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THERMAL REGIME OF THE DOWNSTREAM SHOULDER OF ROCKFILL DAMS (TER--ETC(U)
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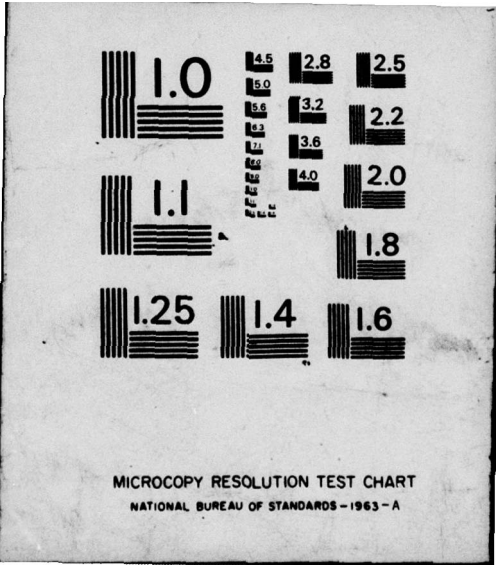
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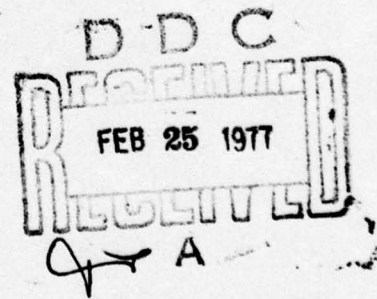
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N.A. Mukhetdinov

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THE THERMAL REGIME OF THE DOWNSTREAM SHOULDER OF ROCKFILL DAMS

A system of equations describing the thermal regime of rockfill dams, one taking natural air convection into account, is presented in the paper. A numerical solution of the equations is given for the profile of the downstream shoulder of the rockfill dam of the Vilyuy Hydroelectric Power Plant, the solution being obtained with the aid of a computer. The results of calculation and of field observations are found to be in close agreement.

This paper is devoted to study of the thermal regime of the downstream shoulder of rockfill dams. Calculation of the thermal regime of the downstream shoulder is necessary in order to establish the thermal state of the apron (core), the base of the dam, and the location of drainage installations. The thermal regime of the downstream shoulder of a rockfill dam is formed as a result of conductive heat transfer and convective heat transfer by the air moving in the pores.

In order for movement of air to occur in the body of the dam it is necessary for the lifting force (due to the heterogeneity of the temperature field) to be greater than or equal to the force of resistance of the rock fill to the movement of the air. The rock fill causes losses due to deformation of the moving air, i.e., losses from contraction and expansion of the air stream and losses due to air friction.

The number of contractions and expansions per unit length of the rock fill depends on the dimensions of the jointings and their geometric shape, i.e., is a function of the diameters of the jointings and of the porosity of the fill.

The force of the resistance due to friction is the most complex in nature. This is to be ascribed not only to the roughness of the surface of the rocks and their varying orientation relative to the flow, but also to the temperature of the moving air itself and of the surface. Hence in order

to determine the force of friction it is necessary in the general case to solve together the problem of the thermal state and that of movement of the air, making allowance for the boundary conditions on the surface of the joint.

In practice it is not possible to solve this problem, especially for a rock fill and its variegated structure. In view of this circumstance it is advisable to introduce quantities characterizing the phenomenon on the average. For example, the force of resistance of a rock fill is regarded as a generalized force which can be determined empirically.

Transfer of heat to the jointings of the fill is effected in various ways. At the places of direct contact between two adjacent jointings and through interlayers filled with stationary air, heat is transferred by thermal conductivity, and by heat exchange in the areas swept by the moving air. Hence the temperature even of a single jointing varies from point to point of the latter. In view of this fact, the variation in the heat content of a rock fill at any point may be expressed through the variation in a certain mean temperature determined from the thermal balance conditions.

The temperature in the core of the moving flow (with the exception of the wall boundary layer) may in practice be assumed to be the same over the entire cross-section, since the air moves in porous channels of infinitely varying cross-section creating the conditions for mechanical displacement.

Hence a flow of air depending on the thermal regime of the dam itself is formed in the downstream shoulder of a rockfill dam. Consequently, a flow of air variable in time will be observed until the temperature of the downstream shoulder of the rockfill dam is everywhere near the temperature of the external air.

The known methods of calculating the thermal regime of the downstream shoulder of rockfill dams are based on utilization of reduced (effective) coefficients of thermal and temperature conductivity, which were derived earlier for one-dimensional problems [1], [2], [3], i.e., for a condition such that the vectors of the thermal flow and of the filtration speed of the air in the pores coincide in the region under consideration.

The geometric dimensions of the downstream shoulder of any rockfill dams are found to be such that a moving flow of air cannot be described as one-dimensional, but rather must be described in at least two spatial coordinates (x,y). Owing to the two-dimensionality of the flow in the pores of the fill, the thermal regime of the higher lying jointings will be determined by thermal calculation of all the lower lying layers, and vice versa.

As is demonstrated by experience acquired in erecting rockfill dams, the grain-size distribution of a rock fill varies over a wide range. The larger pores of a fill usually are filled with smaller fractions, i.e., the area of the particles filling the unit of volume of the fill is a large one. Consequently, the forces of friction will be larger than the forces of inertia of

the moving air. Hence the flow of air in the pores may be regarded as a filtration process [4].

1. Mathematical Statement of the Problem

If it is assumed that the rock fill is isotropic, the porosity does not vary with time, phase shifts are absent from the water vapor forming part of the components of the moving air, the conductive thermal conductivity of the air in the direction of movement is small in comparison to the convective heat transfer and may be disregarded, the volumetric heat capacity of the rock fill is constant and does not depend on the coordinates, other forms of mass transfer (such as water filtration or infiltration) are absent, and the variation in air density in time and the vertical variation in barometric pressure are small [5], then the following relations are valid for the moving air per unit volume:

$$-\frac{\partial P}{\partial x} - F(u) = 0; \quad (1)$$

$$-\frac{\partial P}{\partial y} - F(v) - m\beta\gamma\Delta T = 0; \quad (2)$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0; \quad (3)$$

$$p = \frac{\rho_0}{1 + \beta\theta}, \quad (4)$$

where $\frac{\partial P}{\partial x}$, $\frac{\partial P}{\partial y}$ are the pressure gradients respectively along the ox and the oy axis;

$F(u)$, $F(v)$ are the pressure drops due to air friction in the fill respectively along the ox and the oy axis;

m is the porosity of the fill;

β is the coefficient of volumetric expansion of air;

γ is the volumetric weight of air;

$\Delta T = \theta - \tau_{\text{ext}}$ is the temperature difference of the pore and the external air;

ρ is air density;

ρ_0 is air density at zero temperature;

θ is the temperature of the pore air;

x , y are coordinates;

u , v are the speed of air respectively along the ox and the oy axis;

t_{ext} is the temperature of the external air.

Let us reduce equations (1) and (2) to a form convenient for calculation. For this purpose we differentiate equation (1) with respect to y and equation (2) with respect to x .

Then we obtain

$$-\frac{\partial^2 P}{\partial x \partial y} - \frac{\partial F(u)}{\partial y} = 0; \quad (5)$$

$$-\frac{\partial^2 P}{\partial x \partial y} - \frac{\partial F(v)}{\partial x} - \frac{\partial (m\beta\gamma\Delta T)}{\partial x} = 0, \quad (6)$$

or

$$\frac{\partial F(u)}{\partial y} - \frac{\partial F(v)}{\partial x} = \frac{\partial (m\beta\gamma\Delta T)}{\partial x}. \quad (7)$$

A larger number of papers have been devoted to the problem of determination of the pressure drops in porous media on movement of gases and liquids through them. Many authors regard flow through a porous layer as an internal problem (flow in channels between particles [6], [7]), and others as an external one (flow of air around a sphere [8]).

On the basis of these papers various empirical and semiempirical formulas have been proposed, ones generalizing a large number of experiments conducted with particles of small diameter in one-dimensional flow. The rate of filtration enters these relations in the second power; this is not acceptable for describing losses in a two-dimensional region, since a linear relationship must exist between the velocity vector and its components in a two-dimensional region [9].

Hence it is necessary to adopt the general expression of the Darcy law [10] to describe the losses due to friction:

$$F(u) = -k(\rho u) \quad \text{and} \quad F(v) = -k(\rho v), \quad (8)$$

where $k = \nu/k'$;

ν is the coefficient of kinematic viscosity of air;

k' is the coefficient of permeability of the fill.

Inserting (8) into (7), we obtain

$$\frac{\partial (k\rho u)}{\partial y} - \frac{\partial (k\rho v)}{\partial x} = -\frac{\partial (m\beta\gamma\Delta T)}{\partial x}. \quad (9)$$

Equation (9) must be considered in conjunction with equation of discontinuity (3). It is convenient for practical calculations to reduce equation system (9) and (3) to a single equation.

For this purpose it is necessary to express the mass velocities through the flow line function:

$$\rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial x} \quad (10)$$

Equation (9) then assumes the form

$$\frac{\partial}{\partial y} \left(k \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial x} \left(k \frac{\partial \psi}{\partial x} \right) = -\frac{\partial (m\beta\gamma\Delta T)}{\partial x} \quad (11)$$

This is an elliptical equation with variable coefficients. Equation (11) is also valid for an anisotropic medium if the axes of the anisotropy coincide with the coordinate axes, but with the porosity constant. The equation of discontinuity may subsequently be disregarded, since it is converted to identity.

Thus we obtain the following system of equations with boundary conditions (Figure 1) with which to determine the thermal regime of the downstream shoulder of a rockfill dam:

$$\left. \begin{aligned} \frac{\partial t}{\partial \tau} &= a \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right) + \frac{a_v}{C_{vol}} (\theta - t); & (12) \\ \frac{\partial \theta}{\partial \tau} + \frac{1}{\rho m} \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{1}{\rho m} \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} &= \frac{a_v}{C_{vol}} (t - \theta); & (13) \\ \frac{\partial}{\partial y} \left(k \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial x} \left(k \frac{\partial \psi}{\partial x} \right) &= -\frac{\partial (m\beta\gamma\Delta T)}{\partial x}; & (14) \\ \rho &= \frac{\rho_0}{1 + \beta\theta}. & (15) \end{aligned} \right\} (1)$$

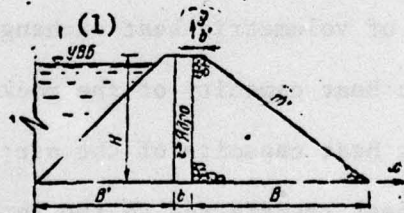


Figure 1. Illustrating determination of boundary conditions: 1. UVB.
2. Core

(1) the temperature on the day of the downstream shoulder

$$\frac{\partial t_{sur}}{\partial \tau} = \frac{a_v}{C_{vol}} (t_{sur} - t_{sur}) + \frac{1}{C_{vol}} q \text{ when } x = x_{sur} \text{ } y = y_{sur} \quad (16)$$

(2) on the boundaries of conjugation of the downstream shoulder with other airtight materials

$$\lambda \frac{\partial t}{\partial n} = \lambda_{res} \frac{\partial t_{res}}{\partial n}; \quad t = t_{res} \quad (17)$$

(3) the temperature of the moving air on the boundaries of the downstream shoulder are:

$$(a) \theta(x, y, \tau) = \theta(x_{sur}, y_{sur}, \tau) \text{ when } x = x_{sur}, y = y_{sur}, \quad (18)$$

$$(b) \theta(x, y, \tau) = \theta(x_{res}, y_{res}, \tau) \text{ when } x = x_{res}, y = y_{res};$$

(4) the flow line function is

$$(a) \psi = 0, \text{ when } 0 \leq x \leq B, y = 0 \text{ and } 0 \leq y \leq H, x = 0, \quad (19)$$

$$(b) \psi(x, y, \tau) = \psi(x_{sur}, y_{sur}, \tau) \text{ when } x = x_{sur}, y = y_{sur} \quad (20)$$

With the initial conditions

$$\left. \begin{aligned} t(x, y, \tau) &= t(x, y, 0); \\ \theta(x, y, \tau) &= \theta(x, y, 0); \\ \psi(x, y, \tau) &= \psi(x, y, 0). \end{aligned} \right\} \quad (21)$$

Here t is the mean temperature of the rock fill;

t_{sur} is the temperature on the surface of the rock fill;

θ_{sur} is the temperature of the moving air on the surface of the dam;

τ is time;

α_v is the coefficient of volumetric heat exchange;

C_{vol} is the volumetric heat capacity of the rock fill;

C_{vol} is the volumetric heat capacity of the air;

q is the amount of heat transferred to the interior of the dam by thermal conductivity;

$\lambda \frac{\partial t}{\partial n}$ is the thermal flux from the direction of the downstream shoulder toward the line of conjugation;

$\lambda_{res} \frac{\partial t_{res}}{\partial n}$ is the thermal flux toward the line of conjugation from the direction of the base and core (apron);

λ is the coefficient of thermal conductivity of the rock fill without convection;

x_{res}, y_{res} are the coordinates of conjugation of the downstream shoulder with the base and core;

λ_{res} is the coefficient of thermal conductivity of the base and apron;

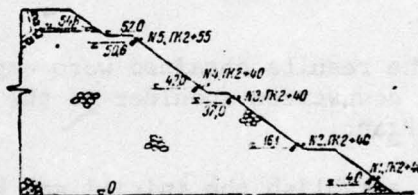
x_{sur}, y_{sur} are the coordinates of the day of the dam.

In order to determine $\theta(x_{sur}, y_{sur}, \tau)$ and $\psi(x_{sur}, y_{sur}, \tau)$ on the day, together with the equations of system I it is necessary to consider equations describing the velocity and temperature fields of the air outside the dam.

It is a characteristic of the process of natural convection in the downstream shoulder of a rockfill dam that the warm air emerging is pressed against the core (apron) of the dam. This is explained by the fact that the core (apron) of the dam has a higher temperature than do the other parts of the downstream shoulder. Hence escape of air always takes place on the crest of the dam (if there is no obstacle inside the dam in the form of airtight interlayers). The warm air escaping from the dam has no appreciable effect on the temperature regime of the surrounding external air of the downstream shoulder of the dam, as has been established by observations at the Vilyuy Hydroelectric Power Plant.

Figure 2. Arrangement of thermistors on the downstream slope of the dam of the Vilyuy Hydroelectric Power Plant

o -- thermistors; x -- mercury thermometers

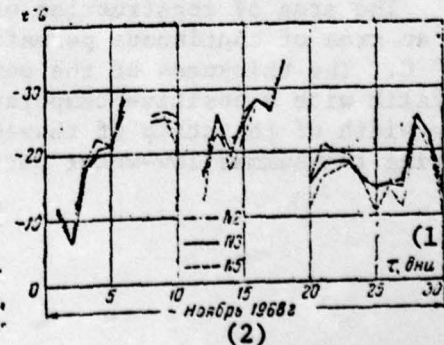


The results of the observations are illustrated in Figure 3. As is to be seen from Figure 3, similar variations take place in the air temperature on the surface of the dam. However, at point No. 2 (Figure 2) there is observed an appreciable movement of air into the interior of the dam, the temperature of which equals the external air temperature.

At a distance of 1.0-1.5 m from the point of escape, the temperature of the air moving in the pores is near the external air temperature (Figure 3); the variation in the former temperature does not exceed 4° . Hence the value of $\frac{\partial(m\beta\gamma\Delta T)}{\partial x}$ on the day may be determined from the difference between

the air temperature on the surface of the downstream shoulder and the external air temperature. An error has an effect at a short distance from the point of escape of the warm air from the dam, within a range of 5 m.

Figure 3. Temperature of external air on dam surface according to readings of mercury thermometers. 1. τ , days. 2. November, 1968.



In this case the velocity field of the air on the day of the downstream shoulder of the dam can be determined from the physical nature of the problem, which assumes that the velocity of approach (departure) of the external air equals the filtration velocity of the air in the pores on the day of the downstream shoulder. This condition is fulfilled if it is assumed that $\partial\psi/\partial n = 0$, i.e., the movement of the air is always normal to the surface on the day of the downstream shoulder. The assumptions adopted permit solution of the problem of the thermal regime of the downstream shoulder of a rockfill dam without analysis of the velocity and temperature fields of the air outside the dam. It should be noted that the velocity and temperature fields of the air outside the dam depend on many meteorological factors which we have not as yet succeeded in allowing for.

Equation system I is also valid for regions having a more complex geometric shape, and in particular for the downstream shoulder of a rockfill dam with an apron. In this case as well the external air temperature, i.e., $\Delta T = \theta - t_{\text{ext}}$, must be taken as the origin of the temperature level [13].

The results obtained were employed to calculate the thermal regime of the downstream shoulder of the rockfill dam of the Vilyuy Hydroelectric Power Plant.

To establish the initial and boundary conditions it is necessary to know the climatic and cryopedologic conditions of the transit line of construction of the dam, the filtration properties of the base, the coarseness of the jointings and the porosity of the rockfill, and the structure of the dam itself.

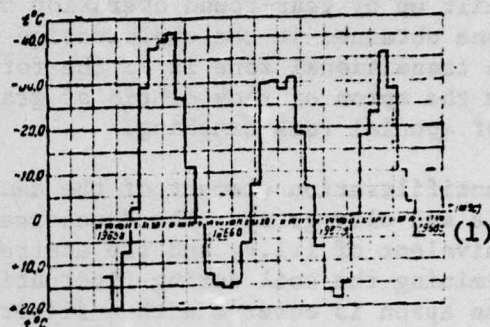
2. Climatic and Cryopedologic Conditions of the Area of Construction

The area of construction of the Vilyuy Hydroelectric Power Plant is characterized by a long cold winter with little snow and a hot summer. The amplitude of the annual temperature variations reaches 100°C , the minimum temperature in winter -60°C , and the maximum temperature in summer $+36^{\circ}\text{C}$. The winter is up to seven months long. The mean annual long-term temperature of the area is -8.2°C . The thickness of the snow cover does not exceed 250-300 mm. The annual distribution of the external air temperature is illustrated in Figure 4.

The area of construction of the water engineering system is situated in an area of continuous permafrost, with prevailing temperatures of -3 -- -5°C . The thickness of the permafrost rocks is estimated at 250-300 m. A talik with a positive temperature can be traced in the bed of the Vilyuy. The width of the strip of thawed rocks corresponds to the width of the river during the summer low-water period [12].

Figure 4. Graph of external air temperature.

1. hours



The original slopes of the Vilyuy valley differ widely from each other in permafrost conditions. The left bank is made up of perennially frozen rocks the temperature of which ranges from -4 to -5° C.

The rocks of the right-bank slope are characterized by low temperatures reaching -7° C. The depth of seasonal thawing in the rock ranges from 3 to 4 m [19].

3. Geological Engineering Conditions of the Construction Site

According to the data of the S. Ya. Zhuk Gidroyekt, the bed portion and the lower slopes of the Vilyuy River in the section line are characterized by constant geological engineering conditions. The normal diabases are widely distributed here. The best preserved and most monolithic rocks occur under the bed and the portion of the right bank adjoining it.

Jointing is associated with the primitive rock, being due to chilling of the intrusion under conditions of tectonic stress. The fissures divide the diabasic massif into columnar, sheet, and slab jointings. The opening of the fissures is small, from fractions of a millimeter to 1.0-1.5 cm. From the viewpoint of jointing the rocks are classified as low-jointed, i.e., with less than 10 cracks per running water. The coefficient of fissure cavitation is small, falling within the limits of 0.5-1.0%.

According to the data of experimental pumpings, the coefficient of filtration of the base is estimated at 0.1 to 2 m/day. The coefficient of filtration of the adjoining left-bank area may reach 10 m/day.

The base of the dam is represented by firm diabases with a small coefficient of filtration; on the riverbank sections they are covered with friable deposits 1.5-2 m thick. The friable deposits were removed when the dam was erected [19].

4. The Rockfill Dam of the Vilyuy Hydroelectric Power Plant

The blind rockfill dam has a height of 74 m and a length along the crest of around 600 m [19]. The horizontal equivalent of the slopes varies in height. Berms used for temporary passages and permanent roads are installed at intervals of 10-15 m on the slopes. The rubble mound of the

dam was built up by year-round operation by the pioneer method with unsorted stone obtained in the exploitation of useful cuts or special quarries. The transitional zone is in the form of two-layers of reverse filters below the apron of rock debris of grain sizes 0-40 and 0-150 mm obtained by means of special rock crushing.

The antifiltration element of the dam is represented by an apron of rock debris and clay gruss. The downstream slope of the apron has a horizontal equivalent of 1:1.4, and the upstream slope one of 1:1.7. To prevent undermining the soil during fluctuation of the head bay, the upstream edge of the apron is covered with a filter above the apron, of a natural sand and gravel mixture. To ensure static stability of the apron from the direction of the head bay, the apron is weighted down with a rock fill.

Conjugation of the apron with the rock base is effected in the form of a lightly reinforced concrete plate with a cementation culvert. Cementation of the talik below the bed from the culvert is schedule before the reservoir is filled.

The grain-size composition of the fill is sufficiently homogenous, according to the data of the "Orgenergostroy" Institute, being determined chiefly by the jointing of the mountain mass.

According to the data of the construction laboratory, the fines content of the blasted rock is 12-15%. The fill of the Vilyuy dam was filled with stone from a height of 10 to 15 m [12], with the exception of the areas adjoining the banks. With a filled layer of this height foliation of the stones is observed which is the more pronounced, the more heterogeneous is the material from the viewpoint of size.

The upper layers of the tier are denser, and in some cases even airtight. An airtight layer was formed on PK 2-40 at an elevation of 54.0 m. This is confirmed by escape of warm air beyond the berm at an elevation of 50.6 m. The fill is anisotropic within the limits of one layer, i.e., the mean diameters and porosity of the fill are not equal in the upper and lower parts of the layer. However, the mean porosity value of the dam of the Vilyuy Hydroelectric Power Plant is in our opinion approximately 30%.

The material of the fill possesses the following physicommechanical properties; volumetric weight approximately 2.97 g/cm^3 , specific gravity 3.025 g/cm^3 , and temporary compressive strength on the average 1700 kg/cm^2 .

The thermophysical characteristics of the fill material, as determined by the two-point method with a cylindrical sample ($R = 0.032 \text{ m}$, $z = 0.055 \text{ m}$), are as follows: coefficient of temperature conductivity (α) $0.00468 \text{ m}^2/\text{h}$, and coefficient of thermal conductivity (λ_{sk}) $2.8 \text{ kcal/m}\cdot\text{h}\cdot\text{deg}$.

The values given for the parameters will be used in what follows in determination of the coefficients appearing in the calculations.

5. System of Calculation of the Thermal Regime of the Rockfill Dam of the Vilyuy Hydroelectric Power Plant

The following system of calculation was adopted to determine the thermal regime of the downstream shoulder of the rockfill dam of the Vilyuy Hydroelectric Power Plant.

The rock fill is isotropic, the porosity is constant, the rock base does not filter, and the latter is frozen. An exception is made by the upstream side of the apron and the upstream weighting, where thawed soil with a moisture content of 20% to a depth of 4.5 m was assumed. The warming effect of snow on the slopes of the downstream shoulder is disregarded, the snow not representing an obstacle to movement of air into the interior of the dam. The data of the mean monthly air temperatures were employed in the calculations. The coefficient of volumetric heat exchange is assumed to be constant and independent of the coordinates and time. The profile of the dam along the downstream slope is assumed to be straight, in contrast to the actual one, which is in the form of a broken line. No analysis is made of the velocity and temperature fields of the external air outside the dam, it being assumed that at a short distance from the surface the temperature of the air equals the temperature of stationary distribution at a great distance from the dam.

It was assumed that the variation in the density and the volumetric weight of the air in the pores as a function of the coordinates will not play a significant role in the velocity distribution. The coefficient of permeability in equation (14) is taken to be constant and to characterize the entire downstream shoulder as a whole, since the granulometric composition of the fill and its filtration properties cannot be definitely indicated [12].

Equations (13), (14) of system I then assume the form

$$\frac{\partial \theta}{\partial \tau} + \frac{1}{m} \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{1}{m} \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{\alpha_v}{C_{vol} m} (t - \theta); \quad (22)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = - \frac{m \beta g}{k} \frac{\partial \theta}{\partial x}, \quad (23)$$

where g is the acceleration of gravity.

Boundary conditions:

(1) temperature on the surface of the downstream shoulder:

$$\frac{\partial t_{sur}}{\partial \tau} = \frac{\alpha_v}{C_{vol}} (\theta_{sur} - t_{sur}) + \frac{1}{C_{vol}} q \text{ when } x = x_{sur}; \quad y = y_{sur} \quad (24)$$

(2) on the edges of conjugation of the downstream shoulder with the other materials, boundary conditions of the fourth kind must be fulfilled, i.e.,

no allowance for convection can be determined on the basis of field observations and laboratory studies.

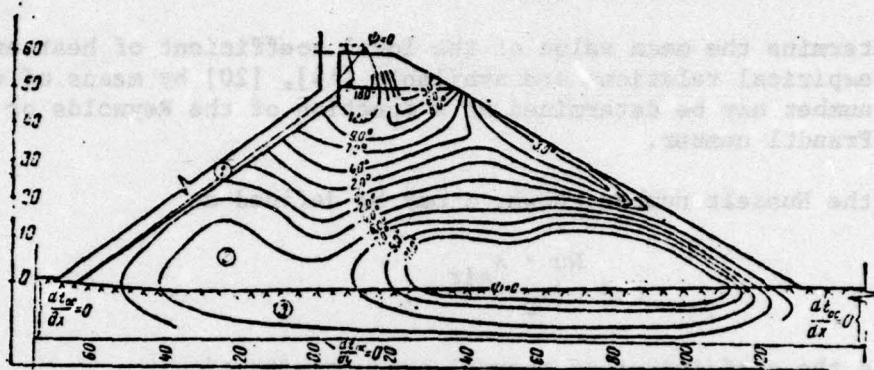


Figure 5. Initial temperature field of downstream shoulder as of 1 November 1967, with no allowance for water-table elevation of the tail bay.

6. Determination of the Coefficients Entering Into the Calculation

Extant recommendations elaborated for other processes, but ones approximating the phenomenon in question, may be utilized for preliminary calculations [14], [15], [16], [20].

The area of the airswept surface participating in heat exchange is defined as [16]:

$$F = \frac{7.5(1-m) \cdot V}{D}, \quad (28)$$

where V is volume;
 F is area;

$D = \sum_{i=1}^x x_i D_i$ is the reduced diameter;

x_i is the volumetric portion of particles of number i ;

D_i is the diameter of particles of number i ;

x is the amount of soil having particles of the same diameter per unit volume.

The volumetric coefficient of heat exchange may be defined as

$$\alpha_v = \alpha F \varphi \quad (29)$$

where α is the local coefficient of heat exchange between the air and the jointings of the fill;

φ is a coefficient allowing for the difference between the mean temperature of the rock fill at a given point and the temperature of the surface of a jointing.

To determine the mean value of the local coefficient of heat exchange a number of empirical relations are available [14], [20] by means of which the Nusselt number may be determined as a function of the Reynolds or the Grashof and Prandtl number.

With the Nusselt number known, α may be defined as

$$\alpha = \frac{\text{Nu} \cdot \lambda_{\text{air}}}{D},$$

where λ_{air} is the coefficient of thermal conductivity of air.

Coefficient φ , which allows for the difference between the mean temperature of the rock fill at a given point and the surface temperature of a jointing, may be defined as:

$$\varphi = \frac{1}{\sqrt{\text{Bi}_v^2 + 1,437\text{Bi}_v + 1}}, \quad (29a)$$

where Bi_v is the Biot number, defined as $\text{Bi}_v = \frac{\alpha \cdot R}{\lambda_{\text{sk}}}$;

R_v is the characteristic dimension of a body, equalling the ratio of the volume to its area.

λ_{sk} is the coefficient of thermal conductivity of the fill material.

It is here assumed that a regular regime of thermal interaction between the moving air and the rock fill arises. The rock fill represents a two-phase statistical mixture of jointings of rock and air. The following formula has been proposed for determination of the coefficient of thermal conductivity of such mixtures [17]:

$$\lambda = \frac{(2-3m)\lambda_{\text{sk}} + (3m-1)\lambda_{\text{air}}}{4} + \sqrt{\left[\frac{(2-3m)\lambda_{\text{sk}} + (3m-1)\lambda_{\text{air}}}{4} \right]^2 + \frac{\lambda_{\text{sk}}\lambda_{\text{air}}}{2}} \quad (30)$$

The volumetric heat capacity of the rock fill is found from the formula

$$C_{\text{vol}} = (1-m)C_r \cdot \gamma_r + mC_{\text{vol}} \quad (31)$$

where γ_r is the volumetric weight of the rock fill jointings;

C_r is the specific heat capacity of a fill jointing.

The coefficient of temperature conductivity may be defined as the derivative value

$$a = \frac{\lambda}{C_{vol}} \quad (32)$$

Relations (28), (29), (30), (31), and (32) given in the foregoing permit determination of all the parameters required for the calculations. They characterize a rock fill for which the thermal regime problem is solved.

7. Method of Solving the Problem

To determine the thermal regime of the downstream shoulder of a rock-fill dam (simplified model) it is necessary to solve equations (12), (22), and (23) concurrently. Physical quantities β , ν , ρ are here taken to be constant for the entire downstream shoulder and correspond to the mean temperature value of air in movement.

System of equations (12), (29), and (23) can be solved by the numerical method. For this purpose we introduce into the region under study a rectangular network consisting of points $x = x_0 + n\Delta x$, $y = y_0 + j\Delta y$, $\tau = k \cdot \Delta \tau$.

Then we obtain the following finite-difference systems for time $(k + 1) \cdot \Delta \tau$.

1. For the equations of the mean rock fill temperature (explicit system):

$$t_{n,j}^{k+1} = \left[1 - 2a\Delta\tau \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) - \frac{a\Delta\tau}{C_{vol}} \right] t_{n,j}^k + \frac{a\Delta\tau}{\Delta x^2} (t_{n+1,j}^k + t_{n-1,j}^k) + \frac{a\Delta\tau}{\Delta y^2} (t_{n,j+1}^k + t_{n,j-1}^k) + \frac{a\Delta\tau}{C_{vol}} \theta_{n,j}^k \quad (33)$$

The temperature of the points situated on the inclined surface of the dam is determined from the formula

$$t_{sur}^{k+1} = t^k + \frac{2a\Delta\tau}{\Delta x^2} (t_{sy}^k - t_{sur}^k) + \frac{2a\Delta\tau}{\Delta y^2} (t_{sy}^k - t_{sur}^k) + \frac{\Delta\tau}{C_{vol}} (\theta_{sur}^k - t_{sur}^k), \quad (34)$$

where $t_{\Delta x}^k$, $t_{\Delta y}^k$ is the temperature of the points adjoining the surface along ox and oy .

2. For the temperature of the air moving in the pores (implicit system).

$$\theta_{n,j}^{k+1} = \frac{1}{L_{n,j}^{k+1}} \left[\theta_{n,j}^k - \eta A_{n,j}^{k+1} \theta_{n+1,j}^{k+1} + (1 - \eta) B_{n-1,j}^{k+1} \theta_{n-1,j}^{k+1} - \chi C_{n,j}^{k+1} \theta_{n,j-1}^{k+1} + (1 - \chi) D_{n,j}^{k+1} \theta_{n,j+1}^{k+1} + \frac{a\Delta\tau}{C_{vol}} t_{n,j}^k \right] \quad (35)$$

where

$$\left. \begin{aligned}
 A_{n,j}^{k+1} &= B_{n,j}^{k+1} = \frac{\Delta\tau}{m \Delta x \Delta y} (\psi_{n,j}^{k+1} - \psi_{n,j-1}^{k+1}); \\
 C_{n,j}^{k+1} &= D_{n,j}^{k+1} = \frac{\Delta\tau}{m \Delta x \Delta y} (\psi_{n,j}^{k+1} - \psi_{n-1,j}^{k+1}); \\
 L_{n,j}^{k+1} &= 1 + \left| \frac{\Delta\tau}{m \Delta x \Delta y} (\psi_{n,j}^{k+1} - \psi_{n,j-1}^{k+1}) \right| + \\
 &+ \left| \frac{\Delta\tau}{m \Delta x \Delta y} (\psi_{n,j}^{k+1} - \psi_{n-1,j}^{k+1}) \right| + \frac{\alpha \Delta\tau}{C_{\text{val}} m}. \\
 \eta &= 1, \quad \text{if } (\psi_{n,j}^{k+1} - \psi_{n,j-1}^{k+1}) / \Delta y < 0; \\
 \eta &= 0, \quad \text{if } (\psi_{n,j}^{k+1} - \psi_{n,j-1}^{k+1}) / \Delta y \geq 0; \\
 \chi &= 1, \quad \text{if } (\psi_{n,j}^{k+1} - \psi_{n-1,j}^{k+1}) / \Delta x < 0; \\
 \chi &= 0, \quad \text{if } (\psi_{n,j}^{k+1} - \psi_{n-1,j}^{k+1}) / \Delta x \geq 0.
 \end{aligned} \right\} \quad (36)$$

3. For determination of the flow function

(a) in the internal nodes:

$$\psi_{n,j}^{k+1} = M (\psi_{n-1,j}^{k+1} + \psi_{n+1,j}^{k+1}) + N (\psi_{n,j+1}^{k+1} + \psi_{n,j-1}^{k+1}) + F_{n,j}^k, \quad (38)$$

where

$$\left. \begin{aligned}
 F_{n,j}^k &= \frac{g \beta m (\theta_{n+1,j}^k - \theta_{n-1,j}^k)}{4k \Delta x \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)}, \\
 M &= \frac{1}{2\Delta x^2 \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)}, \quad N = \frac{1}{2\Delta y^2 \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)}
 \end{aligned} \right\} \quad (39)$$

(b) for determination of the flow function on the boundary nodes of the day (Figure 6):

$$-\frac{\psi_0^{k+1} - \psi_1^{k+1}}{\Delta x} \cos \alpha_0 + \frac{\psi_2^{k+1} - \psi_0^{k+1}}{\Delta y} \sin \alpha_0 = 0, \quad (40)$$

where α_0 is the angle (of inclination) between positive axis ox and that normal to the day of the downstream shoulder.

Figure 6. Illustrative of determination of the normal derivative on the day of the downstream shoulder.



(1) Day

When the number of nodes in the region in question is sufficiently large, it is virtually impossible to solve the algebraic system obtained in mutual connection. If in determination of one of the unknown functions for a given time use is made of the values of the remaining functions from the preceding time layer, the system of algebraic equations is found to be linear. The concurrent physical processes of transfer of air and transfer of heat by the moving air, as well as the equation of propagation of heat, are in this case considered to be discrete for each time step. The time steps and the spatial steps are selected from the conditions of stability and convergence of the finite-difference equations obtained, (33), (34), (35), and (38).

Equations (33) and (34) converge and are stable if the following condition is satisfied:

$$1 - \left(\frac{2a \Delta\tau}{\Delta x^2} + \frac{2a \Delta\tau}{\Delta y^2} + \frac{a \Delta\tau}{C_{vol}} \right) \geq 0 \quad (41)$$

at all the points of the region under consideration. If equation (13) is approximated by an explicit finite-difference system, it should satisfy the following condition of stability and convergence [21] at any point in time:

$$\left\{ 1 - \left[\frac{1}{m \Delta x \Delta y} \max(\psi_{n,j} - \psi_{n,j-1}) \right] + \left[\frac{1}{m \Delta x \Delta y} \max(\psi_{n,j} - \psi_{n-1,j}) \right] \right\} \geq 0 \quad (42)$$

Equations (38) and (40) converge and are stable for any time step.

The condition of stability and convergence of the system is thus defined as

$$\left\{ \begin{aligned} & \left[1 - \left(\frac{2a \Delta\tau}{\Delta x^2} + \frac{2a \Delta\tau}{\Delta y^2} + \frac{a \Delta\tau}{C_{vol}} \right) \right] > 0, \\ & \left\{ 1 - \left[\frac{\Delta\tau}{m \Delta x \Delta y} \max(\psi_{n,j} - \psi_{n,j-1}) \right] + \left[\frac{\Delta\tau}{m \Delta x \Delta y} \max(\psi_{n,j} - \psi_{n-1,j}) \right] \right\} \geq 0 \end{aligned} \right. \quad (43)$$

The minimum time step value determined from condition (43) is sufficient for the stability and convergence of the entire system.

If equation (13) is approximated by an implicit finite-difference system (stable at any value $\Delta\tau$), the time step for solving equations (33), (34), (35), and (36) must be selected from condition (41) only.

Equations (35), (38), and (40) were solved on each time layer by the method of simple iteration.

Using the distribution of the temperature of the air in the pores at time $k \cdot \Delta\tau$, we determine the flow function for the following time step from the formulas:

(a) on the internal nodes:

$$\psi_{n,j}^{k+1+s+1} = M(\psi_{n-1,j}^{k+1+s} + \psi_{n+1,j}^{k+1+s}) + N(\psi_{n,j+1}^{k+1+s} + \psi_{n,j-1}^{k+1+s}) + F_{n,j}^k; \quad (44)$$

(b) on the boundary nodes of the day of the downstream shoulder (Figure 6):

$$\psi_2^{k+1+s+1} = \psi_0^{k+1+s} \left(1 + \frac{\Delta y \cos \alpha_n}{\Delta x \sin \alpha_0} \right) - \psi_1^{k+1+s} \left(\frac{\Delta y \cos \alpha_n}{\Delta x \sin \alpha_0} \right), \quad (45)$$

where s is the number of iterations in the layer.

The convergence of the process was determined by estimating the norm of the difference between two consecutive approximations at all points of the spatial network, i.e.,

$$\|\psi_{n,j}^{k+1+s+1} - \psi_{n,j}^{k+1+s}\| \leq \varepsilon, \quad (46)$$

where ε is the assigned congruence constant.

The method of solving equation (41) is similar to the foregoing one. Coefficients $A_{n,j}^{k+1}$, $B_{n,j}^{k+1}$, $C_{n,j}^{k+1}$ and $L_{n,j}^{k+1}$ were determined in accordance with the values obtained for function $\psi_{n,j}^{k+1}$. The mean temperature of the rock fill adopted in this case was that corresponding to time $k\Delta\tau$.

The solution of equation (35) was assumed to have been obtained on a given time layer if

$$\|\psi_{n,j}^{k+1+s+1} - \psi_{n,j}^{k+1+s}\| \leq \varepsilon, \quad (47)$$

i.e., if a condition analogous to (46) was satisfied.

The value of the air temperature in the pores can be used to obtain the temperature of the rock fill from (33) and (34).

The temperature at the junction of any two regions was determined by applying the heat balance equation, allowance being made for the difference in the thermophysical characteristics of the materials. To calculate the

thermal regime of the upstream weighting of the dam and base use was made of the Fourier equation for solids with constant coefficients. We will not cite the finite-difference analogies of these equations; they are to be found in the corresponding literature [18].

The calculation work was accomplished at the computer center of the Krasnoyarsk aluminum plant, on an M-20 computer. The assigned program of the computer called for input of the initial conditions for the temperature of the rock fill from punch cards and referring of the initial temperature field of the rock fill to the temperature field of the moving air, since they were assumed to be equal at the initial time. The boundary conditions for the moving air were assigned in the form of the external air temperatures in the nodes of the spatial network at a distance of one step from the day.

The physical parameters were introduced as a component of the program they had the following numerical values:

$$\begin{aligned} D &= 0.5 \text{ m;} \\ m &= 0.3; \\ R_v &= 0.03 \text{ m;} \\ \alpha_v &= 21.7 \text{ kcal/h} \cdot \text{deg;} \\ \gamma_v &= 1.293 \text{ kg/m}^3; \\ \beta &= 1/273 \text{ 1/deg;} \\ \lambda &= 2.8 \text{ kcal/m} \cdot \text{h} \cdot \text{deg;} \\ a_{sk} &= 0.005 \text{ m/h;} \\ \lambda_{sk} &= 2.1 \cdot 10^{-2} \text{ kcal/h} \cdot \text{deg;} \\ \lambda_{air} &= 1.9 \text{ kcal/m} \cdot \text{h} \cdot \text{deg;} \\ \lambda_{eq} &= 1.9 \text{ kcal/m} \cdot \text{h} \cdot \text{deg;} \\ \Delta x &= 4.8 \text{ m;} \\ \Delta y &= 3.0 \text{ m;} \\ g &= 9.81 \text{ m/sec}^2; \\ w &= 0.2 \text{ (moisture content of apron and upper weighting);} \\ \mu &= 1.574 \cdot 10^{-6} \frac{\text{kg} \cdot \text{sec}}{\text{m}^2}; \\ k' &= 5.5 \cdot 10^{-6} \text{ m}^2; \\ \rho_0 &= 0.127 \frac{\text{kg} \cdot \text{sec}^2}{\text{m}^4}; \\ C'_{vol} &= 0.31 \text{ kcal/m}^3 \cdot \text{deg;} \\ C_{vol} &= 420 \text{ kcal/m}^3 \cdot \text{deg;} \\ \epsilon &= 0.01. \end{aligned}$$

External air temperature: November -20.1°C , December -25°C ;
January -36.1°C , February -33.4°C . Water temperature of head bay: $+2^\circ\text{C}$.

The constant coefficients of basic finite-difference equations (33), (34), (35), (38), and (40) were computed at the beginning of calculation and were stored in the computer memory. They were retrieved during operation by application to the corresponding memory cell.

It took the longest in the calculations to determine the flow function, i.e., the velocity of the air stream in the rock fill, especially at the initial time of calculation and on change in the boundary conditions for air in movement.

The temperature of the rock fill and the moving air are determined relatively rapidly at each time step $\Delta\tau = 20$ hours.

Owing to the lengthy period of operation, the possibility of malfunction of the computer is not excluded. Hence the intermediate values of the calculation and the program were recorded at intervals of 360 hours on a magnetic drum.

In the event of malfunction, solution began with use of the most recent recording. The results of the solution were printed out, the rock fill temperature at intervals of 180 hours and the flow function and moving air temperature at intervals of 360 hours.

Figure 7 shows the theoretical temperature field of the downstream shoulder of the rockfill dam of the Vilyuy Hydroelectric Power Plant. As is to be seen from the drawing, the downstream shoulder of the rockfill dam is subject to deep chilling.

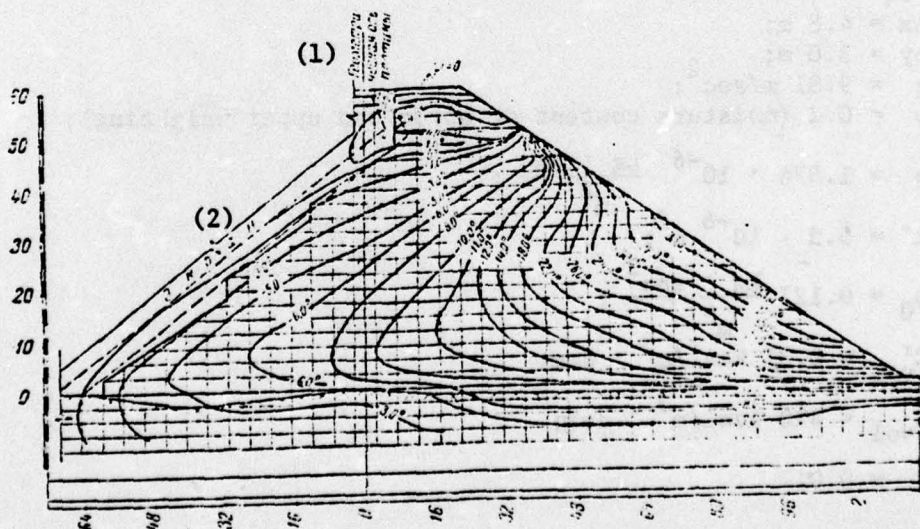


Figure 7. Theoretical temperature field of downstream shoulder as of 22 February 1967 (with no allowance for water-table elevation).
1. Geometric axis of dam. 2. Apron.

A low temperature with large gradients is observed around the base of the dam; this favors rapid chilling of the base of the dam. Use may be made of this circumstance in erecting nonfiltering rockfill dams on soft soils. The elimination of latent heat during phase conversion of the moisture in the apron and in the top weighting, as well as the presence of a horizontal airtight interlayer, exert a warming effect on the thermal regime of the top of the downstream shoulder of the Vilyuy rockfill dam.

A substantial influence is exerted by the airtight interlayer, which reduces the area of formation of the air stream resulting from natural convection.

Figure 8 shows the flow lines of the air stream in the pores of the fill on 22 February 1968. The concentration of the flow line at the point of escape of warm air demonstrates that the warm air escapes within a limited section on the slope. This is confirmed by the results of observations conducted at the Vilyuy dam (Figure 3).

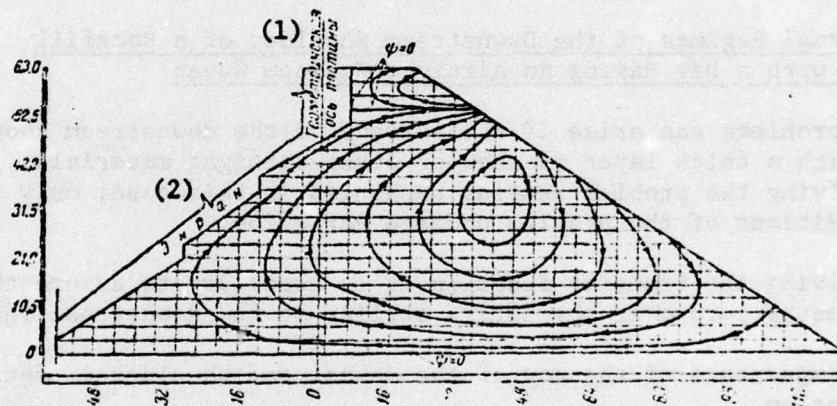


Figure 8. Flow line of air stream in downstream shoulder of dam.

1. Geometric axis of dam.
2. Apron.

Figures 9a, b illustrate the variation in temperature with time and compares the figures with the data of field observations at individual points. The greatest difference between the theoretical and the observed values is observed in the surface zone of the shoulder.

The reason for the difference between the theoretical and the observed data in the surface zone is to be sought in the influence of the variations in external air temperature, the initial conditions of the problem, the heterogeneity of the rock fill, and the influence of the heat insulating and airtight properties of snow on the downstream slope of the dam.

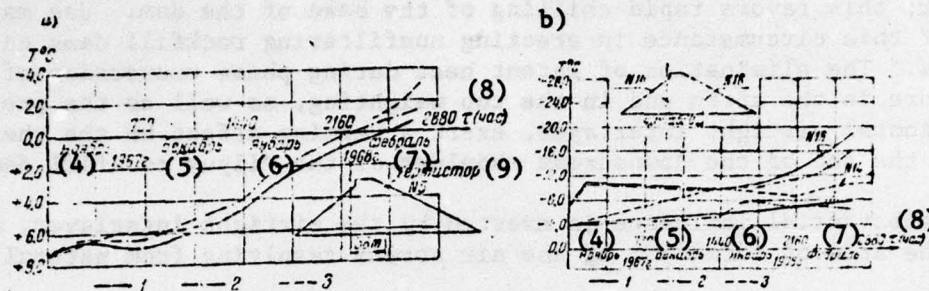


Figure 9. Variation in temperature at various points of the downstream shoulder with time. (a) at point N° 5; (b) at points N° 14 and 18; 1. theoretical curve without allowance for water-table elevation of tail bay. 2. same with allowance made for water-table elevation of tail bay. 3. according to data of field observations. 4. November 1967. 5. December. 6. January. 7. February 1968. 8. hours. 9. thermistor.

9. Thermal Regimes of the Downstream Shoulder of a Rockfill Dam with a Day Having an Airtight Surface Cover

Such problems can arise if the surface of the downstream shoulder is covered with a thick layer of snow or other airtight materials. The method of solving the problem remains unchanged in this case; only the boundary conditions of the problem undergo variation.

In solving the transfer function it is necessary to assume the flow function to equal zero over the entire outline of the downstream shoulder.

The temperature of the day of the downstream shoulder is determined from the equation

$$\lambda \frac{\partial t}{\partial n} = \alpha_{ef}(t_{ext} - t), \quad (54)$$

where λ is the coefficient of thermal conductivity of the rock fill without convection;

$\alpha_{ef} = \frac{1}{\frac{1}{\alpha} + \frac{1}{\lambda_{iz}}}$ is the effective coefficient of heat exchange of the rock fill with air [13];

α is the coefficient of heat exchange of the fill with air;

l is the airtightness thickness of the material;

λ_{iz} is the coefficient of thermal conductivity of the airtight material.

The temperature of the pore air is everywhere determined by finite-difference equation (35). The temperature of the air in the pores of the day of the downstream shoulder is to be adopted in this instance as the origin of the temperature level.

The numerical values of the parameters employed in the calculations were assumed to be the same as those previously used, with the exception of the coefficient of permeability ($k' = 0.34 \cdot 10^{-5} \text{ m}^2$) and the time step ($\Delta t = 10$ hours).

The thermal regime of the base of the dam was not considered. The boundary temperature in the base was assumed to equal zero degrees. The initial condition of the problem is also slightly different (Figure 10).

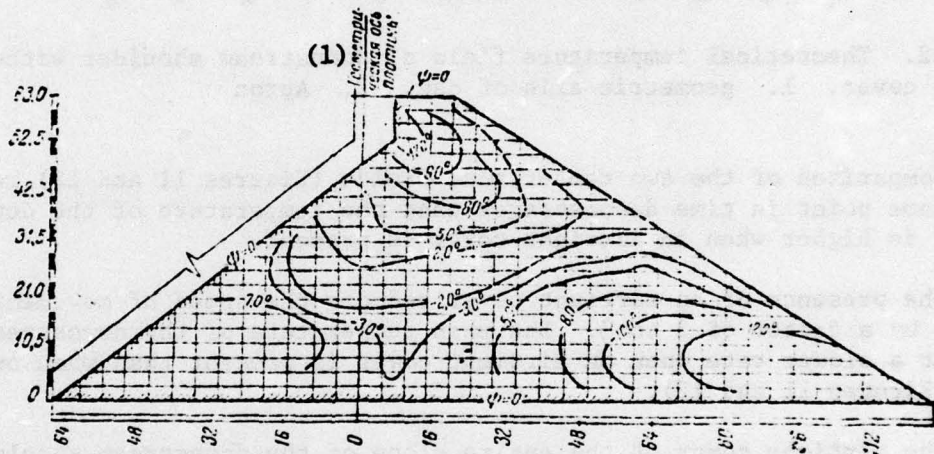


Figure 10. Initial temperature field of downstream shoulder.
1. geometric axis of dam.

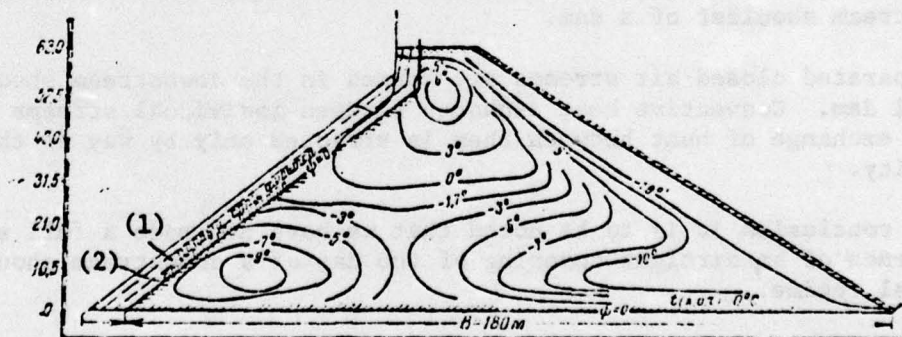


Figure 11. Theoretical temperature field of downstream shoulder with airtight cover. 1. First layer of filter. 2. Second layer of filter

Figure 11 illustrates the theoretical temperature field of the downstream shoulder when an airtight cover is present, and Figure 12 the temperature field problem solved with no airtight cover.

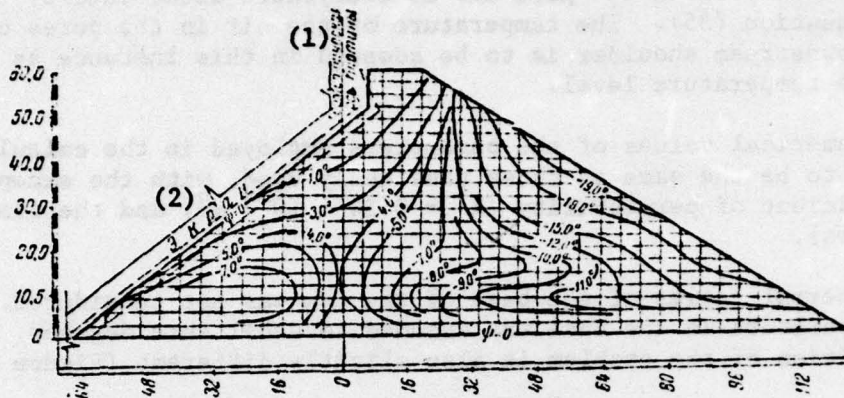


Figure 12. Theoretical temperature field of downstream shoulder without airtight cover. 1. geometric axis of dam. 2. Apron

Comparison of the two temperature fields (Figures 11 and 12) referred to the same point in time demonstrates that the temperature of the downstream shoulder is higher when an airtight cover is present.

The presence of an airtight cover reduces the speed of movement of the pore air by a factor of 2 to 3. The mean temperature of the downstream shoulder varies at a slower rate when an airtight cover is present than when one is absent (Figures 11 and 12).

The airtight cover of the entire slope of the downstream shoulder can be utilized for thermal regulation of rockfill dams. The presence of an air insulating cover during the winter period alone fosters general elevation of the temperature, and during the summer period lowering of the temperature, of the downstream shoulder of a dam.

Separated closed air streams are formed in the downstream shoulder of a rockfill dam. Convective heat exchange between individual streams is absent, and exchange of heat between them is effected only by way of thermal conductivity.

In conclusion it is to be noted that we have not made a full study of the influence of an airtight covering of the day of a downstream shoulder on the thermal regime.

The problem must be solved for a more prolonged period (2-3) years in order to obtain more reliable results.

Conclusions

1. A study has been made of the thermal regime of the downstream shoulder of the rockfill dam of the Vilyuy Hydroelectric Power Plant on the basis of design relationships (12), (22), and (23).

2. The results of calculation and the data of field observations have been found to be in close agreement.

3. Certain differences between them are to be ascribed to determination by way of approximation of the parameters entering into the design relations (porosity of the fill, coefficient of permeability of the rock fill, diameter of the jointings, etc).

4. The results of the study can be employed in the design of rock-fill dams.

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