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THEORY OF TURBULENT VISCOSITY IN WAVE MOTION (K TEORII TURBULEN--ETC(U)
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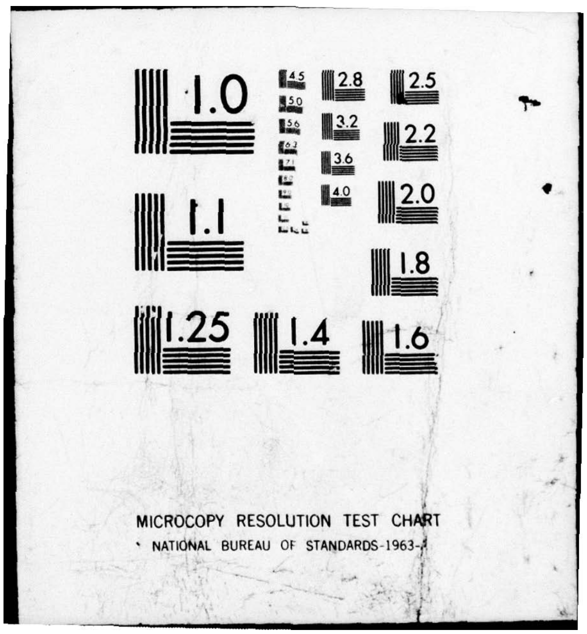
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THEORY OF TURBULENT VISCOSITY IN WAVE MOTION

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At the initial stage of development of wind waves in the sea, the energy dissipation is determined by the molecular viscosity. In this case, the dissipated energy averaged over the wave period may be calculated from the formula (see for example Ref. 1): /773

$$E_{molec} = 2\rho\nu k^3 c^2 a^2, \tag{1}$$

where ρ is the density of the liquid, ν is the kinematic coefficient of molecular viscosity, $k = \frac{2\pi}{\lambda}$, λ is the wavelength, c is the wave propagation velocity, and a is the wave amplitude. As the wind waves develop further, the principal role in their energy change due to scattering begins to be played by turbulence. The magnitude of the energy dissipated in turbulent wave motion may be calculated from the formula

$$E_{turb} = \rho \int_V \overline{\nu_{turb} \left(\frac{du}{dz}\right)^2} dV, \tag{2}$$

where

$$u = a\omega e^{-kz} \cos(kx - \omega t). \tag{3}$$

The bar in Eq. (2) denotes averaging over the wave period, and V is the volume. For the coefficient of turbulent viscosity ν_{turb}, the following expression common in* semiempirical theory may be used in Eq. (2):

$$\nu_{turb} = l^2 \frac{du}{dz}, \tag{4}$$

where ℓ is a quantity representing the turbulence scale, or the Prandtl mixing length.

It is convenient for later purposes to introduce the coefficient of integral turbulent viscosity $\bar{\nu}_{turb}$, given by the relation

$$E_{molec} = E_{turb} \text{ for } \nu = \bar{\nu}_{turb}. \tag{5}$$

Thus, to calculate the dissipation energy, it is necessary to know the expression for the turbulence coefficient $\bar{\nu}_{turb}$. Since turbulent viscosity characterizes the properties of turbulent motion in a wave, $\bar{\nu}_{turb}$ should be determined by all the wave parameters (wave height h = 2a, wavelength λ and wave period T). It is easy to see that these characteristics are exhaustive, since the integral turbulent viscosity $\bar{\nu}_{turb}$ is independent of the coordinates, and one can therefore write

$$\bar{\nu}_{turb} = f(h, \lambda, T). \tag{6}$$

*Numbers in the right margin indicate pagination in the original text.

Considering the dimensions of the quantities in Eq. (6) and denoting by δ the dimensionless parameter $\frac{h}{\lambda}$ representing the wave steepness, we can obtain the most general relation for the coefficient $\bar{\nu}_{turb}$:

$$\bar{\nu}_{turb} = \frac{h\lambda}{T}\psi(\delta). \qquad (7)$$

Concerning the type of the function $\psi(\delta)$ in Eq. (7), the following statement may be made: the steeper the waves (the larger δ), the greater the velocity gradients and energy dissipation. Therefore $\bar{\nu}_{turb}$ and hence, $\psi(\delta)$ should increase with increasing δ, i.e.,

$$\psi'(\delta) > 0. \qquad (8)$$

In order to be able to discuss various theoretical models of turbulence in wave motion and compare them with existing indirect and empirical data, we will postulate that the functional relation (6) may be written in the form of a power /774
law:

$$\bar{\nu}_{turb} = nh^{\alpha}\lambda^{\gamma}T^{p}, \qquad (9)$$

where n is a dimensionless constant coefficient. On the basis of dimensional considerations, it follows from (9) that

$$\alpha + \gamma = 2; \; p = -1. \qquad (10)$$

Using the well-known relations of wave theory

$$\frac{\lambda}{T} = c; \quad \lambda = \frac{2\pi}{g}c^2, \qquad (11)$$

and introducing the notation

$$\beta = \frac{c}{W}; \quad K = 2\pi n, \qquad (12)$$

where W is the velocity of the wind which generated the waves, considering Eqs. 10, we can write Eq. (9) in the form

$$\bar{\nu}_{turb} = K\delta^{\alpha}\beta^{3}\frac{W^3}{g}. \qquad (13)$$

If use is made of the experimental fact that

$$\delta = f(\beta), \qquad (14)$$

one can represent Eq. (13) in the form

$$\bar{\nu}_{turb} = K\frac{W^3}{g}\varphi(\beta), \qquad (15)$$

where $\phi(\beta)$ will depend on both the type of relation (14) and on the magnitude of the exponent α in Eq. (13). Relation (15) obtained shows that the turbulence coefficient depends on the wind velocity W and on the stage of wave development β (β is frequently called the wave age). The latter fact leads to the important conclusion that the question of selection of the exponent α is not a formal one, since α represents the degree of influence of the scale of wind-driven waves on turbulent exchange in the upper layer of the sea. It follows from Eq. (15) that in the case of fully developed waves ($\beta \simeq 1$), the turbulence coefficient depends only on the wind velocity. In contrast to the previously known Ekman-Rossby[2] quadratic dependence $\bar{\nu}_{turb}(W)$, we obtained a cubic one. In Ref. 3, such a dependence was obtained empirically. The data given in Ref. 4 may also be regarded as confirmation of the cubic dependence $\bar{\nu}_{turb}(W)$.

Since, as has been shown experimentally, the wave height and length increase with increasing β, $\bar{\nu}_{turb}$ should also increase with β, i.e.,

$$\bar{\nu}_{turb}(\beta_1) > \bar{\nu}_{turb}(\beta_2) \text{ for } \beta_1 > \beta_2. \qquad (16)$$

If the form of function (14) is considered to be reliably established empirically, condition (16) may serve as a criterion for selecting α. On the other hand, as we have noted above, the selection of α in Eq. (13) should also be consistent with the most general dependences (7), (8).

We will go directly to the determination of the function $\phi(\beta)$ in Eq. (15) for different values of α in Eq. (13). We will use the relationship, known from field observations, between the wave steepness δ and wave age β. Such a relationship was first studied Sverdrup and Munk.[5] This empirical relationship was later refined by Neumann[3] and may be written as follows (Neumann's data, obtained for so-called "substantial" waves, were converted for medium waves according to statistical wave theory[6]):

$$\begin{aligned} &\delta = 0{,}062 \text{ for } \beta \leq \frac{1}{3}, \\ &\delta = 0{,}1075e^{-1{,}667\beta} \text{ for } \frac{1}{3} \leq \beta \leq 1. \end{aligned} \qquad (17)$$

Recently, Krylov[7] obtained a theoretical dependence of the wave steepness δ on β, written in the same form (we use the rounded values given in Ref. 7): /77!

$$\begin{aligned} &\delta = 0{,}06 \text{ for } \beta \leq 0{,}375, \\ &\delta = \frac{0{,}0225}{\beta} \text{ for } 0{,}375 \leq \beta \leq 1. \end{aligned} \qquad (18)$$

Figure 1 shows both of these curves along with the experimental data of Sverdrup and Munk, Neumann, and Vilenskiy and Glukhovskiy.[8] It is evident that the agreement of the curves is completely satisfactory, and therefore either one can be used for our purposes. We shall subsequently use relation (18), since it was obtained theoretically and is written in the form most convenient for calculations. In Fig. 1, the region least studied experimentally is the range of β values from 0 to 0.4 (more accurately, from $\beta_{min} = \frac{c_{min}}{W}$, where $c_{min} = 0.23$ m/sec; calculations show that β_{min} is close to zero). Some experimental results indicate that when $\beta < 0.4$, the wave steepness increases sharply. To simplify further calculations, we will consider the waves in the interval $0 < \beta < 0.4$ to be turbulent ones with a constant

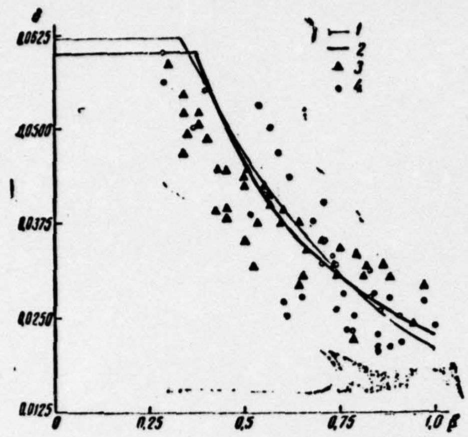

Fig. 1. Relationship between wave steepness δ and wave age β:
1 - Neumann's empirical curve;[3]
2 - Krylov's theoretical curve;[7]
3 - empirical data of Vilenskiy and Glukhovskiy;[8]
4 - empirical data of Sverdrup and Munk.[5]

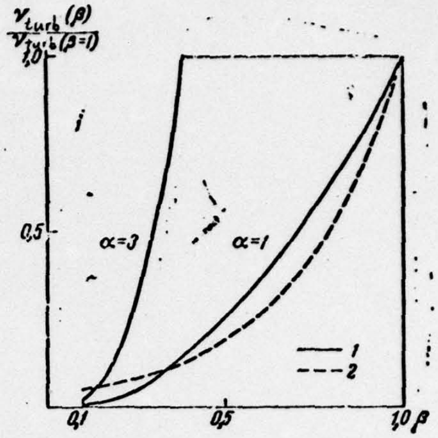

Fig. 2. Turbulent viscosity coefficient vs wave age β:
1 - curves corresponding to different values of α in Eq. (13), relation (19) being used;
2 - Neumann's empirical curve.[3]

steepness. Since the case of developed waves ($\beta > 0.4$) is of principal interest in determining the turbulent viscosity in the upper layer of the sea, this assumption will not appreciably affect the results obtained. In addition to relations (17) and (18), there are also the data of Darbyshire[9,10] pertaining to the dependence of the wave steepness δ on β. The structure of the relationship $\delta(\beta)$ in them is the same as in relation (18); therefore, in determining the function $\bar{\nu}_{turb}(\beta)$, the use of Darbyshire's data will give the same results as when formula (18) is employed. The use of the universal relation $\delta = f(\beta)$ instead of all the other data on waves makes it possible to find from (13) the dependence of the turbulence coefficient on the wave age β. In view of Eqs. (13) and (18), relations of this kind with $\alpha = 1$ and $\alpha = 3$ in Eq. (13) are shown in Fig. 2 along with Neumann's empirical curve. Neumann's curve was obtained by allowing for the wave energy balance. Thus, a comparison of formula (13) with indirect data leads to the conclusion that α should be close to unity. On the other hand, the value of exponent α in Eq. (13) can be found by using expressions (2, 3, 4) and some hypothesis on the turbulence scale ℓ in Eq. (4) for the case of wave motion. The general relations (7), (8), (15), and (16) which we obtained make it possible to analyze some of these hypotheses.

Hypothesis 1. It may be written as follows:

$$l \sim h \text{ or } l \sim he^{-kz}. \qquad (19)$$

In this case, using (2), (3) and (4), an expression corresponding to $\alpha = 3$ in Eq. (13) is obtained for the coefficient of integral turbulent viscosity $\bar{\nu}_{turb}$.

As is evident from Fig. 2, the form of the function $\phi(\beta)$ is inconsistent with condition (16) in this case. Thus, hypothesis (19) inadequately describes the turbulence conditions in a wave, since it leads to the conclusion that as the scale of wind-driven waves increases, the turbulence coefficient remains constant. /776

Hypothesis 2* is related to the assumption that

$$\ell \sim \lambda. \tag{20}$$

It is easy to see from Eq. (3) that this case corresponds to the Karman similarity hypothesis, according to which

$$l \sim \frac{\partial u}{\partial z} \Big/ \frac{\partial^2 u}{\partial z^2}. \tag{21}$$

Using (2), (3), (4), and (20), one can obtain for $\overline{\nu}_{turb}$ an expression corresponding to $\alpha = 1$ in Eq. (13). It is evident from Fig. 2 that this case satisfies condition (16). However, the case $\alpha = 1$, as can be readily seen from formulas (7), (9), and (10), leads to violation of condition (8), since when $\alpha = 1$, in formula (7)

$$\psi(\delta) = \text{const}. \tag{22}$$

The significant deficiencies of hypothesis (20) are chiefly due to the following. Near a free surface, the vertical turbulence scale cannot be greater than the wave height. Present concepts and data on turbulent fluctuations at a free surface indicate their fairly high isotropy. Thus, the only assumption would be to consider that near a free surface, the mixing length should be proportional to the wave height, not the wave length. In addition, the free surface has a damping effect on turbulent fluctuations. This effect is somewhat analogous to that of a solid wall, namely, with increasing distance from the free surface, the turbulence coefficient should increase in some layer. As will be seen below, this fact is confirmed by certain indirect empirical data. However, in the case of (20) and (21), such a layer cannot be obtained, since what then occurs is only the attenuation of turbulence from the physical liquid surface itself, together with the attenuation of wave motion. In this connection, Levich[13] proposed the following hypothesis of the turbulence scale in wave motion:

Hypothesis 3. The quantity ℓ in Eq. (4) is proportional to the wave height and increases with the distance from the free surface according to the expression

$$\ell \sim h + z. \tag{23}$$

Using (2), (3), (4), and (23), one can obtain (see Ref. 14) the following expression for the coefficient $\overline{\nu}_{turb}$:

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*Such a case was discussed in Refs. 7 and 11. Hypothesis (20) was first used by Dobroklonskiy,[12] but the results he obtained do not pertain to the characteristics of integral turbulent viscosity $\overline{\nu}_{turb}$, since he did not calculate the energy dissipation in trochoidal waves. For this reason, the results obtained in Ref. 12 are not analyzed in the present paper.

$$\bar{\nu}_{turb} = K_1 \frac{h\lambda}{T} \psi(\delta), \tag{24}$$

where K_1 is some constant coefficient, and

$$\psi(\delta) = \left(1 + 3\pi\delta + \frac{9}{2}\pi^2\delta^2\right). \tag{25}$$

We see that hypothesis (23) leads to results consistent with general formulas (7) and (8), in contrast to hypothesis (20). It is easy to see from Eq. (25) that if we neglect the first two and last two terms in Eq. (25), we obtain the results flowing correspondingly from hypotheses (19) and (20), whose deficiencies we have already noted.* On the other hand, it follows that when Eq. (24) and (25) are used, the dependence $\bar{\nu}_{turb} = \phi(\beta)$ will be represented in Fig. 2 by a curve occupying an intermediate position between the curves $\alpha = 1$ and $\alpha = 3$, i.e., will satisfy the requirement formulated above for the form of the function $\phi(\beta)$ (16), and will be similar to Neumann's empirical curve. In addition, it follows from expressions (3), (4) and (23) that in a certain layer ($z < \lambda$), the turbulence coefficient will increase with depth. This result may be obtained in another way, which is of interest, since expression (23) does not flow directly from dimensional analysis (in wave motion, three parameters have the dimension of length: h, λ, and z). The transformations (9)-(14) which we made showed that the turbulence coefficient may always be represented as a function of three parameters:

$$\bar{\nu}_{turb} = \Phi(W, g, \beta). \tag{26}$$

One can consider waves with fixed β (at a given stage of development) and /777
assume that the turbulence coefficient also depends on the vertical coordinate z. Then

$$\nu_{turb} = \Phi(W, g, z). \tag{27}$$

In view of (15), dimensional analysis makes it possible to write, by (27),

$$\nu_{turb} = \frac{W^3}{g} \Phi\left(\frac{gz}{W^2}\right). \tag{28}$$

It is evident from Eq. (28) that when analyzing processes of turbulent exchange in the surface layer of the sea, which is included in the waves, one must consider the dimensionless parameter gz/W^2 (Froude number). This distinguishes the problem of turbulence in the upper layer of a sea of homogeneous density from the analogous problem for the ground layer of the atmosphere.

In the case $z < \frac{W^2}{g}$, one can confine oneself to the first terms of the power series expansion of the function $\phi\left(\frac{g^z}{W^2}\right)$ and set in Eq. (28)

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*Levich[13] assumed that the first two terms in Eq. (25) are small in comparison with the third. Actually, they are rather large, but, as we have shown, strictly speaking, only one first term in Eq. (25) cannot be left either.

$$\Phi\left(\frac{gz}{W^2}\right) = 1 + A\frac{gz}{W^2}. \qquad (29)$$

where A is some constant, which in the general case may be regarded as a function of β. Comparing Eq. (29) with the experiment, we will indicate the paper of Pivovarov,[15] who, on the basis of data on propagation of diurnal temperature waves in the upper layer of the sea, obtained a linear dependence of the mixing coefficient on depth, and found that $A > 0$. Unfortunately, no data are given in this paper on the wave and wind regime for this case. Such data are given in Ref. 4, but the latter does not explain for what level the values of the mixing coefficient were calculated. Therefore, it should be emphasized at this point that the relations (28) and (29), obtained from dimensional considerations, together with the indirect data of Ref. 15 confirm hypothesis (23), according to which the turbulence coefficient should increase in a certain surface layer with increasing distance from the free surface of the liquid. Obviously, below this layer, because of the rapid attenuation of the wave motion itself with depth, the turbulence should also be attenuated. Thus, the presence of a maximum of the turbulent mixing coefficient at some intermediate depth in the upper layer of the sea is physically more justified. In the general case, the position of this maximum should depend on the wind velocity and stage of wave development β. These facts are essential in solving certain problems of turbulent mixing in the upper layer of the sea.

In conclusion, the author considers it his duty to express his thanks to V. G. Levich for his counsel.

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