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PHOTOMETRIC REDUCTIONS: THEORY AND PRACTICE

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ABSTRACT

This report discusses sundry topics in the theory and practice of the reduction of photometric data. The topics included are monochromatic extinction, wide-band extinction with all second-order terms, transformations from the instrumental system to the standard system, error analysis, and observing procedures. In addition, the approximate computation of an artificial satellite's standard magnitude from extremely wide-band data is discussed.

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I. INTRODUCTION

This note is concerned with some aspects of photometric reductions necessary to obtain accurate, internally consistent, magnitudes and colors. A large number of time dependent, temperature dependent, azimuthally dependent, etc., problems are not treated here. These effects are usually small ($\simeq 0^m.01$). Furthermore, problems peculiar to artificial satellites are not discussed. The remaining areas considered represent those topics capable of a theoretical discussion and a mathematical formulation. Except for Sec. VI B this note represents a review and summary of the relevant astronomical literature.

In each instance the theoretical discussion is presented first and then the working equations. Several simple examples are presented in great detail so that either analytical techniques or systematic errors can be clearly delineated.

II. NOTATION AND DEFINITIONS

$A(\lambda)$ = transmission of the atmosphere = $\mathrm{dex}[-0.4X\varepsilon(\lambda)]$

c, C = colors inside the atmosphere

c_o, C_o = colors outside the atmosphere

d = constant depending on units, telescope aperture, etc.

D = photometric deflection = $d \int_o^\infty R(\lambda)S(\lambda)d\lambda$

$E(\lambda)$ = emitted energy distribution

$F(\lambda)$ = filter transmission curve

$G = d\ell nE(\lambda)/d\ell n\lambda$

h = height above earth's surface

H = height of top of atmosphere

$\mathcal{H}$ = scale height of the atmosphere

m, M = apparent magnitudes inside the atmosphere

= $-2.5\ \ell og(D)$ + constant

m_o, M_o = apparent magnitudes outside the atmosphere

$n = -d\ell n\varepsilon(\lambda)/d\ell n(\lambda)$

$P(\lambda)$ = photometer response function

R_e = radius of the earth

$R(\lambda)$ = instrumental response function = $F(\lambda)P(\lambda)T(\lambda)$;

bandwidth = $\int_o^\infty R(\lambda)d\lambda$

$S(\lambda)$ = source function = $A(\lambda)E(\lambda)$

$T(\lambda)$ = telescope transmission function

$$W = \int_o^\infty (\lambda - \lambda_o)^2 R(\lambda)d\lambda / \int_o^\infty R(\lambda)d\lambda$$

x, X = air mass

$z(z')$ = true (apparent) zenith distance

ε, $\varepsilon(\lambda)$ = atmospheric extinction measured in magnitudes

λ_o = effective wavelength = $\int_o^\infty \lambda R(\lambda) d\lambda / \int_o^\infty R(\lambda) d\lambda$

$\rho(h)$ = atmospheric density

III. PRELIMINARIES

Before we can intelligently deal with the deflection of the photometer we must understand what happens to a beam of light between the time it enters the earth's atmosphere and the time our photometer records a deflection. To this end we first consider which quantities the deflection depends on and then we describe the atmospheric extinction.

A. The Fundamental Equation of Photometry

The ultimate observational datum of photoelectric photometry is the meter deflection D. The photometer has converted, according to its response curve $P(\lambda)$, the light energy falling on the photocathode into an electric current. The light reaching the photocathode has, most recently, passed through a filter with transmission $F(\lambda)$, a telescope with transmission $T(\lambda)$, and through the atmosphere with transmission $A(\lambda)$. Since it originally had $E(\lambda)$ photons at wavelength λ it follows that

$$D = d\int_0^\infty E(\lambda)A(\lambda)T(\lambda)F(\lambda)P(\lambda)d\lambda = d\int_0^\infty E(\lambda)A(\lambda)R(\lambda)d\lambda. \qquad (1)$$

The constant d is fixed once the unit of length, the telescope aperture, etc., are determined. As far as the instrumental system (photometer, filter, and telescope) is concerned the effective source had a distribution

$$S(\lambda) = E(\lambda)A(\lambda).$$

Astronomical magnitude scales are determined by the logarthmic response of the human eye (Pogson's Law) and history. Likewise magnitudes are related to the logarithm of the deflection. The proportionality constant is -2.5 and the zero of the scale is arbitrary. Hence,

$$m = -2.5 \,\ell og(D/d) + \text{constant}. \tag{2}$$

Once the instrumental system is fixed the additive constant is too.

B. Air Mass and Extinction

Consider the passage of a beam of light through the atmosphere. Let the beam have intensity $I(\lambda, h)$ at wavelength λ and height h above the earth's surface. As the beam passes through a thickness of atmosphere ds it is reduced by the amount dI where

$$dI(\lambda, h) = -\kappa(\lambda, h)I(\lambda, h)ds. \tag{3}$$

The function $\kappa(\lambda, h)$ is the atmospheric opacity. If the atmosphere is plane-parallel then ds = seczdh where z is the zenith distance of the incoming beams (see Fig. 1a). With this expression for ds we can solve for I at the ground (h = 0), viz,

$$I(\lambda, 0) = I(\lambda, H)\exp[-\sec z \int_0^H \kappa(\lambda, h)dh], \tag{4}$$

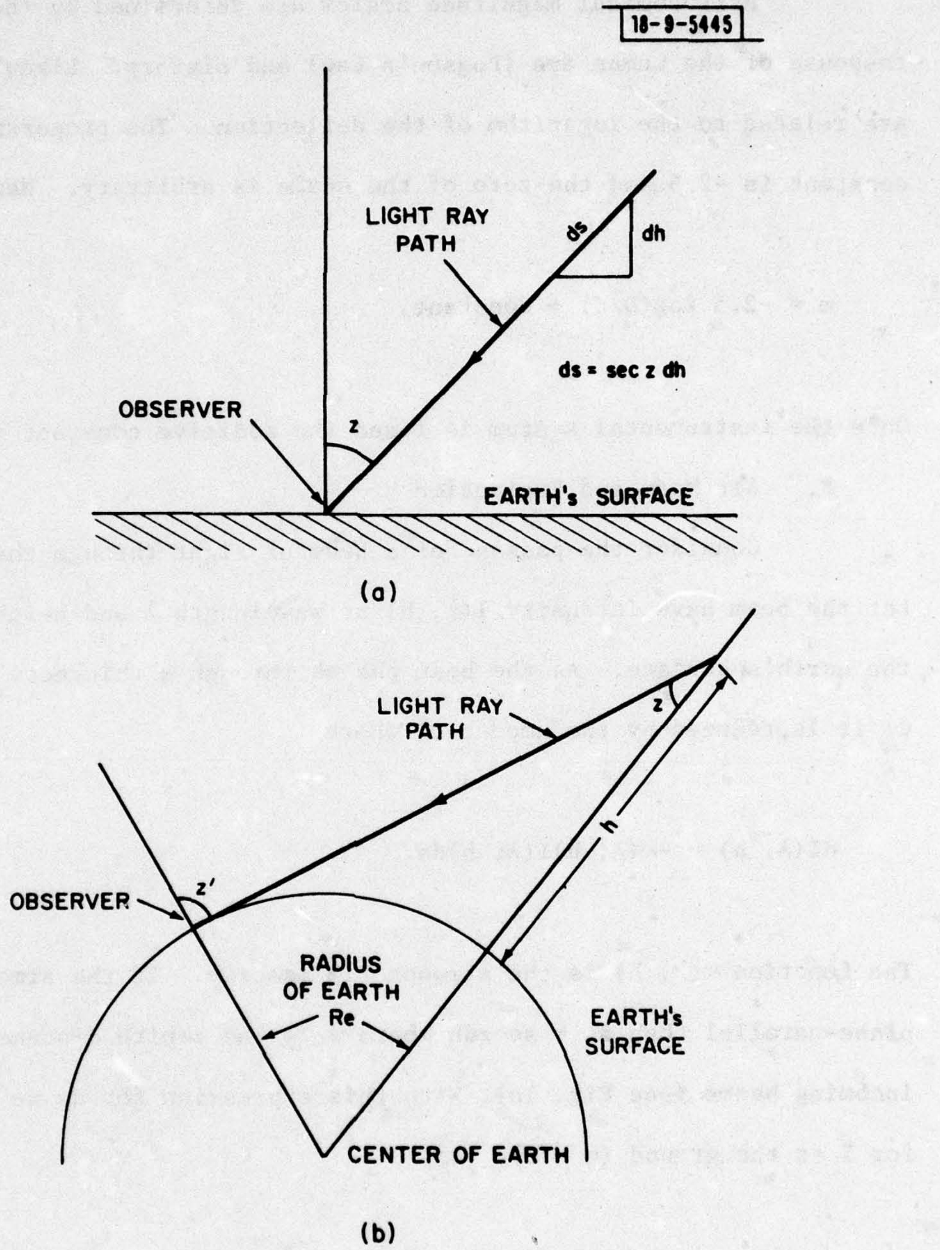

Fig. 1. (a) Path of light ray in plane-parallel atmosphere, (b) Diagram for calculation of air mass, not to scale.

where $\rho(h) = 0$ if $h \geq H$. If $R(\lambda) = r\delta(\lambda - \Lambda)$ then the relation between the inside the atmosphere (m) and outside the atmosphere (m_o) monochromatic magnitudes is

$$
\begin{aligned}
m_o - m &= -2.5(\ell oge)\sec z \int_o^H \kappa(\Lambda, h)dh, \\
&= -1.086\sec z \int_o^H \kappa(\Lambda, h)dh, \\
&= -\varepsilon(\Lambda)\sec z. \qquad (5)
\end{aligned}
$$

The quantity $\varepsilon(\Lambda)$ is the monochromatic extinction at wavelength $\lambda = \Lambda$. The more familiar version of Eq. (5) is

$$m = m_o + \varepsilon X \qquad (6)$$

where X is the air mass.

The expression "air mass" for X refers to the fact that X is the total path length, measured in units relative to the zenith (where X = 1), that the light beam must pass through. In general, $X \propto \int_o^H \kappa(\lambda, h)\sec z dh$. The opacity can be decomposed into a product of the opacity per gram, a, and the density, ρ. Thus,

$$X \propto a(\lambda) \int_o^H \rho(h)\sec z dh. \qquad (7)$$

From Fig. 1b we have, by the law of sines,

$$\sin z/R_e = \sin z'/(R_e + h), \quad (8)$$

so

$$X \propto a(\lambda) \int_0^H \rho(h)(1 + h/R_e)dh/[\cos^2 z' + 2h/R_e + (h/R_e)^2]^{1/2}. \quad (9)$$

As long as the scale height of the atmosphere, $\mathcal{H}$, is small compared to R_e, then the integral in Eq. (9) can be approximated by

$$X \propto a(\lambda)\rho(0)\sec z'[1- (\mathcal{H}/R_e)\sec^2 z'], \quad (10)$$

whence,

$$X = \sec z'[1- (\mathcal{H}/R_e)\sec^2 z']. \quad (11)$$

Equation (11) demonstrates two things. First, it is the apparent zenith distance, not the true zenith distance, that determines X. Second, X is not a linear function of sec z'. However, Eq. (11) is not yet accurate because the curvature of the ray path due to refraction has been neglected and Eq. (7) assumes the atmosphere to be well mixed. The latter assumption is demonstrably false for aerosols, ozone, water vapor, etc., which contribute to the opacity by means other than Rayleigh scattering. Nonetheless, tables of X(z') (cf. Hardie 1962) are used because there is no practical alternative. One should not use them beyond $z' \simeq 75^\circ$ (i.e., $X \simeq 4$).

IV. MONOCHROMATIC EXTINCTION

The term monochromatic is used here to mean $F(\lambda) = F\delta(\lambda - \Lambda)$ where $\delta(u)$ is the Dirac delta function. Although this is an idealized case, it allows us to formulate and solve some problems which would be too complex to handle otherwise. The treatment of monochromatic extinction also lays the groundwork for the more complex wide-band case. Hence, it introduces the methods (but not reduction procedures) necessary in the ab initio construction of photometric systems. The best photometric system for artificial satellites will be discussed elsewhere.

A. The High-Low Method

This represents the simplest procedure and does not assume that the extra-atmospheric magnitude is known. Moreover, it is necessary to observe only a single star. Let the star be observed at air masses $X = X_1, X_2$ with corresponding magnitudes $m = m_1, m_2$. Since,

$$m_j = m_o + \varepsilon X_j, \ j = 1, 2, \tag{12}$$

clearly

$$\varepsilon = (m_1 - m_2)/(X_1 - X_2). \tag{13}$$

If $|X_1 - X_2|$ is small then $|m_1 - m_2|$ will be also. Hence, the accuracy with which ε is known would be poor. On the other hand, a large difference in the air masses implies several hours have elapsed between the two observations.

Therefore, problems associated with instrumental drift, time varying extinction, etc., are encountered. Ignoring these difficulties, it is still not wise to maximize $|X_1 - X_2|$ because we want to minimize σ_ε, the estimate for the standard deviation of the extinction.

Let us expand upon this point further. It is a common (and incorrect) practice to assign equal weights to all observations when determining ε. However, it is known that the standard error of a measurement depends on the air mass. Stock (1968) has found a quadratic air mass dependence. Thus, if σ_1 is the standard error of a measurement made at the zenith (X = 1), the weight for a measurement made at any other air mass will be

$$w \propto 1/\sigma^2 = (\sigma_1 X^2)^{-2}. \tag{14}$$

Consider the case $X_1 = 1$, $X_2 = X$. Then,

$$\sigma_\varepsilon^2 = \sigma_1^2 (X^4 + 1)/(X - 1)^2. \tag{15}$$

The extinction will be determined most accurately when σ_ε is a minimum. This implies $X^4 - 2X^3 - 1 = 0$ or $X \simeq 2.10692$. Thus, if X = secz, the off zenith position corresponds to $z = 61^\circ 40'$ and $\sigma_\varepsilon = 4.111\sigma_1$. The effect of choosing another air mass can be seen in Fig. 2 where $(\sigma_\varepsilon/\sigma_1)^2$ is plotted versus X.

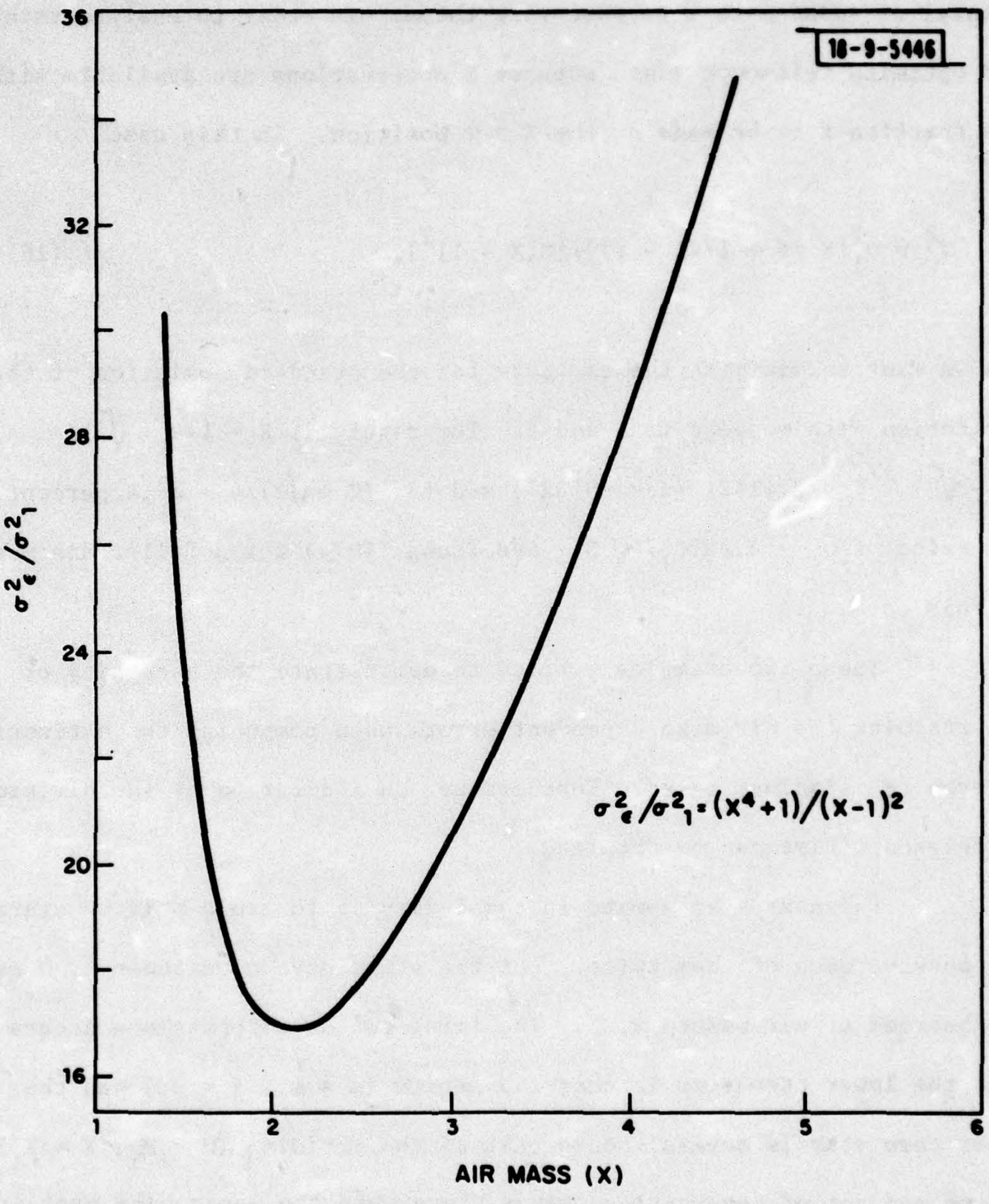

Fig. 2. Variance of extinction coefficient at air mass X relative to zenith (X = 1) assuming weights proportional to $1/X^2$.

As large air mass observations carry low weight, it would be natural to spend more time observing there. In order to analyze this, and optimize telescope time, suppose N observations are available with the fraction f to be made at the X > 1 position. In this case

$$\sigma_\varepsilon^2 = \sigma_1^2[X^4/f + 1/(1 - f)]/[N(X - 1)^2], \tag{16}$$

and we want to minimize the estimate for the standard deviation of the extinction with respect to X and f. The result is $X = [(2 + \sqrt{2})/(2 - \sqrt{2})]^{1/2} \simeq 2.41421$ ($z = 65^\circ 32'$) and $f = (2 + \sqrt{2})/4 = 85.4$ percent. The value of $\sigma_\varepsilon = 4.828\sigma_1/\sqrt{N}$. See Young (1974) for a fuller discussion of this case.

These two examples suffice to demonstrate the necessity of ascertaining the air mass dependent errors when computing the extinction in even the simplest cases. Furthermore, an indication of the division of telescope time can be obtained.

The next step upward in complexity is to use a pair of stars and observe each of them twice. Let the stars have magnitudes m, M and be observed at air masses x, X. The first set of observations occurs when the lower case star is near the zenith ($m = m_1$, $x = x_1$) and the upper case star is several hours east of the meridian ($M = M_1$, $X = X_1$). The second set of observations takes place when the upper case star is near the zenith (M_2, X_2) and, consequently, the lower case star is several hours west of the meridian (m_2, x_2). Then,

$$\varepsilon = [(m_1 - M_1) - (m_2 - M_2)/[(x_1 - X_1) - (x_2 - X_2). \quad (17)$$

This procedure has the advantage of eliminating time dependent effects in the telescope-photometer system because the individual observations within a set are nearly simultaneous. Also, the extra-atmospheric magnitudes m_o, M_o need not be known.

Once more than two stars are used to determine ε an algebraic solution is no longer possible and least squares must be used.

B. Simultaneous Standard Stars

Let us suppose that several stars are chosen to be standards and observed over enough nights to determine good values for m_o. Good means that their extra-atmospheric magnitudes have been calculated sufficiently accurately to rule out the possibility of variability. Then, to determine the extinction, we observe several of these stars over a range of air masses. We now want to minimize

$$\sum w[m - m_o - \varepsilon X]^2 / \sum w$$

with respect to ε. The w's are the air mass dependent weights.

However, if the extra-atmospheric magnitudes are unknown, iterative procedures are available (e.g., Weaver 1952) to simultaneously determine both ε and $\{m_o\}$. For this purpose observations are performed over several nights on the same set of stars. Of course, unless the absolute sensitivity of the equipment can be established by independent means, nightly zero point corrections must be included. Thus, for the j'th observation on the k'th night of a star

$$m_{jk} = m_o + \varepsilon_k X_{jk} + Z_k. \quad (18)$$

C. Monochromatic Colors

In astronomical photometry a color (e.g., color index) is the difference between two magnitudes. That is, if the two monochromatic magnitudes of a star are m, M then c = m - M is a color. From Eq. (4) and the definition of c it follows that

$$c = c_o - \varepsilon_c X. \quad (19)$$

Hence, a high-low method can be used to determine ε_c. We note that sensitivity drift does not affect ε_c. Usually m refers to a shorter wavelength than M.

V. WIDE-BAND EXTINCTION

When the instrumental response function, $R(\lambda)$, has a bandwidth ($\geq$ 100 Å) sufficiently large that the extinction varies across it, the determination of the effective extinction is more complicated. It is obvious from the appearance of the sun at noon and sunset that $\varepsilon(\lambda_{blue}) > \varepsilon(\lambda_{red})$. Hence, the use of a constant ε in Eq. (6) introduces two different errors. The first is, because $d\varepsilon/d\lambda \neq 0$, a blue star suffers more extinction than a red one (Stebbins, et al. 1950, Eggen 1950). The second is, that since $X \neq \sec z'$, as a star's altitude decreases it becomes redder and the effective extinction becomes smaller (Forbes 1842). To ascertain the importance of these complications we follow King (1952) and correctly calculate, through terms of the second-order, the analog of Eq. (6).

A. Theory

The photometer deflection, D, is given by

$$D = d \int_0^\infty S(\lambda) R(\lambda) d\lambda. \tag{20}$$

Since $R(\lambda)$ peaks at some wavelength near $\lambda = \lambda_o$ let us expand S about λ_o.

$$\begin{aligned} D &\simeq d \int_0^\infty [S(\lambda_o) + (\lambda - \lambda_o)S'(\lambda_o) + (\lambda - \lambda_o)^2 S''(\lambda_o)/2] R(\lambda) d\lambda, \\ &= d[S_o + W\lambda_o^2 S_o''/2] \int_0^\infty R(\lambda) d\lambda, \\ &= dE_o A_o [1 + W\lambda_o^2 S_o''/2S_o] \int_0^\infty R(\lambda) d\lambda, \end{aligned} \tag{21}$$

where $S_o = S(\lambda_o)$ etc. The under the atmospheric magnitude $m = -2.5 \log(D/d)$ plus constant and the magnitude outside the atmosphere, m_o, is given by a similar expression with $A(\lambda) = 1$. Hence,

$$m = m_o - 2.5 \log A_o - 2.5 \log\{[1 + W\lambda_o^2 S_o''/2S_o]/[1 + W\lambda_o^2 E_o''/2E_o]\},$$
$$\simeq m_o + \varepsilon_o X - 1.25W\lambda_o^2(\log e)[S_o''/S_o - E_o''/E_o], \tag{22}$$

where $\varepsilon_o X = -2.5 \log A_o$. Since $S = EA$ this can be rewritten as

$$m - m_o = \varepsilon_o X[1 - n_o GW + n_o(n_o + 1)W/2 - W\lambda_o n_o'/2]$$
$$- W\ln 10[n_o \varepsilon_o X]^2/5. \tag{23}$$

The various terms on the right hand side of Eq. (23) represent (i) the monochromatic extinction evaluated at the effective wavelength, (ii) a correction which depends on the color of the star through the gradient of its energy distribution, (iii) a second correction, present for $E(\lambda)$ = constant, which depends on the color of the atmosphere through the gradient of its absorption, (iv) a third air mass dependent correction which depends on the square of the atmospheric color, and (v) a non-linear term in εX (e.g., the Forbes effect). In order to establish (heuristically) that G depends on the stellar color we approximate $E(\lambda)$ by the black body distribution.

The black body spectrum is

$$E(\lambda) = c_1\lambda^{-5}/[\exp(c_2/\lambda T) - 1], \tag{24}$$

where c_1 and c_2 are the first and second radiation constants and T is the absolute temperature. A straightforward calculation yields ($y = c_2/\lambda_o T$),

$$G = -5 + ye^y/(e^y - 1). \tag{25}$$

For the majority of the stars we observe the effective wavelength will be near the wavelength satisfying the Wien displacement law. Thus, $y \simeq 5$, $e^y >> 1$, and

$$G \simeq -5 + y. \tag{26}$$

The star's color, c_o, is given by $c_o = m_o(\lambda') - m_o(\lambda'')$. Thus,

$$\begin{aligned} c_o &\simeq -1.25W(\ell oge)[(\lambda')^2E''(\lambda')/E(\lambda') - (\lambda'')^2E''(\lambda'')/E(\lambda'')] \\ &\simeq -1.25W(\ell oge)[-12(y' - y'') + (y')^2 - (y'')^2] \\ &\simeq -30W(\ell oge)y_o\Delta\lambda/\lambda_o, \end{aligned} \tag{27}$$

where $\lambda' = \lambda_o + \Delta\lambda$, $\lambda'' = \lambda_o - \Delta\lambda$. Thus, comparing Eqs. (26, 27)

$$G \simeq a + bc_o, \tag{28}$$

since y_o, $\Delta\lambda$, and W are fixed.

For n = 4 (Rayleigh scattering), W = 10^{-2} and ε = $0\overset{m}{.}25$. The Forbes effect term varies by only $0\overset{m}{.}01$ as X varies from 1 to 3. If ε = $0\overset{m}{.}5$ then over the same range of air mass it changes by $0\overset{m}{.}15$. Hence, on a good night it represents no problem but on a poor night it results in a systematic error in m_o (or alternatively ε_o). The n_o' term is difficult to evaluate since it depends on those components of the atmosphere whose composition is the most variable. Thus, its systematic neglect will reappear in the large standard deviation of ε_o for stars of widely different color.

B. Practice

It is standard astronomical practice to replace the exact higher order terms in Eq. (23) with observable quantities. Thus, one replaces Eqs. (6, 19) by

$$m = m_o + (\varepsilon' + \varepsilon'' c_o)X, \tag{29a}$$

$$c = c_o + (\varepsilon_c' + \varepsilon_c'' c_o)X, \tag{29b}$$

where ε', ε_c' are referred to as the principal (or first-order) extinction coefficients and ε", ε_c'' are referred to as the second-order extinction coefficients. Sometimes the color index under the atmosphere, c, is used in place of c_o in Eqs. (29). From Eq. (23) we see that both the stellar energy distribution and the atmospheric absorption affect the effective extinction coefficients. Hence, if c_o is used in Eqs. (29) only the stellar component is represented whereas if c is used both the stellar and atmospheric components are included.

C. Determining the Extinction

Consider two stars, of different color, close together on the sky. Then if one star has lower case magnitudes and colors while the other star has upper case ones, we can write

$$m_o - M_o = m - M - \varepsilon''(c - C)X, \qquad (30a)$$

$$c_o - C_o = c - C - \varepsilon''_c(c - C)X. \qquad (30b)$$

Several measurements performed at different air masses allow the least squares determination of the second-order terms (with m_o, C_o, etc., unknown). Once these have been determined, an optical double suffices to compute the first-order terms by the high-low method. Finally, since the second-order coefficients vary (night to night) less than the principal coefficients do, an analog of the high-low method will quickly yield the first-order coefficients once the second-order ones are fixed.

VI. MAGNITUDE AND COLOR TRANSFORMATIONS

A. Theory and Practice

When one attempts to do photometry on a standard system one is faced with the problem of reducing one's own magnitudes and colors to the standard system. This is best accomplished by matching $R(\lambda)$, reduction procedures, observing procedures, etc., as closely as possible. In the end, the observer has a magnitude m and color c in his system that should be close to the corresponding standard quantities, M and C. From the analysis in Sec. V it can easily be shown that (for black bodies) the following linear transformations suffice:

$$M = m + \alpha c + \beta = m + AC + B, \quad (31a)$$

$$C = \gamma c + \delta. \quad (31b)$$

Clearly $\beta(B)$ and δ are zero-point constants and $\alpha(A)$ and γ are scale factors. The values of γ and α relative to unity and zero reflect the sizes of the two bandwidths and the relative position of their effective wavelengths. Thus, if $\gamma < 1$ the standard system has a narrower bandwidth than the instrumental system while if $\alpha < 0$ the effective wavelength of the standard system is redder than that of the instrumental system.

A mismatch in $R(\lambda)$ that is easy to deal with is when W remains constant but λ_o changes. Let $R(\lambda)$ be the instrumental response function of the standard system with effective wavelength Λ_o. Then if $r(\lambda)$ is the observer's response function with effective wavelength λ_o,

$$
\begin{aligned}
D &= d\int_0^\infty S(\lambda)r(\lambda)d\lambda \\
&= d\int_0^\infty [S(\lambda_o) + (\lambda - \lambda_o)S'(\lambda_o)]r(\lambda)d\lambda \\
&= dS_o[1 + (\Lambda_o - \lambda_o)S_o'/2S_o]\int_0^\infty r(\lambda)d\lambda \\
&\simeq dS_o\{1 + \frac{(\Lambda_o - \lambda_o)}{\lambda_o}[G + 0.4n_o\varepsilon_o X\ell n10]\}\int_0^\infty r(\lambda)d\lambda. \qquad (32)
\end{aligned}
$$

Hence,

$$
M = m - 2.5\frac{(\Lambda_o - \lambda_o)}{\lambda_o}[G + 0.4n_o\varepsilon_o X\ell n10], \qquad (33)
$$

and we already know [cf. Eq. (28)] G is a function of the color. The additional, air mass dependent, term varies over $0^m\!.11$ as X varies from 1 to 3 for a 100 Å mismatch at 5000 Å with Rayleigh scattering and $0^m\!.5$ extinction per air mass. This can dominate the color term and furthermore, it is clearly systematic in X.

Finally, we consider the correct least squares determination of the constants in Eqs. (31). We assume that the standard values are without error. Then one should use the regression line of c on C to determine γ and δ. Hence,

$$
\gamma = \sigma_C^2/\text{cov}(c, C) \neq \text{cov}(c, C)/\sigma_c^2, \qquad (34a)
$$

where $\sigma_C^2(\sigma_c^2)$ is the variance of C(c) in the sample and cov(c, C) is the covariance of c and C. The value of δ is calculated from

$$\delta = \langle C\rangle - \gamma\langle c\rangle, \tag{34b}$$

where angular brackets denote a sample mean. When computing A and B it must be recognized that, since C depends M, the errors in M and C are correlated. Hence,

$$A = [\sigma_M^2 - \mathrm{cov}(m, M)]/\mathrm{cov}(M, C) \neq \mathrm{cov}(M - m, C)/\sigma_C^2, \tag{35a}$$

while

$$B = \langle M\rangle - \langle m\rangle - A\langle C\rangle. \tag{35b}$$

It would be simple to obtain estimates for the standard deviations of these constants and, thence, formal estimates for the errors in M and C. It is, however, more meaningful to observe the same stars on several nights, reduce each night separately, and then calculate the internal variance of M and C. These quantities are frequently used as estimates of the external errors of the transformations.

B. S-20 Transformations

The principal artificial satellite detector for GEODSS is an S-20 photocathode. To maximize the signal-to-noise ratio, and hence the ability to confidently detect faint satellites, one does not want to use filters in the light path. Nonetheless, for some considerations, it would be advantageous if at least a magnitude could be reported, in a standard astronomical system, for satellites. We therefore construct two analogs of Eq. (31a). The first observes that the S-20 bandwidth is so large ($\lambda_o = 4850$ Å, $\int_0^\infty R(\lambda)d\lambda = 0.04628$, $\sqrt{W} = 1094$ Å for the S-20 alone), Eq. (31a) is extended by adding more colors. The standard, common, astronomical system that spans the S-20 response region is the UBVRI system. Furthermore, the effective wavelength of the B filter (4420 Å) is closest to that of the S-20. Thus, we write

$$B = m_{20} + a_u(U - B) + a_v(V - B) + a_r(R - B) + a_i(I - B) + a_z . \quad (36)$$

In order to calculate the constants in Eq. (36) the following procedure would be followed: (i) Assume that the colors of artificial satellites are nearly those of the sun [e.g., $(U - B)_\odot = 0\overset{m}{.}13$, $(V - B)_\odot = -0\overset{m}{.}65$, $(R - B)_\odot = -1\overset{m}{.}17$, $(I - B)_\odot = -1\overset{m}{.}46$], (ii) determine, by the reduction procedures described in Sec. V, m_{20} values for stars with similar colors, (iii) perform a least square fit to determine $a_u - a_z$. The quantity W_B,

$$W_B = |(U - B) - (U - B)_\odot| + |(V - B) - (V - B)_\odot| + |(R - B) - (R - B)_\odot| + |(I - B) - (I - B)_\odot| , \quad (37)$$

usefully discriminates against non-solar color stars. Table 1 contains a list of stars (labeled by their number in the Bright Star Catalog) which have $W_B \leq 0\overset{m}{.}5$. Furthermore, when performing the least square

TABLE 1

STARS WITH NEARLY SOLAR TYPE SPECTRAL DISTRIBUTIONS

B. S. #	$100W_V$	$100W_B$	B. S. #	$100W_V$	$100W_B$
77	30	33	4707	24	68
98	13	10	4708	34	39
219	32	29	4785	26	20
244	32	44	4845	40	42
321	38	50	4833	23	25
370	24	23	4979	51	50
424	29	44	4983	24	31
458	41	52	5019	44	50
483	14	9	5235	23	44
618	57	45	5260	22	31
641	39	38	5304	44	55
788	14	22	5338	44	58
937	14	15	5384	20	14
996	29	32	5409	46	55
1017	36	87	5868	16	17
1101	24	32	5914	34	42
1242	46	73	5986	47	60
1674	50	63	6098	53	37
1729	11	6	6978	35	39
2047	19	23	7172	49	61
2693	49	49	7503	17	20
2906	38	52	7504	21	18
3064	24	19	7796	50	48
3391	16	13	7955	36	47
3591	43	43	8131	27	66
3871	30	75	8170	42	54
3881	12	13	8334	41	28
4112	49	62	8729	20	22
4540	31	41	8737	12	11
			8817	36	35
			8905	14	8

fitting, a weighting $\propto$ 1/(monotonic function of W_B) would be appropriate. A standard G2 V star has $W_B = 0^m.13$.

There is no doubt about the success of this procedure. However, unless max $(|a_u|, |a_v|, |a_r|, |a_i|) \simeq 10^{-2}$ it will not yield useful information. Finally, if V were desired one would construct a W_V analogously. Table 1 also includes stars for which $W_V \leq 0^m.5$.

Since Eq. (36) is an extension of a relationship for black bodies, a more fundamental approach may yield better results. In terms of extra-atmospheric values,

$$D_{20} = d \int_0^\infty E(\lambda) P_{20}(\lambda) T(\lambda) d\lambda, \tag{38}$$

$$D_b = d \int_0^\infty E(\lambda) F_b(\lambda) P_{20}(\lambda) T(\lambda) d\lambda, \text{ etc.} \tag{39}$$

Thus, we can regard D_{20} as given by a five-point quadrature formula,

$$D_{20} = A_u D_u + A_b D_b + A_v D_v + A_r D_r + A_i D_i. \tag{40}$$

This implies

$$\begin{aligned} B = m_{20} + b_z - 2.5\ \ell og\{ & b_u \operatorname{dex}[-0.4(U - B)] + b_b \\ & + b_v \operatorname{dex}[-0.4(V - B)] + b_r \operatorname{dex}[-0.4(R - B)] \\ & + b_i \operatorname{dex}[-0.4(I - B)]\}. \end{aligned} \tag{41}$$

The procedure outlined above would again be followed but the least square problem is more complicated. Moreover, the systematic accuracy of the B magnitudes now depends not only on small values for $|b_u|$, etc., but all the colors as well.

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This report discusses sundry topics in the theory and practice of the reduction of photometric data. The topics included are monochromatic extinction, wide-band extinction with all second-order terms, transformations from the instrumental system to the standard system, error analysis, and observing procedures. In addition, the approximate computation of an artificial satellite's standard magnitude from extremely wide-band data is discussed.

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