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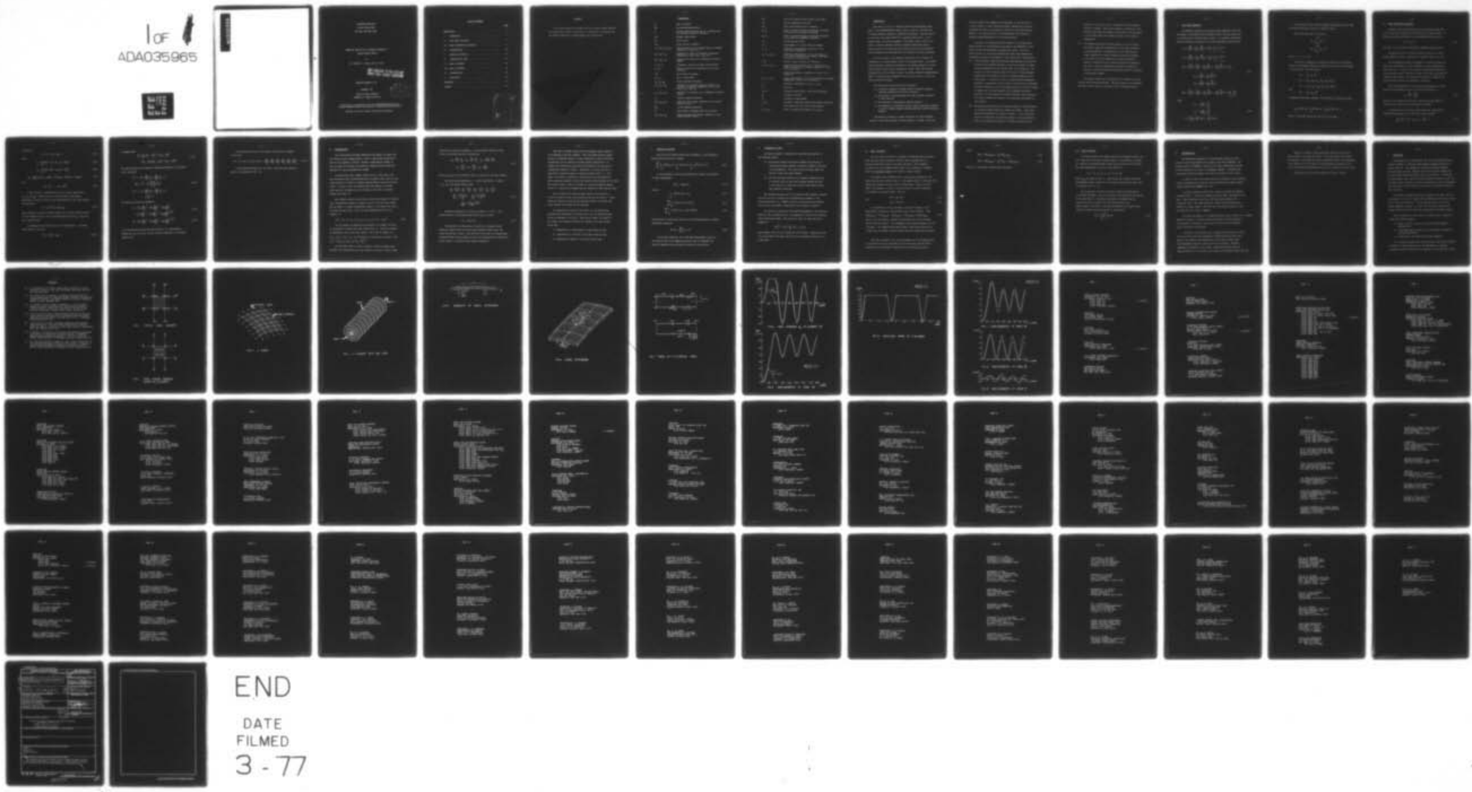

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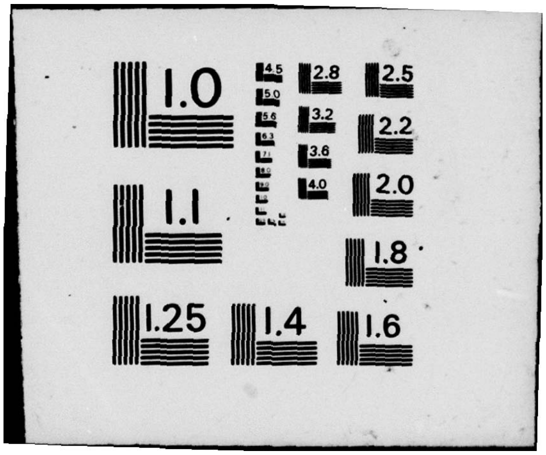
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NUMERICAL ANALYSIS OF THE DYNAMIC RESPONSE OF ELASTO-PLASTIC SHELLS

by

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ABSTRACT

An efficient numerical procedure for the transient dynamic analysis of elasto-plastic shells is introduced. A simple shell is analyzed, and the results achieved are compared against an existing code.

NOMENCLATURE

A_i	Area of element i
$[B]$	Strain-displacement matrix
c_1	Distance between neutral axis of a stiffener and the reference surface of the shell.
$[D]$	Tangent moduli matrix
$\{e\}$	Strain vector
$\{e\}_i$	Strain vector in element i
$\{e_1\},\{e_2\},\{e_3\},\{e_4\}$	Strain vectors in the 4 regions within an element; components of vector $\{e_1\}$.
e_{xx}, e_{yy}, e_{xy}	Components of strain in cartesian coordinates; components of vector $\{e_1\}$, $\{e_2\}$, etc.
e_{11}, e_{22}, e_{12}	Components of strain in an orthogonal coordinate system.
F, F_0, F_L	Subsequent, initial, and limit yield functions
F_s, F_M	Absolute values of the gradients of the yield function F.
$\{F\}_i$	Force vector of element i
h_1, h_2	Metric coefficients
I_N, I_M, I_{NM}	Stress resultant invariants
k_{xx}, k_{yy}, k_{xy}	Components of curvature within an element, in cartesian coordinates; components of the strain vector $\{e_1\}$, $\{e_2\}$ etc.
k_{11}, k_{22}, k_{12}	Components of curvature in an orthogonal coordinate system.
L_1	Distance between stiffeners
M_{11}, M_{22}, M_{12}	Moment per unit length; components of the stress-resultant vector
M^*_{ij}	Strain hardening parameters
N	Total number of elements with the structure.
N_{11}, N_{22}, N_{12}	Normal force per unit length; components of the stress-resultant vector.

$\{P\}$	Vector of external forces acting on the nodes
$\{p\}$	Surface loading per unit area
$\{q\}_i$	Nodal displacement vector of element i.
R_x, R_y	Radii of curvature along the principal directions of the shell in cartesian coordinates.
R_1, R_2	Radii of curvature along the principal directions of the shell in orthogonal coordinates.
$\{s\}$	Stress-resultant vector
$\{u\}$	Displacements of a point within an element.
u_1, u_2	Tangential displacements; components of the vector $\{u\}$ in section II.
u_1, u_2, u_3, u_4	Tangential displacements in the x direction of the 4 nodes contiguous to an element; components of vector $\{q\}$ in section IV.
$\{v\}_j^n$	Velocity vector of node j at time step n.
v_1, v_2, v_3, v_4	Tangential displacements in the y direction of 4 nodes contiguous to an element; component of vector $\{q\}$ in section IV.
w	Normal displacements; components of vector $\{u\}$ in section II.
w_1, $w_2 \ldots w_{12}$	Normal displacements of the 12 nodes around an element; components of vector $\{q\}$ in section IV.
x, y	Cartesian coordinates $x = h_1 \xi_1$, $y = h_2 \xi_2$
Δt	Time step
Δx, Δy	Distance between nodes in cartesian coordinates.
δe	Increment in strain
δu	Increment in displacement
ξ_1, ξ_2	Orthogonal coordinates along the principal curvatures.
ρ	Mass density per unit area of shell surface.
ϕ, ϕ_1, ϕ_2	Twist, rotations with respect to ξ_1 and ξ_2 .

I INTRODUCTION

This report is part of a combined theoretical-experimental study, under a joint DNA/NAVSEA/ONR program, which is aimed at evaluating and increasing submarine hardness to underwater explosions. Previous reports in the series were aimed primarily at studying shock effects, in the elastic range, on internal equipment. The present report is part of an effort to analyze submarine lethality problems, where large elasto-plastic motions of the hull may occur under the action of long duration full envelopment shock loadings, which are produced by nuclear explosions.

The first report in the lethality series, Ref. [1], involves the formulation of an elasto-plastic theory for stiffened shells. The present report describes a numerical procedure for the dynamic analysis of elasto-plastic shells using a direct integration in time. While the examples presented here are for shells in vacuo, the Doubly Asymptotic Approximations (DAA) is currently being implemented into the code in order to treat the fluid interaction problem.

The objectives of this work are the following:

1) A realistic modeling of complex shells of arbitrary geometry, including stiffeners and internal structure.

2) The analysis of shock phenomena with high frequency components in their spectrum.

3) The modeling of elasto-plastic material behavior.

4) The capability of taking into account large displacement gradients in order to analyze dynamic buckling conditions and post-buckling behavior.

The realistic modeling of complex structures for shock phenomena results in very large problems, with many degrees of freedom. This fact and the nonlinearities implied by the conditions (3) and (4) point to a finite element or finite difference method, together with the direct integration in time of the equations of motion, as the most promising approach. The nonlinearities of the problem makes the normal-mode expansion method unsuitable for the present purposes.

A survey of the existing numerical methods of analyzing elasto-plastic shells for shock problems, undertaken at the beginning of this work, revealed that additional development was needed. There are several aspects to the existing methods which make them inappropriate in meeting the analysis requirements of the present work. Specifically:

(a) A refined element, employing high-order approximating functions, has proven to be an efficient approach to the static analysis of shells and the determination of low natural modes and frequencies. The accuracy of such an element allows for the use of larger elements, reducing the total number of degrees of freedom required for the entire shell. In contrast to this, in shock and wave propagation problems, a large number of mass points is the only way of achieving an accurate solution, even if the element itself is relatively simple. It appears that the shell elements with "condensed" stiffness matrices, i.e. with massless and loadless internal nodes eliminated by a static condensation procedure, are especially unsuitable for this purpose.

(b) The masses associated with the rotational degrees of freedom present in the finite element method are usually very small. This causes numerical difficulties in an explicit scheme. If the "rotational masses" are neglected, the solution consists of the integration in time of the translational degrees of freedom, together with the solution of a system of "static" equations for the rotational degrees of freedom. Since these equations are nonlinear, their solution (which must be repeated at each step of the integration in time) would require an enormous increase in the number of computations.

(c) For problems involving a structure subject to cyclic loads in the plastic range, the determination of the stress resultants in existing methods is accomplished by first computing the stress components at several locations through the thickenss of the shell, and then computing the stress resultants by numerical integration of the stresses through the thickness of the shell. While the computations involved are simple, the amount of stored data becomes prohibitively large for structures involving many elements.

The approach adopted in the present work is believed to be free from the above mentioned drawbacks. The main aspects of this approach and their current status are discussed in the following sections.

II BASIC SHELL EQUATIONS

The kinematic equations of the shell theory employed in this work correspond to the Donnell-Vlasov nonlinear theory, with the option open of refining them to achieve Sander's theory. In orthogonal coordinates along the principal curvatures, the strain-displacement relations read

$$e_{11} = \frac{\partial u_1}{h_1 \partial \xi_1} + \frac{1}{h_1 h_2} \frac{\partial h_1}{\partial \xi_2} u_2 + \frac{w}{R_1} + \underline{\frac{1}{2} \phi_1{}^2} + \underline{\frac{1}{2} \phi^2}$$

$$e_{22} = \frac{\partial u_2}{h_2 \partial \xi_2} + \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial \xi_1} u_1 + \frac{w}{R_2} + \underline{\frac{1}{2} \phi_2{}^2} + \underline{\frac{1}{2} \phi^2}$$

$$2e_{12} = \frac{\partial u_2}{h_1 \partial \xi_1} + \frac{\partial u_1}{h_2 \partial \xi_2} - \frac{1}{h_1 h_2} \frac{\partial h_1}{\partial \xi_2} u_1 - \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial \xi_1} u_2 + \underline{\phi_1 \phi_2}$$

$$k_{11} = \frac{\partial \phi_1}{h_1 \partial \xi_1} + \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial \xi_1} \phi_2 \qquad (2.1)$$

$$k_{22} = \frac{\partial \phi_2}{h_2 \partial \xi_2} + \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial \xi_2} \phi_1$$

$$2k_{12} = \frac{\partial \phi_2}{h_1 \partial \xi_1} + \frac{\partial \phi_1}{h_2 \partial \xi_2} - \frac{1}{h_1 h_2} \frac{\partial h_1}{\partial \xi_2} \phi_1 - \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial \xi_1} \phi_2 + \underline{\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$

where

$$\phi_1 = - \frac{\partial w}{h_1 \partial \xi_1} + \underline{\frac{u_1}{R_1}}$$

$$\phi_2 = - \frac{\partial w}{h_2 \partial \xi_2} + \underline{\frac{u_2}{R_2}} \qquad (2.2)$$

$$\phi = \frac{1}{2} \frac{1}{h_1 h_2} \left[\frac{\partial}{\partial \xi_1} (h_2 u_2) - \frac{\partial}{\partial \xi_2} (h_1 u_1)\right]$$

The underlined terms represent geometric nonlinearities; the terms with broken underlines are those of Sander's theory.

The stress resultants are defined as

$$N_{ij} = \int_{-h/2}^{h/2} s_{ij} \, dz$$

$$M_{ij} = \int_{-h/2}^{h/2} s_{ij} \, z \, dz$$

with i = 1, 2; j = 1, 2.

The theory is completed by writing the equations of equilibrium which, for the present purposes, are assumed in the form of the Principle of Virtual Work. With the notation

$$\{u\} = (u_1, u_2, w)^T$$

$$\{s\} = (N_{11}, N_{22}, N_{12}, M_{11}, M_{22}, M_{12})^T$$

$$\{e\} = (e_{11}, e_{22}, 2e_{12}, k_{11}, k_{22}, 2k_{12})^T$$

with

$$\{p\} = (p_1, p_2, p_3)^T$$

standing for the surface loading. The Principle of Virtual Work reads

$$\int_S \{s\}^T\{\delta e\} \, dS - \int_S \{p\}^T\{\delta u\} \, dS + \int_S \rho\{\ddot{u}\}^T\{\delta u\} \, dS = 0 \tag{2.3}$$

where ρ is the mass density per unit area of the shell.

III SHELL CONSTITUTIVE EQUATIONS

The shell constitutive equations are the relations between the rates of the stress resultants and the rates of the shell strains. In the matrix notation introduced in the preceeding section, they are

$$\{\dot{s}\} = [D] \{\dot{e}\} \qquad (3.1)$$

where [D] is the so-called elasto-plastic tangent stiffness matrix.

The explicit from of the above relation is based on an elasto-plastic theory for shells presented in a separate report (Ref. [1]).

The aforementioned theory is analogous to classical theories of plasticity; consisting of a yield condition, a strain hardening rule, and a flow rule. It differs from classical elasto-plastic theories in that the yield surface is defined in terms of the stress resultants {s} instead of the stresses. This avoids the necessity of computing and storing stresses through the thickness of the shell.

The stress components at the top and bottom surfaces of a solid shell can be expressed in terms of the stress resultants as:

$$\sigma_{ij} = \frac{N_{ij}}{h} \pm \frac{6M_{ij}}{h^2} \qquad (3.2)$$

where h is the thickness of the shell, and the plus sign applies to the top and the minus sign to the bottom of the shell.

An expression for the initial yield surface is constructed by Substituting expression (3.2) into Mises' yield condition

$$\frac{1}{\sigma_0^2} (\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11} \sigma_{22} + 3 \sigma_{12}^2) = 1 \qquad (3.3)$$

resulting in

$$F_0 = I_N + I_M \pm 2\, I_{NM} = 1 \tag{3.4}$$

where

$$I_N \equiv \frac{1}{N_0^2}(N_{11}^2 + N_{22}^2 - N_{11}\, N_{22} + 3N_{12}^2) \tag{3.5}$$

$$I_M \equiv \frac{1}{M_0^2}(M_{11}^2 + M_{22}^2 - M_{11}\, M_{22} + 3M_{12}^2) \tag{3.6}$$

$$I_{NM} \equiv \frac{1}{N_0 M_0}(N_{11}\, M_{11} + N_{22}M_{22} - \frac{1}{2}\, N_{11}N_{22} - \frac{1}{2}\, N_{22}M_{11} + 3N_{12}M_{12}) \tag{3.7}$$

and

$$N_0 = \sigma_0 h \quad , \quad M_0 = \sigma_0 h^2/6 \tag{3.8}$$

A limit surface is assumed which is also a linear combination of I_N, I_M, and I_{NM}. Coefficients for the three terms are determined empirically in order to produce a good approximation for the limit surface. The expression,

$$F_L = I_N + \frac{4}{9}\, I_M \pm \frac{2}{3\sqrt{3}}\, I_{NM} \tag{3.9}$$

which represents the limit condition exactly for the three extreme cases of (1) only membrane forces, (2) only bending moments and (3) $N_{11} = N_{22}$ with $M_{11} = M_{22}$, was chosen.

A hardening rule is generated in the following manner. A variable yield surface of the form

$$F \equiv I_N + I_M^* \pm \alpha\, I_{NM} = 1 \tag{3.10}$$

is assumed where

$$I_M^* = \frac{1}{M_0^2} [M_{11} - M_{11}^*)^2 + (M_{22} - M_{22}^*)^2$$
$$- (M_{11} - M_{11}^*)(M_{22} - M_{22}^*) + 3(M_{12} - M_{12}^*)^2] \qquad (3.11)$$

The quantities M_{ij}^*, which represent "hardening parameters", are defined by the following:

If $F = 1$ and $\frac{\partial F}{\partial N_{ij}} \dot{N}_{ij} + \frac{\partial F}{\partial M_{ij}} \dot{M}_{ij} > 0$:

$$dM_{ij} = 2(1 - F_L) \frac{M_0}{k_0} \frac{F_s^2}{F_M^2} dk_{ij}'' \qquad (3.12)$$

If $F < 1$ or $\frac{\partial F}{\partial N_{ij}} \dot{N}_{ij} + \frac{\partial F}{\partial M_{ij}} \dot{M}_{ij} \leq 0$:

$$d\, M_{ij}^* = 0$$

The symbols F_s and F_M are defined as

$$F_s = [(N_0 \frac{\partial F}{\partial N_{11}})^2 + (N_0 \frac{\partial F}{\partial N_{22}})^2 + (N_0 \frac{\partial F}{\partial N_{12}})^2$$
$$+ (M_0 \frac{\partial F}{\partial M_{11}})^2 + (M_0 \frac{\partial F}{\partial M_{22}})^2 + (M_0 \frac{\partial F}{\partial M_{12}})^2]^{1/2} \qquad (3.13)$$

$$F_M = [(M_0 \frac{\partial F}{\partial M_{11}})^2 + (M_0 \frac{\partial F}{\partial M_{22}})^2 + (M_0 \frac{\partial F}{\partial M_{12}})^2]^{1/2} \qquad (3.14)$$

F_s is the absolute value of the vector grad F, in a dimensionless formulation; F_M is the part of grad F which corresponds to the bending moments only.

An associated flow rule for the plastic strain rates is assumed of the form:

$$(e_{11}, e_{22}, e_{12}, k_{11}, k_{22}, k_{12}) = \lambda\left(\frac{\partial F}{\partial N_{11}}, \frac{\partial F}{\partial N_{22}}, \frac{\partial F}{\partial N_{12}}, \frac{\partial F}{\partial M_{11}}, \frac{\partial F}{\partial M_{22}}, \frac{\partial F}{\partial M_{12}}\right) \tag{3.15}$$

A more detailed discussion of the theory, along with some numerical results, are presented in Ref. [1].

IV DISCRETIZATION

The discretization procedure adopted has two aspects in common with the classical finite element method. First, a variational derivation is used for the equations of motion. Second, a quadrilateral element is used, in which the stresses and strains are computed from the nodal displacements of nodes surrounding the element.

A quadrilateral shell element defined by four corner nodes, with each node having three translational and no rotational terms, does not have enough degrees of freedom to represent bending behavior (second derivative terms). In order to have the required additional degrees of freedom, eight nodes not contiguous with the element are also used in computing bending terms (Fig. 1).

Each element accesses twelve nodes and has twenty degrees of freedom; three translational degrees of freedom for each of the four inner nodes, and one degree of freedom (displacement normal to the surface) for each of the eight exterior nodes. Thus, the nodal displacement vector of an element i is

$$\{q\}_i = (u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4, w_1, w_2 \ldots\ldots w_{12})^T \qquad (4.2)$$

For the purpose of computing the stiffness of an element, the area of the element is divided into four regions (Fig. 2). One set of strains is computed for each of the four regions. The strain in element i is $\{e\}_i = (\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\})^T$ where $\{e_1\}$ is the strain in region 1, and $\{e_1\} = (e_{xx}, e_{yy}, 2e_{xy}, k_{xx}, k_{yy}, 2k_{xy})^T$.

At the present time, in order to analyze a series of simple check problems, the discretization has been limited to the case of small strains expressed in cartesian coordinates. In this simplest version of shell theory, the strains have the following form:

$$e_{xx} = \frac{\partial u}{\partial x} + \frac{w}{R_x}\ , \quad e_{yy} = \frac{\partial v}{\partial y} + \frac{w}{R_y}\ , \quad e_{xy} = \frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]$$

$$k_{xx} = \frac{\partial^2 w}{\partial x^2}\ , \quad k_{yy} = \frac{\partial^2 w}{\partial y^2}\ , \quad k_{xy} = \frac{\partial^2 w}{\partial x \partial y} \tag{4.2}$$

where R_x and R_y are the principal radii of curvature of the shell surface.

The discrete approximations to the above expressions, in region 1 of A_i, are (for equally spaced nodes)

$$\frac{\partial u}{\partial x} = \frac{u_2 - u_1}{\Delta x}\ , \quad \frac{\partial v}{\partial y} = \frac{v_4 - v_1}{\Delta y}\ , \quad \frac{\partial u}{\partial y} = \frac{u_4 - u_1}{\Delta y}\ , \quad \frac{\partial v}{\partial x} = \frac{v_2 - v_1}{\Delta x}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{w_2 - 2w_1 - w_5}{\Delta x^2}\ , \quad \frac{\partial^2 w}{\partial y^2} = \frac{w_4 - 2w_1 + w_6}{\Delta y^2} \tag{4.3}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{w_3 - w_2 - w_4 + w_1}{\Delta x \Delta y}$$

Analogous expressions are formed for regions 2, 3 and 4. This process defines a strain-displacement matrix $[B]_i$ such that

$$\{e\}_i = [B]_i\ \{q\}_i \tag{4.4}$$

The expression for derivatives is basically a staggered finite difference scheme, with first derivatives computed between nodes, and second derivatives at nodes. One should note that there is no displacement function assumed over the element area for the determination of derivatives, as one finds in a classical finite element formulation.

This type of element, which uses non-contiguous nodes, results in an overlapping of adjacent elements. This overlapping produces complications at boundaries similar to those encountered in finite difference formulations, but are absent from finite element formulations. To minimize the difficulties associated with boundaries, the structure is conceptually divided into sheets. Each sheet is a curved section of shell with an arbitrary number of nodes and elements (Fig. 3). The shape of the sheet is limited to a surface that can be described by a smooth continuous function without any interior discontinuities in its slope. The elements within a sheet are limited to a purely rectangular organization (i.e. exactly four elements must be connected to each interior node).

Thus a cylinder with end caps would consist of three sheets; a circular cylindrical sheet, and a sheet for each end cap (Fig. 4). Three sheets are required to specify this structure because of the edge that occurs between the cylinder and the end caps.

By organizing the structure into sheets, all the difficulties associated with boundaries are isolated into a set of artificial nodes around the perimeter of the sheet. When several sheets are connected at an edge, the following conditions are imposed on the nodes along a shared edge:

1) Compatability of displacements of nodes along the edge.

2) Compatability of rotations of the nodes along the edge.

3) Equilibrium of moments at the nodes along an edge.

V EQUATIONS OF MOTION

With the shell surface divided into N elements A_i, the Principle of Virtual Work from Section 2 becomes

$$\sum_{i=1}^{i=N} [\int_{A_i} \{s\}_i^T\{\delta e\}_i \; dA - \int_{A_i} \{p\}^T\{\delta u\} \; dA + \int_{A_i} \rho\{\ddot{u}\}^T\{\delta u\} \; dA = 0 \tag{5.1}$$

If the variations of strain are expressed in terms of the variation of nodal displacements

$$\{\delta e\}_i = [B]_i\{\delta q\}_i \tag{5.2}$$

and with

$$\int_{A_i} \{s\}_i^T \; [B]_i \; dA = \{F\}_i \tag{5.3}$$

$$\sum_{i=1}^{i=N} \int_{A_i} \{p\}^T\{\delta u\} \; dA = \{p\}^T\{\delta q\} \tag{5.4}$$

$$\sum_{i=1}^{i=N} \int_{A_i} \rho\{\ddot{u}\}^T\{\delta u\} \; dA = ([m] \; \{\ddot{q}\})^T\{\delta q\} \tag{5.5}$$

the Principle of Virtual Work results in the following system of ordinary differential equations

$$[m] \; \{\ddot{q}\} = -\sum_{i=1}^{i=N} \{F\}_i + \{P\} \tag{5.6}$$

In the above equations, {q} is the nodal displacement vector for the structure, [m] is the lumped mass matrix, and {P} represents the vector of external forces acting on the nodes of the structure.

VI INTEGRATION IN TIME

An explicit scheme of integration in time has been selected for the following reasons:

(a) An implicit scheme would require assembly and inversion of large matrices whose elements, in general, change at each step of one integration. This aspect would seriously impair the ability to deal with large problems.

(b) The more favorable stability of an implicit scheme would not be utilized, since in most shock loading problems the size of the time step is limited by accuracy requirements rather than stability conditions.

The intended incorporation of the method of shell analysis resulting from this work into a program for three-dimensional dynamics of the interacting medium (e.g., TRANAL) provides an additional strong argument for the direct and explicit integration of the equations of motion.

The system of equations for the nodal displacements of the structure, derived in the preceding section, is integrated in time with the following central difference scheme. The velocity of node j at time step n+1 is computed by

$$\{\dot{q}\}_j^{n+1} = \{\dot{q}\}_j^{n} + \frac{\Delta t}{m_j} \left(\{P\}_j - \sum_i \{F\}_i\right)$$

where element forces $\{F\}_i$ are summed over all elements i framing into node j, m_j is the mass of the node, and $\{P\}_u$ are the externally applied forces on the nodes.

VII SMALL STIFFENERS

The term "small stiffeners" is applied to relatively small stiffeners whose spacing is small enough to allow for their representation as a continuous additional layer of the shell structure. This assumption is identical to the assumption which leads to the treatment of stiffened shells as orthotropic shells in the theory of elastic shells.

An element of the stiffened shell is conceptually divided into the shell sheet and the stiffener (Fig. 5). The middle surface of the shell sheet remains the reference surface of the stiffened shell. Within the shell sheet, the increment in strain and the increment in stress resultants are computed in the usual manner. That is,

$$\{de\} = [B]\ \{dq\} \tag{7.1}$$

and

$$\{ds\} = [D]\ \{de\} \tag{7.2}$$

For stiffeners in the ξ_1 direction, the strain increment in the shell sheet is referred to the centroidal axis of the stiffener. Thus,

$$(dk_{11})_{stiff} = (dk_{11})_{sheet}\ ,\ (de_{11})_{stiff} = (de_{11})_{sheet} - c_1 (dk_{11})_{sheet} \tag{7.3}$$

where "c_1" is the distance between the centroidal axis of the stiffener and the reference surface. The increments in the stress resultants for the stiffeners are computed using elasto-plastic constitutive equations for a beam, which are simply a reduced form of the shell constitutive equations

The total increment of the stress resultants for the stiffened shell is the sum of the stress resultants for the shell sheet and the stress resultants of the stiffeners referred back to the reference surface.

Thus,

$$dN_{11} = (dN_{11})_{sheet} + \frac{1}{L_1}(dN_{11})_{stiff} \tag{7.4}$$

$$dM_{11} = (dM_{11})_{sheet} + \frac{1}{L_1}(dM_{11} - c_1 dN_{11})_{stiff} \tag{7.5}$$

where L_1 is the distance between small stiffeners.

VIII LARGE STIFFENERS

For large stiffeners the element mesh must be arranged in such a way that a mesh line runs along a large stiffener. The stiffener is treated as a distinct curved beam element lying along the mesh line of the shell. The nodal displacement vector for a beam element is

$$\{q\}_j = (u_1, u_2, v_1, v_2, w_1, w_2, w_3, w_4) \tag{8.1}$$

where u_1-w_4 are shown in Fig. 6. Beam strains are computed from nodal displacements using a reduced set of relations consistent with those used in the shell (Eq. 4.3, 4.4).

From this point, the beam is treated in exactly the same manner as the small stiffener. That is, the strains are referred to the centroid of the beam (Eq.7.3). The stress resultants are computed from the strains, and these resultants are then referred back to the middle surface of the shell. The beam contribution to the nodal forces along $\{q\}_j$ follows from the principle of virtual work.

$$\{F\}_i^T = \int_{S_j} \begin{Bmatrix} N \\ M \end{Bmatrix}_j^T [B]_j \, dS \tag{8.2}$$

IX IMPLEMENTATION

The formulation summarized in the preceeding sections is in the process of being incorporated into a comptuer code called SHELPLAS. The present version of SHELPLAS is limited to the small displacement analysis of shells modeled by one sheet of rectangular elements. Stiffeners and elasto-plastic material behavior have been incorporated in this version.

In order to verify the overall approach incorporated in this analysis, some test problems were run and the results obtained were compared against results computed with DYNAPLAS (Ref. [2]).

A circular cylindrical shell with a radius of 16.8125" was modeled as shown in Fig. 7. Forty five elements were used along the length of the shell. Small stiffeners, spaced at 5.625 inches, were modeled as an orthotropic shell. The large stiffeners, spaced at 50.625 inches, were modeled as a single element. The left end of the shell was built in, and the right end was a plane of symmetry.

The model was subject to a triangular load (in time), see Fig. 8, of uniform pressure (in space). The peak pressure was 670 psi, and the duration of the load pulse (.5 millesec.) was close to the period of the breathing mode of the shell (.55 millisec).

A history of hoop stresses (σ_θ) at element #28 was plotted in Fig. 8. One can see that there was considerable yielding during the first cycle. Figure 9 is a graph of the displacements versus time from both codes. Peak displacements agree to within less than one percent. Excellent agreement was achieved for all points a sufficient distance from the large stiffeners, as can be seen from a graph of the displaced shape (Fig. 10).

Figures 11 through 13 show displacement histories at the three nodes closest to the large stiffeners. As one approaches the stiffener, the gradient of the moment M_x becomes considerably larger. The differences in the results of the two codes, as one approaches the stiffener, were due to the different approximations of M_x in the elements of the two codes.

The execution time for both codes was less than 2 minutes.

X CONCLUSIONS

As mentioned in the introduction, this work is concerned with the inelastic response of submarines under dynamic loadings generated by a nuclear explosion. The complexities involved in modeling submarine hulls with major internal components require that careful consideration be given toward optimizing computer usage in the analysis. To this end, an elastó-plastic shell theory has been developed and reported in Ref. [1]. The present report describes a numerical procedure and a direct integration code which uses the shell theory in the analysis of stiffened cylindrical shell structures.

The excellent agreement achieved in comparison with other codes for the analysis of a series of simple check problems has verified the correctness of the overall approach. For the eventual analysis of the very complex submarine problems, it is felt that the present numerical procedures will prove much more efficient than presently available methods.

The code presented in this report is currently being augmented to include the following aspects:

1. Fluid - structure interaction using the Doubly Asymptotic Approximation.
2. Large displacement capability to study dynamic buckling and postbuckling behavior.
3. Complex shell structures with internal components.

It is currently planned that this work will be sufficiently advanced to enable us to make predictions for the experiments on stiffened cylindrical structures which will be conducted in the second half of 1977.

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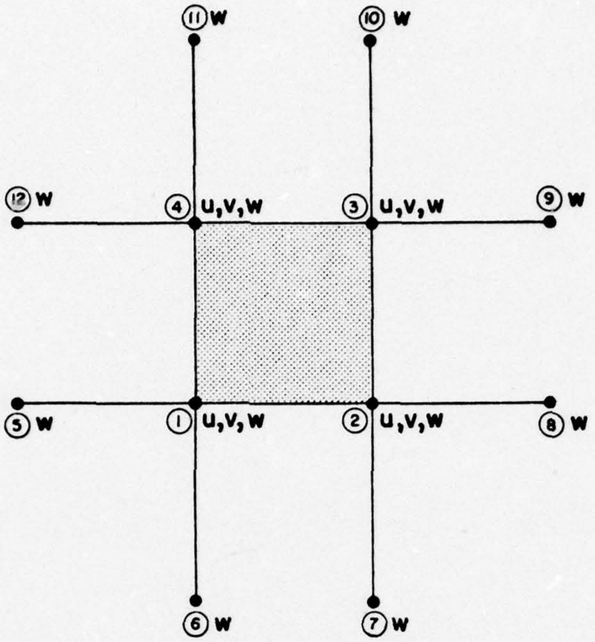

FIG. 1 TYPICAL SHELL ELEMENT

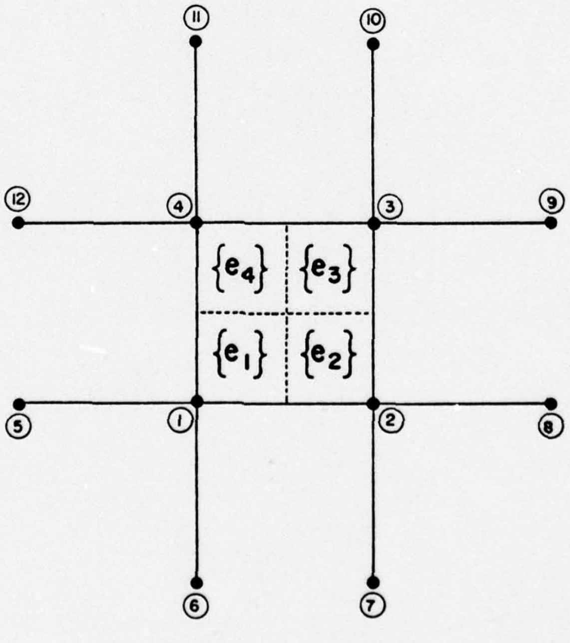

FIG. 2 FOUR STRESS REGIONS WITHIN AN ELEMENT

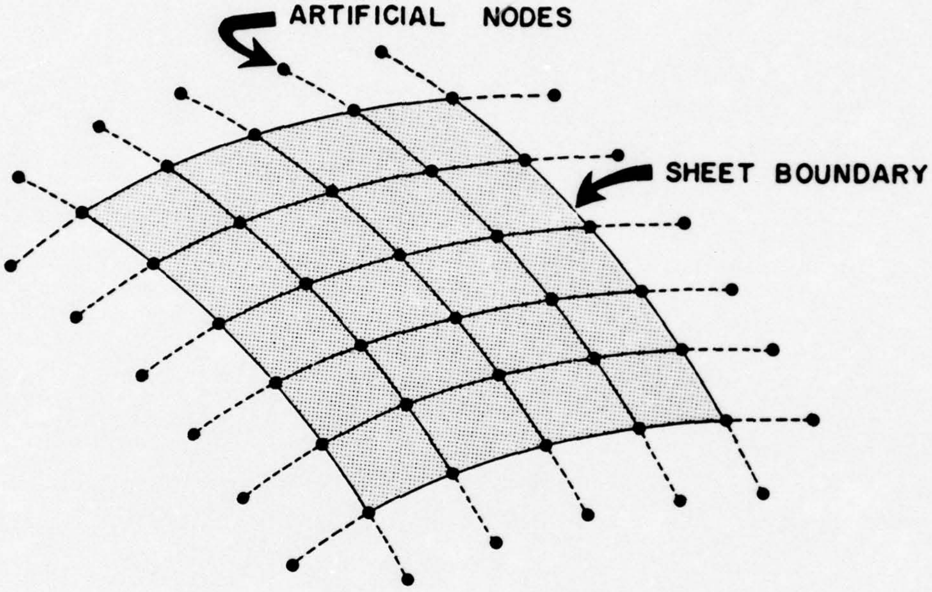

FIG. 3 A SHEET

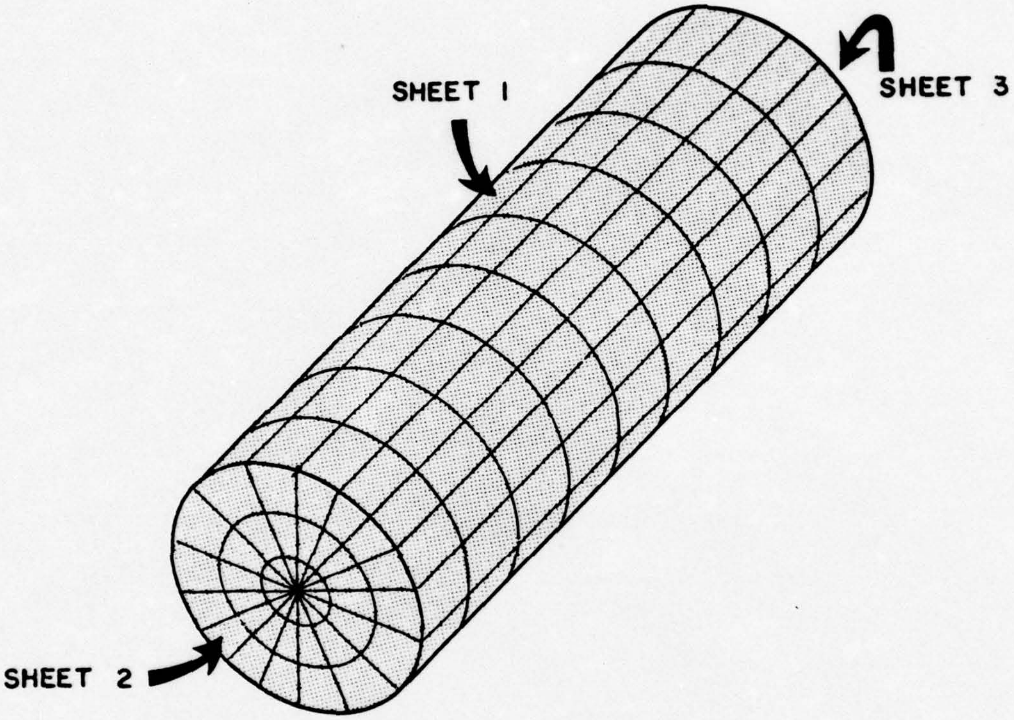

FIG.4 A CYLINDER WITH END CAPS

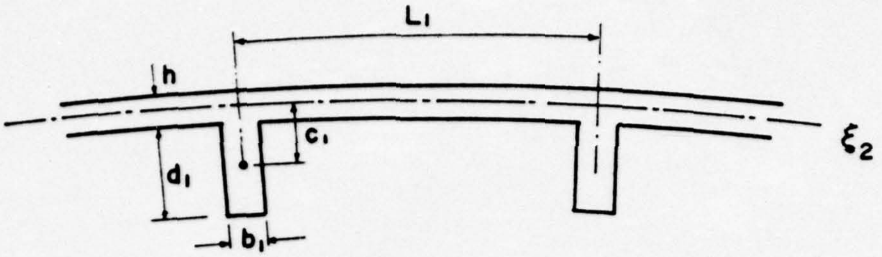

FIG. 5 GEOMETRY OF SMALL STIFFENERS

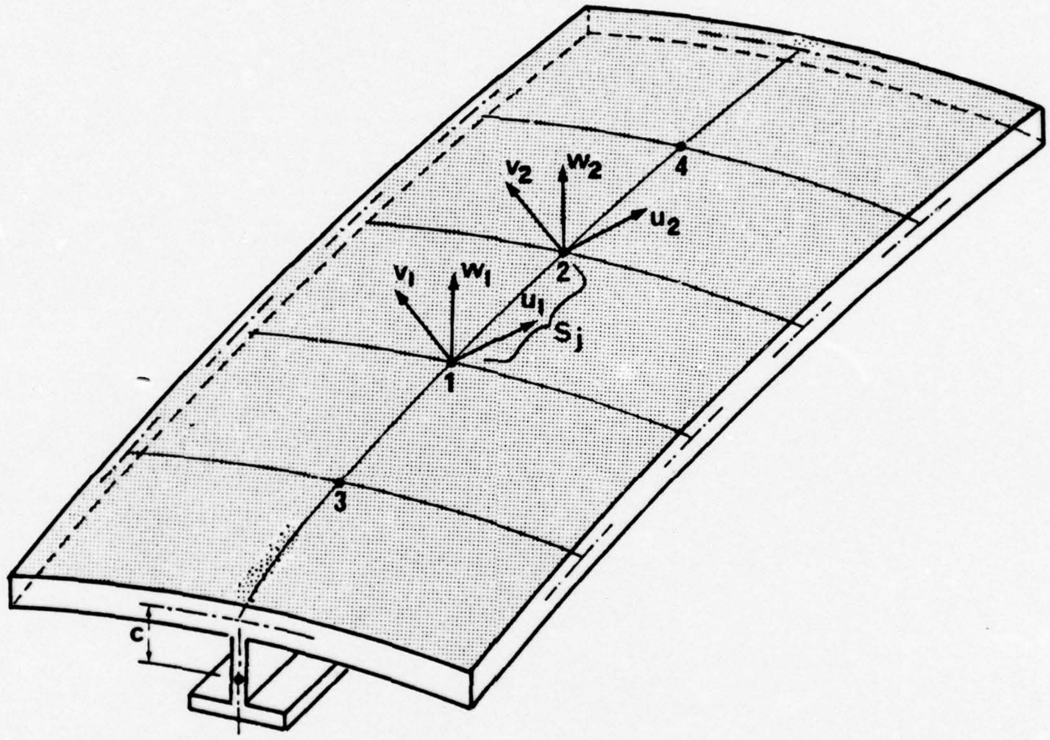

FIG. 6 LARGE STIFFENERS

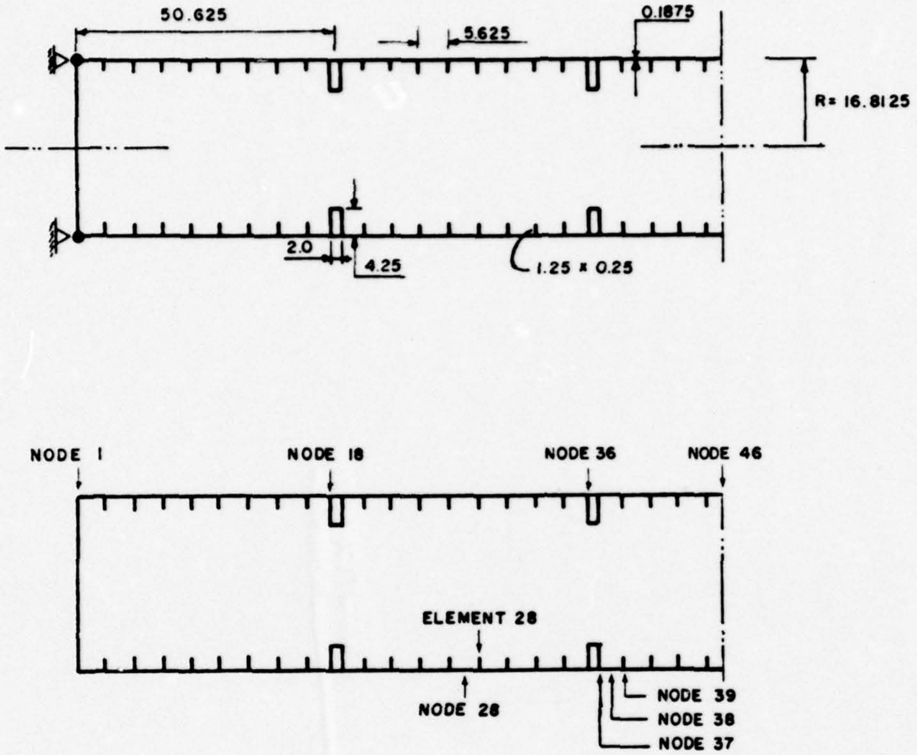

FIG. 7 MODEL OF A CYLIDRICAL SHELL

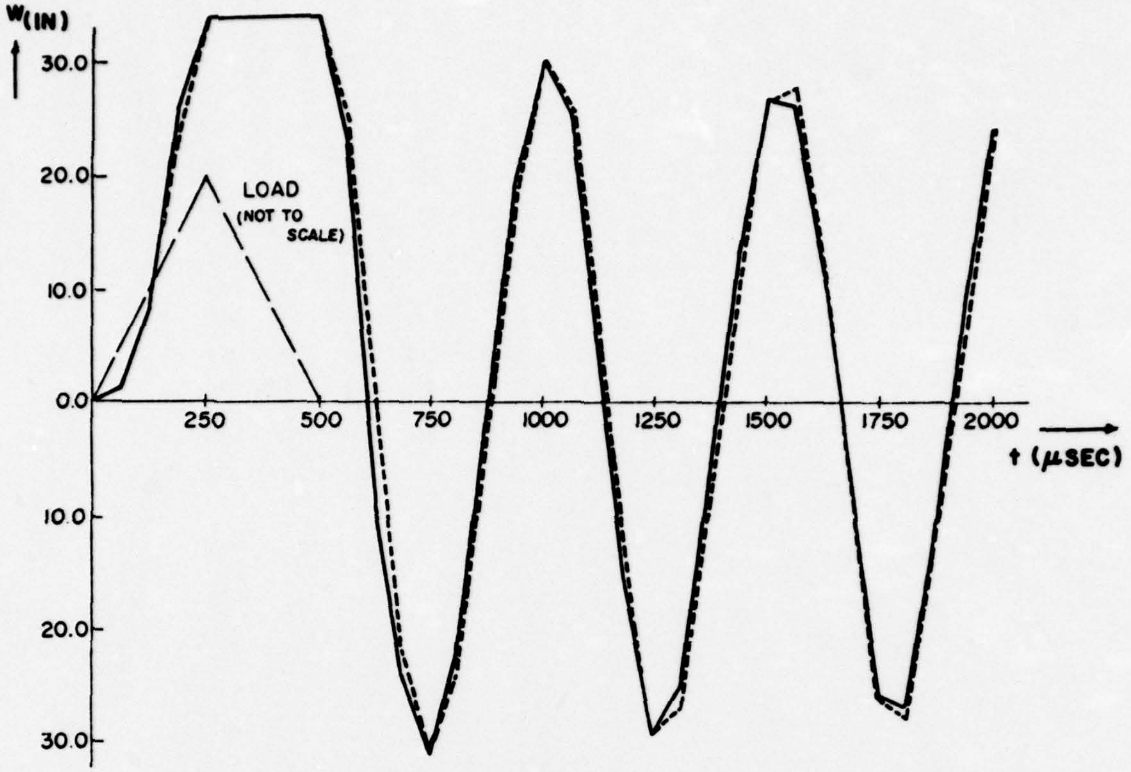

FIG. 8 HOOP STRESSES σ_θ AT ELEMENT 28

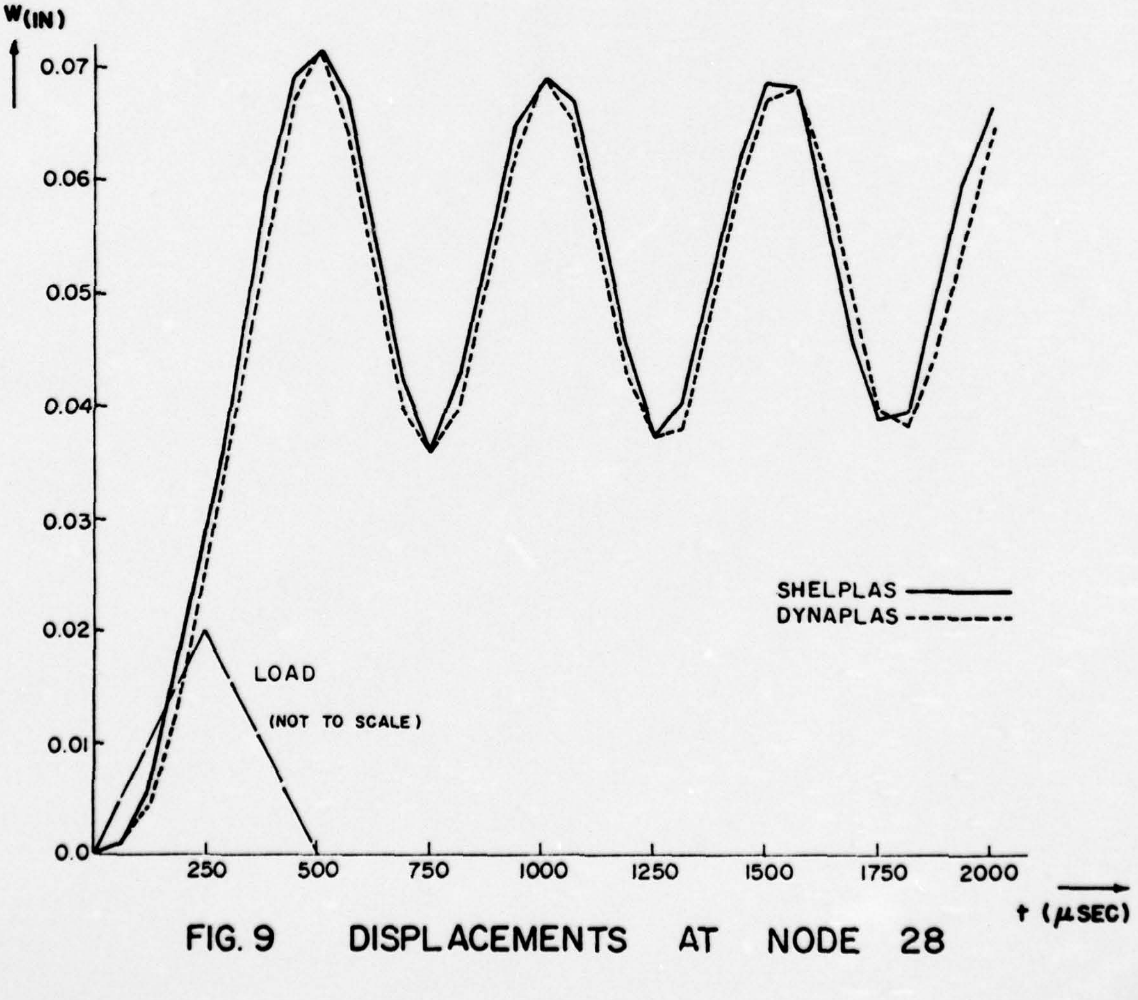

FIG. 9 DISPLACEMENTS AT NODE 28

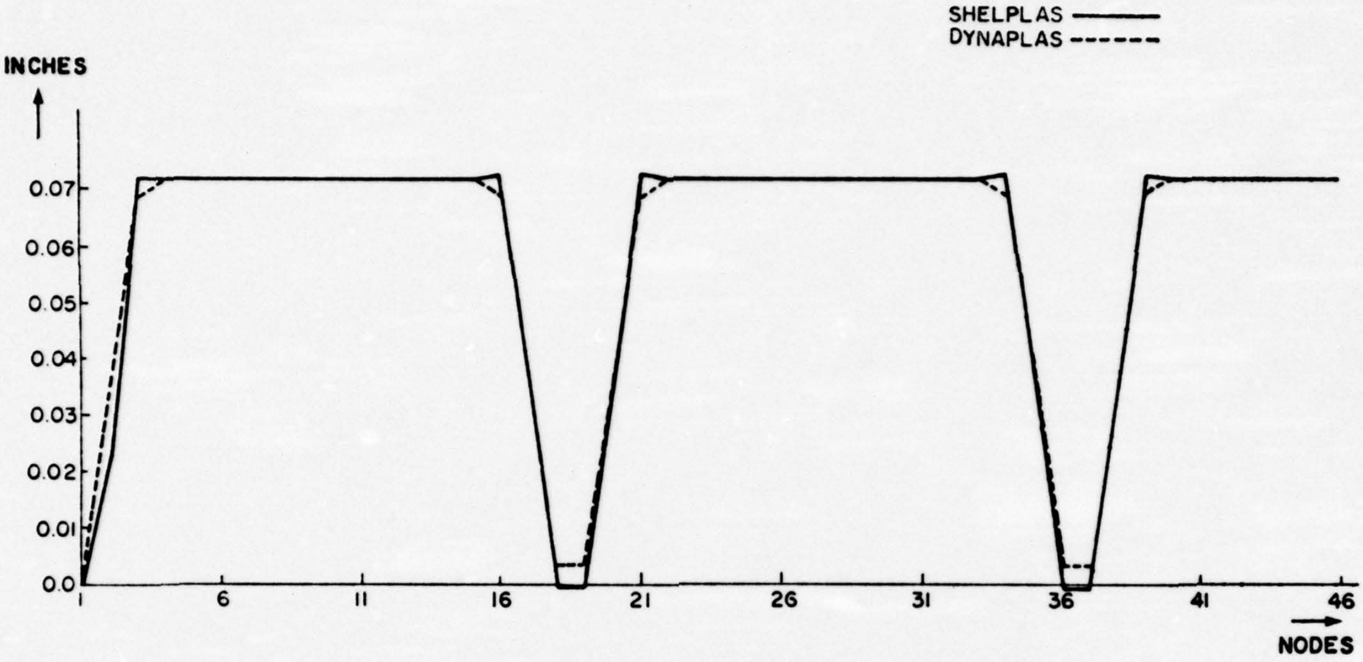

FIG. 10 DISPLACED SHAPE AT T = 0.5 MSEC.

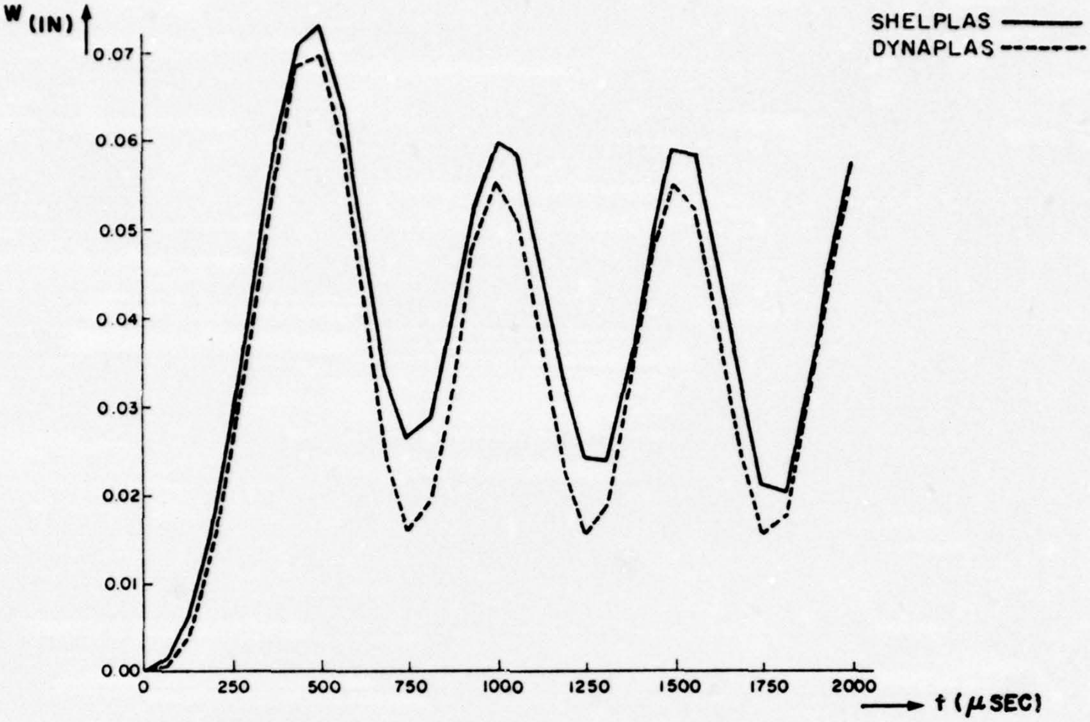

FIG. 11 DISPLACEMENTS AT NODE 39

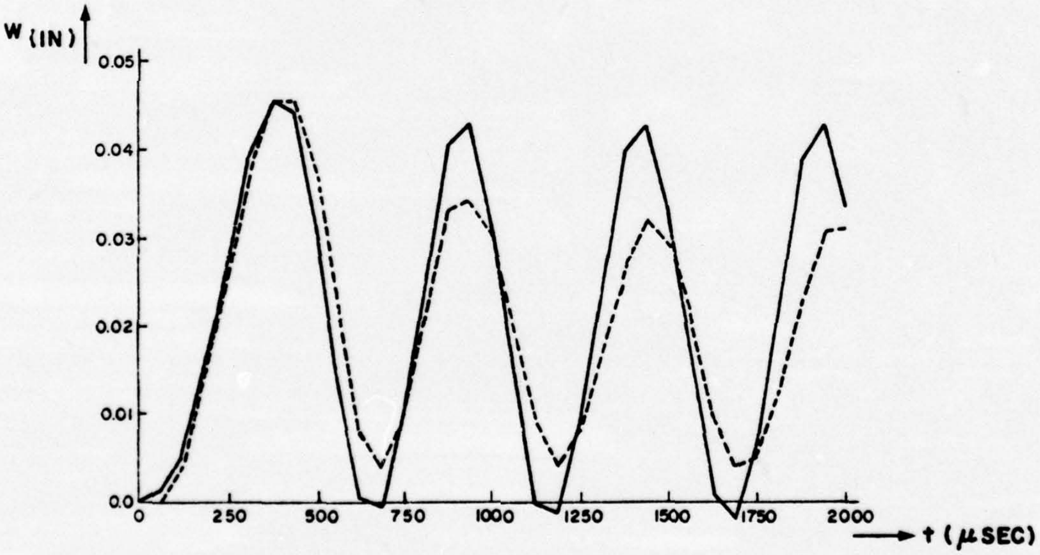

FIG. 12 DISPLACEMENTS AT NODE 38

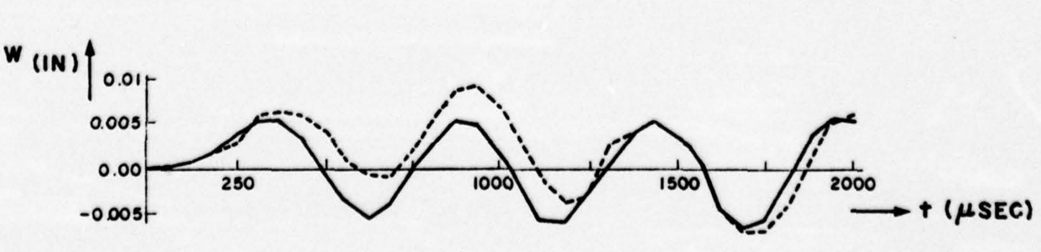

FIG. 13 DISPLACEMENTS AT NODE 37

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