AD-A036 720 TEXAS TECH UNIV LUBBOCK DEPT OF ELECTRICAL ENGINEERING F/G 12/1
CONTINUATION METHODS FOR STABILITY ANALYSIS OF MULTIVARIABLE FE--ETC(U)
AUG 76 R SAEKS, K S CHAO, E C HUANG AF-AFOSR-2631-74

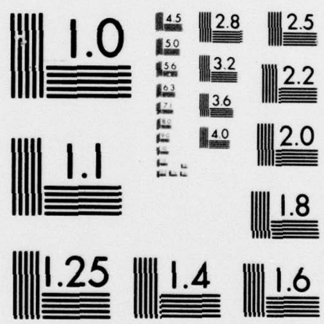
UNCLASSIFIED AFOSR-TR-77-0070 NL

| OF |

AD
A036720

END

DATE
FILMED
4 — 77


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

AFOSR - TR - 77 - 0070

19th MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS

AUGUST 16-17, 1976

See 1473

Edited by
J. D. McPherson

THE UNIVERSITY OF WISCONSIN
MILWAUKEE

Continuation Methods for Stability Analysis of Multivariable Feedback Systems*

R. Saeks, K. S. Chao and E. C. Huang
Department of Electrical Engineering
Texas Tech University
Lubbock, Texas 79409

Abstract

Techniques for implementation of a Nyquist stability result for a linear time invariant multivariable feedback system are described. The approach is based on continuation methods for computing the system's eigenvalue loci.

I. INTRODUCTION

The classical Nyquist stability criterion for single-input single-output, linear time-invariant feedback systems has only recently been generalized to multivariable feedback systems [1,2]. Stability theorems are expressed in terms of the eigenvalue loci of the open loop transfer function G(s) of the system. In particular if G(s) is stable, i.e., G(s) has no poles in the right half of the s-plane or on the $j\omega$-axis, then a linear time-invariant multivariable feedback system with n inputs and n outputs is stable if and only if its generalized Nyquist plots (union of eigenvalue loci) does not pass through or encircle the (-1, 0) point [1]. In order to apply the multivariable Nyquist criterion, it is thus necessary to compute the eigenvalue loci as a function of frequency. For a given frequency, the eigenvalues can be calculated by using classical techniques. Since the eigenvalues are functions of frequency, normally one would have to repeat the entire computational procedure for each frequency. In the actual stability analysis, this repetition is however, impractical. Our approach to the stability analysis of multivariable feedback systems is based on continuation methods. The basic idea of all continuation methods is to convert the solution of a parameterized family of algebraic problems into the solution of a differential equation. Then if one can find the solution of an initial problem by using classical methods the solutions to the other problems can be obtained by integrating the associated differential equation with the initial solution as an initial condition.

II. EIGENVECTOR APPROACH

Our first method is based on the approach described by Faddeev and Fadeeva [3] and Van Ness et. al. [4]. A differential equation is written with the eigenvalues as dependent variables and the frequency as variable parameter. We then compute a set of initial eigenvalues by classical analysis techniques and integrate the resulted differential equation to obtain the required eigenvalues for each frequency. The eigenvalues $\lambda_i(\omega)$ of $G(j\omega)$ and their complex conjugates $\bar{\lambda}_i(\omega)$ satisfy

$$G(j\omega)X_i(\omega) = \lambda_i(\omega)X_i(\omega) \qquad i=1,2,\ldots,n \tag{1}$$

and

$$G^*(j\omega)V_i(\omega) = \bar{\lambda}_i(\omega)V_i(\omega) \qquad i=1,2,\ldots,n \tag{2}$$

where $X_i(\omega)$ and $V_i(\omega)$ are the corresponding eigenvectors of $\lambda_i(\omega)$ and $\bar{\lambda}_i(\omega)$ respectively, and $G^*(j\omega)$ is the complex conjugate transpose matrix of $G(j\omega)$.

*This research was supported in part by NSF Grants GK-36223 and ENG75-09074 and AFOSR Grant 74-2631 .

We differentiate (1) with respect to ω to yield

$$\frac{d\lambda_i}{d\omega} = \frac{\langle \frac{dG}{d\omega} X_i, V_i \rangle}{\langle X_i, V_i \rangle}, \quad i = 1, 2, \ldots, n. \qquad (3)$$

The differential equations involving X_i and V_i are obtained as

$$\frac{dX_i}{d\omega} = \sum_{j=1}^{n} \alpha_{ij} X_j, \quad i = 1, 2, \ldots, n. \qquad (4)$$

$$\frac{dV_i}{d\omega} = \sum_{j=1}^{n} \beta_{ij} V_j, \quad i = 1, 2, \ldots, n. \qquad (5)$$

where

$$\alpha_{ii} = 0, \quad \alpha_{ij} = \frac{\langle \frac{dG}{d\omega} X_i, V_j \rangle}{(\lambda_i - \lambda_j)\langle X_j, V_j \rangle} \quad i \neq j. \qquad (6)$$

$$\beta_{ii} = 0, \quad \beta_{ij} = \frac{\langle \frac{dV_i}{d\omega}, X_j \rangle}{\langle V_j, X_j \rangle} \quad i \neq j. \qquad (7)$$

Starting with a set of predetermined initial conditions $\lambda_i(0) = \lambda_{io}$, $X_i(0) = X_{io}$ and $V_i(0) = V_{io}$ for $i = 1, 2, \ldots, n$, we integrate (3), (4) and (5) to obtain the required eignevalues for each frequency. The eigenvalue loci are computed in a continuous manner by numerical integration.

III. JACOBIAN METHOD

For an nth order system, the above algorithm requires the numerical integration of a set of 3n equations and the computation of two sets of unwanted variables--namely the eigenvectors X_i and V_i. These disadvantage, can easily be avoided if the characteristic equation for the multivariable feedback system can be predetermined. A much simpler method can be formulated based on the approach for finding multiple solutions for a nonlinear equation developed by Chao et. al. [5].

Let the characteristic equation of $G(j\omega)$ be given by an nth order polynomial in eigenvalue λ with complex coefficients

$$f[\lambda(\omega)] = |\lambda I - G(j\omega)| = 0. \qquad (8)$$

Instead of solving (8) directly for each frequency, we consider two simultaneous differential equations of the form

$$\frac{df}{dt} = -f(t) \qquad f(0) = f[\lambda(\omega_0)] = 0$$
$$\frac{d\omega}{dt} = \pm 1 \qquad \omega(0) = \omega_0. \qquad (9)$$

Assuming the nonsingularity of the Jacobian Matrix

$$J = \begin{bmatrix} \frac{\partial f}{\partial \lambda} & \frac{\partial f}{\partial \omega} \\ \frac{\partial \omega}{\partial \lambda} & \frac{\partial \omega}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial \lambda} & \frac{\partial f}{\partial \omega} \\ 0 & 1 \end{bmatrix}, \qquad (10)$$

in the x-ω space the algorithm (9) reduces to

$$\begin{bmatrix} \frac{d\lambda}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = J^{-1} \begin{bmatrix} -f \\ \pm 1 \end{bmatrix}; \quad \begin{bmatrix} f(0) \\ \omega(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_0 \end{bmatrix}. \qquad (11)$$

It is seen from the solution of (9)

$$f(t) = 0e^{-t} \equiv 0 \qquad (12)$$
$$\omega = \pm t.$$

that for any admissible pair of ω_0 and $\lambda(\omega_0)$ satisfying (8), the corresponding trajectory will remain on the solution curve f=0 as ω changes. The + or - sign is chosen depending on whether one would like to increase or decrease ω. Equation (11) may now be solved by any numerical integration techniques and the eigenvalue loci can be traced automatically by integrating only a second order differential system.

IV. EXAMPLE

To illustrate the approaches presented, consider a linear time-invariant, multivariable feedback system with open loop transfer function characterized by

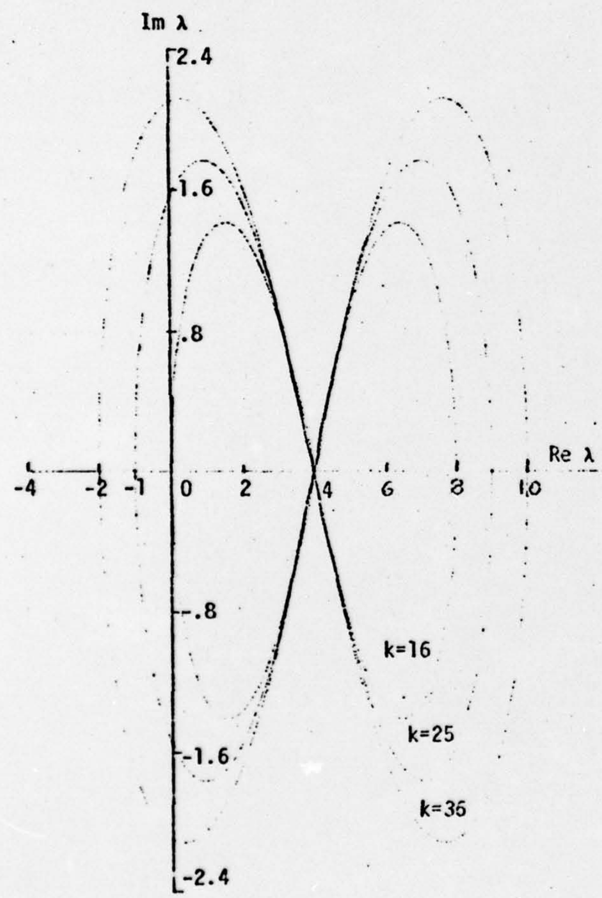
$$G(s) = \begin{bmatrix} 4 & \frac{k}{s+2} \\ \frac{s+2}{s+1} & 4 \end{bmatrix} \qquad (13)$$

for which the characteristic equation is given by

$$f[\lambda(\omega)] = \lambda^2 - 8\lambda + \frac{16 - k + 16\omega^2}{1 + \omega^2} + j\frac{k\omega}{1 + \omega^2} = 0. \qquad (14)$$

The generalized Nyquist plots shown in the accompanied figure for the cases where k=16 and 36 are obtained by applying the eigenvector approach where as in the critical case, k=25, the Jacobian method has been used.

In all three cases, the equations are integrated using Euler's method with a step size of 0.01. It is seen from the figure that the system is stable for k<25 since the generalized Nyquist plots do not encircle -1 point.



FIGURE

REFERENCES

[1]. J. F. Barman and J. Katzenelson, "A Generalized Nyquist-Type Stability Criterion for Multivariable Feedback Systems," Memo No. ERL-383, Electronics Research Laboratory, Univ. of California at Berkeley, 1973 (also to appear in Int. J. Contr).

[2]. R. Saeks, "On the Encirclement Condition and its Generalization, IEEE Trans. on Circuits and Systems, Vol. CAS-22, No. 10, pp. 780-785, Oct. 1975.

[3]. D. K. Faddeev and V. N. Faddeva, Computational Methods of Linear Algebra, Freeman, San Francisco, CA., 1963.

[4]. J. E. Van Ness, J. M. Boyle and F. Imad, "Sensitivities of Multiloop Control Systems," IEEE Trans. on Auto. Cont., Vol. AC-10, pp. 308-315, 1965.

[5]. K. S. Chao, D. K. Liu and C. T. Pan, "A Systematic Search Method for Obtaining Multiple Solutions of Simultaneous Nonlinear Equations," IEEE Trans. on Circuits and Systems, Vol. CAS-22, No. 9, pp. 748-753, Sept. 1975.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19 REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

18 1. REPORT NUMBER: AFOSR - TR - 77 - 0070	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER: 9	
6 4. TITLE (and Subtitle): CONTINUATION METHODS FOR STABILITY ANALYSIS OF MULTIVARIABLE FEEDBACK SYSTEMS.	5. TYPE OF REPORT & PERIOD COVERED: Interim rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 R. Saeks, K. S. Chao, E. C. Huang	8. CONTRACT OR GRANT NUMBER(s): AFOSR 74-2631
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas Tech University, Department of Electrical Engineering, Lubbock, Texas 79409	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2304/A6
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, DC 20332	11 12. REPORT DATE: Aug 1976
	13. NUMBER OF PAGES: 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 5p.	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

15 AF-AFOSR-2631-74, NSF-GK-36223

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

16 2304 17 A6

18. SUPPLEMENTARY NOTES

19th MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, pp 346-348, Aug 76

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nyquist Test Engenvalue Loci
Continuation Methods
Multivariable Feedback System

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Techniques for implementation of a Nyquist stability result for a linear time invariant multivariable feedback system are described. The approach is based on continuation methods for computing the system's eigenvalue loci.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE UNCLASSIFIED

406820

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)