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MATHEMATICAL INVESTIGATIONS OF DYNAMICAL SYSTEMS.(U)

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MATHEMATICAL INVESTIGATIONS OF DYNAMICAL SYSTEMS

Final Report
Contract Grant No. ONR N00014-76-C-0073

Submitted to:

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Mathematical and Information Science Division
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Department of the Navy
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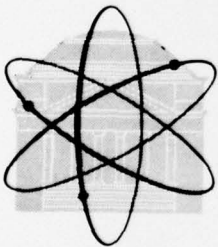
Submitted by:

D. L. Lukes
Associate Professor

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UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

Report No. UVA/525318/AMCS76/101

September 1976

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ABSTRACT

This final report summarizes the work accomplished within the Department of Applied Mathematics and Computer Science at the University of Virginia through support of the United States Office of Naval Research under Contract No. N00014-76-C-0073 which expired July 31, 1976. (See Report No. AMCS-3560-102-74, "Mathematical Investigations of Dynamical Systems," for an account of the results obtained under the previous ONR Contract No. N00014-69-A-0060-0010.)

1. Lukes, D. L., "The Geometry of Differential Equations on Banach Space," Calculus of Variations and Control Theory, Academic Press (1976), pp. 93-138.

Summary.

$F(X)$ denotes the class of bounded C^1 vector fields with bounded derivative on an arbitrary Banach space X . An appropriate Banach space $Z(X)$ is selected so as to contain X^F , the collection of solutions X^f to the ordinary differential equations $\dot{x} = f(x)$ on $(-T, T) \times X$ for $f \in F(X)$. This article studies the dependence of X^f upon f and the geometry of X^F in $Z(X)$.

All but the last chapter deal with the problem in which T is finite. The first chapter defines the spaces $F(X)$, $Z(X)$, X^F and studies an appropriate nonlinear operator on $Z(X)$ as a foundation for later sections.

Chapter 2 proves that the map $X^{(\cdot)}: F(X) \rightarrow Z(X)$ (which applied mathematicians call "solving the differential equation"), sending $f \rightarrow X^f$, is a closed embedding of $F(X)$ into $Z(X)$. In particular $X^{(f)}$ has a continuous (Frechet) derivative $X_f^{(f)}$ and X^F is a closed, nonlinear, submanifold of $Z(X)$, modelled upon a Banach space. A corollary shows that $X_f^{(f)}(t, x)$ and the partial derivative $X^{(f)}(t, x)_f$ agree and a variation of parameters formula for computing $X_f^{(f)}$ is thereby attained.

The geometric nature of the embedding is clarified by chapter 3 which recognizes $X^{(\cdot)}$ as a section of a vector bundle $d: Z(X) \rightarrow F(X)$. A host of related vector bundle isomorphisms appear.

Chapter 4 describes how the differential equations (i.e. $F(X)$) as well as their solutions lie naturally embedded in $Z(X)$. They are a pair of closed submanifolds of $Z(X)$ whose intersection is given five equivalent geometric characterizations. One of these identifies the intersection with the norm-invariant elements of $F(X)$ under the embedding $X^{(\cdot)}$ and another with the solutions to a quasi-linear partial differential equation in $F(X)$. A general solution to this equation is computed for the cases where X is a finite dimensional or separable Hilbert space. Finally the equation is shown to be the steady state form of a partial differential equation, initial value problem in X which is then solved and shown to satisfy a maximum principle.

The point of view taken in Chapter 5 is that of the calculus of variations. The invariance of curve length under the map $X^{(\cdot)}:F(X) \rightarrow Z(X)$ is studied for the situation wherein the vector space $Z(X)$ is turned into a complete Finsler manifold by a change in the measure of curve length which does not disrupt the norm topology of $Z(X)$.

The results of Chapter 6 indicate the existence of some interesting relationships between differential equations on X with those on $F(X)$. This problem brings out the functorial properties of the constructed bundle $d:Z(X) \rightarrow F(X)$ and leads to a natural transformation (in the sense of category theory).

Examples in the last chapter show that a loss of differentiability and continuity of $X^{(\cdot)}$ occurs at some points in $F(X)$ when $T = \infty$.

2. Young, D. F., "Delay Equations in Banach Space and the Control of Linear Volterra-Stieltjes Equations," Ph.D. dissertation in the Department of Applied Mathematics and Computer Science, University of Virginia, August 1975, 112 pages.

Summary.

In this dissertation, we study families of linear operators $\{K_\tau : \tau \in J\}$ and functional equations

$$(1) \quad x = K_\tau x + g$$

where J is any real interval, V is a Banach space and $K_\tau : C(J, V) \rightarrow C(J, V)$.

The properties of $\{K_\tau\}$ abstract properties of the operator family $\hat{K}_\tau : C(J, R^n) \rightarrow C(J, R^n)$ given by

$$(\hat{K}_\tau x)(t) = \int_\tau^+ [d_s \mu(t, s)] x(s) + \alpha(t, \tau) x(\tau)$$

for each $\tau \in J$, so that (1) reduces to a Volterra-Stieltjes equation when $V = R^n$. Sufficient conditions on $\{K_\tau\}$ are given which guarantee the existence and uniqueness of a solution to (1). When $V = C([-r, 0], R^n)$ and K_τ is defined appropriately, (1) is shown to include as a special case the class of neutral functional equations studied by Hale and Meyer. In this case, our results broaden Hale and Meyer's existence and uniqueness results. In the case $V = R^n$, whenever (1) has a unique solution for all τ and all g , this solution is shown to be of the form

$$(2) \quad x(t) = \Phi_1(t, \tau) g(\tau) + \int_\tau^+ \Phi_2(t, s) dg(s).$$

The representation (2) is used to study controllability and time-optimal control of (1), with $g(t) = x_0 + \int_\tau^+ (v(s) + B(s)u(s)) ds$, where x_0 is the initial state, u is the control function and $v(t)$ and $B(t)$ are given. With compact, but not necessarily convex, control restraints, the relationship between local controllability and global controllability is studied, and a weak form of the bang-bang principle is derived. Under certain conditions (examples illustrate that this does not hold for all cases), it is shown that $\Phi_1(t, s) = \Phi_2(t, s)$ and a sufficient matrix criterion for controllability is derived. With compact convex control restraints, the existence of a time-optimal control function is proven for certain special cases of (1).

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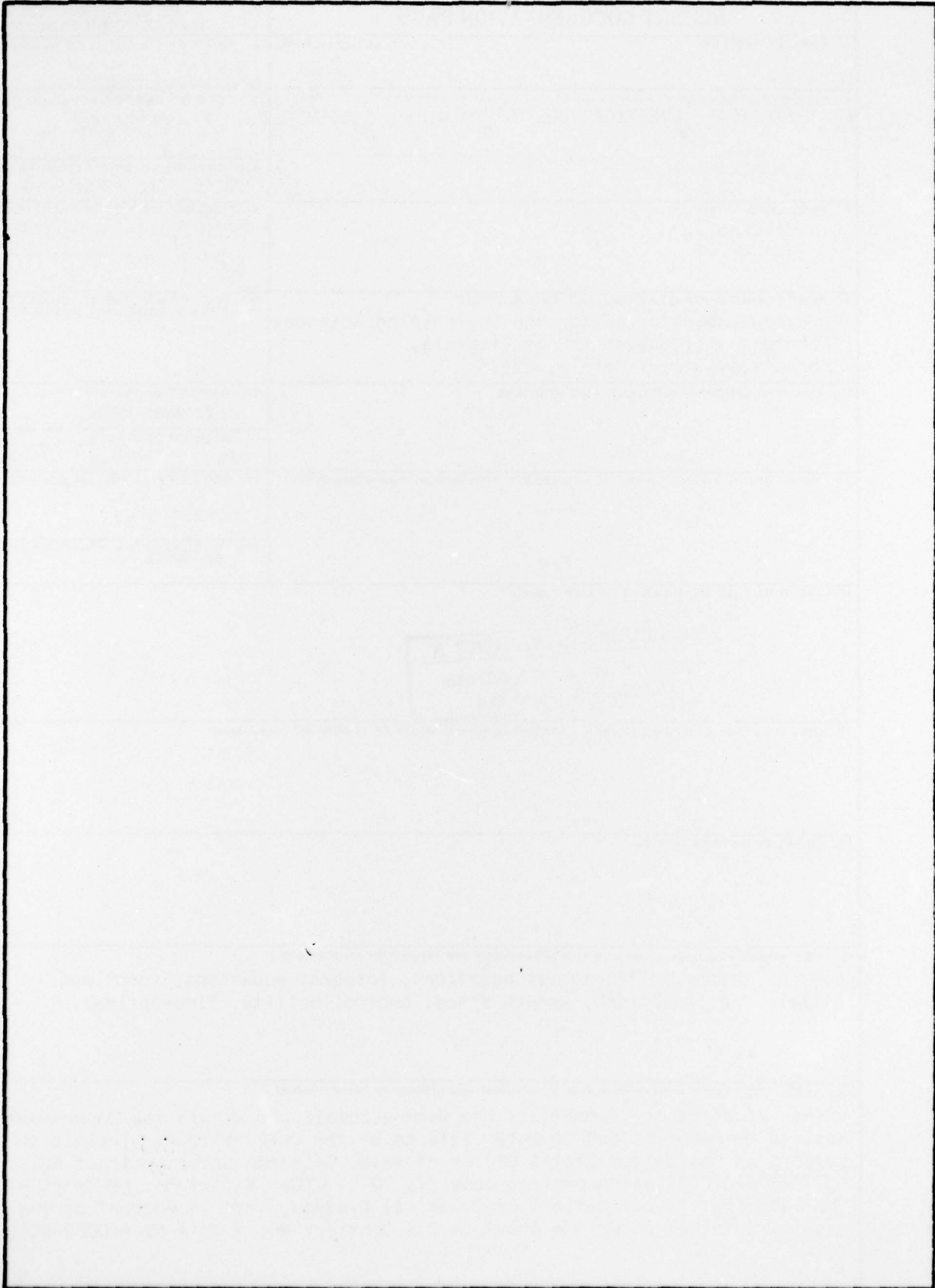
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
6. TITLE (and Subtitle) 6. MATHEMATICAL INVESTIGATIONS OF DYNAMICAL SYSTEMS,		9. TYPE OF REPORT & PERIOD COVERED Final Report.
7. AUTHOR(s) 10. D. L. Lukes		14. PERFORMING ORG. REPORT NUMBER UVA/525318/AMCS76/101
9. PERFORMING ORGANIZATION NAME AND ADDRESS Research Laboratories for the Engineering Sciences Thornton Hall, University of Virginia, Charlottesville, Virginia, 22903		8. CONTRACT OR GRANT NUMBER(s) ONR N00014-76-C-0073
11. CONTROLLING OFFICE NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 11. September 1976
		13. NUMBER OF PAGES eight 12 19p.
		15. SECURITY CLASS. (of this report) unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) control theory, differential equations, integral equations, functional differential equations, Banach space, controllability, time-optimal.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This final report summarizes the work accomplished within the Department of Applied Mathematics and Computer Science at the University of Virginia through support of the United States Office of Naval Research under Contract No. N00014-76-C-0073 which expired July 31, 1976. (See Report No. AMCS-3560-102-74, "Mathematical Investigations of Dynamical Systems," for an account of the results obtained under the previous ONR Contract No. N00014-69-A-0060-0010.)		

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School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,000 students with a graduate enrollment of 350. There are approximately 120 faculty members, a majority of whom conduct research in addition to teaching.

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