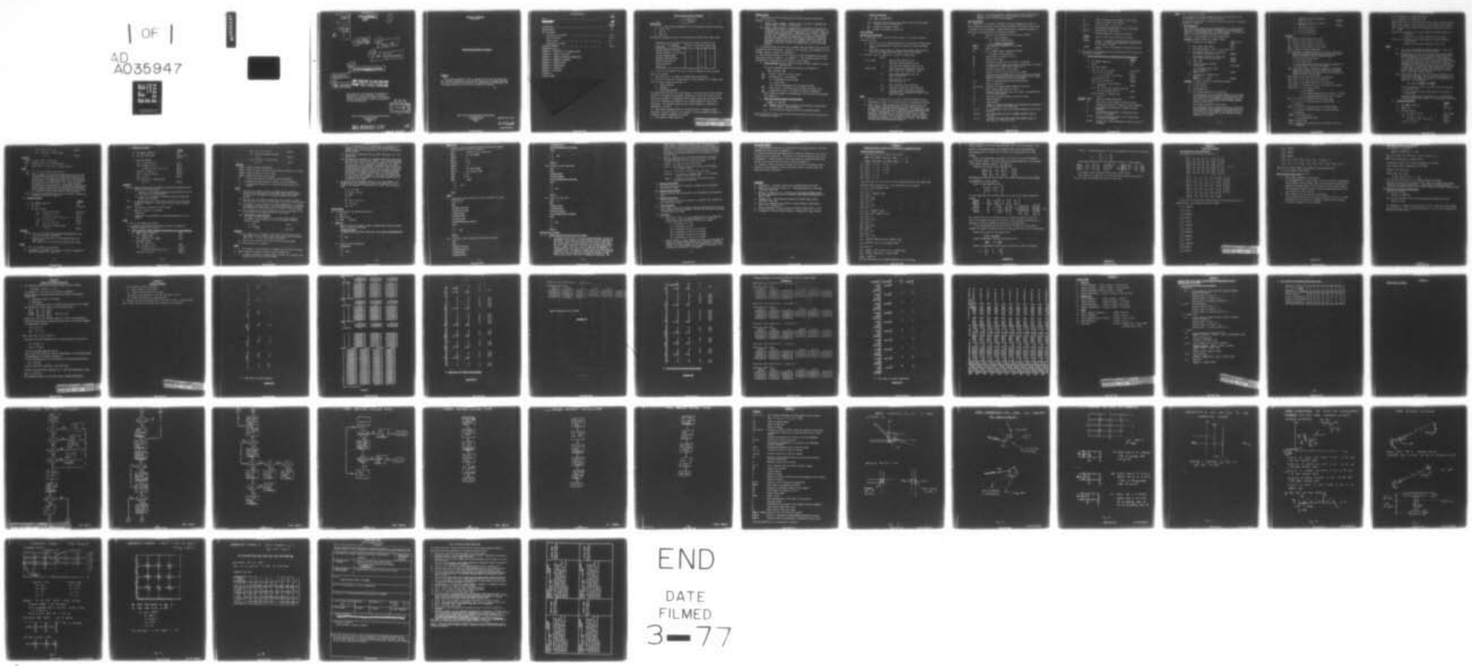
AD-A035 947   NAVAL CONSTRUCTION RESEARCH ESTABLISHMENT DUNFERMLIN--ETC F/G 20/11
PLANE GRILLAGE SUITE OF PROGRAMS,(U)
AUG 76 W J SIMONS

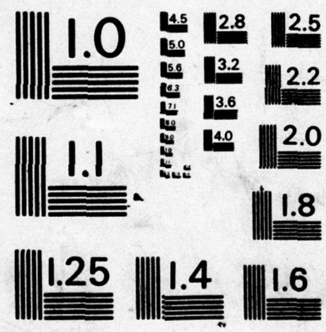
UNCLASSIFIED   NCRE/R637   DRIC-BR-55362   NL

| 1 OF 1 |

AD
A035947



END
DATE
FILMED
3-77



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

UNLIMITED

REPORT NO NCRE/R637 (14)

(1)

(12) 58p.

(11) Aug 76

(10) W. J. Simons

(6) PLANE GRILLAGE SUITE OF PROGRAMS (Item 6A2)

(18) DRIC

(19) BR-55362

Naval Construction Research Establishment
St Leonard's Hill
Dunfermline
Fife

LB

248 200

REPORT NO NCRE/R637
(Item 6A2)

PLANE GRILLAGE SUITE OF PROGRAMS

ABSTRACT

The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

Naval Construction Research Establishment
St Leonard's Hill
Dunfermline
Fife.

Approved for Issue

J. F. Coates

Superintendent

CONTENTS

PLANE GRILLAGE SUITE OF PROGRAMS

by

W J Simons

INTRODUCTION

1. A suite of plane grillage programs has been implemented on three computers:-

   a. UNIVAC 1108
   b. KDF - 9
   c. IBM 1130

The features available on each of the computers are shown in the table below.

Features \ Computer	UNIVAC 1108	KDF-9	IBM 1130
Stress Analysis (in-core)	✓	✓	✓
Stress Analysis (out of core)		✓	
Lowest Buckling Mode	✓	✓	✓
General Buckling Modes	✓		✓
Natural Frequencies	✓		✓
Data Generation		✓	✓
Data Plot		✓	

2. On all of the computers full use is made of symmetry to enable problem size to be reduced.

3. The main text of the report is divided into two sections.

4. The first section of the report gives a brief summary of the methods used for the various solutions required, ie

   a. Stress Analysis
   b. Buckling
   c. Natural Frequencies

The second deals with the use of the program. This gives details of the ideal-isation of the structure considered, preparation of the data for each of the facilities available, a description of the control cards that are required for each of the computers and details of the output produced for each of the facilities. The detailed theory for the various analyses is contained in Appendices A, B and C.

5. This report supersedes R481 by S B Kendrick and J L McKeeman, which describes an earlier version of the program. It was originally developed on a Pegasus computer programmed in autocode.

6. The program is NCRE number N21.

PRECEDING PAGE BLANK-NOT FILMED

GENERAL METHOD

7. A plane grillage can be idealised into two basic types of finite beam elements. These are:-

i. Double - ended elements - characterised by the need to consider the displacements, or forces, at both ends.

ii. Single - ended elements - characterised by the need to consider the displacements or forces at only one end (normally the second end is at a boundary and so has prescribed deflections and stresses). Where two or more of these elements meet, a reference point is chosen, called a hub position. Normally the finite size of the beam cross-section is ignored, and in this case the deflections and rotations of the beam are equal to those at the hub. The program, though also allows for the general case when the beams are separated by a rigid area, (see figure 1) or when one or more of the beams axes do not pass through the hub.

8. A stiffness matrix for each beam is formed using Timoshenko beam theory (1)* allowing for shear deflections. Torsion is also included. The formation of this stiffness matrix is detailed in Appendix A.

9. The stiffness matrices for the elements are then transformed to the global coordinate system and combined. See Appendix A. The resulting matrix is used (and modified if necessary) by each feature of the program

a. Stress Analysis (with or without in-plane end loads coupled to initial deformations)

The set of equations built up and solved by the computer is

$$\{F\} = [A]\{\delta\} + \{B\}$$

$[A]$ - overall structure stiffness matrix

$\{B\}$ - loading matrix - can include:-

i. distributed pressure loads

ii. point loads

iii. initial deformations plus in-plane end loads

$\{F\}$ - force vector - externally applied moments and shear force

$\{\delta\}$ - displacement vector - rotations and normal displacement

This equation is solved for displacements, and stresses are calculated (see Appendix A)

b. Lowest Buckling and General Buckling Modes

$$[A]\{\delta\} = \lambda_1 [B_1]\{\delta\}$$

$[B_1]$ - matrix representing the relative distribution of axial forces in the element. (see Appendix B)

λ_1 - is the axial load factor at which the grillage buckles

*( ) = References on Page 16

c. Natural Frequencies

$$[A_1] \left\{\delta\right\} = (\lambda_2)^2 [M] \left\{\delta\right\}$$

$[A_1]$ - reduced stiffness matrix after Guyan reduction (5) has been applied to $[A]$ (see Appendix C)

$[M]$ - lumped mass diagonal matrix

λ_2 - natural frequency in radians/sec

USE OF PROGRAM

Idealising the Structure

10. The grillage must be idealised into a number of structural elements connected at hubs.

a. Each hub is given a number starting at 1 and numbered consecutively.

b. Double ended elements are numbered at each end using consecutive numbers.

c. Each single ended element is given a label number, which must be different from any other label number used for an element.

11. Different types of element are available as shown below.

	Type No	
Double ended	1	General beam between 2 hubs
	2	Beam with symmetry about one end
	3	Beam with anti symmetry about one end
Single ended	11	Beam elastically restrained at one end ($M_B = -a_1 V_B$, $T_B = -b_1 \tau_B$) $S_B = -c_1 W_B$)
	12	Beam simply supported at one end ($M_B = T_B = W_B = 0$)
	13	Beam clamped at one end ($V_B = \tau_B = W_B = 0$)
	14	Beam symmetrical about mid-point
	15	Beam anti symmetrical about mid-point
	29	A pillar support restraining transverse deflections but producing no moments at the point of support

NOTES

a. Type 11 - The values of a_1, b_1 and c_1 should be obtained from an approximate estimate of the flexibility of the supporting structure.

b. Types 2 and 3 - These elements are included to increase the size of the grillage that can be analysed, so that only a half or a quarter of the structure need be considered. The loading must also be symmetrical about any axis of symmetry and antisymmetrical about any axis of antisymmetry. In using these elements, it is only permissible for them to have one image element. Every element of the complete structure must be accounted for in the idealisation either as an element or as the image of an element. For examples of permissible idealisations see figure 3.

c. Type 29 - The element produces a normal force to the grillage but no moments. If the parameters for this element are set at zero the program assumes a rigid simple support.

Data Preparation

12. A great deal of the input is common to all the programs but because of minor variations individual specifications are given. The notation given below is common to all the programs. Special notation for an individual program is given with the notation for that particular program.

NT	parameter to determine the version of the program NT = 1:- stress analysis = 2:- lowest buckling mode = 3:- general buckling modes 4:- natural frequencies
NBEAM	total number of elements in the grillage
JOINT	number of hubs
Kp	Kp > 0 prints element stiffness matrices = 0 suppress such printing
N1	label number of element (at end 0)
N2	element type number
H1	hub number to which end 0 of element is attached
Hc	if double ended element, hub number to which end B of element is attached, if single ended element Hc = 0
A	effective shear area of the element
I	moment of inertia of cross section of element
Z	distance of the outermost fibre of the element cross section from the neutral axis (positive if measured from the flange, negative if measured from the faying flange)
b, f	width of faying flange and flange respectively.
d	depth of frame web
hb, hf, hw	thickness of faying flange, flange and frame web
θ	angle element makes with the x axis
J	torsional constant (twist per unit length = Torque/GJ where G is the shear modulus
ϕ	the angle in degrees between element and image
E	Young's modulus
G	Shear modulus
B	length of element (the element is considered to be labelled 0 at one end and B at the other end)
X1, Y1	X, Y coordinates of end 0 of element relative to hub at the same end
X2, Y2	X, Y coordinates of end B of element relative to hub at same end
a, b, c	specified for elastically restrained elements (type 11) $M_B = aV_B$, $T_B = -b\tau_B$, $S_B = -cW_B$

A^1	Cross sectional area of support, type 29 only
B^1	length of support, type 29 only
Vx, Vy	rotations about x and y axis
W	displacement perpendicular to plane of grillage
Mx, My	moments about x and y axis
S	force perpendicular to plane of grillage
SIGMAO, SIGMAB	stress at end 0 (or end B) of the element
SIGMAM	stress at a maximum in bending moment distribution, if any. (If there are two maxima the larger of the two stresses is taken)
SIGMAP	stress at point of application of concentrated load if any

NB The values for the parameters can be in any units but they must be consistent throughout.

i. Stress Analysis (with or without initial deformations)

			Format
a.		NT, NBEAM, JOINT, KP	4I10
		NLOAD	I1o
b.		For each element	
	(i)	N1, N2, H1, HC, NX	4I10, F10.0
	(ii)	A, I, Z , if N1>0	3F10.0
		b, hb, d, hw, f, hf if N1<0	6F10.0
	(iii)	θ, J, Ø, E, G	5F10.0
	(iv)	B, X1, Y1, X2, Y2	5F10.0
	(v)	p, w, m, for each loading condition	3F10.0
	(vi)	One set for each loading condition	
		VO, WO, VB, WB If NX ≠ 0	4F10.0
	(vii)	For type 11 elements only	
		A, B, C	3F10.0

NOTATION

NLOAD	number of loading conditions
NX	Axial force in the element, tension positive
p	Uniform load applied to element, positive downwards
w	Concentrated load applied to element, positive downwards
m	A concentrated load acts at a distance mB from end 0 of the element
VO, VB	initial out-of-plane slope at end 0 (and end B) of element (in degrees)
WO, WB	initial out of plane deflection at end 0 (and end B) of element

NOTES NT = 1 for stress analysis

If an element has the same loading data as the one previous to it, set N2 negative and specify up to b (iv) only

An example illustrating this facility is given in Appendix D, numerical example (i), figure 7.

ii. **Data Generation**

This routine cuts down the data preparation for the common special case of a rectangular grillage of othogonally intersecting beams parallel to the coordinate axes. The data produced is suitable for input to the stress analysis section of the program.

There are 2 sub-divisions in the data preparation.

Section 1 - Equal elements at uniform spacing.

a.	NX, NY, XL, YL, NLOAD, KP	2I1Ø, 2F1Ø.Ø, 2I1Ø,
b.	IBC1, IBC2, IBC3, IBC4	4I1Ø
c.	NS1, NS2 if IBC2 or IBC3 = 3 or 4 only	2I1Ø
d.	For elements parallel to the x axis A, I, Z or b, hb, d, hw, f, hf	6F1Ø.Ø
	For elements parallel to the y axis A, I, Z or b, hb, d, hw, f, hf	6F1Ø.Ø
e.	SIGX, SIGY	2F1Ø.Ø
f.	For each loading condition ICOND, PLOAD	I1Ø, F1Ø.Ø
	for concentrated loads only (ie if ICOND = -1) WLOAD, XX, YY	3F1Ø.Ø
g.	Specify only if SIGX ≠ 0 and SIGY ≠ 0 AMD(N), N = 1, NLOAD	6F1Ø.Ø

Section 2 - Unequal elements at non-uniform spacing.

a. As for Section 1

b. As for Section 1

c. As for Section 1

d. For elements parallel to the x-axis:-

As for Section 1, but if elements parallel to the x-axis have different cross-sectional or frame parameters from each other, set A or B negative and specify A, I, Z or b, hb, d, hw, f, hf for each of the remaining elements parallel to the x axis.

For elements parallel to the y-axis:-

As for Section 1, but if elements parallel to the y-axis have different cross-sectional or frame parameters from each other, set A or b negative and specify A, I, Z or b, hb, d, hw, f, hf for each of the remaining elements parallel to the y-axis.

e. specify only if XL is negative
ELX(I), I = 1,N 6F10.0

f. specify only if YL is negative
ELY (I), I = 1, N 6F10.0

g. As (e) for Section 1

h. As (f) for Section 1

j. As (g) for Section 1

NOTATION

NX - number of beams parallel to x-axis

NY - number of beams parallel to y-axis

±XL - length of beam parallel tothe x-axis

±YL - length of beam parallel to the y-axis

NLOAD - number of loading conditions

KP - parameter determining printing of stiffness coefficients from stress analysis program and listing of nodal and element coordinates from the plotting program

= 0 do not print coefficients

= 1 Print coefficients

= 2 print coefficients from stress analysis program and list coordinates from plotting program

IBC1, IBC2, IBC3, IBC4 - parameters indicating the boundary conditions for each of the four sides of the grillage symmetry or asymmetry may only occur about the upper and right hand edge of the grillage

IBC1 refers to the left hand edge of the grillage

IBC2 refers to the upper edge of the grillage

IBC3 refers to the right hand edge of the grillage

IBC4 refers to the bottom edge of the grillage (see figure 4)

Parameters:- = 1 for simple support

= 2 for clamped

= 3 for symmetry/asymmetry about centre of element

= 4 for symmetry/asymmetry about end of element

NS1 = 1 if there is symmetry about the upper edge

= -1 if there is asymmetry about the upper edge

= 0 otherwise

NS2 = 1 if there is symmetry about the right hand edge

= -1 if there is asymmetry about the right hand edge

= 0 otherwise

ICOND = 1 for uniform load

= -1 for concentrated load

PLOAD = uniform load

WLOAD = concentrated load applied to the element, positive downwards

XX = x coordinate of concentrated load

YY = y coordinate of concentrated load

SIGX = axial stress in beams parallel to the x axis, tension positive

SIGY = axial stress in beams parallel to the y-axis, tension positive

AMD = out of plane deflection at mid-point of grillage (see notes below)

ELX (I), I=1, N where N is the total number of elements parallel to the x-axis

= lengths of all elements parallel to the x-axis

ELY (I), I=1, N where N is the total number of elements parallel to the y-axis

= lengths of all elements parallel to the y axis

NOTES

(i) the x and y coordinates are measured from the origin where the first hub is considered to be situated

(ii) For Section 2 (d), (e), (f) the data for all double ended elements parallel to the x (or y) axis should be specified first starting from the beam parallel to the x axis at y = 0 (or parallel to the y axis at x = 0) and specifying the elements in order of x (or y) increasing. Repeat for all subsequent beams parallel to the x axis in the order of y increasing (or beams parallel to the y axis in the order of x increasing). The data for single ended elements parallel to the x axis (or single ended elements parallel to the y axis) should then be specified for each beam in the same order as for double ended elements.

(iii) The parameter AMD gives the out of plane deflection of the centre of the grillage. The first hub of the grillage is the origin of the coordinate system. The deflection at any point x, y is determined by:-

$$WD = AMD \ Sin \ \frac{\pi(x+X1)}{XL} \ Sin \ \frac{\pi(y+Y1)}{YL}$$

X1 is the distance from the left hand side of the grillage to the first hub

Y1 is the distance from the lower edge of the grillage to the first hub

The out of plane slope at any point x, y is determined by differentiating WD with respect to the appropriate direction.

iii **Lowest Buckling Mode**

		Format
a.	NT, NBEAM, JOINT, KP	4I10
b.	MAXIT, ACC	I10, F10.0
c.	For each element	
	(i) N1, N2, H1, HC, Nx	4I10, F10.0
	(ii) If N1>0, A, I, Z	3F10.0
	If N1<0, b, hb, d, hw, f, hf	6F10.0
	(iii) θ, J, ϕ, E, G	5F10.0

(iv) B, X1, Y1, X2, Y2 5F10.0

(v) For type 11 elements only

A, B, C 3F10.0

NOTATION

MAXIT = maximum number of iterations

ACC = accuracy test for the buckling load

Nx = axial force in the element, tension positive

NOTES

(i) NT = 2 for the lowest buckling mode.

(ii) Under in-plane end loading ignoring buckling, each element in the grillage will develop a longitudinal force Nx. For a given set of end loads these forces can be calculated by other programs. (eg NCRE Program N22). If all the end loads are increased by a constant factor the values of Nx will all increase by the same factor. For a given relative distribution of values of Nx this program will calculate the actual values of Nx (having the same relative distribution as the specified Nx) at which the grillage will first buckle. The associated buckling mode shape is also calculated.

(iii) An example illustrating this facility is given in Appendix D, numerical example (ii), figure 8.

iv. General Buckling

		Format
a.	NT, NBEAM, JOINT, KP	4I10
b.	GP, GQ, M	2F10.0,I10
c.	For each element	
(i)	N1, N2, H1, HC, Hx	4I10,F10.0
(ii)	If N1>0; A, I, Z	3F10.0
	If N1<0; b, hb, d, hw, f, hf	6F10.0
(iii)	θ, J, ∅, E, G	5F10.0
(iv)	B, X1, Y1, X2, Y2	5F10.0
(v)	For type 11 elements only	
	A, B, C	3F10.0

NOTATION

GP, GQ - these are the limits within which buckling Factors (λi) are required (order immaterial)

M = number of buckling loads to be calculated within the limits GP, GQ
mode shapes are also calculated if M>0 but not if M<0

NOTES

i. NT = 3 for general buckling modes

ii. An example illustrating this facility is given in Appendix D, numerical example (iii), figure 8.

v. Natural Frequencies

		Format
a.	NT, NBEAM, JOINT, KP	4I10
b.	GP, GQ, M, EX, GRAV	2F10.0, I10, 2F10.0
c.	For each element	
	(i) N1, N2, H1, HC	4I10
	(ii) If N1>0; A, I, Z	3F10.0
	If N1<0; b, hb, d, hw, f, hf	6F10.0
	(iii) θ, J, Ø, E, G	5F10.0
	(iv) B, X1, Y1, X2, Y2	5F10.0
	(v) μ, w, m	3F10.0
	(vi) For type 11 elements only	
	A, B, C	3F10.0

NOTATION

GP, GQ - Limits within which the squares of the natural frequencies are required (order immaterial)

M
- i. If M<0; up to M natural frequencies are calculated, but the mode shapes are not calculated.
- ii. If M>0; up to M natural frequencies and their associated shapes are calculated

EX - A scaling factor, (if left at zero a value of 3×10^7 is set within the program) Its value must be of the order of Young's Modulus

GRAV - acceleration of gravity eg 386.4 in/sec^2

μ - Mass/unit length of an element

w - Lumped mass

m - Lumped mass is positioned at a distance mB from end 0 of the element

NOTES

i. NT = 4 for natural frequencies

ii. An example illustrating this facility is given in Appendix D, numerical example (iv), figure 9.

vi. Stress Analysis, Normal Loading, Disk Solution for Larger Problems

a.	NBEAM, NLOAD, JOINT, KP	4I10
b.	(i) IBAND, MAXIB	2I10
	(ii) KBAND(I), I=1, IBAND	6I10
c.	For each element	
	(i) N1, N2, H1, HC	4I10
	(ii) If N1>0; A, I, Z	3F10.0
	If N1<0, b, hb, d, hw, f, hf	6F10.0
	(iii) θ, J, Ø, E, G	5F10.0

(iv) B, X1, Y1, X2, Y2 5F10.0

(v) For each loading condition

p, w, m 3F10.0

(vi) For type 11 elements only

A, B, C 3F10.0

NOTATION

NLOAD - number of loading conditions

IBAND - number of bands into which the grillage is divided (see below)

MAXIB - largest number of hubs in a band

KBAND - highest hub number in each band

p - uniform load applied to element, positive downwards

w - concentrated load applied to element, positive downwards

m - a concentrated load acts at a distance mB from end 0 of the element

NOTES

i. The equation solution routine uses banded matrix techniques. Storage of the equations on disk backing store enables solution of up to about 240 degrees of freedom (3 degrees of freedom per hub)

ii. The hubs must be numbered in bands such that a beam in the nth band can only link to hubs in that band or hubs in the (n+1)th band.

iii. The program is most efficient for smaller bandwidths. For example, a rectagular grillage would be most efficiently banded if the hubs were numbered in strips taken along the shorter of the two sides.

iv. If an element has the same loading data as the one previous to it, set N2 negative and specify as far as c (iv) only.

vii. Plotting of a Plane Grillage

The input for this program is in the form of data used for i. and vi. At the end of the data the following parameters must be inserted.

a. ± PL F10.0

b. If PL<0

A, IT A8, I10

NOTATION

PL - The length in cm. to which the user wants the grillage plotted. This length does not include space for notation on the plot which takes up to 4 cm. of the usuable paper width.

A - Name of the file which will hold the plotting instructions

IT - The plotting time in minutes

NOTES

i. This program is used chiefly for checking that the data is correct.

13. There follows some general comments on data preparation.

a. In order to specify the minimum amount of data, use is made of the sign and value of N2, the type number.

i. N2 = 0 is specified for an element which is identical in all respects to the previous element (either double ended or single ended). In this case only N1, 0, H1, HC are specified for the element.

ii. N2 positive is specified for an element whose data is in the normal form.

iii. N2 negative is specified for an element which has some of its data the same as the previous element. The loading conditions must all be the same as the previous element if this facility is to be used. It is necessary to specify only up to and including the last value which is different from the previous element with the exception that A, I, Z or b, hb, d, hw, f, hf are considered as groups and if any of the group is different from that of the previous element then the whole group must be specified. A special case of this is to specify only N1, N2, H1, HC where N2 is negative. This gives an element which is identical in all respects to the previous element but has a different type number.

b. If required certain standard values are set in the program. The standard value of a parameter is used in the program if a zero value of the parameter is specified in the data. The values used are:-

$E = 3 \times 10^7$ psi

$G = 1.154 \times 10^7$ psi

$\phi = 180^\circ$

$J = 0$

$X1 = X2 = Y1 = Y2 = 0$

$A1 = 10$ sq in

$B1 = 1$ in

Control Cards

14. A description of the control cards for:-

a. IBM 1130

b. UNIVAC 1108

c. KDF-9 follows

a. IBM 1130

i. Control for stress analysis, lowest buckling mode, general buckling modes or natural frequencies

```
//bXEQb KCH1bbbbb1
*LOCALKCH1,CSE1,CSE2,CSE11,CSE12,CSE14,CSE15,CSE29,CSFRM,CBIGM,CNCSO
.
.   Data
.
.
```

ii. Control for data generation

```
//bXEQbDN21D
.
.
.   Data
.
.
.
```

b. UNIVAC 1108

i. Control for stress analysis, lowest buckling mode, general buckling modes or natural frequencies

```
@RUN          Jobname, Acc No, Project No
@ASG,T        T., U9, XXXXXR
@ASG,T        Z.
@MOVE         T.,6
@COPIN        T.,Z.
@FREE         T.
@ASG,T        F1.,F4Ø ///1ØØØ
@ASG,T        F2., F4Ø ///' 1ØØØ
@USE       2, F1.
@USE       3, F2.
@XQT          Z.ABS
 . )
 . }
 . }      Data
 . }
 . )
@FIN
```

c. KDF-9

i. Control for stress analysis (in-core solution), and lowest buckling mode

```
*JOB ------
*XEQ
*IODUNIT1/DISC/3ØØ
*IODUNIT2/DISC/3ØØ
*IODUNIT5/READ
*IODUNIT6/PRINT
*STORAGE//26400
*DISCPROGRAMNCREN21/NCREUSE1
*DATA
 . )
 . }
 . }      Data
 . }
 . )
*ENDJOB
```

ii. Control for stress analysis (out-of-core solution)

```
*JOB
*XEQ
*IODUNIT1/DISC/300
*IODUNIT2/DISC/300
*IODUNIT5/READ
*IODUNIT6/PRINT
```

```
*STORAGE//26400
*DISCPROGRAMNCREN21B//NCREUSE1
*DATA
 . )
 . )
 . )  Data
 . )
 . )
*ENDJOB
```

iii. Control for Data Generation

```
*JOB - - - -
*XEQ
*IODUNIT5/READ
*IODUNIT6/PRINT
*DISCPROGRAMNCREN21D//NCREUSE1
*DATA
 . )
 . )
 . )  Data
 . )
 . )
*ENDJOB
```

iv. Control for Data Plot

```
*JOB - - - - -
*XEQ
*IODUNIT5/READ
*IODUNIT6/PRINT
*SUBFILEARLLIBRARY
*DISCPROGRAMNCRE21H//NCREUSE1
*DATA
 . )
 . )
 . )  Data
 . )
 . )
 . )
*ENDJOB
```

Description of Output

15. i. Stress Analysis (In Core and Out of Core)

a. Displacement and forces - for each loading condition the hub displacements (Vx, Vy, W) are printed preceded by their appropriate hub number. Next follow the internal beam forces at hubs (Mx, My, S) preceded by the appropriate hub and element label numbers. For each loading condition a similar table of displacements and forces is given. For hubs on one or more axes of symmetry or antisymmetry the hub forces and moments due to the image are also included so that, for example, moments about an axis of symmetry appear zero (or very small) even although the moments in the beams on each side of the axis of symmetry are non-zero. The shear forces at such positions are double the values for the single beams. If moments or shear forces are required at such positions they are easily calculated from the forces and moments at the other end of the beam and the loads acting on the beam. At axes of antisymmetry the situation is reversed and moments are doubled whereas shear forces appear as zero or very small.

b. Stress - After the tables of hub displacements and forces for every loading condition a table of stresses (tension positive) is given in five columns as follows

ELEMENT - the label at end 0 of the beam

SIGMA 0 - the stress at the end of the beam corresponding to the label number

SIGMA B - the stress at the other end of the beam

SIGMA Max - the stress at a maximum (dM/dx = 0) in bending moment distribution, if any

SIGMA P - the stress at the point of application of the concentrated load, if any

ii. <u>Lowest Buckling Mode</u>

The lowest buckling load is printed, followed by the associated buckling mode shape.

iii. <u>General Buckling Modes</u>

Up to M buckling loads are printed, each followed by its associated shape.

iv. <u>Natural Frequencies</u>

Up to M natural frequencies (rad/sec) are printed, each followed by their associated shapes.

v. <u>Data Generation</u>

Line printer output of the prepared data is produced for use with the stress analysis (in core) program. The IBM 1130 version also produces a punched tape of the data.

vi. <u>Data Check</u>

a. Line printer output:- If the parameter KP in the initial data is set equal to 2, the nodal coordinates (X, Y) and the element coordinates (XØ, YØ, XB, YB) will be listed.

X - X coordinate of a hub

Y - Y coordinate of a hub

XO- X coordinate of end 0 of an element

YO- Y coordinate of end 0 of an element

XB- X coordinate of end B of an element

YB- Y coordinate of end B of an element

b. Plotter Output:- Lines, denoting the beams, are drawn with hub numbers and element type numbers at the boundaries. Uniform loading is shown by 2 small 'v's positioned at each end of the element, and a concentrated load is shown by one large 'v' at a distance mB from end 0 of the element.

CONCLUDING REMARKS

16. Sample problems have been run and timed on the various computers and these running times can be found in Appendix E.

17. There are different limitations for the different computers on the size of problem that can be analysed. Restrictions on the size of problem are set out in Appendix F. Certain elements are not available for certain facilities and details of this restriction can also be found in Appendix F.

18. Appendix G gives a general flow chart for the suite of programs, giving details of stress analysis, lowest buckling and general buckling loads and modes and natural frequencies.

REFERENCES

1. Timoshenko S. "Strength of Materials" Macmillan and Co Ltd 1940.
2. Frazer R A, Duncan W J, Collar A R. "Elementary Matrices", Cambridge University Press 1947
3. Martin R S, Wilkinson J H. "Reduction of the Symmetric Eigen problem $Ax = \lambda Bx$ and Related Problems to Standard Form" Numerische Mathematik 11, 99-110, 1968
4. Wilkinson J H. "The Algebracic Eigenvalue Problem" Pages 290-299 Clarendon Press 1965
5. Guyan, R J, "Reduction of Stiffness and Mass Matrices" AIAA Journal, Vol 3, No 2, February 1965
6. Macneal-Schwendler "Nastran Theoretical Manual" Pages 7.2-3 to 7.2-8
7. Gallagher R H "Finite Element Analysis - Fundamentals" Pages 388-395.

APPENDIX A

THEORY FOR STRESS ANALYSIS AND FORMATION OF STIFFNESS MATRICES

Element Stiffness Matrices

1. The equations to be solved are as follows.

$$\left\{Mh\right\} = [A_1] \left\{\delta h\right\} + [A_2] \left\{\delta_c\right\} + \left\{B_1\right\} \quad - - - - (1)$$

$$\left\{Mc\right\} = [A_3] \left\{\delta h\right\} + [A_4] \left\{\delta c\right\} + \left\{B_2\right\} \quad - - - - (2)$$

$A_1 = (a_{ij}) \quad i = 1,3; \quad j = 1,3$

$A_2 = (a_{ij}) \quad i = 1,3; \quad j = 4,6$

$A_3 = (b_{ij}) \quad i = 1,3; \quad j = 4,6$

$A_4 = (b_{ij}) \quad i = 1,3; \quad j = 1,3$

$B_1 = (a_{i7}) \quad i = 1,3$

$B_2 = (b_{i7}) \quad i = 1,3$

(a_{ij}) and (b_{ij}) are the stiffness matrices whose coefficients are formed from Timoshenko beam theory (see 1). The coefficients are as follows.

$a_{11} = b_{11} = EI\ (4+12\beta)/B(1+12\beta)$

$a_{12} = b_{12} = 0$

$a_{13} = -b_{13} = 6EI/B^2(1+12\beta)$

$a_{21} = b_{21} = 0$

$a_{22} = b_{22} = GJ/B$

$a_{23} = b_{23} = 0$

$a_{31} = -b_{31} = a_{13}$

$a_{32} = b_{32} = 0$

$a_{33} = b_{33} = 12EI/B^3\ (1+12\beta)$

$a_{14} = b_{14} = 2EI\ (1-6\beta)/B(1+12\beta)$

$a_{15} = b_{15} = 0$

$a_{16} = -b_{16} = -a_{13}$

$a_{24} = b_{24} = 0$

$a_{25} = b_{25} = -a_{22}$

$a_{26} = b_{26} = 0$

$a_{34} = b_{34} = a_{13}$

$a_{35} = b_{35} = 0$

$a_{36} = b_{36} = -a_{33}$

$a_{17} = -(pB^2/12) - WB(1-m)\ m[(1-m) + 6\beta]/(1+12\beta)$

$b_{17} = (pB^2/12) + WB\ (1-m)\ m\ [\ m+6\beta]/(1+12\beta)$

$a_{27} = b_{27} = 0$

$a_{37} = -(pB/2) - W(1-m)\ [(1+2m)\ (1-m) + 12\beta]/(1+12\beta)$

$b_{37} = -(pB/2) - Wm[m(3-2m) + 12\beta]/(1+12\beta)$

where $= EI/B^2\ GA$

NB The B matrices are the loading conditions on the grillage.

$\{M_h\}$ and $\{M_c\}$ are the force vectors at ends h and c of a beam element. They consist of the bending force, the torsional force and the shearing force. $\{\delta_h\}$ and $\{\delta_c\}$ are the displacement vectors at the ends h and c of a beam element.

Forces and displacements specified so far refer to the local coordinate system of each element. The local forces and displacements must be transformed to the global coordinate system. See figure 5.

Equations (1) and (2) can be combined into

$$\begin{bmatrix} M_h \\ M_c \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} \delta_h \\ \delta_c \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Define $\{M_x\}$ as being the force vector in the global coordinate system and $\{\delta x\}$ as displacements in that system.

Define a transformation matrix

$$T = \begin{bmatrix} \text{Cos } \theta & \text{Sin } \theta & 0 \\ -\text{Sin } \theta & \text{Cos } \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

θ is the angle that the beam makes with the x- axis of the global coordinate system.

$$\Rightarrow \begin{bmatrix} Mxh \\ Mxc \end{bmatrix} = \begin{bmatrix} TA_1 & TA_2 \\ TA_3 & TA_4 \end{bmatrix} \begin{bmatrix} \delta h \\ \delta c \end{bmatrix} + \begin{bmatrix} TB_1 \\ TB_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} Mxh \\ Mxc \end{bmatrix} = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} T^{-1} & 0 \\ 0 & T^{-1} \end{bmatrix} \begin{bmatrix} \delta xh \\ \delta xc \end{bmatrix} + \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \qquad (3)$$

The equations are put in a form which can be solved by equating to zero the sums of the moments and forces at every hub. This provides a sufficient set of equations to find the unknown hub rotations and displacements.

If the hub positions are not at the end of the beam it is necessary to transfer the origin to points distance $x_1 y_1$ and $x_2 y_2$ from ends h and c respectively.

Define force vectors at the offset hub as

$\{MxH\}$ and $\{MxC\}$

Define displacement vectors at the offset hub as

$\{\delta xH\}$ and $\{\delta xC\}$

Define a transformation matrix for the forces at the ends of the beam

$$T_1 = \begin{vmatrix} 1 & 0 & -x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{vmatrix}$$

Define a transformation matrix for the displacements at the ends of the beam.

$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x & -y & 1 \end{bmatrix}$$

These transformations are then performed on equation (3) which gives.

$$\begin{bmatrix} MxH \\ MxC \end{bmatrix} = \begin{bmatrix} T_1 T & 0 \\ 0 & T_1 T \end{bmatrix} \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} (T_2 T)^{-1} & 0 \\ 0 & (T_2 T)^{-1} \end{bmatrix} \begin{bmatrix} \delta xH \\ \delta xC \end{bmatrix} + \begin{bmatrix} T_1 T & 0 \\ 0 & T_1 T \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

See figure 6 which illustrates how the hubs can be offset.

This equation is again put in a form that can be solved by equating to zero the sums of the moments and forces at every hub.

APPENDIX B

THEORY FOR BUCKLING

i. The Differential Stiffness Matrix

The equation to be solved for buckling is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{Bmatrix} v_{xh} \\ v_{yh} \\ w_h \\ v_{xc} \\ v_{yc} \\ w_c \end{Bmatrix} = \lambda_1 \quad X$$

$$\begin{bmatrix} b^1_{11} & b^1_{12} & b^1_{13} & b^1_{14} & b^1_{15} & b^1_{16} \\ b^1_{21} & b^1_{22} & b^1_{23} & b^1_{24} & b^1_{25} & b^1_{26} \\ b^1_{31} & b^1_{32} & b^1_{33} & b^1_{34} & b^1_{35} & b^1_{36} \\ b^1_{41} & b^1_{42} & b^1_{43} & b^1_{44} & b^1_{45} & b^1_{46} \\ b^1_{51} & b^1_{52} & b^1_{53} & b^1_{54} & b^1_{55} & b^1_{56} \\ b^1_{61} & b^1_{62} & b^1_{63} & b^1_{64} & b^1_{65} & b^1_{66} \end{bmatrix} \begin{Bmatrix} v_{xh} \\ v_{yh} \\ w_h \\ v_{xc} \\ v_{yc} \\ w_c \end{Bmatrix}$$

($b^1 ij$) is the differential stiffness matrix. For details of how it is formed see (6). The coefficients are as follows

$b^1_{11} = 2NxB/15$

$b^1_{13} = Nx/10$

$b^1_{14} = -NxB/30$

$b^1_{16} = -Nx/10$

$b^1_{31} = Nx/10$

$b^1_{33} = 6Nx/5B$

$b^1_{34} = Nx/10$

$b^1_{36} = -6Nx/5B$

$b^1_{41} = -NxB/30$

$b^1_{43} = Nx/10$

$b^1_{44} = 2NxB/15$

$b^1_{46} = -Nx/10$

$b^1_{61} = -Nx/10$

$b^1_{63} = -6Nx/5B$

$b^1_{64} = -Nx/10$

$b^1_{66} = 6Nx/5B$

$b^1_{12} = b^1_{15} = b^1_{21} = b^1_{22} = b^1_{23} = b^1_{24} = b^1_{25} = b^1_{26} = 0$

$b^1_{32} = b^1_{35} = b^1_{41} = b^1_{42} = b^1_{43} = b^1_{44} = b^1_{45} = b^1_{46} = b^1_{62} = b^1_{65} = 0$

where Nx is the axial force in the element, tension positive and
B is the length of the element.

Errors in the Critical Load (see 7)

a. Beam simply supported at either end:-
If only one element is used to model the beam an error of approximately 20% is present in the critical load. In the case of this program this problem is academic because of the facility of single ended elements. Two elements are needed to model a simply supported beam and for this case the error is less than 1%.

b. Beam clamped at one end and free at the other.
For one element the error in the critical load was found to be 0.735% (according to (7) the error is 0.752%). For more than one element the error was found to be less than 1%.

ii. <u>Evaluation of Lowest Buckling Mode</u>

The equation to be solved is

$$[\underline{A}]\ [\delta] = \lambda 1 [B^D]\ [\delta] \text{ - - - } (1)$$

$[A]$ is the same as the $[A]$ matrix derived in Appendix A.

$[B^D]$ is the differential stiffness matrix

To solve the equation pre multiply throughout (1) by $[A]^{-1}$

$([I] - [U]\ \lambda_1)\ [\delta] = 0$ where $[U] = [A]^{-1}\ [B^D]$

and $[I]$ is the unit matrix

Let $\beta = 1/\lambda_1$ then we get

$$(\beta\ [I] - [U])\ [\delta] = 0 \text{ - - - } (2)$$

This has now been reduced to the standard eigenvalue problem.

The root of interest of the equation $\beta\ [I] - [U]$ is the dominant root since this gives the smallest value of λ_1. The dominant root can be found by repeated by pre-multiplication of an arbitary column by $[U]$. (For theory see (2))

iii. <u>Evaluation of General Buckling Modes</u>

Again we are solving

$$[A]\ [\delta] = \lambda_1\ [B^D]\ [\delta]$$

This can be transformed into the standard symmetric eigenvalue problem

$$[E]\ [x] = \lambda\ [x]$$

by carrying out a Choleski decomposition of B into $B = LL^T$ (see 3) and setting $E = L^{-1}AL^{-T}$ and $x = L^T\delta$. This standard problem is solved in the program by an HGW type of method (see 4).

APPENDIX C

THEORY FOR NATURAL FREQUENCIES

1. The vibration problem can be written in matrix form as follows.

$$W^2 [M][\delta] = [A] [\delta] \quad - - - (1)$$

The [A]matrix is the same as that derived in Appendix A

The [M] matrix is the same as the [B] matrix derived in Appendix A except that:-

p = mass/unit length of an element

w = lumped mass

m = the lumped mass acts at a distance mB from end 0 of the element

The equation (1) can be written in the following form.

$$\begin{bmatrix} 0 \\ -m\ddot{\underset{\sim}{W}} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} \underset{\sim}{\theta} \\ \underset{\sim}{W} \end{bmatrix} \qquad \text{where } A_3 = A_2^T$$

where $\underset{\sim}{\theta}$ is a vector of rotations and $\underset{\sim}{W}$ is a vector of displacements.

Rotary inertias have been neglected because they are very small compared to translational inertias.

$$A_1\theta + A_2 W = 0$$

$$\Rightarrow \theta = - A_1^{-1} A_2 W$$

also $$m \ddot{\underset{\sim}{W}} + [A_4 - A_3 A_1^{-1} A_2] \underset{\sim}{W} = 0$$

Rotations have been eliminated and the final equation to be solved is

$$(E - \omega^2 m) \underset{\sim}{W} = 0$$

$$[E] \underset{\sim}{W} = \omega^2 [m] \underset{\sim}{W}$$

This is the Guyan reduction (see 5)

The $\underset{\sim}{W}$ vector now defines the relative displacements in the vibration mode corresponding to the natural frequency, ω

As in the buckling case we can factorise [E] by Choleski Factorisation

$$[E] = [L] [L]^T$$

$$\Rightarrow ([L]^{-1} [m] [L]^{-T}) ([L]^T [W]) = 1/\omega^2 ([L]^T [W])$$

Therefore the matrix has eigenvalues $\lambda i = 1/\omega i^2$ and eigenvalues qi such that $qi = [L]^T [Wi]$

The eigenvalue problem is now solved using a standard HGW method.

PRECEDING PAGE BLANK-NOT FILMED

APPENDIX D

NUMERICAL EXAMPLES

1. Four numerical examples are shown:-
   - i. Stress analysis for grillage
   - ii. Lowest buckling mode for a grillage
   - iii. General buckling modes for the same grillage as in ii.
   - iv. Natural frequencies of a free-free beam

2. Figures 7-9 show the idealisation used for each of these. Listing of the data, and the results obtained from these examples are as follows:-

```
  1     1     1
  5   300     2

 60
  1

  1
  3     2     2
  5   300     2

 60
  1

  1
  5    13     1
  5   300     2
180
 60
  1

  1
  6    13     1
2.7    75    -3
270
 50
  1

  1
  7     0     2
  8     0     3
  9    14     1
2.7    75    -3
 90
 50
  1

  1
 10     0     2
 11    14     3
2.7    75    -3
 90
 50
  1
  0     1   0.5
  1     1   0.5
  1     9     3          1
  3
```

i. Data Input for Stress Analysis

LOADING CONDITION 1

HUB	VX	VY	W
1	0.10868754E-04	-0.91554011E-05	0.53711724E-03
2	0.60793354E-05	-0.18281407E-04	0.10725096E-02
3	0.00000000E 00	-0.21264997E-04	0.12475471E-02

HUB	ELEMENT	MX	MY	S
1	1	0.15245990E 03	0.00000000E 00	-0.38865081E 02
1	5	-0.15246231E 03	0.00000000E 00	0.39425827E 02
1	6	0.00000000E 00	0.61565417E 03	0.24439224E 02
1	9	0.00000000E 00	-0.61565258E 03	-0.25000003E 02
2	2	-0.68436633E 03	0.00000000E 00	-0.21134922E 02
2	3	0.68435937E 03	0.00000000E 00	-0.27584713E 02
2	7	0.00000000E 00	0.14369948E 04	0.73719650E 02
2	10	0.00000000E 00	-0.14369929E 04	-0.25000003E 02
3	4	-0.53944165E 03	0.00000000E 00	-0.32415298E 02
3	8	0.00000000E 00	0.17055180E 04	0.89831070E 02
3	11	0.00000000E 00	-0.17055161E 04	-0.25000003E 02

LOADING CONDITION 2

HUB	VX	VY	W
1	0.13653952E-07	-0.62919838E-08	0.36913030E-06
2	0.25840016E-07	-0.27558222E-07	0.16167496E-05
3	0.00000000E 00	-0.76121381E-07	0.29071470E-05

HUB	ELEMENT	MX	MY	S
1	1	-0.24476518E 01	0.00000000E 00	-0.20658083E-01
1	5	0.24476475E 01	0.00000000E 00	-0.13318514E-01
1	6	0.00000000E 00	0.56628072E 00	0.33976808E-01
1	9	0.00000000E 00	-0.56627857E 00	0.00000000E 00
2	2	0.12081675E 01	0.00000000E 00	0.20658079E-01
2	3	-0.12081775E 01	0.00000000E 00	-0.16947266E 00
2	7	0.00000000E 00	0.24802417E 01	0.14881446E 00
2	10	0.00000000E 00	-0.24802398E 01	0.00000000E 00
3	4	-0.89601841E 01	0.00000000E 00	0.16947266E 00
3	8	0.00000000E 00	0.60093021E 00	0.16105571E 00
3	11	0.00000000E 00	-0.60092365E 00	-0.50000012E 00

LOADING CONDITION 3

HUB	VX	VY	W
1	0.10882407E-04	-0.91616930E-05	0.53748628E-03
2	0.61051760E-05	-0.18308965E-04	0.10741264E-02
3	0.00000000E 00	-0.21341118E-04	0.12504542E-02

HUB	ELEMENT	MX	MY	S
1	1	0.15001074E 03	0.00000000E 00	-0.38885749E 02
1	5	-0.15001425E 03	0.00000000E 00	0.39412483E 02
1	6	0.00000000E 00	0.61621985E 03	0.24473182E 02
1	9	0.00000000E 00	-0.61621887E 03	-0.25000003E 02
2	2	-0.68315930E 03	0.00000000E 00	-0.21114261E 02
2	3	0.68315112E 03	0.00000000E 00	-0.27754138E 02
2	7	0.00000000E 00	0.14394751E 04	0.73868484E 02
2	10	0.00000000E 00	-0.14394731E 04	-0.25000003E 02
3	4	-0.54840173E 03	0.00000000E 00	-0.32245872E 02
3	8	0.00000000E 00	0.17061191E 04	0.89992126E 02
3	11	0.00000000E 00	-0.17061169E 04	-0.25500003E 02

ELEMENT	SIGMA O	SIGMA B	SIGMA MAX	SIGMA P
1	0.101639E 01	0.456242E 01	0.605137E 01	0.000000E 00
1	-0.163176E-01	-0.805444E-02	0.000000E 00	0.000000E 00
1	0.100007E 01	0.455436E 01	0.604040E 01	0.000000E 00
3	0.456239E 01	0.359627E 01	0.709878E 01	0.000000E 00
3	-0.805451E-02	0.597345E-01	0.000000E 00	0.000000E 00
3	0.455433E 01	0.365599E 01	0.712197E 01	0.000000E 00
5	0.101641E 01	-0.267539E 02	0.000000E 00	0.000000E 00
5	-0.163176E-01	-0.109902E-01	0.000000E 00	0.000000E 00
5	0.100009E 01	-0.267648E 02	0.000000E 00	0.000000E 00
6	-0.246261E 02	0.742522E 02	0.000000E 00	0.000000E 00
6	-0.226512E-01	0.453023E-01	0.000000E 00	0.000000E 00
6	-0.246487E 02	0.742975E 02	0.000000E 00	0.000000E 00
7	-0.574797E 02	0.139959E 03	0.000000E 00	0.000000E 00
7	-0.992096E-01	0.198419E 00	0.000000E 00	0.000000E 00
7	-0.575789E 02	0.140157E 03	0.000000E 00	0.000000E 00
8	-0.682207E 02	0.161441E 03	0.000000E 00	0.000000E 00
8	-0.240372E-01	0.298074E 00	0.000000E 00	0.000000E 00
8	-0.682447E 02	0.161739E 03	0.000000E 00	0.000000E 00
9	-0.246260E 02	-0.246260E 02	-0.371260E 02	0.000000E 00
9	-0.226511E-01	-0.226511E-01	0.000000E 00	0.000000E 00
9	-0.246487E 02	-0.246487E 02	-0.371487E 02	0.000000E 00
10	-0.574796E 02	-0.574796E 02	-0.699797E 02	0.000000E 00
10	-0.992095E-01	-0.992095E-01	0.000000E 00	0.000000E 00
10	-0.575789E 02	-0.575789E 02	-0.700789E 02	0.000000E 00
11	-0.682206E 02	-0.682206E 02	-0.807206E 02	0.000000E 00
11	-0.240369E-01	-0.240368E-01	0.000000E 00	-0.524036E 00
11	-0.682446E 02	-0.682446E 02	0.000000E 00	-0.812446E 02

Results for Stress Analysis

Appendix D3

2	24	9	0	
20	.0001			
1	1	1	2	1.0
200	4.745	.125		
0	1			
24				
3	0	2	3	1.0
5	0	4	5	1.0
7	0	5	6	1.0
9	0	7	8	1.0
11	0	8	9	1.0
13	1	1	4	1.0
200	4.745	.125		
90	1			
24				
15	0	4	7	1.0
17	0	2	5	1.0
19	0	5	8	1.0
21	0	3	6	1.0
23	0	6	9	1.0
25	13	9	0	1.0
200	4.745	.125		
0	1			
24				
26	0	6	0	1.0
27	0	3	0	1.0
28	13	3	0	1.0
200	4.745	.125		
270	1			
24				
29	0	2	0	1.0
30	0	1	0	1.0
31	13	1	0	1.0
200	4.745	.125		
180	1			
24				
32	0	4	0	1.0
33	0	7	0	1.0
34	13	7	0	1.0
200	4.745	.125		
90	1			
24				
35	0	8	0	1.0
36	0	9	0	1.0

ii. Data Input for Lowest Buckling Modes

Appendix D4

BUCKLING LOAD COEFFICIENT = 0.6275E 06

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
0.1633E-01	-0.1645E-01	0.2509E 00	-0.1173E-03	-0.3225E-01	0.4988E 00
-0.1625E-01	-0.1625E-01	0.2481E 00	0.3241E-01	-0.1173E-03	0.5047E 00
-0.2511E-03	-0.2512E-03	0.1000E 01	-0.3225E-01	-0.1174E-03	0.4988E 00
0.1653E-01	0.1653E-01	0.2536E 00	-0.1173E-03	0.3241E-01	0.5048E 00
-0.1645E-01	0.1633E-01	0.2509E 00			

Lowest buckling mode of grillage

<u>Appendix D5</u>

3	24	9	1	
-10	100000000	10		
1	1	1	2	1.0
200	4.745	.125		
0	1			
24				
3	0	2	3	1.0
5	0	4	5	1.0
7	0	5	6	1.0
9	0	7	8	1.0
11	0	8	9	1.0
13	1	1	4	1.0
200	4.745	.125		
90	1			
24				
15	0	4	7	1.0
17	0	2	5	1.0
19	0	5	8	1.0
21	0	3	6	1.0
23	0	6	9	1.0
25	13	9	0	1.0
200	4.745	.125		
0	1			
24				
26	0	6	0	1.0
27	0	3	0	1.0
28	13	3	0	1.0
200	4.745	.125		
270	1			
24				
29	0	2	0	1.0
30	0	1	0	1.0
31	13	1	0	1.0
200	4.745	.125		
180	1			
24				
32	0	4	0	1.0
33	0	7	0	1.0
34	13	7	0	1.0
200	4.745	.125		
90	1			
24				
35	0	8	0	1.0
36	0	9	0	1.0

iii. <u>Data Input for General Buckling Modes</u>

<u>Appendix D6</u>

General Buckling of the grillage with only the first 5 modes shown

Appendix D7

BUCKLING LOAD COEFFICIENT = 0.6275E 06

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
-0.5310E-04	0.5310E-04	-0.8126E-03	0.8705E-10	0.1047E-03	-0.1625E-02
0.5310E-04	0.5310E-04	-0.8126E-03	-0.1047E-03	0.1045E-09	-0.1625E-02
0.1440E-09	0.3765E-09	-0.3239E-02	0.1047E-03	0.2296E-09	-0.1625E-02
-0.5310E-04	-0.5310E-04	-0.8126E-03	-0.1228E-09	-0.1047E-03	-0.1625E-02
0.5310E-04	-0.5310E-04	-0.8126E-03			

BUCKLING LOAD COEFFICIENT = 0.1143E 07

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
-0.3935E-04	0.5025E-04	-0.8819E-03	0.4777E-04	0.1909E-04	-0.7247E-03
0.8901E-05	-0.3249E-04	0.2321E-03	-0.3273E-04	-0.2786E-04	-0.1242E-02
0.1042E-03	-0.6077E-04	0.5468E-09	-0.3273E-04	-0.2786E-04	0.1242E-02
0.8901E-05	-0.3249E-04	-0.2321E-03	0.4777E-04	0.1909E-04	0.7247E-03
-0.3935E-04	0.5025E-04	0.8820E-03			

BUCKLING LOAD COEFFICIENT = 0.1143E 07

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
0.3233E-04	0.9099E-05	0.2287E-03	0.2805E-04	-0.3265E-04	0.1239E-02
-0.5021E-04	-0.3948E-04	0.8829E-03	-0.1922E-04	0.4766E-04	-0.7296E-03
0.6118E-04	0.1039E-03	0.3820E-08	-0.1922E-04	0.4766E-04	0.7296E-03
-0.5021E-04	-0.3948E-04	-0.8829E-03	0.2805E-04	-0.3265E-04	-0.1239E-02
0.3233E-04	0.9099E-05	-0.2287E-03			

BUCKLING LOAD COEFFICIENT = 0.1316E 07

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
0.2505E-04	-0.2504E-04	0.9091E-03	-0.7760E-04	-0.3561E-10	0.3910E-08
0.2504E-04	0.2504E-04	-0.9091E-03	-0.6973E-10	0.7760E-04	-0.3397E-09
-0.9565E-10	0.4211E-09	-0.2895E-08	0.1387E-09	-0.7759E-04	-0.2248E-08
-0.2505E-04	-0.2504E-04	-0.9091E-03	0.7759E-04	-0.2163E-09	-0.6521E-08
-0.2504E-04	0.2504E-04	0.9091E-03			

BUCKLING LOAD COEFFICIENT = 0.1917E 07

BUCKLING MODE SHAPE

THETAX	THETAY	W	THETAX	THETAY	W
-0.6553E-05	0.7871E-05	-0.9093E-03	0.4309E-04	-0.8281E-05	0.2607E-03
-0.2517E-04	0.2555E-04	-0.2361E-04	0.7862E-05	-0.4539E-04	0.2474E-03
-0.2235E-04	0.2355E-04	-0.9060E-08	0.7863E-05	-0.4539E-04	-0.2475E-03
-0.2517E-04	0.2555E-04	0.2361E-04	0.4309E-04	-0.8282E-05	-0.2606E-03
-0.6554E-05	0.7870E-05	0.9093E-03			

4	10	11	0
-1000000.	10000000.	11	10000.0
1	1	1	2
.9610	10.3308		
0.0			
4.7244			
10.1727			
3	1	2	3
2.5885	77.1206		
0.0			
5.3150			
24.7420			
5	1	3	4
2.5885	94.8991		
0.0			
5.0000			
38.1394			
7	1	4	5
2.0925	78.8023		
0.0			
7.0079			
20.2122			
9	1	5	6
2.5885	96.1004		
0.0			
7.0079			
16.0439			
11	1	6	7
2.5885	70.1533		
0.0			
6.4961			
16.7310			
13	1	7	8
2.5885	68.7118		
0.0			
5.5118			
15.4391			
15	1	8	9
2.5885	62.4652		
0.0			
6.4961			
18.3430			
17	1	9	10
2.5885	68.7118		
0.0			
5.0000			
23.6552			
19	1	10	11
.9455	6.7270		
0.0			
5.0000			
10.9347			

iv. Data Input to Natural Frequencies

Appendix D8

```
4
NATURAL FREQUENCY(RADIANS/SEC) =  0.81722E 04
MODE SHAPES=
  0.12185E-01 -0.14983E-01  0.17029E-01 -0.35620E-01  0.16268E 00 -0.52647E 00
  0.10919E 01 -0.11247E 01  0.78093E 00 -0.60114E 00  0.36982E 00
NATURAL FREQUENCY(RADIANS/SEC) =  0.73902E 04
MODE SHAPES=
 -0.17208E 01  0.13126E 01 -0.79777E 00  0.39668E 00 -0.21572E 00  0.23392E 00
 -0.18819E 00 -0.37705E-01  0.24241E 00 -0.29810E 00  0.24627E 00
NATURAL FREQUENCY(RADIANS/SEC) =  0.72761E 04
MODE SHAPES=
  0.57574E 00 -0.41303E 00  0.21210E 00 -0.21017E-01 -0.29088E 00  0.70423E 00
 -0.86421E 00  0.72959E-01  0.83142E 00 -0.11251E 01  0.97085E 00
NATURAL FREQUENCY(RADIANS/SEC) =  0.62160E 04
MODE SHAPES=
 -0.10798E 01  0.37257E 00  0.23148E 00 -0.58421E 00  0.90195E 00 -0.76598E 00
 -0.25479E 00  0.71679E 00  0.22648E-01 -0.61153E 00  0.88034E 00
NATURAL FREQUENCY(RADIANS/SEC) =  0.56724E 04
MODE SHAPES=
 -0.18652E 01  0.35810E 00  0.66281E 00 -0.70653E 00 -0.87695E-01  0.67535E 00
 -0.16208E 00 -0.60157E 00  0.19030E 00  0.46139E 00 -0.93582E 00
NATURAL FREQUENCY(RADIANS/SEC) =  0.48016E 04
MODE SHAPES=
 -0.10880E 01 -0.34478E-01  0.45056E 00  0.49822E-01 -0.81225E 00  0.14362E 00
  0.76808E 00  0.27085E 00 -0.70974E 00 -0.39925E 00  0.19163E 01
NATURAL FREQUENCY(RADIANS/SEC) =  0.38504E 04
MODE SHAPES=
 -0.14490E 01 -0.36626E 00  0.43666E 00  0.56343E 00 -0.54303E 00 -0.77320E 00
 -0.97065E-01  0.53497E 00  0.55011E 00 -0.99558E-01 -0.15995E 01
NATURAL FREQUENCY(RADIANS/SEC) =  0.27578E 04
MODE SHAPES=
 -0.14840E 01 -0.66506E 00  0.12377E 00  0.67367E 00  0.57737E 00 -0.88565E-01
 -0.67050E 00 -0.76041E 00 -0.24467E 00  0.40616E 00  0.13344E 01
NATURAL FREQUENCY(RADIANS/SEC) =  0.13807E 04
MODE SHAPES=
  0.11640E 01  0.75417E 00  0.30187E 00 -0.13134E 00 -0.62834E 00 -0.80266E 00
 -0.65505E 00 -0.30122E 00  0.29961E 00  0.79171E 00  0.13269E 01
NATURAL FREQUENCY(RADIANS/SEC) =  0.20528E 01
MODE SHAPES=
  0.96830E 00  0.89083E 00  0.80368E 00  0.72169E 00  0.60678E 00  0.49187E 00
  0.38536E 00  0.29498E 00  0.18847E 00  0.10649E 00  0.24509E-01
NATURAL FREQUENCY(RADIANS/SEC) =  0.63377E 01
MODE SHAPES=
  0.61039E 00  0.45743E 00  0.28536E 00  0.12348E 00 -0.10340E 00 -0.33029E 00
 -0.54061E 00 -0.71907E 00 -0.92940E 00 -0.10912E 01 -0.12531E 01
```

APPENDIX E

RUNNING TIMES

1. IBM 1130

a. Stress Analysis:- 3 hubs, 9 beams - 5 mins 20 secs

b. Lowest Buckling:- 9 hubs, 24 beams - 37 mins 46 secs

c. General Buckling:- 9 hubs, 24 beams - 15 mins

d. Natural Frequencies:- 9 hubs, 24 beams - 6 mins 34 secs

2. UNIVAC 1108

a. Stress Analysis:- 3 hubs, 9 beams - 1 CP sec

b. Lowest Buckling:- 9 hubs, 24 beams - 1.5 CP secs

c. General Buckling:- 9 hubs, 24 beams - 2.5 CP secs

d. Natural Frequencies:- 10 hubs, 11 beams - 1.3 CP secs

3. KDF-9

a. Stress Analysis (in-core):- 4 hubs - 38 secs

b. Lowest Buckling:- 9 hubs - 23 secs

c. Stress Analysis (out of core):- 30 hubs - 2 min 24 secs

d. Data Generation:- 12 hubs - 20 secs

e. Data Plot:- 18 hubs - running time - 2 min 23 sec
plotting time- 11 min

APPENDIX F

RESTRICTIONS ON THE SIZE OF THE PROBLEM AND RESTRICTIONS ON THE ELEMENTS THAT CAN BE USED

1. Restriction on the Size of the Problem

a. IBM 1130

a. i. Stress analysis, lowest buckling, general buckling, natural frequencies

Maximum number of hubs = 20

Maximum number of elements = 40

Maximum number of loading conditions = 6

a.ii. Data Generation

Maximum number of hubs = 12

Maximum number of elements = 30

Maximum number of loading conditions = 5

b. UNIVAC 1108

b.i. Stress Analysis, lowest buckling, general buckling, natural frequencies

Maximum number of hubs = 30

Maximum number of elements = 80

Maximum number of loading conditions = 6

c. KDF-9

c.i. Stress Analysis (in-core solution)

$\mathrm{NBEAM}\,(21+17\mathrm{NLOAD}) + 18\mathrm{NLOAD} + 3\mathrm{JOINT}\,(3\mathrm{JOINT}+\mathrm{NLOAD}) < 1600$

c.ii Lowest buckling mode

$20\,\mathrm{NBEAM} + 2(3\mathrm{JOINT})^2 < 1600$

c.iii. Stress Analysis (out of core solution)

$\mathrm{NBEAM}(56+19\mathrm{NLOAD}) + 6\mathrm{NLOAD} + 9\mathrm{JOINT} + 2\mathrm{IBAND} + 3\mathrm{MAXIB}\,(9\mathrm{MAXIB} + \mathrm{NLOAD}) < 1600$

and

$3\mathrm{JOINT}\,(3\mathrm{JOINT} + \mathrm{NLOAD}) < 200000$.

c.iv. Data Generation

$26\mathrm{NBEAM} + 7\mathrm{NBEAM}\,\mathrm{NLOAD} + \mathrm{NLOAD} + 2\mathrm{JOINT} < 16000$

c.v. Data Plot

$7\mathrm{NBEAM}\,(3 + \mathrm{NLOAD}) < 16000$

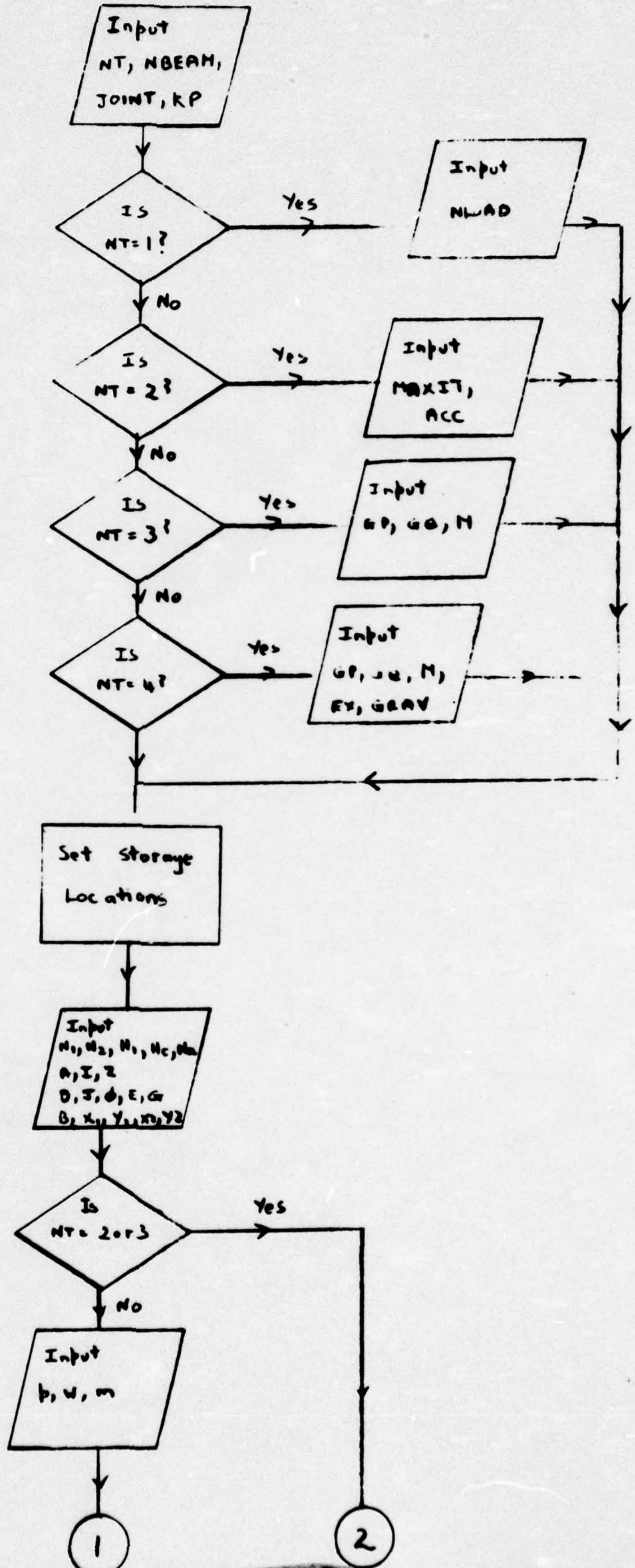
2. **Restrictions on the Elements that can be Used**

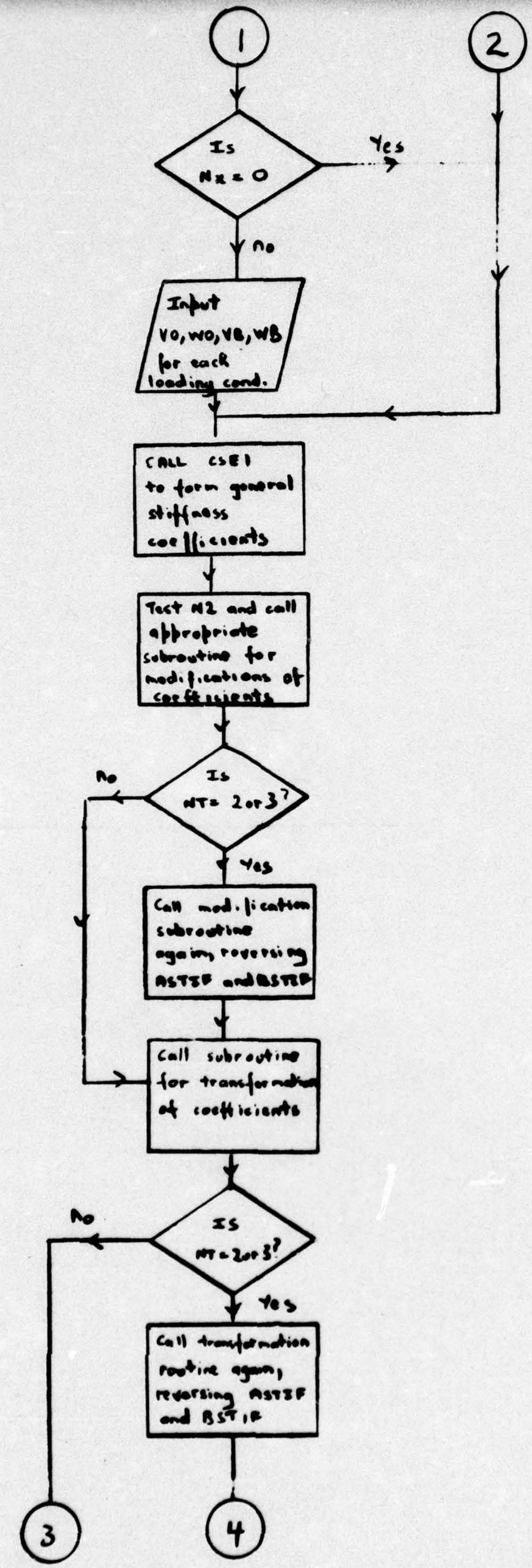
Facility \ Element Type	1	2	3	11	12	13	14	15	29
Stress Analysis	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lowest Buckling Mode	✓	✓	✓	✓	✓	✓	✓	✓	✓
General Buckling Mode	✓	✓	✓	✓	✓	✓			✓
Natural Frequencies	✓	✓		✓	✓	✓			✓

APPENDIX G

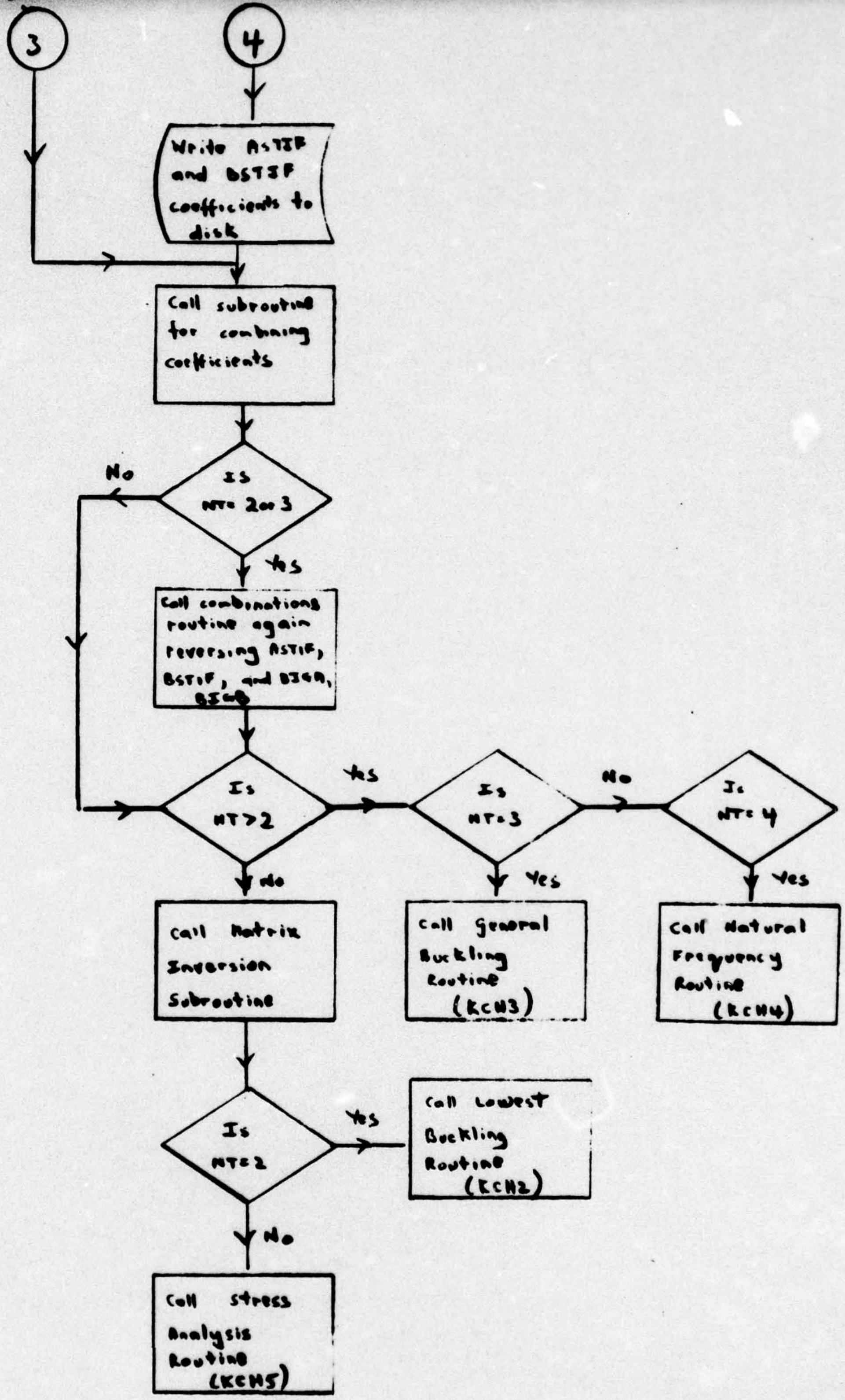
FLOW CHART FOR PROGRAM

FLOWCHART FOR SUITE OF PROGRAMS




1
2
Is
NZ = 0
Yes
No
Input
VO, WO, VB, WB
for each
loading cond.
CALL CSE1
to form general
stiffness
coefficients
Test N2 and call
appropriate
subroutine for
modifications of
coefficients
Is
NT= 2 or 3?
No
Yes
Call modification
subroutine
again, reversing
ASTIF and BSTIF
Call subroutine
for transformation
of coefficients
IS
NT=2 or 3?
No
Yes
Call transformation
routine again,
reversing ASTIF
and BSTIF
3
4

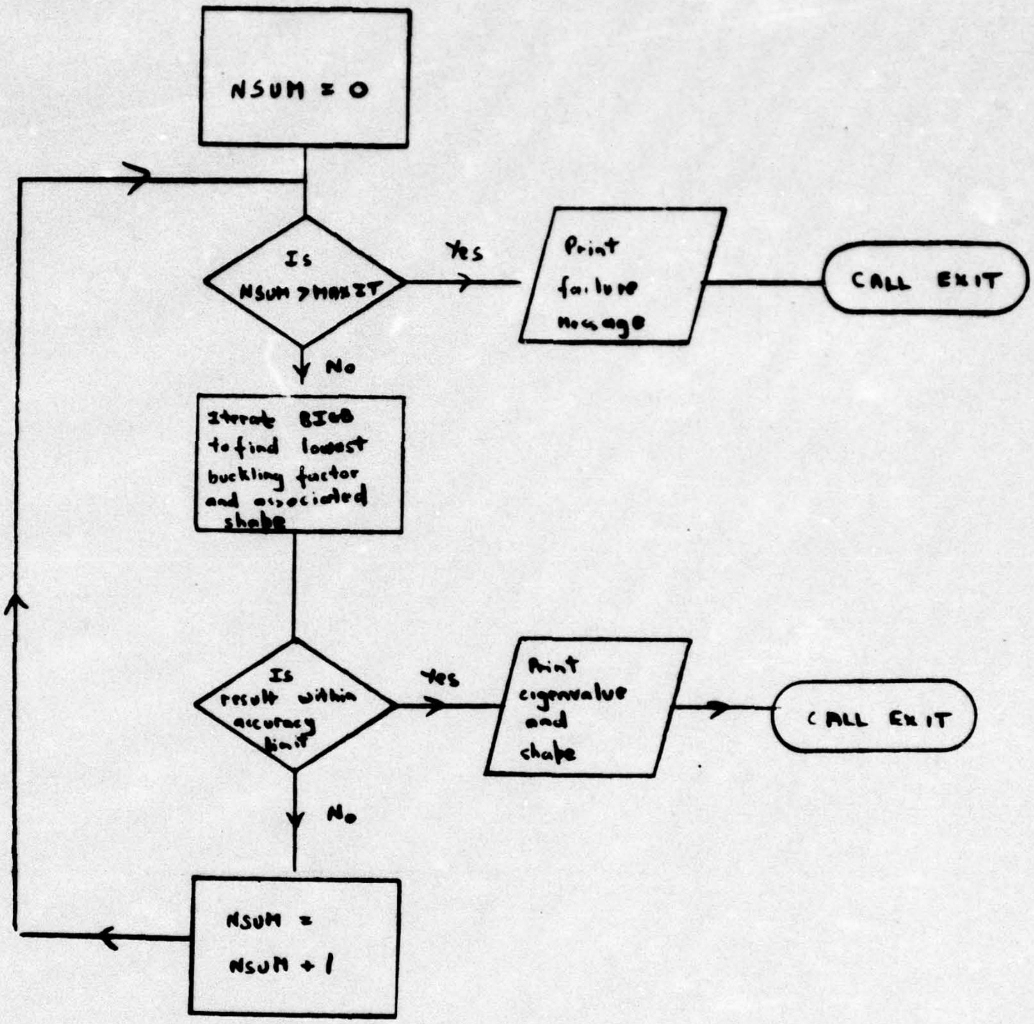
R637 APP G



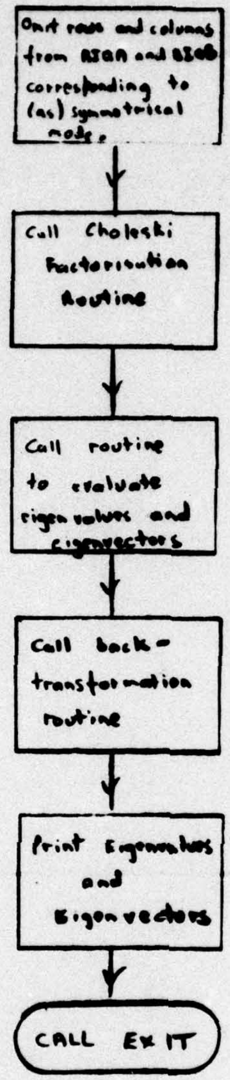
3
4
Write ASTIF and BSTIF coefficients to disk
Call subroutine for combining coefficients
Is NT= 2 or 3
No
Yes
Call combinations routine again reversing ASTIF, BSTIF, and BIGA, BIGB
Is NT>2
Yes
No
Is NT=3
No
Is NT=4
Yes
Yes
Call Matrix Inversion Subroutine
Call General Buckling Routine (KCH3)
Call Natural Frequency Routine (KCH4)
Is NT=2
Yes
No
Call Lowest Buckling Routine (KCH2)
Call Stress Analysis Routine (KCH5)

R637 APP G

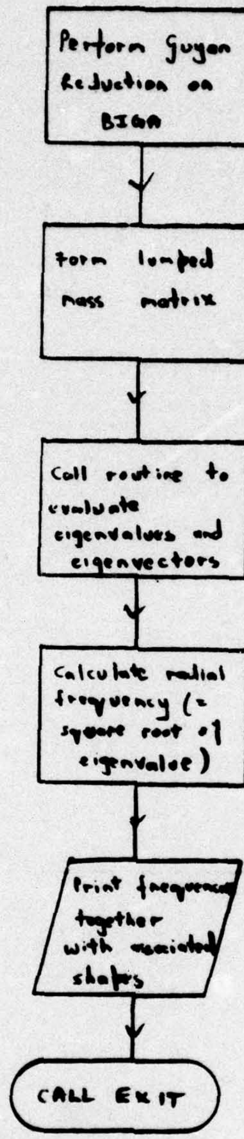
LOWEST BUCKLING ROUTINE (KCH2)



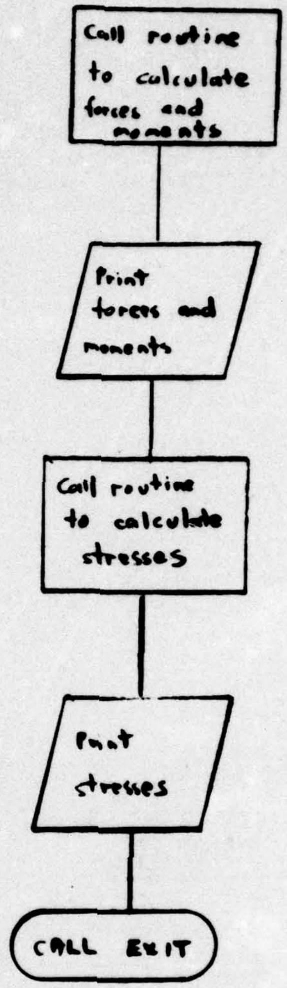
GENERAL BUCKLING ROUTINE (KCH3)



NATURAL FREQUENCY ROUTINE (KCH4)



R637 APP G


Call routine to calculate forces and moments
Print forces and moments
Call routine to calculate stresses
Print stresses
CALL EXIT

APPENDIX H

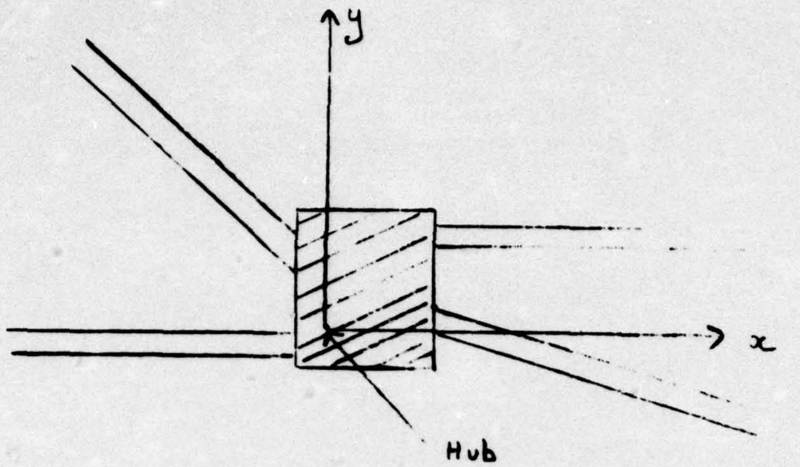
NOTATION

$a^1, b^1, c^1,$	End restraint constants for single ended in plane element $(M_B = -a^1 V_B, T_B = -b^1 \gamma_B, S_B = -c^1 W_B)$
*b	width of faying flange
*d	depth of frame web
*f	width of flange
*hb, hf, hw	thickness of faying flange, flange and frame web respectively
m	A concentrated load acts at a distance mB from end 0 of the element
p	Uniform load applied to the element, positive downwards
Vx, Vy	Rotations in x and y directions
W	Displacement perpendicular to the plane of the grillage, positive downwards
*x, y	Rectangular coordinate axes at each hub point
$*x_1, y_1$	Distances from hub to end 0 of element
$*x_2, y_2$	Distances from hub to end B of element
*Z	Distance of the outermost fibre of the beam cross-section from the neutral axis
A	Effective shear area of element
A^1, B^1	Cross sectional area and length of pillar support
*B	Length of an element
E	Young's Modulus
G	Modulus of shear
I	Moment of inertia of cross-section for bending out of the plain
J	Torsional constant
Mx, My	Moment in x and y directions respectively
NBEAM	Total number of elements in grillage
NT	Parameter to determine version of the program
*N1	Label number of element (at end 0)
N2	Element type number
JOINT	Number of hubs
S	Force perpendicular to the plane of the grillage
T	Torsional moment
W	Concentrated load applied to the element, positive downwards
θ	Angle element makes with x axis
$*\phi$	Angle between element and image
SIGMA 0, SIGMA B	Stress at end 0 (or end B) of element
SIGMA M	Stress at a maximum in bending moment distribution if any
SIGMA P	Stress at point of application of concentrated load if any.

Notation marked with * is illustrated in figure 6

BEAMS SEPARATED BY A RIGID AREA

(a) General Case



(b) Common Practical Cases

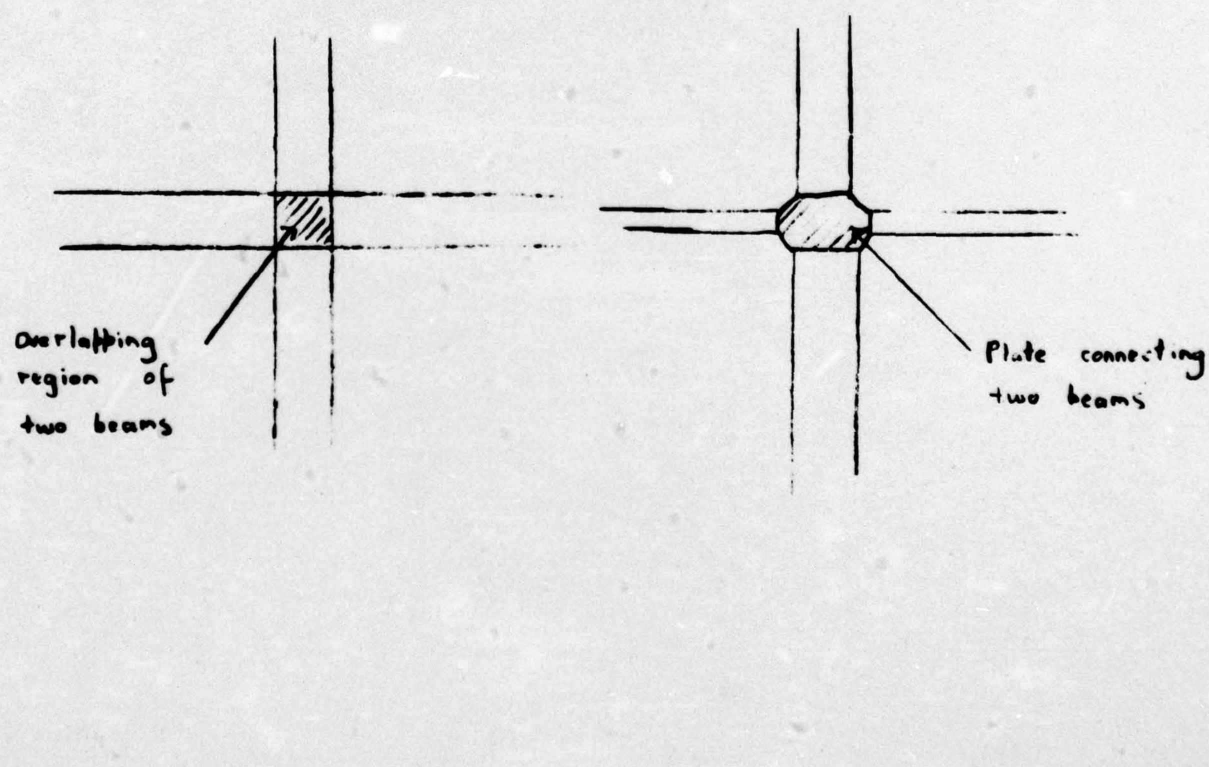
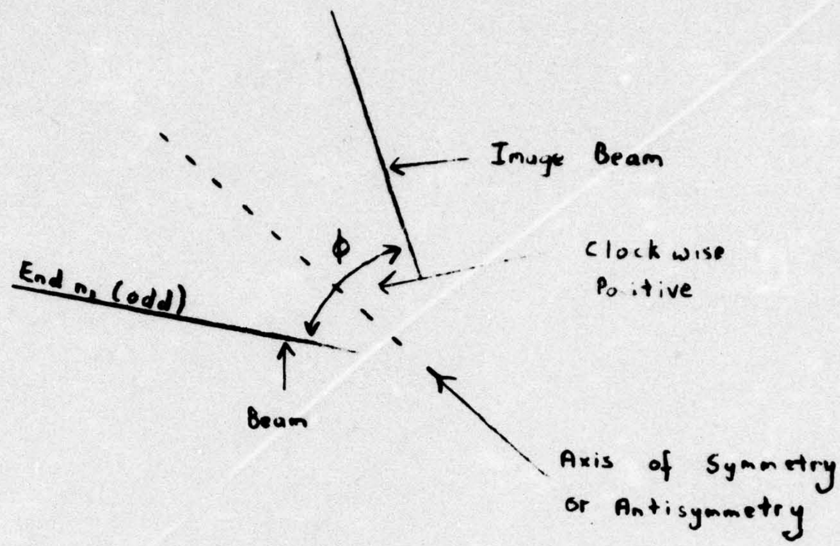


Fig. 1

SIGN CONVENTION FOR AXES OF SYMMETRY OR ANTISYMMETRY



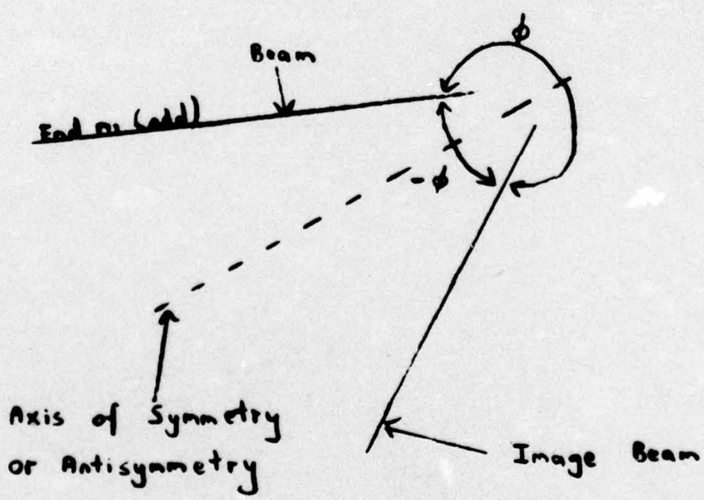
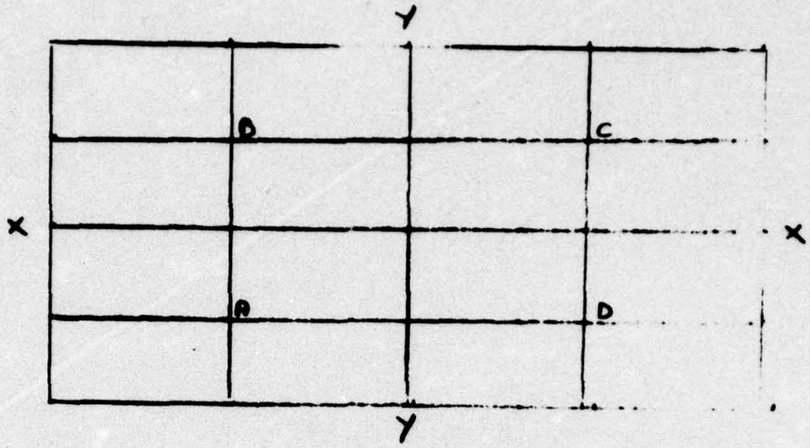
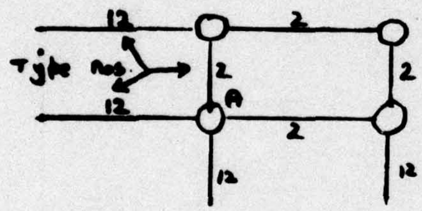


Fig. 2

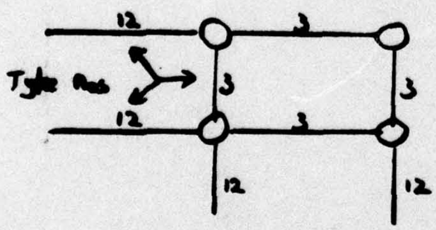
EXAMPLE OF AXES OF SYMMETRY



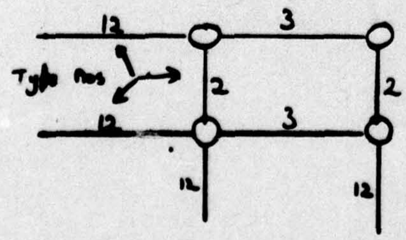
Simple Support at All Edges



(a) Positive Load p at A, B, C, D
2 Axes of Symmetry about XX and YY.



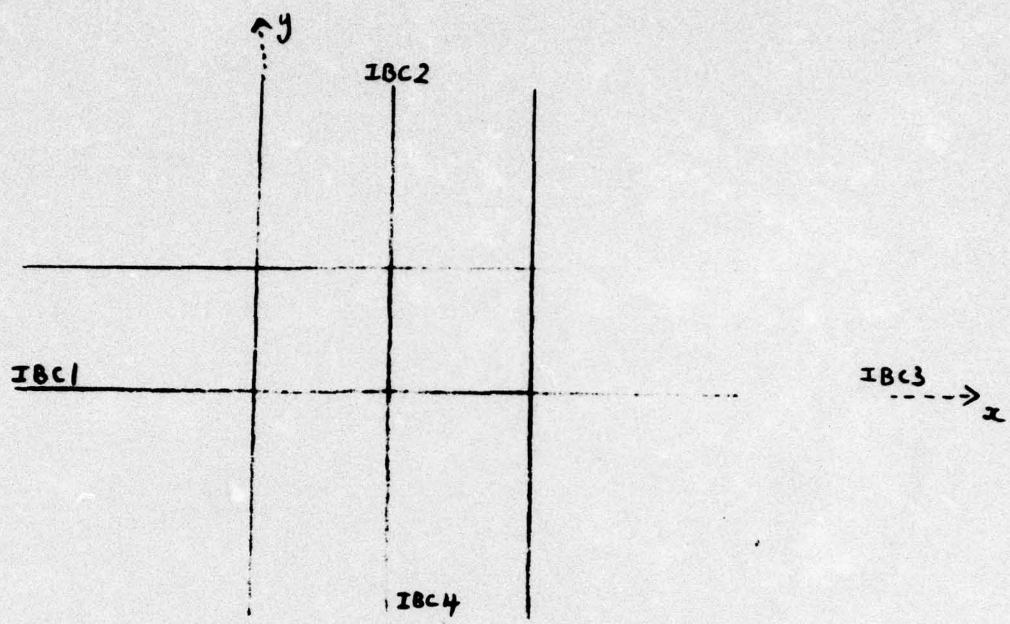
(b) Positive Load p at A and C
Negative Load p at B and D
2 Axes of Antisymmetry about XX and YY



(c) Positive Load p at A and B
Negative Load p at C and D
Axis of Symmetry about XX
Axis of Antisymmetry about YY

Fig. 3

DEFINITION OF IBC2 AND IBC3 FOR DATA GENERATION PROGRAM

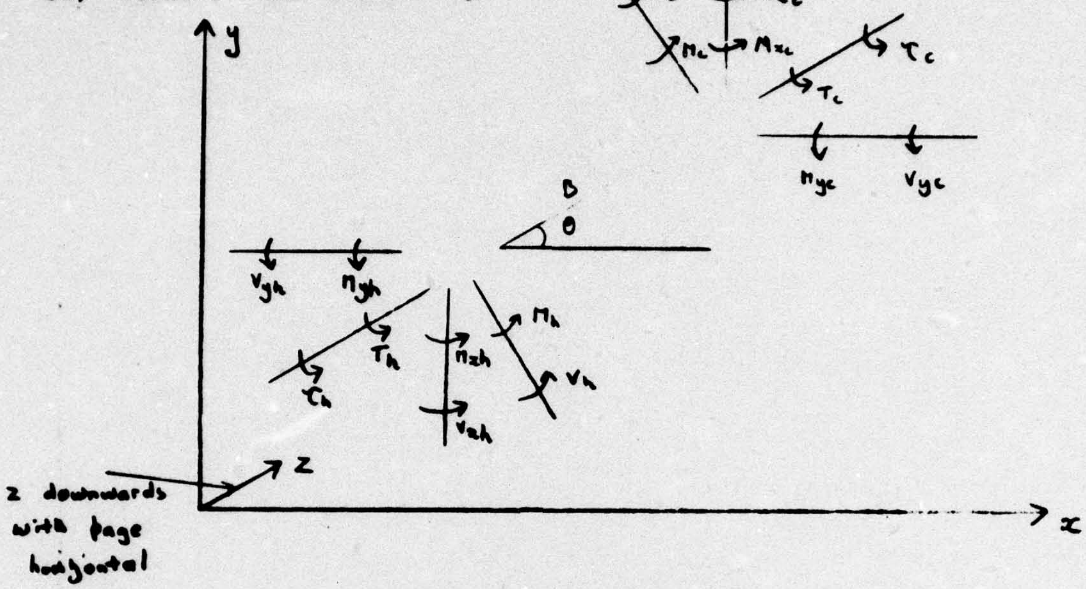


Symmetry or Asymmetry may only occur about IBC2 and IBC3

Fig 14

SIGN CONVENTIONS FOR FORCES AND DISPLACEMENTS (SHOWING LOCAL AND GLOBAL COORDINATE SYSTEMS)

(a) Moments and Rotations



V_c, M_c etc are rotations and moments at end c of the beam in the local coordinate system

V_{zc}, M_{zc} etc are rotations and moments at end c of the beam in the global coordinate system

V_h, M_h etc are rotations and moments at end h of the beam in the local coordinate system

V_{zh}, M_{zh} etc are rotations and moments at end h of the beam in the global coordinate system

M and T are moments on beams. Moments on hubs are of opposite sign

(b) Shear Force and lateral displacement

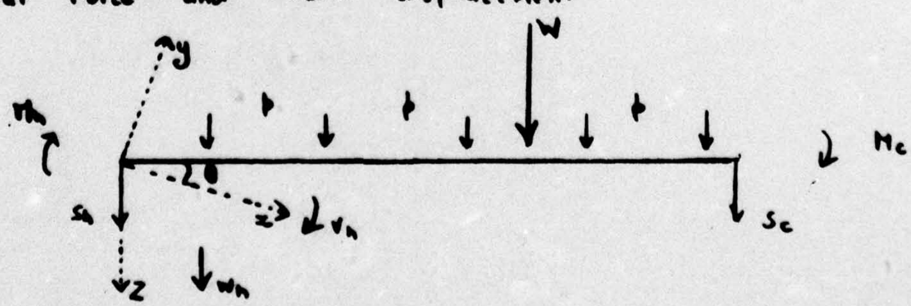
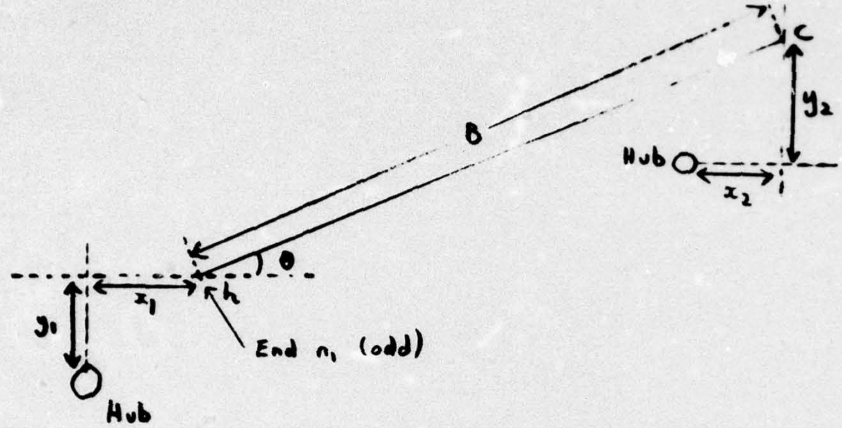


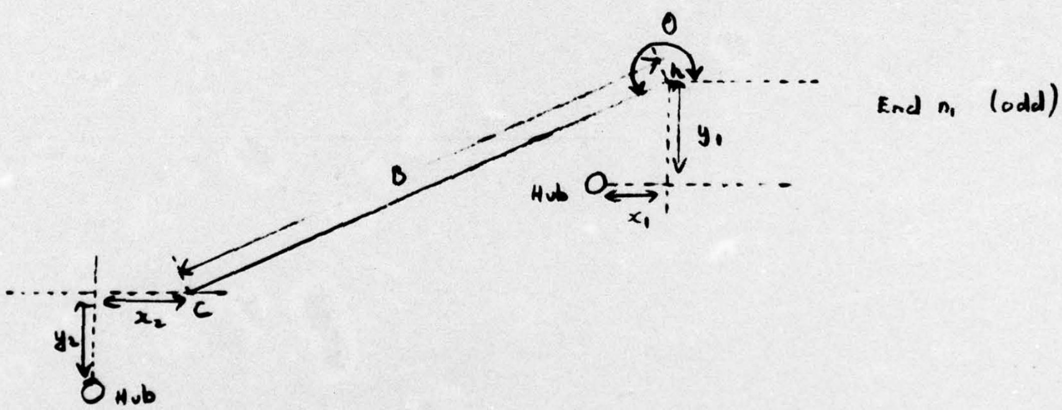
Fig 5

BEAM GEOMETRY NOTATION



Uniform lateral load p, downwards positive

Concentrated load W, distance mB from h, downwards positive



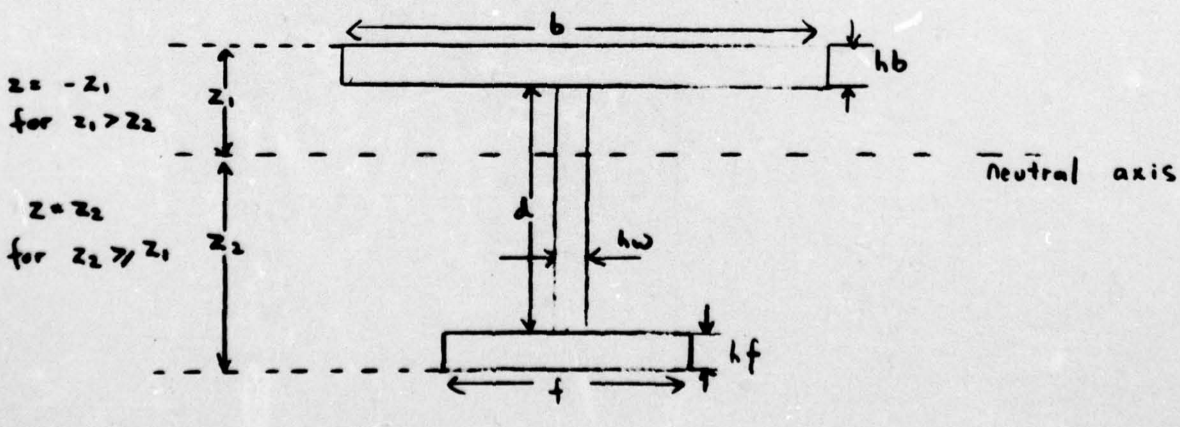
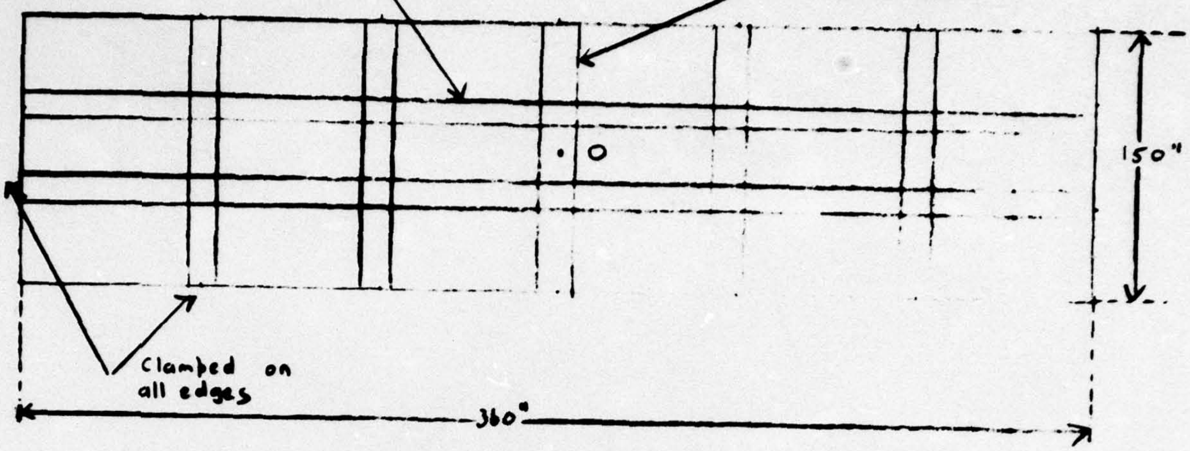


Fig 6

NUMERICAL EXAMPLE 1 (Stress Analysis)

(a) Grillage Geometry



Longitudinal beams	Transverse beams
$A = 5\ in^2$	$A = 2.7\ in^2$
$I = 300\ in^4$	$I = 75\ in^4$
$Z = 2\ in$	$Z = -3\ in$
$J = 0$	$J = 0$

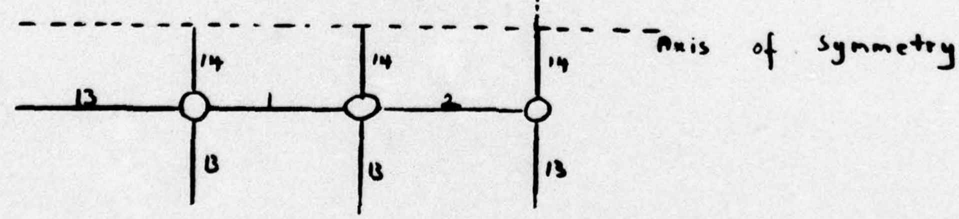
Loading :- There are three separate loading conditions

(i) Uniform ~~loading~~ on all the beams

(ii) A concentrated load at the centre of the grillage ie at point O

(iii) The combined effect of (i) and (ii)

(b) Element type numbers



(c) Label and hub numbers

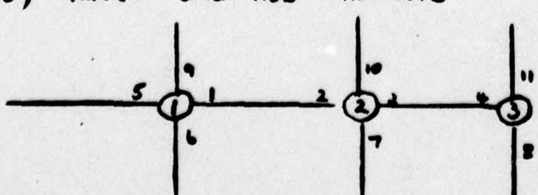
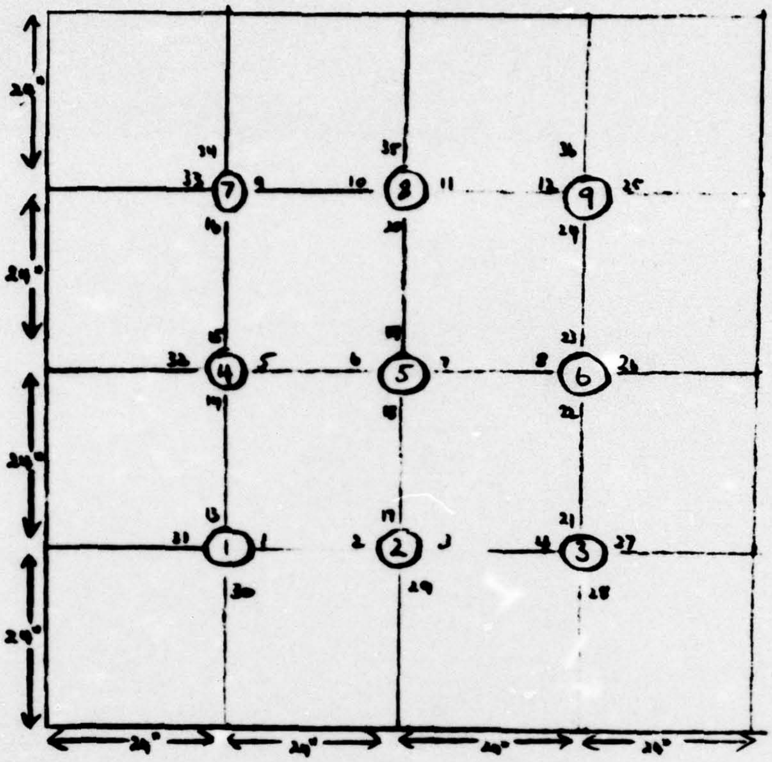


Fig 17

NUMERICAL EXAMPLES 2 AND 3 (Lowest and General Buckling modes)



All single ended elements are type 13

All double ended elements are type 1

For each element ..

A = 200 in^2

I = 4·745 in^4

Z = ·125 in

J = 1·0

The axial force in each element is 1 lb

Fig 3

NUMERICAL EXAMPLE 4 (Natural Frequencies of a 'Free – Free' Beam)

All elements used are type 1

There are no constraints at either end of the beam

Properties of beam

Property \ Elements Label No	1	3	5	7	9	11	13	15	17	19
(in^2) Shear Area	·961	2·5885	2·5885	2·0925	2·5885	2·5885	2·5885	2·5885	2·5885	·9455
(in^4) Moment of Inertia	10·33	77·12	94·90	78·80	96·10	70·15	68·712	62·465	68·71	6·727
(in) Length	4724	5·315	5·0	7·0079	7·0079	6·496	5·5118	6·4961	5·0	5·0
(lbs/in) Mass / unit length	10·1727	27·742	38·139	20·212	16·044	8·731	15·439	18·343	23·655	10·935

Fig. 9

DOCUMENT CONTROL SHEET

(Notes on completion overleaf)

Overall security classification of sheet .... UNLIMITED ....

(As far as possible this sheet should contain only unclassified information. If is is necessary to enter classified information, the box concerned must be marked to indicate the classification eg (R),(C) or (S)).

1. DRIC Reference (if known)	**2. Originator's Reference**	**3. Agency Reference**	**4. Report Security Classification**
	NCRE/R637		UNLIMITED

5. Originator's Code (if known)	**6. Originator (Corporate Author) Name and Location**
	Naval Construction Research Establishment St Leonard's Hill DUNFERMLINE
5a. Sponsoring Agency's Code (if known)	**6a. Sponsoring Agency (Contract Authority) Name and Location**

7. Title

PLANE GRILLAGE SUITE OF PROGRAMS

7a. Title in Foreign Language (in the case of translations)

7b. Presented at (for conference papers). Title, place and date of conference

8. Author 1, Surname, initials	**9a Author 2**	**9b Authors 3, 4...**	**10. Date**	**pp**	**ref**
Simons W J			7.1976	60	7

11. Contract Number	**12. Period**	**13. Project**	**14. Other References**

15. Distribution statement

[redacted]

Descriptors (or keywords)

Plane grillage - computer programs

Abstract The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

NOTES ON COMPLETION OF DOCUMENT CONTROL SHEET

This Document Process Sheet is designed specifically for MOD reports and reports produced by Contractors.

Boxes marked* below need be completed only if the information is readily available.

*1. DRIC reference: Enter DRIC reference (BR number) if one has been assigned.

2. Originator's Reference: Enter the report number by which the document is identified by the originator of the report, in the form in which it appears on the cover.

3. Agency reference: Enter reference number allocated by sponsoring agency (contract authority) in the case of contract reports.

4. Report Security Classification: Enter security classification or marking which limits the circulation of the report, or enter UNLIMITED when this applies.

*5. Originator's Code: Code number for the DRIC-standardised form of the entry appearing in Box 6.

*5a. Sponsoring Agency's Code: Code number for the DRIC-standardised form of the entry appearing in Box 6a.

6. Originator (corporate author): Enter name and location of the organisation preparing the report.

6a. Sponsoring Agency (Contract Authority): Enter the name of the monitoring MOD Branch or Establishment in the case of Contractor's reports. If an MOD report covers work funded by an outside agency, enter the name of that agency.

7. Title: Enter the complete report title in capital letters but omitting initial definite or indefinite articles. If the report covers a specific period, enter this after the title, eg (1.1.1972-31.3.1972).

7a Title in Foreign Language: In the case of translations, enter the foreign language title (transliterated if necessary) and the translated English title in Box. 7.

7b. Conference Papers: If 7 is the title of a paper presented at a Conference, or a Conference proceedings, enter the Conference Title, where it was held and the date.

8. Author 1: Enter the name of the first author, followed by his initials.

9a. Author 2: Enter the name of the second author, followed by his initials.

9b. Authors 3,4...: Enter third and following authors' names.

10. Date: Enter the month (in figures) and the year of the report (Dec., 1969 is written 12.1969). If the report is undated but a period covered by the report is indicated, enter the date at the end of the period. pp.ref. Enter the inclusive number of pages in the report containing information, i.e. including appendices, tables and illustrations, and the total number of references cited.

11. Contract Number: Enter the number of the contract or grant under which the report was written.

12. Period: (always associated with the Contract Number). Only to be used for reports covering a specific period, e.g. quarterly, annual or final reports. Enter QR-1, AR, FR, as appropriate.

13. Project: Enter project name or number.

14. Other Reference: Enter any reference, other than those in Boxes 2 or 3, by which the report may be identified.

15. Distribution statement. Enter any limitations on the distribution of the document. If distribution is limited to particular groups eg MOD, MOD and its Contractors, etc. it should be stated. If the distribution is the responsibility of another authority eg a HQ Directorate (Technical Policy Authority) enter "Distribution controlled by MOD Technical Policy Authority". Enter "via DRIC" after "Authority" where appropriate and name the Technical Policy Authority if possible.

Descriptors: Any number of descriptors (or key-words) can be entered. If selected from a published thesaurus, eg The Thesaurus of Engineering and Scientific Terms (TEST), this should be indicated.

Abstract: The abstract should preferably not exceed 150 words, i.e. it can be considerably shorter than the Abstract to be provided on the Title Page of the Report. Information available in the report title need not be included in the abstract.

NCRE/R637 UNLIMITED

PLANE GRILLAGE SUITE OF PROGRAMS

by W J Simons, 1976

The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

SUBJECT INDEX

Plane grillage - computer programs

NCRE/R637 UNLIMITED

PLANE GRILLAGE SUITE OF PROGRAMS

by W J Simons, 1976

The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

SUBJECT INDEX

Plane grillage - computer programs

NCRE/R637 UNLIMITED

PLANE GRILLAGE SUITE OF PROGRAMS

by W J Simons, 1976

The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

SUBJECT INDEX

Plane grillage - computer programs

NCRE/R637 UNLIMITED

PLANE GRILLAGE SUITE OF PROGRAMS

by W J Simons, 1976

The report describes a suite of programs for calculating deflections and stresses, buckling loads and natural frequencies of a plane grillage. The suite has additional facilities for generating and checking data, and for larger problems, an out of core solution for stresses.

SUBJECT INDEX

Plane grillage - computer programs