients, will be referred to as the feedback loop equation for the complex weights. An exact solution of this equation in the general case is difficult to obtain because the equations are coupled and $R_X(t)$ is time-varying. However, various techniques have been used to obtain meaningful approximate solutions. Usually, solutions are obtained by replacing $R_X(t)$ by its expectation or its time-average value. In general, the weight solutions to Eq. (14) are non-stationary random processes. Each weight has a mean value and a fluctuating ("jitter") component. If the weight vector is statistically independent of $R_X(t)$, one can take the expectation of Eq. (14) and invoke the relation $E\{R_X(t)W(t)\} = E\{R_X(t)\}E\{W(t)\}$. This procedure reduces Eq. (14) to a system of equations for $E\{W(t)\}$ with constant coefficients (if $R_X(t)$ is stationary), which is easily solved. Although the statistical independence of $R_X(t)$ and $W(t)$ appears difficult to prove (in fact, they cannot be strictly independent), experimental results seem to indicate that the mean values of the weights are correctly predicted by neglecting the time-varying components of $R_X(t)$.[†] That approach will be taken in this study.

[†] The fluctuating components of the weights appear more difficult to analyze. Brennan and Reed first obtained approximate results[12] Koleszar[13] has developed a perturbation approach that allows one to calculate the first-order effects of the time-varying components of $R_X(t)$ on the weights. Also, Miller[14] has studied the effects of weight jitter on bit error probability for an adaptive array in a digital communications system.

Let us consider a two-element array that is receiving a CW desired signal and a CW interference signal. We suppose the array elements are one-half wavelength apart at the desired signal frequency. Let the desired signal $D(t)$, the interference signal $I(t)$ and the reference signal $R(t)$ be given in complex form by

$$(15) \quad \begin{cases} D(t) = A e^{j\omega_D t} \\ I(t) = B e^{j\omega_I t} \\ R(t) = C e^{j\omega_D t} \end{cases}$$

where ω_D and ω_I are the radian frequencies of the respective signals. (Note that the frequencies of the desired and reference signals are the same.) With these signals, the solution to Eq. (15) when the time-varying components of $R_X(t)$ are neglected, is of the form

$$(16) \quad W_i(t) = \gamma_i + \sum_{j=1}^2 \alpha_{ij} e^{-2K_A \lambda_j t} + \beta_i e^{-j\omega_\Delta t}$$

where

$W_i(t)$ = complex weight in the i th element,

$\gamma_i, \alpha_{ij}, \beta_i$ = complex constants,

λ_j = j th eigenvalue of the time-average value of $R_X(t)$, and

$$\omega_\Delta = \omega_I - \omega_D$$

The dc terms γ_i in this solution result from the dc terms in $2K_A X(t)R^*(t)$ and the oscillating terms result from the difference frequency terms in $2K_A X(t)R^*(t)$. The exponentials are the homogeneous solution to Eq. (14) and are adjusted to satisfy the initial conditions on $W_i(t)$.

Because of the form of Eq. (14), we may view the weight vector W as the output response of a linear lowpass filter to an input vector $2K_A X(t)R^*(t)$. For the sinusoidal signals in Eq. (15), it is clear that the coefficients β_i in Eq. (16) will be small if

$$(17) \quad \omega_\Delta \gg 2K_A \lambda_{MAX}$$

where $\lambda_{MAX} = \max\{\lambda_j\}$, because then the component of $2K_A X(t)R^*(t)$ at frequency ω_Δ will be above the (highest) 3 dB cutoff, $2K_A \lambda_{MAX}$, of the linear lowpass filter. In general, for arbitrary signals $X(t)$ and $R(t)$, spectral components of $2K_A X(t)R^*(t)$ above the cutoff frequency $2K_A \lambda_{MAX}$ have little effect on the weights. On the other hand, dc components of $2K_A X(t)R^*(t)$ determine the dc values of the weights, and non-zero frequency components

of $2K_A X(t)R^*(t)$ below the cutoff frequency $2K_A \lambda_{MAX}$ produce weight fluctuation or jitter.

For the special case of CW signals as given in Eq. (15), the desired signal-reference signal products in $2K_A X(t)R^*(t)$ produce dc terms that control the steady-state values of the weights. The interference signal-reference signal products are at frequency ω_Δ . If $\omega_\Delta \gg 2K_A \lambda_{MAX}$ these components are filtered out and do not affect the weights. In this situation the array nulls the interference. If $\omega_\Delta < 2K_A \lambda_{MAX}$ the ω_Δ terms produce oscillating weights. In general, the array does not null the interference, but instead amplitude or phase modulates both the desired and interference signals.[†]

Now suppose the reference signal has a slightly different frequency than the desired signal. Instead of Eqs. (15), we assume the signals are given by

$$(15') \quad \begin{cases} D(t) = A e^{j\omega_D t} \\ I(t) = B e^{j\omega_I t} \\ R(t) = C e^{j\omega_R t} \end{cases}$$

where ω_R may be different from ω_D . In this case, the product $2K_A X(t)R^*(t)$ will contain frequency components at^{††}

$$(18) \quad \omega_{\Delta 1} = \omega_D - \omega_R$$

as well as at

$$(19) \quad \omega_{\Delta 2} = \omega_I - \omega_R$$

[†] It must be kept in mind that the solution in Eq. (16) is only approximate. It is based on the assumption that the time-varying components of $R_X(t)$ are ignored. However, these components also contain terms oscillating at the difference frequency ω_Δ , which will contribute to the weight fluctuations too.

^{††} Note that the time-varying components of $R_X(t)$ now contain terms oscillating at $\omega_{\Delta 3} = \omega_I - \omega_D$.

If $\omega_{\Delta 1} \gg 2K_A \lambda_{MAX}$ and $\omega_{\Delta 2} \gg 2K_A \lambda_{MAX}$, the array will null both the desired and interference signals. But if $\omega_{\Delta 1} < 2K_A \lambda_{MAX}$ (and $\omega_{\Delta 2} \gg 2K_A \lambda_{MAX}$), the array will null the interference, but not the desired signal. The array weights will oscillate because of the $\omega_{\Delta 1}$ components, and this oscillation will produce FM on the desired signal.

This situation may be understood by noting that the array feedback attempts to match the array output to the reference signal in both amplitude and phase. A small frequency offset between the desired and reference signals is equivalent to a phase mismatch that increases linearly with time. If this phase difference changes slowly enough that the array weights can follow it (i.e., if $\omega_{\Delta 1} < 2K_A \lambda_{MAX}$) the weights simply roll the phase of the desired signal continuously backward to make it match the reference signal phase. In this process, the in-phase and quadrature weights on each element oscillate in quadrature, and the desired signal undergoes a frequency shift. Such desired signal FM has been discussed by DiCarlo and Compton in a slightly different context [15].

Now let us consider what happens with more general types of signals. In a radio communications system, the signals $X(t)$ and $R(t)$ will be band-limited signals centered at a non-zero carrier frequency. Hence the product $2K_A X(t)R^*(t)$ will contain frequencies in a band centered around zero frequency. If the reference signal is properly designed, the desired signal-reference signal product in $2K_A X(t)R^*(t)$ will contribute a large dc term, which will control the steady-state weights and will cause the array to retain the desired signal in the array output. The interference signal-reference signal product should contain no dc term; most of its spectral components should fall outside the cutoff frequency $2K_A \lambda_{MAX}$. The portion that falls inside contributes to the weight jitter and results in desired signal modulation.

In the present context, we say that two signals (such as the interference signal and the reference signal) are "uncorrelated" so long as most of the power in the spectral products they contribute to $2K_A X(t)R^*(t)$ lies outside the feedback loop bandwidth, $2K_A \lambda_{MAX}$. Two signals are strongly "correlated" when the power in these spectral components is mostly within the loop bandwidth. In general, the reference signal should be designed so the desired signal-reference signal products in $2K_A X(t)R^*(t)$ produce a strong dc component, and so the interference-reference signal products have as little power as possible within the feedback loop bandwidth $2K_A \lambda_{MAX}$.

Now consider what happens when the reference and desired signals contain the phase switching as in Eqs. (3) and (4), and the interference is CW. A CW interference signal will contribute terms of the form

$$(20) \quad C_{IR}(t) = I(t)R^*(t) = BC p(t)e^{-j\omega_{\Delta}t}$$

to the product $2K_A X(t)R^*(t)$,[†] where

$$(21) \quad p(t) = e^{j\phi(t)},$$

as defined in Eq. (5) and Fig. 3. When $p(t)$ is a square wave as in Fig. 4, its Fourier Transform is as shown in Fig. 5, so the spectrum of $C_{IR}(t)$ is similar to that in Fig. 5 but with all frequency components shifted to the left by ω_Δ . We see immediately that if

$$(22) \quad \omega_\Delta = \pm\omega_\rho, \pm 3\omega_\rho, \pm 5\omega_\rho, \dots$$

a spectral line of $C_{IR}(t)$ will fall at dc. Thus, for certain interference frequencies, the interference has strong dc correlation with the reference signal. Moreover, even if ω_Δ does not have exactly one of the values above, when ω_Δ is within $2K_A \lambda_{MAX}$ of any of these frequencies, a spectral line will fall within the feedback loop bandwidth and will cause weight oscillations.

Whether this situation is a problem for the array depends on what fraction of the total power in $C_{IR}(t)$ falls within the feedback loop bandwidth (and also on whether the desired signal is present^{††}). Clearly, if $p(t)$ is a square-wave, the least desirable situation occurs if $\omega_\Delta = \pm\omega_\rho$, because the frequency components of $p(t)$ are strongest at $\pm\omega_\rho$. If, however, $p(t)$ is a maximal length pseudonoise (PN) code, instead of a square-wave, the performance degradation is less than with a square wave. Such a code is a periodic waveform with period $T = (2^n - 1)T_p$, where T_p is the duration of a single bit and n is the number of bits in the shift register generating the code [10]. Since the fundamental period of the PN code is longer than that of the square wave by the ratio $(2^n - 1)/2$, the frequency components of the PN code will be at integral multiples of

[†]It will also contribute terms of the form

$$C_{ID}(t) = I(t)D^*(t) = AB[1 + mf(t)]p(t)e^{-j\omega_\Delta t}$$

to the components of $R_X(t)$.

^{††}If the desired signal is not present, the array feedback will match the CW interference signal at the array output with the corresponding spectral line in the reference signal. It will not null the interference. However, when the desired signal is not present, there is nothing to "receive" anyway, so the interference does not matter. When the desired signal is present, the error signal is minimized by matching the desired signal to the reference signal and nulling the interference, because matching the desired and reference signals matches all the discrete spectral lines of the reference signal, not just the one at the interference frequency.

$$(23) \quad \omega_0 = \frac{1}{2^n - 1} \omega_p$$

i.e., much more closely spaced than for the square wave. Thus, with a PN code on the reference signal, there will be many more frequency components in the product $C_{IR}(t)$, but each one will contain a smaller proportion of the total power.

The performance of the array against CW interference has been examined with computer simulations and the results are discussed in a companion report[16]. It appears that, although the array does not null interference quite as effectively for certain interference frequencies, the overall performance is nevertheless satisfactory for reliable communications.

Finally, let us briefly discuss what happens with other types of interference besides CW. Some typical other types are: (1) conventionally modulated signals such as AM, FSK, etc., whose spectral components fall within the frequency band of the desired signal (we refer to this type as friendly interference); (2) broadband noise interference; and (3) pulsed interference. The last two are typical of intentional jamming.

First, we note that with broadband noise interference, there are no critical frequencies, because $C_{IR}(t)$ will not contain any discrete frequency components. Second, with conventional modulation and pulse jamming, a variety of situations are possible. In general, discrete spectral lines will occur in $C_{IR}(t)$ only if the interference spectrum contains discrete spectral lines. When discrete spectral lines do occur, we may expect some performance degradation at certain interference frequencies. The effect on array performance will depend on the specific waveform. The more closely the interference spectrum matches that of the reference signal, the more power $C_{IR}(t)$ will contain within the feedback loop bandwidth and the more performance degradation will result.

V. DESIGN TRADEOFFS FOR THE PROPOSED AM SYSTEM

In this section, design relations and bounds for the proposed AM adaptive array system will be discussed. As noted in the last section, good array performance requires the reference signal to be highly correlated with the desired signal but uncorrelated with the interference, where the degree of correlation between two signals refers to the spectral power in their product inside the feedback loop bandwidth, $2K_A \lambda_{MAX}$.

First, consider the correlation between the desired and reference signals. From Eqs. (3) and (4), these signals may be written in complex form as

$$(24) \quad D(t) = A[1 + mf(t)] e^{j[\omega_c t + \phi(t)]}$$

$$(25) \quad R(t) = C e^{j[\omega_c t + \phi(t)]}$$

where the phase constant θ in Eq. (3) has been set equal to zero for convenience and where we have assumed for the moment that the code estimate $\hat{\phi}(t)$ in the reference signal is perfect, i.e., $\hat{\phi}(t) = \phi(t)$. (Note that the signals in Eqs. (24) and (25), although complex, are not analytic, because the bandwidth of $e^{j\phi(t)}$ is infinite. However, under the assumption that ω_c is very much greater than the switching frequency of $\phi(t)$, little inaccuracy will result for our purposes from considering these as analytic.) The product $D(t)R^*(t)$ is

$$(26) \quad C_{DR}(t) = D(t)R^*(t) = AC + AC mf(t)$$

Let us assume the audio waveform $f(t)$ has a Fourier Transform $F(\omega)$ as shown in Fig. 8. Let $F(\omega)$ be strictly bandlimited, with ω_{\min} the minimum and ω_{\max} the maximum audio frequencies, respectively.

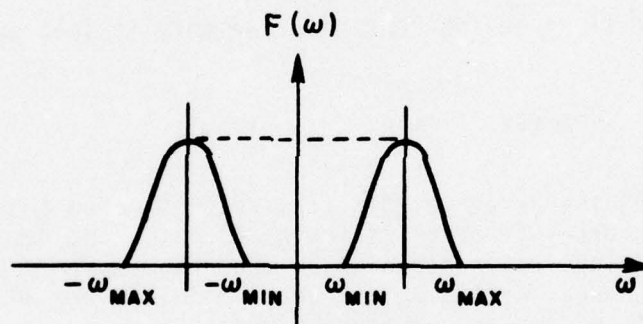


Fig. 8. Spectrum of a typical audio waveform $f(t)$.

The Fourier Transform of $C_{DR}(t)$ is then

$$(27) \quad C_{DR}(\omega) = AC[2\pi\delta(\omega) + mF(\omega)]$$

as illustrated in Fig. 9.

$C_{DR}(\omega)$ contains an impulse at dc and the audio sidebands of $f(t)$. The dc impulse results from the correlation between the carrier component of the desired signal and the reference signal; it represents the desired correlation between the two signals. The sideband components represent undesired signal products.

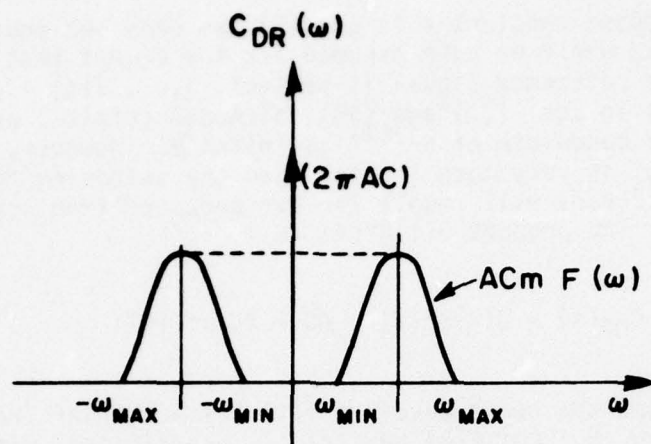


Fig. 9. Spectrum $C_{DR}(\omega)$.

The first design constraint we note is that we must choose

$$(28) \quad 2K_A \lambda_{MAX} \ll \omega_{min} \quad .$$

to prevent the array weights from responding to the audio modulation. Since the array feedback is trying to match the desired signal to a constant envelope reference signal, the weights, if capable of changing rapidly enough, will simply inverse modulate the AM signal, i.e., convert it to a constant envelope signal.

Next, we note that the speed of response of the array is limited by the smallest eigenvalue of $R_X(t)$ in Eq. (16). We therefore define the time constant of the array to be

$$(29) \quad \tau_A = \frac{1}{2K_A \lambda_{MIN}}$$

where

$$\lambda_{MIN} = \min_j \{ \lambda_j \} \quad .$$

In a typical design we would like the array to be fast enough to track changes of the signal directions in space. The resulting maximum value

for τ_A , say τ_{MAX} , will dictate a minimum value of λ_{MIN} . However, since $\lambda_{MIN} \leq \lambda_{MAX}$, we now have

$$(30) \quad \frac{1}{\tau_{MAX}} = 2K_A \lambda_{MIN} \leq 2K_A \lambda_{MAX} \ll \omega_{min}$$

so we find that the lowest audio frequency limits the fastest speed of response that can be attained. Moreover, this limitation can be a severe one, because the eigenvalues λ_j in the array are dependent on the incoming signal power. (See the definition of $R_X(t)$ below Eq. (14).) Normally the array has to be designed to operate over a range of signal powers. The inequality for λ_{MAX} in Eq. (30) must be satisfied for the strongest interference signal to be handled, while for λ_{MIN} we must use the weakest desired signal of interest. The result is that with wide dynamic signal levels there will be several orders of magnitude between λ_{MIN} and λ_{MAX} . Hence the array will have to have a slow response if typical audio values are used for ω_{min} .

The next parameter limitation is one that has already been mentioned in Section III in connection with phase-envelope coupling. As discussed there, bandlimiting the phase-switched AM signal will produce envelope distortion, especially in the region of a bit transition where the envelope "rolls over". The speed at which the envelope rolls over is determined by the system bandwidth, i.e., by the highest audio frequency transmitted, ω_{MAX} . The envelope rollover speed is independent of the code frequency ω_p ($\omega_p = \pi/T_p$, where T_p is the bit duration). To make the envelope rollover occur as infrequently as possible, we want the bit duration as long as possible, or ω_p as small as possible. In a typical audio system with, say, $\omega_{min} = 2\pi(100 \text{ Hz})$ and $\omega_{max} = 2\pi(4 \text{ kHz})$, if we choose $\omega_p = \omega_{min}$, the bit durations will be approximately 40 times as long as the envelope rollover region, so severe envelope distortion will occur only a few percent of the time. For this reason, to minimize envelope distortion, we choose

$$(31) \quad \omega_p \approx \omega_{min} \ll \omega_{max}$$

Finally, we note that regardless of the relative values of ω_p and ω_{min} , the inequality

$$(32) \quad 2K_A \lambda_{MAX} \ll \omega_p$$

must always be enforced, in addition to the inequality in Eq. (30). If $2K_A \lambda_{MAX}$ is not small compared to ω_p , the feedback loops will be capable of tracking the switching rate of the biphase modulation. This situation is undesirable because, for example, the array weights could modulate a CW interference signal to make it match the biphase modulated reference

signal. The inequality in Eq. (32) is equivalent to having the integration time window of the array feedback long compared to the bit interval T_D . If that condition is not enforced, the phase switching will not decorrelate the reference signal from the interference.

When the inequalities in Eqs. (30), (31), and (32) are combined, the following ordering of parameter values results:

$$(33) \quad \frac{1}{\tau_{MAX}} = 2K_A \lambda_{MIN} \leq 2K_A \lambda_{MAX} \ll \min\{\omega_p, \omega_{min}\} \ll \omega_{max} \quad .$$

VI. RELATIONS BETWEEN THE ANALOG ARRAY AND ITS DIGITAL SIMULATION

In the next section some initial results of an adaptive array simulation with phase modulated AM signals are presented. The simulation is based on the discrete form of the LMS algorithm. In this section, we briefly discuss the relation between the discrete and analog forms of the LMS algorithm.

The adaptive array is simulated by using the complex discrete LMS algorithm[11], in which an iterative correction to the weights is made every T seconds. The discrete algorithm is

$$(34) \quad W(J+1) = W(J) + 2K_D(J) \epsilon^*(J)$$

where J = sample index

K_D = digital feedback loop gain constant .

Equation (34) is a forward difference approximation of the analog LMS algorithm in Eq. (13). Equation (34) can be manipulated into the form

$$(35) \quad W(J+1) = [2K_D R_X(J) - 1] W(J) = 2K_D X(J) R^*(J) \quad ,$$

which is the control loop difference equation for the weight vector $W(J)$. For a two-element array with CW desired, interference and reference signals,

$$(36) \quad \begin{cases} D(J) = A e^{j\omega_D J T} \\ I(J) = B e^{j\omega_I J T} \\ R(J) = C e^{j\omega_D J T} \end{cases}$$

the solution of Eq. (35) when the time-varying components of $R_X(J)$ are neglected is

$$(37) \quad W_i(J) = \gamma_i + \sum_{j=1}^2 \xi_{ij} [1 - 2K_D \lambda_j]^J + q_i e^{j\omega_\Delta J T}$$

or

$$(38) \quad W_i(J) = \gamma_i + \sum_{j=1}^2 \xi_{ij} e^{-\left(-\frac{\ln[1 - 2K_D \lambda_j]}{T}\right) J T} + q_i e^{j\omega_\Delta J T}$$

where

$\gamma_i, \xi_{ij}, q_i = \text{complex constants}$

$\lambda_j = j\text{th eigenvalue of the (time-averaged) matrix } R_X(J). \text{ (Note that the } \lambda_j \text{ are the same for both the analog and discrete array.)}$

$$\omega_\Delta = \omega_I - \omega_D$$

In these equations it is assumed that

$$0 < \max_i \{2K_D \lambda_j\} < 1$$

so the frequency response of the difference equation Eq. (35) models a lowpass filter similar to Eq. (14). (For

$$1 < \max_j \{2K_D \lambda_j\} < 2,$$

the difference equation Eq. (35) is stable but does not have a lowpass characteristic). Moreover, we assume $\omega_\Delta T \ll 2\pi$ so ω_Δ is within the first period of the (periodic) frequency response of Eq. (35).

By analogy with Eqs. (16) and (17), we note that for $\omega_\Delta \gg B_D$, where

$$(39) \quad B_D = -\frac{1}{T} \ln[1 - 2K_D \lambda_{MAX}]$$

(but still $\omega_\Delta \ll 2\pi/T$) the coefficients q_i will be small, because the components of $2K_D X(J)R^*(J)$ at frequency ω_Δ will be outside the lowpass cutoff of the weights. This behavior is similar to the behavior of the β_i in Eq. (16) for the analog loop.

If Eqs. (14) and (35) and their solutions are compared, we find the following relationships. First, the two solutions have the same steady state values (γ_j). Second, the transient responses of the analog and discrete algorithms will not be the same unless they are properly scaled. It is possible to do this scaling in two ways:

(1) By Matching the Bandwidths

If we choose K_A and K_D such that

$$(40) \quad 2K_A \lambda_{MAX} = -\frac{1}{T} \ln[1 - 2K_D \lambda_{MAX}] \quad ,$$

which requires

$$(41) \quad K_A = -\frac{\ln[1 - 2K_D \lambda_{MAX}]}{2\lambda_{MAX}T}$$

or

$$(42) \quad K_D = \frac{1 - e^{-2K_A \lambda_{MAX}T}}{2\lambda_{MAX}}$$

the bandwidths of the analog and discrete arrays will be identical. However, in this case the speeds of the two arrays will not usually be the same. The speeds of the analog and discrete arrays are given in terms of their time constants, which are related to the minimum eigenvalue,

$$(43) \quad \tau_A = \frac{1}{2K_A \lambda_{MIN}} \quad ,$$

and

$$(44) \quad \tau_D = -\frac{T}{\ln[1 - 2K_D \lambda_{MIN}]} \quad .$$

In general, once K_A and K_D are chosen to satisfy Eqs. (41) or (42), τ_A and τ_D will not be the same.

(2) By Matching the Speed

Matching the speeds requires that K_A and K_D satisfy

$$(45) \quad 2K_A \lambda_{MIN} = -\frac{1}{T} \ln[1 - 2K_D \lambda_{MIN}] \quad ,$$

or

$$(46) \quad K_A = - \frac{\ln[1 - 2K_D \lambda_{\text{MIN}}]}{2\lambda_{\text{MIN}}T} ,$$

and

$$(47) \quad K_D = \frac{1 - e^{-2K_A \lambda_{\text{MIN}}T}}{2\lambda_{\text{MIN}}} .$$

In this case, the speeds of the two arrays are the same, but not their bandwidths.

In the special case of a two element array (so there are only two eigenvalues) it may be possible to match both the bandwidths and the speeds, if the sampling interval T is properly chosen.

To do this, both Eqs. (40) and (45) have to be satisfied, which requires that K_D , λ_{MIN} and λ_{MAX} satisfy

$$(48) \quad (1 - 2K_D \lambda_{\text{MAX}})^{\lambda_{\text{MIN}}} = (1 - 2K_D \lambda_{\text{MIN}})^{\lambda_{\text{MAX}}}$$

and then the sampling time T must be

$$(49) \quad T = - \frac{\ln[1 - 2K_D \lambda_{\text{MAX}}]}{2K_A \lambda_{\text{MAX}}} .$$

Also, K_A will be given by

$$(50) \quad K_A = \frac{\ln[1 - 2K_D \lambda_{\text{MIN}}]}{-2\lambda_{\text{MIN}}T} = - \frac{\ln[1 - 2K_D \lambda_{\text{MAX}}]}{2\lambda_{\text{MAX}}T} .$$

Usually in a computer simulation it is more important to match bandwidths than to match time constants, so that the effects of partial correlations between the interference, desired and reference signals are properly modelled. That procedure has been used in the next section.

Finally, we note that for the discrete algorithm, the parameter relations in Eq. (33) take the form

$$(51) \quad \frac{1}{T_{\text{MAX}}} = - \frac{\ln[1 - 2K_D \lambda_{\text{MIN}}]}{T} \leq - \frac{\ln[1 - 2K_D \lambda_{\text{MAX}}]}{T} \\ \ll \min\{\omega_p, \omega_{\text{min}}\} \ll \omega_{\text{max}} \ll 2\pi/T .$$

The last inequality in Eq. (51), $\omega_{\max} \ll 2\pi/T$, is necessary in a discrete simulation to prevent the signal spectra from overlapping into the next period of the discrete frequency response, which is periodic with period $2\pi/T$.

VII. TYPICAL SIMULATION RESULTS

In this section, we present some simulation results for a two-element adaptive array with phase modulated AM signals and CW interference. The curves shown are intended merely to illustrate typical results. A more complete discussion of the simulation results showing the effects of various system parameters such as interference and switching frequencies, etc. is contained in a companion report currently in preparation[16].

In the simulations described below, the following signals have been assumed:

$$d(t) = \text{desired signal} = A[1 + m \cos\omega_m t] p(t) \cos\omega_c t$$

$$i(t) = \text{interference signal} = B \cos\omega_I t$$

$$r(t) = \text{reference signal} = C p(t) \cos\omega_c t$$

where $p(t)$ is a square wave with switching frequency ω_p . The array elements are assumed isotropic and spaced a half wavelength apart at frequency ω_c . The parameter values used are shown in Table I. Table I also shows the feedback loop bandwidth and array time constant as obtained from Eqs. (39), (44), (54) and (55). The values of A , m and B in Table I result in an input signal-to-interference ratio[†] of -22.3 dB[17]. The desired signal was incident on the array from broadside and the interference was incident from an angle θ_I off broadside.

The frequencies shown in Table I are scaled frequencies chosen to allow a reasonable computer running time in the simulations. If realistic frequencies are used with the digital LMS algorithm, impractically long computer times usually result. To illustrate this problem, it is interesting to calculate the speed of the analog and digital array for a typical set of signal parameters:

[†]Defined as Total incident desired signal power (including sidebands) divided by incident interference power.

TABLE 1
PARAMETER VALUES OF THE SIMULATED ADAPTIVE ARRAY

Parameter	Value
ω_c	$2\pi (100 \times 10^3)$ rad/sec.
ω_I	$2\pi (100 \times 10^3)$ rad/sec.
ω_m	$2\pi (8 \times 10^3)$ rad/sec.
ω_p	$2\pi (20 \times 10^3)$ rad/sec.
A	0.75
B	10.
C	1.
m	0.3333
K_D	0.00015
T	2.5×10^{-6} seconds
B_D	$2\pi (4.8 \times 10^3)$ rad/sec.
\dot{D}	1666T

Consider:

$$\begin{aligned}
 \omega_c &= 2\pi (20 \times 10^6) \text{ rad/sec} \\
 \omega_I &= 2\pi (10 \times 10^6) \text{ rad/sec} \\
 A &= 0.75 \\
 B &= 10. \\
 (52) \quad C &= 1. \\
 m &= 0.3333 \\
 T &= 12.5 \times 10^{-9} \text{ sec} \\
 \theta_D &= 0^\circ \\
 \theta_I &= 60^\circ
 \end{aligned}$$

The estimates of the eigenvalues are

$$(53) \quad \begin{cases} \lambda_{\text{MIN}} = 2 \\ \lambda_{\text{MAX}} = 242 \end{cases}$$

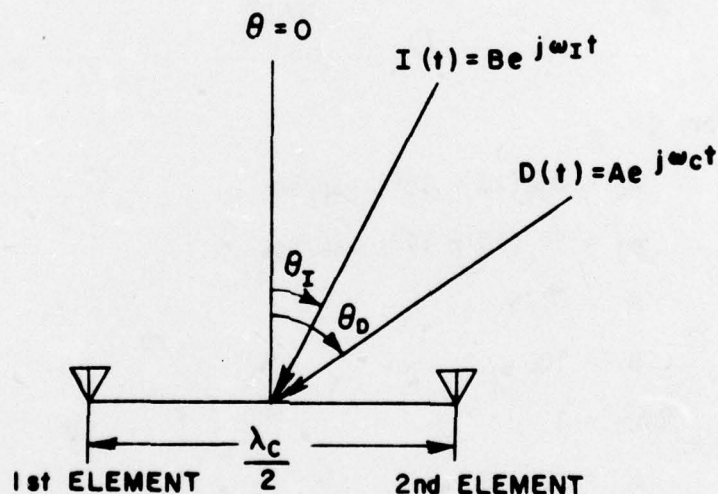
The eigenvalues shown have been estimated as follows. Since $\lambda_{\text{MAX}} \gg \lambda_{\text{MIN}}$, λ_{MAX} can be approximated by

$$(54) \quad \lambda_{\text{MAX}} = \text{Trace } R_X(t) = |X_1(t)|^2 + |X_2(t)|^2$$

and in order to have a worst case estimate, $|X_1(t)|^2 + |X_2(t)|^2$ has been replaced by its maximum instantaneous value. λ_{MIN} is the actual eigenvalue for the time-averaged $R_X(t)$ with CW signals, viz.,

$$(55) \quad \lambda_{\text{MIN}} = (A^2+B^2) - \sqrt{(A^2+B^2)^2 - 2A^2B^2 \left(1 - \cos\left(\pi \sin\theta_D - \frac{\omega_I}{\omega_C} \sin\theta_I\right)\right)}$$

where θ_I and θ_D are the angles of arrival of the interference and desired signals, respectively, as shown in Fig. 10.



λ_c = WAVELENGTH AT THE DESIRED SIGNAL
CARRIER FREQUENCY

Fig. 10. Direction of arrival of the input signals.

Satisfying the bandwidth requirements of Eq. (30) gives,

$$(56) \quad 2K_A(242) \ll 2\pi 100$$

or

$$K_A \ll 1.298$$

For $K_A=1$, we find from Eq. (42) that $K_D = 0.0000012497$ and the time constants of the two arrays are,

$$(57) \quad \tau_A = \frac{1}{2K_A \lambda_{\text{MIN}}} = 0.25 \text{ (sec)}$$

$$(58) \quad \tau_D = \frac{-T}{\ln(1-2K_D \lambda_{\text{MIN}})} \approx 200,050T = 200,050 \text{ iterations}$$

Thus the response time (say 5 time constants) of the analog array is 1.25 sec. For the digital array it is 1,000,250 computer iterations. With the size of the computer programs needed for the simulations, this number of iterations requires an impractical amount of computer time. To increase the array speed, large values of K_D must be used. However, using large values of K_D causes the bandwidth to be large; and to satisfy Eqs. (33) and (51), large value of ω_p and ω_{min} are needed. The frequencies shown in Table I thus represent a compromise to allow reasonable computer times.

Figure 11 shows a typical set of weight transients that result when the interference angle θ_1 is 60° , where the two complex weights are written

$$(59) \quad W_1(J) = W_{11}(J) + j W_{12}(J)$$

$$(60) \quad W_2(J) = W_{21}(J) + j W_{22}(J)$$

and initial values of the weights were arbitrarily assumed to be $W_{11}(0) = 1$, $W_{12}(0) = 0$, $W_{21}(0) = 0$ and $W_{22}(0) = 0$.

From the weights, the array response to each signal can be calculated. In general, the voltage response of the array with weights $W_1(J)$ and $W_2(J)$ to a signal at angle θ and frequency ω is

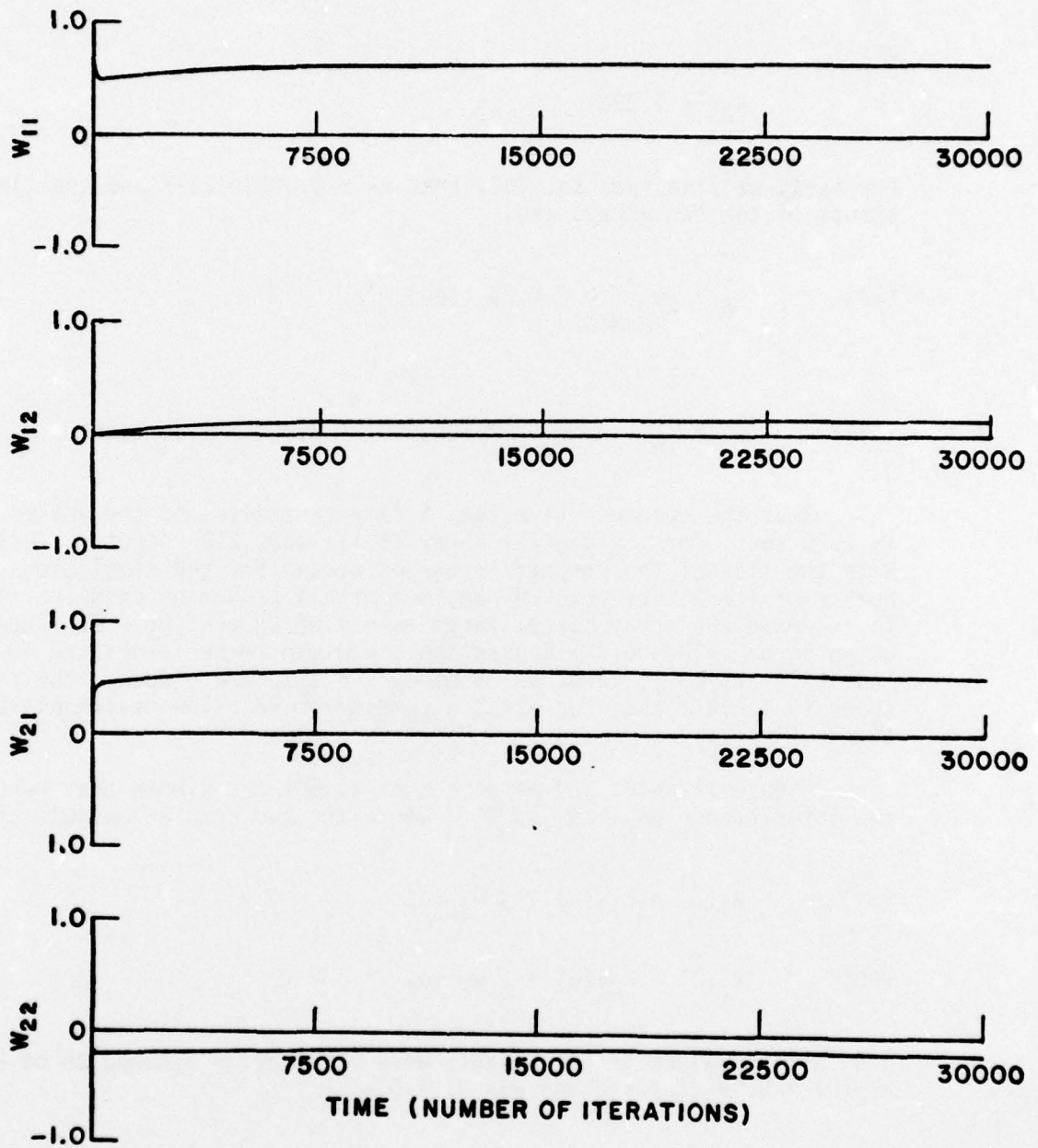


Fig. 11. Transient response of the complex weights ($\theta_I=60^\circ$).

From the weights, the array response to each signal can be calculated. In general, the voltage response of the array with weights $W_1(J)$ and $W_2(J)$ to a signal at angle θ and frequency ω is

$$(61) \quad AF(\theta, \omega, J) = W_1^*(J) + W_2^*(J) e^{j\pi \frac{\omega}{\omega_c} \sin \theta}$$

The magnitude and phase of the array response to the desired signal are then

$$(62) \quad GAIN_{\theta_D}(J) = |AF(\theta_D, \omega_c, J)|$$

and

$$(63) \quad PHASE_{\theta_D}(J) = \angle AF(\theta_D, \omega_c, J)$$

and for the interference, the amplitude response is

$$(64) \quad GAIN_{\theta_I}(J) = |AF(\theta_I, \omega_I, J)|$$

(Since phase shift for the interference signal is of little interest, $PHASE_{\theta_I}(J)$ is not discussed.)

Figure 12 shows the transient behavior of $GAIN_{\theta_D}(J)$, $PHASE_{\theta_D}(J)$ and $GAIN_{\theta_I}(J)$ corresponding to the weight transients shown in Fig. 11. Also shown in Fig. 12 is the Gain Ratio, defined as the ratio of the desired signal response to the interference signal response:

$$(65) \quad \text{Gain Ratio} = \frac{|AF(\theta_D, \omega_c, J)|}{|AF(\theta_I, \omega_I, J)|}$$

The behavior of these quantities after the initial weight transients have ended is shown on a more expanded scale in Fig. 13. As may be seen, small fluctuations occur in these quantities, particularly in the interference response $GAIN_{\theta_I}$. The reason for these fluctuations is that the products $C_{DR}(t) = D(t)R^*(t)$ and $C_{IR}(t) = I(t)R^*(t)$ contain spectral lines that cause the array weights to have a small fluctuating component. The fluctuations are more noticeable on the interference than on the desired signal, because the interference is in a pattern null. The bounds for the fluctuation in each parameter are given in Table 2. Also listed in Table 2 are the corresponding bounds for the output signal-to-interference ratio.

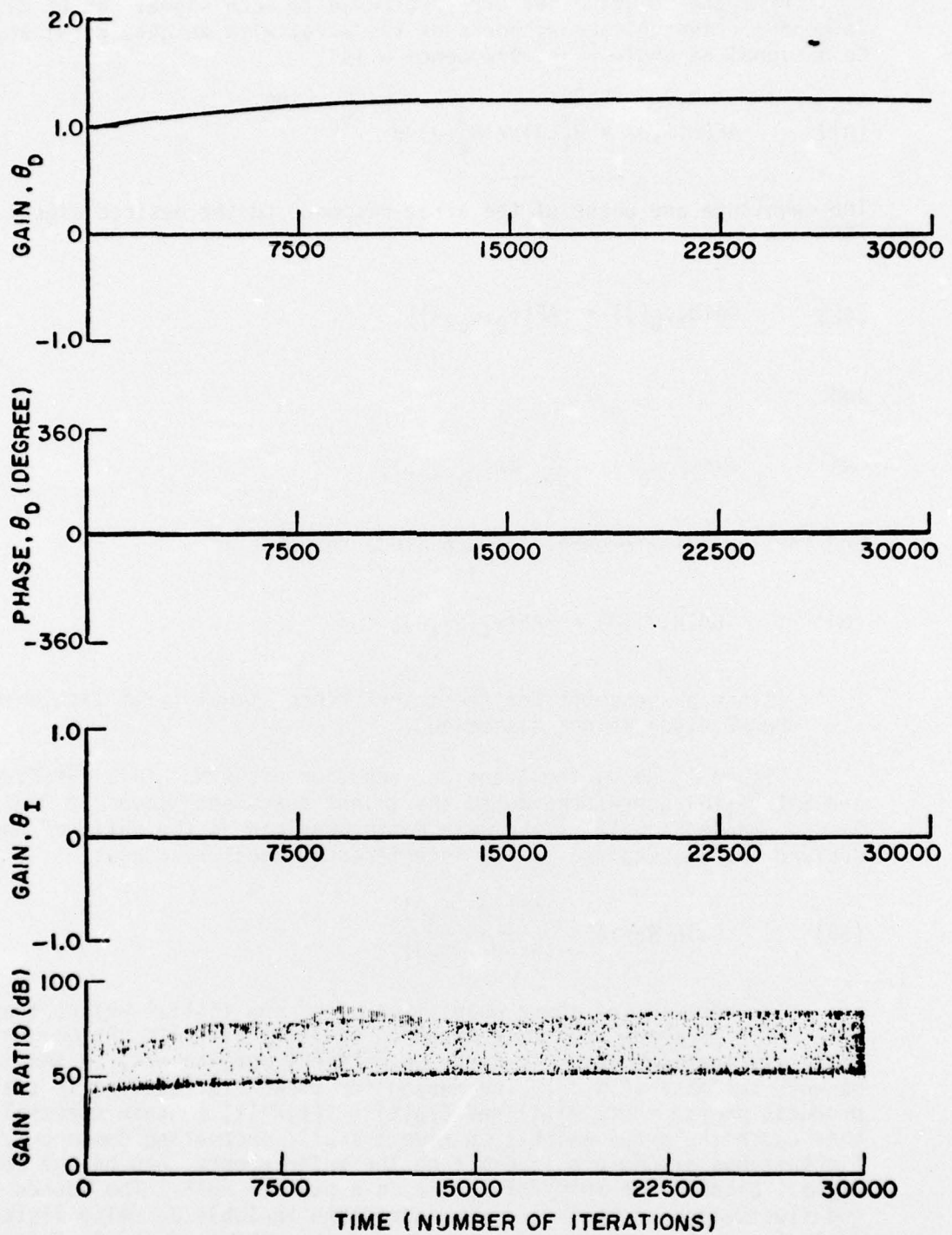


Fig. 12. Transient response of the array gains and phase ($\theta_I=60^\circ$).

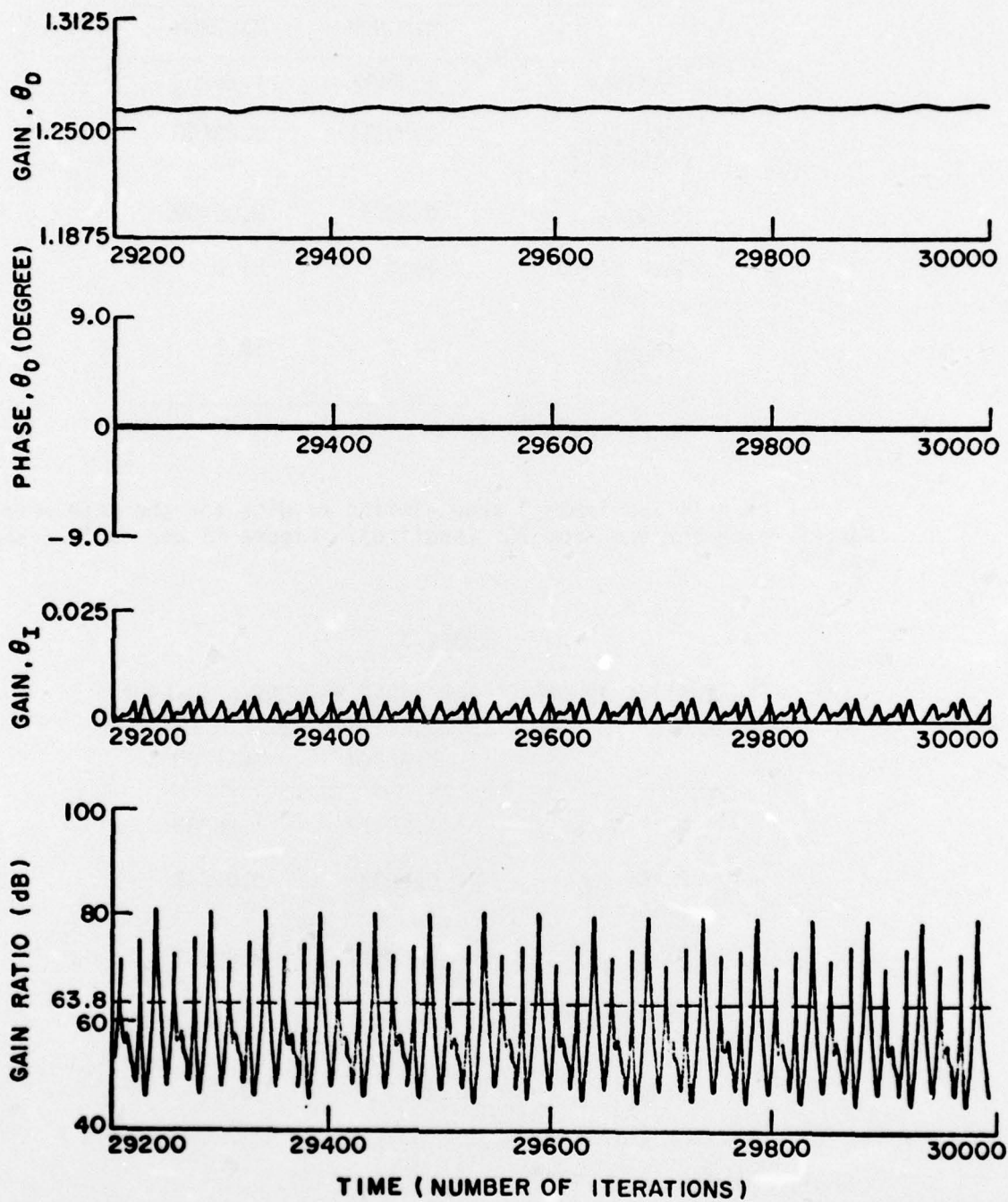


Fig. 13. Steady state array gains and phase ($\theta_I=60^\circ$).

TABLE 2
 FLUCTUATION BOUNDS OF THE ARRAY RESPONSES ($\theta_I=60^\circ$)

	MINIMUM	MAXIMUM
GAIN, θ_D	1.26086	1.26203
PHASE, θ_D (DEGREES)	0.00360	0.08640
GAIN, θ_I	0.00011	0.00459
GAIN RATIO (DB)	46.6	81.0
SIR _{OUT} (DB)	24.3	58.7

Note: $\theta_I=60^\circ$

Figure 14 and Table 3 show similar results for the case where the interference arrives from 90° (endfire). Figure 15 and Table 4 shows $\theta_I=30^\circ$.

TABLE 3
 FLUCTUATION BOUNDS OF THE ARRAY RESPONSES ($\theta_I=90^\circ$)

	MINIMUM	MAXIMUM
GAIN, θ_D	1.26172	1.26283
PHASE, θ_D (DEGREE)	0.00000	0.00000
GAIN, θ_I	0.00001	0.00591
GAIN RATIO (DB)	46.6	100.0
SIR _{OUT} (DB)	22.3	77.7

Note: $\theta_I = 90^\circ$

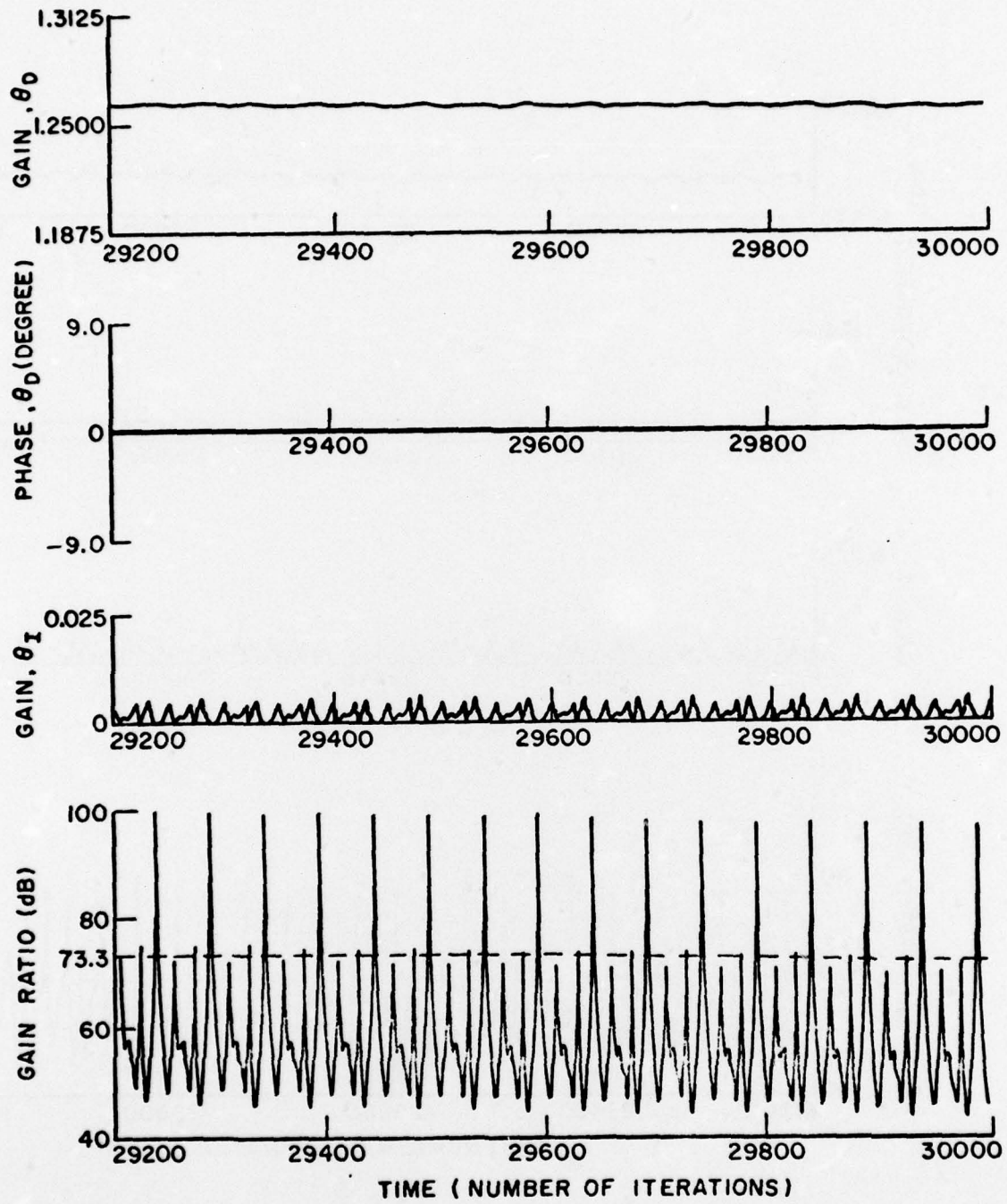


Fig. 14. Steady state gains and phase ($\theta_1=90^\circ$).

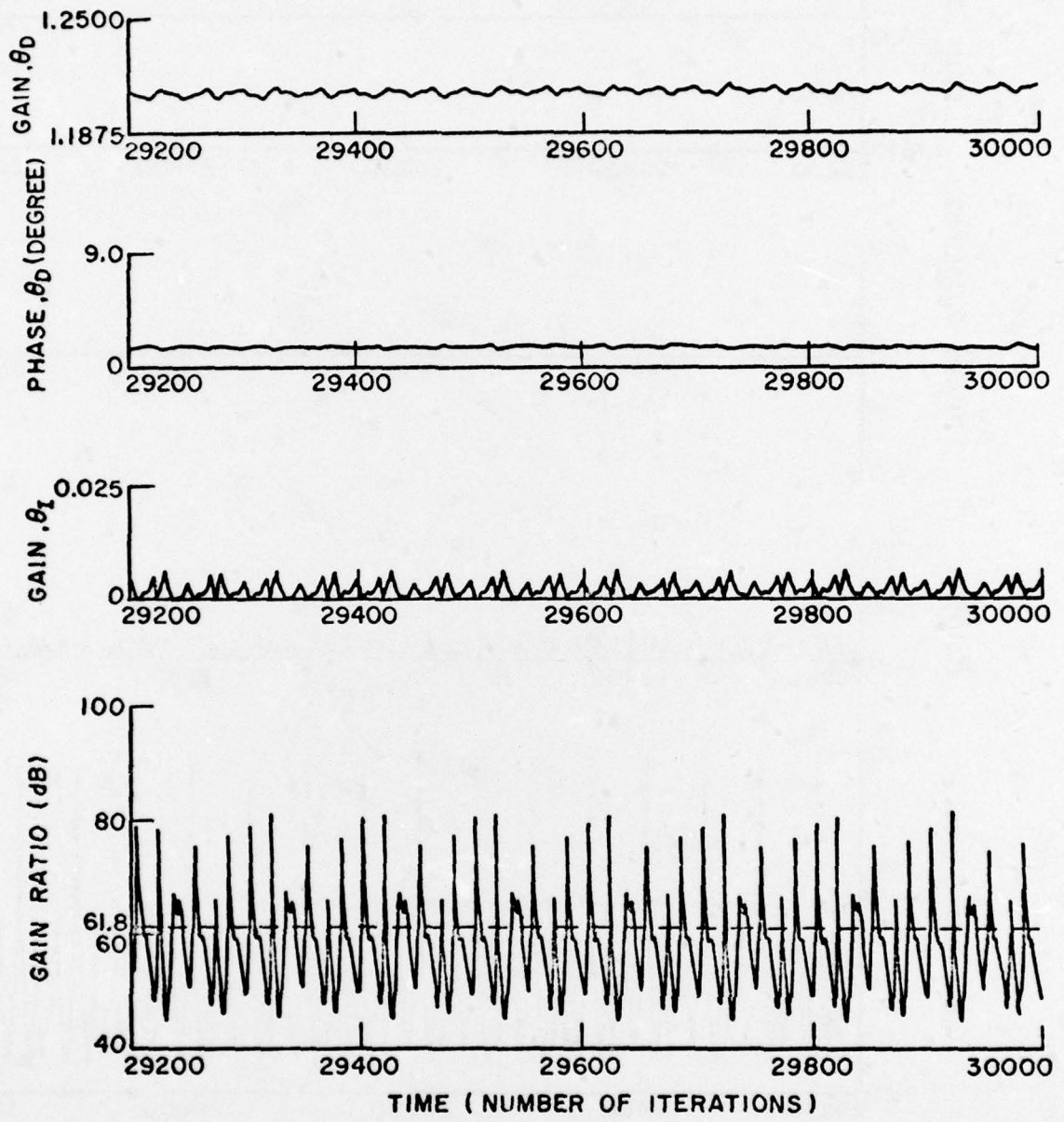


Fig. 15. Steady state gain and phase ($\theta_I=30^\circ$).

TABLE 4
 FLUCTUATION BOUNDS OF THE ARRAY RESPONSES ($\theta_I = 30^\circ$)

	MINIMUM	MAXIMUM
GAIN, θ_D	1.21039	1.21627
PHASE, θ_D (DEGREE)	1.404	1.68840
GAIN, θ_I	0.00016	0.00512
GAIN RATIO (DB)	46.2	77.4
SIR _{OUT} (DB)	23.9	55.1

Note: $\theta_I = 30^\circ$

Finally, Fig. 16 shows a curve of the gain ratio and the array output signal-to-interference ratio as a function of the interference arrival angle θ_I . The value plotted is the average of the minimum and maximum values obtained during the weight fluctuation cycle.

VIII. SUMMARY AND CONCLUSIONS

In this report a technique for integrating adaptive arrays into conventional AM communication systems has been discussed. This technique utilizes an extra biphase modulation on the desired signal, in addition to the AM. The biphase modulation is derived from a pseudonoise code. The reference signal for the adaptive array consists of a constant envelope signal biphase modulated with the same PN code. A method for locking and tracking the code is available.

The signals described here would be transmitted in the same bandwidth as conventional AM signals and hence would be compatible with existing AM systems. Inclusion of the biphase modulation on the AM signal produces some envelope distortion, but this distortion does not appear to be serious if the code modulation frequency is chosen near the low frequency end of the audio band.

Digital simulations of such a system have been done and a few of the results are shown in this report. These results indicate that the array will provide suitable interference protection with such signals. A companion report[16] containing extensive simulation results and showing the effects of various system parameters on performance is currently in preparation.

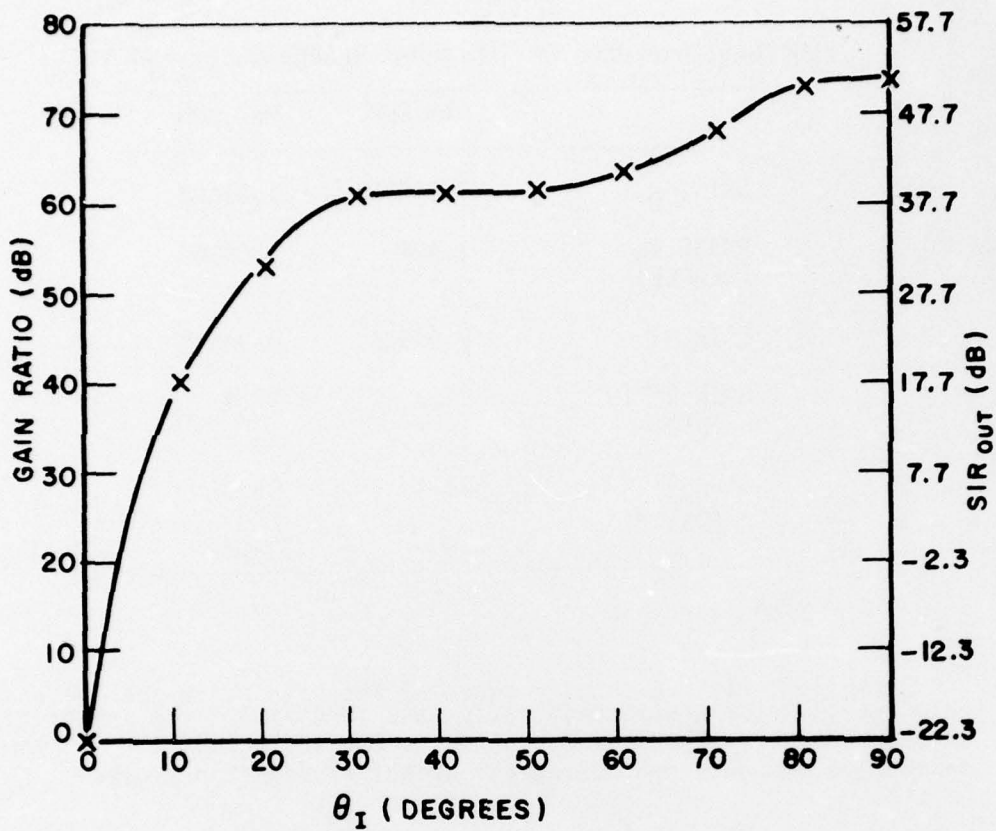


Fig. 16. Gain ratio and SIR_{OUT} vs θ_I .

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