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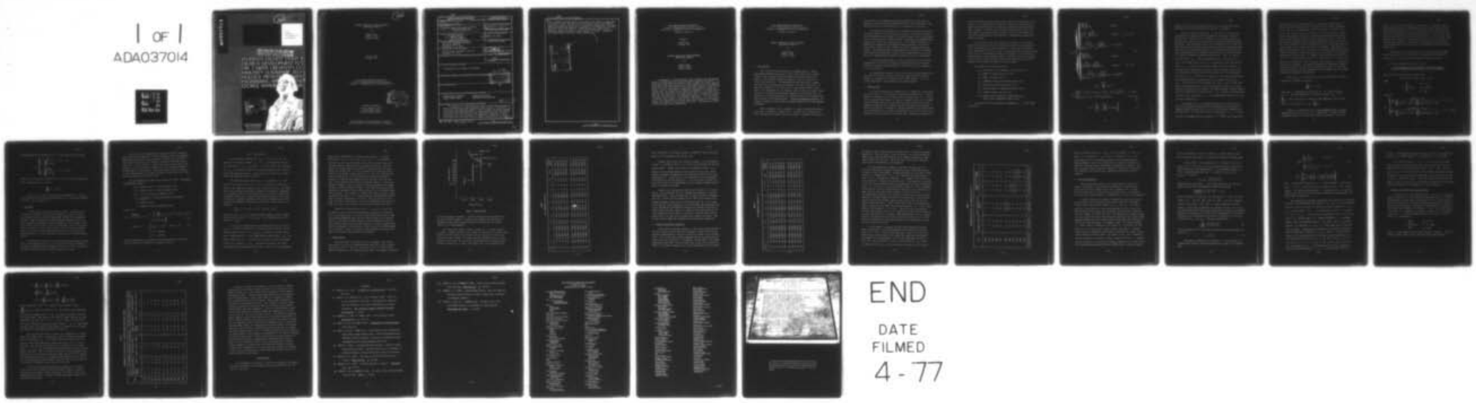

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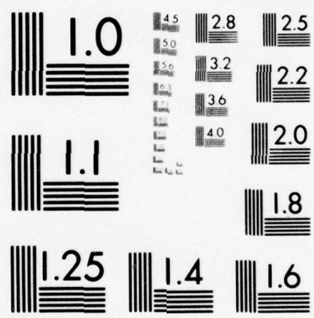

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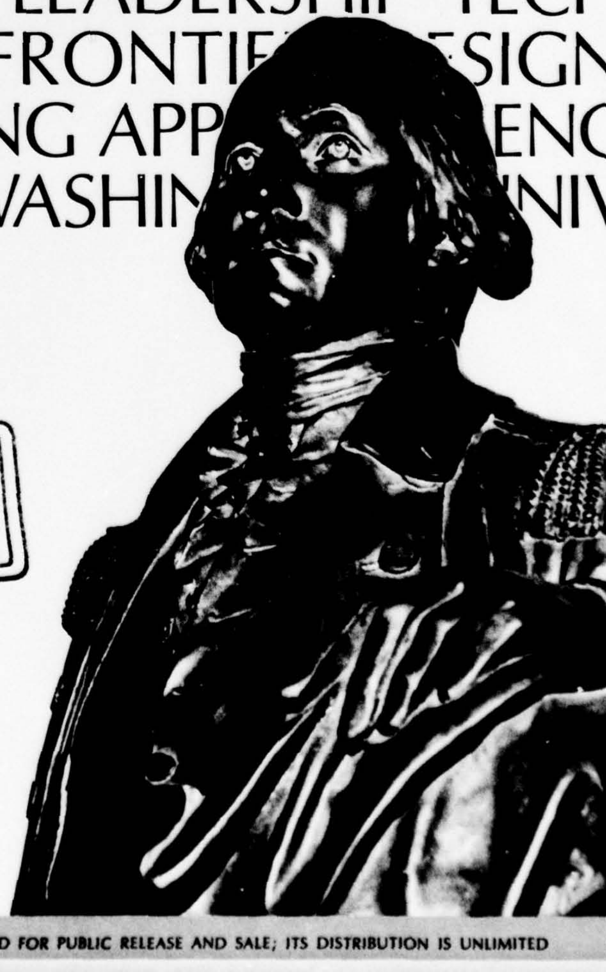

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(12)

QUEUEING MODELS FOR SPARES INVENTORY AND REPAIR CAPACITY

by

Donald Gross
Henry D. Kahn
Joseph D. Marsh

Serial T-344
5 January 1977

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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A population of items which break down at random times and require repair is studied (the classic "machine repair problem with spares"). It is desired to determine the number of repair channels and spares required over a multi-year planning horizon in which population size and component reliability vary, and a service level constraint is imposed. When an item fails, a spare (if available) is immediately dispatched to replace the failed item. The failed item is removed, transported to the repair
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depot, repaired, and then placed in the spares pool (which is constrained to be empty not more than 10% of the time) unless there is a backlog of requests for spares in which case it is dispatched immediately. The first model considered treats removal, transportation, and repair as one service operation. The second model is a series queue which allows for the separate treatment of removal, transportation, and repair. Breakdowns are assumed Poisson and repair times, exponential.

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1. Introduction

One of the earliest applications of queueing theory to spares provisioning problems was the work of Taylor and Jackson [12], in which a finite source queueing model with spares was used to determine the number of spare engines required to maintain a fleet of aircraft at a certain efficiency level. A bibliography of some recent work on queueing approaches to provisioning type problems is given in Lureau [6]. Of particular interest are [2], [8], and [9]. The problem treated in this paper is the determination of an adequate number of spares and repair lines (servers) for replacing and repairing components which randomly fail, assuming that the failed components are replaced by spares (if available) and once repaired, in turn become spares. A multi-year planning horizon is considered, allowing for growth both in component population size and component reliability.

When a component fails, a request for a spare is immediately placed. If no spare is available, a delay occurs. A service level constraint of 10% is imposed on such delays; that is, we desire a capability such that at at least 90% of the requests for spares are immediately filled from on-hand spares inventory (at most 10% backordering of spares requests). This service criterion is often referred to as availability or fill rate.

The objective then becomes one of minimizing expenditures for spares and servers subject to a 90% fill rate constraint. The classical machine repair queueing model with spares is the driving model in that it determines fill rate given a specific number of spares and servers, the component population size, failure rate and service rate. Times between failures are assumed to be exponentially distributed random variables as are service times (which include the time required for removal, transportation, and repair). This queueing model is then embedded in a heuristic cost optimization model which determines a "good" mix of spares and servers for each year in the planning horizon, while satisfying the fill rate constraints.

An alternate queueing model is also presented which treats removal, transportation, and repair as separate serving stations, but which must assume an "infinite" calling population. The accuracy of this assumption is also investigated.

The methodology presented here has been used for provisioning servers and spares for a fleet of marine gas turbine engine ships (each engine having two components--a gas generator and a power turbine). It is, of course, applicable to other similar provisioning problems.

2. Queueing Model

In order to determine the probability of a request for a spare being met without delay, it is necessary to calculate the probability of various numbers of components in the repair system at any particular time. Changes in the population with respect to size and reliability take place at random times throughout each year so that unless (or until) population size and component reliability stop changing, steady state cannot be approached. It appears analytically intractable to calculate transient state probabilities. As an approximation, we consider the population to be in steady state at its average size and reliability for an entire year, changing instantaneously to a new steady-state position at new average values at the beginning of each new year. In situations where failures are frequent, transient effects should die out quickly and our steady-state approximations should be adequate. However, if failures are infrequent, the transient effects will take longer to disappear and might present a problem. Nevertheless, for most provisioning purposes, the assumption of instantaneous steady state should be adequate.

At any point in time, the population is composed of units which may have different failure rates, since units added to the population in later years generally are more reliable due to technological learning. It is assumed here that for each year, all components have identical failure rates equal to the population average, which changes on a yearly basis, as described above. This assumption that all components operate at the population average failure rate was investigated in Reference [5], and the results will be briefly described later in this paper.

We introduce the following notation:

$p_{n,i}$ = steady-state Pr{n components in repair, year i}

c_i = number of repair facilities, year i

y_i = number of spares, year i

λ_i = component failure rate (Poisson mean), year i

$\bar{\lambda}_i$ = population average failure rate, year i

$\bar{R}_i$ = expected number of components repaired, year i

N_i = component population size, year i

$1/\mu_i$ = average service (removal, transportation, and repair) time, (exponential mean), year i.

The equations for the steady-state probability of n items "down" are given as

$$
p_{n,i} = \begin{cases}
\begin{cases}
\dfrac{N_i^n}{n!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (0 \le n \le c_i) \\[2ex]
\dfrac{N_i^n}{c_i^{\,n-c_i}\, c_i!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (c_i \le n \le y_i) \\[2ex]
\dfrac{N_i^{y_i} N_i!}{(N-n+y_i)!\; c_i^{\,n-c_i}\; c_i!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (y_i < n \le y_i + N_i)
\end{cases} & \langle c_i \le y_i \rangle \\[8ex]
\begin{cases}
\dfrac{N_i^n}{n!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (0 \le n \le y_i) \\[2ex]
\dfrac{N_i^{y_i} N_i!}{(N_i-n+y_i)!\; n!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (y_i < n \le c_i) \\[2ex]
\dfrac{N_i^{y_i} N_i!}{(N_i-n+y_i)!\; c_i^{\,n-c_i}\; c_i!}\left(\dfrac{\overline{\lambda}_i}{\mu_i}\right)^n p_{0,i} & (c_i < n \le y_i + N_i)
\end{cases} & \langle c_i > y_i \rangle
\end{cases} \tag{1}
$$

$$
p_{0,i} : \sum_{n=0}^{N_i+y_i} p_{n,i} = 1
$$

$$
\overline{\lambda}_i = \begin{cases}
\left\{ (N_i - N_{i-1})\lambda_i + \overline{R}_{i-1}\lambda_{i-1} + (N_{i-1} - \overline{R}_{i-1})\overline{\lambda}_{i-1} \right\} \Big/ N_i , & N_i \ge N_{i-1} \\[1ex]
\left\{ \overline{R}_{i-1}\lambda_{i-1} + (N_{i-1} - \overline{R}_{i-1})\overline{\lambda}_{i-1} \right\} \Big/ N_{i-1} , & N_i \le N_{i-1}
\end{cases} \tag{2}
$$

$$
\overline{R}_i = \overline{\lambda}_i \left[N_i - \sum_{n=y_i+1}^{N_i+y_i} (n-y_i)\, p_{n,i} \right]. \tag{3}
$$

Equation Set (1) gives the standard formulas for a machine repair model with spares (see Gross and Harris [4], page 123, for example).

Using the average failure rate, $\bar{\lambda}$, for each year allows for the incorporation of changing component reliability as the years progress. Equation Set (2) shows how $\bar{\lambda}$ is computed. It is, perhaps, easiest to explain in the context of the marine gas turbine engine example mentioned previously. Year one begins with the first introduction of a few gas turbine ships and each succeeding year brings into the fleet additional ships until the fleet reaches full strength. The components introduced in the later years generally have improved reliability, either through learning or a conscious component improvement program (CIP). Thus λ_i tends to be smaller than λ_{i-1} . When the population size is increasing $(N_i > N_{i-1})$, the average failure rate over all components in the population for year i, $\bar{\lambda}_i$, is given as the weighted average of the new components introduced into the population at the best current achievable failure rate, λ_i ; those repaired during the past year at that year's best achievable failure rate, λ_{i-1} ; and those old components in the population which were not repaired and are thus operating at the old average failure rate, $\bar{\lambda}_{i-1}$. If the fleet size is decreasing $(N_i < N_{i-1})$, that is, some ships may be retired from service, then the computation is given by the second equation of Equation Set (2), namely, the weighted average of those repaired last year coming in with a failure rate of λ_{i-1} and those not repaired but operating at the old average $\bar{\lambda}_{i-1}$. The average number repaired in a year is given by Equation (3).

The assumption that all components operate at an average failure rate was investigated in Reference [5] by developing the exact model for unequal failure rates with $N=1$, $Y=1$, $c=1$, so that one component has a failure rate of λ_1 while the other has a failure rate of λ_2 . It turns out that (i) assuming both units operate at $\bar{\lambda} = \frac{\lambda_1+\lambda_2}{2}$ gives conservative results with respect to availability, i.e., the actual availability is greater than that computed using $\bar{\lambda}$ and (ii) even for sizable differences in λ_1 and λ_2 , the approximate availability using $\bar{\lambda}$ is close to the actual. The reader is referred to Reference [5] for more detail. Result (i) has some intuitive appeal since components having higher failure rates would be expected to fail (and hence be in repair) more often. Thus, the more reliable components operate a larger proportion of the time so that the actual failure rate for the system would tend to be lower than the arithmetic average of the failure rates over all components. Nevertheless, it appears from Result (ii) that this effect is not highly significant and that using the population average failure rate for each component is an adequate approximation in most cases. For example, using the exact model with $N = Y = c = 1$, it was found that even for λ_2 twice that of λ_1 , the percentage error in availability was no more than 5% when using $\bar{\lambda}$ as the failure rate for each component. In actual applications, the individual component failure rates tend to be much closer together than a factor of two, since reliability growth is gradual, and hence we feel confident that the average failure rate assumption is justifiable.

Once the $p_{n,i}$ are obtained, the fill rate constraint is given by (temporarily dropping the subscript i)

$$\sum_{n=0}^{v-1} p_n \geq 0.90 ,$$

since when n components are down, there are $y-n$ spares available $(n < y)$. Thus the probability of no spares on hand, P_{out} , is $1 - \sum_{n=0}^{y-1} p_n$ and the percentage of requests filled immediately from on-hand spares is $\lambda(1-P_{out})/\lambda = 1 - P_{out} = \sum_{n=0}^{y-1} p_n$.

However, the probability that a failed component finds n in the system should be used in the fill rate computation. In common queueing terminology these are the "arriving customer" probabilities and, in the context of this paper, correspond to the occurrence of component failures which generate requests for spares and thus they shall be referred to as failure point probabilities. For the finite source with spares queue considered in this paper the failure point probabilities are not equal to the general time probabilities given by Equation Set (1). For the finite source-no spares queue (see, e.g., Cooper [1], pp. 82 ff.) the failure point probabilities are equivalent to the general time probabilities for a finite calling population of one less but this relationship does not hold for the spares case.

The failure point probabilities for the finite source with spares queue (denoted by q_n as contrasted to the general time probabilities, denoted by p_n) may be derived as follows. Using Bayes' theorem,

$$q_n \equiv \Pr\{n \text{ in system} \mid \text{failure about to occur}\}$$

$$= \frac{\Pr\{n \text{ in system}\} \Pr\{\text{failure about to occur} \mid n \text{ in system}\}}{\sum_n [\Pr\{n \text{ in system}\} \Pr\{\text{failure about to occur} \mid n \text{ in system}\}]} .$$

Now, since this is a birth-death process with

$$\Pr\{\text{failure in } t, t+\Delta t\} = \lambda_n \Delta t + o(\Delta t) ,$$

where

$$\lambda_n = \begin{cases} N\lambda , & (0 \le n < y) \\ (N-n+y)\lambda , & (y \le n \le y+N) , \\ 0 , & (n > y+N) \end{cases}$$

we obtain

$$q_n = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{p_n[N\lambda\Delta t + o(\Delta t)]}{\sum\limits_{n=0}^{y-1} p_n[N\lambda\Delta t + o(\Delta t)] + \sum\limits_{n=y}^{y+N} p_n[(N-n+y)\lambda\Delta t + o(\Delta t)]} , & (0 \le n < y) \\ \lim\limits_{\Delta t \to 0} \dfrac{p_n[(N-n+y)\lambda\Delta t + o(\Delta t)]}{\sum\limits_{n=0}^{y-1} p_n[N\lambda\Delta t + o(\Delta t)] + \sum\limits_{n=y}^{y+N} p_n[(N-n+y)\lambda\Delta t + o(\Delta t)]} , & (y \le n \le y+N) . \end{cases}$$

Dividing numerator and denominator by Δt and taking the limit yields,

$$q_n = \begin{cases} \dfrac{Np_n}{N - \sum_{n=y}^{y+N} (n-y)p_n}, & (0 \leq n < y) \\[2ex] \dfrac{(N-n+y)p_n}{N - \sum_{n=y}^{y+N} (n-y)p_n}, & (y \leq n \leq y+N) \end{cases},$$

where the p_n are given in Equation (1). Thus, the fill rate constraints must be based on the q_n, that is

$$\sum_{n=0}^{y-1} q_n \geq 0.90 .$$

For the $\overline{R}$ calculation, the general time probability is required so that Equation (1) is used as given, and in fact $\overline{R}$ equals the denominator term of q_n multiplied by λ.

3. Cost Model

The cost model considers four types of costs: (1) purchase cost of spares, (2) purchase cost of service channels, (3) repair costs and (4) investment costs in component improvement (reliability improvement) programs. Annual operating costs associated with running the spares inventory system or operating the service channels are not included. While the variable portion of these costs can be important, they are not considered in this model since the major purpose of such a strategic planning model as this is for capital budgeting and hence it is the purchase expenditures which are of prime concern at this stage of the decision-making process.

The optimization problem is an integer-nonlinear-programming problem with a hard-to-manage objective function and highly complex constraints. An integer programming algorithm approach has not as yet been successful in solving the problem and hence a heuristic method is utilized.

The heuristic cost "optimization" algorithm considers explicitly only the first two categories of costs, although the repair and component improvement program (CIP) costs are always calculated for each year so that sensitivity analyses with respect to various CIP programs, service times, etc., can be performed. It should be noted that even with no CIP program, $\bar{\lambda}$ and N would still change, but the CIP program influences the magnitudes of these changes. The present worth of the cost stream over the planning horizon is always calculated for just such a use. This will be illustrated in the next section.

The cost minimization problem can be stated as follows. Considering the additional notation,

$C_{1,i}$ = purchase cost of a repair channel, year i

$C_{2,i}$ = purchase cost of a spare component, year i

$C_{3,i}$ = cost of repairing a component, year i

$C_{4,i}$ = investment in reliability growth (CIP) program, year i

d = discount factor

k = number of years in planning horizon,

we desire to

$$\underset{c_1,c_2,\ldots,c_k;\ y_1,y_2,\ldots,y_k}{\text{Minimize}} \quad Z = \sum_{i=1}^{k} d^i \Big[C_{1,i}(c_i - c_{i-1})^+ + C_{2,i}(y_i - y_{i-1})^+ + C_{3,i}\,\bar{R}_i + C_{4,i} \Big] \qquad (4)$$

$$\text{Subject to} \quad \sum_{n=0}^{y-1} q_{n,i} \geq 0.90 \ , \quad (i=1,2,\ldots,k) \qquad (5)$$

$$c_i \geq 0 \ ; \quad \text{integer}$$

$$y_i \geq 0 \ ; \quad \text{integer.}$$

The plus superscript on the first two cost terms indicates the term is zero if the factor in parentheses is negative, that is,

$$(a-b)^+ = \max\{(a-b),0\} .$$

The last term in Equation (4), $C_{4,i}$, is the cost of the CIP and does not explicitly depend on c_i or y_i , although indirectly one can argue that if $C_{4,i}$ is increased, λ_i will be smaller thus affecting the $p_{n,i}$ calculation and hence the constraint [Equation (5)]. The heuristic algorithm ignores this. This CIP effect is observed via a sensitivity type analysis referred to above.

The third term, which involves $\bar{R}_i$, is a function of y_i [explicitly; see Equation (3)], and y_i and c_i implicitly through $p_{n,i}$. However, when compared to purchase costs of spares and servers, the annual repair costs should be small and are ignored by the heuristic algorithm. It is included in the cost calculation since it is directly influenced by reliability growth and is needed for any sensitivity study of CIP. Thus the heuristic algorithm looks only at the costs of purchasing spares and servers and furthermore, considers only one year at a time, that is, it attempts to

$$\min_{c_i,y_i} \quad Z_i = C_{1,i}(c_i - c_{i-1})^+ + C_{2,i}(y_i - y_{i-1})^+ ,$$

for each $i=1,2,\ldots,k$, in a sequential manner, using the "best" combination it finds for year i-1 (c_{i-1},y_{i-1}) as the starting point for its search for (c_i,y_i) .

For the year i computations, the algorithm first checks to see if c_{i-1},y_{i-1} satisfy the constraint. If the constraint is not met, the algorithm computes the ratio $\Delta = C_{2,i}/C_{1,i}$ and takes the largest integer value contained therein. (In the applications we have studied, $C_{2,i}$ was always larger than $C_{1,i}$. If this is not the case, the algorithm should be modified to compute $C_{1,i}/C_{2,i}$, and the words "server" and "spare" interchanged in the description that follows.) Thus for an (approximately) equal dollar expenditure, we can purchase one spare or Δ servers. Both points $(c_{i-1} + \Delta, y_{i-1})$ and $(c_{i-1}, y_{i-1} + 1)$ are checked to determine the fill rate percentage via use of the queueing model, and the point with the largest fill rate percentage is chosen. This procedure is continued from the "new points" chosen until the fill rate reaches (or exceeds) 90%. If 90% is exceeded (and it generally will be because of the integer values required for c and y) a reduction procedure is employed to see if any servers can be decreased (since Δ servers at a time have been added) and still maintain a 90% fill rate. Also, if the feasible region has been entered by adding Δ servers (as opposed to adding a spare) in addition to decreasing servers, an attempt is made to reduce the number of spares. This reduction portion of the algorithm is intended to find a solution as near to the constraint boundary as possible. The procedure is schematically shown in Figure 1, which displays all the points (c,y) evaluated by the algorithm along with the path of greatest improvement in fill rate. Note for each point on the path, fill rate is calculated for the points immediately above and to the right, and then a move is made to the one with the largest fill rate.

Should the starting value for year i (c_{i-1},y_{i-1}) be such that the 90% fill rate criterion is exceeded (this may happen if population size is decreasing, that is, units are being retired from service), then the algorithm immediately goes into the reduction mode, first trying to reduce, to the greatest extent possible, spares (assuming that they are more expensive than servers and hence should have a larger salvage value) and then attempting to reduce servers to get as close to the 90% boundary line as possible. Again, if servers are more expensive than spares, the words "server" and "spare" should be interchanged.

4. Sample Results

Using the marine gas turbine fleet as an example, Table 1 shows the results for an 11-year planning horizon, from 1975 to 1985, using an interest rate of 10%. The fleet size starts out at a relatively small number, building up to full strength by year 1985. The component exhibited

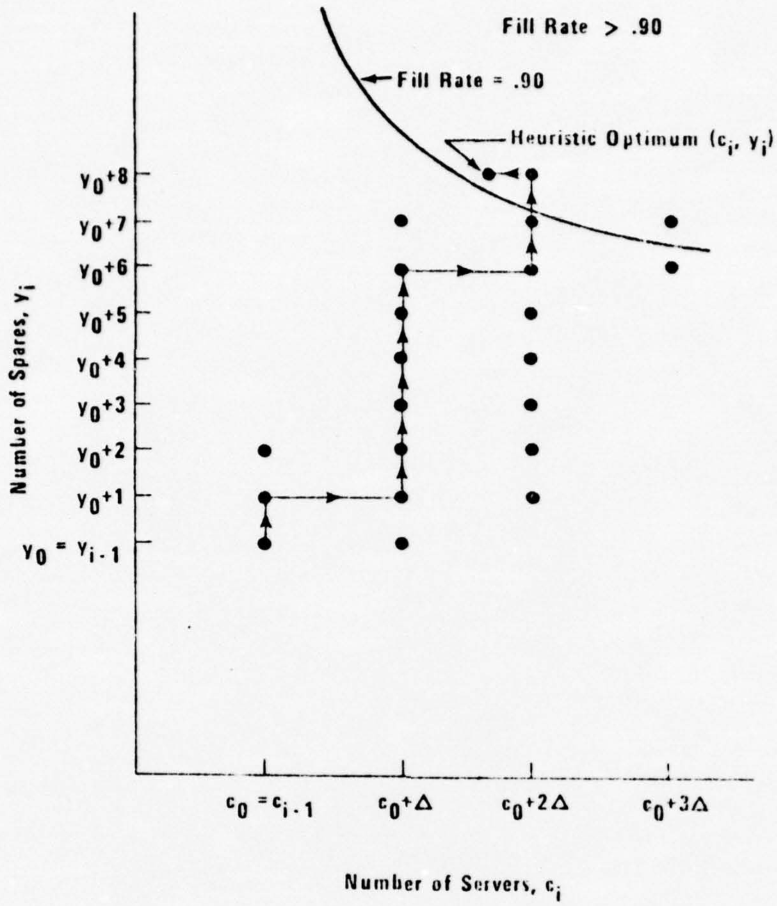

Figure 1 – Heuristic algorithm.

is the gas generator component. Similar calculations were performed for the power turbine component. Generators and turbines are completely independent as they require different repair facilities, and, of course, spares are not interchangeable.

The first nine columns of Table 1 (except the $\overline{\lambda}$ column, Column 4) are input. Column 1 gives the year, Column 2 the anticipated population size, Column 3 the mean failure rate schedule for the particular CIP chosen (in failures per day), Column 5 the average service (including removal, transportation, and repair) time (in days), and Columns 6-9 the purchase cost of servers, purchase cost of spares, unit repair cost, and investment in CIP

TABLE 1

GAS GENERATOR--FULL CIP

Year	N	λ	$\overline{\lambda}$	$1/\mu$	C_1	C_2	C_3	C_4	c	y	$\overline{R}$	Cost	Present Worth
75	10	0.0015	0.0015	65.0	132.0	822.0	49.0	1975.0	3	3	5.3	5097.61	5097.61
76	28	0.0015	0.0015	62.5	132.0	945.0	49.0	2760.0	5	6	15.3	6607.21	11104.17
77	50	0.0014	0.0015	60.0	132.0	1087.0	37.8	3840.0	8	8	26.4	7407.54	17226.10
78	82	0.0010	0.0013	57.5	132.0	1174.0	40.0	3950.0	10	10	37.2	8048.63	23273.15
79	121	0.0008	0.0010	55.0	132.0	1268.0	42.0	2800.0	12	11	45.5	6243.08	27537.26
80	158	0.0007	0.0009	55.0	132.0	1369.0	44.0	1100.0	15	12	51.8	5144.28	30731.45
81	182	0.0006	0.0008	55.0	132.0	1369[illegible]	44.0	350.0	13	13	53.9	4092.71	33041.68
82	208	0.0006	0.0007	55.0	132.0	1369.0	44.0	350.0	14	13	56.3	2957.99	34559.60
83	229	0.0006	0.0007	55.0	132.0	1369.0	44.0	350.0	13	14	58.3	4282.26	36557.30
84	251	0.0006	0.0006	55.0	132.0	1369.0	44.0	350.0	15	14	61.6	3323.18	37966.66
85	256	0.0006	0.0006	55.0	132.0	1369.0	44.0	350.0	15	14	61.6	3059.91	39146.38

cost, respectively, C_1 through C_3 being in thousands of dollars per unit while C_4 is in thousands of dollars per year.

Column 4 and the last five columns are output. It was convenient to show $\bar{\lambda}_i$ (Column 4) next to λ_i so that one can readily see the reliability growth. Columns 10 and 11 give the algorithm's "best" c_i, y_i. Column 12 shows average number of units repaired for each year, while Column 13 provides the expenditures (in thousands of dollars) up to and including year i. Column 14 gives the present worth of the discounted cost stream up to and including year i. The final value in Column 14 is the present worth of all expenditures over the complete planning horizon, again in thousands of dollars.

Another run is presented in Table 2 which differs only in the CIP chosen. This illustrates how one can study the consequence of various alternative CIP's. There we assume that in order to save CIP investment cost (C_4), we will stop the program at year 1978, thus achieving only a minimum component failure rate of 0.001 failures per day. By so doing, we save the two large expenditures in 1979 and 1980, and immediately drop to the \$350,000 per year needed to maintain the achieved reliability of 0.001 failures per day. However, in comparing the present worth of the cost streams for the two cases, we see that it actually costs about \$5 million more to do this since we require more servers and spares. Thus in this case the added CIP investment is well worth the expenditure.

5. Program Perturbation Capability

The heuristic algorithm operates on a year-by-year basis and hence has the limitation that it does not "look ahead." While we believe the heuristic algorithm does a good job in giving an optimal or near optimal point (c,y) for any year, when considering the entire planning horizon it may not prove as effective. For example, consider Table 1 once again, particularly year 1980. We see for that year, a jump from 12 servers in 1979 to 15 in 1980 is required, but in the following year only 13 servers

TABLE 2

GAS GENERATOR—PARTIAL CIP

Year	N	λ	$\bar{\lambda}$	$1/\mu$	C_1	C_2	C_3	C_4	c	y	$\bar{R}$	Cost	Present Worth
75	10	0.0015	0.0015	65.0	132.0	822.0	49.0	1975.0	3	3	5.3	5097.61	5097.61
76	28	0.0015	0.0015	62.5	132.0	945.0	49.0	2760.0	5	6	15.3	6607.21	11104.17
77	50	0.0014	0.0015	60.0	132.0	1087.0	37.8	3840.0	8	8	26.4	7407.54	17226.10
78	82	0.0010	0.0013	57.5	132.0	1174.0	40.0	3950.0	10	10	37.2	8048.63	23272.15
79	121	0.0010	0.0010	55.0	132.0	1268.0	42.0	350.0	11	12	47.6	5016.79	26699.69
80	158	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	13	14	59.0	5947.59	30392.67
81	182	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	16	15	66.7	5049.24	33242.83
82	208	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	17	17	75.0	6548.08	36603.03
83	229	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	19	18	83.4	5651.54	39239.51
84	251	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	19	20	91.9	7129.79	42263.24
85	256	0.0010	0.0010	55.0	132.0	1369.0	44.0	350.0	21	20	94.3	4761.92	44099.16

are needed. Recall that no explicit salvage value is received for decreasing servers. This, in itself, is not too serious, except we note that for 1984 we must increase servers back up to 15, having to purchase two additional servers for that year. Had we kept the 15 from 1981 we would not have needed to make further server purchases.

But more importantly, perhaps we could have avoided going to 15 servers in 1980 had we purchased an additional spare, especially since it is needed for year 1981 anyway. The algorithm allows us to go back to 1980 and use as a starting point $c=12$, $y=13$ (instead of the 1979 solution $c=12$, $y=11$). This may give a different c,y schedule from that year on, which might have a cheaper fixed present worth. This capability to go back to any year in the schedule and use a starting point other than the previous year's c,y we refer to as the perturbation capability. Using a different starting point (if chosen "intelligently") might get us to a different feasible solution point which avoids temporary increases in either c or y . The program then continues its calculation from the year for which the perturbation is desired, adding the new cost stream with the correct discount factor from that year forward, giving us the present worth for the new schedule which may result. Table 3 shows this effect for perturbing the Table 1 solution at year 1980, using the starting point $c=12$, $y=13$ rather than the 1979 solution of $c=12$, $y=11$. We see for the new resulting schedule (which differs only for year 1980) that the total present worth is now about $39.05 million, a savings of approximately $0.1 million. Further possible perturbations are also shown with the resulting costs. No other obvious perturbations are indicated, although earlier purchasing to avoid inflation might be evaluated.

At first glance, it appears this multi-year programming problem might be amenable to a dynamic programming solution where the years are stages and c and y are the state variables. Aside from the fact that there are two state variables (which makes the computations formidable) the problem is not decomposable. The fill rate constraint must hold for each year. This constraint (for year i, say) involves the $q_{n,i}$ which are functions of the $p_{n,i}$ which in turn are functions not only of the

TABLE 3

PERTURBED SOLUTIONS, GAS GENERATOR--FULL CIP

Year	Original Solution		First Perturbation (1980)		Second Perturbations (1982)		Second Perturbations (1983)	
	c	y	c	y	c	y	c	y
1975	3	3	3	3	3	3	3	3
1976	5	6	5	6	5	6	5	6
1977	8	8	8	8	8	8	8	8
1978	10	10	10	10	10	10	10	10
1979	12	11	12	11	12	11	12	11
1980	15	12	12	13	12	13	12	13
1981	13	13	13	13	13	13	13	13
1982	14	13	14	13	13	14	14	13
1983	13	14	13	14	13	14	14	14
1984	15	14	15	14	15	14	15	14
1985	15	14	15	14	15	14	15	14
Present Worth	$39,146,380		$39,051,460		$39,046,080		$38,995,440	

decision variables for year i (c_i, y_i) , but all previous c's and y's since $\overline{\lambda}_i$ is a function of $\overline{\lambda}_{i-1}$ which is a function of $\overline{\lambda}_{i-2}$, etc., [see Equation (2)]. This prevents starting at the last stage and proceeding backward. Only if the reliability were constant throughout the multi-year planning horizon could dynamic programming be attempted. However, we believe that the heuristic algorithm, coupled with the perturbation capability, does provide good, although not necessarily optimal, schedules.

6. Series Queueing Model

The queueing model portion of the program given by Equation (1) assumes that removal, transportation, and repair are a single service action so that if all servers are busy and a component fails, it must wait in a queue prior to removal for an available server. Once removal is initiated, no further queues for transportation or repair are encountered.

In many situations, the removal, transportation, and repair phases may be separate, each having their own "servers" with their own queues. To model this situation would require the $p_{n,i}$ and $q_{n,i}$ to be determined by a closed network or cyclic queueing model which can account for the finiteness of the population, but requires quite involved calculations (see, for example, [3], [10], and [11]). The question then naturally arises as to how crucial the finite source assumption might be, since rather simple queueing formulas exist for infinite source series queues with exponential arrival and service patterns. For example, suppose for component removal, c_1 removal teams are available for the system (c_1 could equal the population size N in which case we have an ample server model). Assuming removal times are exponential and failures are Poisson, the removal portion can be modeled by an $M/M/c_1$ queue. The output of such a queue (assuming an infinite population) is Poisson. Further, suppose there were c_2 transport vehicles available to the system for shipping components to the repair depot. The transportation phase would then be an $M/M/c_2$ queue, assuming transport times are exponential. Finally, assuming c_3 repair channels (again with an infinite population), the repair portion becomes an $M/M/c_3$ queue.

Letting $p_{n,i}^{(j)}$ be the probability of n components at the jth phase $(j=1,2,3)$ for year i, it can be shown (see Gross and Harris [4], page 203) that the joint probability of ℓ in the removal portion, m in the transportation portion, and n in repair for year i (call $p_{\ell,m,n;i}$) is given by

$$p_{\ell,m,n;i} = p_{\ell,i}^{(1)}\, p_{m,i}^{(2)}\, p_{n,i}^{(3)} \; .$$

Dropping the year subscript i for the time being, the fill rate constraint for each year corresponding to Equation (5) becomes

$$\sum_{\ell}\sum_{m}\sum_{n} p_{\ell}^{(1)}\, p_{m}^{(2)}\, p_{n}^{(3)} \geq 0.90 \; ,$$

where the summation is taken over all combinations of ℓ,m,n such that $(\ell+m+n \leq y-1)$. Here the $q_k^{(j)}$ equal the $p_k^{(j)}$, as we are assuming an infinite source model. Since $p_k^{(j)}$ is readily computable for M/M/c models, a series queue representation is possible as long as we can justify approximating the finite source situation by an infinite source model. Assuming for the moment we can (this will be discussed more fully below), then the previous methodology can be used if we substitute for the previously used $q_{n,i}$ and $p_{n,i}$, the sum of probabilities

$$\sum_{\ell+m+k=n} p_{\ell,i}^{(1)}\, p_{m,i}^{(2)}\, p_{k,i}^{(3)} \; , \tag{6}$$

with the arrival rate for this infinite source series queueing model adjusted to be

$$\lambda_i = N_i \overline{\lambda}_i \; . \tag{7}$$

Once again, dropping the year subscript i for convenience, the standard M/M/c formulas (see Gross and Harris [4], page 96, for example) give

$$p_k^{(j)} = \begin{cases} \dfrac{\lambda^k}{k!\ \mu_j^k}\, p_0 , & (1 \le k \le c_j \\[2ex] \dfrac{\lambda^k}{c_j^{\,k-c_j}\ c_j!\ \mu_j^k}\, p_0 , & (k \ge c_j) \end{cases} \tag{8}$$

$$p_0^{(j)} = \left[\sum_{k=0}^{c_{j-1}} \frac{1}{k!} \left(\frac{\lambda}{\mu_j}\right)^k + \frac{1}{c_1!} \left(\frac{\lambda}{\mu_j}\right)^{c_j} \left(\frac{c_j\mu_j}{c_j\mu_j-\lambda}\right) \right]^{-1} , \tag{9}$$

with λ being given by Equation (7), c_j being the number of servers for phase j , and μ_j being the mean service rate for phase j ($j=1$ denotes removal, $j=2$ denotes transportation, $j=3$ denotes repair). In some situations, there may even be a fourth phase consisting of transportation from repair depot to spares pool.

The mathematical programming problem which we attacked via a heuristic algorithm is now considerably more complex if c_1 and c_2 are also decision variables, since we now seek, for each year, the best combination (c_1, c_2, c_3, y) and not merely (c,y) . We no longer have a single Δ , but three Δ's , say $\Delta_1, \Delta_2, \Delta_3$. For equal dollar expenditures, then, we can buy one spare or Δ_1 removal teams or Δ_2 transport vehicles or Δ_3 repairmen. Further, there are combinations such as $\alpha\Delta_1$ removal teams + $\beta\Delta_2$ transport vehicles + $\gamma\Delta_3$ repairmen $(0 \le \alpha,\beta,\gamma \le 1)$, which might equal the purchase cost of a spare. Conceptually, the extension to our heuristic algorithm would be that given a particular c_1, c_2, c_3 and y , calculate the increase in fill rate when moving to all points of equal expenditure, that is, calculate the fill rate for $(c_1, c_2, c_3, y+1)$, $(c_1+\Delta_1, c_2, c_3, y)$, $(c_1, c_2+\Delta_2, c_3, y)$, $(c_1, c_2, c_3+\Delta_3, y)$ and $(c_1+\alpha\Delta_1, c_2+\beta\Delta_2, c_3+\gamma\Delta_3, y)$ for all appropriate α,β,γ . For simplification, the equal expenditure points involving combinations of c_1, c_2, c_3 (those points with the α,

β, and γ terms) might be ignored so that at each step we add either one spare, or Δ_1 removal teams, or Δ_2 transports, or Δ_3 repairmen.

If, on the other hand, c_1 and c_2 are not decision variables (and in our applications they were not), then the algorithm need not be modified since we seek only "optimal points" (c_3, y). Thus, once the p_n's and q_n's are replaced by their series model counterparts as given in Equations (6), (8) and (9), the algorithm is exercised as before, with c_1 and c_2 being fixed values. In the next section we present some results for such a case after first considering the question of accuracy when using an infinite source approximation to a finite population.

7. Accuracy of Infinite Source Approximations

In order to use a series queueing model, the effect of assuming an infinite population for calculating state probabilities when in actuality the population is finite must be investigated. Let us consider now the non-series finite source repairman with spares model given by Equation (1) and its infinite population counterpart, an M/M/c model with an arrival rate adjusted as in Equation (7), that is, $\lambda = N\bar{\lambda}$. In the M/M/c model, λ is always $N\bar{\lambda}$ regardless of the number of units down. In the finite source model, λ is state dependent and in fact is given in Section 2 for the q_n development as

$$\lambda_n = \begin{cases} N\bar{\lambda} & , \quad (0 \le n \le y) \\ (N-n+y)\bar{\lambda} & , \quad (y < n \le y+N) \\ 0 & , \quad (n > y+N) \end{cases} ,$$

where n is the number of units "down," that is, in "repair." Now the overall average arrival rate of the finite source model would be

$$\bar{\bar{\lambda}} = \sum_{n=0}^{\infty} \lambda_n p_n = \sum_{n=0}^{y} N\bar{\lambda} p_n + \sum_{n=y+1}^{y+N} (N-n+y)\bar{\lambda} p_n$$

$$= \sum_{n=0}^{y+N} N\bar{\lambda} p_n - \sum_{n=y+1}^{y+N} (n-y)\bar{\lambda} p_n$$

$$= N\bar{\lambda} - \bar{\lambda} \sum_{n=y+1}^{y+N} (n-y)p_n = \bar{\lambda}\left[N - \sum_{n=y+1}^{y+N} (n-y)p_n\right] ,$$

which incidentally is how $\bar{R}$ of Equation (3) is derived. Thus if $\sum_{n=y+1}^{y+N} (n-y)p_n$ is small in relation to N, the infinite source approximation should be adequate since $\bar{\bar{\lambda}} \doteq N\bar{\lambda}$, the quantity assumed by the infinite source model. This should be the case for systems with not too much congestion (p_n small for $n > y$), y and N large. The fact that a fill rate of 90% must be guaranteed should require c and y large enough such that there is a relatively small amount of congestion in the system, thus making the approximation good even for small N.

The first and third blocks of Table 4 show comparisons for the two cases run in Tables 1 and 2, respectively, with an infinite source M/M/c used in calculating $p_{n,i}$. The results differ only slightly, with the infinite source model requiring an additional server or two in a few of the years. The infinite source model will occasionally require slightly higher c (or y) than the finite source model in order to meet the availability constraint. This is caused by the lack of state dependence in the arrival rate for the infinite source model which has the effect of making the infinite source arrival rate slightly higher than the arrival rate for the corresponding finite source model. Hence the infinite source model is a conservative approximation with respect to fill rate.

Since the infinite source approximation appears to give good results, we have run an infinite source three-stage (removal, transport, and repair) series model, using the same data as in Tables 1 and 2, but assuming removal is M/M/∞, transport is M/M/∞, and repair is M/M/c′ (c to be determined as before).

TABLE 4

COMPARISON OF FINITE SOURCE, INFINITE SOURCE AND SERIES MODELS

Year	Case 1 - Full Component Improvement Program						Case 2 - Partial Component Improvement Program					
	Single-Stage Models				Multi-Stage Model		Single-Stage Models				Multi-Stage Model	
	Finite Source		M/M/c		M/G/∞ → M/G/∞ → M/M/c		Finite Source		M/M/c		M/G/∞ → M/G/∞ → M/M/c	
	c	y	c	y	c	y	c	y	c	y	c	y
1975	3	3	3	3	3	3	3	3	3	3	3	3
1976	5	6	5	6	4	6	5	6	5	6	4	6
1977	8	8	8	8	7	8	8	8	8	8	7	8
1978	10	10	10	10	8	10	10	10	10	10	8	10
1979	12	11	12	11	10	11	11	12	11	12	9	12
1980	15	12	16	12	13	12	13	14	14	14	11	14
1981	13	13	13	13	10	13	16	15	16	15	13	15
1982	14	13	14	13	12	13	17	17	17	17	14	17
1983	13	14	13	14	11	14	19	18	19	18	16	18
1984	15	14	15	14	13	14	19	20	20	20	16	20
1985	15	14	15	14	13	14	21	20	21	20	17	20

Actually, for ample server queues, it is not necessary to assume exponential service since the output of $M/G/\infty$ is also Poisson (see, for example, Mirasol [7]). Thus, these results also hold for $M/G/\infty$ removal and transport phases. The mean single stage repair time is allocated to the multi-stage model so that on the average 5% is consumed in removal, 20% in transportation, and 75% in repair. These results comparing the two models are also shown in Table 4. Here, in many of the years, fewer servers are required to achieve the 90% fill rate when treating removal, transportation, and repair separately. This is as we would expect, since if these three phases are separate, removal and transportation can occur even if there is a queue at the repair facility. By treating the separate phases together as one service operation, as in the single-stage model, we are in essence requiring a free repair facility prior to removal, in which case the repairman would be idle until the component arrived. Thus in the situation where removal and transportation have ample servers and queueing takes place only at repair, conservative results are obtained if we treat the three separate phases as one. If indeed the service portion is made up of separate phases, a more realistic representation of the system is obtained by using the series model in spite of the slight inaccuracy caused by invoking the infinite source assumption when calculating probabilities. With fill rate constraints of 90% or better which guarantee relatively little congestion, it appears that the inaccuracies caused by using the infinite source assumption in the series model are negligible as compared to those that result when using a single-stage finite source model if the service actually is multi-stage.

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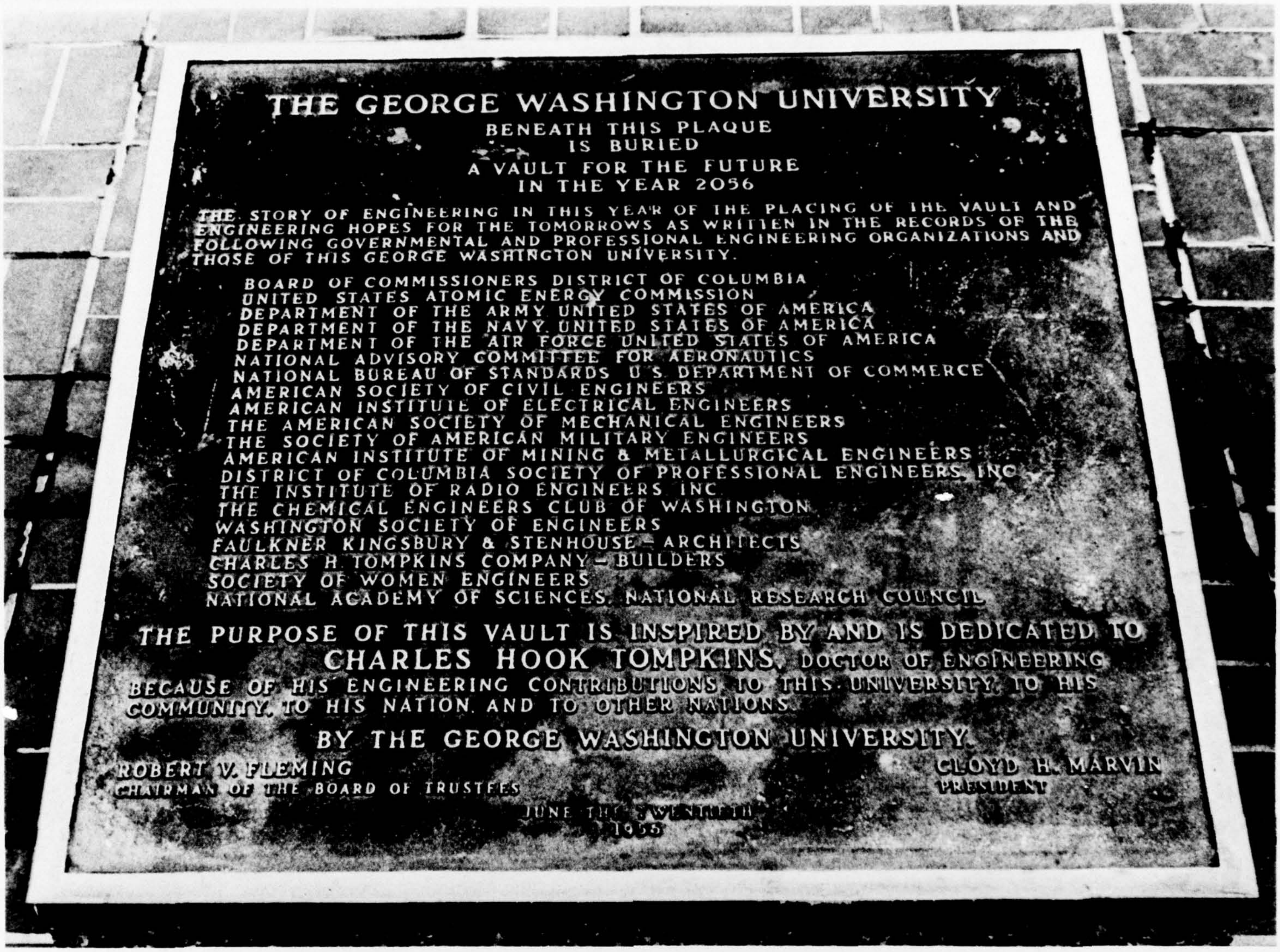

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